5d partition functions with A twist

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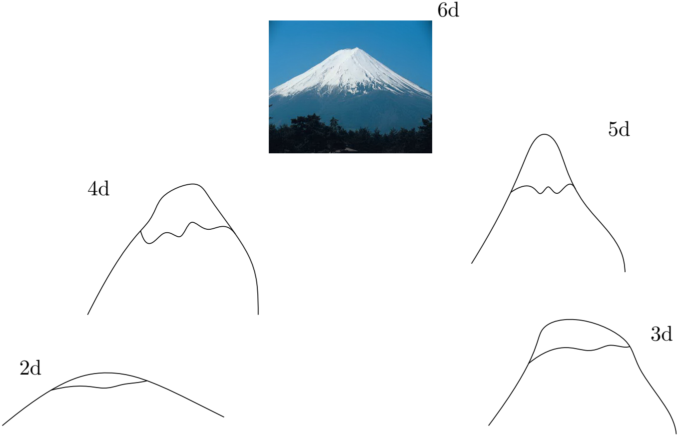
Landscape of SCFTs

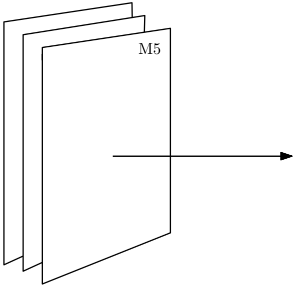
String theory predicts the existence of interacting local SCFTs in $d = 5, 6$.



Arise from worldvolume of D- and M-branes. AdS/CFT $\Rightarrow$ conformal

6d

Most famous example: maximal 6d $(2,0)$ theory

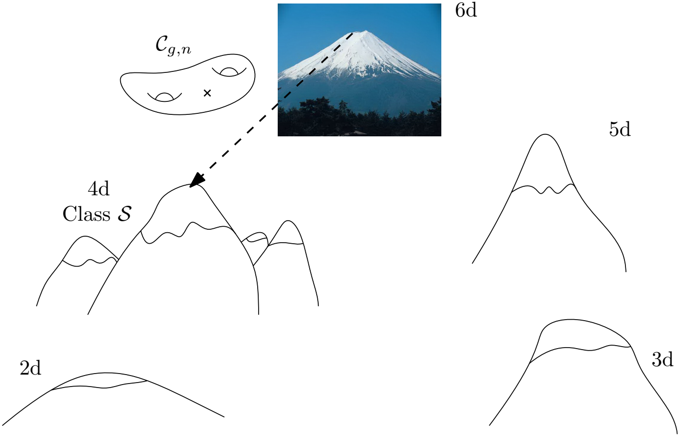


Still remain rather mysterious. Known features include

- ADE classification
- Global symmetry: $SO(6) \times SO(5)_R$
- 't Hooft anomalies
- AdS_7 gravity dual

Compactification: 6d → 4d [Gaiotto]

Our limited knowledge has not stopped us from exploring these theories



Vast space of new 4d $\mathcal{N} = 2$ and $\mathcal{N} = 1$ SCFTs

The power of high dimensions

Number of interesting results and insights:

- Discovery of new theories in 4d **[Gaiotto]**
- Most theories are non-Lagrangian
- New dualities: 4d/2d **[AGT]** and 3d/3d **[Dimofte-Gaiotto-Gukov,..]**
- Interesting interplay with holography **[Gaiotto-Maldacena, BBBW]**

Due to lack of Lagrangians results are rather indirect.

What about 5d?

Outline

I) 5d $\mathcal{N} = 1$ gauge theories

II) Partition function on $M_3 \times \Sigma_{\mathfrak{g}}$

III) Two applications

I) 5d $\mathcal{N} = 1$ gauge theories

5d $\mathcal{N} = 1$ gauge theories (8 SUSYs)

Multiplets of 5d $\mathcal{N} = 1$ SUSY:

$$\text{vector: } V = (A_\mu, \sigma, \lambda^I, D^{IJ}) \qquad \text{hyper: } \Phi = (q^I, \psi)$$

Global symmetry:

$$SO(6) \times SU(2)_R \times G_F$$

We can write Lagrangians, e.g.,

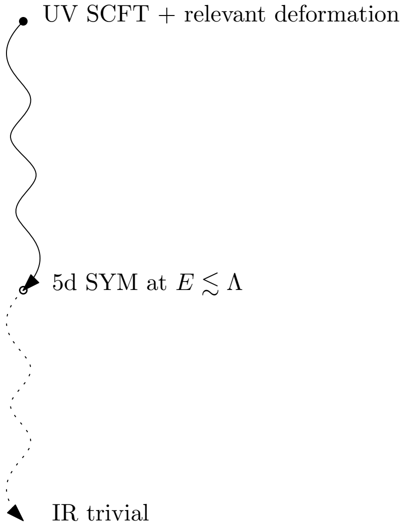
$$S_V = \frac{1}{g_5^2} \mathrm{Tr} \int d^5x \left(\frac{1}{2} F^{\mu\nu} F_{\mu\nu} + i\lambda_I \not{D} \lambda^I - D^\mu \sigma D_\mu \sigma - \lambda_I [\sigma, \lambda^I] - \frac{1}{2} D^{IJ} D_{IJ} \right)$$

- Non-renormalizable: $g_5^2 \sim mass^{-1}$ (always effective theories)
- Coupling decreases at large distances:

$$g_5 \to 0 \qquad \text{as } E \to 0$$

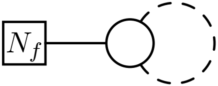
RG flow

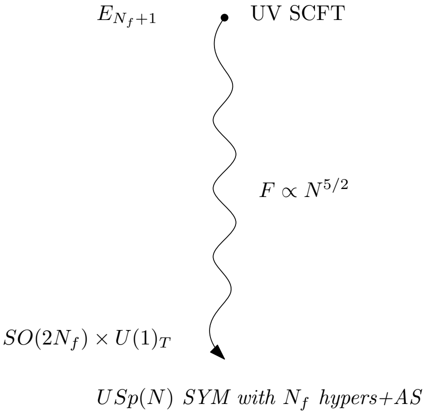
Unlike $d \leq 4$, in $d = 5$ strongly-coupled SCFT at UV:



Example 1: Seiberg theories

Some 5d gauge theories admit UV completions as CFTs [Seiberg]

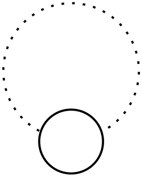


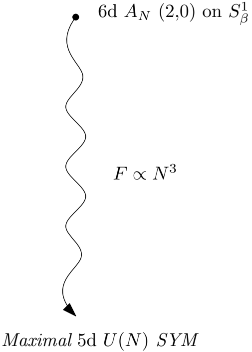


- For $N_f < 8$, gauge theory is the IR description of an SCFT with enhanced E_{N_f+1}.
- Number of d.o.f. grows to $N^{5/2}$

Example 2: Maximal theory

Some 5d theories admit UV completions as 6d theories! **[Douglas, Lambert et al.]**



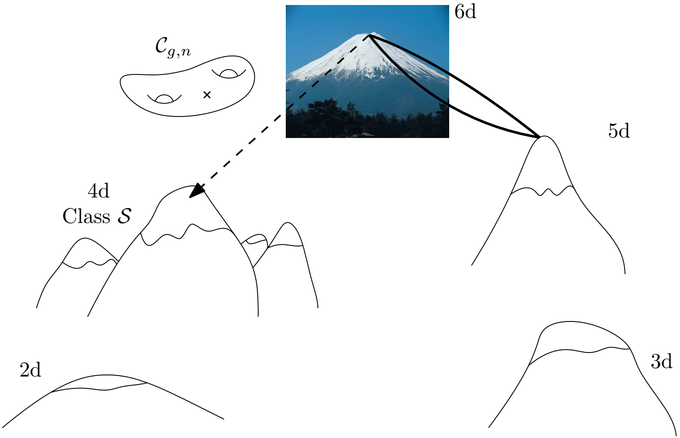


Size of "emergent direction"

$$\beta = \frac{g_5^2}{2\pi},$$

grows at large coupling. (Examples with eight supercharges also exist, e.g., Seiberg theory with $N_f = 8$.)

“5d is the best d”



A (5d) Lagrangian method for (4d) non-Lagrangian theories

Then

$$
\begin{tikzcd}[column sep=small]
 & M_3 \times \mathcal{C}_{g,n} \times S^1_\beta \arrow[dl] \arrow[dr] & \\
M_3 \times S^1_\beta & & M_3 \times \mathcal{C}_{g,n}
\end{tikzcd}
$$

Partition function invariant under RG $\Rightarrow$

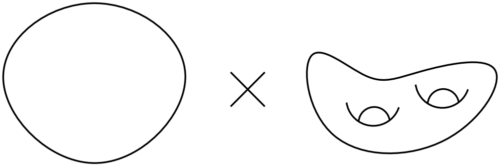
$$\underbrace{Z_{M_3\times S^1_\beta}[\text{Class } \mathcal{S}]}_{Non-Lagrangian} \quad = \quad \underbrace{Z_{M_3\times \mathcal{C}_{g,n}}[\text{5d SYM}]}_{Lagrangian}$$

Match can be seen as:

- Precision test of *5d maximal SYM* $\Leftrightarrow$ *6d* $(2,0)$ conjecture
- Computational tool for non-Lagrangian theories

(Basic idea considered previously in **[Fukuda-Kawano-Matsumiya]**)

II) Partition function on



$M_3 \qquad\qquad \mathcal{C}_{g,n}$

(In this talk we will take $M_3 = S_b^3$ and $\mathcal{C}_{g,n} = \Sigma_g$ for simplicity)

Supersymmetric localization

Basic localization argument **[Witten, Pestun]**

$$Z_{M_d} = \int [D\Phi] e^{-S[\Phi]} \quad \to \quad Z_{M_d}(t) = \int [D\Phi] e^{-(S[\Phi] + t\mathcal{Q}V[\Phi])}$$

If $\mathcal{Q}V[\Phi]$ positive semidefinite and $\mathcal{Q}$-invariant, then

$$\frac{d}{dt} Z_{M_d}(t) = 0\,.$$

For $t \to \infty$ contributions only from BPS locus, $\{\Phi_0 \,|\, \mathcal{Q}V[\Phi_0] = 0\}$,

$$Z_{M_d} = \int d\Phi_0 \, e^{-S[\Phi_0]} \, Z^{1-loop}(\Phi_0) Z^{inst}(\Phi_0) \qquad \text{Exact!}$$

BPS locus depends on M_d (e.g., $\mathcal{Q}$ depends on M_d)

The 5d background

First thing to check is if SUSY can be preserved on $M_5 = S_b^3 \times \Sigma_{\mathfrak{g}}$.

However, space has no Killing spinors. Problem is Riemann surface:

$$(\partial_\mu + \frac{1}{4}\omega_\mu)\epsilon^{(2d)} \neq 0$$

Problem solved by "topological twist." **[Witten]** If global R-symmetry then:

$$(\partial_\mu + \frac{1}{4}\omega_\mu + \underbrace{A_\mu^{R,\,back}}_{-\frac{1}{4}\omega_\mu})\epsilon^{(2d)} = \partial_\mu \epsilon^{(2d)} = 0$$

Then, take

$$\zeta^{(5d)} = \zeta^{(3d)} \otimes \epsilon_0^{(2d)}$$

with $\zeta^{(3d)}$ Killing spinor of S_b^3 and $\epsilon_0^{(2d)}$ constant. **[Kawano-Matsumiya]**

Other 5-manifolds possible; classification in

[Alday-Genolini-Fluder-Richmond-Sparks]

The approach

The now standard approach to localization is:

- Couple QFT to supergravity and take rigid limit **[Festuccia-Seiberg]**
- Solve for background
- Compute 1-loop determinants in background

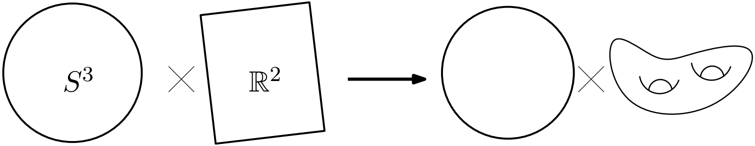
Approach is systematic and powerful, but every background requires calculation from scratch

Our approach will be instead to “glue” together basic building blocks

(Related results on $M_4 \times S^1$ by **[Hosseini-Yaakov-Zaffaroni]**)

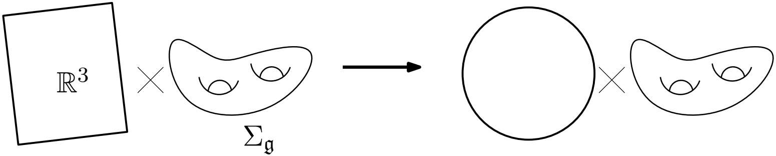
Three perspectives

1) Reduction on S^3: A-model on $\Sigma_{\mathfrak{g}}$



$$Z_{S^3\times\Sigma_{\mathfrak{g}}} = Z_{A-model}$$

2) Reduction on $\Sigma_{\mathfrak{g}}$: Direct sum of 3d theories



$$Z_{S^3\times\Sigma_{\mathfrak{g}}} = \bigoplus_{\mathfrak{m}\in\Lambda_G} (Z_{S^3})^{\mathfrak{m}}$$

3) Uplift from 4d: Nekrasov partition function

The final answer

By either method, one finds **[MC-Jain-Willett]**

$$\boxed{Z_{S_b^3\times\Sigma_{\mathfrak{g}}}(\nu)_{\mathfrak{n}} = \sum_{\hat{u}:\ \Pi_a(\hat{u})=1} \Pi_i(\hat{u},\nu)^{\mathfrak{n}_i}\mathcal{H}(\hat{u},\nu)^{\mathfrak{g}-1}}$$

with

$$\underbrace{\Pi_i = e^{2\pi i\partial_{\nu_i}\mathcal{W}}}_{flavor-flux\ op.},\quad \underbrace{\Pi_a = e^{2\pi i\partial_{u_a}\mathcal{W}}}_{gauge-flux\ op.},\qquad \underbrace{\mathcal{H} = e^{2\pi i\Omega}\det_{a,b}\partial_{u_a}\partial_{u_b}\mathcal{W}}_{handle-gluing\ op.},$$

determined by the two functions:

$$\underbrace{\mathcal{W}}_{Bethe\ potential} = \mathcal{W}_{pert} + \mathcal{W}_{inst}\,,\qquad \underbrace{\Omega}_{effective\ dilaton} = \Omega_{pert} + \Omega_{inst}$$

$\mathcal{W}_{pert}$, Ω_{pert} computed by a standard 1-loop calculation

Perturbative contribution

Reducing on $\Sigma_{\mathfrak{g}}$, non-zero modes cancel leaving

$$Z^{pert}_{S^3_b\times\Sigma_{\mathfrak{g}}} = \sum_{\mathfrak{m}\in\Lambda_G} \oint du \left(Z^{chiral}_{S^3_b}(u,\Delta)\right)^{n_R-n_L} \left(Z^{vector}_{S^3_b}(u)\right)^{n_R-n_L}$$

where [Kapustin-Willett-Yaakov] [Hama-Hosomishi-Lee] [Jafferis]

$$Z^{chiral}_{S^3_b}(u) = \prod_{j,k\geq 0} \frac{(j+\frac{1}{2})b + (k+\frac{1}{2})b^{-1} - Qu}{(j+\frac{1}{2})b + (k+\frac{1}{2})b^{-1} + Qu} \equiv s_b(-iQu) \qquad (0.1)$$

and $Z^{chiral}_{S^3_b}(u,\Delta) = Z^{chiral}_{S^3_b}(u + i(\Delta - \frac{1}{2}))$, $Q \equiv \frac{1}{2}(b+b^{-1})$. By 2d index theorem:

$$n_R - n_L = \frac{1}{2\pi}\int (F_{gauge} + F_{flavor} + F_R) = \mathfrak{m} + \mathfrak{n} + (\mathfrak{g}-1)(r-1)$$

$\mathfrak{m}\in\Lambda_G$ is a gauge magnetic flux on $\Sigma_{\mathfrak{g}}$ one must sum over

Collecting terms

$$Z^{pert}_{S^3_b \times \Sigma_{\mathfrak{g}}} = \sum_{\mathfrak{m}} \oint du \, \Pi_a^{pert}(u,\nu)^{\mathfrak{m}_a} \, \Pi_i^{pert}(u,\nu)^{\mathfrak{n}_i} \, \mathcal{H}^{pert}(u,\nu)^{\mathfrak{g}}$$

where $\nu = 1 - \Delta$ and

$$\Pi_i^{pert} = e^{2\pi i \partial_{\nu_i} \mathcal{W}^{pert}} \,, \quad \Pi_a^{pert} = e^{2\pi i \partial_{u_a} \mathcal{W}^{pert}}$$

with

$$\begin{aligned} \mathcal{W}^{pert}(\mu,\nu) &= \sum_{\rho \in R} g_b(\rho(u)+\nu) - \sum_{\alpha \in Ad(G)} g_b(\alpha(u)-1) \\ \Omega^{pert} &= \sum_{\rho \in R} (r-1) \, l_b(\rho(u)+\nu) \\ g_b(u) &= \frac{1}{2\pi i} \int du \log s_b(-iQu) \,, \quad l_b(u) = \frac{1}{2\pi i} \log s_b(-iQu) \end{aligned}$$

Performing sum over $\mathfrak{m}$ and contour integral gives expression above.

Relation to 5d Nekrasov partition function

The full nonperturbative answer is:

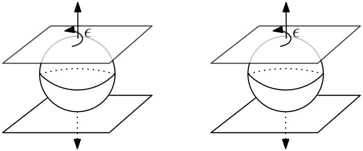
$$\mathcal{W} = \mathcal{W}_{NS}^{(5d)}\,, \qquad \Omega = \Omega_{NS}^{(5d)}$$

where these are read off from the *5d Nekrasov partition function*:

5d Ω-background: $\displaystyle\lim_{\epsilon_2\to 0} Z_{\mathbb{R}_{\epsilon_1}\times\mathbb{R}_{\epsilon_2}\times S^1} = e^{\frac{1}{\epsilon_2}\mathcal{W}_{NS}^{(5d)}(u,\epsilon_1)+\Omega_{NS}^{(5d)}(u,\epsilon_1)+\mathcal{O}(\epsilon_2)}$

and identification $\epsilon_1 = \frac{1}{2}(b-b^{-1})$. To see this we "glue" four copies:

$$Z_{S^2\times S^2\times S^1} = \prod_{l=1}^{4} Z^{(l)}_{\mathbb{R}_{\epsilon_1^{(l)}}\times\mathbb{R}_{\epsilon_2^{(l)}}\times S^1}$$



and finally fiber the S^1 over one of the S^2's to get S^3_b. (With no fibration gives exact result for $S^2\times S^2\times S^1$)

Summary

The final result of localization is:

$$Z_{S_b^3\times\Sigma_{\mathfrak{g}}}(\nu)_{\mathfrak{n}} = \sum_{\hat{u}:\ \Pi_a(\hat{u})=1} \Pi_i(\hat{u},\nu)^{\mathfrak{n}_i}\mathcal{H}(\hat{u},\nu)^{\mathfrak{g}-1}$$

Main features:

- Formula is exact; includes all instanton contributions
- Has the form of an A-twisted 2d TQFT $(\mathcal{W},\Omega)$
- Determined fully by 5d Nekrasov partition function; $\mathcal{W}=\mathcal{W}_{NS}^{(5d)}$ and $\Omega=\Omega_{NS}^{(5d)}$

Next, I discuss applications of this formula

III) Two Applications

Applications

1) Maximal SYM: Superconformal index of 4d class $\mathcal{S}$

2) Seiberg theory and holography

I will mostly focus on 2)

1) Maximal $U(N)$ 5d SYM

First, solve Bethe equations

$$\Pi_a(u) = e^{2\pi i \partial_{u_a}(\mathcal{W}^V(u)+\mathcal{W}^H_{Ad}(u,\nu_{Ad}))} = 1$$

Difficult but simplified for $\nu_{Ad} = \frac{i}{2}(b - b^{-1})$ (Schur limit) $\Rightarrow$

$$Z^{\mathcal{N}=2\ \mathrm{SYM}}_{S^3_b\times\Sigma_\mathfrak{g}} = \sum_{\hat{u}:\ \Pi_a(\hat{u})=1} \Pi_i(\hat{u},\nu_{Ad})^{\mathfrak{n}_i}\mathcal{H}(\hat{u},\nu_{Ad})^{\mathfrak{g}-1} = e^{-\beta E_C}\mathcal{I}(p,q)$$

with

$$\mathcal{I}(p,q)=\left(\frac{p^{\frac{1}{12}N(N^2-1)}V(p)}{(p;p)^{N-1}}\right)^{-\ell_1}\left(\frac{q^{\frac{1}{12}N(N^2-1)}V(q)}{(q;q)^{N-1}}\right)^{-\ell_2} \times \sum_{\lambda\in\Lambda^+_{cr}} \dim_p(R_\lambda)^{-\ell_1}\dim_q(R_\lambda)^{-\ell_2}$$

where $\ell_1 = \mathfrak{g}-1+\mathfrak{n}\,, \ell_2 = \mathfrak{g}-1-\mathfrak{n}$, identifications

$$p = e^{2\pi b\gamma^{-1}}, \qquad q = e^{2\pi b^{-1}\gamma^{-1}}, \quad \gamma = -\frac{2\pi Q}{{g_5}^2}$$

$V(p)$ polynomial and $\dim_p(R_\lambda)$ the "quantum dimension" of a representation with highest weight λ. Matches **[Rastelli et al.][Beem-Gadde]**

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Comments

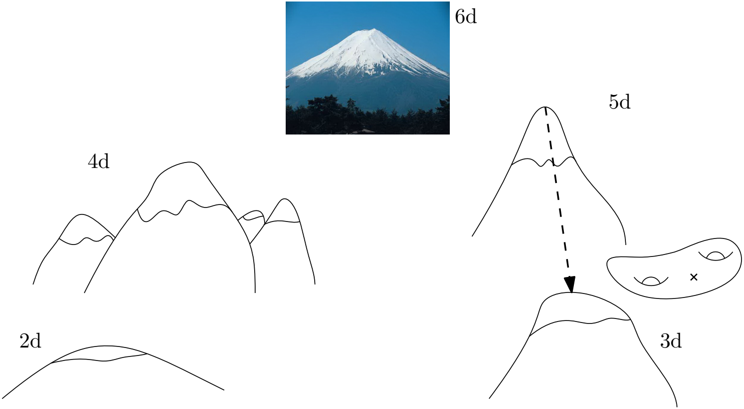
- Perfect match with previous results **[Rastelli et al.][Beem-Gadde]** in the Schur limit

- Index known only for this and other special limits

- In principle, 5d partition function gives full answer (might be hard to explicitly solve the Bethe equations in general)

2) Seiberg theory and holography

The Seiberg theory has a 5d UV SCFT completion. Compactifying gives rise to a 3d theory (instead of 4d):



New SCFTs in 3d

Compactification from 5d provides a map:

$$\mathcal{T}^{(5d)}, \Sigma_{\mathfrak{g}}, \mathfrak{n} \qquad \Rightarrow \qquad \mathcal{T}^{(3d)}_{\mathfrak{g},\mathfrak{n}}$$

Assuming RG flow ends in a nontrivial fixed point:

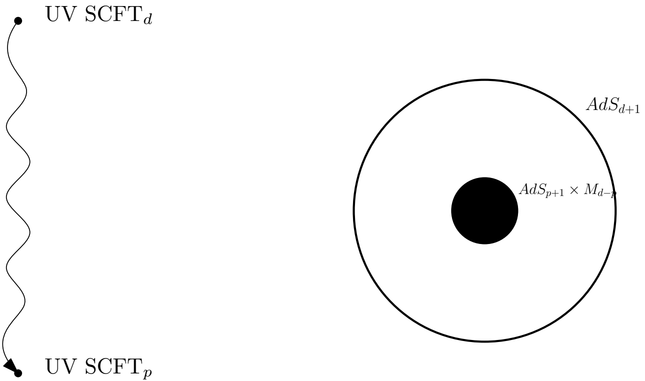
Infinite family of 3d SCFTs labeled by $\mathfrak{g}, \mathfrak{n}$

Their partition function is explictily given by

$$Z_{S^3}[\mathcal{T}^{(3d)}_{\mathfrak{g},\mathfrak{n}}] = Z_{S^3\times\Sigma_{\mathfrak{g}}}[\mathcal{T}^{(5d)}]_{\mathfrak{n}}$$

How do we know RG flows end in interacting SCFTs?

According to AdS/CFT **[Maldacena-Núñez]**



Extremal black p-branes in asymptotically locally AdS are flows across dimensions!

Universal RG flow

Idea: Use holography to construct 5d $\to$ 3d RG flows explicitly

Consider 6d $F(4)$ gauged supergravity (unique theory with AdS_6 vacuum) **[Romans]**

$$\text{Bosonic content:} \qquad \{g_{\mu\nu}, A^I_\mu, A_\mu, B_{\mu\nu}, \phi\}$$

Extremal 2-brane solution **[Núñez et al., Naka]**

$$ds_6^2 = e^{2f(r)} ds^2(\mathbb{R}^4) + e^{2g(r)} ds^2(\Sigma_{\mathfrak{g}>1}) \,, \quad dA^I = \delta^{I3} \mathrm{dvol}(\Sigma_{\mathfrak{g}}) \,, \quad \phi = \phi(r) \,.$$

Indeed interpolates between AdS_6 and AdS_4. But

Is this what we are looking for?

Universal RG flow

Topological twist implemented by A^I only (dual to $SU(2)_R$). In field theory this means

$$\mathfrak{n} = 0$$

From sugra solution it follows that [Bobev-MC]

$$F^{sugra}_{\text{AdS}_4} = -\frac{8}{9}(\mathfrak{g}-1)F^{sugra}_{\text{AdS}_6}$$

Using AdS/CFT dictionary

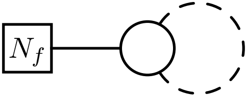
$$F^{\text{QFT}}_{S^3} = -\frac{8}{9}(\mathfrak{g}-1)F^{\text{QFT}}_{S^5}$$

Sugra solution can be uplifted to 10d massive IIA with a generic internal M_4 $\Rightarrow$

Relation is universal

Back to field theory

We can test the holographic prediction for the Seiberg theory:



Then, the Bethe potential is:

$$
\begin{aligned}
\mathcal{W}^{pert} = & -\sum_{i<j} \left[g_b(1 \pm (u_i - u_j)) + g_b(1 \pm (u_i + u_j)) \right] - \sum_i g_b(1 + 2u_i) \\
& + \sum_{i<j} \left[g_b(\nu_{AS} \pm (u_i - u_j)) + g_b(\nu_{AS} \pm (u_i + u_j)) \right] \\
& + \sum_{I=1}^{N_f} \sum_i g_b(\nu_I + u_i)
\end{aligned}
$$

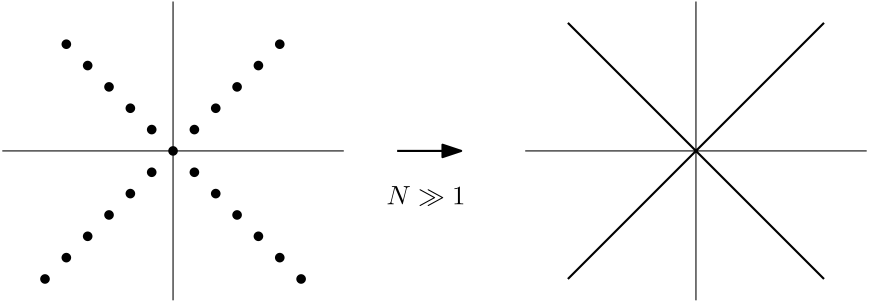
Perturbative accuracy sufficient at large N

Continuum method

Need to solve Bethe equations:

$$e^{2\pi i \partial_{u_i} \mathcal{W}^{pert}} = 1$$

At large N:



Continuum method **[Herzog-Klebanov-Pufu-Tesileanu]**

$$u_i \to i N^\alpha x + y(x)\,, \qquad \sum_i \to N \int dx\, \rho(x)\,, \tag{0.2}$$

In our case $y(x) = 0$ and

$$\mathcal{W}^{pert} \approx \frac{(8 - N_f)}{6} Q^2\, N^{1+3\alpha} \int dx\, \rho(x)\, |x|^3 - \frac{1}{2} Q^2 \left(1 - \nu_{AS}^2\right) N^{2+\alpha} \int_{y<x} dx\, dy\, \rho(x)\rho(y) \left(|x+y| + |x-y|\right) .$$

Balancing of two terms requires

$$\alpha = \frac{1}{2}$$

Extremizing with respect to density ρ (and ν_{AS}), and evaluating free energy $F \equiv -\log|Z|$ on-shell (and $b = 1$):

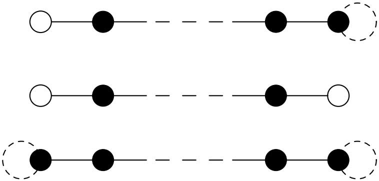
$$F_{S_b^3 \times \Sigma_\mathfrak{g}} \simeq -\frac{8}{9}(\mathfrak{g} - 1) \underbrace{\left(-\frac{9\sqrt{2}\pi N^{5/2}}{5\sqrt{8 - N_f}} \right)}_{F_{S^5}\ \text{[Jafferis–Pufu]}}$$

Exactly reproduces holographic prediction

More general theories

5d quiver gauge theories for $Sp(N)$ and $U(N)$ nodes with gravity duals

[Bergman-Rodriguez]



In all cases explicit computation yields (always $\mathfrak{n} = 0$)

$$F_{S_b^3 \times \Sigma_{\mathfrak{g}}} \simeq -\frac{8}{9}(\mathfrak{g} - 1)\, F_{S^5}$$

- Universal prediction satisfied for all quivers
- Strong evidence for existence of 3d SCFTs
- Free energy scales as $N^{5/2}$

Generalizations

For nonzero $\mathfrak{n}$ field theory calculation yields

$$F_{S^3_b \times \Sigma_\mathfrak{g}} = \frac{8}{9}(\mathfrak{g}-1)\left(\frac{\sqrt{2}\left|\hat{\mathfrak{n}}^2-\kappa^2\right|^{3/2}\left(\sqrt{\kappa^2+8\hat{\mathfrak{n}}^2}-\kappa\right)}{\kappa\left|4\hat{\mathfrak{n}}^2-\kappa^2+\kappa\sqrt{\kappa^2+8\hat{\mathfrak{n}}^2}\right|^{3/2}}\right) F_{S^5}$$

where $\kappa = 0, \pm 1$. Holographic solutions recently constructed [Bah-Passias-Weck] [Hosseini, Hristov, Passias, Zaffaroni] with perfect agreement.

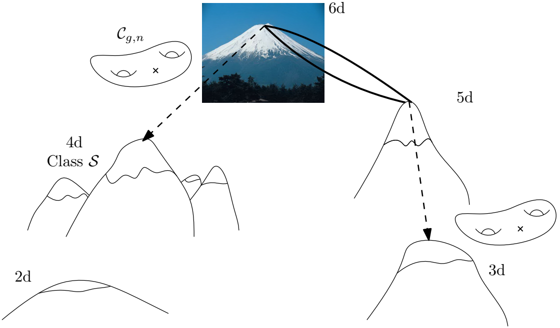
Other applications for case $\Sigma_\mathfrak{g} \times \Sigma_{\mathfrak{g}'} \times S^1$ include computation of BH entropy in AdS_6 [Hosseini-Yaakov-Zaffaroni] [Fluder-Hosseini-Uhlemann]

To summarize:

- Large class of 3d SCFTs: $\mathcal{T}^{(3d)}_{\mathfrak{g},\mathfrak{n}}$
- Exact partition functions calculable as 2d TQFT
- Holographic duals confirm flows (at least at large N)

Summary

Argued that 5d is a fruitful vantage point:



- Can access theories in $d = 6$ as well as $d < 5$
- Explicit computations are possible by localization
- Computed the exact partition function on $S^3_b \times \Sigma_\mathfrak{g}$
- Precision tests of holography and field theory

Outlook

- General expression for 4d superconformal index of class $\mathcal{S}$
- Include punctures on $\Sigma_{\mathfrak{g}}$. More general M_3.
- M_4 partition function of class $\mathcal{S}$?

- Purely three-dimensional description of 3d SCFTs?
- 3d/2d correspondence?

Thank you