Non-standard electroweak phase transitions in extensions to the standard model: Monopoles[1,2] and Scale invariance[3]

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IPMU, February 2019

[1] SA & A. Kobakhidze, 1702.04068 Eur. Phys. J. C (2017) 77: 444
[2] SA, D. Collison and A. Kobakhidze, [arXiv:1810.10696]
[3] SA, A. Kobakhidze, C.Lagger, S. Liang and A.Zhou 1709.10322 PLB 776 (2018)

Outline

1. Motivation
2. Electroweak monopoles and the electroweak phase transition
3. The Standard Model with hidden scale invariance
4. Conclusion

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Motivation

- There is a significant asymmetry between the matter and antimatter abundance in the universe.
- Sakharov conditions:
  - Baryon number violation
  - C and CP violation
  - Out of equilibrium processes
- One possible mechanism is electroweak baryogenesis
- Baryon asymmetry is generated via sphaleron mediated scattering processes which violate $B + L$.
- Requires a first order electroweak phase transition for departure from equilibrium.
- Needs to be strong enough to suppress sphaleron processes in the broken phase.

The Electroweak Phase transition

- SM high temperature effective potential:

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 - \frac{1}{4}\lambda_T\phi^4$$

- curvature at the origin changes at $T = T_0$
- the nature of the transition depends on the values of the SM parameters.

First order phase transition

- The minima become degenerate before T_0
- Bubbles of the broken phase form
- collisions lead to gravitational waves, baryogenesis etc.

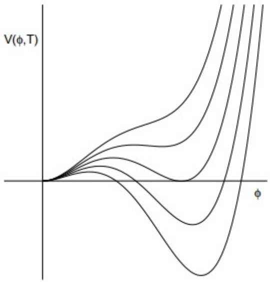

Figure: First order phase transition (Petropoulos,2003)

Second order phase transition

- the universe rolls homogeneously into the broken phase
- predicted by SM parameters

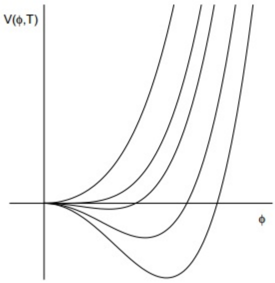

Figure: Second order phase transition (Petropoulos,2003)

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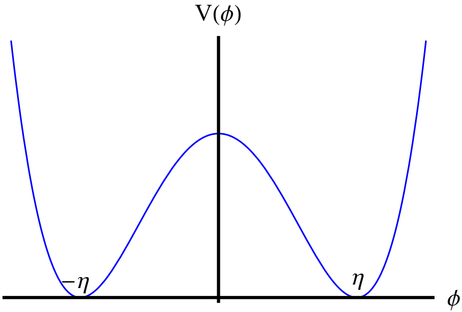
1-D topological defects

- Consider a 1D potential:

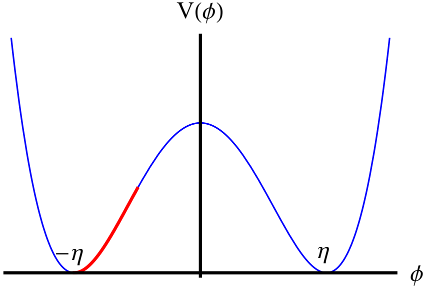
$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$



- For $\int_{-\infty}^{\infty} V(\phi)dx < \infty, \phi(\pm\infty) \rightarrow \pm\eta$

1-D topological defects

- Suppose $\phi(\infty) = \phi(-\infty) = -\eta$



- Decays to the constant solution

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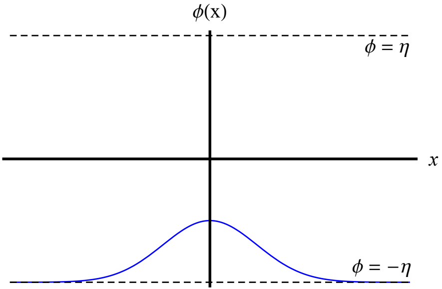

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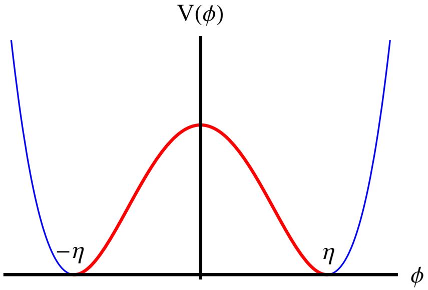

- Heuristically requires an infinite amount of energy to transition to constant solution.
- Topological stability from disconnected vacuum manifold
- $\pi_0(M_{vac}) \neq 0$.

1-D topological defects

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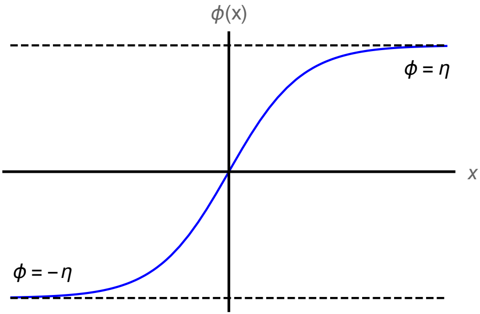

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Monopoles

- Monopoles are an extension of this idea to 3 spatial dimensions
- Spatial infinity is described by a 2-sphere
- Finite energy requires $\phi : S^2_{\infty} \to M_{vac}$.
- Topologically non-trivial solutions exist when $\pi_2(M_{vac}) \neq 0$
- For the standard model, $M_{\text{vac}} = (SU(2)_L \times U(1)_Y)/U(1)_{EM}$
- $\pi_2(M_{\text{vac}}) = \pi_2(S^3) = 0$
- No electroweak monopoles?

The Ansatz

- Cho and Maison (1997) found electroweak monopoles through the ansatz:

$$
\begin{aligned}
\phi &= \frac{1}{\sqrt{2}}\rho\xi \\
\rho &= \rho(r) \\
\xi &= i\begin{pmatrix}\sin(\theta/2)e^{-i\varphi} \\ -\cos(\theta/2)\end{pmatrix} \\
A_\mu &= \frac{1}{g}A(r)\partial_\mu t\hat{r} + \frac{1}{g}(f(r)-1)\hat{r}\times\partial_\mu\hat{r} \\
B_\mu &= -\frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1-\cos\theta)\partial_\mu\varphi
\end{aligned}
$$

Why is this stable?

$$\xi = i \begin{pmatrix} \sin(\theta/2) e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}$$

$$B_\mu = -\frac{1}{g'}(1 - \cos\theta)\partial_\mu \varphi$$

- Gauge invariance under $U(1)$ implies that the vacuum manifold is defined up to a phase.
- String singularities in both fields at $\theta = \pi$
- Can be removed using a Wu-Yang construction
- Each hemisphere maps onto $\mathbb{C}^1$
- By definition, this corresponds to the Riemann sphere, $\mathbb{CP}^1$
- $\pi_2(M_{\text{vac}}) = \mathbb{Z}$

The energy

$$E = E_0 + E_1$$

$$E_0 = 4\pi \int_0^\infty \frac{dr}{2r^2} \left\{ \frac{1}{g'^2} + \frac{1}{g^2}(f^2 - 1)^2 \right\}$$

$$\begin{aligned} E_1 = 4\pi \int_0^\infty dr \Bigg\{ & \frac{1}{2}(r\dot{\rho})^2 + \frac{1}{g^2}\left(\dot{f}^2 + \frac{1}{2}(r\dot{A})^2 + f^2 A^2\right) \\ & + \frac{1}{2g'^2}(r\dot{B})^2 + \frac{\lambda r^2}{8}(\rho^2 - \rho_0^2)^2 \\ & + \frac{1}{4} f^2 \rho^2 + \frac{r^2}{8}(B - A)^2 \rho^2 \Bigg\} \end{aligned}$$

- The first term of E_0 is divergent at the origin.

Regularisation

- Cho, Kim and Yoon(2015) proposed a regularisation of the form:

$$g' \to \frac{g'}{\sqrt{\epsilon}}$$

$$\epsilon = \left(\frac{\phi}{\phi_0}\right)^n$$

- However, g' becomes non-peturbative as $\phi \to 0$.
- This is undesirable in an EFT framework.
- We instead propose a Born-Infeld modification for the $U(1)_Y$ kinetic term.

Born-Infeld modification

- We regularise the $U(1)_Y$ kinetic term by replacing it with:

$$\beta^2\left[1-\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{\beta}B_{\mu\nu}\right)}\right]$$

$$=\beta^2\left[1-\sqrt{1+\frac{1}{2\beta^2}B_{\mu\nu}B^{\mu\nu}-\frac{1}{16\beta^4}(B_{\mu\nu}\tilde{B}^{\mu\nu})^2}\right]$$

- As $\beta\to\infty$, the SM is recovered.
- The corresponding energy is

$$\int_0^\infty dr\,\beta^2\left[\sqrt{(4\pi r^2)^2+\left(\frac{4\pi}{g'\beta}\right)^2}-4\pi r^2\right]$$

$$=\frac{4\pi^{5/2}}{3\Gamma\left(\frac{3}{4}\right)^2}\sqrt{\frac{\beta}{g'^3}}\approx 72.8\sqrt{\beta}$$

- Hence, β acts as a mass parameter for the monopoles.

- Extend the $SU(2)$ sector as well with an independent Born-Infeld term:

$$\beta_1^2\left[1-\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{\beta_1}B_{\mu\nu}\right)}\right]+\beta_2^2\left[1-\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{\beta_2}F_{\mu\nu}\right)}\right]$$

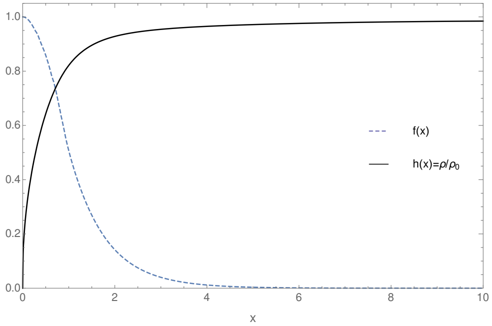
- Constrained by light by light scattering results (Ellis et al. 2017):

$$\sqrt{\beta_{EM}}=\frac{\sqrt{\beta_2}}{\sqrt[4]{\sin^4\theta_W+\cos^4\theta_W\left(\frac{\beta_2}{\beta_1}\right)^2}}\gtrsim 100\text{GeV}$$

- For $\beta_2 >> \beta_1$ (perturbative unitarity) gives a lower bound for monopole mass of $\sim 9-11$TeV

Solution

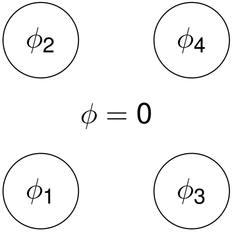
- Simple solution: $A = B = 0$
- $h = \frac{4\pi}{e}$



- A new analytical solution has been found with non-monotonic behaviour for $f(x)$. (Mavromatos and Sarkar, 2018)

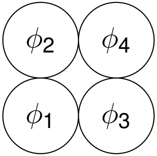
The Kibble Mechanism (Kibble, 1976)

- At $T = T_c$, domains of the broken phase will appear
- The higgs field in each domain takes independent directions on the vacuum manifold



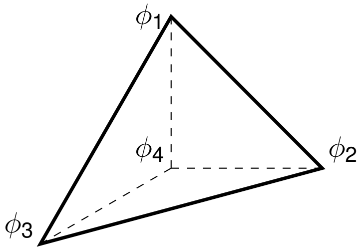
The Kibble mechanism (Kibble, 1976)

- As the Higgs field is continuous, it must be interpolated at the intersections.
- Consider an intersection of four of these domains:



The Kibble mechanism (Kibble, 1976)

- In field space, these points form the vertices of a tetrahedron.
- This tetrahedron should be shrunk to a point at the intersection.
- If these cannot be shrunk to a point continuously, a topological defect in the form of a monopole which continuously joins the two minima.
- The tetrahedron is homotopically equivalent to S^2.
- Therefore, $\pi_2(\mathbb{CP}^1) = \mathbb{Z}$ implies the existence of monopoles



Sphaleron Processes

- Sphaleron mediated scattering processes occur in the unbroken phase
- They violate $B + L$ in units of $\Delta B = \Delta L = 3$
- If unsuppressed, they washout any pre-existing baryon number.
- Supression in the broken phase requires a 1st order EWPT with $\frac{\phi_c}{T_c} \gtrsim 1$.

The electroweak phase transition

- The Gibbs free energy:

$$G_u = V(0)$$
$$G_b = V(\phi_c(T)) + E_{\text{monopoles}}$$

- At the critical temperature:

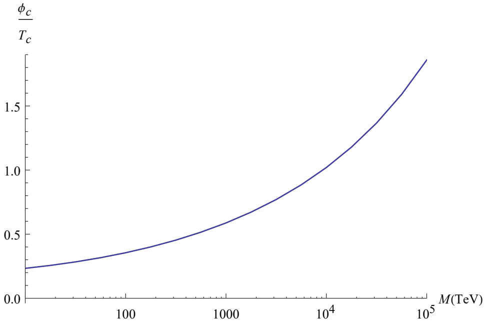
$$V(0) = V(\phi_c(T_c)) + E_{\text{monopoles}}$$

- Assuming $T << M$,the monopoles are decoupled and $E_{monopoles} = M \times n_M$

The initial density

- $n_M \approx \frac{1}{d^3}$ where d is the separation of two uncorrelated monopoles.
- This is chosen to be the Coulomb capture distance.
- Hence, $n_M \approx \left(\frac{4\pi}{h^2}\right)^3 T^3$

Results



- Sphaleron processes are suppressed for $M > 0.9 \cdot 10^4$ TeV.

The constraint

- The monopole density should not dominate the universe at the time of helium synthesis. This implies:
- $\frac{n}{T^3}\Big|_{T=1\text{MeV}} < \frac{1\text{MeV}}{M}$
- Hence, the evolution of the number density over time must be considered.

The number density at lower temperatures

- Consider monopoles drifting towards anti monopoles in a plasma of charged fermions.
- Scattering cross-section: $\sigma_{q_i M} = (hq_i/4\pi)^2 T^{-2}$
- After $\sim \frac{M}{T}$ collisions, the monopole is scattered at a large angle and drifts towards the antimonopole.
- This yields a mean free path of:

$$\lambda \approx \frac{v_{\text{drift}}}{\sum_i n_i \sigma_i} \frac{M}{T} \approx \frac{1}{B} \left(\frac{M}{T^3} \right)^{1/2}$$

- $B = \frac{3}{4\pi^2} \zeta(3) \sum_i (hq_i/4\pi)^2$

$$\frac{dn_M}{dt} = -Dn_M^2 - 3Hn_M$$

- Annihilation ends when $\lambda \approx \frac{h^2}{4\pi T}$, the Coulomb capture radius.
- This occurs at $T_f \approx \left(\frac{4\pi}{h^2}\right)^2 \frac{M}{B^2}$
- For $T < T_f$, the monopole density simply dilutes as $n \propto T^3$.

Nucleosynthesis constraint

- Solving the Boltzmann equation, one obtains (Preskill, 1979)

$$\frac{n}{T^3} = \frac{1}{Bh^2}\left(\frac{4\pi}{h^2}\right)^2 \frac{M}{CM_{pl}}, \; (T > T_f)$$

- $C = (45/4\pi^3 N)^{1/2}$
- This constrains the mass of the monopole to $M \lesssim 2.3 \cdot 10^4$ TeV.

Sakharov conditions

- In 1967, Andrei Sakharov proposed three conditions for baryogenesis to occur:
  1. Baryon number violation
  2. C and CP violation
  3. Departure from thermal equilibrium- 1st order EWPT.

C and CP-violation

- Consider the θ- terms:

$$\mathcal{L}_\theta = \theta_2 F^a_{\mu\nu}\tilde{F}^{a\mu\nu} + \theta_1 B_{\mu\nu}\tilde{B}^{\mu\nu}$$

- In the ususal case:
    - hypercharge sector is topologically trivial, and hence, θ_1 is unphysical
    - θ_2 can be rotated away by a $B + L$-rotation of quarks and leptons.
    - no CP-violation

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- In the ususal case:
  - hypercharge sector is topologically trivial, and hence, θ_1 is unphysical
  - θ_2 can be rotated away by a $B+L$-rotation of quarks and leptons.
  - no CP-violation
- With electroweak monopoles:
  - Monopoles gain an electric charge proportional to θ_{EM} through the WItten effect
  - Supports θ_1
  - Only one can be rotated away
  - a new source of CP violation

$B+L$-violation

$$\mathcal{L}_\theta = \theta_{ew} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \ ,$$

- Topologically inequivalent vacuum configurations related by large gauge transformations $g \in SU(2)_L$ give rise to the θ_{ew}-vacuum structure.

$$|M, \theta_{ew}\rangle = \sum_{n=-\infty}^{n=+\infty} e^{in\theta_{ew}} (U[g])^n |M, 0\rangle \ .$$

- monopole-antimopole pair that carries $\Delta n = 1$ topological charge, would annihilate into 9 quarks and 3 leptons, giving rise to $\Delta B = \Delta L = 3$.
- not suppressed even at zero temperature (Callan, 1982) (Rubakov, 1981)

Baryon asymmetry of the universe

$$\frac{d\bar{n}_B}{dt} = -\kappa\theta \frac{dn_M}{dt}$$

- $\bar{n}_B$ is the difference in the number densities of matter and antimatter
- κ describes the asymmetry generated in each collision
- for monopoles, $n_{M0} >> n_{Mf}$.
- Hence,

$$\bar{n}_B \approx \kappa\theta n_0 = \kappa\theta\alpha_{EM}^3 T_c^3$$

Baryon asymmetry of the universe

- The asymmetry parameter, η_B, can now be evaluated:

$$\eta_B = \frac{\bar{n}_B}{s} = \kappa\theta \frac{45\alpha_{\text{EM}}^3 T_c^3}{2\pi^2 g_\star T_f^3}$$

- $1.6 \times 10^{-8}\kappa\theta \leq \eta_B \leq 2.5 \times 10^{-7}\kappa\theta$.
- Empirical values for the asymmetry parameter $\eta_B \approx 10^{-10}$ can be accommodated for with $\kappa\theta_{ew} \sim 10^{-3} - 10^{-2}$.

Summary

- Finite energy monopoles exist in the Standard model with a Born-Infeld extension.
- The mass is related to the Born-Infeld parameters
- Sphaleron mediated processes can be made ineffective in the broken phase while remaining under the nucleosynthesis constraints.
- This occurs for monopoles with a mass of $(0.9 - 2.3) \cdot 10^4$TeV.
- Baryon asymmetry of the universe can be accounted for through this mechanism

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Motivation

- Scale invariance is an attractive framework for addressing the problem of the origin of mass and hierarchies of mass scales.
- Quantum fluctuations result in a mass scale via dimensional transmutation
- Dimensionless couplings are responsible for generating mass hierarchies.
- Scale (conformal) invariance is an essential symmetry in string theory
- What is the nature of the EWPT in this framework?

The model

- Consider the SM as a low energy Wilsonian effective theory with cutoff Λ:

$$V(\Phi^\dagger\Phi) = V_0(\Lambda) + \lambda(\Lambda)\left[\Phi^\dagger\Phi - v_{ew}^2(\Lambda)\right]^2$$

- Assume the fundamental theory exhibits conformal invariance which is spontaneously broken down to Poincare invariance
- Promote dimensionful parameters to the dilaton field, the scalar Goldstone boson.

$$\Lambda \to \alpha\chi, \quad v_{ew}^2(\Lambda) \to \frac{\xi(\alpha\chi)}{2}\chi^2, \quad V_0(\Lambda) \to \frac{\rho(\alpha\chi)}{4}\chi^4 \,, \qquad (1)$$

The model

- Impose the following conditions:
  - $\left.\frac{dV}{d\phi}\right|_{\Phi=v_{ew},\chi=v_\chi} = \left.\frac{dV}{d\chi}\right|_{\Phi=v_{ew},\chi=v_\chi} = 0$ (Existence of the electroweak vev)
  - $V(v_{ew}, v_\chi) = 0$ (Cosmological constant)
- Implications:
  - $\rho(\alpha v_\chi) = \beta_\rho(\alpha v_\chi) = 0$
  - $\xi(\alpha v_\chi) = \frac{v_{ew}^2}{v_\chi^2}$
  - $m_\chi^2 \simeq \frac{\beta'_\rho(\Lambda)}{4\xi(\Lambda)} v_{ew}^2 \simeq (10^{-8}\text{eV})^2$ for $\alpha\chi \sim M_P$

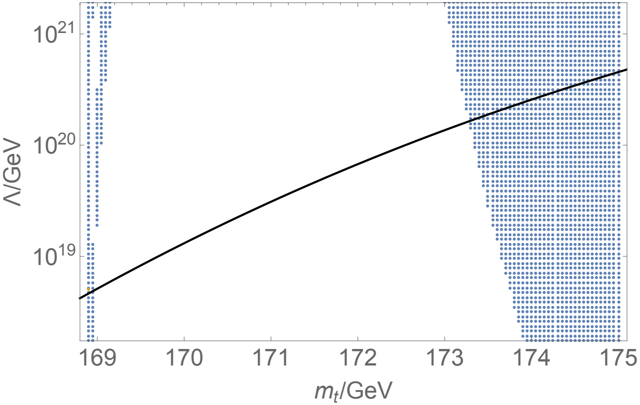


Figure: Plot of the allowed range of parameters (shaded region) with $m_\chi^2(v_{ew}) > 0$, i.e., the electroweak vacuum being a minimum. The solid line displays the cut-off scale Λ as function of the top-quark mass m_t for which the conditions are satisfied.

The thermal effective potential

- At high temperatures:

$$V_T(h,\chi) = \frac{\lambda(\Lambda)}{4}\left[h^2 - \frac{v_{ew}^2}{v_\chi^2}\chi^2\right]^2 + c(h)\pi^2 T^4 - \frac{\lambda(\Lambda)}{24}\frac{v_{ew}^2}{v_\chi^2}\chi^2 T^2$$
$$+\frac{1}{48}\left[6\lambda(\Lambda) + 6y_t^2(\Lambda) + \frac{9}{2}g^2(\Lambda) + \frac{3}{2}g'^2(\Lambda)\right] h^2 T^2$$

- Minimising this potential w.r.t. χ :

$$\chi^2 \approx \frac{v_\chi^2}{v_{ew}^2}\left(h^2 + \frac{T^2}{12}\right)$$

Thermal effective potential

The effective potential in this direction is given by:

$$V_T(h, \chi(h)) = \left[c(h)\pi^2 - \frac{\lambda(\Lambda)}{576}\right] T^4 + \frac{1}{48}\left[4\lambda(\Lambda) + 6y_t^2(\Lambda) + \frac{9}{2}g^2(\Lambda) + \frac{3}{2}g'^2(\Lambda)\right] h^2 T^2$$

Standard model

- SM high temperature effective potential:

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 - \frac{1}{4}\lambda_T\phi^4$$

- curvature at the origin changes at $T = T_0$
- the nature of the transition depends on the values of the SM parameters.

The scale-invariant model

- Along the flat direction, $T_0 = 0$
- Furthermore, the minima are degenerate only at $T = 0$
- No phase transition???

Chiral phase transition

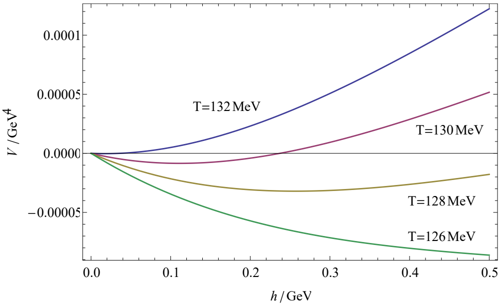
- Consider the Yukawa term:

$$y \langle \bar{q}q \rangle_T \phi$$

- At $T \sim 132$MeV, chiral condensates form.
- This term is given by (Gasser & Leutwyler, 1987):

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle \left[1 - (N^2 - 1) \frac{T^2}{12 N f_\pi^2} + \mathcal{O}\left(T^4\right) \right]$$

The electroweak phase transition

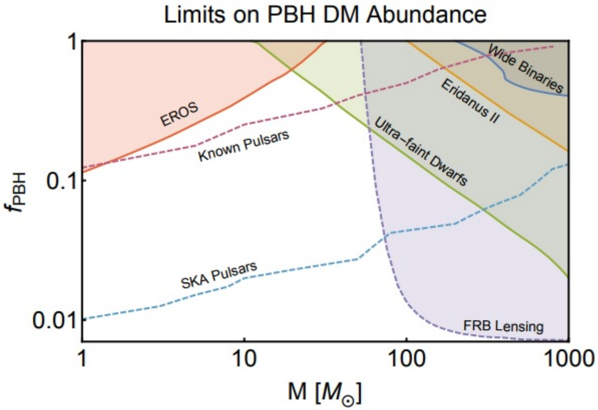


- The linear term shifts the minimum from the origin
- at $T \sim 127$MeV, the minimum disappears and the EWPT is triggered
- The EWPT is 2nd order.

Implications

- 6 relativistic quarks at the critical temperature indicates a 1st order chiral PT. (Pisarski& Wilczek, 1983)
- Gravitational waves with peak frequency $\sim 10^{-8}$ Hz, potentially detectable by means of pulsar timing (EPTA, SKA...)

- Production of primordial black holes with mass $M_{BH} \sim M_{\odot}$



(Schutz & Liu, 2016)

Implications

- Scale invariant theories predict a light feebly coupled dilaton.
- Electroweak phase transition driven by the QCD chiral phase transition and occurs at $T \sim 130$ MeV.
- QCD phase transition could be strongly first order $\implies$ gravitational waves, black holes, QCD baryogenesis.
- Detection of a light scalar particle + the above astrophysical signatures will provide strong evidence for the fundamental role of scale invariance in particle physics and cosmology.

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Conclusion

- Electroweak monopoles
  - Cho-Maison monopoles have infinite energy in the SM
  - This can be be regularised using a Born-Infeld extension
  - Production increases the energy of the broken phase at the EWPT
  - Results in a strong electroweak phase transition while consistent with BBN results.
  - Monopole Baryogenesis
- Scale Invariance
  - Chiral phase transition occurs before EWPT
  - 6 massless quarks and therefore it is first order
  - Potentially leads to GW, primordial black holes etc.)