Alternative Fayet-Iliopoulos terms in supergravity

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Supergravity, R-invariance and Fayet-Iliopoulos terms

- Rigid supersymmetry can be broken spontaneously using a U(1) vector multiplet with a Fayet-Iliopoulos (FI) term.

  P. Fayet & J. Iliopoulos (1974)

- Locally supersymmetric extension of the FI term is achieved by gauging *R*-symmetry.

  D. Freedman (1977)

- "...In order for a U(1) gauge theory with a FI term to be consistently coupled to supergravity, preserving gauge invariance, superpotential must be R invariant. A supersymmetric cosmological term and therefore an explicit mass-like term for the gravitino is forbidden by gauge invariance."

  R. Barbieri, S. Ferrara, D. Nanopoulos & K. Stelle (1982)

FI terms in supergravity without gauged R-symmetry

Generalised Fayet-Iliopoulos terms in supergravity, which do not require gauged R-symmetry, were proposed in:

- N. Cribiori, F. Farakos, M. Tournoy & A. Van Proeyen (22 December, 2017) [arXiv:1712.08601]

$$\mathfrak{J}_{\rm FI}^{(1)} = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \, E \, \Upsilon \mathfrak{V}_1 \, , \quad \mathfrak{V}_1 := -4 \frac{W^2 \bar W^2 \mathcal{D} W}{(\mathcal{D}^2 W^2)(\bar{\mathcal{D}}^2 \bar W^2)}$$

- SMK (15 January, 2018) [arXiv:1801.04794]

$$\mathfrak{J}_{\rm FI}^{(n)} = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \, E \, \Upsilon \mathfrak{V}_n \, , \quad \mathfrak{V}_n := -4 \frac{W^2 \bar W^2 (\mathcal{D} W)^{4n-3}}{\big[(\mathcal{D}^2 W^2)(\bar{\mathcal{D}}^2 \bar W^2)\big]^n}$$

  Υ conformal compensator of any off-shell supergravity.

- arXiv:1801.04794 was a natural extension of SMK, I. McArthur & G. Tartaglino-Mazzucchelli [arXiv:1702.02423].
- It is possible that Van Proeyen was looking for such a generalisation ever since his 1983 work with Ferrara, Girardello & Kugo.

Weyl-invariant formulation for Einstein's gravity

Einstein-Hilbert action with a cosmological term

$$S_{\rm EH} = \frac{1}{2\kappa^2}\int {\rm d}^4x\, e\, R - \Lambda \int {\rm d}^4x\, e$$

Weyl-invariant reformulation

S. Deser (1970)
B. Zumino (1970)

$$S = \frac{1}{2}\int {\rm d}^4x\, e\left(\nabla^a\varphi\nabla_a\varphi + \frac{1}{6}R\varphi^2 - \lambda\varphi^4\right) ,$$

where φ is a nowhere vanishing conformal compensator.
Weyl transformations

$$\delta\nabla_a = \sigma\nabla_a + (\nabla^b\sigma)M_{ba} , \qquad \delta\varphi = \sigma\varphi ,$$
$$\nabla_a := e_a{}^m\partial_m + \frac{1}{2}\omega_a{}^{bc}M_{bc} , \qquad [\nabla_a, \nabla_b] = \frac{1}{2}R_{ab}{}^{cd}M_{cd}$$

Weyl invariance is part of the gauge freedom of conformal gravity.
In the case of Weyl-invariant formulation for Einstein's gravity, imposing Weyl gauge $\varphi = \frac{\sqrt{6}}{\kappa} = {\rm const}$ takes us back to the original action.

Off-shell formulations for supergravity: a review

- Pure 4D $\mathcal{N} = 1$ supergravity can be realised as conformal supergravity coupled to a compensating supermultiplet.

  M. Kaku & P. Townsend (1978)
  T. Kugo & S. Uehara (1983)

- Different off-shell formulations for supergravity correspond to different compensators.

  W. Siegel & J. Gates (1979)
  S. Ferrara, L. Girardello, T. Kugo & A. Van Proeyen (1983)

- The simplest way to describe $\mathcal{N} = 1$ conformal supergravity in superspace is to make use of the geometry proposed by

  R. Grimm, J. Wess & B. Zumino (1978)

  This superspace geometry was used in the very first published work on the old minimal formulation for $\mathcal{N} = 1$ supergravity:

  J. Wess and B. Zumino, Phys. Lett. B **74**, 51 (1978)

  Old minimal supergravity was independently developed by

  K. Stelle & P. West, Phys. Lett. B **74**, 330 (1978)
  S. Ferrara & P. van Nieuwenhuizen, Phys. Lett. B **74**, 333 (1978)

Grimm-Wess-Zumino superspace geometry

Superspace covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot\alpha})$ have the form

$$\mathcal{D}_A \,=\, E_A{}^M \partial_M + \Omega_A{}^{\beta\gamma} M_{\beta\gamma} + \bar{\Omega}_A{}^{\dot\beta\dot\gamma} \bar{M}_{\dot\beta\dot\gamma} \ .$$

Graded commutation relations

$$\begin{aligned} \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} &= -2\mathrm{i}\mathcal{D}_{\alpha\dot\alpha} \ , \\ \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} \,=\, -4\bar{R} M_{\alpha\beta} \ , &\qquad \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 4R\bar{M}_{\dot\alpha\dot\beta} \ , \\ \Big[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}\Big] \,=\, &\mathrm{i}\varepsilon_{\alpha\beta}\Big(\bar{R}\,\bar{\mathcal{D}}_{\dot\beta} + G^\gamma{}_{\dot\beta}\mathcal{D}_\gamma - (\mathcal{D}^\gamma G^\delta{}_{\dot\beta})M_{\gamma\delta} + 2\bar{W}_{\dot\beta}{}^{\dot\gamma\dot\delta}\bar{M}_{\dot\gamma\dot\delta}\Big) \\ &+\mathrm{i}(\bar{\mathcal{D}}_{\dot\beta}\bar{R})M_{\alpha\beta} \ . \end{aligned}$$

Torsion superfields R, $G_{\alpha\dot\alpha} = \bar{G}_{\alpha\dot\alpha}$ and $W_{\alpha\beta\gamma}$ obey the Bianchi identities:

$$\bar{\mathcal{D}}_{\dot\alpha} R = 0 \ , \qquad \bar{\mathcal{D}}_{\dot\alpha} W_{\alpha\beta\gamma} = 0 \ , \qquad \bar{\mathcal{D}}^{\dot\alpha} G_{\alpha\dot\alpha} = \mathcal{D}_\alpha R$$

R, $G_{\alpha\dot\alpha}$ and $W_{\alpha\beta\gamma}$ are supergravity analogues of the scalar curvature, traceless Ricci tensor and self-dual Weyl tensor, respectively.

Super-Weyl transformations

P. Howe & R. Tucker (1978)

$$\delta_\sigma \mathcal{D}_\alpha = (\bar\sigma - \frac{1}{2}\sigma)\mathcal{D}_\alpha + (\mathcal{D}^\beta \sigma)\, M_{\alpha\beta} \ ,$$

$$\delta_\sigma \bar{\mathcal{D}}_{\dot\alpha} = (\sigma - \frac{1}{2}\bar\sigma)\bar{\mathcal{D}}_{\dot\alpha} + (\bar{\mathcal{D}}^{\dot\beta}\bar\sigma)\bar{M}_{\dot\alpha\dot\beta} \ ,$$

$$\delta_\sigma \mathcal{D}_{\alpha\dot\alpha} = \frac{1}{2}(\sigma + \bar\sigma)\mathcal{D}_{\alpha\dot\alpha} + \frac{\mathrm{i}}{2}(\bar{\mathcal{D}}_{\dot\alpha}\bar\sigma)\mathcal{D}_\alpha + \frac{\mathrm{i}}{2}(\mathcal{D}_\alpha\sigma)\bar{\mathcal{D}}_{\dot\alpha}$$
$$+(\mathcal{D}^\beta{}_{\dot\alpha}\sigma)M_{\alpha\beta} + (\mathcal{D}_\alpha{}^{\dot\beta}\bar\sigma)\bar{M}_{\dot\alpha\dot\beta} \ ,$$

where σ is an arbitrary covariantly chiral scalar superfield, $\bar{\mathcal{D}}_{\dot\alpha}\sigma = 0$.
The torsion tensors transform as follows:

$$\delta_\sigma R = 2\sigma R + \frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\bar\sigma \ ,$$

$$\delta_\sigma G_{\alpha\dot\alpha} = \frac{1}{2}(\sigma + \bar\sigma)G_{\alpha\dot\alpha} + \mathrm{i}\mathcal{D}_{\alpha\dot\alpha}(\sigma - \bar\sigma) \ ,$$

$$\delta_\sigma W_{\alpha\beta\gamma} = \frac{3}{2}\sigma W_{\alpha\beta\gamma} \ .$$

Off-shell formulations for supergravity: a review

- Old minimal supergravity
  Its conformal compensator is a chiral scalar superfield S_0, $\bar{\mathcal{D}}_{\dot\alpha} S_0 = 0$, with the super-Weyl transformation

$$\delta_\sigma S_0 = \sigma S_0$$

  Pure supergravity action

$$S_{\rm OMSG} = -\frac{3}{\kappa^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\, \bar{S}_0 S_0 + \left\{ \frac{\mu}{\kappa^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\, \mathcal{E}\, S_0^3 + \mathrm{c.c.} \right\} ,$$

  where $E^{-1} = \mathrm{Ber}(E_A{}^M)$ and $\mathcal{E}$ is the chiral density.
- New minimal supergravity
  Its conformal compensator is a real linear superfield, $\bar{\mathbb{L}} - \mathbb{L} = (\bar{\mathcal{D}}^2 - 4R)\mathbb{L} = 0$, with the super-Weyl transformation

$$\delta_\sigma \mathbb{L} = (\sigma + \bar\sigma)\mathbb{L}$$

  Pure supergravity action (no cosmological terms is allowed)

$$S_{\rm NMSG} = \frac{3}{\kappa^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\, \mathbb{L} \ln\frac{\mathbb{L}}{|S_0|^2}$$

Fayet-Iliopoulos term in off-shell supergravity

Fayet-Iliopoulos term in new minimal supergravity

Consider new minimal supergravity coupled to a nonlinear σ-model and a U(1) vector multiplet with a Fayet-Iliopoulos. The action is

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\,\mathbb{L}\left\{\frac{3}{\kappa^2}\ln\frac{\mathbb{L}}{|S_0|^2} + K(\phi^i,\bar{\phi}^{\bar{i}})\right\} + S[V] ,$$

where $S[V]$ denotes the vector multiplet action,

$$S[V] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\left\{\frac{1}{8}V\mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V - 2f\mathbb{L}V\right\} .$$

All ϕ^i are assumed to be neutral under the super-Weyl transformations, $\delta_\sigma\phi^i = 0$.
The action is invariant under Kähler transformations

$$K(\phi,\bar{\phi}) \to K(\phi,\bar{\phi}) + F(\phi) + \bar{F}(\bar{\phi}) ,$$

with $F(\phi)$ an arbitrary holomorphic function.
It is also invariant under the gauge transformations of V,

$$\delta_\lambda V = \lambda + \bar{\lambda} , \qquad \bar{\mathcal{D}}_{\dot{\alpha}}\lambda = 0 .$$

Fayet-Iliopoulos term in old minimal supergravity

Applying a superfield Legendre transformation to the theory considered above, we end up with a dual formulation whihc describes old minimal supergravity coupled to a nonlinear σ-model and a U(1) vector multiplet with FI term. The resulting action is

$$S = -\frac{3}{\kappa^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \bar{S}_0 \exp\Big(\frac{2}{3} f\kappa^2 V\Big) S_0 \exp\Big(-\frac{\kappa^2}{3}K(\phi,\bar{\phi})\Big) + \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \frac{1}{8} V\mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V\ .$$

Kähler invariance:

$$K(\phi,\bar{\phi}) \to K(\phi,\bar{\phi}) + F(\phi) + \bar{F}(\bar{\phi})\ , \qquad S_0 \to \mathrm{e}^{\frac{\kappa^2}{3}F(\phi)} S_0$$

Gauge invariance

$$V \to V + \lambda + \bar{\lambda}\ , \qquad S_0 \to \mathrm{e}^{-\frac{2}{3}f\kappa^2\lambda} S_0$$

Fayet-Iliopoulos term in new minimal supergravity

We now consider the case of chiral matter with a superpotential $W(\phi^i)$.
Realisation I: Supergravity-matter action is

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\,\mathbb{L}\left\{\frac{3}{\kappa^2}\ln\frac{\mathbb{L}}{|S_0|^2} + K(\phi^i, \mathrm{e}^{q_i V}\bar\phi^{\bar i})\right\} + \left\{\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,W(\phi^i) + \mathrm{c.c.}\right\} + S[V]\ ,$$

where $S[V]$ is the same vector multiplet action,

$$S[V] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\left\{\frac{1}{8}V\mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V - 2f\mathbb{L}V\right\}\ .$$

In the presence of superpotential, ϕ^i and V vary under the super-Weyl transformations (containing local $\mathrm{U}(1)_R$ transformations),

$$\delta_\sigma\phi^i = q_i\sigma\phi^i\ , \qquad \delta_\sigma V = -\sigma - \bar\sigma\ ,$$

with real $\mathrm{U}(1)_R$ charges q_i. The $K(\phi,\bar\phi)$ and $W(\phi)$ have the properties

$$\mathrm{Im}\sum_i q_i\phi^i\partial_i K = 0\ , \qquad \sum_i q_i\phi^i\partial_i W = 3W\ .$$

Fayet-Iliopoulos term in new minimal supergravity

We now consider the case of chiral matter with a superpotential $W(\phi^i)$.
Realisation II: Supergravity-matter action is

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\,\mathbb{L}\left\{\frac{3}{\kappa^2}\ln\frac{\mathbb{L}}{|S_0|^2} + K\Big(\frac{\phi^i}{\mathbb{L}^{q_i/2}}, \frac{\bar{\phi}^{\bar{i}}}{\mathbb{L}^{q_i/2}}\Big)\right\}$$
$$+\Big\{\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,W(\phi^i) + \mathrm{c.c.}\Big\} + S[V]\ ,$$

where $S[V]$ is the same vector multiplet action,

$$S[V] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\left\{\frac{1}{8}V\mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V - 2f\mathbb{L}V\right\}\ .$$

In the presence of superpotential, ϕ^i vary under the super-Weyl transformations (containing local $\mathrm{U}(1)_R$ transformations),

$$\delta_\sigma\phi^i = q_i\sigma\phi^i\ , \qquad \delta_\sigma V = 0\ ,$$

with real $\mathrm{U}(1)_R$ charges q_i. The $K(\phi,\bar{\phi})$ and $W(\phi)$ have the properties

$$\mathrm{Im}\sum_i q_i\phi^i\partial_i K = 0\ , \qquad \sum_i q_i\phi^i\partial_i W = 3W\ .$$

Fayet-Iliopoulos term in old minimal supergravity

"...The new minimal auxiliary field formulation is equivalent to the restricted class of old minimal formulation, namely the one with R symmetry. This symmetry is a necessary and sufficient condition for the Fayet-Iliopoulos term to be introduced."

S. Ferrara, L. Girardello, T. Kugo & A. Van Proeyen (1983)

Nilpotent real scalar supermultiplet

Nilpotent real scalar supermultiplet

$\mathcal{N} = 1$ Goldstino superfield model proposed in
SMK, I. McArthur & G. Tartaglino-Mazzucchelli [arXiv:1702.02423]
is described in terms of a real scalar superfield V with properties:
(i) it is super-Weyl invariant, $\delta_\sigma V = 0$; and (ii) it is constrained by

$$V^2 = 0 \ , \qquad V\mathcal{D}_A\mathcal{D}_B V = 0 \ , \qquad V\mathcal{D}_A\mathcal{D}_B\mathcal{D}_C V = 0 \ .$$

In order for V to serve as a Goldstino supermultiplet, the real descendant $\mathcal{D}W := \mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot\alpha}\bar{W}^{\dot\alpha}$ be nowhere vanishing, with

$$W_\alpha := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V \ , \qquad \bar{\mathcal{D}}_{\dot\beta}W_\alpha = 0 \ .$$

Dynamics is governed by the super-Weyl invariant action

$$S[V] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\left\{\frac{1}{8}V\mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V - 2f\mathbb{L}V\right\} \ .$$

$S[V]$ also describes a U(1) vector multiplet with the FI term
in the case when V is unconstrained. Then $S[V]$ is gauge invariant.

Nilpotent real scalar supermultiplet

- Super-Weyl transformation laws:

$$\delta_\sigma V = 0 \ , \qquad \delta_\sigma W_\alpha = \frac{3}{2}\sigma W_\alpha \ , \qquad \delta_\sigma(\mathcal{D}W) = (\sigma + \bar\sigma)\mathcal{D}W \ .$$

- Constraints $V^2 = 0 \ , \quad V\mathcal{D}_A\mathcal{D}_B V = 0 \ , \quad V\mathcal{D}_A\mathcal{D}_B\mathcal{D}_C V = 0$ imply

$$V = -4\frac{W^2\bar W^2}{(\mathcal{D}W)^3} \ , \qquad W^2 := W^\alpha W_\alpha \ .$$

- Important by-product:
  Consider a massless vector supermultiplet described by gauge-invariant chiral field strength W_α such that $\mathcal{D}W \neq 0$.
  Then the following composite

$$\mathfrak{V} = -4\frac{W^2\bar W^2}{(\mathcal{D}W)^3} = \bar{\mathfrak{V}}$$

is super-Weyl invariant, $\qquad \delta_\sigma \mathfrak{V} = 0$.

Constructing alternative FI terms in supergravity

Constructing alternative FI terms in supergravity

Consider a massless vector supermultiplet coupled to conformal supergravity. It is described by a real prepotential V with properties:

- It is defined modulo gauge transformations

$$\delta_\lambda V = \lambda + \bar{\lambda} \ , \qquad \bar{\mathcal{D}}_{\dot{\alpha}} \lambda = 0$$

- It is super-Weyl inert, $\delta_\sigma V = 0$.
- Top component of V (D-field) is assumed to be nowhere vanishing,

$$\mathcal{D}W := \mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \neq 0 \ , \qquad W_\alpha := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V$$

Example: Vector supermultiplet model with FI term in new minimal supergravity with action

$$S[V] = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, E \left\{ \frac{1}{8} V \mathcal{D}^\alpha (\bar{\mathcal{D}}^2 - 4R) \mathcal{D}_\alpha V - 2f \mathbb{L} V \right\} .$$

Equation of motion for V: $\mathcal{D}W = -2f\mathbb{L} \neq 0$.

Constructing alternative FI terms in supergravity

Since $\mathcal{D}W$ is nowhere vanishing, we can introduce real scalar composite

$$\mathfrak{V} := -4\frac{W^2\bar{W}^2}{(\mathcal{D}W)^3} , \qquad W^2 := W^\alpha W_\alpha .$$

The properties of $\mathfrak{V}$ are as follows:

- $\mathfrak{V}$ is gauge invariant, $\delta_\lambda \mathfrak{V} = 0$;
- $\mathfrak{V}$ is super-Weyl invariant, $\delta_\sigma \mathfrak{V} = 0$;
- $\mathfrak{V}$ obeys the nilpotency conditions

$$\mathfrak{V}^2 = 0 , \quad \mathfrak{V}\mathcal{D}_A\mathcal{D}_B\mathfrak{V} = 0 , \quad \mathfrak{V}\mathcal{D}_A\mathcal{D}_B\mathcal{D}_C\mathfrak{V} = 0$$

and, therefore, $\mathfrak{V}$ may be interpret as a Goldstino superfield.

We can use $\mathfrak{V}$ to construct a super-Weyl invariant functional

$$\mathfrak{I} = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \Upsilon \mathfrak{V} ,$$

Υ may be identified with a compensator: (i) $\Upsilon = \bar{S}_0 S_0$ in OMSG; and (ii) $\Upsilon = \mathbb{L}$ in NMSG. More general choices are possible.

$\mathfrak{V}$ as Goldstino supermultiplet

Nilpotency conditions

$$\mathfrak{V}^2 = 0\ , \quad \mathfrak{V}\mathcal{D}_A\mathcal{D}_B\mathfrak{V} = 0\ , \quad \mathfrak{V}\mathcal{D}_A\mathcal{D}_B\mathcal{D}_C\mathfrak{V} = 0$$

imply

$$\mathfrak{V} := -4\frac{\mathfrak{W}^2\bar{\mathfrak{W}}^2}{(\mathcal{D}\mathfrak{W})^3}\ , \qquad \mathfrak{W}_\alpha := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha\mathfrak{V}$$

Interpretation of $\mathfrak{V}$ as a Goldstino superfield is consistent provided its D-field is nowhere vanishing (equivalently, $(\mathcal{D}\mathfrak{W})^{-1}$ exists). This holds if $\mathcal{D}^2 W^2$ is nowhere vanishing.

$\mathfrak{V}$ has only two independent component fields: photino/Goldstino $\psi_\alpha \propto \mathfrak{W}_\alpha|_{\theta=0}$ and auxiliary scalar $D \propto \mathcal{D}^\alpha\mathfrak{W}_\alpha|_{\theta=0}$.

Component analysis

Let's analyse the component content of $\mathfrak{V}$ following the component reduction procedure described, e.g., in

SMK & S. McCarthy [hep-th/0501172]

- Component fields of the vector supermultiplet

$$W_\alpha| = \psi_\alpha \ , \quad -\frac{1}{2}\mathcal{D}^\alpha W_\alpha| = D \ , \quad \mathcal{D}_{(\alpha} W_{\beta)}| = 2\mathrm{i}\hat{F}_{\alpha\beta} \ = \mathrm{i}(\sigma^{ab})_{\alpha\beta}\hat{F}_{ab}$$

Bar-projection $U|$ means switching off the Grassmann variables $\theta, \bar{\theta}$.

- U(1) field strength

$$\begin{aligned}
\hat{F}_{ab} \ &= \ F_{ab} - \frac{1}{2}(\Psi_a\sigma_b\bar{\psi} + \psi\sigma_b\bar{\Psi}_a) + \frac{1}{2}(\Psi_b\sigma_a\bar{\psi} + \psi\sigma_a\bar{\Psi}_b) \ , \\
F_{ab} \ &= \ \nabla_a V_b - \nabla_b V_a - \mathcal{T}_{ab}{}^c V_c \ ,
\end{aligned}$$

with $V_a = e_a{}^m(x)\, V_m(x)$ the gauge one-form, and $\Psi_a{}^\beta$ the gravitino.

- ∇_a denotes spacetime covariant derivative with torsion.

Component analysis

∇_a denotes spacetime covariant derivative with torsion

$$[\nabla_a, \nabla_b] = \mathcal{T}_{ab}{}^c \nabla_c + \frac{1}{2} \mathcal{R}_{abcd} M^{cd} ,$$
$$\mathcal{T}_{abc} = -\frac{\mathrm{i}}{2}(\Psi_a \sigma_c \bar{\Psi}_b - \Psi_b \sigma_c \bar{\Psi}_a) .$$

where $\mathcal{R}_{abcd}$ is the curvature tensor and $\mathcal{T}_{abc}$ is the torsion tensor. Component expressions:

$$-\frac{1}{4}\mathcal{D}^2 W^2| = D^2 - 2F^{\alpha\beta}F_{\alpha\beta} + \text{fermionic terms} .$$

The D component field of $\mathfrak{V}$ is

$$-\frac{1}{2}\mathcal{D}\mathfrak{W}| = D\left|1 - 2\frac{F^{\alpha\beta}F_{\alpha\beta}}{D^2}\right|^2 + \text{fermionic terms} ,$$

where $\mathfrak{W}_\alpha := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha \mathfrak{V}$

Component analysis

$$
\begin{aligned}
\mathfrak{I} &= 2\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\,\Upsilon\mathfrak{V} \\
&\approx \int \mathrm{d}^4x\, e\, D\left|1 - 2\frac{F^{\alpha\beta}F_{\alpha\beta}}{D^2}\right|^2 + \text{fermionic terms}
\end{aligned}
$$

Constructing alternative FI terms in supergravity

Our construction of

$$\mathfrak{V} := -4\frac{W^2\bar{W}^2}{(\mathcal{D}W)^3} , \qquad W^2 := W^\alpha W_\alpha .$$

can naturally be generalised by introducing a gauge invariant and super-Weyl invariant composite operator

$$\mathfrak{V}_n := \mathfrak{V}\frac{(\mathcal{D}W)^{4n}}{\left[\mathcal{D}^2W^2\bar{\mathcal{D}}^2\bar{W}^2\right]^n} = -4\frac{W^2\bar{W}^2(\mathcal{D}W)^{4n-3}}{\left[(\mathcal{D}^2W^2)(\bar{\mathcal{D}}^2\bar{W}^2)\right]^n}$$

for a real parameter n, with $\mathfrak{V}_0 = \mathfrak{V}$.
Super-Weyl invariance follows from the following observation: Given a nowhere vanishing real scalar U with super-Weyl transformation law

$$\delta_\sigma U = (\sigma + \bar{\sigma})U ,$$

the composite operator

$$\left(\mathcal{D}^2 - 4\bar{R}\right)\frac{W^2}{U^2}$$

is super-Weyl invariant. S. Cecotti & S. Ferrara (1987)

Component analysis

$$\mathfrak{I}_n = 2\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \Upsilon \mathfrak{V}_n$$
$$\approx \int \mathrm{d}^4x\, e\, D\left|1 - 2\frac{F^{\alpha\beta}F_{\alpha\beta}}{D^2}\right|^{2-2n} + \text{fermionic terms}$$

Choice $n = 1$ is special:

$$\mathfrak{I}_1 = 2\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \Upsilon \mathfrak{V}_1 \approx \int \mathrm{d}^4x\, e\, D + \text{fermionic terms}$$

N. Cribiori, F. Farakos, M. Tournoy & A. Van Proeyen [1712.08601]

$\mathfrak{V}_n$ as Goldstino supermultiplet

Nilpotency conditions

$$\mathfrak{V}_n\mathfrak{V}_n = 0 \ , \quad \mathfrak{V}_n\mathcal{D}_A\mathcal{D}_B\mathfrak{V}_n = 0 \ , \quad \mathfrak{V}_n\mathcal{D}_A\mathcal{D}_B\mathcal{D}_C\mathfrak{V}_n = 0$$

imply

$$\mathfrak{V}_n := -4\frac{\mathfrak{W}_n^2\bar{\mathfrak{W}}_n^2}{(\mathcal{D}\mathfrak{W}_n)^3} \ , \qquad \mathfrak{W}_n{}^\alpha := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}^\alpha\mathfrak{V}_n$$

$\mathfrak{V}$ has only two independent component fields: photino/Goldstino $\psi^\alpha \propto \mathfrak{W}_n{}^\alpha|_{\theta=0}$ and auxiliary scalar $\tilde{D} \propto \mathcal{D}\mathfrak{W}_n|_{\theta=0}$.

Constructing alternative D^2 terms in supergravity

- Introduce super-Weyl invariant kinetic-like term

$$
\begin{aligned}
K_n &= \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \frac{1}{8}\mathfrak{V}_n \mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V \\
&= -\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \mathfrak{V}_n \mathcal{D}W
\end{aligned}
$$

- Component reduction

$$
K_n \approx \frac{1}{2}\int \mathrm{d}^4x\, e\, D^2 \left| 1 - 2\frac{F^{\alpha\beta}F_{\alpha\beta}}{D^2} \right|^{2-2n} + \text{fermionic terms}
$$

- Functional K_n can be added to the action only with a non-negative overall coefficient since K_n contains the fermionic kinetic terms.

U(1) duality invariant models and FI-type terms

U(1) duality invariant models and FI-type terms

General family of U(1) duality invariant models for a massless vector supermultiplet coupled to off-shell supergravity, old minimal or new minimal.

SMK & S. McCarthy (2005)

$$S[V;\Upsilon] = \frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,W^2 \,+\, \frac{1}{4}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,E\,\frac{W^2\,\bar{W}^2}{\Upsilon^2}\,\Lambda\Big(\frac{\omega}{\Upsilon^2},\frac{\bar{\omega}}{\Upsilon^2}\Big)$$

Here $\omega := \frac{1}{8}\mathcal{D}^2 W^2$, and $\Lambda(\omega,\bar{\omega})$ is a real analytic function satisfying the differential equation SMK & S. Theisen (2000)

$$\mathrm{Im}\Big\{\Gamma - \bar{\omega}\,\Gamma^2\Big\} = 0\ , \qquad \Gamma := \frac{\partial(\omega\,\Lambda)}{\partial\omega}\ .$$

Supersymmetric Born-Infeld action corresponds to (g coupling constant)

$$\Lambda_{\rm SBI}(\omega,\bar{\omega}) \,=\, \frac{g^2}{1+\frac{1}{2}\,A\,+\sqrt{1+A+\frac{1}{4}\,B^2}}\ ,$$

$$A \,=\, g^2(\omega+\bar{\omega})\ , \qquad B = g^2(\omega-\bar{\omega})$$

U(1) duality invariant models and FI-type terms

In the flat-space limit, the fermionic sector of

$$S[V] = \frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\, W^2 \; + \; \frac{1}{4}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, W^2\, \bar{W}^2\, \Lambda(\omega,\bar{\omega})$$

proves to coincide, modulo a nonlinear field redefinition, with the Volkov-Akulov action, under the mild restriction

$$\Lambda_{\omega\bar{\omega}}(0,0) = 3\Lambda^3(0,0)$$

Explanation of such ubiquitous appearance of the Volkov-Akulov action: SMK (2010)

If the FI term is added to the action,

$$S[V] \to \frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\, W^2 + \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\left\{\frac{1}{4}W^2\,\bar{W}^2\,\Lambda(\omega,\bar{\omega}) - 2f\,V\right\}$$

then auxiliary D-field develops a non-vanishing expectation value, in general.

U(1) duality invariant models and FI-type terms

- Supersymmetric Born-Infeld action

$$S_{\rm SBI}[V] = \frac{1}{2}\int {\rm d}^4x{\rm d}^2\theta\, W^2 + \frac{1}{4}\int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta\, W^2\, \bar W^2\, \Lambda_{\rm SBI}\,(\omega,\bar\omega)$$

  describes partial $\mathcal{N}=2 \to \mathcal{N}=1$ SUSY breaking.
  J. Bagger & A. Galperin (1997)

- Standard FI term is invariant under the second nonlinearly realised supersymmetry of the rigid supersymmetric Born-Infeld action.
  I. Antoniadis, J. Derendinger & T. Maillard (2009)

- Deformed supersymmetric BI action, $S_{\rm SBI}[V] - 2f\int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta\, V$, also describes partial $\mathcal{N}=2 \to \mathcal{N}=1$ SUSY breaking, as well as possesses U(1) duality invariance.
  SMK (2010)

- Last two properties are not preserved by alternative FI terms.

Recent developments

- I. Antoniadis, A. Chatrabhuti, H. Isono and R. Knoops [arXiv:1803.03817].
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- F. Farakos, A. Kehagias and A. Riotto [arXiv:1805.01877].
- Y. Aldabergenov, S. V. Ketov and R. Knoops [arXiv:1806.04290].
- H. Abe, Y. Aldabergenov, S. Aoki and S. V. Ketov [arXiv:1808.00669].
- N. Cribiori, F. Farakos and M. Tournoy [arXiv:1811.08424].
- H. Abe, Y. Aldabergenov, S. Aoki and S. V. Ketov [arXiv:1812.01297].