A nAttractor for AdS$_2$ Quantum Gravity

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Ubiquity of AdS$_2$

- ***Extremal*** black holes are important: they have the smallest possible mass for given charges so they are ***ground states***.

- Spherically symmetric asymptotically flat extremal black holes in $D = 4$ have ***horizon geometry AdS$_2$*** $\times S^2$.

- In fact ***all*** extremal black holes include an AdS$_2$ factor (a theorem).

- This motivates interest in ***AdS$_2$ quantum gravity***.

$\mathbf{AdS_2/CFT_1}$ Holography?

- $\text{AdS}_{d+1}/\text{CFT}_d$ correspondence is confusing for $d = 1$.

- No finite energy excitations possible in AdS_2:

  Their ***backreaction spoil asymptotic AdS$_2$*** boundary conditions.

- Also many other (related) unpleasantries.

- So $\text{AdS}_2/\text{CFT}_1$ holography is not yet well understood.

$\text{nAdS}_2/\text{nCFT}_1$ Holography.

- Recently developed version $\text{AdS}_2/\text{CFT}_1$ holography: duality between ***nearly*** AdS_2 geometry and ***nearly*** CFT_1.

- ***Conformal symmetry is broken*** spontaneously and explicitly.

- Interesting nCFT_1's realize the symmetry breaking pattern: ***SYK*** ,....

- Motivation and ***outlook***: develop ***SUSY breaking*** in setting with ***precision and detail***.

This Talk: The Scales

- $\text{nAdS}_2/\text{nCFT}_1$ holography is ***not scale invariant***.

- So: what physical scale(s) appear does the theory depend on?

- Inspiration: the ***extremal*** AdS_2 geometry (including its matter) is determined by an ***Attractor Mechanism***.

- Result: the ***near***extremal AdS_2 geometry (and matter supporting it) is determined by a ***nAttractor Mechanism***.

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A Canonical Setting

- 4D $\mathcal{N} = 2$ ungauged SUGRA with n_V vector multiplets.

- The black hole parameters are:

  Mass M

  Charges (p^I, q_I), $I = 0, ...n_V$

  Asymptotic value of complex scalars z^i_∞, $i = 1, ...n_V$.

- For ***extreme*** black holes the mass M is not independent: it is ***minimal*** (a function of the other parameters).

- Extremal black holes in this setting have been studied extensively.

The Extremal Attractor Mechanism

- A radial ***flow***: the scalars z^i ***evolve*** from infinity to the horizon.

- The ***attractor mechanism***: scalar fields ***at the horizon*** are independent of their "initial" value at infinity.

- So the horizon theory is ***universal***: independent of moduli, including the coupling constants,....

- The attractor mechanism ***determines the attractor values*** for the scalars (as function of black hole charges).

- "Practical" aspect: ***no need to analyze the black hole solutions***.

Preview: nAttractor Mechanism

- We want to determine the scales characterizing the nAdS$_2$ region.

- They will depend on the black hole charges (p^I, q_I) and the moduli z^i_∞.

- A ***nAttractor mechanism***: these scales are computed by a generalization of the ***extremal*** attractor mechanism

- "Practical" aspect: ***no need to analyze nonextremal black hole solutions***.

A Physical Scale: the Specific Heat

- The ***extremal*** black hole entropy is a ground state entropy

$$S_0 = \frac{A}{4G_4} = \frac{1}{4G_2}$$

There is ***no scale, just a large dimensionless number***.

- The ***nearly*** extreme black hole entropy has ***small temperature***:

$$S = S_0 + \frac{1}{2}\pi L T$$

The length L is the ***symmetry breaking scale***.

- It is essentially the ***specific heat*** $C = T\partial_T S$.

Near Extreme Black Holes

The “near” of ***n***AdS$_2/$***n***CFT_1 appears in ***two ways***:

1. Black holes only ***nearly*** extremal. So scalars at the horizon depart from their extremal attractor value.

2. Also: nAdS$_2$/nCFT$_1$ considers the entire ***near horizon region***. So the scalars are ***not constant***.

We consider these two challenges in turn.

Non-Extreme Black Holes

- General ***non-extreme*** black holes depends on a single ***radial function*** $R(r)$:

$$ds_4^2 = -\frac{r^2 - r_0^2}{R^2(r)}dt^2 + \frac{R^2(r)}{r^2 - r_0^2}dr^2 + R^2(r)d\Omega_2^2$$

- There is an event horizon at $r = r_0$.

- Entropy and temperature are encoded in the radial function:

$$S = \frac{\pi R^2(r_0)}{G_4} \; .$$

$$T = \frac{r_0}{2\pi R^2(r_0)} \; .$$

- The extremal limit is $r_0 \to 0$ ***with charges and moduli fixed***.

Near-Extreme Black Holes

- The near extreme entropy ***depends on M***:

$$\Delta S = \frac{\partial S}{\partial M}\Delta M$$

Estimates: $\Delta S \sim T$ but $\partial_M S \sim T^{-1}$ (1st law) so $\Delta M \sim T^2$.

- The radial function $R(r)$ depends on r and ***also on M***.

$$\Delta S = \frac{\pi}{G_4}\left(\frac{\partial R^2}{\partial M}\Delta M + \frac{\partial R^2}{\partial r}\Delta r\right)$$

Estimates: $\Delta S \sim T$ from $\Delta r \sim r_0 \sim T$. $\partial_M R^2$ is ***subleading***.

- ΔS follows from R^2 ***at extremality*** but at ***a new position*** $r = r_0$.

- This is a ***major simplification***.

The Symmetry Breaking Scale

- The symmetry breaking scale only depends on ***moving away from the horizon*** (but not on the solution being non-extreme):

$$L = \frac{2}{\pi}\frac{\Delta S}{T} = \frac{2\pi}{G_4}\frac{\partial R^4}{\partial r}\bigg|_{\rm hor} .$$

- Moreover, the dependence is extremely simple: just a radial derivative.

- Highlight: the function $R(r)$ is that of the ***extremal*** black hole.

The Extremal Attractor

- For fixed charges, the $F_{\mu\nu}F^{\mu\nu}$-type terms in the Lagrangian subject the scalars z^i to an ***effective potential*** V.

- The scalars z^i are ***constant*** on the AdS$_2 \times S^2$ attractor geometry.

- So the ***effective potential*** V ***is extremized***: $\partial_i V = 0$.

- The extremum value of the potential gives: $R^2(0) = G_4 V_{\text{ext}}$.

- This procedure is identical to the ***entropy function formalism***.

Results of Extremization

- Notation for the resulting radial function on $\mathrm{AdS}_2 \times S^2$:

$$R^4(0) = I_4(P^I, Q_I)$$

- The ***generating*** function I_4 is ***quartic*** in the charges.

- Example ($\mathcal{N} = 4$ SUGRA): $I_4(p^I, q_I) = \vec{p}^2 \vec{q}^2 - (\vec{p}\vec{q})^2$.

- The ***scalar*** values at the horizon are ***also encoded in*** I_4:

$$\begin{pmatrix} X^I_{\mathrm{hor}} \\ F^{\mathrm{hor}}_I \end{pmatrix} = \begin{pmatrix} p^I \\ q_I \end{pmatrix} - i \begin{pmatrix} -\partial_{q_I} \\ \partial_{p^I} \end{pmatrix} I_4^{1/2}(p^I, q_I)$$

  Symplectic section (X^I, F_I) represents scalars ***projectively***: $z^i = X^i / X^0$.

Moving Away from the Horizon

- The radial function ***at the horizon*** depends only on charges.

- It depends on ***scalars at infinity*** away from the horizon.

- Parametrize scalars at infinity through “charges” p^I_∞, q^∞_I:

$$\begin{pmatrix} X^I_\infty \\ F^\infty_I \end{pmatrix} = \begin{pmatrix} p^I_\infty \\ q^\infty_I \end{pmatrix} - i \begin{pmatrix} -\partial_{q^\infty_I} \\ \partial_{p^I_\infty} \end{pmatrix} I_4^{1/2}(p^I_\infty, q^\infty_I)$$

- So: parametrize scalars ***at infinity*** using the charge/scalar relation determined ***at the horizon***.

- The ***full attractor flow*** has the radial function

$$R^4(r) = I_4(P^I + r p^I_\infty, Q_I + r q^\infty_I)$$

The Symmetry Breaking Scale

- The ***radial*** derivative of R^4 gives the symmetry breaking scale.

- It is equivalent to ***a derivative in charge space***

$$L = \frac{2\pi}{G_4}\left(p^I_\infty \frac{\partial}{\partial P^I} + q_I^\infty \frac{\partial}{\partial Q_I} \right) I_4(P^I, Q_I) \ .$$

- So the ***nAttractor behavior*** follows from ***attractor geometry***.

- I_4 is quartic in the charges; L is ***cubic in charges*** and linear in moduli.

- The derivative replaces a charge by its corresponding modulus.

Explicit Example: The STU Model

- The "four-charge" solution has one electric charge q_0 and three magnetic ones p^1, p^2, p^3.

- The effective potential

$$V = \frac{1}{8y^1y^2y^3}\left(q_0^2 + (p^1y^2y^3)^2 + (p^2y^3y^1)^2 + (p^3y^1y^2)^2\right) \ .$$

  The y^i (with $i = 1, 2, 3$) are scalar fields.

- The ***extremal*** attractor gives scalar fields y^i ***at the horizon*** as

$$y^i_{\mathrm{hor}} = \sqrt{\frac{q_0}{p^1p^2p^3}}\, p^i$$

  independently of their asymptotic values.

- The extremal entropy

$$S = 4\pi V_{\mathrm{hor}} = 2\pi\sqrt{q_0p^1p^2p^3}$$

A nAttractor Mechanism

- Present moduli ***at infinity*** as "charges" by inverting

$$y^i_\infty = \sqrt{\frac{q_0^\infty}{p^1_\infty p^2_\infty p^3_\infty}}\, p^i_\infty$$

- The ***symmetry breaking scale***/specific heat:

$$L = \frac{2\pi}{G_4}\left(p^i_\infty \frac{\partial}{\partial P^i} + q_0^\infty \frac{\partial}{\partial Q_0} \right) I_4$$

$$= 2\pi q_0 p^1 p^2 p^3 R_{11} \left(\frac{1}{q_0} + \frac{1}{p^1 y^2_\infty y^3_\infty} + \frac{1}{p^2 y^3_\infty y^1_\infty} + \frac{1}{p^3 y^1_\infty y^2_\infty} \right)$$

- It ***depends on moduli at infinity***: R_{11}, $y^{1,2,3}_\infty$.

- It depends on ***non-trivial combinations of charges***.

The Long String Scale

- In the ***dilute gas regime*** the electric charge is ***small*** compared to magnetic background charges.

- Then the symmetry breaking scale is

$$L = 2\pi p^1 p^2 p^3 R_{11}$$

- This is the ***long string scale*** known from microscopic black hole models.

- Physics: low energy excitations much lighter L^{-1} than naïve geometrical estimate R_{11}^{-1}.

A Flow of Many Fields

- "The" breaking scale is (essentially) the radial derivative of R^2.
- Other scalar fields ***approach*** their fixed value $z^i_{\rm hor}$ at the horizon.
- Their radial derivatives from differentiation in charge space:

$$\frac{dz^i}{dr} = \left(p^I_\infty \frac{\partial}{\partial P^I} + q_I^\infty \frac{\partial}{\partial Q_I}\right) z^i_{\rm hor} \equiv \frac{L_i}{R^2} z^i_{\rm hor}$$

- In general ***each scalar field introduces a scale***.
- STU example: the four apparent terms are independent scales.

Effective Boundary Theory

- So far: symmetry breaking scales of the geometry and thermodynamics ***from the UV theory***.

- Now: the symmetry breaking scale in the ***effective IR action***:

$$-\frac{1}{4}L\int_{\partial D} du\left(-\frac{1}{2}(\frac{\tau''}{\tau'})^2+(\frac{\tau''}{\tau'})'\right)$$

- How does this action ***emerge from the UV theory***?

2D SUGRA Action

- Start from $\mathcal{N} = 2$ SUGRA in 4D

- Dimensional reduction on $\mathcal{M}_2 \times S^2$ gives

$$4G_4\mathcal{L}_2 = R^2\mathcal{R}^{(2)} + 2 + 2(\nabla R)^2 - 2R^2 g_{i\bar{j}}\nabla_\mu z^i \nabla^\mu \bar{z}^{\bar{j}} - \frac{2V}{R^2}$$

- The effective potential V depends on electric and magnetic charges (p^I, q_I) and moduli z^i.

- $\mathcal{M}_2$=AdS$_2$ with $R^2 = V = \ell_2^2$ is solution for constant scalars extremizing the potential $\partial_i V = 0$.

- We want to compute the ***on-shell action of our solution***.

Black Hole Geometry

- The ***complete (Euclidean) geometry*** with $r^2 = r_0^2 \cosh\frac{\rho}{\ell_2}$:

$$ds^2 = \frac{\ell_2^2 \sinh^2\frac{\rho}{\ell_2}}{R^2} d\tau^2 + \frac{R^2}{\ell_2^2} d\rho^2 + R^2 d\Omega_2^2 \, .$$

- For R^2 constant with $R = \ell_2$: geometry is AdS_2 with radius ℓ_2.

- Here: R^2 given by complete solution, ***including flow away from the horizon***.

- Near extreme black holes: there is a near horizon region where:

$$R^2 \sim R_0^2 + (r - r_0)\partial_r R^2 + \ldots$$

with R^2 ***evaluated for the extreme geometry***.

The Boundary

- Introduce a ***boundary at*** $\rho \sim \rho_c$ with ρ_c so $\frac{r_0}{\ell_2} \ll \frac{r_0}{\ell_2} \sinh \frac{\rho_c}{\ell_2} \ll 1$.

- Allow ***general boundary curve*** $(\tau(u), \rho(u))$ but special interest in ***thermal boundary*** $\tau = \frac{R^2(r_0)}{r_0} u$ with $u \in [0, 2\pi]$.

- Extrinsic curvature of boundary curve:

$$\mathcal{K} = \frac{1}{\ell_2} \coth \frac{\rho}{\ell_2} - \frac{r_0}{\ell_2^2} \sinh \frac{\rho}{\ell_2} \partial_r R + \frac{\ell_2}{(\tau')^2 \sinh^2 \frac{\rho}{\ell_2}} \left(\frac{\tau'''}{\tau'} - \frac{3}{2} (\frac{\tau''}{\tau'})^2 \right)$$

- The first two terms are large because $\rho_c \gg \ell_2$.

The Gauss-Bonnet Theorem

- The key terms in the on-shell action

$$\int_M R^2\mathcal{R} + 2\int_{\partial M} R^2\mathcal{K} = 4\pi\chi R^2(r_0) + \ldots$$

- $\chi = 1$ for a topological disc.

- Exact result (omit “dots”) applies when ***explicit*** R^2 constant but the R^2 ***in the geometry*** general.

- All terms in the “dots” are proportional to the ***derivative*** $\partial_r R^2$ of the ***explicit*** R^2.

Effective IR Action from the UV

- With the exception of the Gauss-Bonnet term, ***all terms are proportional to $\partial_r R^2$***:

$$\log Z = -I = \frac{\pi R_0^2}{G_4} + \frac{r_0 \partial_r R^2}{2G_4}\left[2\pi + \int du \left(\frac{\tau'''}{\tau'} - \frac{3}{2}(\frac{\tau''}{\tau'})^2\right)\right]$$

- The scale Schwarzian and Gauss-Bonnet terms are the same.

- Restoring the ***nAttractor scale L***:

$$\log Z = -I = S_0 + \frac{1}{4} LT \left[2\pi + \int du \left(\frac{\tau'''}{\tau'} - \frac{3}{2}(\frac{\tau''}{\tau'})^2\right)\right]$$

- No contribution to the black hole from the Schwarzian: $\tau \sim u$.

Matching in Effective QFT

- UV: boundary curve just ***outside the AdS$_2$ region*** gives Schwarzian with determined coefficient.

- IR: boundary curve just ***inside the AdS$_2$ region*** gives Schwarzian form but undetermined coefficient.

- Effective QFT: UV and IR computations must give same form of the action.

- The effective parameters in the IR theory are determined by ***matching***.

Summary and Outlook

- $nAdS_2/nCFT_1$ holography describes the ***near*** horizon region of ***nearly*** extreme black holes.

- The ***near*** extremality is unimportant: ***near*** horizon aspect is a radial derivative.

- A ***nAttractor*** mechanism computes ***near*** extreme heat capacity and ***near*** horizon scalars in terms of the ***extreme*** attractor.

- Generalizations: $D > 4$, gauged SUGRA, 4D nonBPS branch, rotation, ... (in progress, with Hong and Liu).

- Does SUSY nonrenormalization protect scales and other physics?