Dark energy stars

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Dark energy stars

Dark energy ($p=-\rho$) in a finite volume

Vacuum bubble

A finite region where $p = -\rho$. For example, true and false vacuum bubbles, …

Coleman, De Luccia 1980, Sato, Sasaki, Kodama, Maeda 1981+, Blau, Guendelman, Guth 1987, Farhi, Guth 1987; …

Interior-deSitter/exterior-Schwarzschild geometry

A spherically symmetric geometry that is asymptotically de Sitter as $r \to 0$ and asymptotically Schwarzschild as $r \to \infty$. For example, regular black holes with $p = -\rho$ at the center, smooth gravastars, …

Sakharov 1966, Gliner 1966, Dymnikova 1996+, Catoen, Faber, Visser 2005, Ansoldi, Sindoni 2008, …
{reviews: Dymnikova 2003, Ansoldi 2008, …}

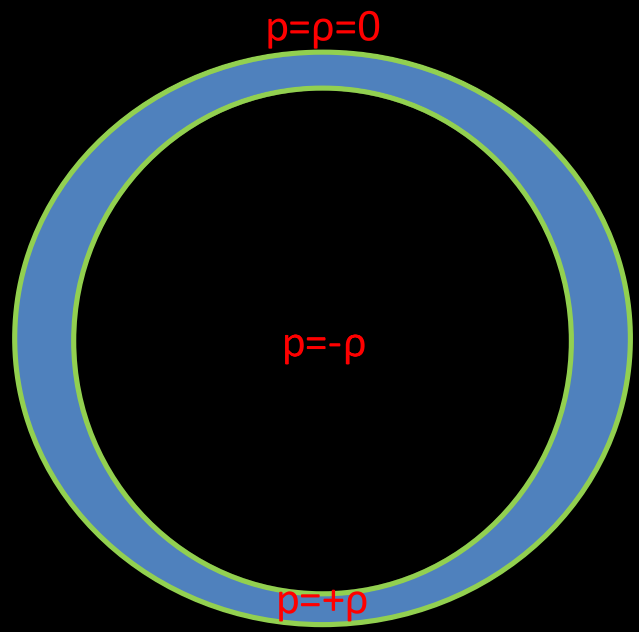
Gravastar

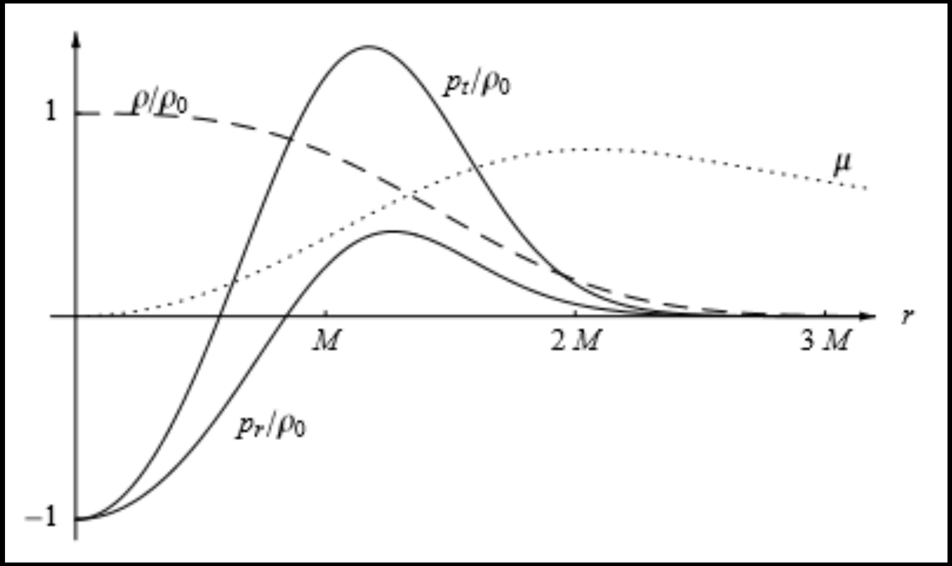
Originally, a star with a $p = -\rho$ interior of volume V matched to a Schwarzschild exterior of mass $M = \rho\, V$ and having no horizons or spacetime singularities.

Mazur, Mottola 2002+, Catoen, Faber, Visser 2005, …

Gravastars

- A center with negative pressure
- Radius > Schwarzschild radius
- Initial model by Mazur and Mottola: a pure dark energy core, stiff matter shell, and vacuum exterior, with infinitesimal boundary layers
- Cattoen, Faber, and Visser replaced boundary layers with anisotropic stress





DeBenedictis, Horvat, Ilijic, Kloster, Viswanathan (2007)

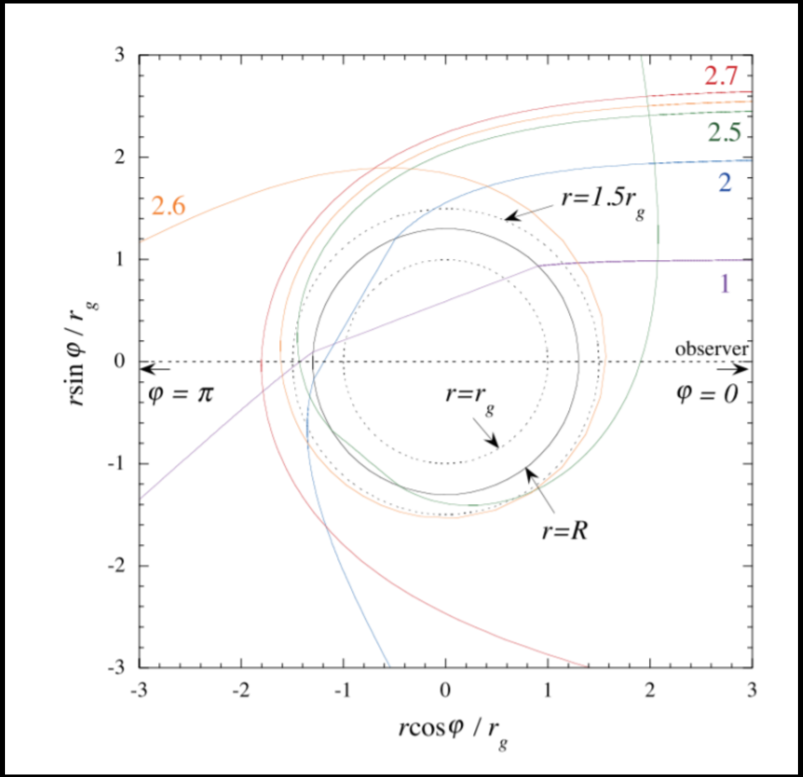
Gravastars

Gravitational lensing

- gravastars do not require event horizons
- it may be possible to have light pass through
- interesting lensing trajectories

Gravitational waves

- matter on surface can possibly give a “seismic” signature in gravitational waves



Sakai, Saida, Tamaki 2014

Gravastars

- "Gravitational condensates": temperature/entropy term must be zero

  Mazur, Mottola 2002

- Numerical simulations indicate (slow) rigid rotation and angular momentum are possible, Schwarzschild interior dark energy star nearly matches Kerr source

  Chirenti, Rezzolla 2008, Posada 2016

- Gravastars can be electrically charged

  Horvat, Ilijic, Marunovic 2008

- Gravastar stability has been studied. Possible to oscillate between radii rather than settle (bounded excursion)

  Chirenti, Rezzolla 2007; Rocha et al 2008

- Formation from normal matter configurations has not been studied

Formation of dark energy stars

Formation of dark energy stars

Beltracchi, Gondolo 2019a

Time-dependent spherically-symmetric anisotropic solution

$$ds^2 = -e^{2\Phi(t,r)}\,dt^2 + \frac{dr^2}{1-\frac{2Gm(t,r)}{r}} + r^2\,d\theta^2 + r^2\,\sin^2\theta\,d\phi^2$$

$$T_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \rho & -S_r & 0 & 0 \\ -S_r & p_r & 0 & 0 \\ 0 & 0 & p_T & 0 \\ 0 & 0 & 0 & p_T \end{pmatrix}$$

pressure anisotropy $\Delta = p_T - p_r$

radial momentum flow S_r

Formation of dark energy stars

Beltracchi, Gondolo 2019
Beltracchi, Gondolo 2019a

Einstein's equations

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho$$

$$\frac{\partial \Phi}{\partial r} = \frac{G\left(m + 4\pi r^3 p_r\right)}{r^2\left(1 - \frac{2Gm}{r}\right)}$$

$$\frac{\partial \rho}{\partial \tau} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\sqrt{1 - \frac{2Gm}{r}}\, S_r\right)$$

$$-\frac{\partial p_r}{\partial r} - \frac{G\left(m + 4\pi r^3 p_r\right)\left(\rho + p_r\right)}{r^2\left(1 - \frac{2Gm}{r}\right)} + \frac{2\Delta}{r} = \sqrt{1 - \frac{2Gm}{r}}\,\frac{\partial}{\partial \tau}\left(\frac{S_r}{1 - \frac{2Gm}{r}}\right)$$

Force equation $F = \mathrm{d}p/\mathrm{d}t$

In the static case, it reduces to the Tolman-Oppenheimer-Volkoff equation plus an anisotropy term

τ is proper time at fixed r, θ, ϕ $\quad (d\tau = e^{\Phi(t,r)}\, dt)$

Formation of dark energy stars

Beltracchi, Gondolo 2019a

Einstein's equations

$$m = \int_0^r \rho \, 4\pi r^2 \, dr$$

$$\Phi = -\int_r^\infty \frac{G\left(m + 4\pi r^3 p_r\right)}{r^2\left(1 - \frac{2Gm}{r}\right)} \, dr$$

$$S_r = -\frac{1}{\sqrt{1 - \frac{2Gm}{r}}} \frac{1}{4\pi r^2} \int_0^r \frac{\partial \rho}{\partial \tau} 4\pi r^2 \, dr$$

$$\Delta = \frac{G\left(m + 4\pi r^3 p_r\right)\left(\rho + p_r\right)}{2r\left(1 - \frac{2Gm}{r}\right)} + \frac{r}{2} \frac{\partial p_r}{\partial r} + \frac{r}{2} \sqrt{1 - \frac{2Gm}{r}} \frac{\partial}{\partial \tau} \left[\frac{S_r}{1 - \frac{2Gm}{r}} \right]$$

τ is proper time at fixed r, θ, ϕ $\quad (d\tau = e^{\Phi(t,r)} \, dt)$

Formation of dark energy stars

Beltracchi, Gondolo 2019a

Einstein's equations

Give $\rho(t,r)$ and $p_r(t,r)$

Find $S_r(t,r)$ and $\Delta(t,r)$

Check energy conditions

$$m = \int_0^r \rho \, 4\pi r^2 \, dr$$

$$\Phi = -\int_r^\infty \frac{G\left(m + 4\pi r^3 p_r\right)}{r^2\left(1 - \frac{2Gm}{r}\right)} \, dr$$

$$S_r = -\frac{1}{\sqrt{1 - \frac{2Gm}{r}}} \frac{1}{4\pi r^2} \int_0^r \frac{\partial \rho}{\partial \tau} 4\pi r^2 \, dr$$

$$\Delta = \frac{G\left(m + 4\pi r^3 p_r\right)\left(\rho + p_r\right)}{2r\left(1 - \frac{2Gm}{r}\right)} + \frac{r}{2}\frac{\partial p_r}{\partial r} + \frac{r}{2}\sqrt{1 - \frac{2Gm}{r}} \frac{\partial}{\partial \tau}\left[\frac{S_r}{1 - \frac{2Gm}{r}}\right]$$

τ is proper time at fixed r, θ, ϕ $\quad (d\tau = e^{\Phi(t,r)} \, dt)$

Energy conditions

Beltracchi, Gondolo 2019a

Weak energy condition

$T_{\mu\nu}k^\mu k^\nu \geq 0$ for all time-like vectors k^μ

$\rho - \dfrac{S_r^2}{p_r} \geq 0$ if $p_r - |S_r| \geq 0$ and $p_T \geq 0$

$\rho + p_r - 2|S_r| \geq 0$ if $p_r - |S_r| \leq 0$ and $p_T \geq p_r - |S_r|$

$\rho + p_T + \dfrac{S_r^2}{p_T - p_r} \geq 0$ if $p_T \leq p_r - |S_r|$ and $p_T \leq 0$

Energy conditions

Beltracchi, Gondolo 2019a

Null energy condition

$T_{\mu\nu} k^\mu k^\nu \geq 0$ for all light-like vectors k^μ

$\rho + p_r - 2|S_r| \geq 0$ if $|p_T - p_r| \leq |S_r|$

$\rho + p_T + \dfrac{S_r^2}{p_T - p_r} \geq 0$ if $|p_T - p_r| \geq |S_r|$

Pile-up model

Beltracchi, Gondolo 2019a

Form a $p_r = p_T = -\rho = \text{const}$ core of increasing radius

surface radius $R = s(\tau)\, R_s$ (empty space outside)

core radius $R_c(\tau) = f(\tau)\, R_s$ (for $f(\tau) > 0$)

relation between f and s from $M = \text{const}$

Evolution parameter $f = f(\tau)$ controls formation of singularities and horizons

$f < 0$ precursor, $0 < f < 1$ dark energy star, $f = 1$ horizon would form

$$\rho(\tau, r) = \rho(r, f)$$

$$p_r(\tau, r) = p_r(r, f)$$

$$f = f(\tau)$$

Pile-up model

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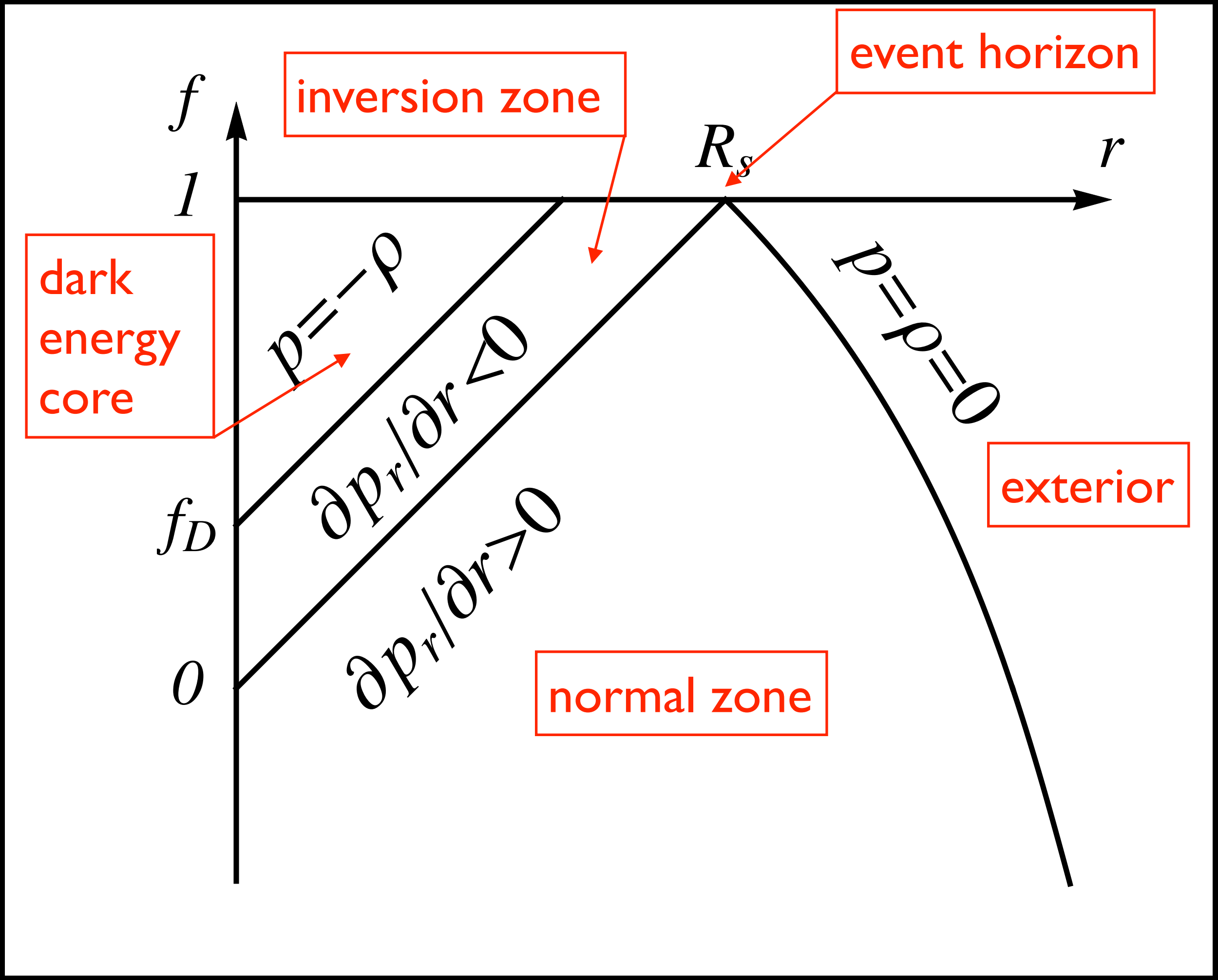


Pile-up model

Beltracchi, Gondolo 2019a

Form a $p_r = p_T = -\rho = \text{const}$ core of increasing radius

specific example

$$\frac{\rho(r,f)}{\rho_S} = \begin{cases} 1 & 0 < x \le f \\ \frac{s-x}{s-f} & 0 < f < x < s \\ \frac{4(s-x)}{s^4} & f < 0 < x < s \\ 0 & s \le x \end{cases}$$

$$R_s \equiv \frac{2GM}{c^2}$$

$$\rho_S \equiv \frac{3M}{4\pi R_s^3}$$

$$x \equiv \frac{r}{R_s}$$

$$\frac{p_r(r,f)}{\rho_S} = \begin{cases} (1+a)\Psi\left(\frac{x-f+f_D}{f_D}\right) - 1 & f > 0, 0 < x \le f \\ a - \frac{f}{f_D}(1+a)\left[1 - \Psi\left(\frac{x}{f}\right)\right] & 0 < x < f \le f_D \\ a \cos^4\left(\frac{\pi(x-f)}{2(s-f)}\right) & 0 < f < x < s \\ a \frac{4}{s^3} \cos^4\left(\frac{\pi x}{2s}\right) & f < 0 < x < s \\ 0 & s \le x \end{cases}$$

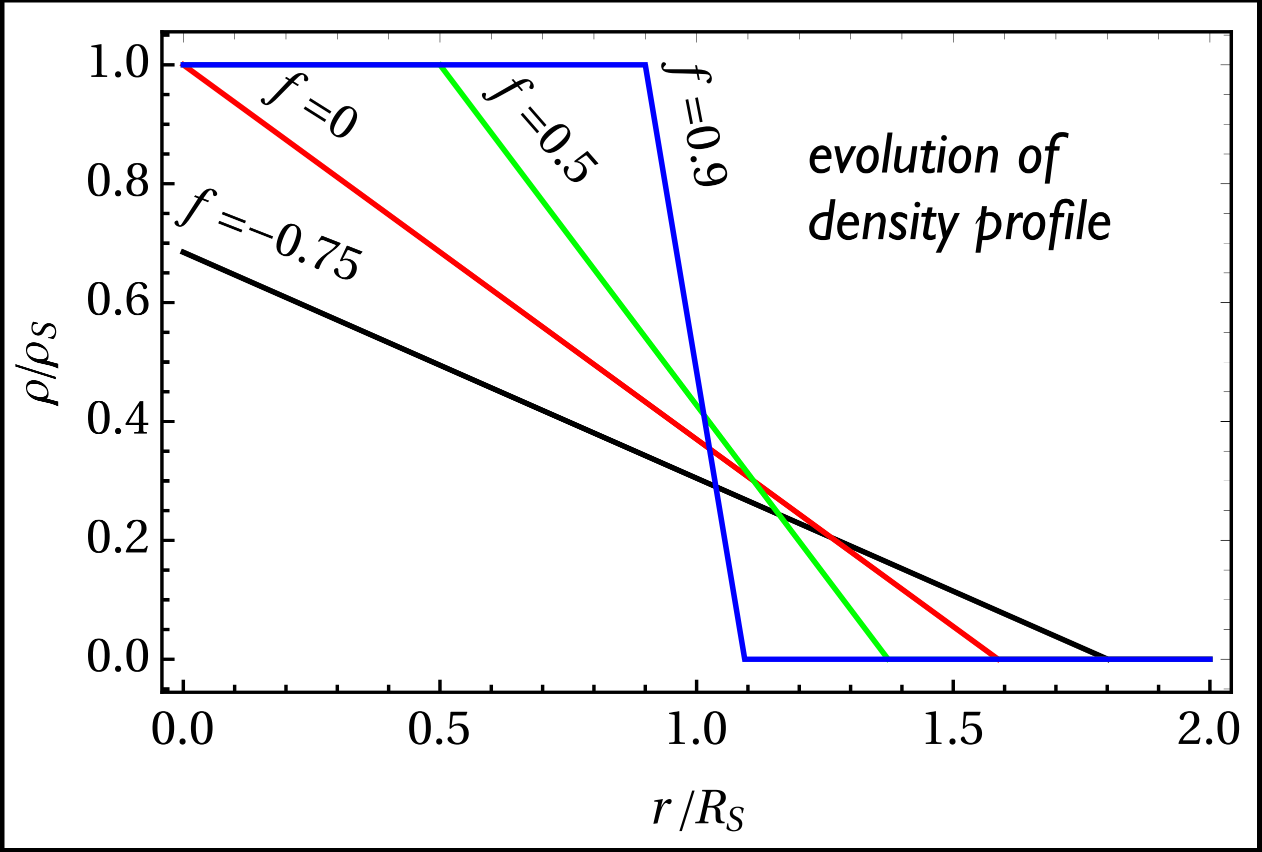
$$f(\tau) = \frac{2f_\infty}{\pi} \arctan\tau$$

$f_\infty < 1$ avoids formation of horizon

Pile-up model

Beltracchi, Gondolo 2019a

Form a $p_r = p_T = -\rho = $ const core of increasing radius



Evolution parameter $f(\tau)$

$f < 0$ precursor

$0 < f < 1$ dark energy star

$f = 1$ horizon would form

Pile-up model

Beltracchi, Gondolo 2019a

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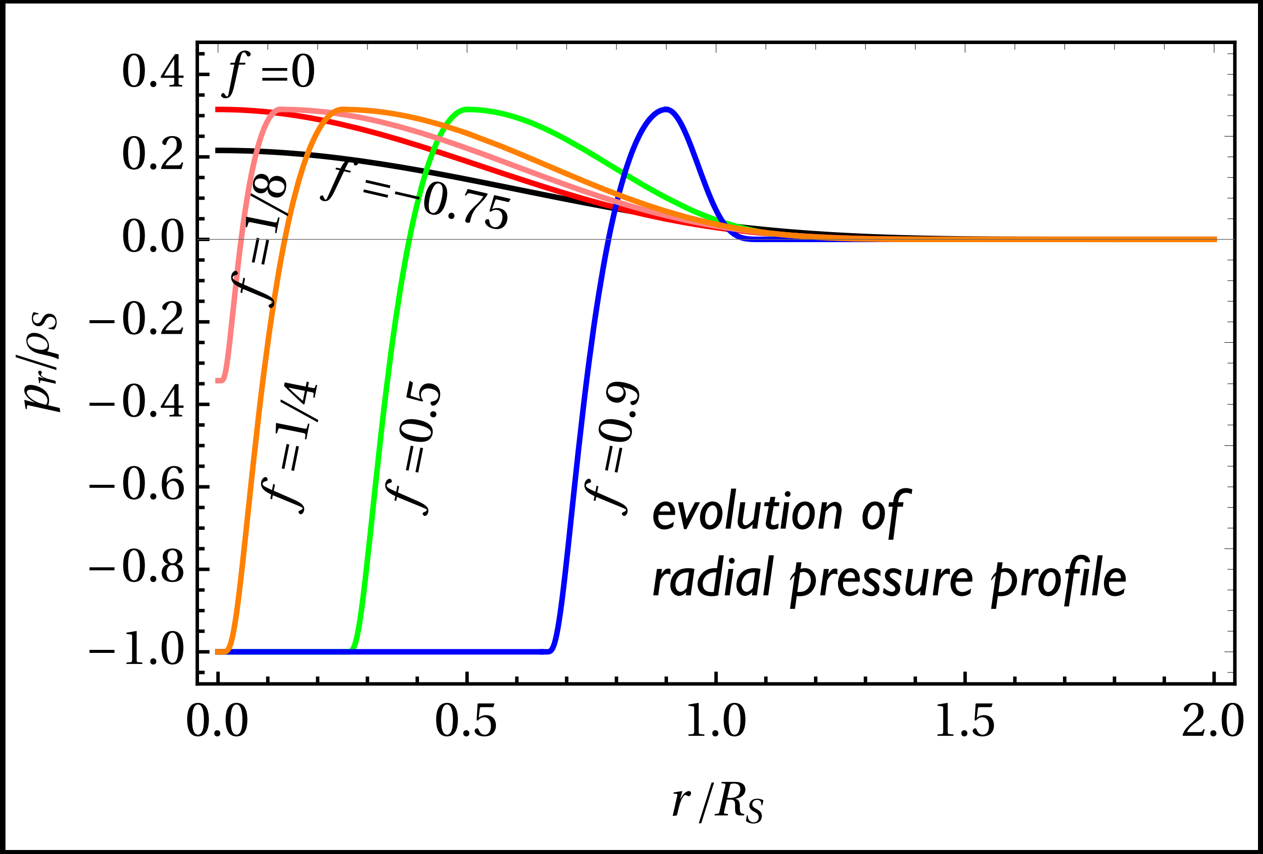

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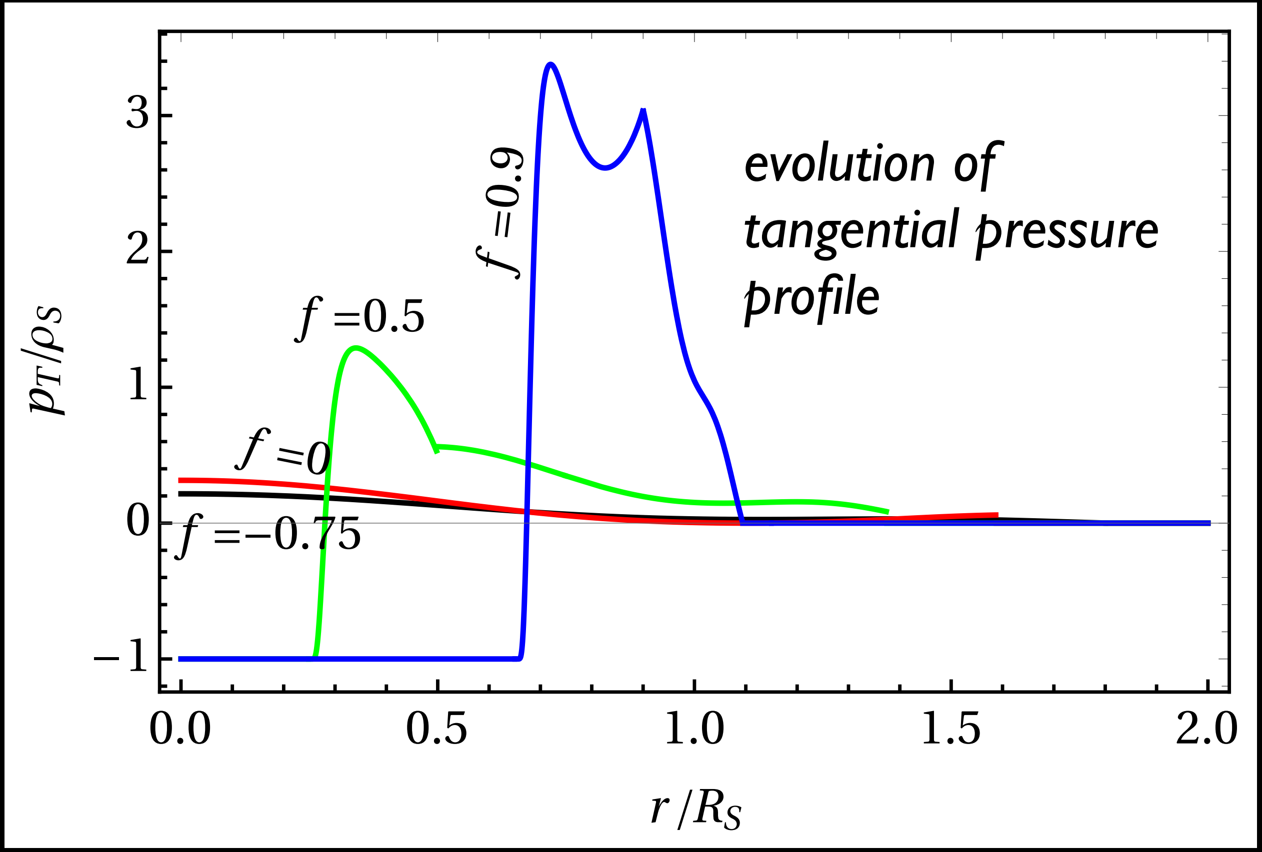

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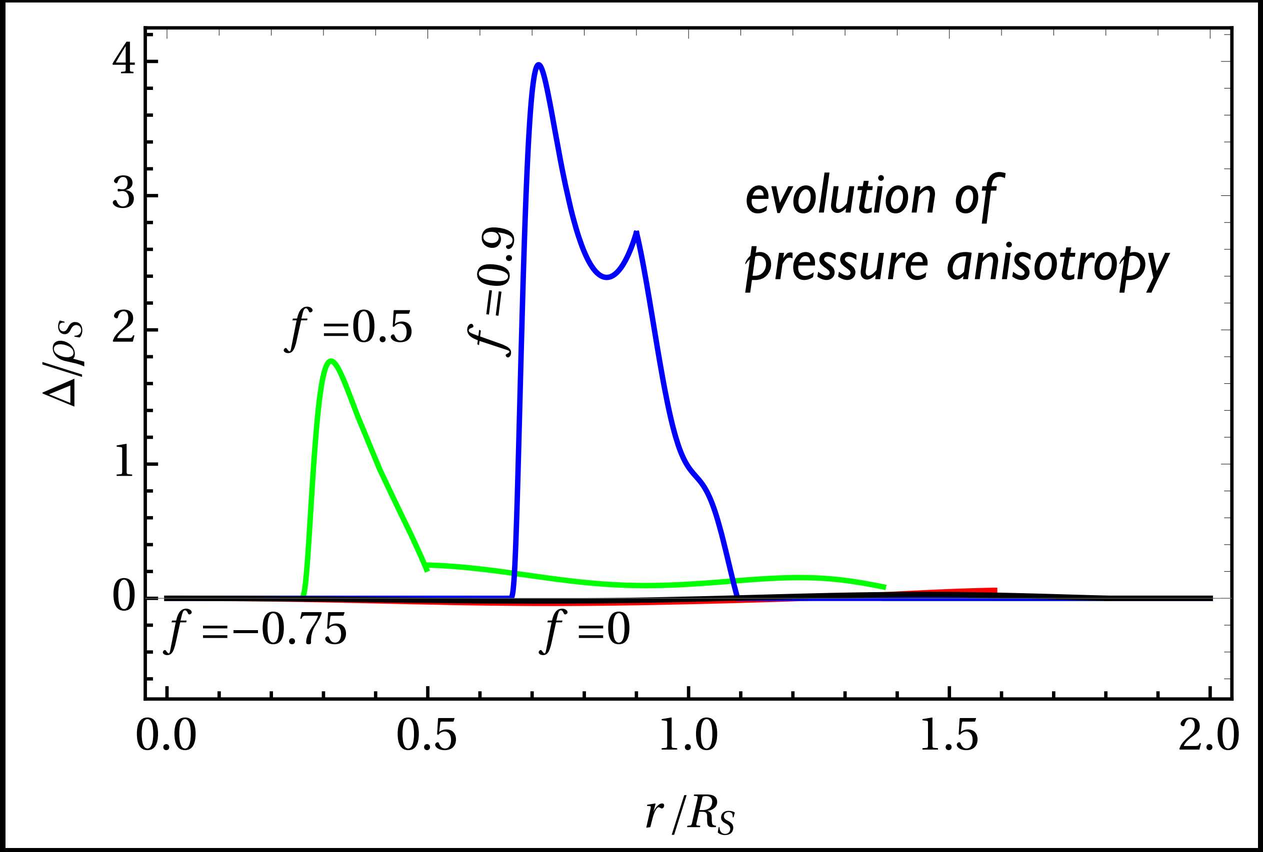

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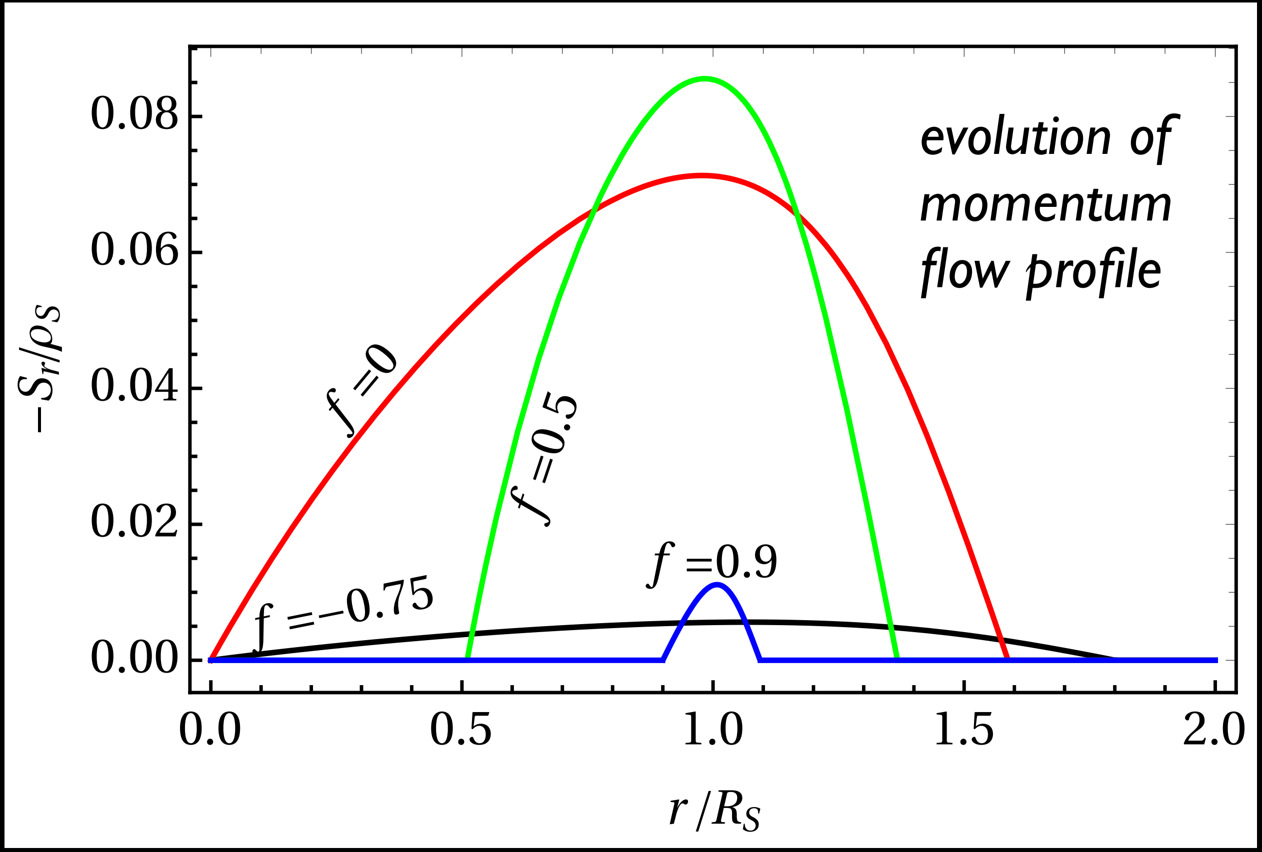

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Pile-up model

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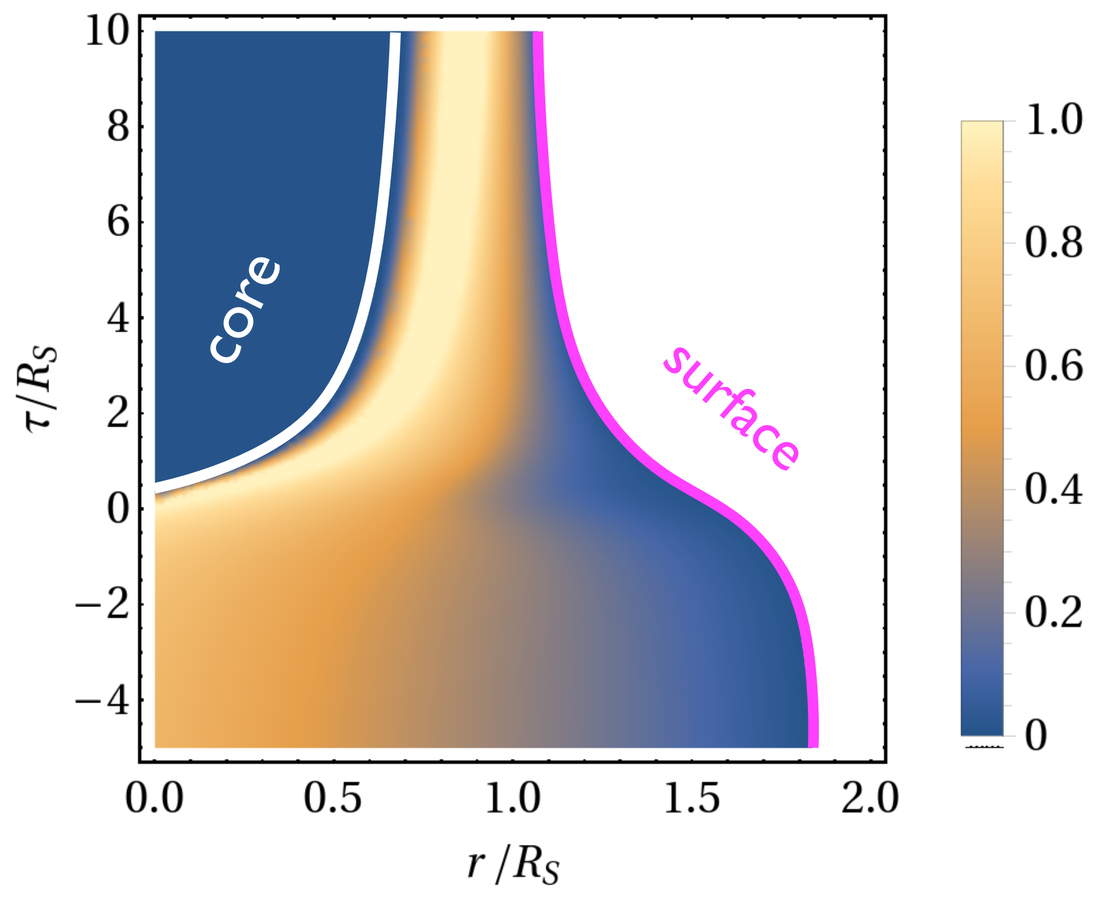


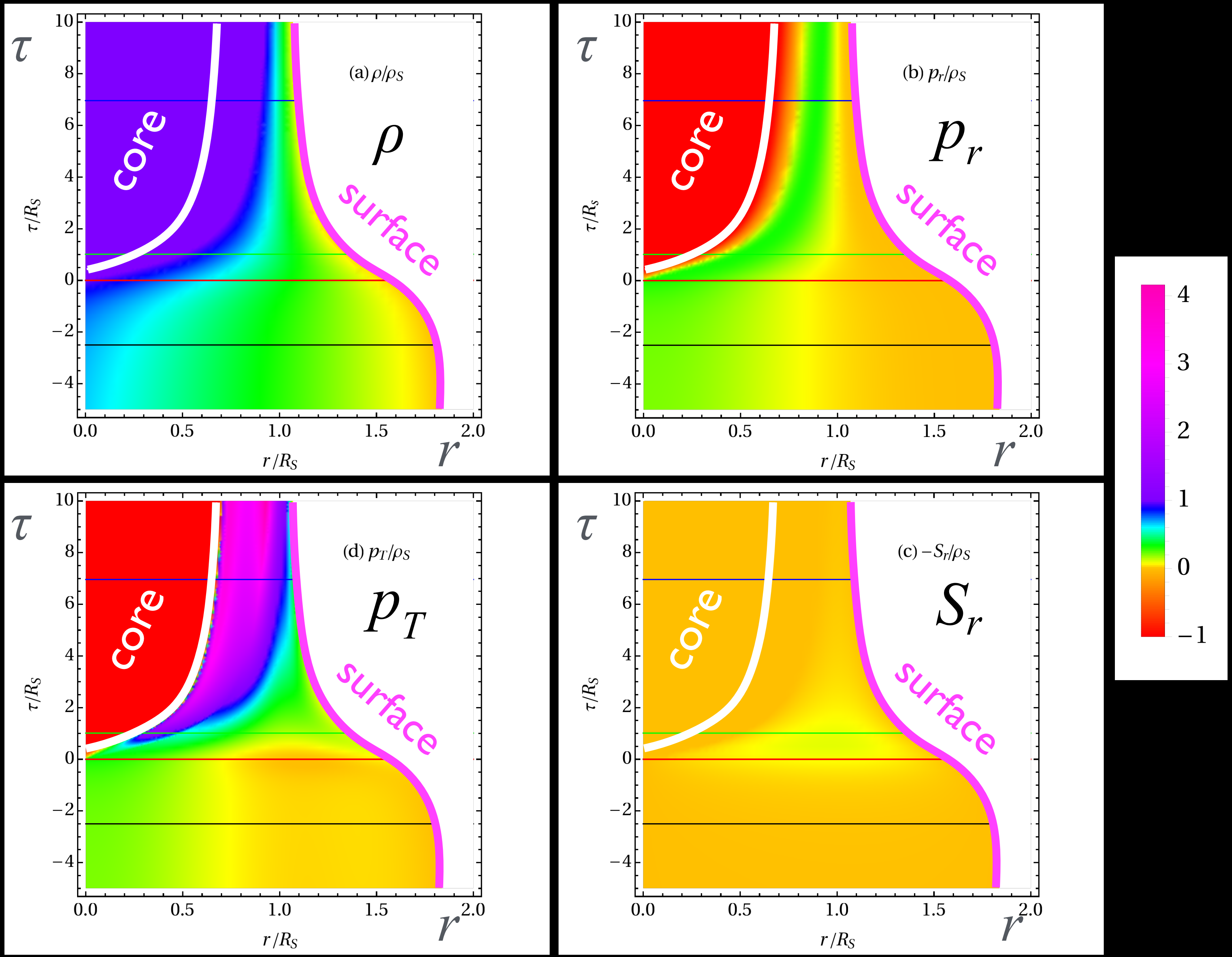
minimum $T_{\mu\nu}k^{\mu}k^{\nu}$ over time-like k^{μ}

weak energy condition satisfied at any t, r

Pile-up model

Beltracchi, Gondolo 2019a

Form a $p_r = p_T = -\rho = \text{const}$ core of increasing radius



Pile-up model

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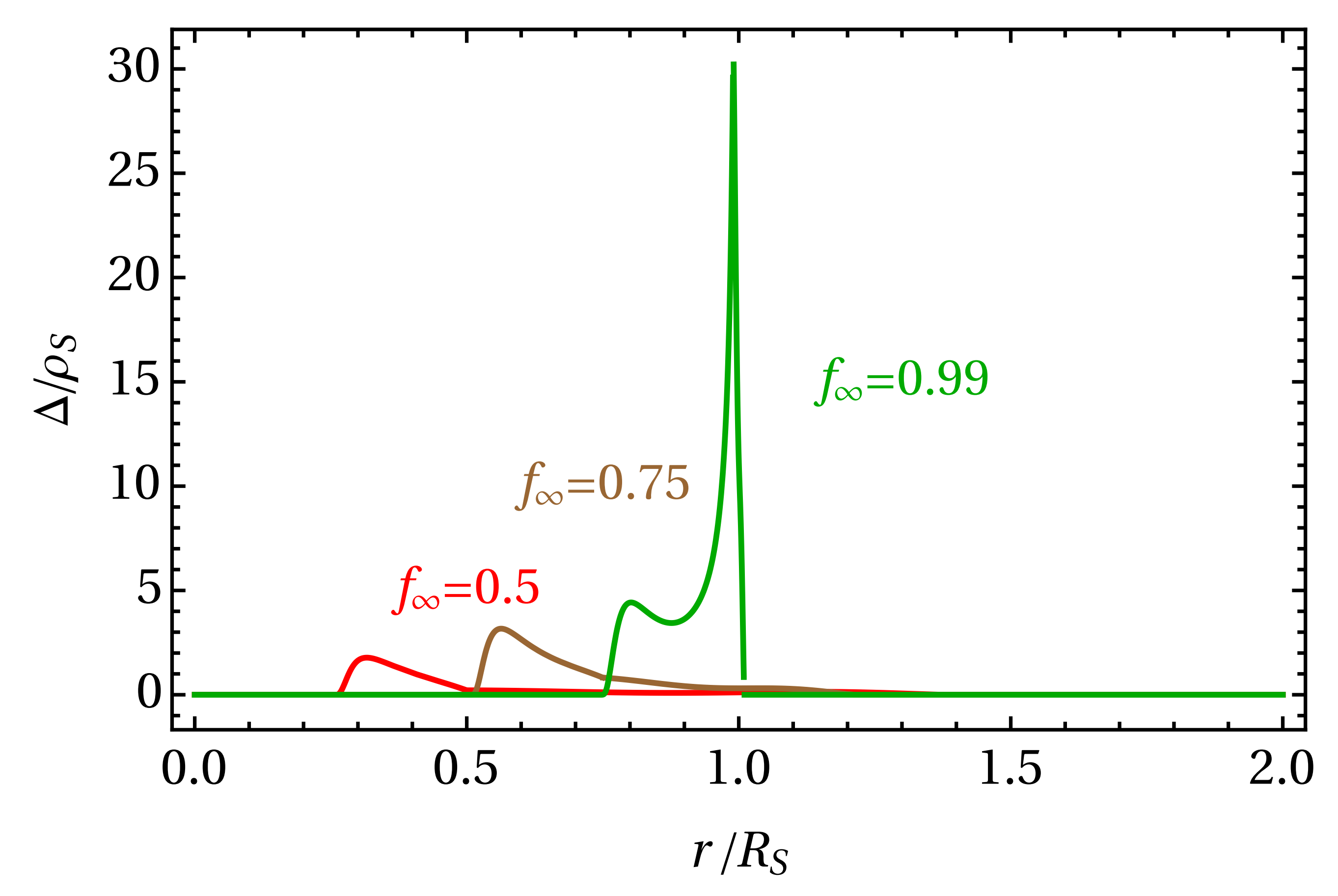
Form a $p_r = p_T = -\rho = \text{const}$ core of increasing radius

Pressure anisotropy in equilibrium configuration



Pile-up model

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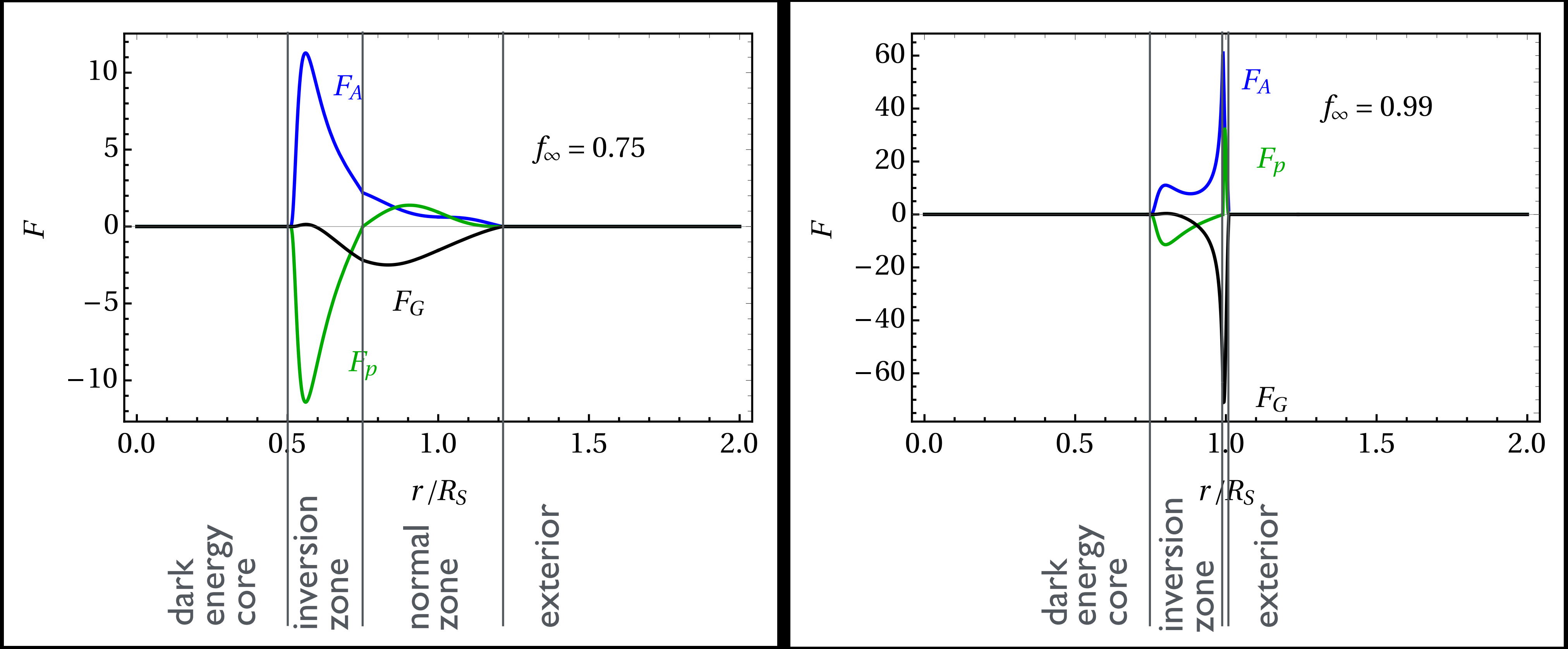
Form a $p_r = p_T = -\rho =$ const core of increasing radius

Force balance in equilibrium configuration



In the inversion zone, the pressure gradient force and the gravitational force point inwards and are balanced by the anisotropy force.

An exact time-dependent interior Schwarzschild solution

Schwarzschild stars

A spherically-symmetric, static, constant density star *Schwarzschild 1916*

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)$$

with

$$f(r) = \begin{cases} \frac{1}{4}(3a-b)^2, & r \leq R, \\ 1 - \frac{R_s}{r}, & r \geq R, \end{cases}$$

$$h(r) = \begin{cases} b^2, & r \leq R, \\ 1 - \frac{R_s}{r}, & r \geq R. \end{cases}$$

where

$$a = \sqrt{1 - \frac{R_s}{R}}, \qquad b = \sqrt{1 - \frac{R_s r^2}{R}}$$

Schwarzschild stars

- The pressure is everywhere finite if $R/R_s > 9/8 = 1.125$ (Buchdahl bound).

- For $R/R_s < 9/8$, the pressure diverges at finite radius $R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}$

- The singularity is integrable in the sense that

$$M_{\mathrm{grav}}(V) = \int_V (\rho + p_x + p_y + p_z)\,\sqrt{-g_{tt}}\,dV$$

is finite in any volume V *Mazur, Mottola 2015*

Formation of a Schwarzschild star

Beltracchi, Gondolo (in prep.)

Time-dependent exact solution of Einstein's equations

Ansatz: the radius of the Schwarzschild star depends on time, $R = R(t)$

Then $T_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \rho & S_r & 0 & 0 \\ S_r & p_r & 0 & 0 \\ 0 & 0 & p_T & 0 \\ 0 & 0 & 0 & p_T \end{pmatrix}$

anisotropic pressure $p_r \neq p_T$
momentum flow S_r

Continuity of p_r and p_T at the surface of the star gives

$$2R\ddot{R}(R_s - R) + \dot{R}^2(R_s + 8R) = 0,$$

which can solved analytically to find

$$\frac{t-t_0}{t_s} = F\left(\frac{R_s}{R}\right)$$

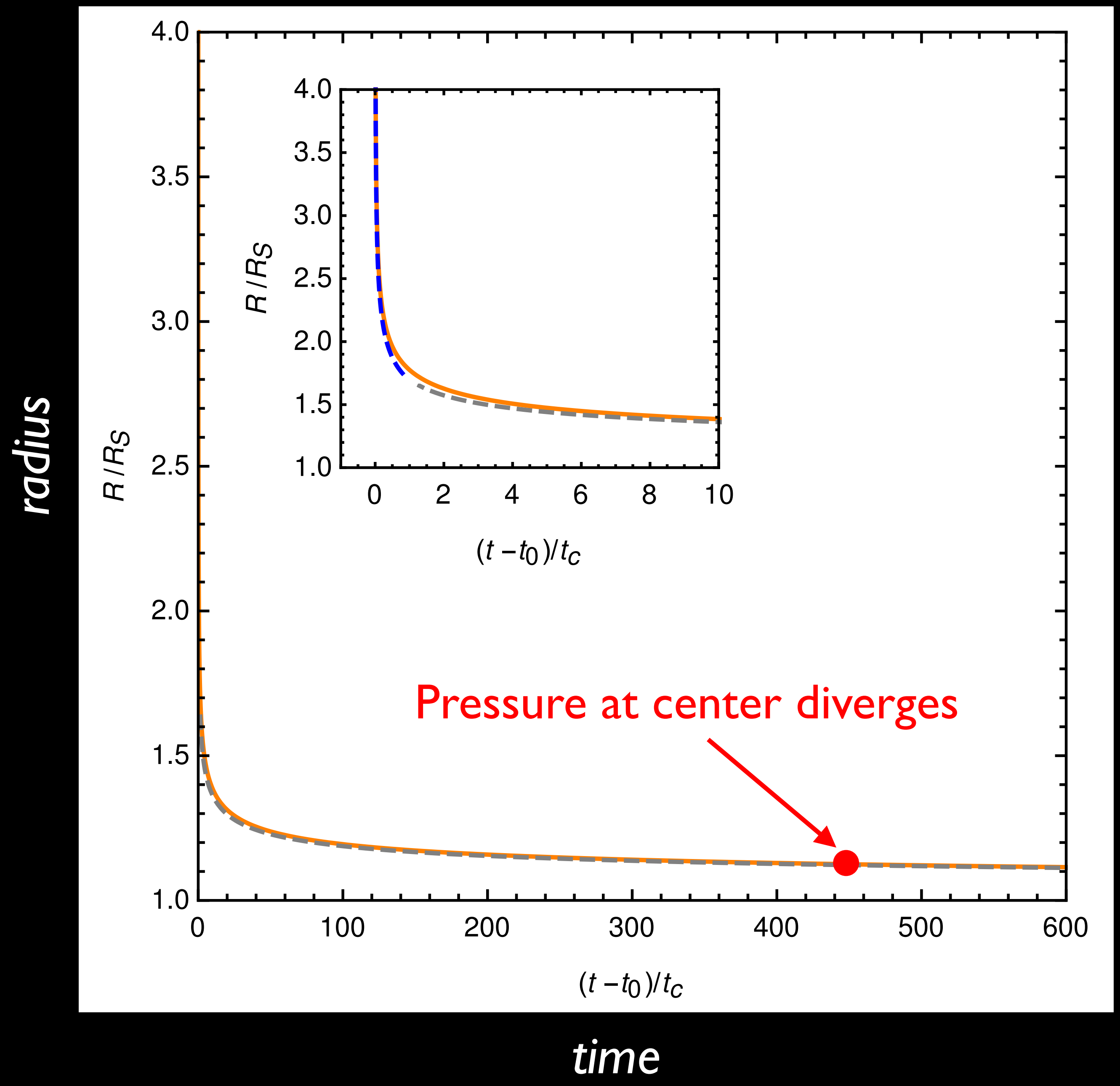
where t_0 and t_s are integration constants and

$$F(x) = \frac{1}{2942}\left(\frac{8-28x+35x^2}{8(1-x)^{7/2}} - 1\right).$$

At t_0, R was infinite.
At $t_0 + t_s$, the pressure becomes singular.

Formation of a Schwarzschild star

Beltracchi, Gondolo 2019b

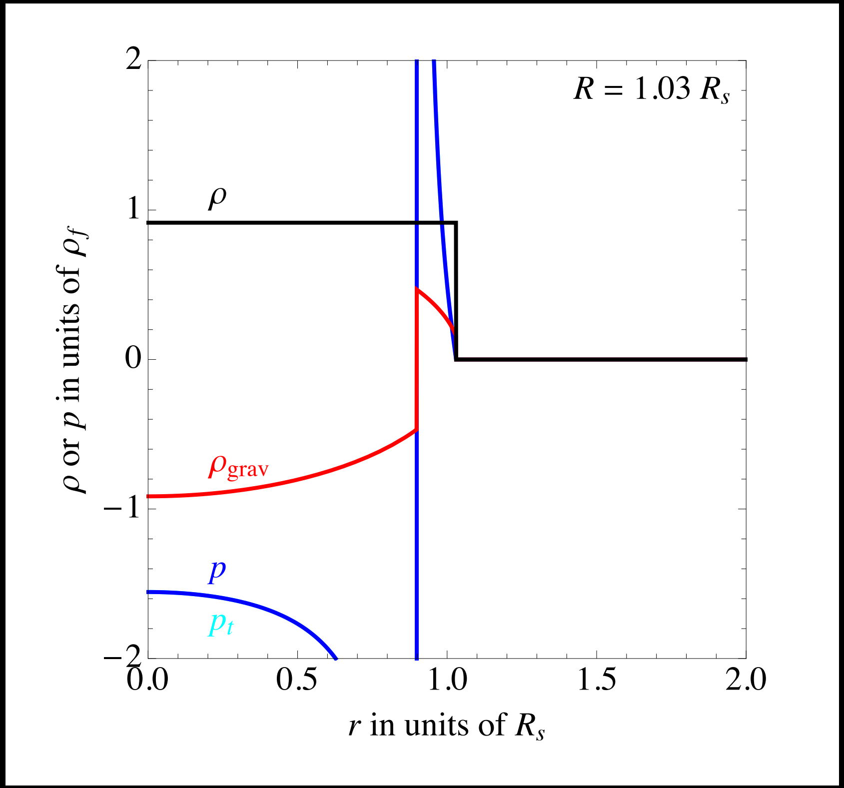


Formation of a Schwarzschild star

Beltracchi, Gondolo 2019b

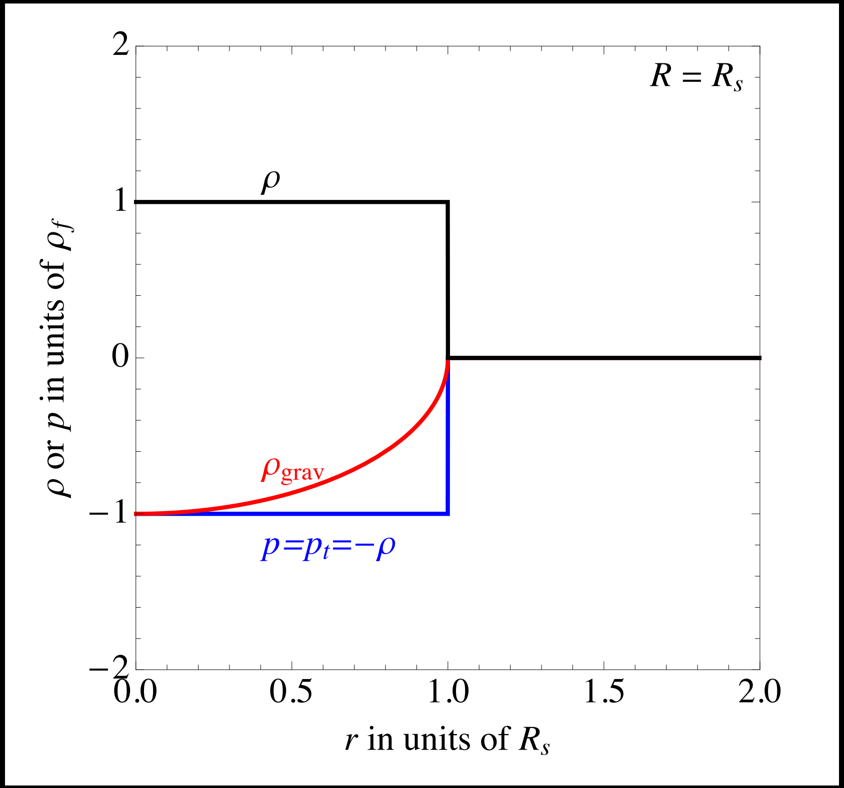
Energy density and pressure profiles

after the pressure diverges



Violates the weak and null energy conditions

at $t = \infty$

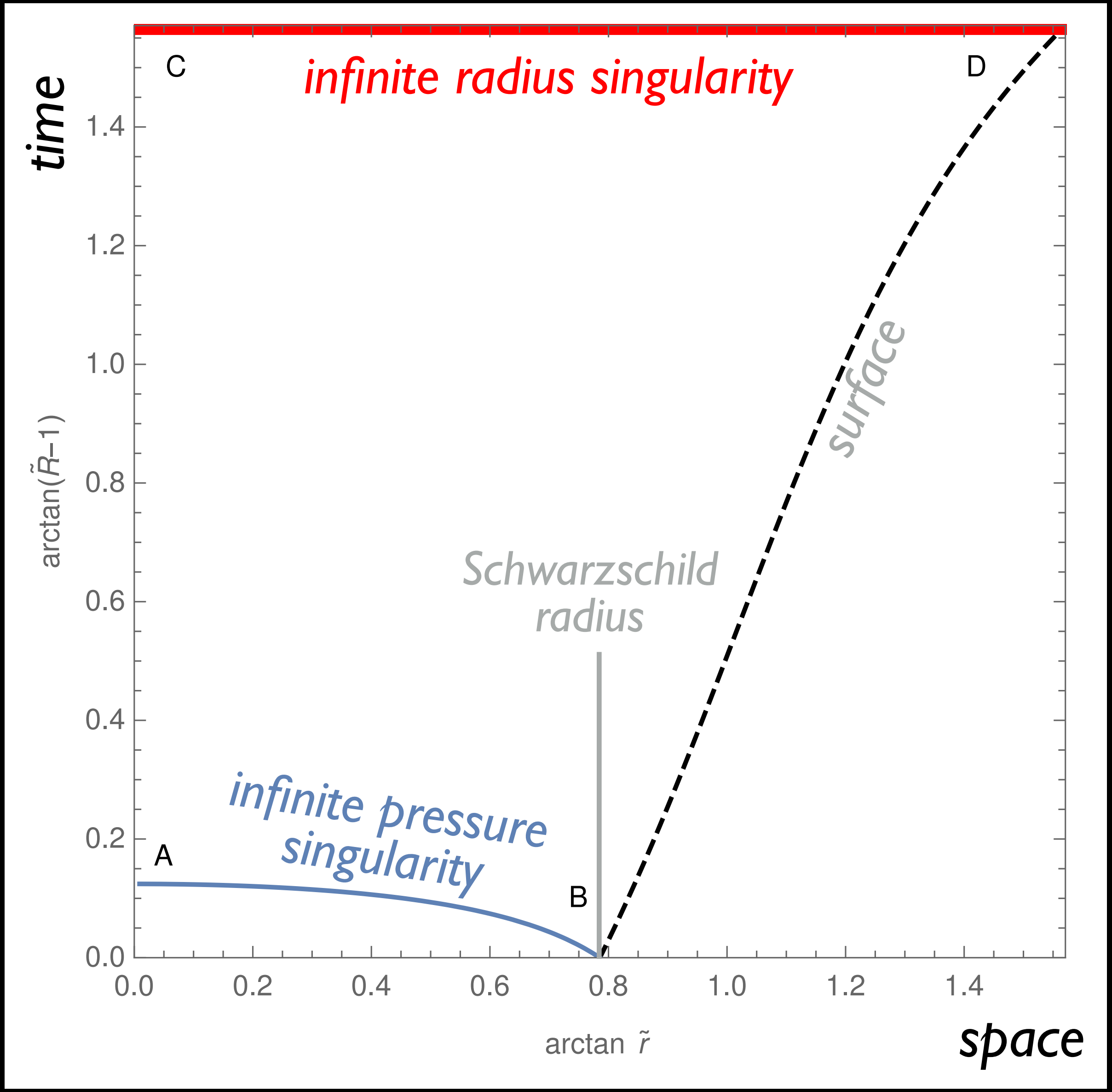


Ends in a gravastar

Formation of a Schwarzschild star

Beltracchi, Gondolo 2019b

Curvature singularities in the spacetime diagram



Conclusions

A dark energy star is a finite region of space with equation of state $p = -\rho$ typical of dark energy.

We found explicit time-dependent semi-analytic solutions of Einstein's equations giving the collapse of a spherical object to a dark energy star.

Our exact solution is interesting because exact time-dependent solutions of Einstein's field equations are rare.

Our "pile-up" solutions have no horizons and no singularities and obey the weak energy condition at all times and positions.