2019/1/15 IPMU

Melonic Supertensor Models

work in progress with Mukund Rangamani
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Melonic tensor model is a new type of solvable model, where the melonic Feynman diagrams dominate in the large N limit. I will also argue later that supersymmetry is inevitable for the model to be interesting in $d \geq 2$.

Motivation and background

Solve QFT:

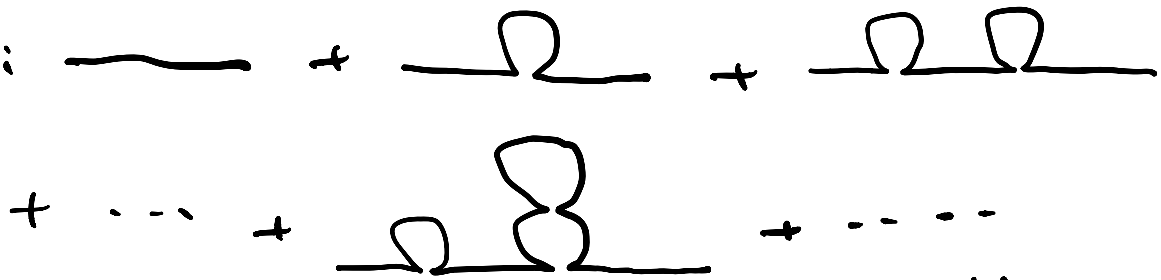
- preturbation theory – Feynman diagram

- resum to all orders
- 't Hooft large N limit

1. vector (-like) models ϕ^i, $i=1,\dots,N$

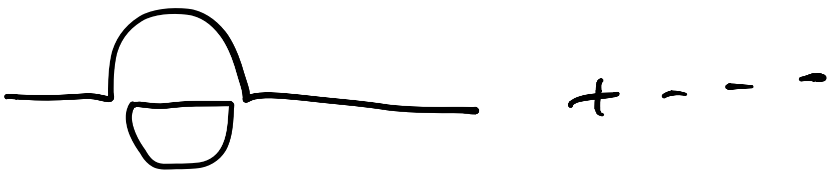
$$L = \frac{1}{2}\partial\phi^i\partial\phi^i + \frac{1}{4}(\phi^i\phi^i)^2$$

N^0:

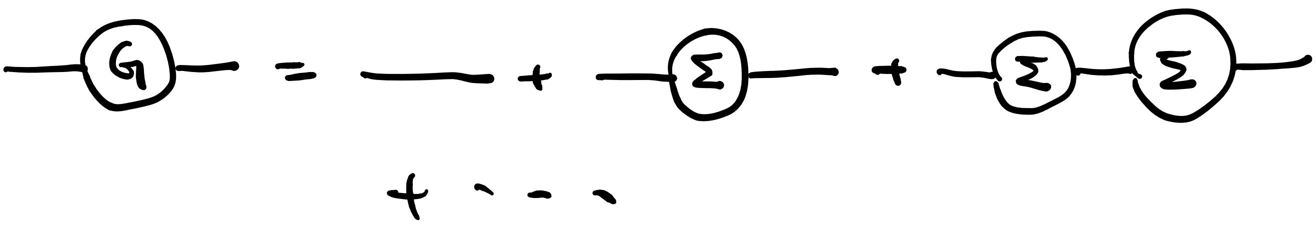


all bubble diagrams

N^1:



Schwinger-Dyson equations



Solvable in any dimensions! ☺

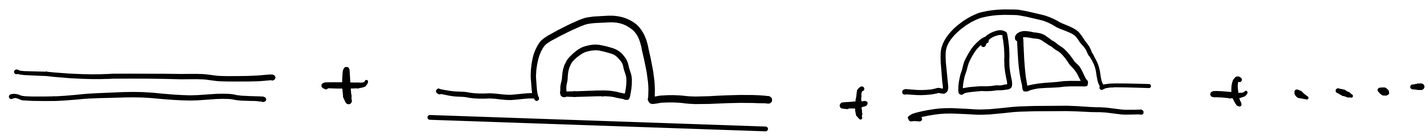
Other examples: Chern-Simons vector models

2. Matrix (-like) models, fixing $\lambda = g^2 N$

ϕ : $N \times N$ matrix

$$L = \frac{1}{2}\mathrm{Tr}(\partial\phi\,\partial\phi) + \frac{1}{4} g\,\mathrm{Tr}(\phi^4)$$

N^0: planar diagrams



SD eqns do not truncate. Only solvable in special cases (by other techniques).

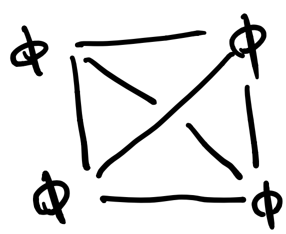
Solvable example: $\mathcal{N}=4$ SYM. $d=0$ matrix model

- New large N limit Gurau - Witten Carrozza - Tanasa - Klebanov - Tarnopolsky

$\phi^{i_1 i_2 \cdots i_{q-1}}$, $\quad i_r = 1, \cdots, N$, $\quad r = 1, \cdots, q-1$

$$\frac{1}{2}(\partial\phi)^2 + \frac{1}{4} g\,[\phi^q]$$

$$q=4 \;:\; [\phi^4] = \phi^{i_1 i_2 i_3}\,\phi^{i_1 j_2 j_3}\,\phi^{j_1 i_2 j_3}\,\phi^{j_1 j_2 i_3}$$

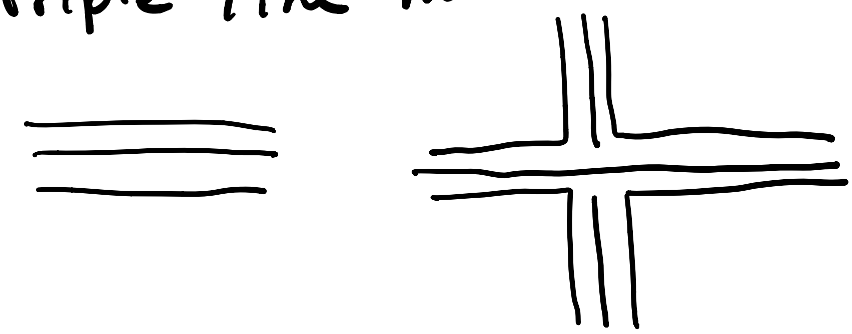


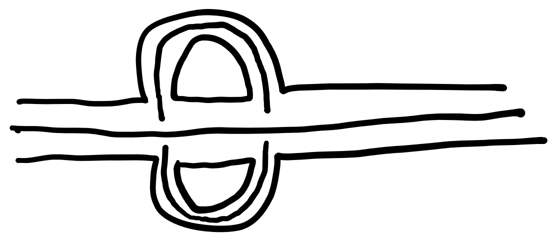
tetrahedron

Each pair of fields has one index contracted between them.

large N with $g^2 N^3 \equiv J^2$ fixed

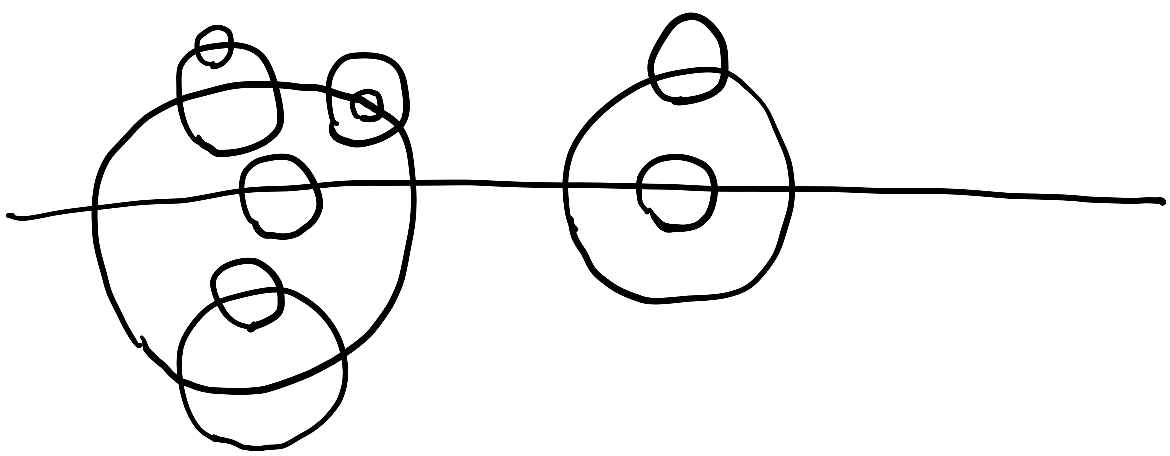
triple line notation:



N^0 :  +  + . . .

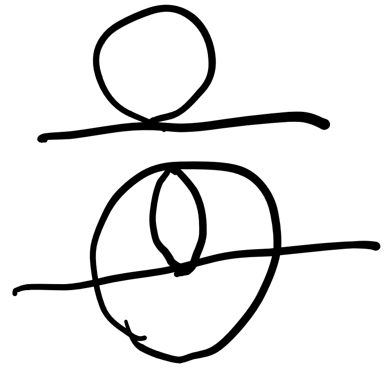
$\sim g^2 N^3 = J$

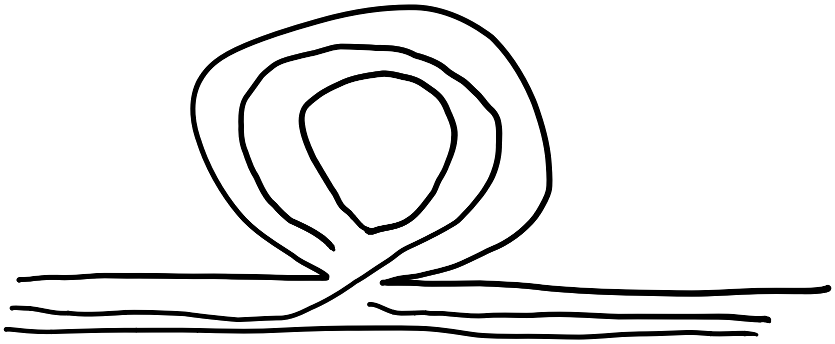
large N limit is dominated by melonic diagrams



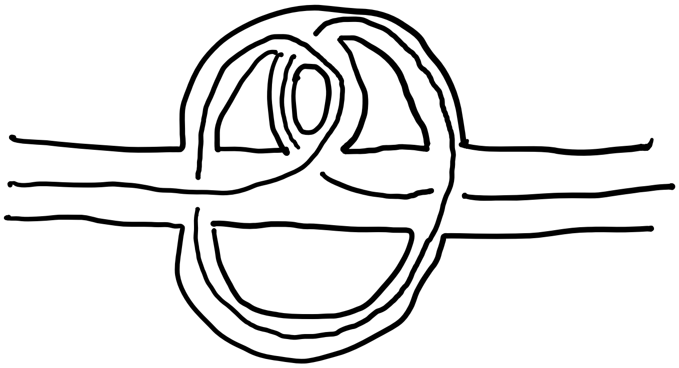
(single line notation)

Nonmelonic diagrams:



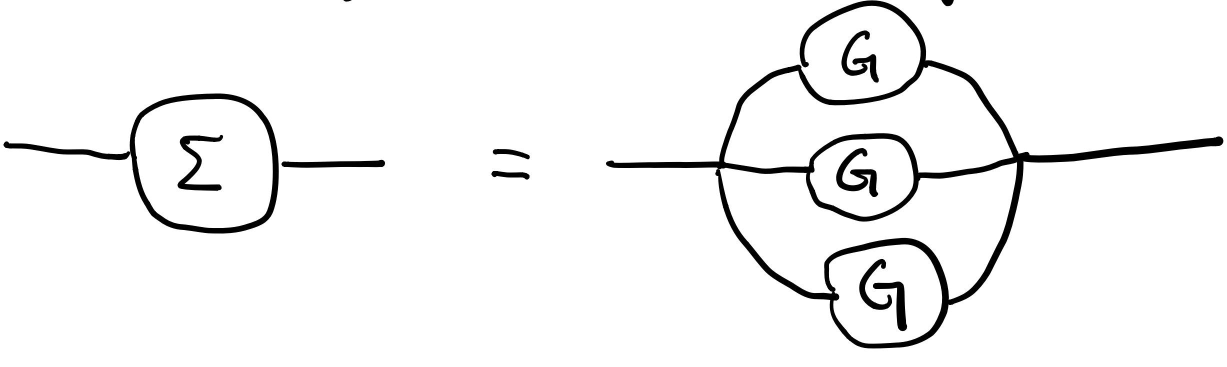


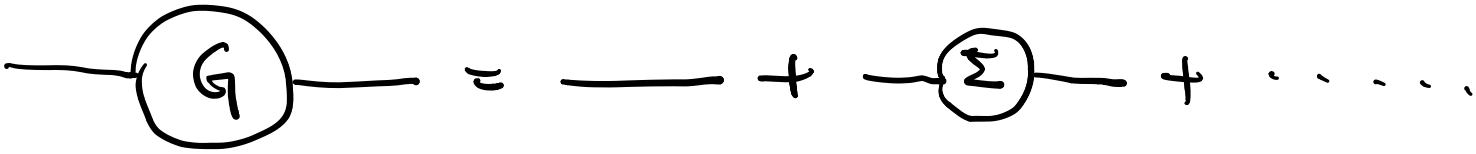
$$\sim gN = \sqrt{\frac{J}{N}}$$



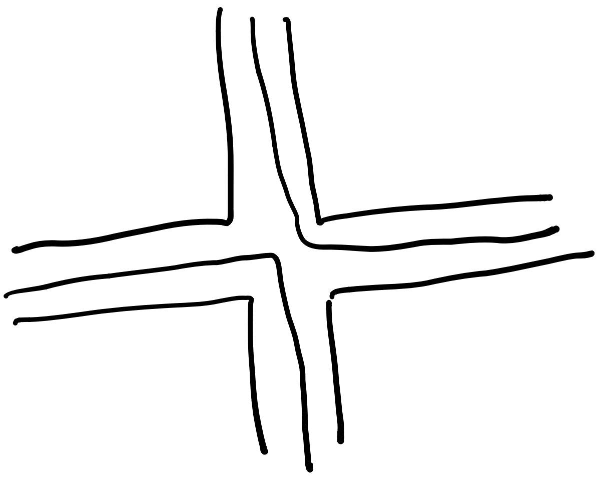
$$\sim g^4 N^4 = \frac{J^2}{N^2}$$

SD equation of 2-pt function

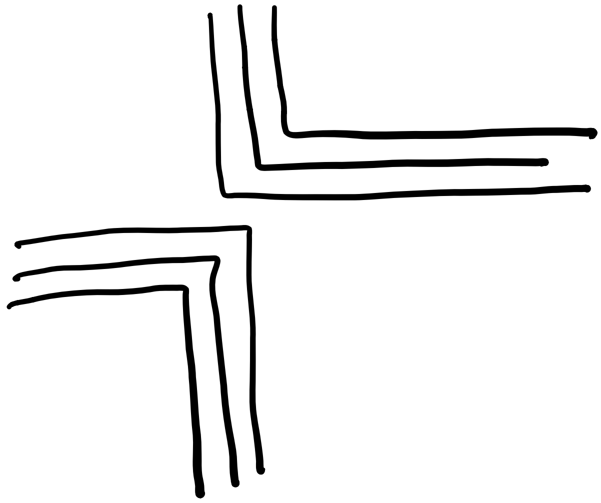




Other type of interactions:

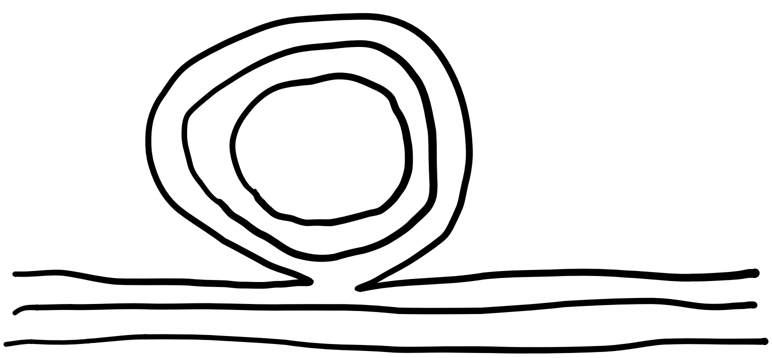


pillow



double-sum

Pillow and double-sum ruin melonic dominance



$$\sim g N^2 = \sqrt{J N}$$

Lore of QFT:

RG flow generates all possible interactions allowed by symmetry.

resolution: supersymmetry!

Non-renormalization theorem :

For theories with 4 supercharges, holomorphy prevents the superpotential being renormalized.

UV theory w/ melonic superpotential $\xrightarrow{RG}$ IR SCFT

want : marginal or relevant couplings

melonic : quartic and higher

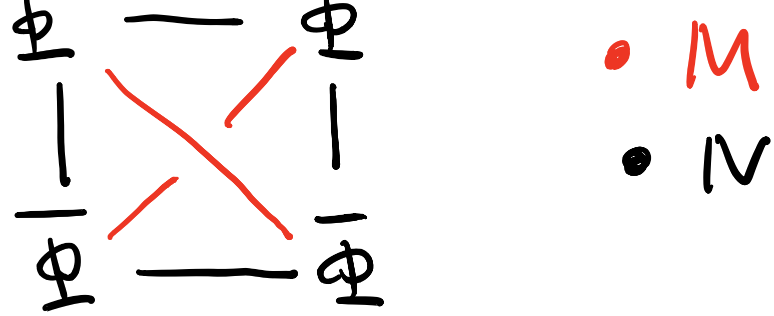
4d : X
3d : quartic → marginally irrelevant X
2d : quartic, sixtic, ...

2d $\mathcal{N} = (2,2)$

chiral superfields : $(\Phi_i)_a{}^b$

$i = 1, \dots, M$, $a, b = 1, \dots, N$

superpotential : $W = g \sum_{i,j} \mathrm{Tr}(\Phi_i \Phi_j \Phi_i \Phi_j)$



large N, M with $J = g^2 N^2 M$ fixed

$\Rightarrow$ melonic dominance

Comments

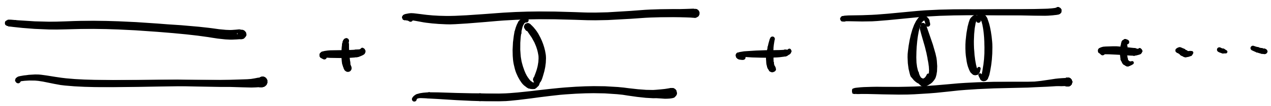
1. Same large N limit as the 2d susy SYK model (with random coupling) studied by Murugan–Stanford–Witten and Bulycheva.
   1706.05362   1801.09006

2. Two-point function:

$$\Delta_\phi = \underset{\substack{= \\ \nwarrow \text{free theory}}}{0} + \frac{1}{4} \qquad \left(\Phi = \phi + \theta^+ \psi_+ + \cdots \right)$$

3. Four-point function solved by ladder

four point function summed by ladder diagrams



central charge : $c = \frac{3}{2} N^2 M$

double-twist operator spectrum :

$$E - J = 2n + 2\Delta_\phi + \varepsilon(n,J)$$

$$\varepsilon(n,J) \sim O(1) \,,\quad \varepsilon(n,J) \xrightarrow[\substack{n\to\infty \\ \text{or } J\to\infty}]{} 0$$

$$\Phi(\partial\bar\partial)^n \partial^m \Phi$$

4. Chaos exponent $\lambda_L \simeq 0.6 \frac{2\pi}{\beta} \leq \frac{2\pi}{\beta}$

$$\langle \Phi(t,0)\,\Phi(0,x)\,\Phi(t,0)\,\Phi(0,x) \rangle \sim f_0 - \epsilon\, e^{\lambda_L t}$$

5. IR SCFT

Bulk dual : AdS_3 string theory with finite tension.

- Noncompact moduli space

$$\frac{\partial W}{\partial \bar{\Phi}_i} = \sum_j \bar{\Phi}_j \bar{\Phi}_i \bar{\Phi}_j \quad \text{(F-term eq)}$$

1. $\bar{\Phi}_1$: nilpotent matrix

$$\bar{\Phi}_2 = \bar{\Phi}_3 = \cdots = \bar{\Phi}_M = 0$$

2. $\bar{\Phi}_1 \propto \sigma_1$, $\bar{\Phi}_2 \propto \sigma_2$

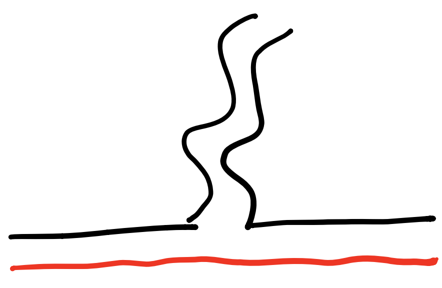
$$\bar{\Phi}_3 = \bar{\Phi}_4 = \cdots = \bar{\Phi}_M = 0$$

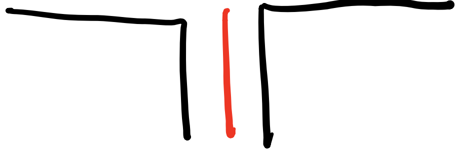
$\Rightarrow$ Continuum in the spectrum of IR CFT

lifting the flat directions requires

1. gauge the SU(N) sym
2. break the O(M) sym by anisotropic deformation

Gauge the SU(N) sym





g

g_{YM}

Melonic coupling : $J = g^2 N^2 M$

't Hooft coupling : $\lambda = g_{YM}^2 N$

$$[g] = [g_{YM}] = 1$$

Choosing $g = g_{YM}$, melonic coupling is much larger than 'tHooft coupling.

$$J = g^2 N^2 M \gg g_{YM}^2 N = \lambda$$

still melonic after gauging !

Anisotropic deformation

$$W = \sum_i a_i \operatorname{tr}(\Phi_i^4) + \sum_{i \neq j} a_{ij} \operatorname{tr}(\Phi_i \Phi_j \Phi_i \Phi_j)$$

a_i, a_{ij} : cx parameters

Melonic dominance : most of $a_{ij} \neq 0$

- large IR conformal manifold.

Higgs branch :

F-term eq : $a_i \Phi_i^3 + \sum_{j \neq i} a_{ij} \Phi^j \Phi^i \Phi^j = 0$

D-term eq : $\sum_i [\Phi_i^\dagger , \Phi_i] = 0$

Higgs branch is a symplectic quotient :

$$\{ \text{F-term eq} , \text{D-term eq} \} / SU(N)$$

holomorphic quotient :

$$\{ \text{F-term eq} \} / SU(N)_{\mathbb{C}}$$

$$(\text{holomorphic quotient}) = M_1 \sqcup \underbrace{M_2 \sqcup M_3}_{\substack{\shortparallel \\ (\text{symplectic quotient})}} \cup \cdots$$

Conjecture: symplectic quotient is compact for generic a_{ij}.

example: $N = M = 2$, $a_1 = a_2 = 1$, $a_{12} = a_{21} = a$

$$\begin{cases} X^3 + a\, YXY = 0 \\ Y^3 + a\, XYX = 0 \end{cases} \qquad a \neq \pm 1, 0, \infty$$

$$\Rightarrow \det X = \det Y = 0$$

$$\begin{cases} (\mathrm{Tr} X^2) X + a (\mathrm{Tr} XY) Y = 0 \\ (\mathrm{Tr} Y^2) Y + a (\mathrm{Tr} XY) X = 0 \end{cases}$$

$\Rightarrow$ 1. $X = Y = 0$

2. $X \propto Y$ and $X^2 = 0$

$$X = x \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad Y = y \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$e^{\eta \sigma_3} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} e^{-\eta \sigma_3} = e^{2\eta} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow \mathbb{CP}^1$

holomorphic quotient $= \mathbb{CP}^1 \sqcup *$

D-term eq : $|x|^2 + |y|^2 = 0$

symplectic quotient $= *$

$$\begin{cases} \text{compact} : \text{generic } a \\ \text{non-compact} : a = 0, \pm 1, \infty \end{cases}$$

Coulomb branch:

twisted chiral superfield

$$\Sigma = \underbrace{\sigma}_{\text{cx scalar}} - i\theta^+ \underbrace{\bar{\lambda}_+ - i\bar{\theta}^- \lambda_-}_{\text{gaugino}} + \theta^+ \bar{\theta}^- (\underbrace{D}_{\text{auxiliary}} - i \underbrace{F_{01}}_{\text{field strength}}) + \cdots$$

$V \sim |D|^2 + |F|^2 + \mathrm{tr}[\sigma, \sigma^+]$

$$+ \ \mathrm{tr}\left([\sigma,\phi][\sigma^{\dagger},\bar{\Phi}] + [\sigma^{\dagger},\bar{\Phi}][\sigma,\phi]\right)$$

classical Coulomb branch :

$$\phi = \bar{\Phi} = 0$$

$$\sigma = \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_N \end{pmatrix}, \quad \sum_i \sigma_i = 0$$

Non-compact , $N-1$ dimension.

assuming : $\sigma_i \neq \sigma_j$ for $i \neq j$

$\sigma_i \neq 0 \quad \forall i$

integrating out W-bosons and charged matters

effective twisted superpotential Hori-Tong 0609032

$$\widetilde{W} = (M+1) \sum_{i \neq j} (\sigma_i - \sigma_j)\left[\log(\sigma_i - \sigma_j) - 1\right]$$

$$= 2i(M+1)\pi \sum_{i=1}^{N-1} (N-i)\,\sigma_i$$

Theta angles for $U(1)^{N-1}$

$$\theta_i = \mathrm{Im}\, \frac{\partial \widetilde{W}}{\partial \sigma_i} = 2(M+1)\pi \; (N-i)$$

$$\theta_i \sim \theta_i + 2\pi$$

$\Rightarrow$ flat directions are preserved.

Kill the flat directions by consider $SU(N)/\mathbb{Z}_N$

For example $SU(2)/\mathbb{Z}_2 \cong SO(3)$

$$\widetilde{W} = \pi i \,(M+1)\, \sigma_1$$ 1104.2853 Hori

compactness : $\theta_{bare} = 0$, M even
$\theta_{bare} = \pi$, M odd

Future directions :

1. compute elleptic genus
(N=2 done)

2. generalization

M_1

$$
\begin{tikzcd}
N_1 \arrow[r, bend left] & N_2 \arrow[l, bend left, "M_2"]
\end{tikzcd}
$$

3. embed in string theory
brane construction ?