Black hole entropy, hyperbolic 3-manifold and analytic torsions

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ArXiv : 1808.02797 with Nakwoo Kim (KyungHee U)
+ Some works in progress

A Magnetically charged AdS4 Black hole

Classically

$$ds^2 = -\left(\rho - \frac{1}{2\rho}\right)^2 dt^2 + \left(\rho - \frac{1}{2\rho}\right)^{-2} d\rho^2 + \rho^2 ds^2(\Sigma_g)$$

$$F = \frac{dx_1 \wedge dx_2}{{x_2}^2}$$ (Magnetic flux for U(1) gauge field along Riemmann surface $\Sigma_{g>0}$)

- BPS Solution for 4D $\mathcal{N}$ =2 minimal gauged supergravity

$$I = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2\right) + (\text{fermions})$$

- Near horizon $\left(\rho = \frac{1}{2^{1/2}}\right)$: $AdS_2 \times \Sigma_g$,

Asymptotically ($\rho \to \infty$) : AdS_4 with asymptotic boundary $\mathbf{R}_t \times \Sigma_g$

- In terms of AdS/CFT, the BH solution describes

RG : (3D $\mathcal{N}$ =2 SCFT on $\mathbf{R}_t \times \Sigma_g$) ⟶ (1D SQM on $\mathbf{R}_t$)

topological twisting : $\left(A^{(b,g)}\right)_R = -\omega(\Sigma_g)$

Superconformal R-symmetry : "*universal twist*"

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From semiclassical analysis

[Bekenstein, Hawking]

$$S_{\mathrm{BH}} = \frac{A}{4G_4} = \frac{(g-1)\pi}{2G_4} + \text{(subleadings in } G_4\text{)}$$

If the BH solution (AdS4 supergravity) can be embedded into an UV complete Quantum Gravity,

We may give a non-perturbative definition of $d_{micro}(\#\ of\ micorstates\ of\ BH)$, which should satisfy

1) $\boldsymbol{d_{micro}}(g, G_4)$ is an non-negative integer (after including all corrections)

2) $S_{\mathrm{BH}} = \log d_{micro}(g, G_4) = \frac{(g-1)\pi}{2G_4} + \text{(subleadings in } G_4\text{)}.$

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In this talk,

- Embedding the BH into **M-theory** on AdS4 x M x S4 (M : hyperbolic 3-manifold)
- $d_{micro}\ (g, G_4)$ using **AdS4/CFT3 and 3d-3d relation ,** $\sum_\alpha (\text{Analytic torsion})^{g-1}$
- Check of 1) integrality at finite N $\sim (G_4)^{-1/3}$

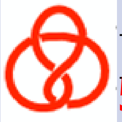
  2) Bekenstein-Hawking + sub-leadings in large N

2) $S_{\text{BH}} = \log d_{micro}\ (g, G_4) = \frac{(g-1)\pi}{2G_4} + (\text{subleadings in } G_4)$.

A Magnetically charged AdS4 BH in M-theory

BH solution with asymptotically AdS4 → Can be studied using AdS4/CFT3

Two classes of well-established AdS4/CFT3 using M-theory

AdS4/CFT3 from M2-branes	AdS4/CFT3 from M5-branes
$R^{1,2}$ × Cone(Y_7) (Y_7 : Sasakian 7-manifold) with N M2-branes on $R^{1,2}$ → $T_N[Y_7]$ 3D $\mathcal{N}$=2 SCFT with global $U(1)_R \subset G$ = ISO(Y_7)	$R^{1,2}$x (T^*M)x R^2 (T^*M_3: cotangent-bundle of 3-manifold M_3) with N M5 branes on $R^{1,2}$x M_3 → $T_N[M_3]$ 3D $\mathcal{N}$=2 SCFT, with global $U(1)_R$
M-theory on AdS4xY_7 ↓ ($G_4 = \sqrt{\frac{27}{8N^3\pi^4}\mathrm{Vol}(Y_7)}$) 4D $\mathcal{N}$=2 Gauged supergravity with G = ISO(Y_7) $S_{\mathrm{BH}} = \frac{(g-1)\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\mathrm{Vol}(Y_7)}}N^{3/2}\pi^3$	M-Theory on Warped AdS4xM_3xS4 (for hyperbolic M_3) [Pernici ;'85] [Gauntlet-Kim-Waldra;00] ↓ ($G_4 = \frac{3\pi^2}{2N^3 vol(M)}$) 4D $\mathcal{N}$ =2 Gauged supergravity with G =U(1) $S_{\mathrm{BH}} = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)vol(M)}{3\pi}N^3$
Field theoretic description of $T_N[Y_7]$ [ABJM;08][HLLLP;08]........ e.g) T_N[S7/Zk]=ABJM model	Field theoretic description of $T_N[M_3]$ [Dimoft-Gukov-Gaiotto;11][DG-Yonekura;18] e.g) $T_{N=2}$[ 5]= (U(1) + Φ with k=-7/2)

Non-perturbative definition of d_{micro} using AdS4/CFT3

$\boldsymbol{Question}$: *Which quantity in CFT3 corresponds to the* d_{micro} *of the BH ?*

Hints:

BH : Asymptotic AdS_4 with $\partial(AdS_4) = \boldsymbol{R}_t \times \Sigma_g$ ⟶ Near horizon $AdS_2 \times \Sigma_g$,

RG : (3D $\boldsymbol{\mathcal{N}}$ =2 SCFT on $\boldsymbol{R}_t \times \Sigma_g$) ⟶ (1D SQM on $\boldsymbol{R}_t$)

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$\boldsymbol{Natural\ Answer}$: $the\ number\ of\ ground\ states\ of\ 3d\ SCFT\ on\ \Sigma_g$

d_{micro}

$= \dim H^{E=0}(\text{3D } \mathcal{N}=2 \text{ SCFT } on\ \Sigma_g)$

= # of supersymmetric ground states of $(\text{3D } \mathcal{N}=2 \text{ SCFT } on\ \Sigma_g)$

$cf)\ \ d_{\text{micro}}^{\text{SUSY}} := \text{Tr}_{H^{E=0}(\text{3D } \mathcal{N}=2 \text{ SCFT } on\ \Sigma_g)} (-1)^R = \text{Tr}_{H(\text{3D } \mathcal{N}=2 \text{ SCFT } on\ \Sigma_g)} (-1)^R e^{-\beta E}$

Twisted index

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Hints:

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$\textbf{\textit{Natural Answer}}$: $\textit{the number of ground states of 3d SCFT on } \Sigma_g$

d_{micro}

$= \dim H^{E=0}\left(\text{3D } \mathcal{N}=2 \text{ SCFT } on\ \Sigma_g\right)$

= # of supersymmetric ground states of (3D $\mathcal{N}$ =2 SCFT $on\ \Sigma_g$)

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Twisted index

Recently people found that [Benini-Zaffaroni ;'16] [Hosseini-Zaffaroni ;'16].......

$$\text{Log}(d_{\text{micro}}^{\text{SUSY}}(T_N[Y_7],\text{g})) \xrightarrow{N \to \infty} \frac{(g-1)\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\text{Vol}(Y_7)}} N^{3/2}\pi^3 + \text{sub-leadings}$$

$$d_{\text{micro}}^{\text{SUSY}}(T_N[Y_7]) = d_{\text{micro}}(T_N[Y_7]) \ ??$$

Twisted index $d^{\text{SUSY}}_{\text{micro}}(g) = \text{Tr}_{H(\text{3D } \mathcal{N} = 2 \text{ SCFT } on\ \Sigma_g)} (-1)^R e^{-\beta E}$

For $g = 1$ $(\Sigma_g = T^2)$ case : It is just usual Witten index [Kim-Kim ;'10] [Seiberg-Intrilligator ;'12]

For $g = 0$ (S^2) case [Benini-Hristov-Zaffaroni ;'15]

For general g [Benini-Zaffaroni ;'16] [Closset-Kim ;'16]

For general $3d$ $N = 2$ theory with gauge G,
the index can be written as finite sum over so called \`Bethe vacua'

$$\boldsymbol{d^{\text{SUSY}}_{micro}(g) = \sum_{\alpha:\text{Bethe}-\text{vacua}} (H^\alpha)^{g-1}},$$ [Closset-Kim-Willet ;'17]

Bethe vacua: solutions of eqn $\exp\left(2\pi i z_i \frac{\partial W}{\partial z_i}\right) = 1,$ for $i = 1,..,\text{rank}(G)$

$W(z_1, \ldots, z_{\text{rank}(G)})$: Twisted superpotential for $2d$ (2,2) theory obtained by S^1 reduction keeping all infinity KK−modes

Chiral field : $\delta W = \text{Li}_2\left(\prod z_i^{-Q_i}\right)$, CS term $\delta W = k_{ij}\text{Log}[z_i]\text{Log}[z_j]$

$H^\alpha(z_1, \ldots, z_{\text{rank}(G)})$: \`handle gluing operator' ,

$$\text{Log}[H] = -\log \text{Det}[\partial_{\log[z_i]}\partial_{\log[z_j]}\text{Log}[W]] + \sum_{\text{Chiral}} \text{Li}_1(z_i^{-Q_i})$$

Most recent studies on AdS4 BH microstates counting are about BH made of M2-branes

$$\text{Log}(d^{\text{SUSY}}_{\text{micro}}(T_N[Y_7],\text{g})) \xrightarrow{N\to\infty} \frac{(g-1)\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\text{Vol}(Y_7)}}N^{3/2}\pi^3 + \text{sub-leadings}$$

Good : Gauge theory description is simple → Matrix model technique

Flavor symmetry other than U(1) R-symmetry → Rich SUSY BHs

Bad : Flavor symmetry other than U(1) R-symmetry

Improperly quantized superconformal R-charge : universal twisting is impossible

Computation of sub-leading seems to be challenging (additional Legendre transformation procedure))

BH made of M5-branes?

$$\text{Log}(d^{\textbf{SUSY}}_{\text{micro}}(T_N[M_3],\text{g})) \xrightarrow{N\to\infty} (g-1)\frac{N^3\text{vol}(M_3)}{3\pi} \;??$$

Bad : UV Gauge theory description is very ugly, no matrix model ($u(1)^{N^3}$ gauge group)

Good : we can use the power of **3d-3d relation**

(Computation of perturbative sub-leadings are doable)

$d^{SUSY}_{micro}(T_N[M_3],g)$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on $R^2 \times S^1) \sim$ (SL(N,C) Chern Simons theory on M_3) , **not duality but a relation**

[Yamazaki-Terashima ;'11][Dimoft-Gukov-Gaiotto;11]
[Yagi;13][Lee-Yamazaki;13][Cordova-Jafferis;13]

M-theoretic derivation : $6d\,A_{N-1}(2,0)$ theory on $(R^2 \times S^1) \times M_3$

$(T_N[M_3]$ on $R^2 \times S^1)$ ← → (SL(N,C) Chern Simons theory on M_3)

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$(T_N[M_3]$ on $R^2 \times S^1)$ ← → $(SL(N,C)$ Chern Simons theory on $M_3)$

Dictionary :

$T_N[M_3]$ on $R^2 \times S^1$	$SL(N,C)$ Chern Simons theory on M_3
Bethe vacuum α	SL(N,C) irreducible flat connection A^α
Handle gluing operator H^α	N $\mathrm{Exp}[-2S^\alpha(1)]$

$$dA^\alpha + A^\alpha \wedge A^\alpha = 0$$

$$\frac{\delta CS[A]}{\delta A} = dA + A \wedge A$$

$$CS[A] = \int_M \mathrm{tr}\left(AdA + \frac{2}{3}A^2\right)$$

Recall that $d_{micro}^{SUSY}(g) = \sum_{\alpha:\mathrm{Bethe-vacua}} (H^\alpha)^{g-1}$,

$S^\alpha(n)$: *n loop perturbative expansion coefficient around a flat connection* A^α

$$\log \int \frac{[d(\delta A)]}{(\mathrm{gauge})} \mathrm{Exp}[\frac{1}{2\hbar} CS[A^\alpha + \delta A]] \longrightarrow \frac{1}{\hbar}S^\alpha(0) + S^\alpha(1) + ..\hbar^{n-1}S^\alpha(n) + \ldots$$

$$S^\alpha(1) = \frac{1}{4}\mathrm{Log}[\frac{\left(\det{}' \Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}' \Delta_1{}^{(\alpha)}\right)}]$$ (Ray-singer torsion)

$\Delta_n{}^{(\alpha)} = * d_A * d_A + d_A * d_A *$, $\quad d_A = d + A^\alpha \wedge$
(Laplacian acting on n-form twisted by A^α)

$d^{\mathrm{SUSY}}_{micro}(T_N[M_3], \mathrm{g})$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on $R^2 \times S^1) \sim (SL(N,C)$ Chern Simons theory on $M_3)$, not duality but a relation

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$(T_N[M_3]$ on $R^2 \times S^1)$ ↙ ↘ $(SL(N,C)$ Chern Simons theory on $M_3)$

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Derivation : From $M = \left(\bigcup_{i=1}^{k} \Delta_i \bigcup_i^m S_i\right)/\sim$, Δ: (*ideal tetrahedron*), S: *solid torus*

[Dimofte-Garoufalidis;12]
[DG-Romo-Yamazaki;12]

Field theoretic description of $T_N[M_3]$ [Dimofte-Gaiotto-Gukov;11][DG-Yonekura;18]

↓ (Localization)

$N\ \mathrm{Exp}[-2S^\alpha(1)]$ $=$ H^α

Observation, not M-theoretic derivation

$d^{SUSY}_{micro}(T_N[M_3],g)$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on $R^2 \times S^1) \sim (\mathrm{SL}(N,C)$ Chern Simons theory on $M_3)$, not duality but a relation

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$$S^\alpha(1) = \frac{1}{4}\mathrm{Log}\left[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}\right]$$ (Ray-singer torsion)

$$d^{SUSY}_{micro}(T_N[M_3],g) = N^{1-g} \sum_\alpha \exp(-2S^\alpha(1)[M_3\ ;N]\ )^{g-1}$$

We reduce the microstates counting of BH to a mathematical problem !!
Use mathematical results to to study BH entropy !!

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on $R^2 \times S^1) \sim (\text{SL}(N,C)$ Chern Simons theory on $M_3)$, not duality but a relation

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$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{1-g} \sum_\alpha \exp(-2S^\alpha(1)[M_3\ ; N]\,)^{g-1}$$

We reduce the microstates counting of BH to a mathematical problem !!
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Using the expression, Let us check followings

1) $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$ is an integer (after including all corrections)

2) $S_{\text{BH}} = \log d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)vol(M)}{3\pi} N^3 + (\text{subleadings in } 1/N).$

Integrality of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

Irreducible flat connection : $dA^\alpha + A^\alpha \wedge A^\alpha = 0$

$$S^\alpha(1)[\mathrm{M};\mathrm{N}] = \frac{1}{4}\mathrm{Log}[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}]$$ (Ray-singer torsion)

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\ ;N]\ )^{g-1}$$

Conjecture (?)

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) \in \boldsymbol{Z}$

Integrality of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

Irreducible flat connection : $dA^\alpha + A^\alpha \wedge A^\alpha = 0$

$$S^\alpha(1)[M;N] = \frac{1}{4}\text{Log}\left[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}\right]$$ (Ray-singer torsion)

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\,;N]\,)^{g-1}$$

Conjecture (?)

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) \in \boldsymbol{Z}$

e.g) $M_3 =$  $_5$, $N=2$

[Computable using tools developed by mathematicians]

$\{\exp(-2S^\alpha(1)[M_3;N]\,)\}_{\alpha=1,2,3,4}$

={-1.90538-0.568995 i,-1.90538+0.568995 i,1.73992, 2.57085} $\quad (\,x^4 - 1/2\,x^3 - 8x^2 + 283/16 = 0\,)$

⟶ $\{d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})\}_{g=0,1,2,..}$ = { 0, 4, 1, 65, 97, 1045,...}

Conjecture (?)

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}=0) \in \mathbf{0}$

No SUSY black hole for **g=0**

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\mathrm{g})$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\mathrm{g}) = N^{g-1} \sum_\alpha \exp(-2S^\alpha(1)[M_3\ ; N]\ )^{g-1}$$

Two canonical irreducible flat connections A^{hyp} *and* $A^{\overline{hyp}}$

$$A^{hyp} = \rho_N[\omega + ie]\ ,\quad A^{\overline{hyp}} = \rho_N[\omega - ie]$$

$\rho_N: \mathrm{su}(2) \to \mathrm{su}(N), N -$ dimensionalirredrepresentation

ω: spin connection
e: vielbein

for unique hyperbolic metric on M satisfying $R_{\mu\nu} = -2g_{\mu\nu}$

Both of them can be locally considered as so(3) valued 1 forms

$\omega \pm ie$: $sl(2, C)$ *valued 1 form satisyfing flat connection equation* $dA + A \wedge A$

These two give dominant contributions to the d^{SUSY}_{micro} in large N (g>1)

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\mathrm{g}) = \left(H^{\text{hyp}}\right)^{g-1} + (c.c)$$
$$+\text{exponentially small in } 1/N$$

$$H^\alpha = N \exp(-2S^\alpha(1)[M_3\ ; N]\ )$$

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

Two canonical flat connections give dominant contributions

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = \left(H^{\text{hyp}}\right)^{g-1} + (c.c) \qquad H^{\alpha} = N\exp(-2S^{\alpha}(1)[M_3\,;N])$$

$$+\text{exponentially small in } 1/N$$

Mathematicians studied [Muller;14]......

$$2\text{Re}\left[S^{\text{hyp}}(1)[M_3;N]\right] \longrightarrow -\frac{(N^3-N)\text{vol}(M_3)}{3\pi} + \text{a}(M_3)(N-1) + \text{b}(M_3) + \text{o}(\text{e}^{-\text{N}})$$

$$a(M) := a_1(M) + a_2(M) \quad \text{where}$$

$$a_1(M) := \log|T_M^{\text{geom}}(\tau_1, N=2)| = \Re\log\frac{(\det{}'\Delta_0)^{3/4}}{(\det{}'\Delta_1)^{1/4}},$$

$$a_2(M) := -\Re\sum_{[\gamma]}\sum_{s=1}^{\infty}\frac{1}{s}\frac{e^{-s\ell_{\mathbb{C}}(\gamma)}}{1-e^{-s\ell_{\mathbb{C}}(\gamma)}} = \sum_{[\gamma]}\sum_{m=1}^{\infty}\log|1-e^{-m\ell_{\mathbb{C}}(\gamma)}|,$$

$$b(M) := \Re\sum_{[\gamma]}\sum_{s=1}\frac{1}{s}\left(\frac{e^{-s\ell_{\mathbb{C}}(\gamma)}}{1-e^{-s\ell_{\mathbb{C}}(\gamma)}}\right)^2$$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = 2\text{Cos}[\theta_N[M_3]]\exp\left((g-1)\left(\frac{N^3\text{vol}(M)}{3\pi} - a\,N - b + \log N + \text{o}(\text{e}^{-\text{N}})\right)\right) + \text{o}(\text{e}^{-\text{N}})$$

(Relative phase) (Bekenstein-Hawking) (Logarithmic correction $\frac{(1-g)}{3}\log G_4$)

[Liu, Pando Zayas, Rathee, Zhao;17]

Summary and future directions

We study microstate counting $d^{\text{SUSY}}_{micro}(T_N[M_3],g)$ for 4d magnetically charged BH made of wrapped N M5-branes on 3-manifold M_3

$T_N[M_3]$ on $R^2 \times S^1$	$SL(N,C)$ Chern Simons $\times$ theory on M_3
Bethe vacuum α	SL(N,C) irreducible flat connection A^α
Handle gluing operator H^α	N $\text{Exp}[-2S^\alpha(1)]$

Using a 3d-3d relation, the counting to a mathematical problem

$$d^{\text{SUSY}}_{micro}(T_N[M_3],g) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\,;N])^{g-1}$$

Then, using known mathematical results

$$d^{\text{SUSY}}_{micro}(T_N[M_3],g) = 2\text{Cos}[\theta_N[M_3]]\exp\left((g-1)(\frac{N^3\text{vol}(M)}{3\pi} - a\,N - b + \log N + o(e^{-N}))\right) + o(e^{-N})$$

(Relative phase) (Bekenstein-Hawking) (Logarithmic correction $\frac{(1-g)}{3}\log G_4$)

Future work : 1) 6D derivation of the 3d-3d relation for twisted index

2) Understanding the perturbative corrections (contributions from M2?)

3) $d^{\text{SUSY}}_{\text{micro}}(T_N[M_3]) = d_{\text{micro}}(T_N[M_3])$??