Mimetic Gravity: Pros and Cons

Mohammad Ali Gorji
IPM, Tehran

In collaboration with
H. Firouzjahi, S. A. Hosseini Mansoori, and S. Mukohyama

IPMU, Tokyo

February 26, 2019

Outline

1. Scalar Field Mimetic Gravity
   - Mimetic Dark Matter
   - Instabilities
   - Stable Extensions: HD Terms and Two-Field Extension
2. Gauge Field Mimetic Gravity
   - p-form Generalization
   - Spatial Curvature via 1-form Case
3. Mimetic SU(2) Cosmology
   - Disentangling Spatial Curvature
4. Summary

Outline

Disformal Transformations

- The disformal transformation (DT) is defined as

$$g_{\mu\nu} = A(\phi, \tilde{X})\tilde{g}_{\mu\nu} + B(\phi, \tilde{X})\partial_\mu\phi\partial_\nu\phi$$

where $g_{\mu\nu}$ is the physical metric, $\tilde{g}_{\mu\nu}$ is the auxiliary metric, and $\tilde{X} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.

Disformal Transformations

- The disformal transformation (DT) is defined as

$$g_{\mu\nu} = A(\phi, \tilde{X})\tilde{g}_{\mu\nu} + B(\phi, \tilde{X})\partial_\mu\phi\partial_\nu\phi$$

where $g_{\mu\nu}$ is the physical metric, $\tilde{g}_{\mu\nu}$ is the auxiliary metric, and $\tilde{X} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.

- DTs neatly classify the higher derivative theories

$$P(\phi, X) \overset{\text{DT}}{\longleftrightarrow} \text{Horndeski} \overset{\text{DT}}{\longleftrightarrow} \text{beyond Horndeski} \overset{\text{DT}}{\longleftrightarrow} \text{DHOST}$$

Disformal Transformations

- The disformal transformation (DT) is defined as

$$g_{\mu\nu} = A(\phi, \tilde{X})\tilde{g}_{\mu\nu} + B(\phi, \tilde{X})\partial_\mu\phi\partial_\nu\phi$$

  where $g_{\mu\nu}$ is the physical metric, $\tilde{g}_{\mu\nu}$ is the auxiliary metric, and $\tilde{X} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.
- DTs neatly classify the higher derivative theories

$$P(\phi, X) \overset{\text{DT}}{\longleftrightarrow} \text{Horndeski} \overset{\text{DT}}{\longleftrightarrow} \text{beyond Horndeski} \overset{\text{DT}}{\longleftrightarrow} \text{DHOST}$$

- For the non-singular (invertible) DT, the number of degrees of freedom does not change.

Disformal Transformations

- The disformal transformation (DT) is defined as

$$g_{\mu\nu} = A(\phi, \tilde{X})\tilde{g}_{\mu\nu} + B(\phi, \tilde{X})\partial_\mu\phi\partial_\nu\phi$$

  where $g_{\mu\nu}$ is the physical metric, $\tilde{g}_{\mu\nu}$ is the auxiliary metric, and $\tilde{X} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.
- DTs neatly classify the higher derivative theories

$$P(\phi, X) \overset{\text{DT}}{\longleftrightarrow} \text{Horndeski} \overset{\text{DT}}{\longleftrightarrow} \text{beyond Horndeski} \overset{\text{DT}}{\longleftrightarrow} \text{DHOST}$$

- For the non-singular (invertible) DT, the number of degrees of freedom does not change.
- What happens in the case of singular transformation?

Mimetic Gravity

- For the conformal transformation with $B = 0$, the singular limit uniquely gives the mimetic transformation [A. Chamseddine and V. Mukhanov (JHEP,2013)]

$$g_{\mu\nu} = \left(\tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\tilde{g}_{\mu\nu}$$

This transformation implies

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1$$

Mimetic Gravity

- For the conformal transformation with $B = 0$, the singular limit uniquely gives the mimetic transformation [A. Chamseddine and V. Mukhanov (JHEP,2013)]

$$g_{\mu\nu} = \left(\tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\tilde{g}_{\mu\nu}$$

  This transformation implies

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1$$

- Therefore, instead of the standard Einstein-Hilbert action, we would have

$$S = \int dt d^3x\sqrt{-g}\Big[\frac{R}{2} + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1)\Big]$$

- The Einstein's Equations then gives

$$G^{\mu\nu} = -2\lambda\partial^{\mu}\phi\partial^{\nu}\phi\,,$$

which shows that the mimetic term induces energy momentum tensor like dust. The auxiliary field λ is the energy density and mimetic field ϕ is the velocity potential.

- The Einstein's Equations then gives

$$G^{\mu\nu} = -2\lambda\partial^{\mu}\phi\partial^{\nu}\phi\,,$$

which shows that the mimetic term induces energy momentum tensor like dust. The auxiliary field λ is the energy density and mimetic field ϕ is the velocity potential.

- Even in the absence of any matter field, mimetic term provide dark matter

$$\rho_{\rm DM} \propto a^{-3}$$

in the spatially flat FLRW background.

Two Problems:

- Because of the attractive behavior of dark matter, the scenario suffers from caustics formations beyond which the scalar field is ill-defined. Since mimetic field supposed to be a fundamental (universal) field, the mimetic scenario breaks down due to the caustics formations!

Two Problems:

- Because of the attractive behavior of dark matter, the scenario suffers from caustics formations beyond which the scalar field is ill-defined. Since mimetic field supposed to be a fundamental (universal) field, the mimetic scenario breaks down due to the caustics formations!

- The sound speed vanishes for the scalar mode and curvature perturbations are non-dynamical. The reason is that the pressure always vanishes in this scenario and we cannot construct, i.e., inflationary model in this framework.

Outline

- In order to have nonzero sound speed for the curvature perturbations, it is suggested to add a higher derivative term to the action [A. Chamseddine, V. Mukhanov, A. Vikman (JCAP,2014)]

$$S = \int dtd^3x\sqrt{-g}\left[\frac{R}{2} + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1) + \frac{1}{2}\gamma(\Box\phi)^2\right]$$

In Newtonian gauge, the Mukhanov-Sasaki equation is

$$\delta\ddot{\phi} + H\delta\dot{\phi} + \dot{H}\delta\phi - \frac{c_s^2}{a^2}\delta\phi = 0,$$

with constant nonzero sound speed $c_s^2 = \frac{\gamma}{2-3\gamma}$.

- In order to have nonzero sound speed for the curvature perturbations, it is suggested to add a higher derivative term to the action [A. Chamseddine, V. Mukhanov, A. Vikman (JCAP,2014)]

$$S = \int dt d^3x \sqrt{-g}\left[\frac{R}{2} + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1) + \frac{1}{2}\gamma(\Box\phi)^2\right]$$

In Newtonian gauge, the Mukhanov-Sasaki equation is

$$\delta\ddot{\phi} + H\delta\dot{\phi} + \dot{H}\delta\phi - \frac{c_s^2}{a^2}\delta\phi = 0,$$

with constant nonzero sound speed $c_s^2 = \frac{\gamma}{2-3\gamma}$.

- This model is, however, unstable, and suffers from gradient/ghost instabilities. [S. Ramazanov, F. Arroja, M. Celoria, S. Matarrese, L. Pilo (JHEP,2016)], [A. Ijjas, J. Ripley, P. J. Steinhardt (PLB,2016)]

- Another version appeared in [A. Chamseddine and V. Mukhanov (JCAP,2017)]

$$S = \int dt d^3x \sqrt{-g} \left[\frac{R}{2} + \lambda (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 1) + f(\Box \phi) \right]$$

- Another version appeared in [A. Chamseddine and V. Mukhanov (JCAP,2017)]

$$S = \int dtd^3x\sqrt{-g}\Big[\frac{R}{2} + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1) + f(\Box\phi)\Big]$$

- For the complicated form of

$$f(\Box\phi) = \chi_m^2\left(1 + \frac{1}{3}\frac{\Box\phi^2}{\chi_m^2} - \sqrt{\frac{2}{3}}\frac{\Box\phi}{\chi_m}\sin^{-1}\left(\sqrt{\frac{2}{3}}\frac{\Box\phi}{\chi_m}\right) - \sqrt{1 - \frac{2}{3}\frac{\Box\phi^2}{\chi_m^2}}\right)$$

with $\Box\phi < \sqrt{\frac{3}{2}}\chi_m$, we find the Friedmann equation

$$H^2 = \frac{1}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right),$$

where $\rho_c = 2\chi_m^2$ is the maximum energy density.

- Another version appeared in [A. Chamseddine and V. Mukhanov (JCAP,2017)]

$$S=\int dtd^3x\sqrt{-g}\Big[\frac{R}{2}+\lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+1)+f(\Box\phi)\Big]$$

- For the complicated form of

$$f(\Box\phi)=\chi_m^2\left(1+\frac{1}{3}\frac{\Box\phi^2}{\chi_m^2}-\sqrt{\frac{2}{3}}\frac{\Box\phi}{\chi_m}\sin^{-1}\left(\sqrt{\frac{2}{3}}\frac{\Box\phi}{\chi_m}\right)-\sqrt{1-\frac{2}{3}\frac{\Box\phi^2}{\chi_m^2}}\right)$$

with $\Box\phi<\sqrt{\frac{3}{2}}\chi_m$, we find the Friedmann equation

$$H^2=\frac{1}{3}\rho\left(1-\frac{\rho}{\rho_c}\right),$$

where $\rho_c=2\chi_m^2$ is the maximum energy density.

- The Big Bang singularity removes in this scenario.

- The Friedmann equation has the same form as Friedmann equation suggested by LQC models. The LQC equations are not obtained from a Lagrangian, but through loop quantization of a Hamiltonian and, therefore, the mimetic scenario suggests effective HD Lagrangian for the LQC equations.

- The Friedmann equation has the same form as Friedmann equation suggested by LQC models. The LQC equations are not obtained from a Lagrangian, but through loop quantization of a Hamiltonian and, therefore, the mimetic scenario suggests effective HD Lagrangian for the LQC equations.
- The quadratic action in Fourier space is given by

$$S^{(2)} = \int dt\, d^3k\, a^3(-c_s^2)\Big[\dot{\mathcal{R}}^2 - \frac{c_s^2 k^2}{a^2}\mathcal{R}^2\Big]$$

with the sound speed $c_s^2 = \frac{f'}{2-3f'}$. Demanding $c_s^2 > 0$ to avoid the gradient instability, the system finds ghost instability! [H. Firouzjahi, M. A. G, S. A. Hosseini Mansoori (JCAP,2017)]

Outline

1. Scalar Field Mimetic Gravity
   - Mimetic Dark Matter
   - Instabilities
   - Stable Extensions: HD Terms and Two-Field Extension
2. Gauge Field Mimetic Gravity
   - p-form Generalization
   - Spatial Curvature via 1-form Case
3. Mimetic SU(2) Cosmology
   - Disentangling Spatial Curvature
4. Summary

Higher Derivative Terms

- Adding higher derivative term $\nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi$ and a coupling between the curvature and second derivative of the scalar field such as $\Box \phi R$ can make the setup stable. [Y. Zheng, L. Shen, Y. Mou, M. Li (JCAP,2017)]

Higher Derivative Terms

- Adding higher derivative term $\nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi$ and a coupling between the curvature and second derivative of the scalar field such as $\Box\phi R$ can make the setup stable. [Y. Zheng, L. Shen, Y. Mou, M. Li (JCAP,2017)]
- It is not clear how we can have mimetic dark matter or bouncing scenario with healthy curvature perturbations.

Higher Derivative Terms

- Adding higher derivative term $\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi$ and a coupling between the curvature and second derivative of the scalar field such as $\Box\phi R$ can make the setup stable. [Y. Zheng, L. Shen, Y. Mou, M. Li (JCAP,2017)]
- It is not clear how we can have mimetic dark matter or bouncing scenario with healthy curvature perturbations.
- By means of the effective field theory methods, it is possible to find mimetic dark matter with healthy propagating perturbations. [S. Hirano, S. Nishi, T. Kobayashi (JCAP,2017)]

Higher Derivative Terms

- Adding higher derivative term $\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi$ and a coupling between the curvature and second derivative of the scalar field such as $\Box\phi R$ can make the setup stable. [Y. Zheng, L. Shen, Y. Mou, M. Li (JCAP,2017)]
- It is not clear how we can have mimetic dark matter or bouncing scenario with healthy curvature perturbations.
- By means of the effective field theory methods, it is possible to find mimetic dark matter with healthy propagating perturbations. [S. Hirano, S. Nishi, T. Kobayashi (JCAP,2017)]
- Using the classified higher derivative terms like $\Box\phi R$ and $\nabla_\mu\nabla_\nu\phi R^{\mu\nu}$ one can find mimetic dark matter with healthy scalar mode. But, we need even quartic and quintic terms like $(\Box\phi)^3 R$ and $(\Box\phi)^4 R$! The setup is complicated if we one to have control on both background and perturbations. [M. A. G, S. A. Hosseini Mansoori, H. Firouzjahi (JCAP,2017)]

$$
\begin{aligned}
&L_1^{(3,2)} = (\Box\phi)^3\, R\,, && L_2^{(3,2)} = \Box\phi\, \phi_{\mu\nu}\phi^{\mu\nu}\, R\,, && L_3^{(3,2)} = \phi_{\mu\nu}\phi^{\nu\alpha}\phi^{\mu}_{\alpha}\, R\,,\\
&L_4^{(3,2)} = (\Box\phi)^2\phi^{\mu}\phi_{\mu\nu}\phi^{\nu}\, R\,, && L_5^{(3,2)} = \Box\phi\, \phi^{\mu}\phi_{\mu\nu}\phi^{\nu\alpha}\phi_{\alpha}\, R\,, && L_6^{(3,2)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_{\alpha}\phi^{\alpha\beta}\phi_{\beta}\, R\,,\\
&L_7^{(3)} = \phi_{\mu}\phi^{\mu\nu}\phi_{\nu\alpha}\phi^{\alpha\beta}\phi_{\beta}\, R, && L_8^{(3,2)} = \phi_{\mu}\phi^{\mu\nu}\phi_{\nu\alpha}\phi^{\alpha}\phi_{\beta}\phi^{\beta\eta}\phi_{\eta} R\,, && L_9^{(3,2)} = \Box\phi\, (\phi^{\mu}\phi_{\mu\nu}\phi^{\nu})^2\, R\,,\\
&L_{10}^{(3,2)} = (\phi^{\mu}\phi_{\mu\nu}\phi^{\nu})^3\, R\,, && L_{11}^{(3,2)} = (\Box\phi)^3\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,, && L_{12}^{(3,2)} = \Box\phi\, \phi_{\mu\nu}\phi^{\mu\nu}\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,,\\
&L_{13}^{(3,2)} = \phi_{\mu\nu}\phi^{\nu\alpha}\phi^{\mu}_{\alpha}\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,, && L_{14}^{(3,2)} = (\Box\phi)^2\phi^{\mu}\phi_{\mu\nu}\phi^{\nu}\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,,\\
&L_{15}^{(3,2)} = \Box\phi\, \phi^{\mu}\phi_{\mu\nu}\phi^{\nu\alpha}\phi_{\alpha}\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,, && L_{16}^{(3,2)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_{\alpha}\phi^{\alpha\beta}\phi_{\beta}\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,,\\
&L_{17}^{(3)} = \phi_{\mu}\phi^{\mu\nu}\phi_{\nu\alpha}\phi^{\alpha\beta}\phi_{\beta}\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,, && L_{18}^{(3,2)} = \phi_{\mu}\phi^{\mu\nu}\phi_{\nu\alpha}\phi^{\alpha}\phi_{\beta}\phi^{\beta\eta}\phi_{\eta}\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,,\\
&L_{19}^{(3,2)} = \Box\phi\, (\phi^{\mu}\phi_{\mu\nu}\phi^{\nu})^2\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,, && L_{20}^{(3,2)} = (\phi^{\mu}\phi_{\mu\nu}\phi^{\nu})^3\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,,\\
&L_{21}^{(3,2)} = \Box\phi\, \phi^{\alpha\mu}\phi^{\beta\nu} R_{\mu\nu\alpha\beta}\,, && L_{22}^{(3,2)} = \phi^{\mu}\phi_{\mu\nu}\phi^{\nu}\, \phi^{\alpha\eta}\phi^{\beta\sigma} R_{\eta\sigma\alpha\beta}\,,
\end{aligned} \tag{43}
$$

and

$$
\begin{aligned}
&L_1^{(1,4)} = \phi^{\mu\nu} R_{\mu\nu} R\,, && L_2^{(1,4)} = \Box\phi\, R^2\,, && L_3^{(1,4)} = \phi^{\mu}\phi_{\mu\nu}\phi^{\nu}\, R^2\,, \qquad (44)\\
&L_4^{(1,4)} = \Box\phi\, \phi^{\mu}\phi^{\nu} R_{\mu\nu} R\,, && L_5^{(1,4)} = \phi_{\alpha}\phi^{\alpha\beta}\phi_{\beta}\phi^{\mu}\phi^{\nu} R_{\mu\nu} R\,, && L_6^{(1,4)} = \phi^{\alpha\beta} R_{\alpha\beta}\, \phi^{\mu}\phi^{\nu} R_{\mu\nu}\,,\\
&L_7^{(1,4)} = \Box\phi\, (\phi^{\mu}\phi^{\nu} R_{\mu\nu})^2\,, && L_8^{(1,4)} = \phi_{\alpha}\phi^{\alpha\beta}\phi_{\beta}(\phi^{\mu}\phi^{\nu} R_{\mu\nu})^2\,, && L_9^{(1,4)} = \Box\phi\, R_{\mu\nu} R^{\mu\nu}\,,\\
&L_{10}^{(1,4)} = \phi_{\alpha}\phi^{\alpha\beta}\phi_{\beta} R_{\mu\nu} R^{\mu\nu}\,, && L_{11}^{(1,4)} = \Box\phi\, R_{\mu\nu\eta\sigma} R^{\mu\nu\eta\sigma} && L_{12}^{(1,4)} = \phi_{\alpha}\phi^{\alpha\beta}\phi_{\beta} R_{\mu\nu\eta\sigma} R^{\mu\nu\eta\sigma}\,.
\end{aligned}
$$

Multiple-Field Extension

- The two-field extension can be obtained by looking at the singular limit of [H. Firouzjahi, M. A. G, S. A. Hosseini Mansoori, A. Karami, T. Rostami (JCAP,2018)]

$$g_{\mu\nu} = A(\phi, \chi, \tilde{X}, \tilde{Y}, \tilde{Z})\tilde{g}_{\mu\nu} \, ,$$

where $\tilde{X} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, $\tilde{Y} = \tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$, $\tilde{Z} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\chi$,
which implies the following constraint

$$g^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi) = -1$$

- The two-field extension can be obtained by looking at the singular limit of [H. Firouzjahi, M. A. G, S. A. Hosseini Mansoori, A. Karami, T. Rostami (JCAP,2018)]

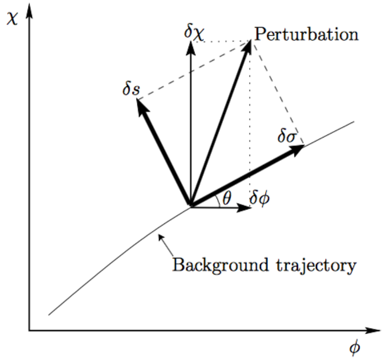
$$g_{\mu\nu} = A(\phi, \chi, \tilde{X}, \tilde{Y}, \tilde{Z})\tilde{g}_{\mu\nu} \, ,$$

where $\tilde{X} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, $\tilde{Y} = \tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$, $\tilde{Z} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\chi$, which implies the following constraint

$$g^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi) = -1$$

- In cosmological background the model provides dark matter much similar to the single field model.

The two-field mimetic scenario only provide entropy perturbations and curvature perturbations still have zero sound speed! [credit of figure by C. Gordon, *et al* (PRD, 2001)]



Outline

Disformal Transformation via Gauge Field

- The whole idea of mimetic gravity is to isolate the conformal mode of gravity by means of a singular transformation. So, we can implement any other field rather than the most simple case of scalar field. [M. A. G, S. Mukohyama, H. Firouzjahi, S. A. Hosseini Mansoori (JCAP,2018)]

Disformal Transformation via Gauge Field

- The whole idea of mimetic gravity is to isolate the conformal mode of gravity by means of a singular transformation. So, we can implement any other field rather than the most simple case of scalar field. [M. A. G, S. Mukohyama, H. Firouzjahi, S. A. Hosseini Mansoori (JCAP,2018)]
- We then look at the singular limit of the transformation which gives

$$g_{\mu\nu} = A(\tilde{K})\,\tilde{g}_{\mu\nu}\,,$$

where $\tilde{K} = -\tilde{g}^{\rho\alpha}\tilde{g}^{\sigma\beta}F_{\alpha\beta}F_{\rho\sigma}$ and we find

$$g_{\mu\nu} = \left(-\tilde{g}^{\rho\alpha}\tilde{g}^{\sigma\beta}F_{\alpha\beta}F_{\rho\sigma}\right)^{\frac{1}{2}}\,\tilde{g}_{\mu\nu} \quad \rightarrow \quad g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}F_{\mu\nu} = -1$$

p-form Generalization

- We extend the setup to the case of a p-form potential $\mathcal{A}$ with the associated field strength $\mathcal{F} = d\mathcal{A}$ so that the action is given by

$$S_p = \frac{1}{2} \int d^4x \sqrt{-g} \left[R - \lambda_p \big(\langle \mathcal{F}, \mathcal{F} \rangle \pm 1 \big) \right]$$

where $\langle \mathcal{F}, \mathcal{F} \rangle$ is the internal product which gives $\partial^\mu \phi \partial_\mu \phi$ and $F_{\mu\nu} F^{\mu\nu}$ for $p = 0$ and $p = 1$ cases respectively. The mimetic constraint is then given by

$$\langle \mathcal{F}, \mathcal{F} \rangle = \mp 1$$

p-form Generalization

- We extend the setup to the case of a p-form potential $\mathcal{A}$ with the associated field strength $\mathcal{F} = d\mathcal{A}$ so that the action is given by

$$S_p = \frac{1}{2}\int d^4x\sqrt{-g}\,\big[R - \lambda_p\big(\langle\mathcal{F},\mathcal{F}\rangle \pm 1\big)\big]$$

where $\langle\mathcal{F},\mathcal{F}\rangle$ is the internal product which gives $\partial^\mu\phi\partial_\mu\phi$ and $F_{\mu\nu}F^{\mu\nu}$ for $p = 0$ and $p = 1$ cases respectively. The mimetic constraint is then given by

$$\langle\mathcal{F},\mathcal{F}\rangle = \mp 1$$

- The case $p = 0$ corresponds to the standard scalar field scenario which provides a dark matter-like component $\rho_{\lambda_0} \to a^{-3}$.

p-form Generalization

- We extend the setup to the case of a p-form potential $\mathcal{A}$ with the associated field strength $\mathcal{F} = d\mathcal{A}$ so that the action is given by

$$S_p = \frac{1}{2}\int d^4x\sqrt{-g}\left[R - \lambda_p\big(\langle\mathcal{F},\mathcal{F}\rangle \pm 1\big)\right]$$

  where $\langle\mathcal{F},\mathcal{F}\rangle$ is the internal product which gives $\partial^\mu\phi\partial_\mu\phi$ and $F_{\mu\nu}F^{\mu\nu}$ for $p = 0$ and $p = 1$ cases respectively. The mimetic constraint is then given by

$$\langle\mathcal{F},\mathcal{F}\rangle = \mp 1$$

- The case $p = 0$ corresponds to the standard scalar field scenario which provides a dark matter-like component $\rho_{\lambda_0} \to a^{-3}$.
- The case $p = 2$ turns out to be dual to the $p = 0$ case with the strong/week like duality $\lambda_0 \leftrightarrow \frac{1}{\lambda_2}$.

Outline

1-form Case

- In order to find isotropic solution, we consider global $O(3)$ symmetry for the internal field space

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{2}R - \lambda\Big(\sum_{a=1}^{3} F^a_{\mu\nu}F_a^{\mu\nu} + 1\Big) + \Lambda\sum_{a=1}^{3} F^a_{\mu\nu}F_a^{\mu\nu}\Big].$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$$

1-form Case

- In order to find isotropic solution, we consider global $O(3)$ symmetry for the internal field space

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{2}R - \lambda\Big(\sum_{a=1}^{3} F^a_{\mu\nu}F_a^{\mu\nu} + 1\Big) + \Lambda \sum_{a=1}^{3} F^a_{\mu\nu}F_a^{\mu\nu}\Big].$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$$

- Thanks to the constraint, the coefficient of the Maxwell term Λ plays the roles of cosmological constant.

1-form Case

- In order to find isotropic solution, we consider global $O(3)$ symmetry for the internal field space

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{2}R - \lambda\Big(\sum_{a=1}^{3} F^a_{\mu\nu}F^{\mu\nu}_a + 1\Big) + \Lambda\sum_{a=1}^{3} F^a_{\mu\nu}F^{\mu\nu}_a\Big].$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$$

- Thanks to the constraint, the coefficient of the Maxwell term Λ plays the roles of cosmological constant.
- In the spatially flat FLRW spacetime with the Ansatz $A^{(a)}_\mu = A(t)\,\delta^a_\mu$, we find

$$3H^2 = \rho_{\lambda_1} + \rho_\Lambda\,, \qquad \textit{with} \qquad \rho_{\lambda_1} = -\frac{3K_{\rm eff}}{a^2} \;\; \textit{and} \;\; \rho_\Lambda = \Lambda$$

where $K_{\rm eff}$ is an integration constant.

- In the case of $p = 1$, the mimetic term provides energy density like the spatial curvature at the background level. We therefore find flat $K_{\rm eff} = 0$, closed $K_{\rm eff} = 1$ and open $K_{\rm eff} = -1$ de Sitter universes even if we consider spatially flat FLRW metric.

- In the case of $p = 1$, the mimetic term provides energy density like the spatial curvature at the background level. We therefore find flat $K_{\rm eff} = 0$, closed $K_{\rm eff} = 1$ and open $K_{\rm eff} = -1$ de Sitter universes even if we consider spatially flat FLRW metric.
- After fixing all gauge freedoms, imposing the mimetic constraint and integrating out the non-dynamical modes, we left with two scalar modes, two vector modes, and four tensor modes

$$S^{(2)} = S_S^{(2)}\big(\delta Q, U\big) + S_V^{(2)}\big(U_a\big) + S_T^{(2)}\big(h_{ij}, t_{ij}\big) \, .$$

- In the case of $p = 1$, the mimetic term provides energy density like the spatial curvature at the background level. We therefore find flat $K_{\rm eff} = 0$, closed $K_{\rm eff} = 1$ and open $K_{\rm eff} = -1$ de Sitter universes even if we consider spatially flat FLRW metric.
- After fixing all gauge freedoms, imposing the mimetic constraint and integrating out the non-dynamical modes, we left with two scalar modes, two vector modes, and four tensor modes

$$S^{(2)} = S_S^{(2)}\big(\delta Q, U\big) + S_V^{(2)}\big(U_a\big) + S_T^{(2)}\big(h_{ij}, t_{ij}\big)\,.$$

- In the case of closed de Sitter universe, the scalar, vector and tensor modes become ghost. For the flat and open case, however, all modes are healthy.

Outline

- In order to compare the spatial curvature coming from the mimetic gauge field model with the standard one, we need to consider $SU(2)$ gauge symmetry in the spatially curved FLRW [H. Firouzjahi, M. A. G, S. Mukohyama, work in progress]

$$S = \int d^4x \sqrt{-g} \Big[\frac{1}{2} R - \lambda \big(F^a_{\mu\nu} F^{\mu\nu}_a + 1 \big) \Big] ,$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\epsilon_{abc} A^b_\mu A^c_\nu \,.$$

- In order to compare the spatial curvature coming from the mimetic gauge field model with the standard one, we need to consider $SU(2)$ gauge symmetry in the spatially curved FLRW [H. Firouzjahi, M. A. G, S. Mukohyama, work in progress]

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{2}R - \lambda\big(F^a_{\mu\nu}F^{\mu\nu}_a + 1\big)\Big],$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\epsilon_{abc}A^b_\mu A^c_\nu\,.$$

- The last term includes the local gauge symmetry effects. In the global limit $g \to 0$, the above definition coincides with its global $O(3)$ counterpart which we have already studied.

- In spatially curved FLRW, the Friedmann equation is

$$\Omega_k + \Omega_\lambda + \Omega_r = 1\,,$$

with

$$\Omega_r = \frac{4\lambda}{H^2 g^2}\frac{(A^2-k)^2}{a^4}\,, \qquad \Omega_\lambda = \frac{2\lambda}{3H^2}\,, \qquad \Omega_k = -\frac{k}{a^2H^2}\,.$$

- The total spatial curvature is determined by the two components: Ω_λ is the spatial curvature coming from the mimetic sector and Ω_k is the standard geometrical spatial curvature:

$$\Omega_k^T = \Omega_k + \Omega_\lambda$$

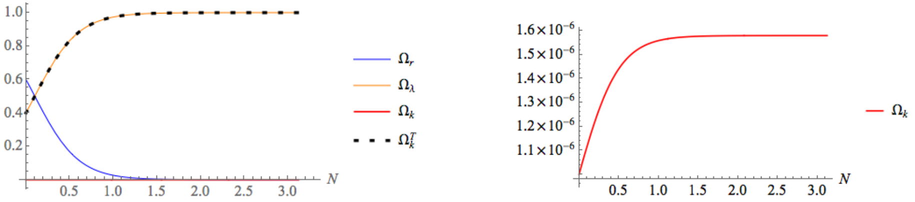


Figure: The negative branch for $k < 0$, $\Omega_r = 0.6$, and $\Omega_k = 10^{-6}$.

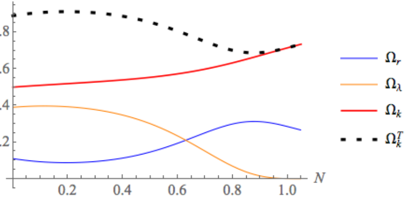


Figure: Positive branch for $k < 0$, $\Omega_r = 0.11$, and $\Omega_k = 0.5$

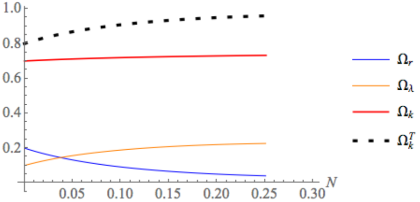


Figure: Negative branch: $k < 0$, $\Omega_r = 0.2$, and $\Omega_k = 0.7$

Summary

- The standard mimetic gravity provides dark matter-like energy density component while it suffers from two problems: 1) Caustics formations 2) Non-dynamical curvature perturbations. We can make the curvature modes to be dynamical by adding some simple higher derivative terms but the setup is unstable. The second problem can be solved if we consider a coupling between the higher derivative terms and curvature but the setup becomes very complicated.

Summary

- The standard mimetic gravity provides dark matter-like energy density component while it suffers from two problems: 1) Caustics formations 2) Non-dynamical curvature perturbations. We can make the curvature modes to be dynamical by adding some simple higher derivative terms but the setup is unstable. The second problem can be solved if we consider a coupling between the higher derivative terms and curvature but the setup becomes very complicated.
- Implementing a gauge field rather than scalar field, we do not have dark matter but there is no caustic in this scenario. The curvature perturbations are also dynamical and stable.

Summary

- The standard mimetic gravity provides dark matter-like energy density component while it suffers from two problems: 1) Caustics formations 2) Non-dynamical curvature perturbations. We can make the curvature modes to be dynamical by adding some simple higher derivative terms but the setup is unstable. The second problem can be solved if we consider a coupling between the higher derivative terms and curvature but the setup becomes very complicated.
- Implementing a gauge field rather than scalar field, we do not have dark matter but there is no caustic in this scenario. The curvature perturbations are also dynamical and stable.
- In the spatially curved FLRW background, we then find extra spatial curvature energy density which coming from the mimetic sector only at the dynamical level. If we have cosmological observations that constraint the geometrical and dynamical spatial curvatures separately, we may find an observational consequence of the model.

Thank You

Two-Field Disformal Transformation

$$g_{\mu\nu} = A\tilde{g}_{\mu\nu} + B\,\phi_{,\mu}\phi_{,\nu} + C\,\psi_{,\mu}\psi_{,\nu} + D\left(\phi_{,\mu}\psi_{,\nu} + \psi_{,\mu}\phi_{,\nu}\right),$$

where A, B, C, D are given functions of ϕ, ψ, X, Y, Z where X, Y, Z are defined as

$$\begin{cases} X \equiv \tilde{g}^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\,, \\ Y \equiv \tilde{g}^{\mu\nu}\psi_{,\mu}\psi_{,\nu}\,, \\ Z \equiv \tilde{g}^{\mu\nu}\phi_{,\mu}\psi_{,\nu}\,. \end{cases}$$

$$\left(\frac{\partial g_{\mu\nu}}{\partial \tilde{g}_{\alpha\beta}} - \lambda^{(n)}\,\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu}\right)\xi^{(n)}_{\alpha\beta} = 0\,,$$