On Lorentz-invariant spin-2 theories

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plan of today's talk

- ✤ introduction
  - cosmic acceleration, dark energy, massive gravity
  - degenerate theory/degenerate kinetic matrix

- ✤ case of spin-1 in degenerate theory
  [1608.07066 & 1803.10876]

- ✤ case of spin-2 in degenerate theory
  [1812.10886]

- ✤ summary

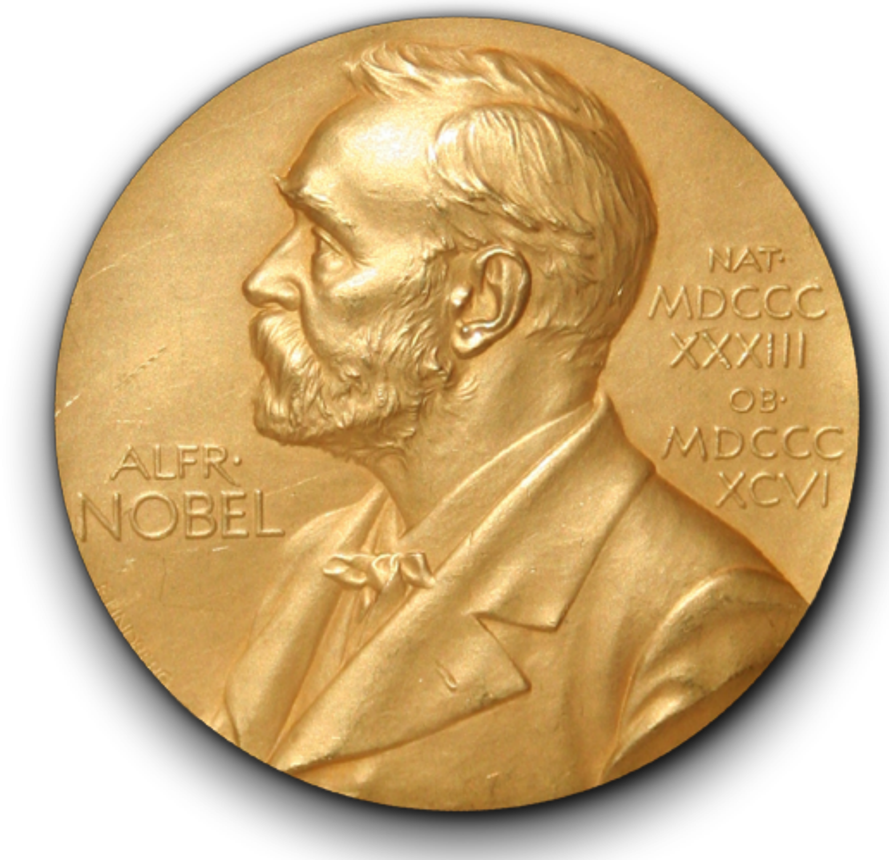
introduction

cosmic acceleration, dark energy, massive gravity

universe is accelerating !!

✓ the current universe is ***accelerating*** !!!

Cosmic Pie



Nobel prize in 2011

for the ***discovery*** of the accelerating expansion of the Universe

cosmological constant or ??

✓ to understand/explain primordial and current **accelerated expansion** of the universe

$$\frac{\ddot{a}}{a} = -\frac{\rho + 3P}{6} \geq 0 \quad \Rightarrow \quad P \leq -\frac{\rho}{3}$$

exotic matter ? *change of law of gravity ??*

① Λ ?? (why so small ? why that value ?)

② Λ [ϕ, A_μ] ?? ⇒ **scalar**/**vector**-tensor theory

e.g. canonical, k-essence, Horndeski, … Proca, …

③ $g_{\mu\nu}$?? (change of gravity's law) = **tensor** theory

e.g. **(dRGT) massive gravity**, bi-gravity…

Fierz-Pauli Theory

- Fierz-Pauli theory (Fierz, Pauli, 1939)

$$S = M_{\rm Pl}^2 \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{4} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \right]$$

Linearized Einstein-Hilbert term

Only allowed mass term which does not have ghost at linear order

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = -\frac{1}{2}(\Box h_{\mu\nu} - \partial_\mu \partial_\alpha h_\nu^\alpha - \partial_\nu \partial_\alpha h_\mu^\alpha + \partial_\mu \partial_\nu h_\alpha^\alpha - \eta_{\mu\nu} \Box h_\alpha^\alpha + \eta_{\mu\nu} \partial_\alpha \partial_\beta h_\beta^\alpha)$$

(1) Lorentz invariant theory

(2) No ghost (5 DOF = 2 tensor + 2 vector + 1 scalar)

(3) **Simple nonlinear extension contains ghost at nonlinear level**

(Boulware-Deser ghost, 6th DOF) (Boulware, Deser, 1971)

dRGT Massive Gravity

- dRGT massive gravity (de Rham, Gabadadze, Tolley 2011)

$$S_{MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}\left(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right)\right] + S_m[g_{\mu\nu},\psi]$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \sqrt{\delta^\mu{}_\nu - H^\mu{}_\nu} = \delta^\mu{}_\nu - \left(\sqrt{g^{-1}\eta}\right)^\mu{}_\nu$$

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$
$$\sqrt{X^\mu{}_\alpha}\sqrt{X^\alpha{}_\nu} = X^\mu{}_\nu$$

$$\mathcal{U}_2 = 2\varepsilon_{\mu\alpha\rho\sigma}\varepsilon^{\nu\beta\rho\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta = 4\left([\mathcal{K}^2] - [\mathcal{K}]^2\right)$$
$$\mathcal{U}_3 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\rho}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta = -[\mathcal{K}]^3 + 3[\mathcal{K}][\mathcal{K}^2] - 2[\mathcal{K}^3]$$
$$\mathcal{U}_4 = \varepsilon_{\mu\alpha\gamma\rho}\varepsilon^{\nu\beta\delta\sigma}\mathcal{K}^\mu{}_\nu\mathcal{K}^\alpha{}_\beta\mathcal{K}^\gamma{}_\delta\mathcal{K}^\rho{}_\sigma = -[\mathcal{K}]^4 + 6[\mathcal{K}]^2[\mathcal{K}^2] - 3[\mathcal{K}^2]^2 - 8[\mathcal{K}][\mathcal{K}^3] + 6[\mathcal{K}^4]$$

cosmological solution is unstable…

instabilities in perturbations…

No BD ghost at full order (5 DOF) (Hassan, Rosen, 2011)

Non-canonical Kinetic Term

(Kimura & Yamauchi, 2013)

- dRGT mass term is uniquely determined
- Is dRGT theory a unique theory describing massive graviton without introducing other fields ?

$$S_{MG} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}(\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4)\right] + S_{int} + S_m[g_{\mu\nu}, \psi]$$

really impossible to extend the kinetic term ??

$$M_{\rm Pl}^2\sqrt{-g}\,\nabla_{\cdot}\nabla_{\cdot}\,H^{\cdot}_{\ \cdot}\,H^{\cdot\cdot},\ M_{\rm Pl}^2\sqrt{-g}\,\nabla_{\cdot}\nabla_{\cdot}\,H^{\cdot}_{\ \cdot}\,H^{\cdot}_{\ \cdot}\,H^{\cdot\cdot},\ \cdots$$

- **No-go theorem** - no derivative interaction cannot be introduced in dRGT theory (de Rham et al. 2013)

introduction

degenerate theory & degenerate kinetic matrix

degenerate theory

or

degenerate kinetic matrix

$\Leftrightarrow$ **magic** to introduce a kinetic term

for ***non-dynamical*** d.o.f.(s)

Ostrogradsky instability

✓ reminder :

$$\mathcal{L} = \frac{1}{2}\dot{q}^2 \qquad \mathcal{H} = \frac{1}{2}p^2 \geq 0 \quad : p = \dot{q}$$

✓ system with higher derivatives ⇔ **2 dofs :**

$$(Q = \dot{q})$$

$$\mathcal{L} = \frac{1}{2}\ddot{q}^2 \qquad \mathcal{H} = \frac{1}{2}P^2 - p\,Q \quad : P = \dot{Q} = \ddot{q}$$

⇒ energy is **not bounded below** ⇔ **unstable**

what about multi field (matter & gravity ?) case ??

non-degenerate system

✓ two fields ***without degeneracy***

$$\mathcal{L} = \frac{1}{2}\,\dot{\phi}_1^2 + \frac{1}{2}\,\dot{\phi}_2^2$$

$\phi 1 \Leftrightarrow \ddot{\phi}$ or **A0**, $\phi 2 \Leftrightarrow$ **$g_{\mu\nu}$, Ai**

✓ conjugate mom. (π_1, π_2) ♻ $(\dot{\phi}_1\,,\ \dot{\phi}_2)$ ***invertible !***

$$\pi_1 \equiv \frac{\delta \mathcal{L}}{\delta \dot{\phi}_1} = \dot{\phi}_1\,, \qquad \pi_2 \equiv \frac{\delta \mathcal{L}}{\delta \dot{\phi}_2} = \dot{\phi}_2\,,$$

$$\begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix} = \mathcal{K}\begin{pmatrix}\dot{\phi}_1\\ \dot{\phi}_2\end{pmatrix} \quad \text{with} \quad \det|\mathcal{K}| \neq 0$$

⇔ ***no primary constraint*** in the language of Hamiltonian analysis

degenerate system

✓ two fields ***with degeneracy***

$\phi 1 \Leftrightarrow$ ~~$\ddot{\phi}$~~ or ~~**A0**~~**,** $\phi 2$

$$\mathcal{L} = \frac{1}{2}\dot{\phi}_1^2 + \frac{1}{2}\dot{\phi}_2^2 + \dot{\phi}_1 \dot{\phi}_2 \quad \rightarrow \quad \frac{1}{2}\dot{\Phi}^2 \qquad \Phi \equiv \phi_1 + \phi_2$$

✓ conjugate mom. (π_1, π_2) ~~⇔~~ $(\dot{\phi}_1\,,\ \dot{\phi}_2)$ ***non-invertible !!***

$$\pi_1 \equiv \frac{\delta \mathcal{L}}{\delta \dot{\phi}_1} = \dot{\phi}_1 + \dot{\phi}_2\,, \qquad \pi_2 \equiv \frac{\delta \mathcal{L}}{\delta \dot{\phi}_2} = \dot{\phi}_1 + \dot{\phi}_2 = \pi_1$$

$$\begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = \mathcal{K} \begin{pmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix} \quad \text{with} \quad \det|\mathcal{K}| \ \cancel{\neq 0} \ = 0$$

⇔ ***a primary constraint*** in the language of Hamiltonian analysis

question

can we construct new massless/massive

spin-1 & 2 theories with degenerate kinetic matrix ??

~ case of spin-1 ~

Maxwell & Proca

✓ A_μ has 4 components = (in maximum) **4 d.o.f.s**

✓ In Maxwell, **A0 is non-dynamical** (no kinetic term)

$$-F_{\mu\nu}F^{\mu\nu} \sim \vec{E}^2 - \vec{B}^2 \sim \dot{A}_i^2 - A_{[i\ ,j]}^2 \not\supset \dot{A}_0^2$$

(gauge sym. kills longitudinal mode → 2 d.o.f.s)

✓ **Proca theory** ($+m^2A^2$, no gauge sym.) ⇔ 3 d.o.f.s

In Maxwell & Proca, **A0 is non-dynamical = no kinetic term**

→ **with magic (degeneracy)**, kinetic term for **A0 ??**

Kimura, **AN**, Yoshida [1608.07066 (2016)]

Extended vector-tensor

✓ action w/ 2 derivatives of Aμ **& 4D general covariance**

$$\mathcal{L} = f(Y)\,R + C^{\mu\nu\rho\sigma}\,(\nabla_\mu A_\nu)\,(\nabla_\rho A_\sigma)$$

$$C^{\mu\nu\rho\sigma} = \alpha_1 g^{\mu(\rho}g^{\sigma)\nu} + \alpha_2 g^{\mu\nu}g^{\rho\sigma} + \frac{1}{2}\alpha_3(A^\mu A^\nu g^{\rho\sigma} + A^\rho A^\sigma g^{\mu\nu})$$
$$+\frac{1}{2}\alpha_4(A^\mu A^{(\rho}g^{\sigma)\nu} + A^\nu A^{(\rho}g^{\sigma)\mu}) + \alpha_5 A^\mu A^\nu A^\rho A^\sigma + \alpha_6 g^{\mu[\rho}g^{\sigma]\nu}$$
$$+\frac{1}{2}\alpha_7(A^\mu A^{[\rho}g^{\sigma]\nu} - A^\nu A^{[\rho}g^{\sigma]\mu}) + \frac{1}{4}\alpha_8(A^\mu A^\rho g^{\nu\sigma} - A^\nu A^\sigma g^{\mu\rho}) + \frac{1}{2}\alpha_9\varepsilon^{\mu\nu\rho\sigma}.$$

— sym.

— asym.

f, αi : functions of $Y = A_\mu A^\mu$

☝ not symmetric under μ ↔ ν (ρ ↔ σ) [cf. $\nabla_\mu A_\nu \rightarrow \nabla_\mu\nabla_\nu\phi$]

☝ for general αi, **6** = **1 (A0)** + 3 (Ai) + 2 (GW) d.o.f.s

degeneracy condition

✓ **degeneracy** cond. ⇔ making **A0 non-dynamical** :

$$0 = |\mathcal{M}_{\rm kin}| = \mathcal{D}_0 + \mathcal{D}_2\, A_*^2 + \mathcal{D}_4\, A_*^4$$

$$0 = \mathcal{D}_0 \propto (\alpha_1 + \alpha_2)\, F(\alpha_i\,, f)$$

→ **case A** : $\alpha_1 + \alpha_2 = 0$

→ **case B** : $F = 0 \;\; (f \neq 0)$

→ **case C** : $F = 0 \;\; (f = 0)$

✓ an example of **4 or 5** dofs = GW & vector (massive/less)

$$\alpha_1 = \alpha_1[\alpha_2\,, \alpha_3\,, \alpha_8] = \frac{-8(2\alpha_2 + Y\alpha_3) - Y\alpha_8(4 + 4Y\alpha_2 - Y^2\alpha_3)}{2Y^2\alpha_8}$$

α1 is not free !!

cosmology
in degenerate theory

no-go for degenerate theory ??

✓ In ***degenerate Scalar****-Tensor theory with* $\ddot{\phi}$, there is no Ostrogradsky instability by construction.. but..

✓ de Rham & Matas found **instabilities in perturbations** in a class of ***degenerate Scalar****-Tensor theory*

$$\mathcal{L}_\Phi = 2\frac{a^3}{\dot{\phi}_0^2}(G - XA_1)\left(3\dot{\phi}_0^2 - 2\frac{G - XA_1}{A_1 + A_2}\right)\dot{\Phi}^2 - 2aG\Phi\nabla^2\Phi\,, \tag{B.6}$$

and that branch of solutions always exhibits a gradient instability (if $G > 0$, the instability is in the scalar mode and had we taken $G < 0$, the instability would have been in the tensor modes).

✓ instability in **vector-tensor** theory as well ??

Kase, Kimura, **AN**, Yoshida [1803.10876]

cosmology in EVT

✓ **The future is bright !!**

✓ cosmology in extended vector-tensor theory

BG : FLRW + $\bar{A}_\mu = \left(\bar{A}_0(t)\,,\mathbf{0}\right)$

✓ There is parameter space in which all the perturbations do not suffer from **ANY instabilities** (even with $c_T^2 = 1$).

➡ instability in scalar theory is **not universal** !!

✓ inflation in EVT (ongoing with **OBT**)

~ case of spin-2 ~

spin-2

✓ fluctuations around Minkowski spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

hμν =10 components in 4D

✓ **GR** [**massless** spin-2] = 2 dynamical dofs ⇄ **GW**

10 hμν − ~~**4 gauge**~~ − ~~**4 const.**~~ = **2 GW**

✓ **Fierz-Pauli** [**massive** spin-2] = no gauge **+ extra const.**

10 - **0** - 4 **- 1** (ghost) = **5 dofs** ⇄ **GW + S&V**

AN, Kimura, Yamauchi [1812.10886]

Lorentz-invariant spin-2

✓ generic spin-2 action with Lorentz invariance

$$S = \int \mathrm{d}^4x \Big(-\mathcal{K}^{\alpha\beta|\mu\nu\rho\sigma} h_{\mu\nu,\alpha} h_{\rho\sigma,\beta} - \mathcal{M}^{\mu\nu\rho\sigma} h_{\mu\nu} h_{\rho\sigma} \Big)$$

$$\mathcal{K}^{\alpha\beta|\mu\nu\rho\sigma} = \kappa_1 \eta^{\alpha\beta}\eta^{\mu\rho}\eta^{\nu\sigma} + \kappa_2 \eta^{\mu\alpha}\eta^{\rho\beta}\eta^{\nu\sigma} + \kappa_3 \eta^{\alpha\mu}\eta^{\nu\beta}\eta^{\rho\sigma} + \kappa_4 \eta^{\alpha\beta}\eta^{\mu\nu}\eta^{\rho\sigma}$$

$$\mathcal{M}^{\mu\nu\rho\sigma} = \mu_1 \eta^{\mu\rho}\eta^{\nu\sigma} + \mu_2 \eta^{\mu\nu}\eta^{\rho\sigma}$$

κi & **μi** : (constant) free parameters

✓ **GR** : $\kappa_2 = -\kappa_3 = 2\kappa_4 = -2\kappa_1$ & **μi** = 0

gauge symmetry = $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

✓ **FP** : κi same as GR & **μ1** = **- μ2** (FP mass tuning)

action under SVT decom.

✓ generic (massive) spin-2 = spin-0 + spin-1 + spin-2

$$h_{00} = h^{00} = \underline{-2\alpha}\,, \qquad h_{0i} = -h^{0i} = \underline{\beta_{,i}} + \underline{B_i}$$

$$h_{ij} = h^{ij} = \underline{2\mathcal{R}\delta_{ij} + 2\mathcal{E}_{,ij}} + \underline{F_{i\,,j} + F_{j\,,i}} + \underline{2H_{ij}}$$

✓ tensor mode (in Fourier space) :

$$S^T[H_{ij}] = 4\int \mathrm{d}t\,\mathrm{d}^3k\left[\kappa_1\dot{H}_{ij}^2 - (\kappa_1 k^2 + \mu_1)H_{ij}^2\right]$$

- tensor exist iff κ1 ≠ 0

- ghost & tachyon free iff κ1 > 0 & μ1 > 0

vector mode

✓ vector mode = **4 d.o.f.s** ⇔ **2** in Bi + **2** in Fi :

$$S^V[B_i\,, F_i] = \int \mathrm{d}t\, \mathrm{d}^3k \Big[\underline{-(2\kappa_1 + \kappa_2)\dot{B}_i^2} + 2\kappa_1 \dot{F}_i^2 + 2\kappa_2 k B_i \dot{F}_i + 2\left(\kappa_1 k^2 + \mu_1\right) B_i^2 - \left(k^2(2\kappa_1 + \kappa_2) - 2\mu_1\right) F_i^2 \Big]$$

ghost !! → demand 2κ1+κ2=0 !!

① case V1 [μ1 = 0] : **0 dof**

(include **GR**, w/ gauge symmetry ~ dif. inv.)

② case V2 [μ1 ≠ 0] : **2 dofs**

(include **FP**, **but more general**)

scalar mode

✓ scalar mode = **4 d.o.f.s** ⇔ α, β, R, E

↪ **β** is **non-dynamical** when $2\kappa_1+\kappa_2=0$

$$\begin{cases} \text{case 1 = 1 primary constraint} \\ \text{case 2 = 2 primary constraints} \end{cases}$$

① case 1 & **$\mu_1 \neq 0$** ⇄ 3 **dofs** *with ghost..*

② case 1 & **$\mu_1 = 0$** ⇄ 1 **dof** **New Theory !!**

2 primary constraints

✓ case 2 = **2 primary constraints**

$$\mathcal{L} = a_1\,\dot{\alpha}^2 + a_2\,\dot{\mathcal{R}}^2 + a_3\,\dot{\mathcal{E}}^2 + \cdots$$

det[ai] = 0 ➡ **another primary constraint**

① **μ1 = μ2 = 0** ⇄ **0 dof**

(include **GR**, w/ gauge symmetry ~ dif. inv.)

② **μ1 = 0** & **μ2 ≠ 0** ⇄ **0 dof** **New Theory !!**

(no gauge symmetry, extra constraints, only tensor)

③ **μ1 = 0** & **μ2 = f(μ1)** ⇄ **1 dof** ⊃ **New Theory !!**

(include **FP** *[μ1=-μ2]*, but more general)

new & relatives

✓ linear **Lorentz-invariant** field transformation :

$$h_{\mu\nu} \to h_{\mu\nu} + (\mathrm{const.}) * \mathrm{tr}[h]\ \eta_{\mu\nu}$$

☞ **preserve classifications** (case1 → case1)

2 dofs = tensor	**3 dofs** = 2T+1S	**5 dofs** = 2T+2V+1S
GR & New	**New**	**FP**

✓ massive gravity with **new** matter coupling ??

$$\mathrm{FP} + \mathcal{L}^m[h\,,\psi] \;\to\; \mathrm{FP}^{new} + \mathcal{L}^m[g\,,\psi] = \mathrm{FP} + \mathcal{L}^m[\tilde{h}\,,\psi]$$

summary

summary of spin-1

- ✓ We have constructed degenerate vector-tensor theory that includes two first derivative of vector field.
- ✓ New theory for massless/massive vector field includes 4/5 (≠6) d.o.f.s = 2/3 massive vector & 2 GW
- ✓ Applying transformations of metric & vector field, we have investigated the stability of classification
- ✓ There exist healthy cosmological solutions finding no instabilities in perturbations ⇔ scalar theory

summary of spin-2

- ✓ considered generic spin-2 theories applying the idea of degenerate theory to spin-2 field for the first time.
- ✓ uncovered new theories with 2 and 3 (and 5) dofs
- ✓ considered a field transformation : $h_{\mu\nu} \to h_{\mu\nu} + \mathrm{Tr}[h]\,\eta_{\mu\nu}$
  all theory with 5 dofs → Fierz-Pauli theory
- ✓ new matter coupling in massive gravity ??

$$FP[h] + \mathcal{L}^m[h] \neq FP[\tilde{h}] + \mathcal{L}^m[h] = FP[h] + \mathcal{L}^m[\tilde{h}]$$

Thank you very much for your attention