Are the small parameters in the quark sector related?

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♣ Sources : arXiv:1705.07784 (PRD) and arXiv:1808.02297 (PRD)

♣ Collaborators : U. K. Dey and P. B. Pal

Towards a world without miracles

- Approximate Reality:

$$m_u \approx 0\,, \qquad m_d \approx 0\,, \qquad V_{\text{CKM}} \approx \begin{pmatrix} \cos\theta_C & -\sin\theta_C & 0 \\ \sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- The zeros are unrelated in the SM.
- Rule of Attraction:

$$\text{Attractiveness of a theory} \propto \frac{1}{(\text{No. of miracles it needs to work})^n}$$

  The value of 'n' depends on personal taste!

- We will use S_3 flavor symmetry to reduce the no. of miracles (hopefully!).

" True beauty comes from symmetry, not chance,
As those move easiest who have learned to dance. "

Alexander Pope

S_3 basics

“Symmetry is what we see at a glance; based on the fact that there is no reason for any difference..."

Blaise Pascal

$$P_{123} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \; ; \; P_{231} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \; ; \; P_{312} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \; ;$$

$$P_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \; ; \; P_{13} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \; ; \; P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} .$$

The three dimensional representation of S_3 is not an irreducible one simply because we can easily construct a linear combination of the elements, $1 + 2 + 3$, which remains unaltered under the permutation of the indices.

S_3 basics

$$P'_{123} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \quad P'_{231} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} ; \quad P'_{312} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} ;$$

$$P'_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \quad P'_{13} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} ; \quad P'_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} .$$

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$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \end{pmatrix} ; \quad P'_X = U P_X U^\dagger ;$$

S_3 basics continued ...

- S_3 has three irreducible representation, $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$.
- Important Rule: $\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$.
- In terms of the components:

  Suppose, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ transform as $\mathbf{2}$ of S_3. Then, for our choice of basis

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \otimes \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{(x_1 y_1 + x_2 y_2)}_{\mathbf{1}} \oplus \underbrace{(x_1 y_2 - x_2 y_1)}_{\mathbf{1}'} \oplus \underbrace{\begin{bmatrix} x_1 y_2 + x_2 y_1 \\ x_1 y_1 - x_2 y_2 \end{bmatrix}}_{\mathbf{2}}$$

S_3 symmetric 2HDM

Quarks transform in the following way under S_3 :

$$\begin{bmatrix} Q_{1L} \\ Q_{2L} \end{bmatrix}, \qquad \begin{bmatrix} u_{1R} \\ u_{2R} \end{bmatrix}, \qquad \begin{bmatrix} d_{1R} \\ d_{2R} \end{bmatrix} \; : \qquad \mathbf{2}$$

$$Q_{3L}\,, \qquad u_{3R}\,, \qquad d_{3R} \; : \qquad \mathbf{1}$$

We also have two $SU(2)_L$ scalar doublets transforming under S_3 as follows:

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \; : \qquad \mathbf{2}$$

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Yukawa Lagrangian for the up-sector (for example):

$$\begin{aligned} \mathscr{L}_Y^{(u)} \;=\; & -A_u\left(\bar{Q}_{1L}\tilde{\phi}_1 + \bar{Q}_{2L}\tilde{\phi}_2\right)u_{3R} - C_u\bar{Q}_{3L}\left(\tilde{\phi}_1 u_{1R} + \tilde{\phi}_2 u_{2R}\right) \\ & -B_u\left\{\left(\bar{Q}_{1L}\tilde{\phi}_2 + \bar{Q}_{2L}\tilde{\phi}_1\right)u_{1R} + \left(\bar{Q}_{1L}\tilde{\phi}_1 - \bar{Q}_{2L}\tilde{\phi}_2\right)u_{2R}\right\} + \text{h.c.} \end{aligned}$$

Mass matrix resulting from this Yukawa Lagrangian:

$$\mathcal{M}_q = v\begin{pmatrix} B_q\sin\beta & B_q\cos\beta & A_q\cos\beta \\ B_q\cos\beta & -B_q\sin\beta & A_q\sin\beta \\ C_q\cos\beta & C_q\sin\beta & 0 \end{pmatrix}, \qquad \tan\beta = \langle\phi_2\rangle\,/\,\langle\phi_1\rangle$$

Diagonalization of the mass matrix

- $U_q(\mathcal{M}_q \mathcal{M}_q^\dagger) U_q^\dagger = D_q^2 = \text{diag}\{m_{1q}^2, m_{2q}^2, m_{3q}^2\}\,. \quad \Rightarrow \quad V_{\text{CKM}} = U_u U_d^\dagger\,.$
  We therefore need to find the diagonalizing matrices U_u and U_d.
- We parametrize $U_q^{(0)}$ as $U_q^{(0)} = \mathcal{O}_q \mathcal{U}$.

$$\mathcal{U} = \begin{pmatrix} 0 & 0 & 1 \\ \sin\beta & -\cos\beta & 0 \\ \cos\beta & \sin\beta & 0 \end{pmatrix}$$

- The matrix $\mathcal{U}$ does not affect the CKM matrix.

$$V_{\text{CKM}}^{(0)} = U_u^{(0)} U_d^{(0)\dagger} \equiv \mathcal{O}_u \mathcal{O}_d^\dagger$$

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- Two things to note:
  1. $\mathrm{Det}(\mathcal{M}_q\mathcal{M}_q^\dagger) = v^6 A_q^2 B_q^2 C_q^2 \sin^2 3\beta$.
  2.

$$M_q^2 = \mathcal{U}\mathcal{M}_q\mathcal{M}_q^\dagger\mathcal{U}^\dagger = v^2 \begin{pmatrix} C_q^2 & -B_q C_q \cos 3\beta & B_q C_q \sin 3\beta \\ -B_q C_q \cos 3\beta & B_q^2 & 0 \\ B_q C_q \sin 3\beta & 0 & A_q^2 + B_q^2 \end{pmatrix}$$

Quark sector at the zeroth order

- To have our desired zeros we require $\sin 3\beta = 0\,, \quad \Rightarrow \beta = \pi/3\,.$
-

$$M_q^2 = \mathcal{U}\mathcal{M}_q\mathcal{M}_q^\dagger\mathcal{U}^\dagger = v^2 \begin{pmatrix} C_q^2 & -B_qC_q & 0 \\ -B_qC_q & B_q^2 & 0 \\ 0 & 0 & A_q^2 + B_q^2 \end{pmatrix}$$

- Define $\tan\theta_q = C_q/B_q$ and

$$\mathcal{O}_q = \begin{pmatrix} \cos\theta_q & -\sin\theta_q & 0 \\ \sin\theta_q & \cos\theta_q & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- Perform the rest of the diagonalization as

$$\mathcal{O}_q M_q^2 \mathcal{O}_q^\dagger = \underbrace{\mathcal{O}_q\mathcal{U}}_{U_q^{(0)}}\,\mathcal{M}_q\mathcal{M}_q^\dagger\,\underbrace{\mathcal{U}^\dagger\mathcal{O}_q^\dagger}_{U_q^{(0)\dagger}} = v^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & B_q^2 + C_q^2 & 0 \\ 0 & 0 & A_q^2 + B_q^2 \end{pmatrix}$$

Quark sector at the zeroth order

- The CKM matrix at the leading order:

$$V_{\text{CKM}}^{(0)} = U_u^{(0)} U_d^{(0)\,\dagger} = \begin{pmatrix} \cos(\theta_u - \theta_d) & -\sin(\theta_u - \theta_d) & 0 \\ \sin(\theta_u - \theta_d) & \cos(\theta_u - \theta_d) & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

- Quark masses are as follows:

$$m_t^2 = (A_u^2 + B_u^2) v^2 \,, \qquad m_c^2 = (B_u^2 + C_u^2) v^2 \,, \qquad m_u^2 = 0 \,,$$
$$m_b^2 = (A_d^2 + B_d^2) v^2 \,, \qquad m_s^2 = (B_d^2 + C_d^2) v^2 \,, \qquad m_d^2 = 0 \,.$$

- All the zeros are results of a single miracle: $\beta = \pi/3$!
- Yukawa hierarchies (clear from the mass eigenvalues):

$$A_u^2 \gg B_u^2, C_u^2 \,, \qquad A_d^2 \gg B_d^2, C_d^2 \,, \qquad A_u^2 \gg A_d^2 \,.$$

How to get closer to the reality?

- How to generate correct nonzero values of the small parameters?
- We can try $\sin 3\beta = \delta \to 0$ and then calculate the small parameters in terms of δ. But it does not work!
- This $S3$-2HDM might be a constituent part of a more elaborate theoretical framework.

" Whoever thinks a faultless piece to see,
Thinks what ne'er was, nor is, nor e'er shall be, "

Alexander Pope

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- Enter left-right symmetry (LRS).

LRS Basics

- Under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$$Q_{iL} \ : \ (2, 1, 1/3)\,, \qquad Q_{iR} \ : \ (1, 2, 1/3)\,, \qquad \phi \ : \ (2, 2, 0)\,.$$

- $\widetilde{\phi} = \sigma_2\, \phi^* \sigma_2$ transforms in the same way as ϕ.
- The Yukawa Lagrangian (quark sector) reads

$$\mathscr{L}_Y = -\sum_{i,j=1}^{3} \left[\Gamma_{ij}\, \bar{Q}_{iL}\, \phi\, Q_{jR} + \Delta_{ij}\, \bar{Q}_{iL}\, \widetilde{\phi}\, Q_{jR} \right]\,.$$

- After SSB

$$\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}\,.$$

- The mass matrices will be

$$M_u = \kappa\, \Gamma + \kappa'\, \Delta\,, \qquad M_d = \kappa'\, \Gamma + \kappa\, \Delta\,.$$

Adding S_3 to LRS

- The left and right handed quark doublets transform under S_3:

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} : \mathbf{2}, \qquad Q_3 : \mathbf{1}.$$

- In order to obtain an acceptable mass pattern, we now need scalars to be in the 2 representation of S_3. This means that we need to add an extra bidoublet.

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} : \mathbf{2}.$$

- Yukawa Lagrangian:

$$\begin{aligned} -\mathscr{L}_Y \;=\;& A\left(\bar{Q}_{1L}\Phi_1 + \bar{Q}_{2L}\Phi_2\right) Q_{3R} + C\bar{Q}_{3L}\left(\Phi_1 Q_{1R} + \Phi_2 Q_{2R}\right) \\ &+B\left[\left(\bar{Q}_{1L}\Phi_2 + \bar{Q}_{2L}\Phi_1\right) Q_{1R} + \left(\bar{Q}_{1L}\Phi_1 - \bar{Q}_{2L}\Phi_2\right) Q_{2R}\right] \\ &+\tilde{A}\left(\bar{Q}_{1L}\tilde{\Phi}_1 + \bar{Q}_{2L}\tilde{\Phi}_2\right) Q_{3R} + \tilde{C}\bar{Q}_{3L}\left(\tilde{\Phi}_1 Q_{1R} + \tilde{\Phi}_2 Q_{2R}\right) \\ &+\tilde{B}\left[\left(\bar{Q}_{1L}\tilde{\Phi}_2 + \bar{Q}_{2L}\tilde{\Phi}_1\right) Q_{1R} + \left(\bar{Q}_{1L}\tilde{\Phi}_1 - \bar{Q}_{2L}\tilde{\Phi}_2\right) Q_{2R}\right] + \text{h.c.}\,. \end{aligned}$$

Mass Matrices

- After SSB:

$$\langle \phi_a \rangle = \begin{pmatrix} \kappa_a & 0 \\ 0 & \kappa'_a \end{pmatrix} , \qquad a = 1, 2.$$

- Quark mass matrices:

$$\mathscr{M}_u = F\kappa_1 + G\kappa_2 + \tilde{F}\kappa'_1 + \tilde{G}\kappa'_2 ,$$
$$\mathscr{M}_d = \tilde{F}\kappa_1 + \tilde{G}\kappa_2 + F\kappa'_1 + G\kappa'_2 ,$$

$$F = \begin{pmatrix} 0 & B & A \\ B & 0 & 0 \\ C & 0 & 0 \end{pmatrix} , \qquad G = \begin{pmatrix} B & 0 & 0 \\ 0 & -B & A \\ 0 & C & 0 \end{pmatrix} .$$

- Renaming:

$$A_u = A , \qquad B_u = B , \qquad C_u = C ,$$
$$A_d = \tilde{A} , \qquad B_d = \tilde{B} , \qquad C_d = \tilde{C} .$$

Large and small terms

- We assume $\kappa_1', \kappa_2' \ll \kappa_1, \kappa_2$ which is not wild because we know W_L-W_R mixing to be very small.
- In the zeroth order

$$\mathscr{M}_q^{(0)} = v \begin{pmatrix} B_q \sin\beta & B_q \cos\beta & A_q \cos\beta \\ B_q \cos\beta & -B_q \sin\beta & A_q \sin\beta \\ C_q \cos\beta & C_q \sin\beta & 0 \end{pmatrix} .$$

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  Same as in S_3-2HDM!
- This simple picture will be perturbed when we turn on small but nonzero values of κ_1', κ_2' and $\sin 3\beta$.
- We will only keep corrections that are proportional to the top-Yukawa.
- We parametrize the corrections to the mass matrices as:

$$\mathscr{M}_q = \mathscr{M}_q^{(0)} + \mathscr{M}_q' .$$

Leading corrections

- Taking $\sin 3\beta = 3\delta$, we may write:

$$\mathscr{M}'_u \approx v\, A_u \begin{pmatrix} 0 & 0 & \frac{\sqrt{3}}{2}\delta \\ 0 & 0 & -\frac{1}{2}\delta \\ 0 & 0 & 0 \end{pmatrix}, \qquad \mathscr{M}'_d \approx A_u \begin{pmatrix} 0 & 0 & \kappa'_1 \\ 0 & 0 & \kappa'_2 \\ 0 & 0 & 0 \end{pmatrix}.$$

- Parametrization:

$$\left(\mathscr{M}'_u\right)_{13} = m_t \epsilon_u \cos\chi_u\,, \qquad \left(\mathscr{M}'_u\right)_{23} = m_t \epsilon_u \sin\chi_u\,,$$

$$\left(\mathscr{M}'_d\right)_{13} = m_t \epsilon_d \cos\chi_d\,, \qquad \left(\mathscr{M}'_d\right)_{23} = m_t \epsilon_d \sin\chi_d\,,$$

$$\epsilon_u = \delta\,, \qquad \chi_u = -\pi/6\,,$$

$$\epsilon_d = \sqrt{\kappa_1'^2 + \kappa_2'^2}\Big/ v\,, \qquad \tan\chi_d = \kappa'_2/\kappa'_1\,.$$

Leading corrections ...

- Determinant:

$$|\det \mathscr{M}_q| = v^2 B_q C_q m_t \, \epsilon_q \sin\left(\frac{\pi}{3} - \chi_q\right) = m_{1q}\, m_{2q}\, m_{3q}\,.$$

- From this eq. extract the first generation masses:

$$m_{1q} = \frac{m_t m_{2q}}{m_{3q}} \sin\theta_q \cos\theta_q \epsilon_q \sin\left(\frac{\pi}{3} - \chi_q\right) = \epsilon'_q m_{2q} \sin\theta_q \cos\theta_q\,,$$

$$\epsilon'_q = \frac{m_t}{m_{3q}} \epsilon_q \sin\left(\frac{\pi}{3} - \chi_q\right).$$

- More cleanly, formula for the light quark masses:

$$\begin{aligned} m_u &= \epsilon'_u m_c \sin\theta_u \cos\theta_u\,, \\ m_d &= \epsilon'_d m_s \sin\theta_d \cos\theta_d\,. \end{aligned}$$

Leading corrections ...

- For diagonalization of $\mathscr{M}_q \mathscr{M}_q^\dagger$, we modify the diagonalizing matrix as:

$$U_q = X_q U_q^{(0)} ,$$

where X_q is supposed to inflict small corrections on $U_q^{(0)}$. We now parametrize X_q by writing

$$X_q = \begin{pmatrix} 1 & 0 & \alpha_q \\ 0 & 1 & \gamma_q \\ -\alpha_q & -\gamma_q & 1 \end{pmatrix} ,$$

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$$\alpha_q = \epsilon'_q \sin\theta_q \,, \qquad \gamma_q = -\epsilon'_q \cos\theta_q \,.$$

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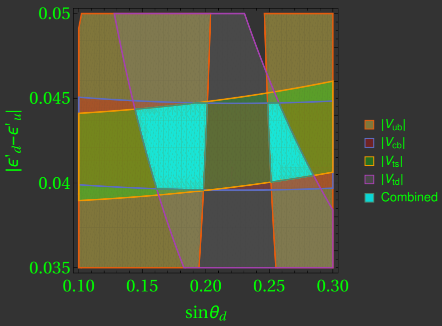
-

$$\begin{aligned} V_{\rm CKM} &= X_u U_u^{(0)} U_d^{(0)\,\dagger} X_d^\dagger = X_u V_{\rm CKM}^{(0)} X_d^\dagger \\ &\approx \begin{pmatrix} \cos\theta_C & -\sin\theta_C & -(\epsilon'_d - \epsilon'_u)\sin\theta_u \\ \sin\theta_C & \cos\theta_C & (\epsilon'_d - \epsilon'_u)\cos\theta_u \\ (\epsilon'_d - \epsilon'_u)\sin\theta_d & -(\epsilon'_d - \epsilon'_u)\cos\theta_d & 1 \end{pmatrix} . \end{aligned}$$

Results



Sample benchmark:

$$\epsilon'_d = 0.072\,, \qquad \epsilon'_u = 0.028\,, \qquad \sin\theta_d = 0.26\,.$$

We get,

$$|V_{ub}| \approx 0.002\,, \qquad |V_{cb}| \approx 0.044\,, \qquad |V_{td}| \approx 0.011\,, \qquad |V_{ts}| \approx 0.042\,,$$
$$m_u \approx 1.2\text{ MeV}\,, \qquad m_d \approx 2.0\text{ MeV}\,.$$

Conclusions

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We are here?!

The road to progress: Polytheism ⟶ Monotheism ⟶ Atheism

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We are here?!

The road to progress: Polytheism $\longrightarrow$ Monotheism $\longrightarrow$ Atheism

People who believe in hidden patterns in quark masses and mixings

Conclusions

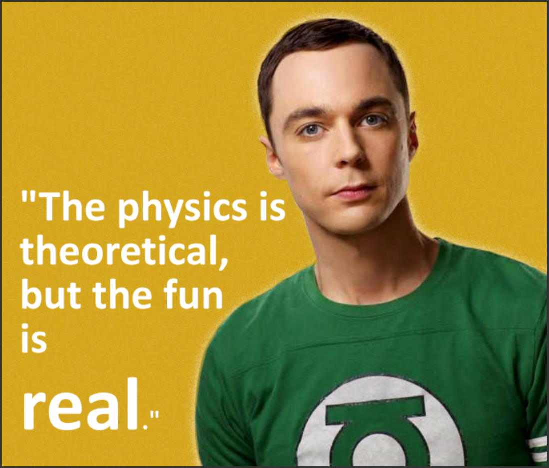
We are here?!

The road to progress: Polytheism $\longrightarrow$ Monotheism $\longrightarrow$ Atheism

People who believe in hidden patterns in quark masses and mixings

Believing in hidden pattern in the fermionic sector and being a reasonable intelligent adult might not be mutually exclusive!

Conclusions



THANK YOU !