

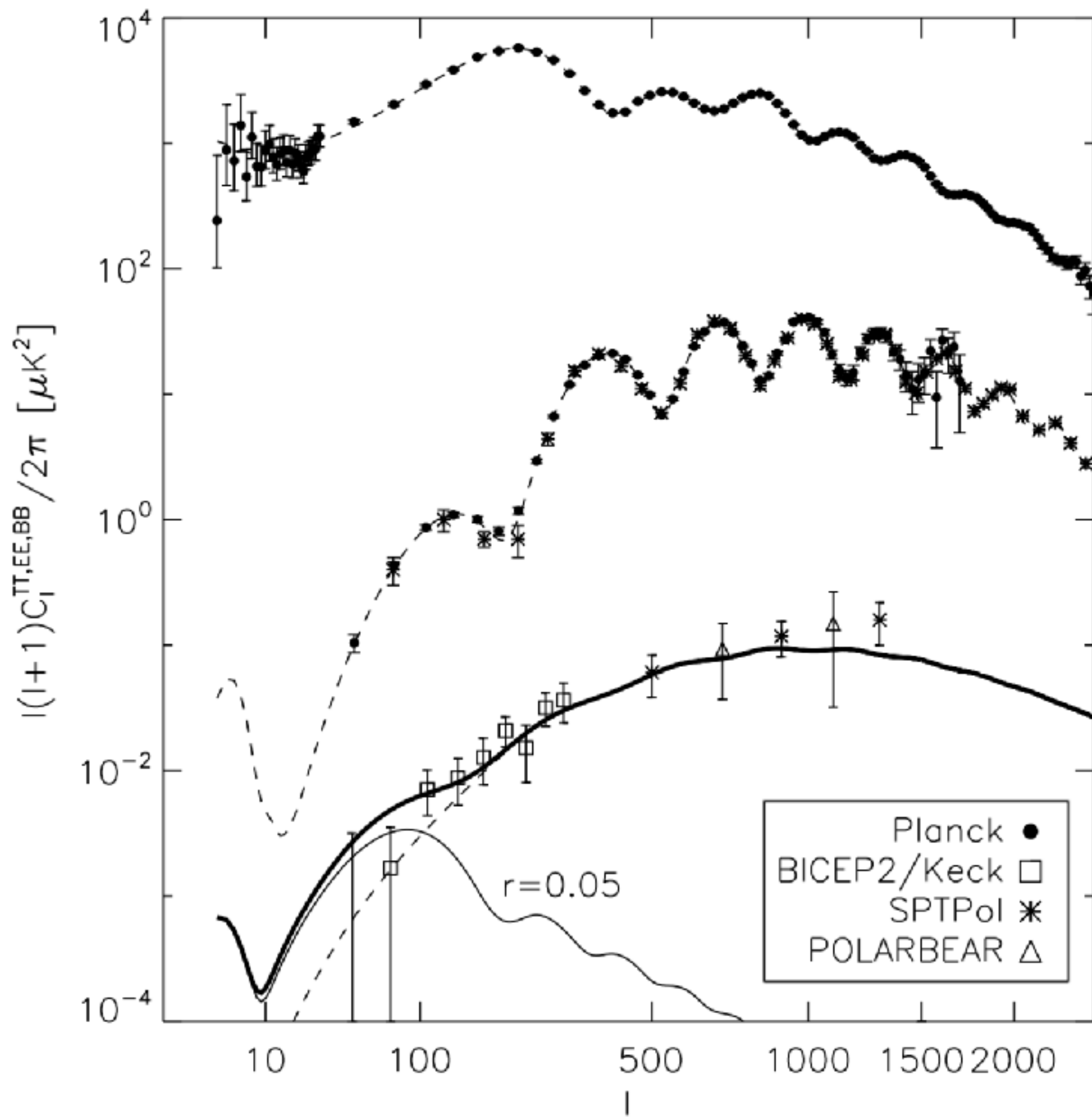
# **Non-Gaussian gravitational waves from inflation**

Eiichiro Komatsu

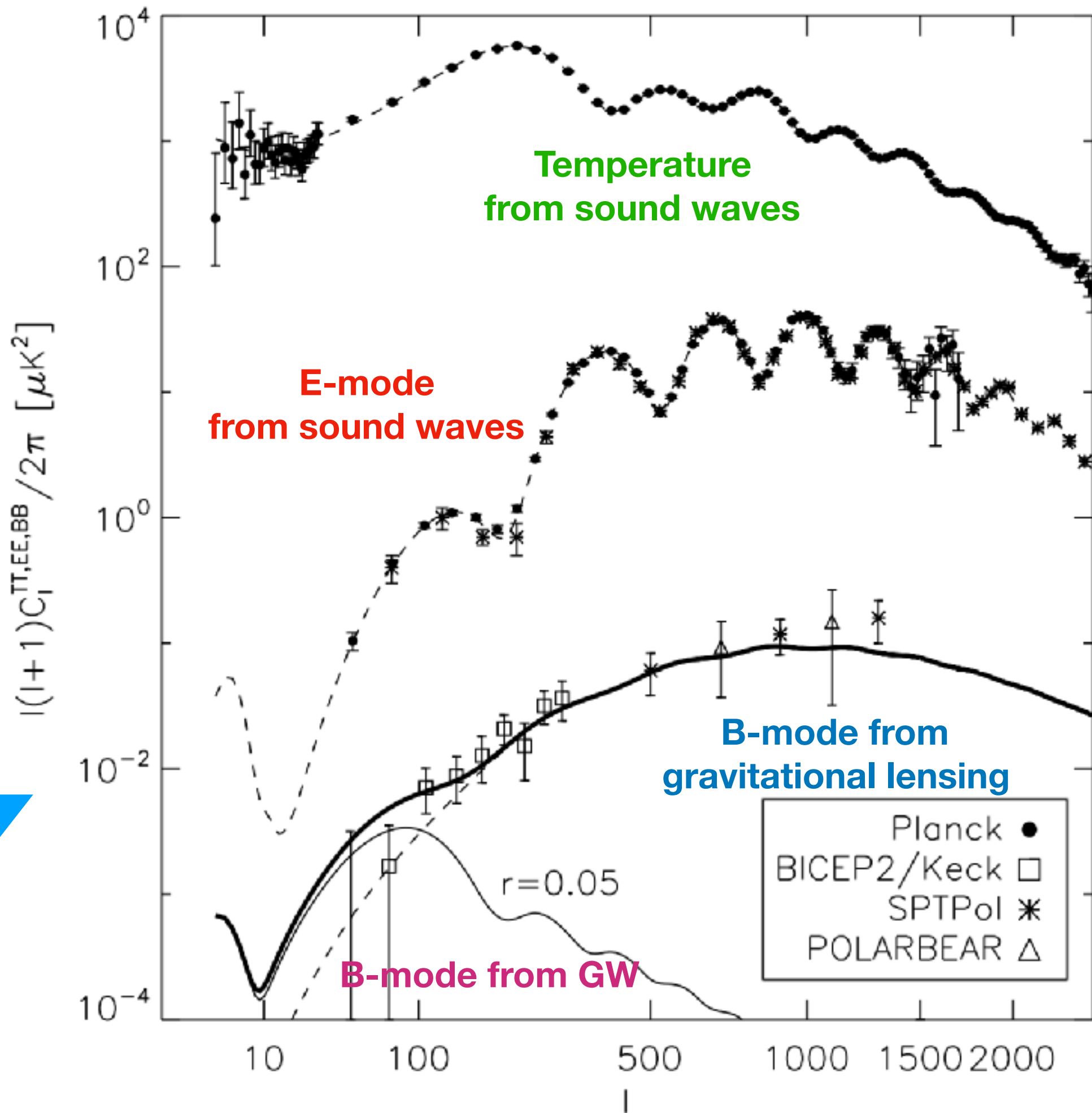
[Max Planck Institute for Astrophysics / Kavli IPMU]

APEC Seminar, Kavli IPMU

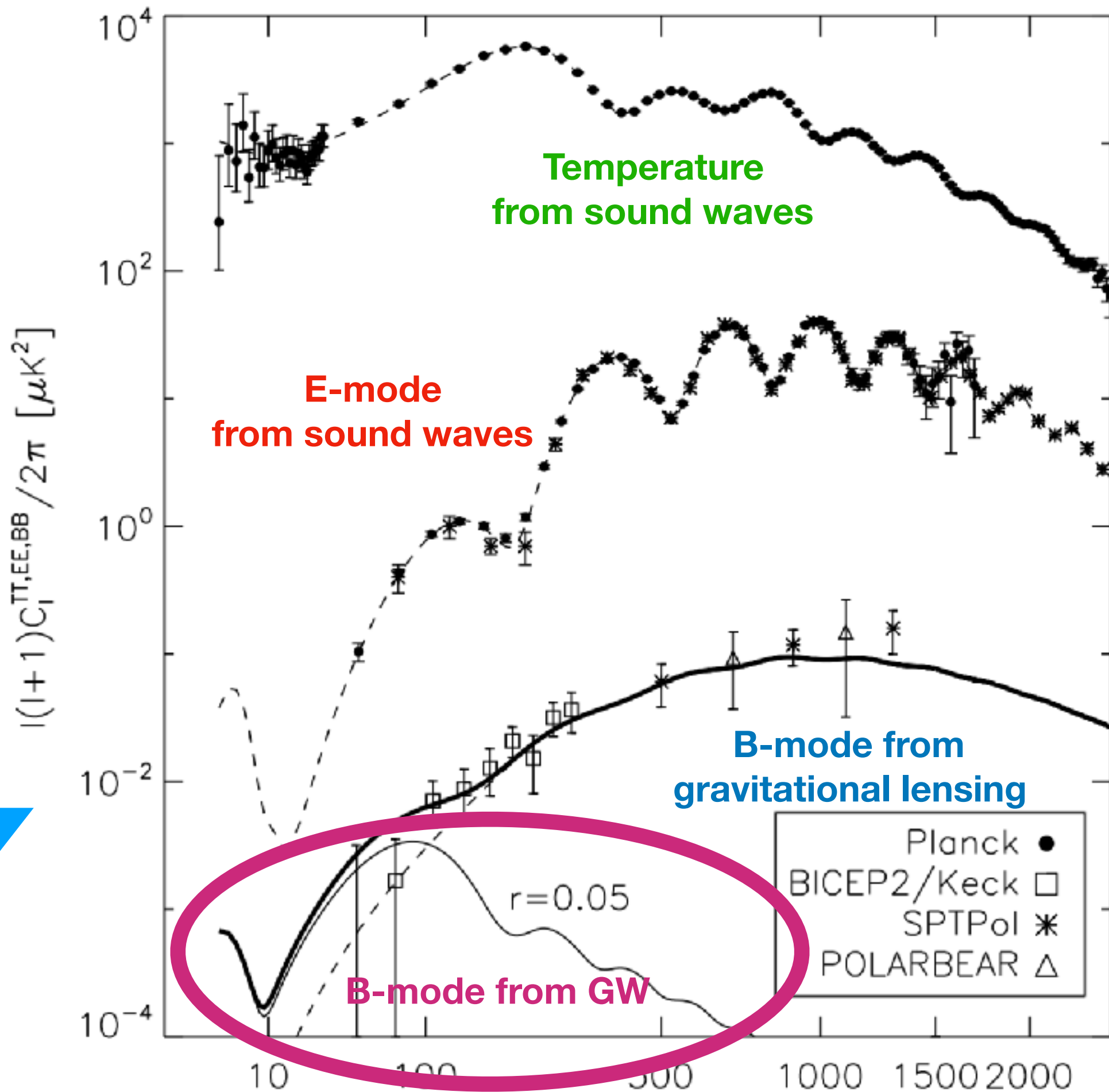
April 23, 2019



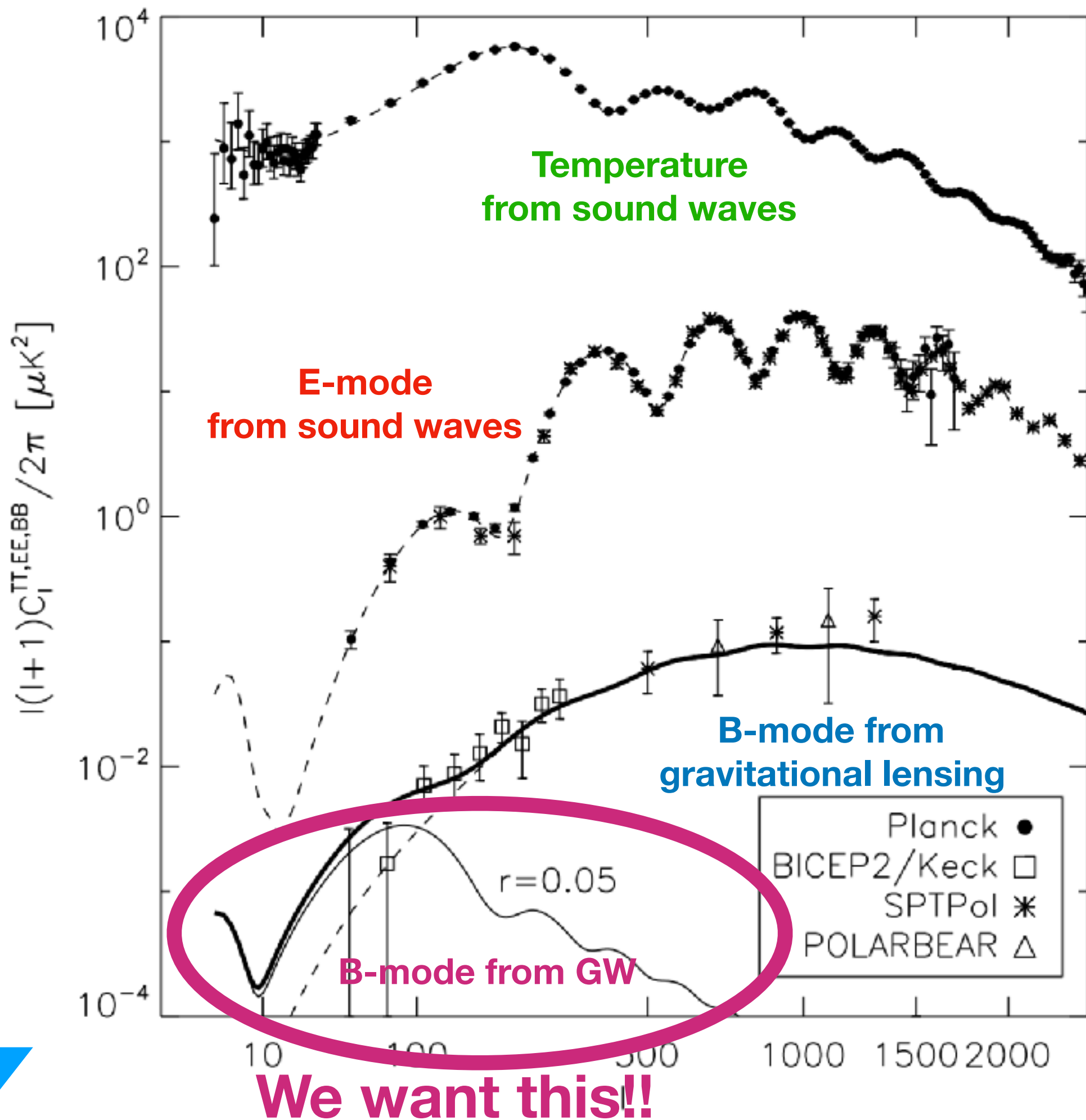
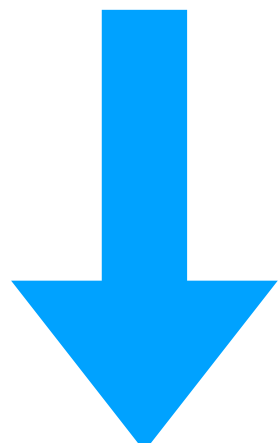
Seven orders of magnitude in power  
in “just” 25 years



Seven orders of magnitude in power  
in “just” 25 years



Another two orders of magnitude  
in the next 10–15 years

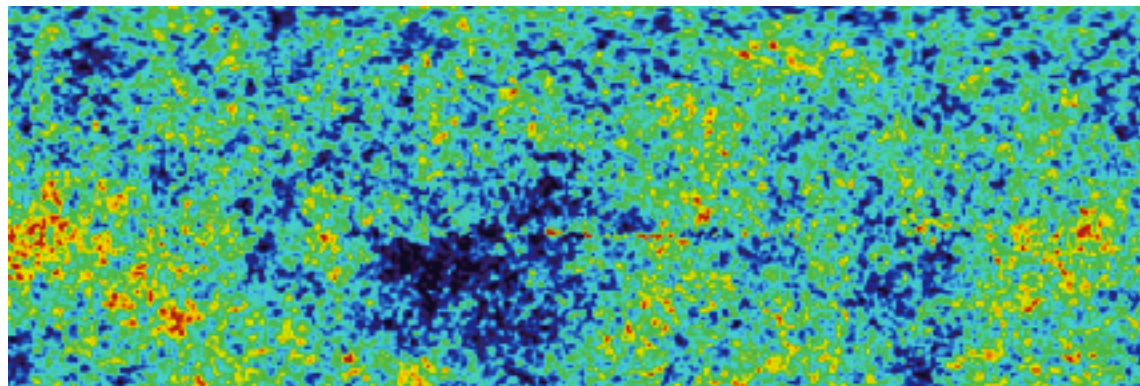
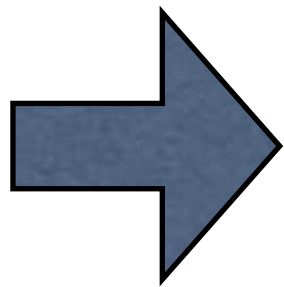


# Key Predictions

 $\zeta$ 

scalar  
mode

- Fluctuations we observe today in CMB and the matter distribution originate from quantum fluctuations during inflation

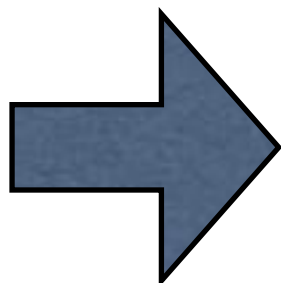


*Mukhanov&Chibisov (1981)*  
*Guth & Pi (1982)*  
*Hawking (1982)*  
*Starobinsky (1982)*  
*Bardeen, Steinhardt&Turner (1983)*

 $h_{ij}$ 

tensor  
mode

- There should also be *ultra long-wavelength* gravitational waves generated during inflation



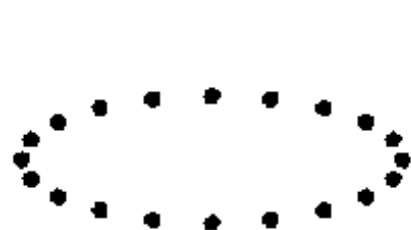
*Grishchuk (1974)*  
*Starobinsky (1979)*

# We measure distortions in space

- A distance between two points in space

$$d\ell^2 = a^2(t)[1 + 2\zeta(\mathbf{x}, t)][\delta_{ij} + h_{ij}(\mathbf{x}, t)]dx^i dx^j$$

- $\zeta$  : “curvature perturbation” (scalar mode)
  - Perturbation to the determinant of the spatial metric
- $h_{ij}$  : “gravitational waves” (tensor mode)
  - Perturbation that does not alter the determinant

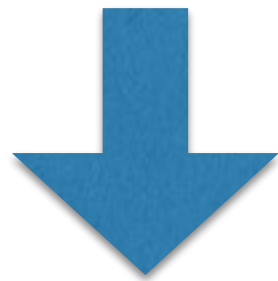


$$\sum_i h_{ii} = 0$$

# Measuring GW

- GW changes distances between two points

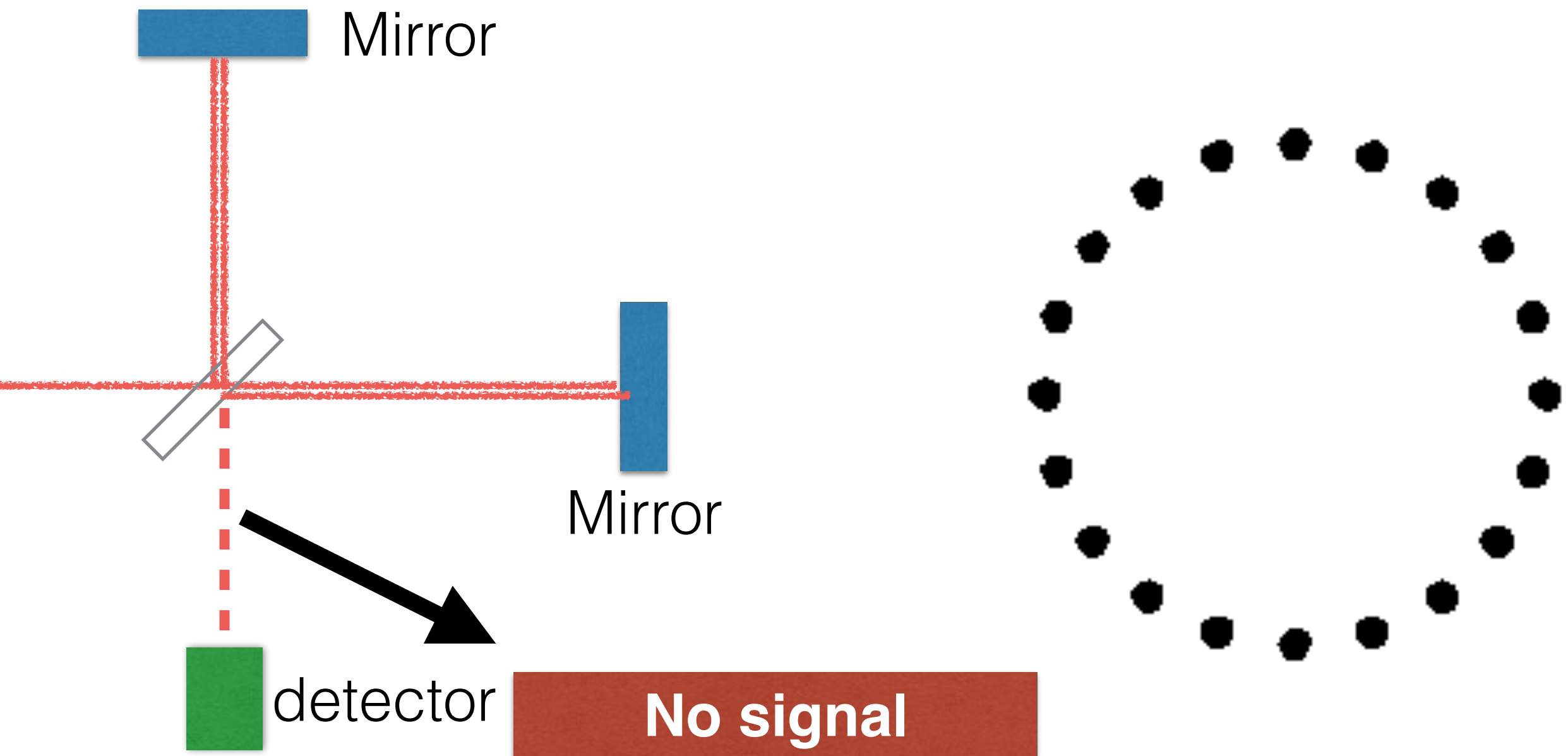
$$d\ell^2 = d\mathbf{x}^2 = \sum_{ij} \delta_{ij} dx^i dx^j$$



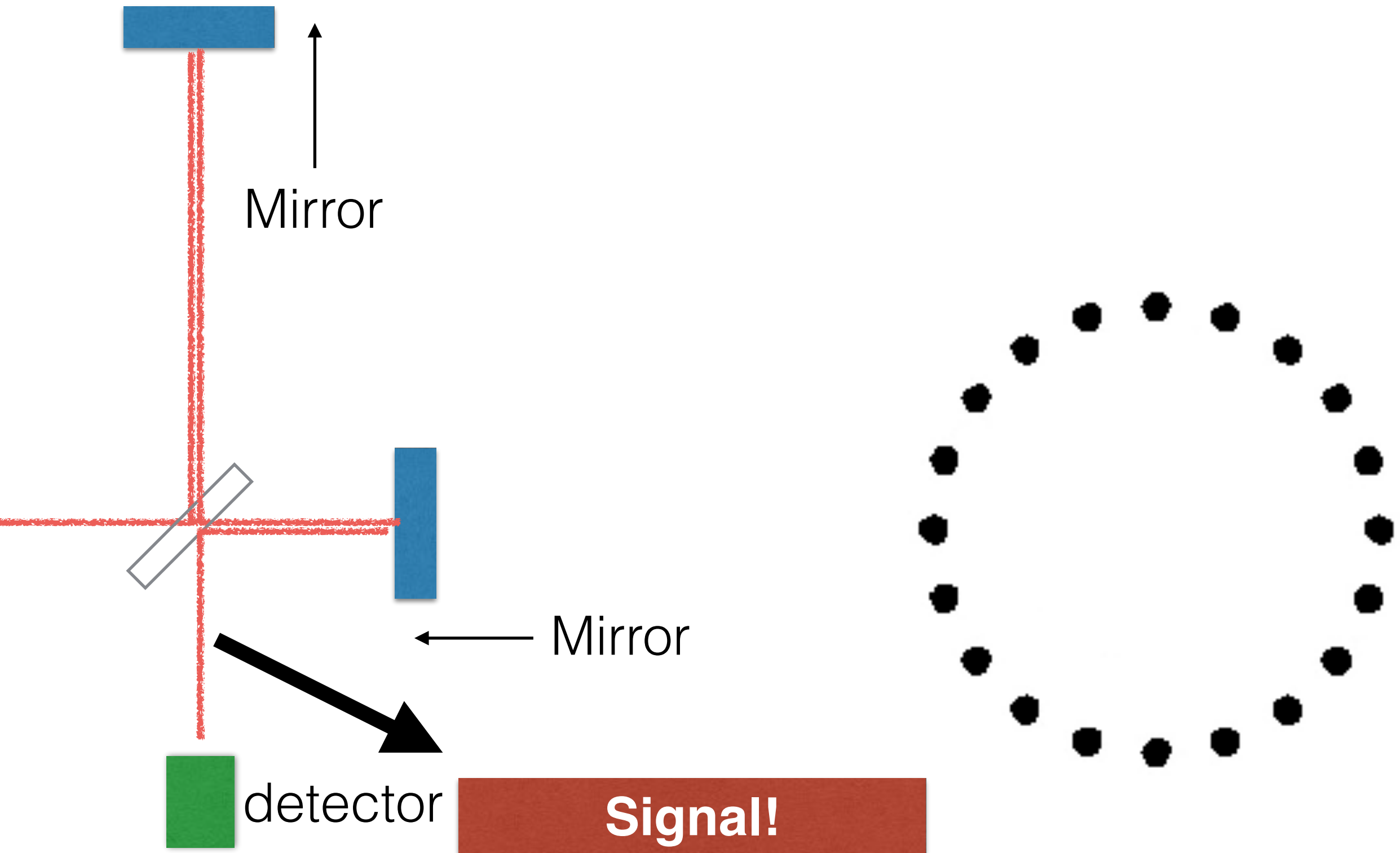
$$d\ell^2 = \sum_{ij} (\delta_{ij} + \textcolor{red}{h_{ij}}) dx^i dx^j$$



# Laser Interferometer



# Laser Interferometer



LIGO detected GW from a binary blackholes, with the wavelength of thousands of kilometres

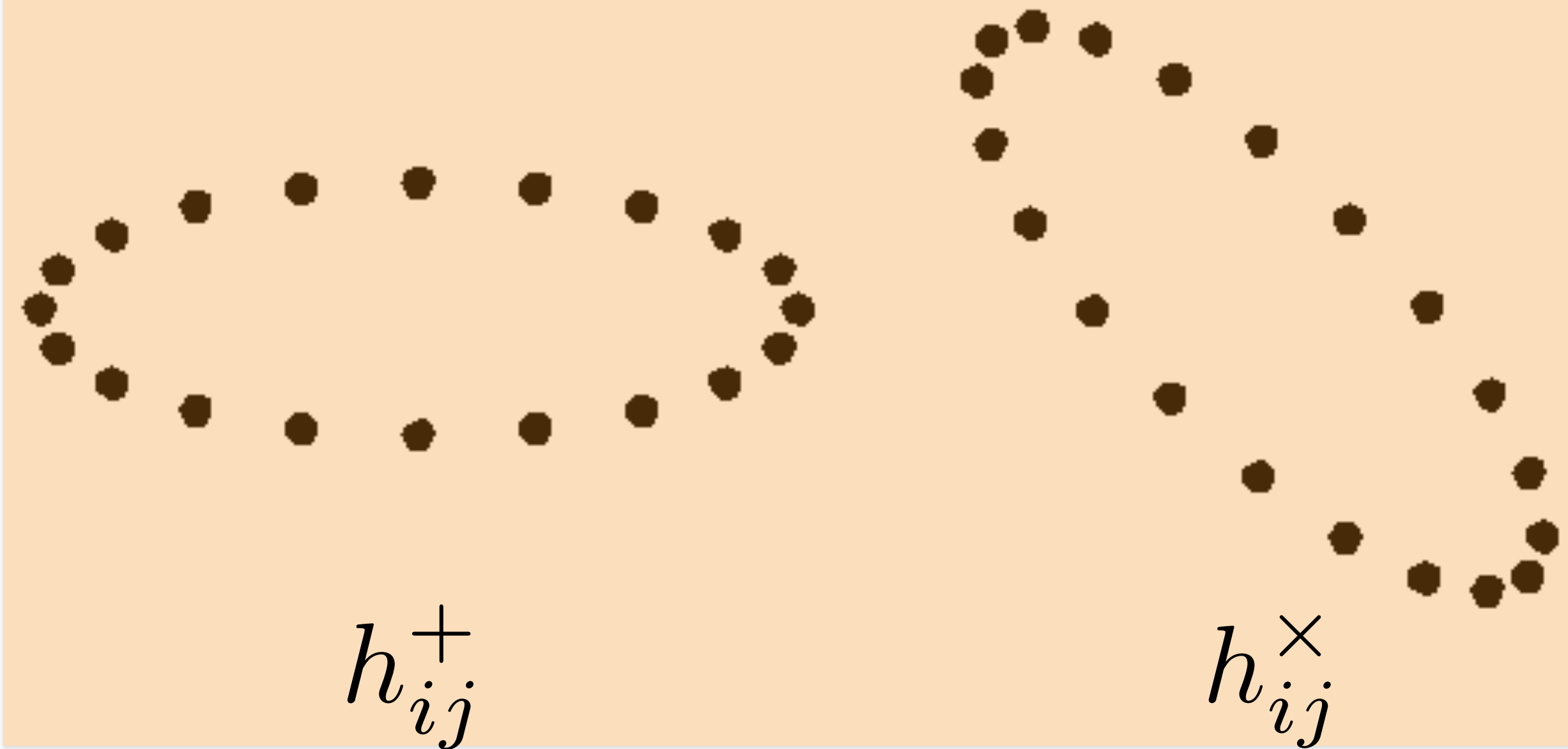
But, the primordial GW affecting the CMB has a wavelength of **billions of light-years!!** How do we find it?

# Detecting GW by CMB

Isotropic electro-magnetic fields

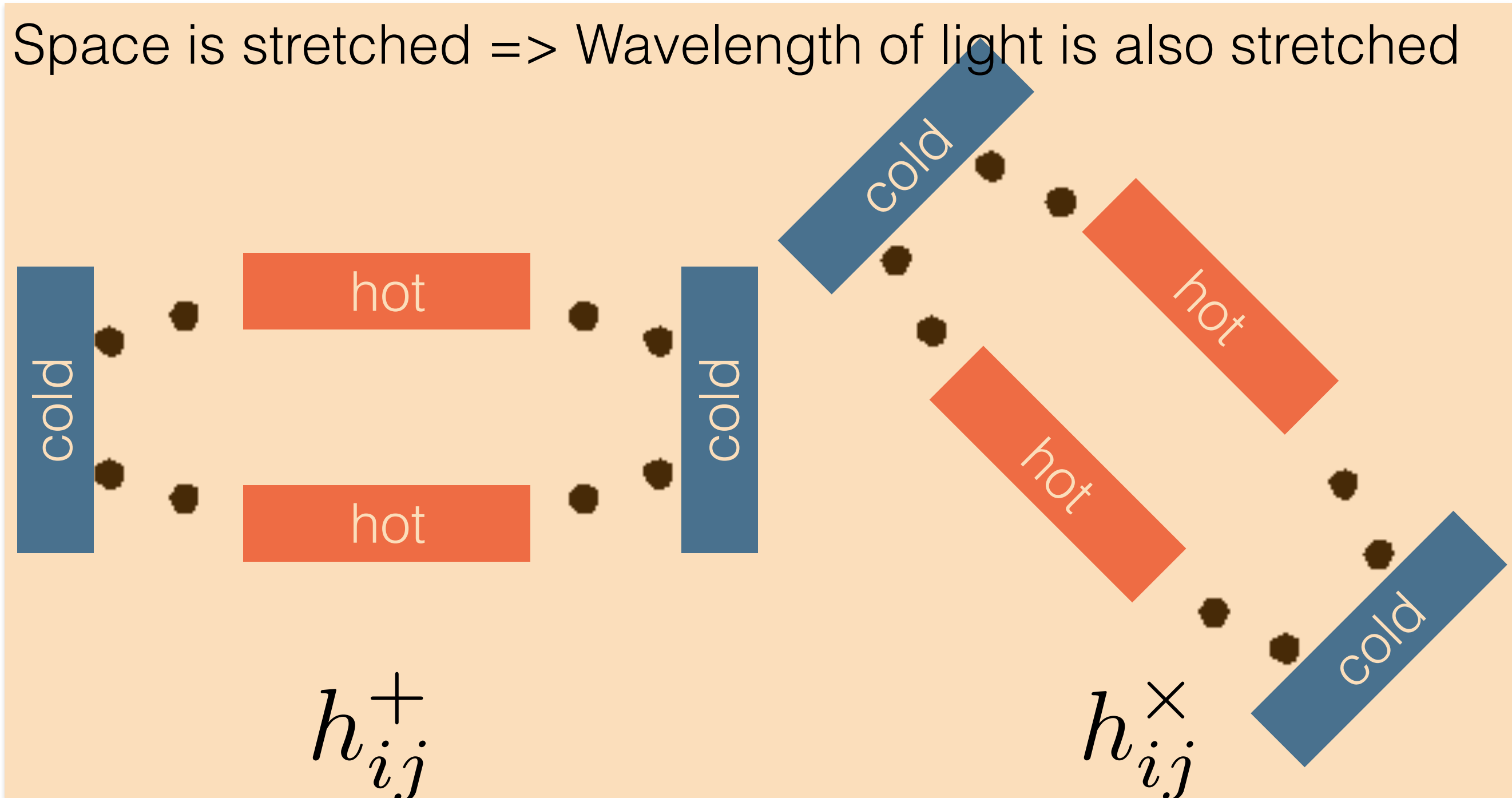
# Detecting GW by CMB

GW propagating in isotropic electro-magnetic fields



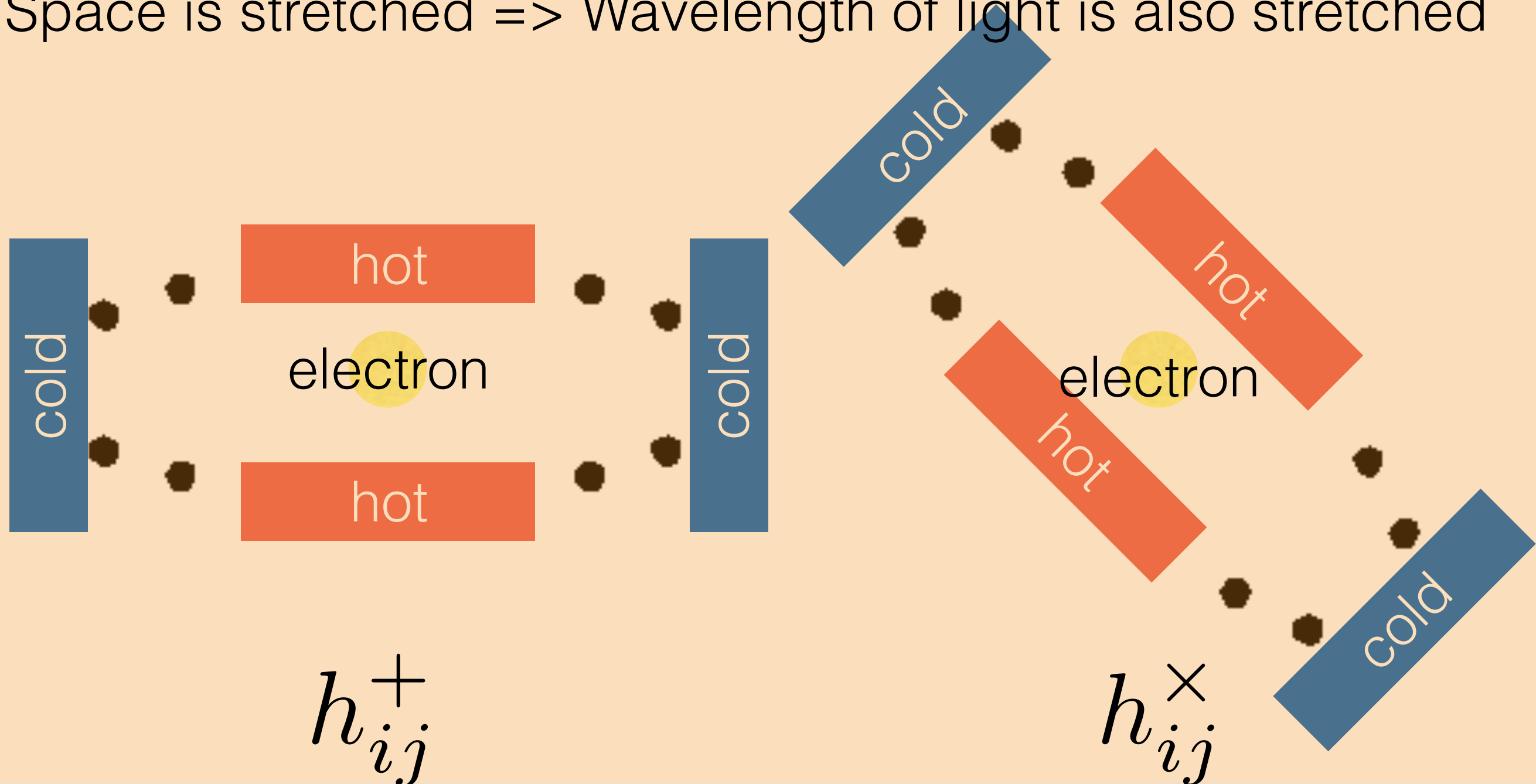
# Detecting GW by CMB

Space is stretched => Wavelength of light is also stretched



# Detecting GW by CMB **Polarisation**

Space is stretched => Wavelength of light is also stretched



# Detecting GW by CMB

## Polarisation

Space is stretched => Wavelength of light is also stretched

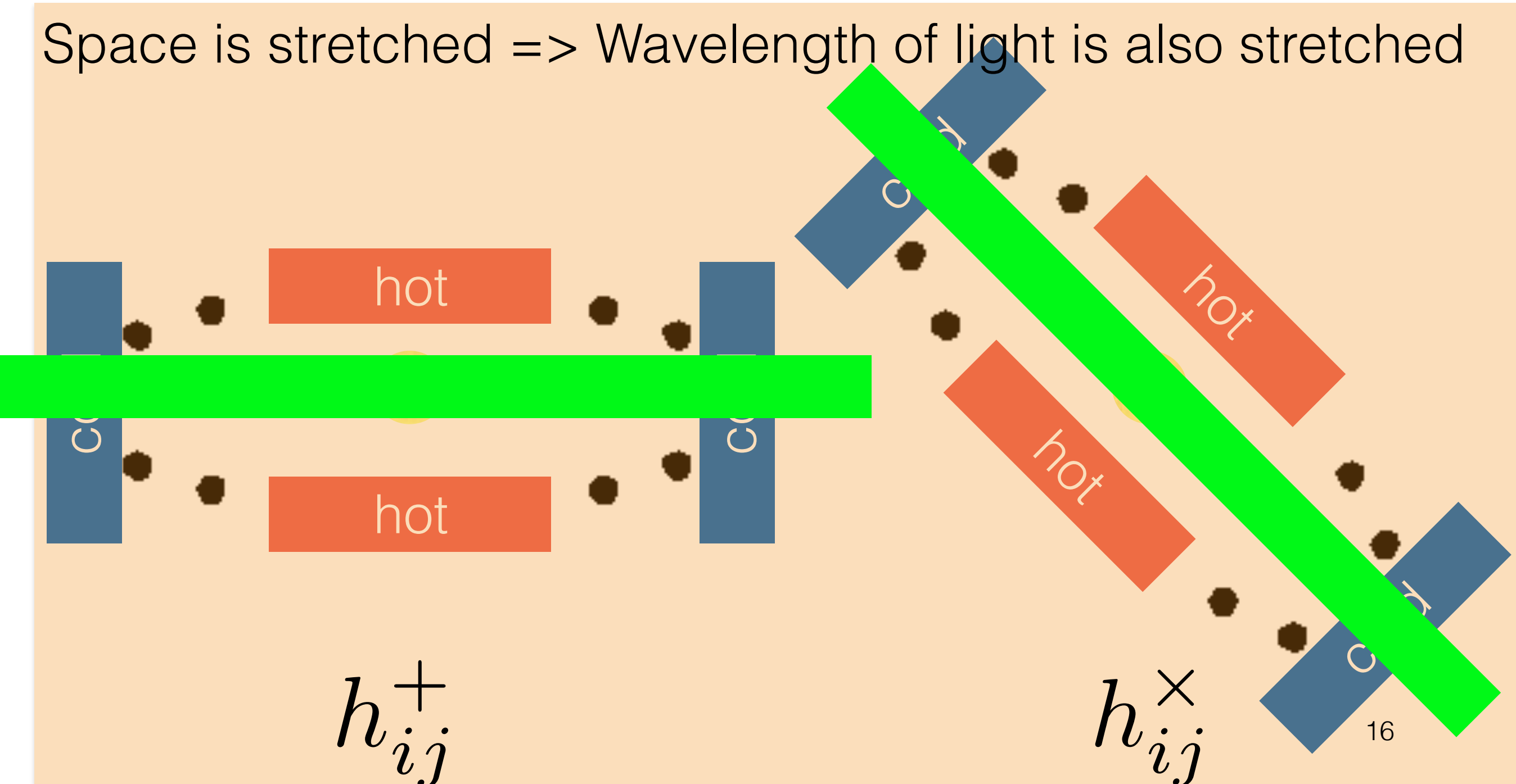


Photo Credit: TALEX



horizontally polarised

Photo Credit: TALEX



# Gravitational waves as the quantum vacuum fluctuation in spacetime

- Quantising the gravitational waves in de Sitter space in **vacuum**

$$\square h_{ij} = 0$$

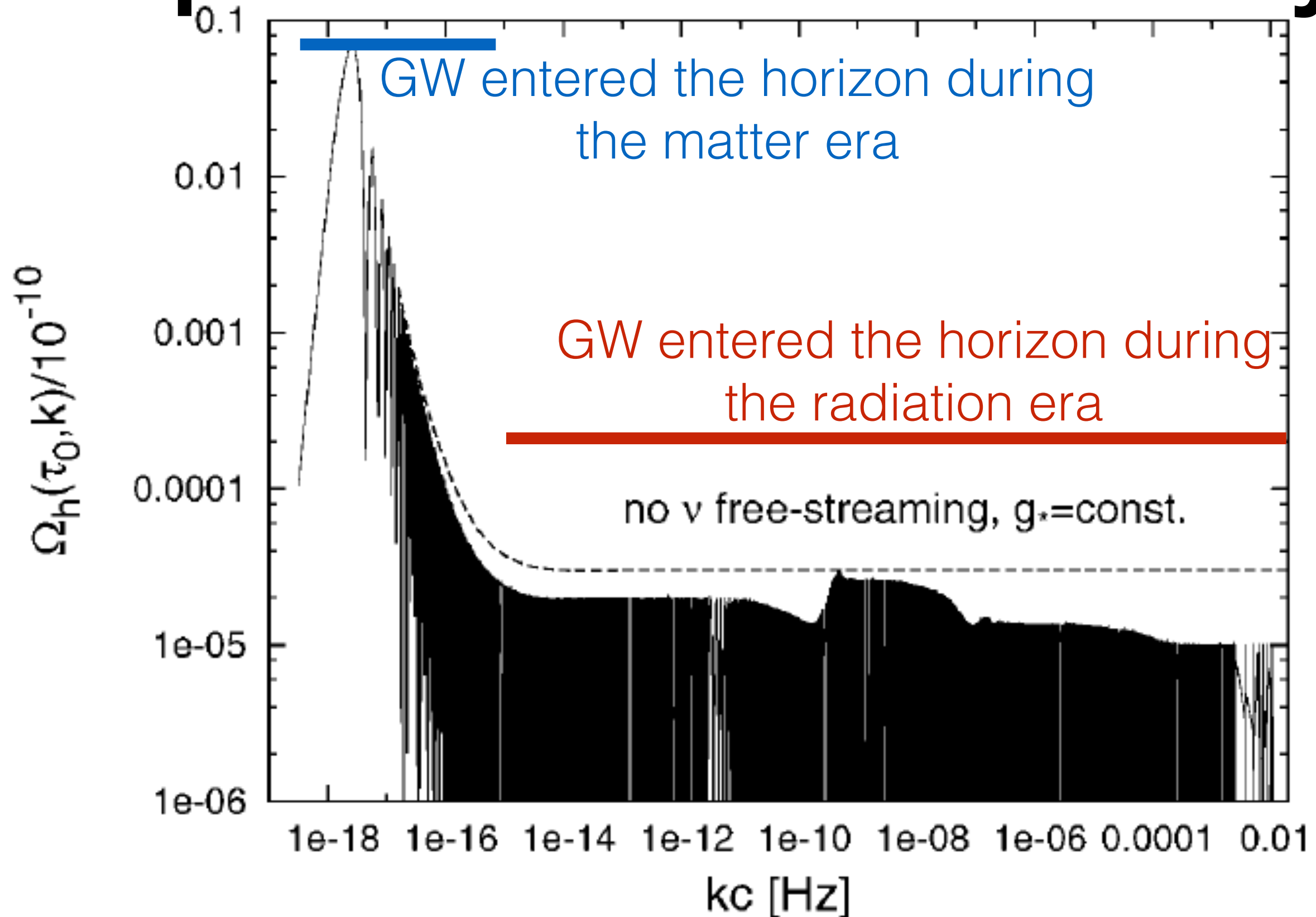
gives

$$k^3 \langle h_{ij}(\mathbf{k}) h^{ij*}(\mathbf{k}') \rangle$$

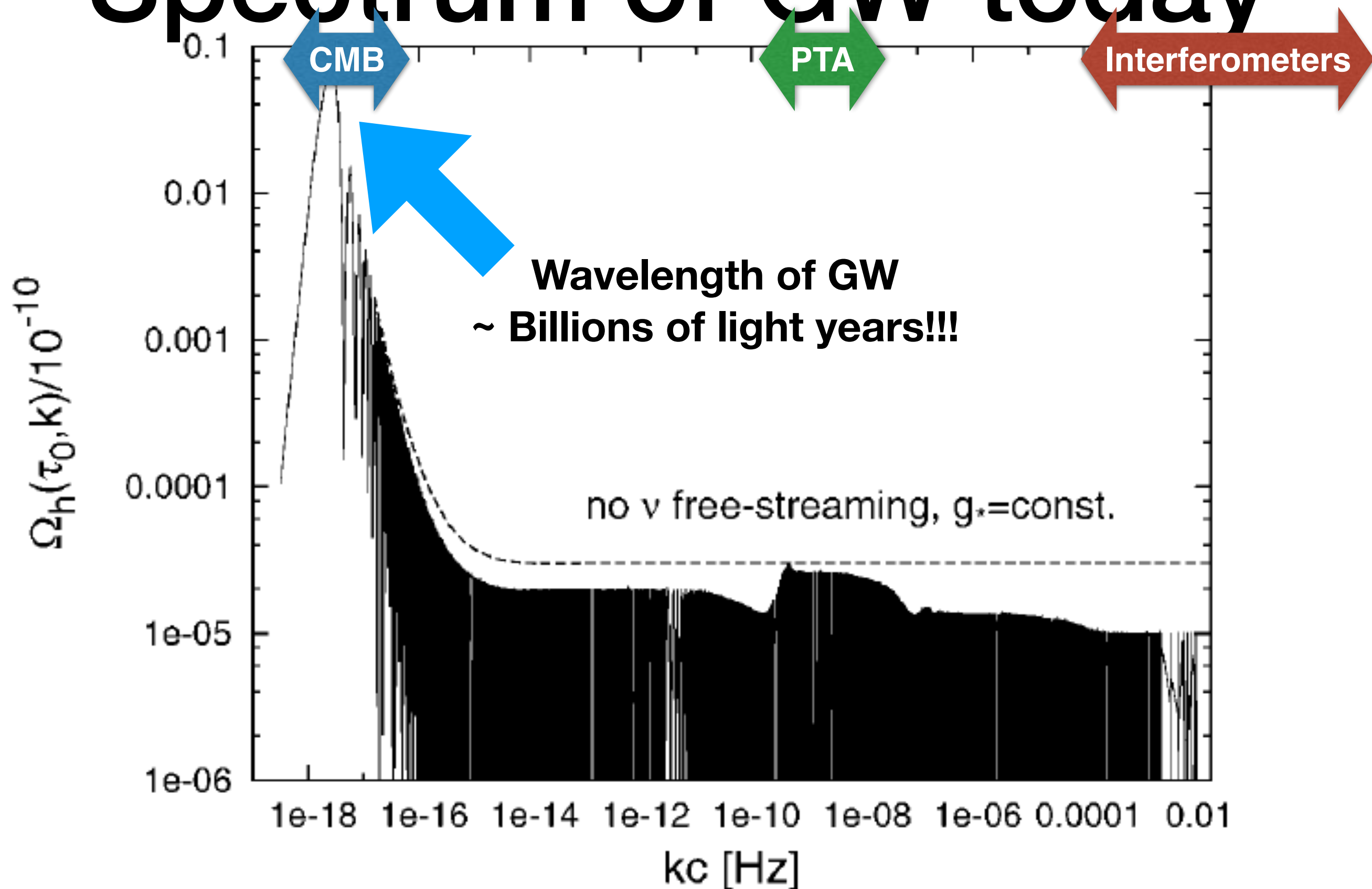
$$= (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') \frac{8}{M_{\text{pl}}^2} \left( \frac{H}{2\pi} \right)^2$$

scale-invariant spectrum

# Theoretical energy density Spectrum of GW today



# Theoretical energy density Spectrum of GW today

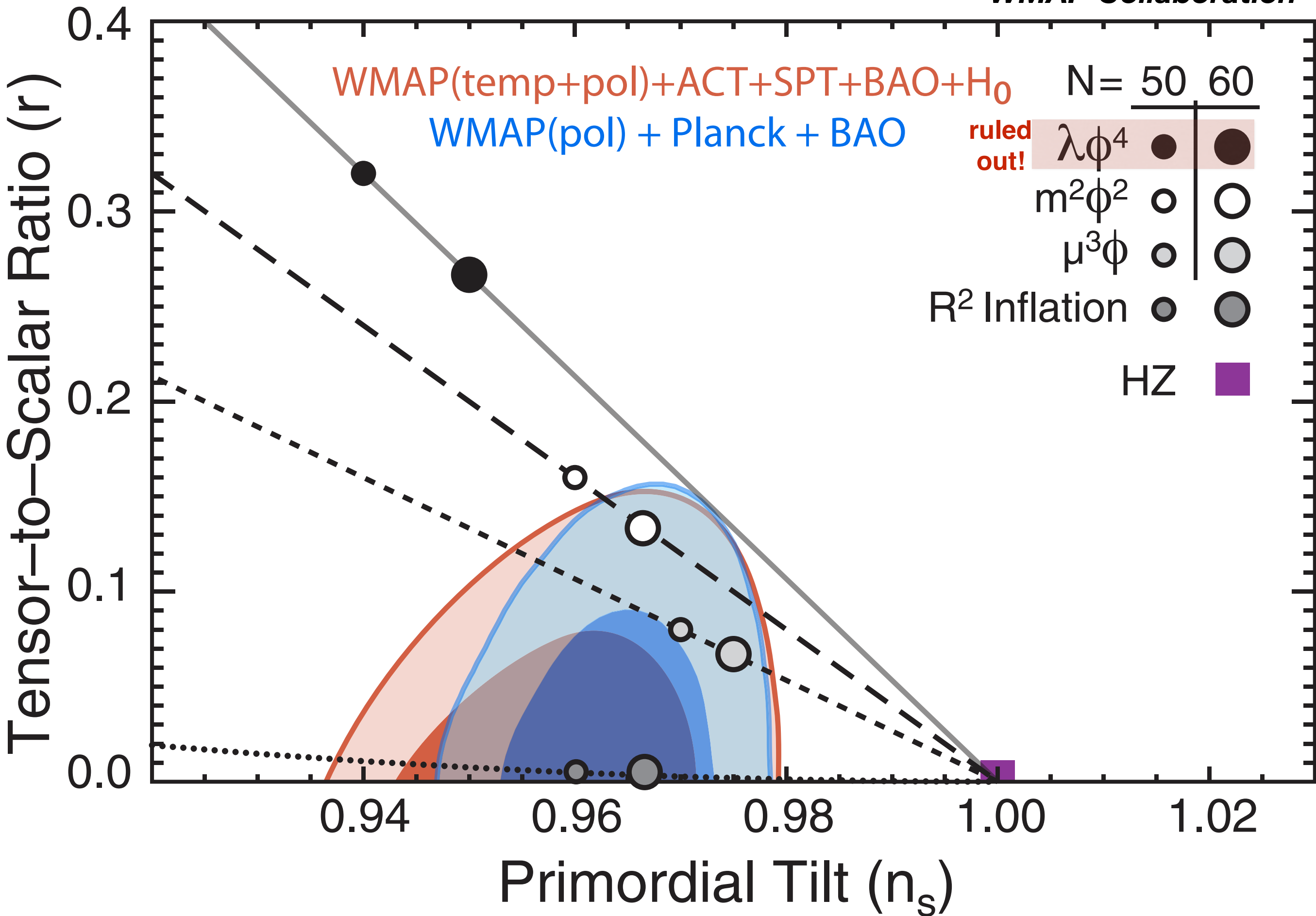


# Tensor-to-scalar Ratio

$$r \equiv \frac{\langle h_{ij} h^{ij} \rangle}{\langle \zeta^2 \rangle}$$

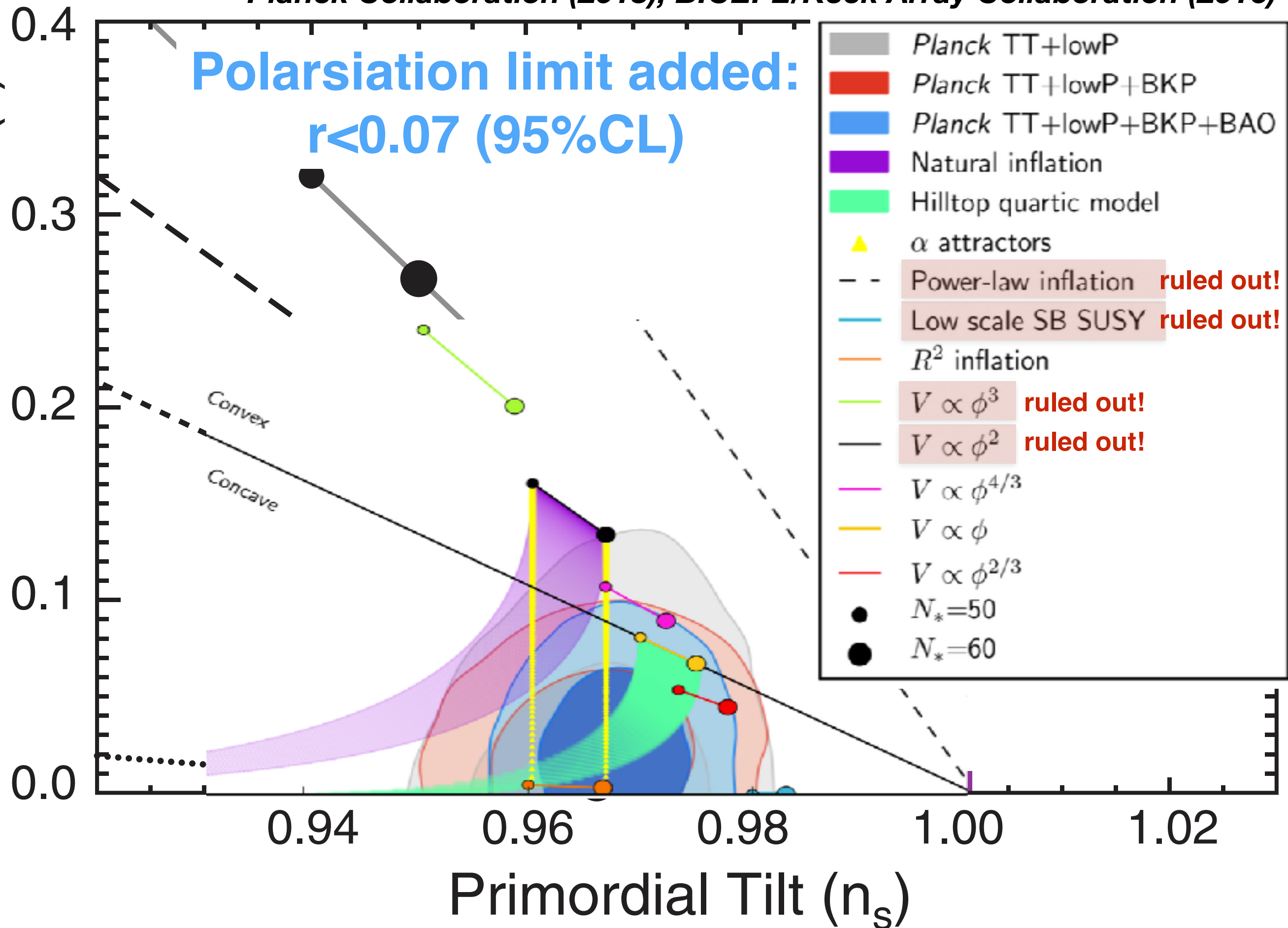
- We really want to find this! The current upper bound is  **$r < 0.06$**  (95%CL)

BICEP2/Keck Array Collaboration (2018)



Tensor-to-Scalar Ratio ( $r$ )

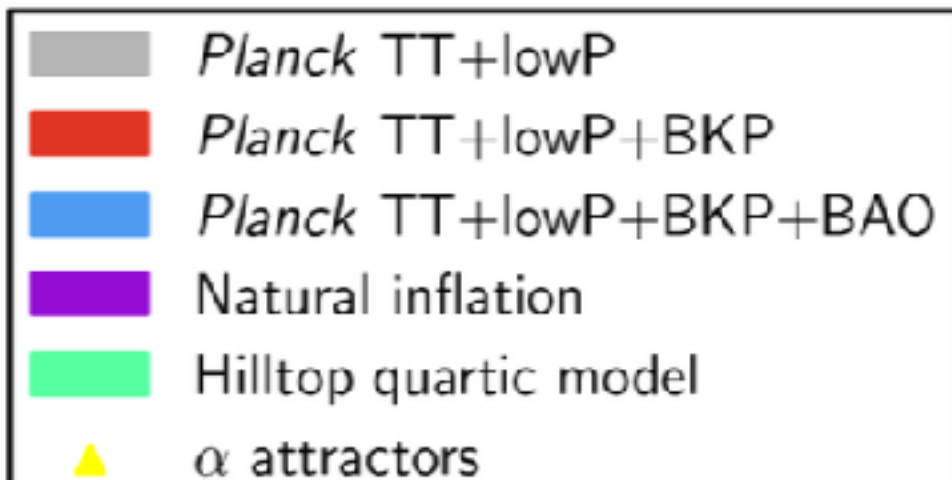
Polarisation limit added:  
 $r < 0.07$  (95%CL)



Planck Collaboration (2015); BICEP2/Keck Array Collaboration (2016)

Tensor-to-Scalar Ratio ( $r$ )

Polarisation limit added:  
 $r < 0.07$  (95%CL)



0.4  
0.3  
0.2  
0.1  
0.0

Convex  
Concave

Convex  
Concave

Planck TT+ $\tau$  prior+lensing+BAO

+BK15

2018

$r < 0.06$   
(95%CL)

BICEP2/Keck Array  
Collaboration (2018)

0.94

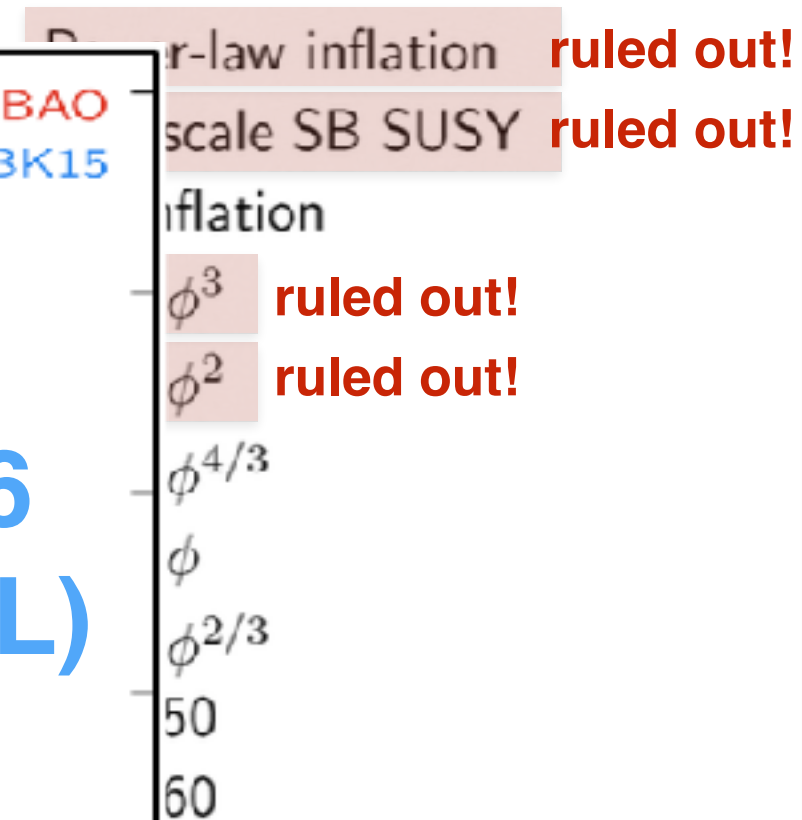
0.96

0.98

1.00

1.02

Primordial Tilt ( $n_s$ )



**But, wait a minute...**

# Are GWs from vacuum fluctuation in spacetime, or from sources?

$$\square h_{ij} = -16\pi G \pi_{ij}$$

- **Homogeneous solution:** “GWs from vacuum fluctuation”
- **Inhomogeneous solution:** “GWs from sources”
  - Scalar and vector fields cannot source tensor fluctuations at linear order (possible at non-linear level)  
Many papers by Sorbo, Peloso, and others
  - SU(2) gauge field can!

Maleknejad & Sheikh-Jabbari (2013); Dimastrogiovanni & Peloso (2013);  
Adshead, Martinec & Wyman (2013); Obata & Soda (2016); ...

# Important Message

$$\square h_{ij} = -16\pi G \pi_{ij}$$

- Do not take it for granted if someone told you that detection of the primordial gravitational waves would be a signature of “quantum gravity”!
- Only the homogeneous solution corresponds to the vacuum tensor metric perturbation. **There is no *a priori* reason to neglect an inhomogeneous solution!**
- Contrary, we have several examples in which detectable B-modes are generated by **sources** [U(1) and SU(2)]

# Experimental Strategy

## Commonly Assumed So Far

1. Detect CMB polarisation in multiple frequencies, to make sure that it is from the CMB (i.e., Planck spectrum)
2. Check for scale invariance: Consistent with a scale invariant spectrum?
  - Yes => Announce discovery of the vacuum fluctuation in spacetime
  - No => WTF?

# New Experimental Strategy: New Standard!

1. Detect CMB polarisation in multiple frequencies, to make sure that it is from the CMB (i.e., Planck spectrum)
  2. Consistent with a scale invariant spectrum?
  3. Parity violating correlations consistent with zero?
  4. Consistent with Gaussianity?
- If, and **ONLY IF** Yes to **all** => Announce discovery of the vacuum fluctuation in spacetime

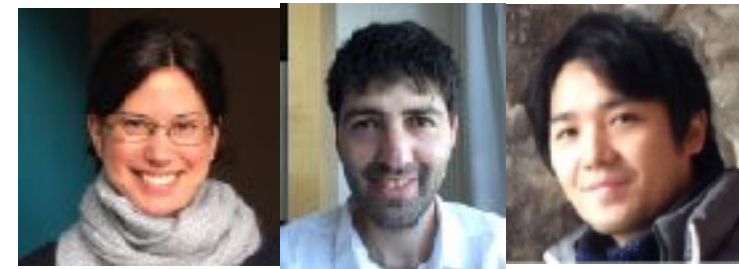
# If not, you may have just discovered new physics during inflation!

2. Consistent with a scale invariant spectrum?
  3. Parity violating correlations consistent with zero?
  4. Consistent with Gaussianity?
- If, and **ONLY IF** Yes to **all** => Announce discovery of the vacuum fluctuation in spacetime

# Further Remarks

- “*Guys, you are complicating things too much!*”
- **No.** These sources (eg., gauge fields) should be ubiquitous in a high-energy universe. They have every right to produce GWs if they are around
- Sourced GWs with  $r \gg 0.001$  can be phenomenologically more attractive than the vacuum GW from the large-field inflation [requiring super-Planckian field excursion]. Better radiative stability, etc
- Rich[er] phenomenology: Better integration with the Standard Model; reheating; baryon synthesis via leptogenesis, etc. **Testable using many more probes!**

# GW from Axion-SU(2) Dynamics



$$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_\phi + \mathcal{L}_\chi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\lambda \chi}{4f} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$$

- $\phi$ : inflaton field => Just provides quasi-de Sitter background  
[I don't want to touch this sector because I don't understand inflaton]
- $\chi$ : pseudo-scalar “axion” field. Spectator field (i.e., negligible energy density compared to the inflaton)

- Field strength of an SU(2) field  $A_\nu^a$  :

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon^{abc} A_\mu^b A_\nu^c$$

*self-interaction term*

[a=1,2,3;  $\mu=0,1,2,3$ ]

## A well-defined set up:

Axion-SU(2) gauge field dynamics in a given de-Sitter background.

Everything is calculable!

- $\phi$ : inflaton field => Just provides quasi-de Sitter background  
[I don't want to touch this sector because I don't understand inflaton]
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*self-interaction term*

[a=1,2,3;  $\mu=0,1,2,3$ ]

# Background and Perturbation



A. Maleknejad  
(MPA)

- In an inflating background, the SU(2) field has an **isotropic** background solution:

$$A_i^a = [\text{scale factor}] \times Q \times \delta_i^a$$

$$Q \equiv (-f \partial_\chi U / 3g\lambda H)^{1/3}$$

**U: axion potential**

- Perturbations contain a tensor (spin-2) mode (as well as S&V)

$$\delta A_i^a = t_{ai} + \dots$$

$$t_{ii} = \partial_a t_{ai} = \partial_i t_{ai} = 0$$

# Scenario

- The SU(2) field contains 1 tensor, 2 vectors, and 3 scalars (9 DOF =  $12 - 3$ )
- The tensor components are amplified strongly by a coupling to the axion field
- Only one helicity is amplified  $\Rightarrow$  GW is **chiral** (well-known result, also for U(1))
- New result: **GWs sourced by this mechanism are strongly non-Gaussian!**

*Agrawal, Fujita & EK, PRD, 97, 103526 (2018); JCAP 1806, 027 (2018)*

# Gravitational Waves

- Defining canonically-normalised circular polarisation modes as

$$\psi_{L,R} \equiv (aM_{\text{Pl}}/2)(h_+ \pm ih_\times)$$

- The equations of motion for L and R modes are

$$\square \psi_{L,R} \neq 0$$

# Gravitational Waves

- Defining canonically-normalised circular polarisation modes as

$$\psi_{L,R} \equiv (aM_{\text{Pl}}/2)(h_+ \pm ih_\times)$$

- The equations of motion for L and R modes are ( $x \equiv k/aH$ )

$$\partial_x^2 \psi_{R,L} + \left(1 - \frac{2}{x^2}\right) \psi_{R,L} = \frac{2\sqrt{\epsilon_E}}{x} \partial_x t_{R,L}^{\text{spin-2 field}} + \frac{2\sqrt{\epsilon_B}}{x^2} (m_Q \mp x) t_{R,L}^{\text{spin-2 field}}$$

$$\left( \begin{array}{l} m_Q \equiv gQ/H = \text{a few} \\ \epsilon_B \equiv g^2 Q^4 / (H M_{\text{Pl}})^2 \ll 1 \\ \epsilon_E \equiv (H\dot{Q} + \dot{Q})^2 / (\dot{H} M_{\text{Pl}})^2 \ll 1 \end{array} \right.$$

# Spin-2 Field from SU(2)

- The equations of motion for L and R modes of SU(2) are

$$\partial_x^2 t_{R,L} + \left[ 1 + \frac{2}{x^2} (m_Q \xi \mp x(m_Q + \xi)) \right] t_{R,L}$$

the minus sign gives an instability -> exponential amplification of  $t_R$ !

$$= -\frac{2\sqrt{\epsilon_E}}{x} \partial_x \psi_{R,L} + \frac{2}{x^2} [(m_Q \mp x) \sqrt{\epsilon_B} + \sqrt{\epsilon_E}] \psi_{R,L}$$

$$\left( \begin{array}{l} \xi \equiv \frac{\lambda}{2fH} \dot{\chi} \simeq m_Q + \frac{1}{m_Q} \\ m_Q \equiv gQ/H = \text{a few} \\ \epsilon_B \equiv g^2 Q^4 / (H M_{\text{Pl}})^2 \ll 1 \\ \epsilon_E \equiv (H\dot{Q} + \dot{Q})^2 / (\dot{H} M_{\text{Pl}})^2 \ll 1 \end{array} \right.$$

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- The produced gravitational waves are totally chiral!**
- The solution (when all the parameters are constant and the terms on the right hand side are ignored):

$$t_R(x) = \frac{1}{\sqrt{2k}} i^\beta W_{\beta,\alpha}(-2ix) \quad \begin{pmatrix} \alpha \equiv -i\sqrt{2m_Q\xi - 1/4} \\ \beta \equiv -i(m_Q + \xi) \end{pmatrix}$$

[Whittaker function]

# Gravitational Waves

- Defining canonically-normalised circular polarisation modes as

$$\psi_{L,R} \equiv (aM_{\text{Pl}}/2)(h_+ \pm ih_\times)$$

- The equations of motion for L and R modes are (  $x \equiv k/aH$  )

$$\partial_x^2 \psi_{R,L} + \left(1 - \frac{2}{x^2}\right) \psi_{R,L} = \frac{2\sqrt{\epsilon_E}}{x} \partial_x t_{R,L} + \frac{2\sqrt{\epsilon_B}}{x^2} (m_Q \mp x) t_{R,L}$$

- Inhomogeneous solution:

$$\lim_{x \rightarrow 0} \psi_R^{(s)}(x) = \frac{1}{\sqrt{2kx}} \left[ \mathcal{F}_E \sqrt{\epsilon_E} + \mathcal{F}_B \sqrt{\epsilon_B} \right]$$

$\mathcal{F}_E, \mathcal{F}_B$ : some complicated functions

# Power Spectrum!

$$\mathcal{P}_h^{(s)}(k) = \frac{H^2}{\pi^2 M_{\text{Pl}}^2} \left| \sqrt{2kx} \lim_{x \rightarrow 0} \psi_R^{(s)}(x) \right|^2 = \frac{\epsilon_B H^2}{\pi^2 M_{\text{Pl}}^2} \mathcal{F}^2$$

$$\mathcal{F}^2 \equiv \left| \mathcal{F}_B + \sqrt{\epsilon_E/\epsilon_B} \mathcal{F}_E \right|^2 \approx \exp(3.6 m_Q)$$

- This exponential dependence on  $m_Q$  makes it possible to have  $\mathbf{P_{sourced} \gg P_{vacuum} = (2/\pi^2)H^2/M_{Pl}^2}$
- **New Paradigm**

# Phenomenology

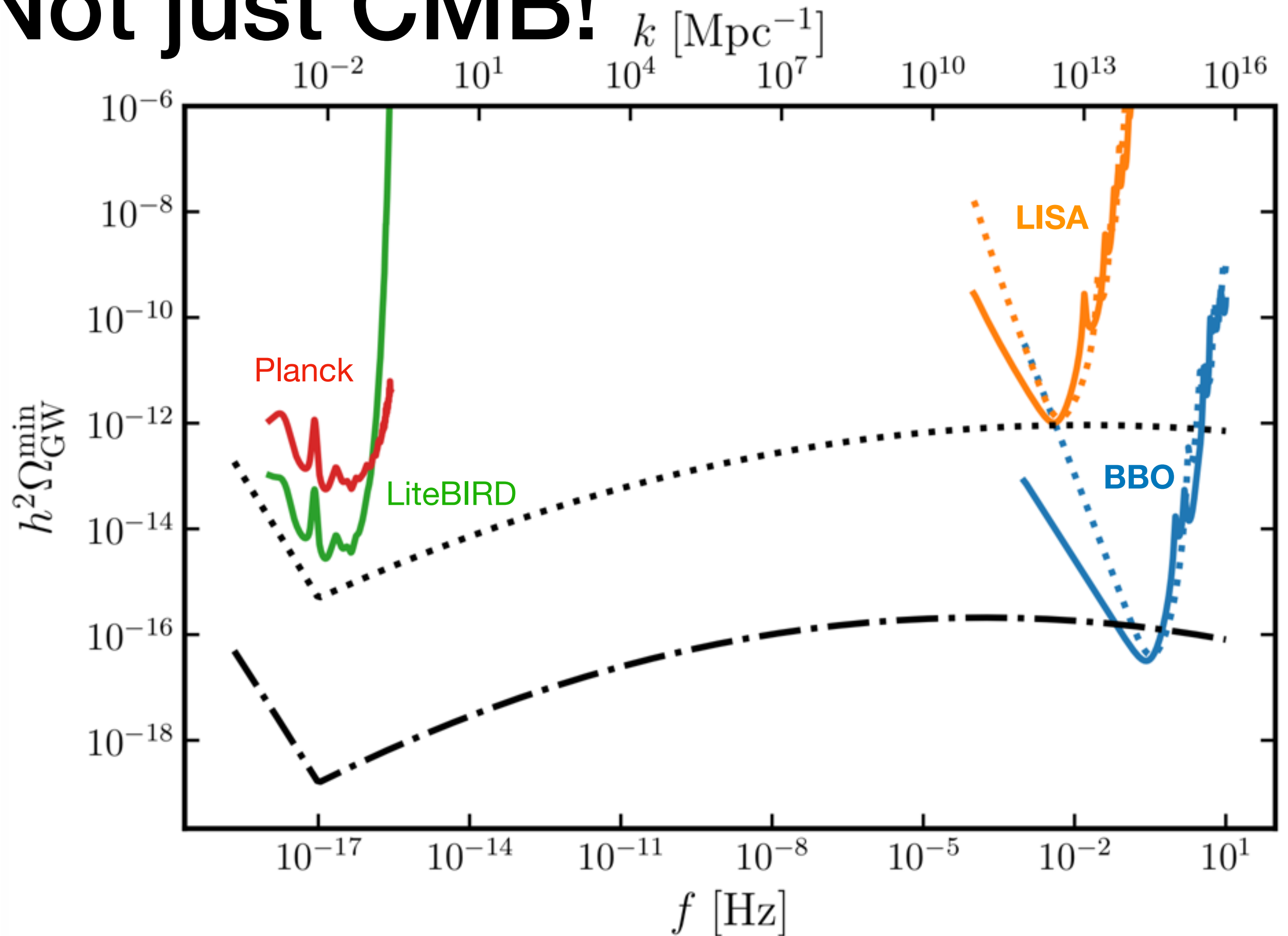
$$\partial_x^2 t_{R,L} + \left[ 1 + \frac{2}{x^2} (m_Q \xi \mp x(m_Q + \xi)) \right] t_{R,L} = \dots$$

the minus sign gives an instability -> exponential amplification of  $t_R$ !

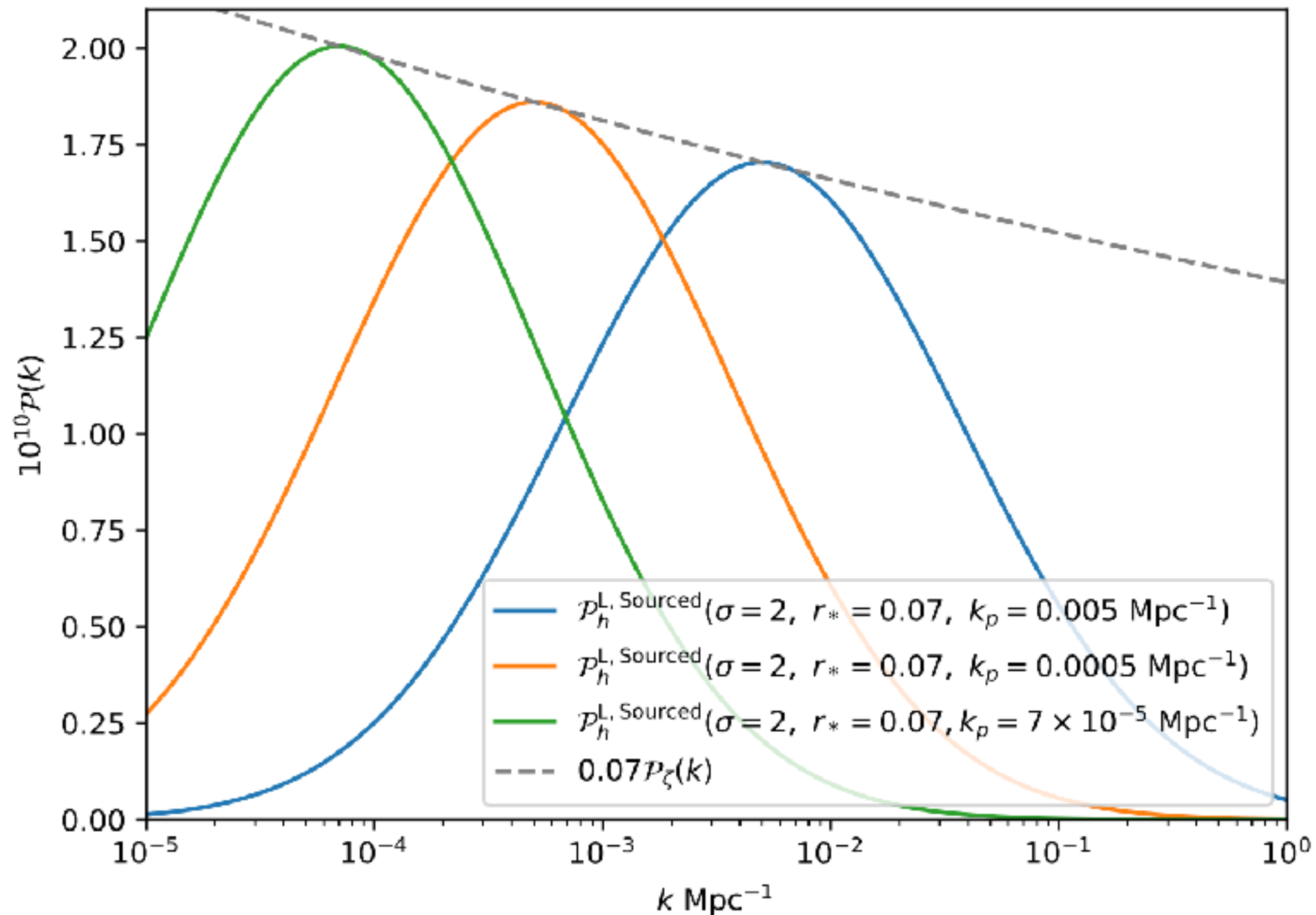
$$\left( \begin{array}{l} \xi \equiv \frac{\lambda}{2fH} \dot{\chi} \simeq m_Q + \frac{1}{m_Q} \\ m_Q \equiv gQ/H = \text{a few} \end{array} \right.$$

- The scale-dependence of the produced tensor modes is determined by how  $m_Q$  changes with time
- E.g., Axion rolling faster towards the end of inflation: BLUE TILTED power spectrum! Therefore...

# Not just CMB!



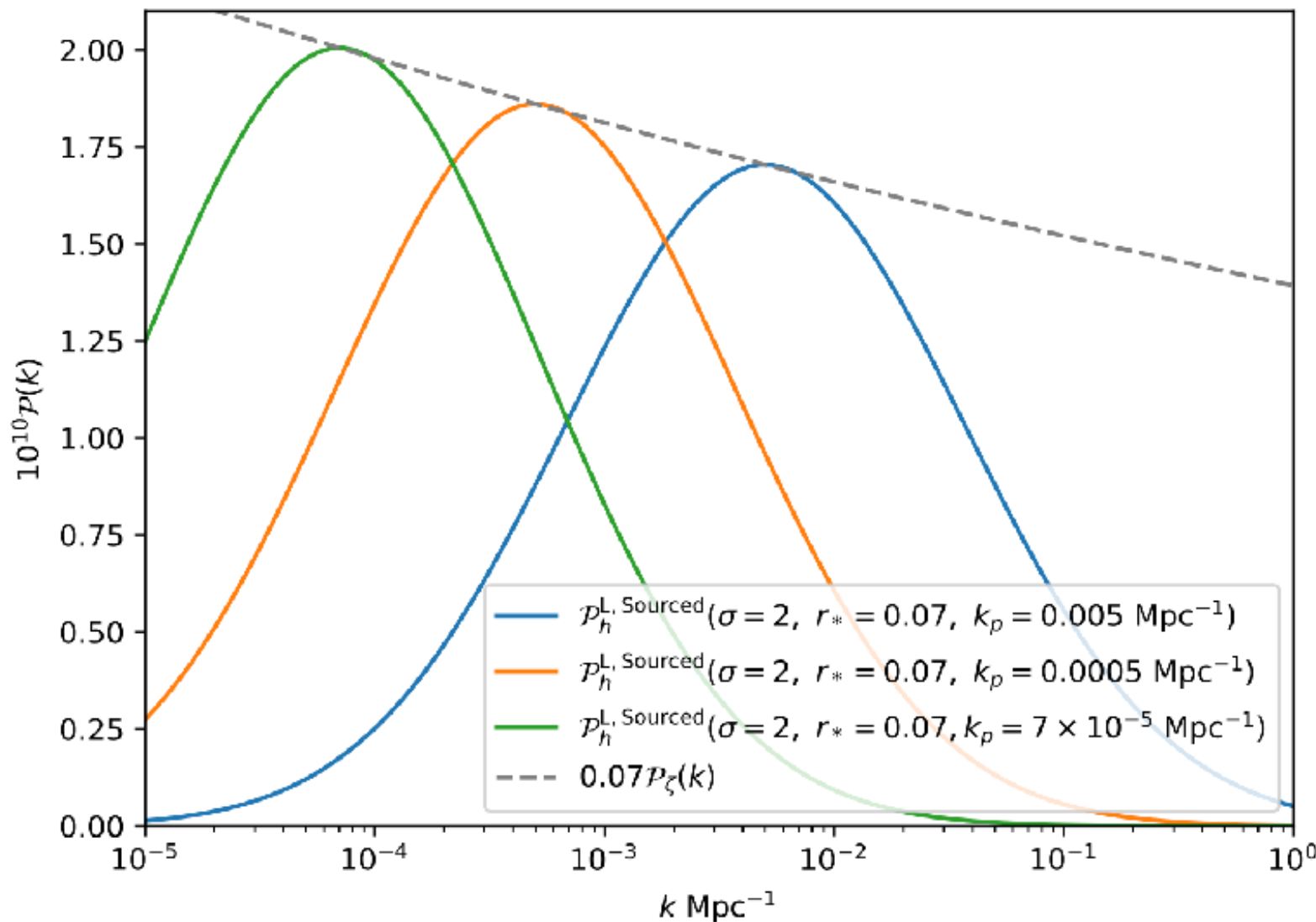
# Example Tensor Spectra



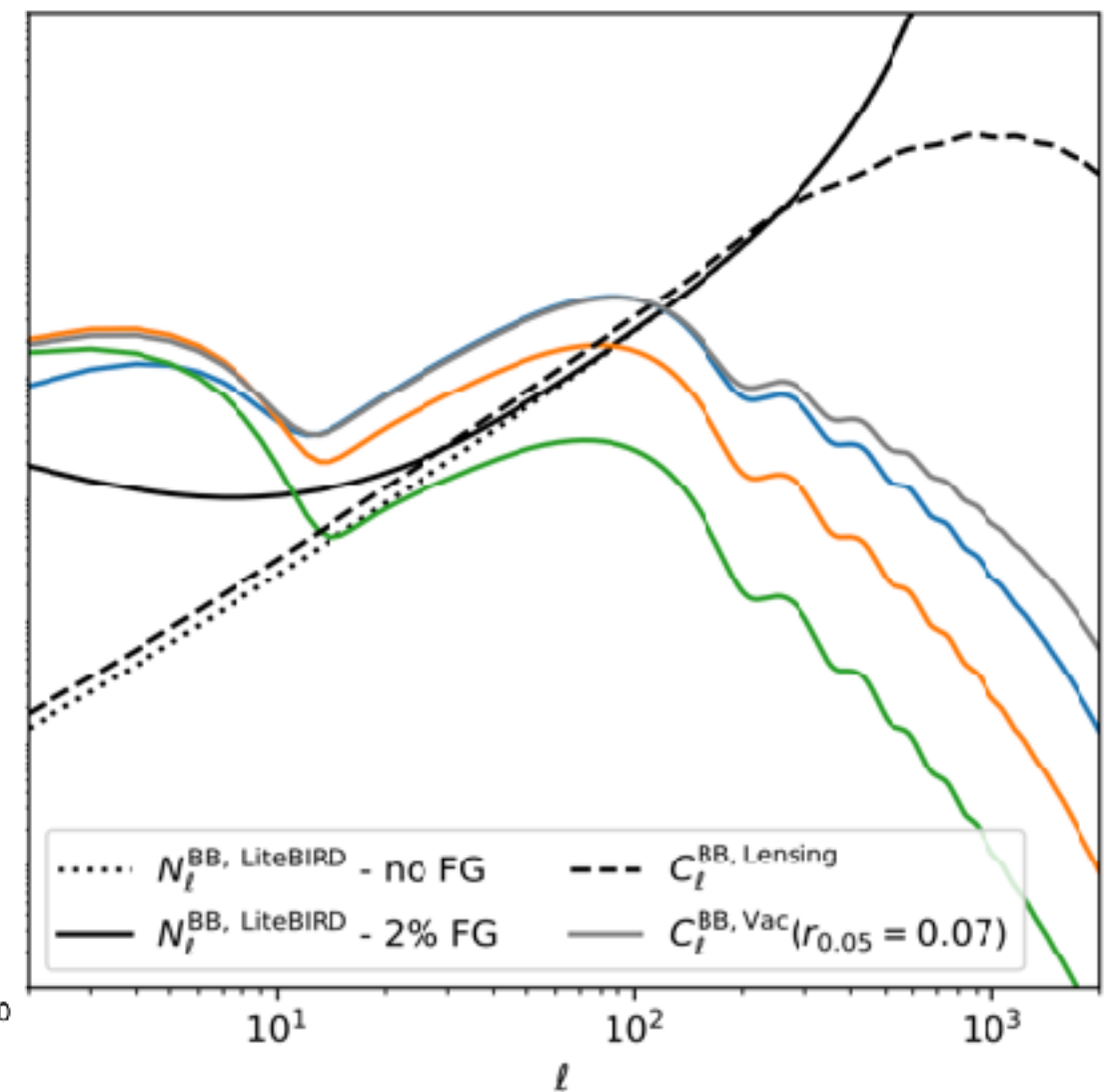
- Sourced tensor spectrum can also be bumpy

# Example Tensor Spectra

Tensor Power Spectrum,  $P(k)$

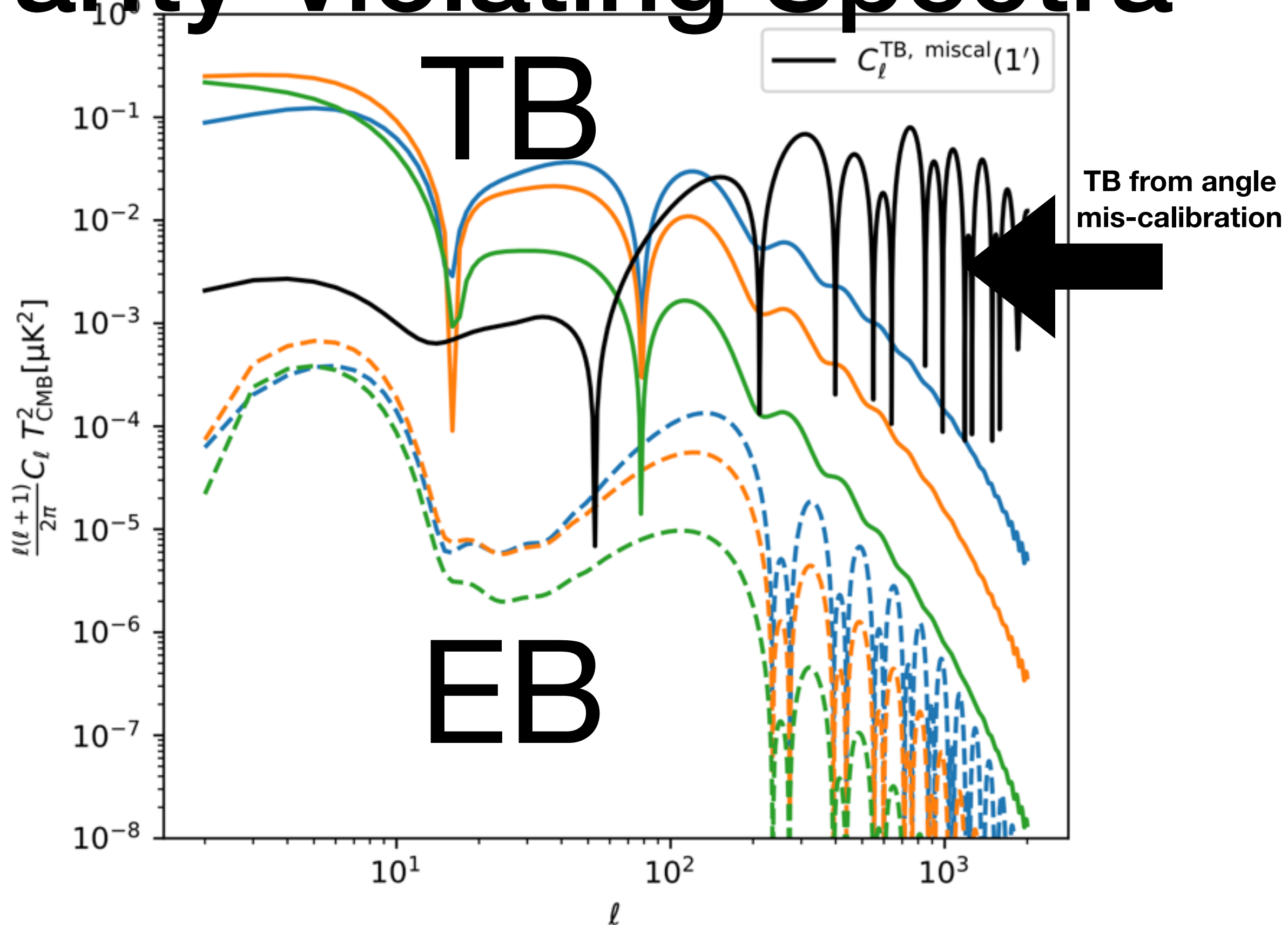


B-mode CMB spectrum,  $C_l^{\text{BB}}$



- The B-mode power spectrum still looks rather normal

# Parity-violating Spectra



- Angle mis-calibration can be distinguished easily!

# Large bispectrum in GW from SU(2) fields



Aniket Agrawal  
(MPA)

$$\frac{B_h^{RRR}(k, k, k)}{P_h^2(k)} \approx \frac{25}{\Omega_A}$$



Tomo Fujita  
(Kyoto)

$$\langle \hat{h}_R(\mathbf{k}_1) \hat{h}_R(\mathbf{k}_2) \hat{h}_R(\mathbf{k}_3) \rangle = (2\pi)^3 \delta \left( \sum_{i=1}^3 \mathbf{k}_i \right) B_h^{RRR}(k_1, k_2, k_3)$$

- $\Omega_A \ll 1$  is the energy density fraction of the gauge field
- **$B_h/P_h^2$  is of order unity for the vacuum contribution**  
[Maldacena (2003); Maldacena & Pimentel (2011)]
- **Gaussianity offers a powerful test of whether the detected GW comes from the vacuum or sources**

# NG generated at the tree level

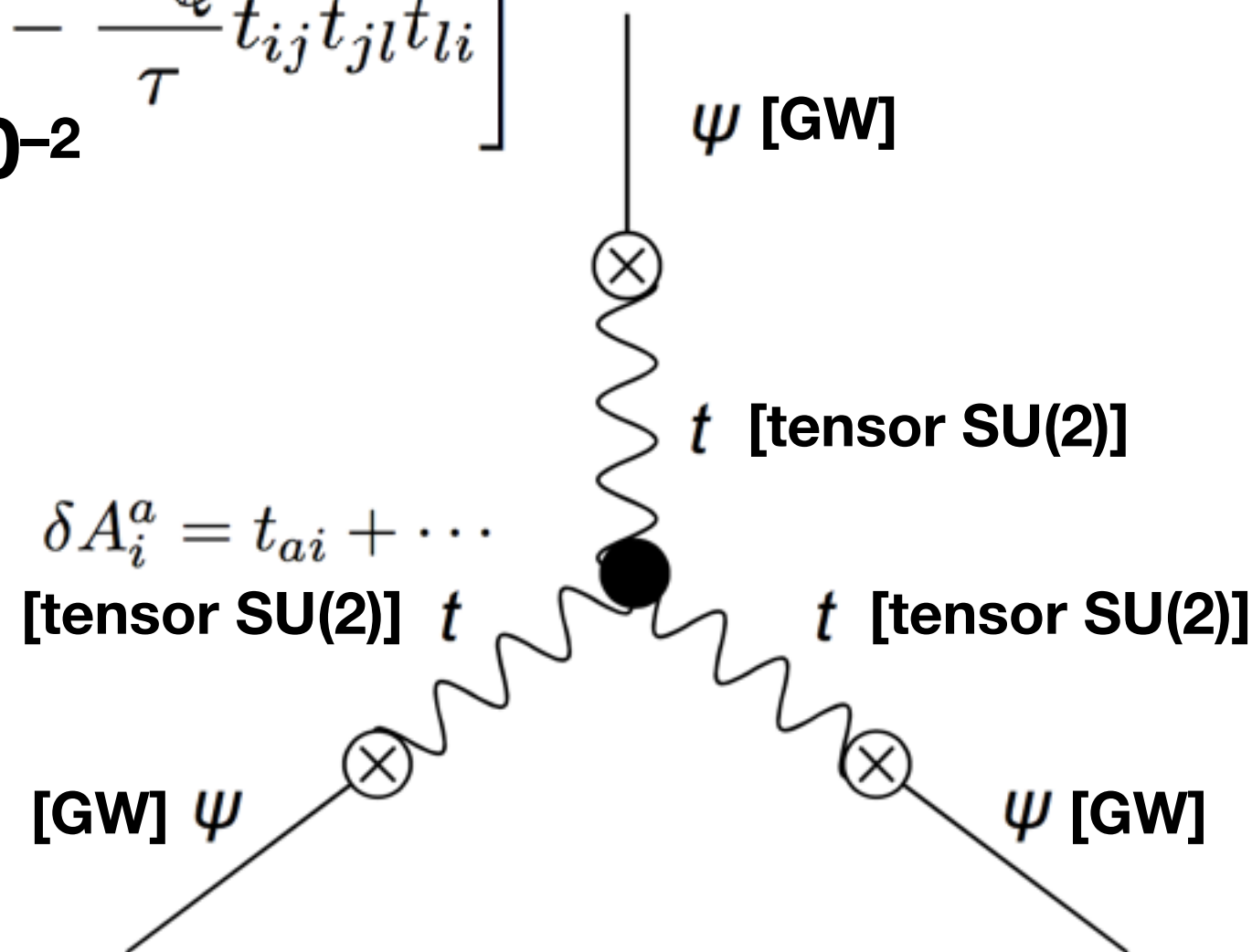
$$L_3^{(i)} = c^{(i)} \left[ \epsilon^{abc} t_{ai} t_{bj} \left( \partial_i t_{cj} - \frac{m_Q^2 + 1}{3m_Q \tau} \epsilon^{ijk} t_{ck} \right) \right.$$

$$\left. - \frac{m_Q}{\tau} t_{ij} t_{jl} t_{li} \right] \\ c^{(i)} = g = m_Q^2 H / \sqrt{\epsilon_B} M_{\text{Pl}} \sim 10^{-2}$$

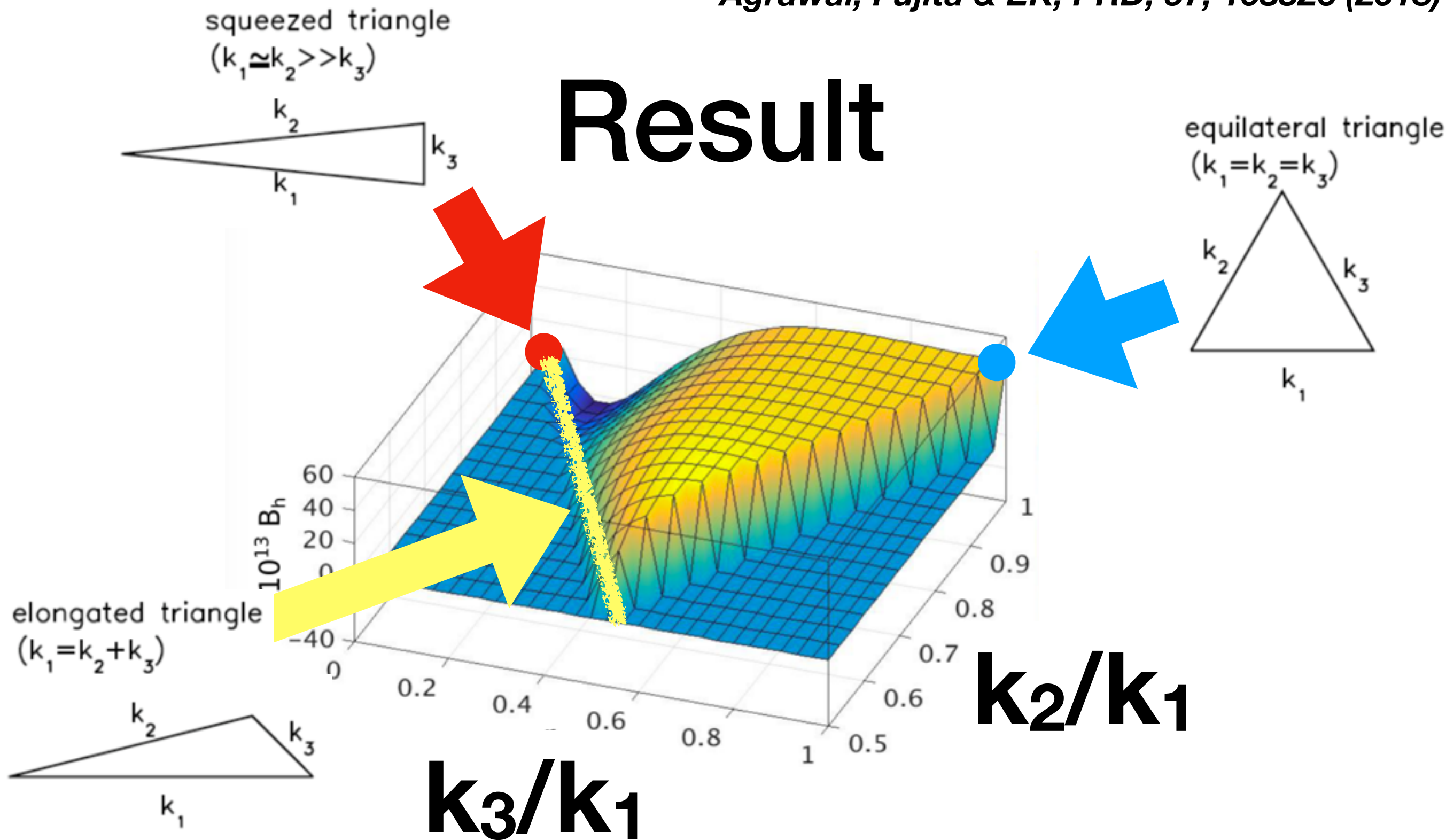
$$\epsilon_B \equiv \frac{g^2 Q^4}{H^2 M_{\text{Pl}}^2} \simeq \frac{2\Omega_A}{1 + m_Q^{-2}} \ll 1$$

$$m_Q \equiv gQ/H \quad [m_Q \sim \text{a few}]$$

- This diagram generates second-order equation of motion for GW



# Result



- This shape is similar to, but not exactly the same as, what was used by the Planck team to look for tensor bispectrum

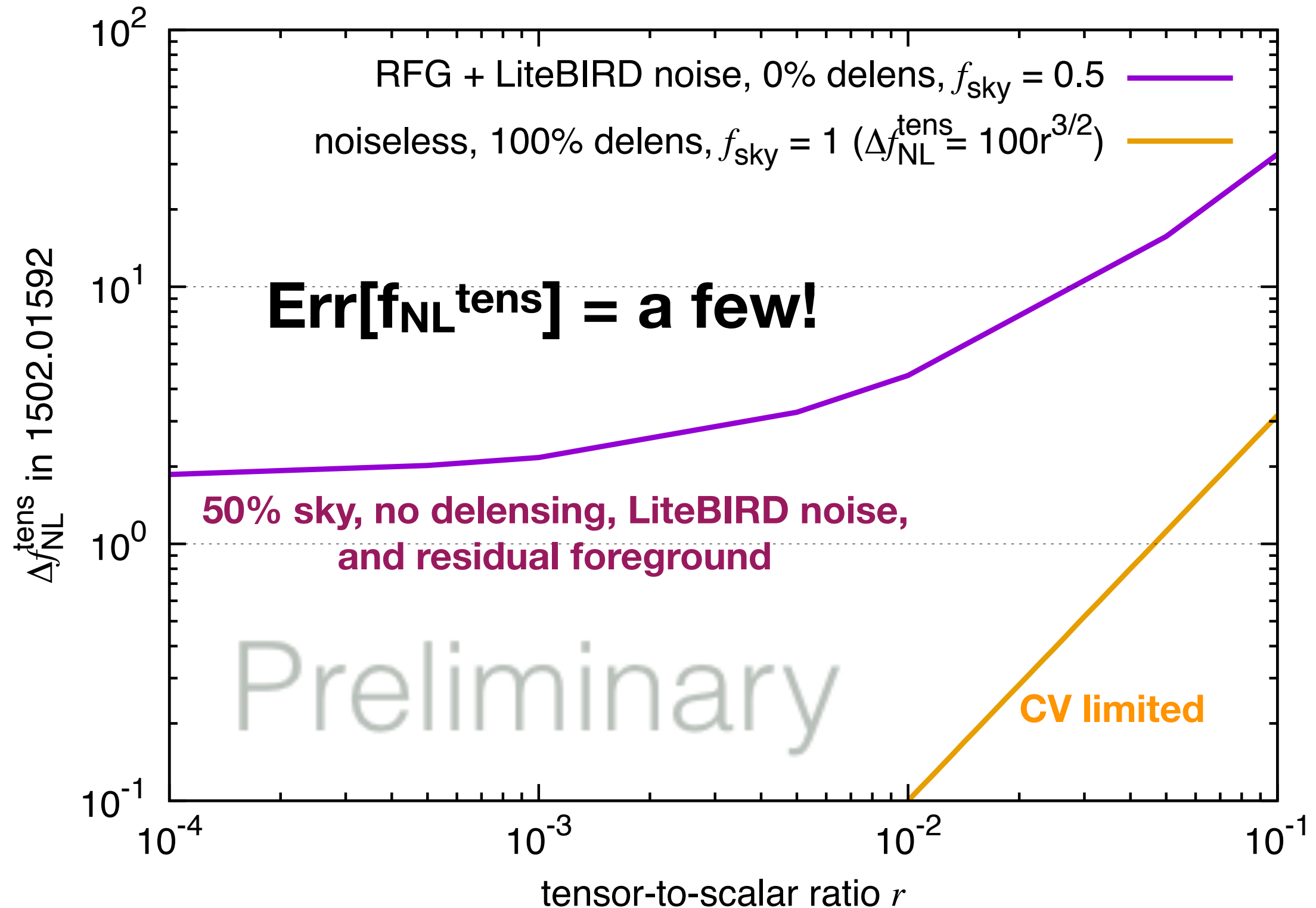
# Current Limit on Tensor NG

- The Planck team reported a limit on the tensor bispectrum in the following form:

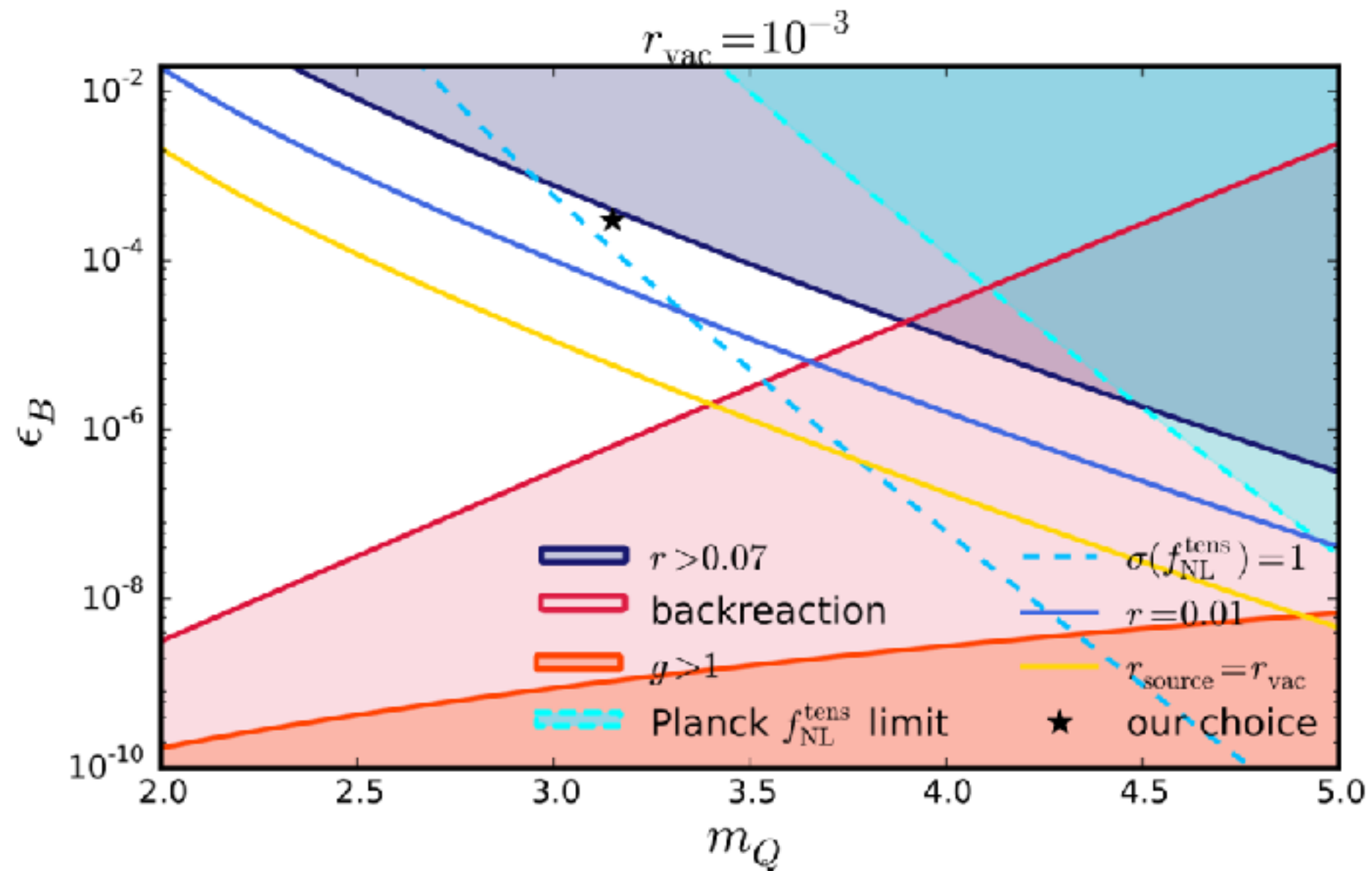
$$f_{\text{NL}}^{\text{tens}} \equiv \frac{B_h^{+++}(k, k, k)}{F_{\text{scalar}}^{\text{equil.}}(k, k, k)}$$

- The denominator is the **scalar** equilateral bispectrum template, giving  $F_{\text{scalar}}^{\text{equil.}}(k, k, k) = (18/5)P_{\text{scalar}}^2(k)$
- The current 68%CL constraint is  $f_{\text{NL}}^{\text{tens}} = 400 \pm 1500$

# LiteBIRD would nail it!



# Parameter Scan



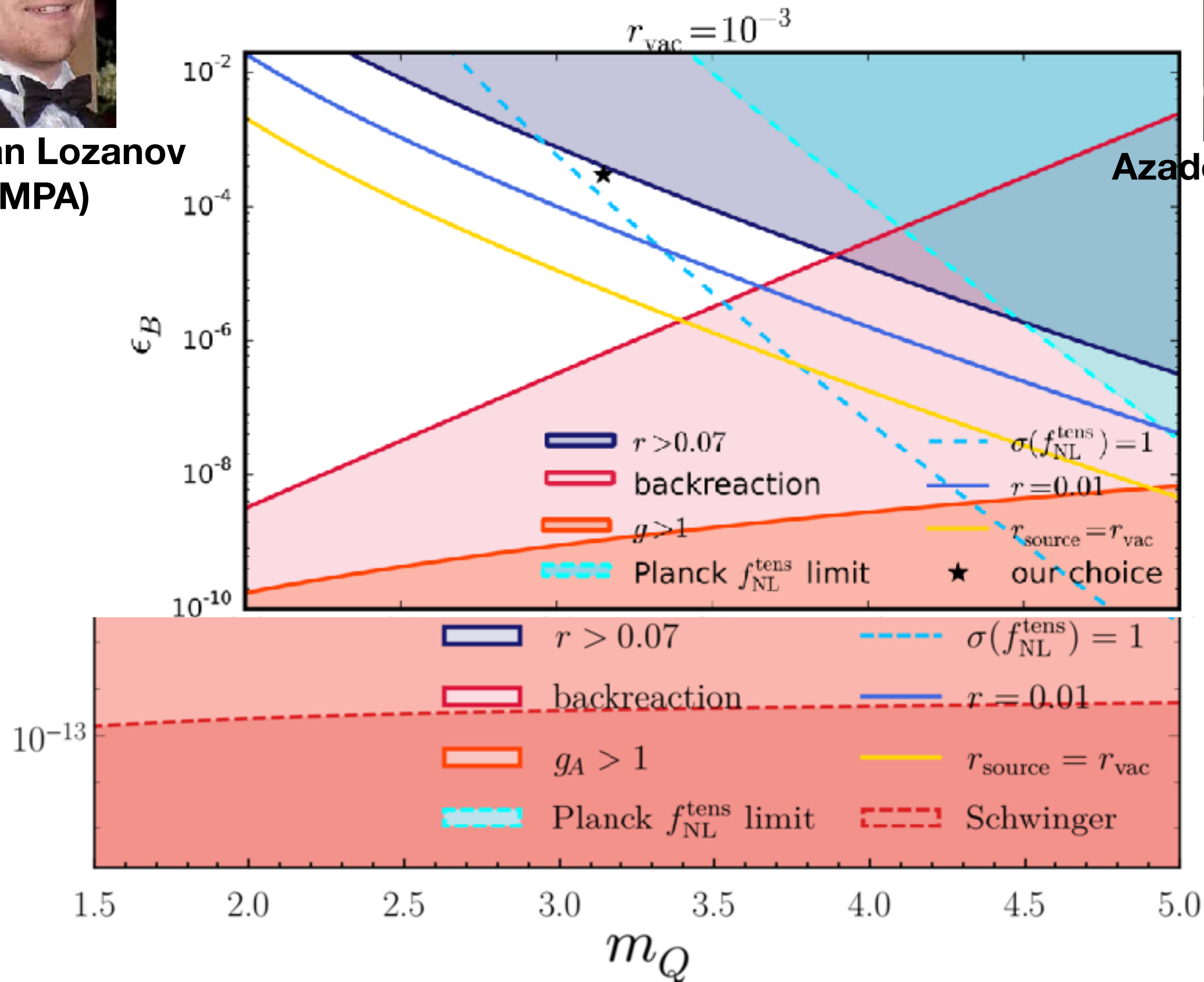
# Schwinger Effect



Kaloian Lozanov  
(MPA)



Azadeh Maleknejad  
(MPA)

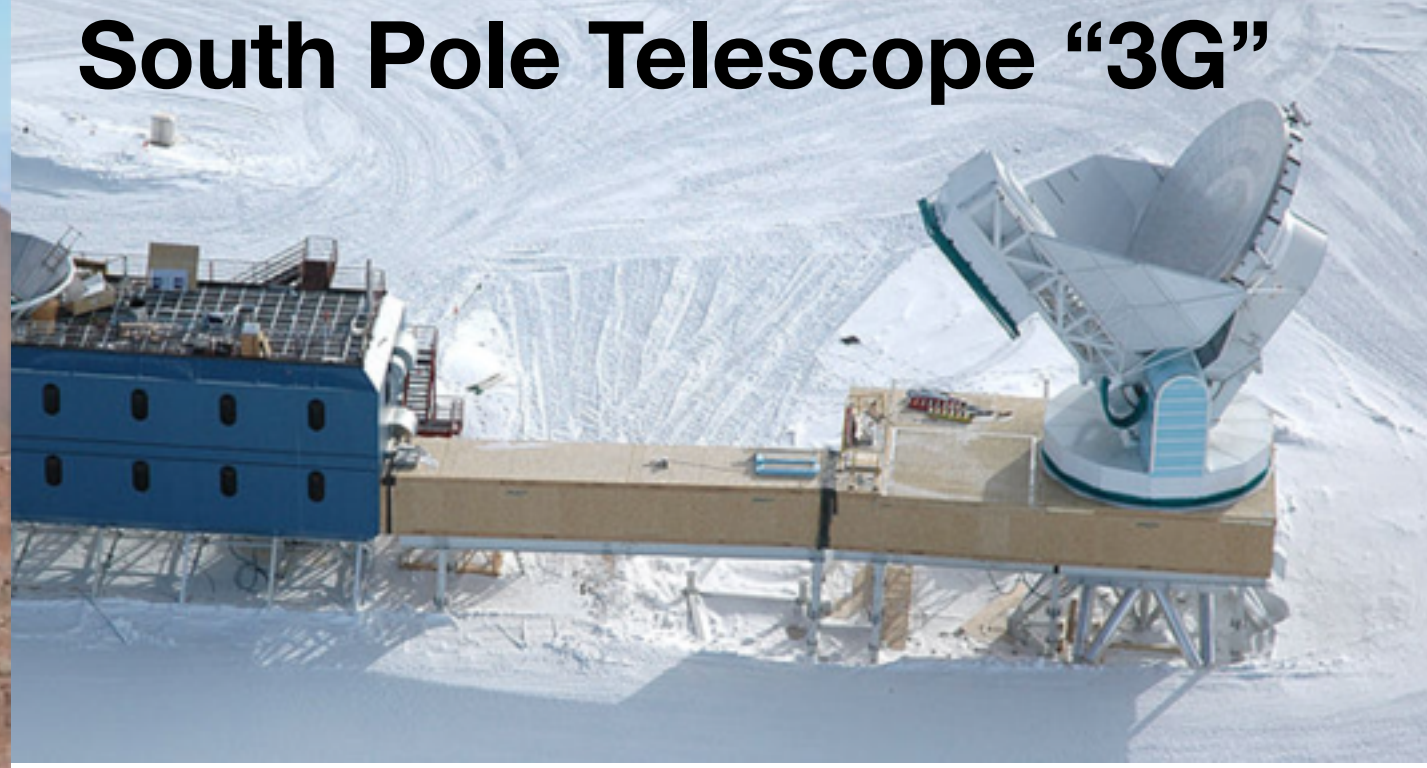


# Experimental Landscape

# Advanced Atacama Cosmology Telescope

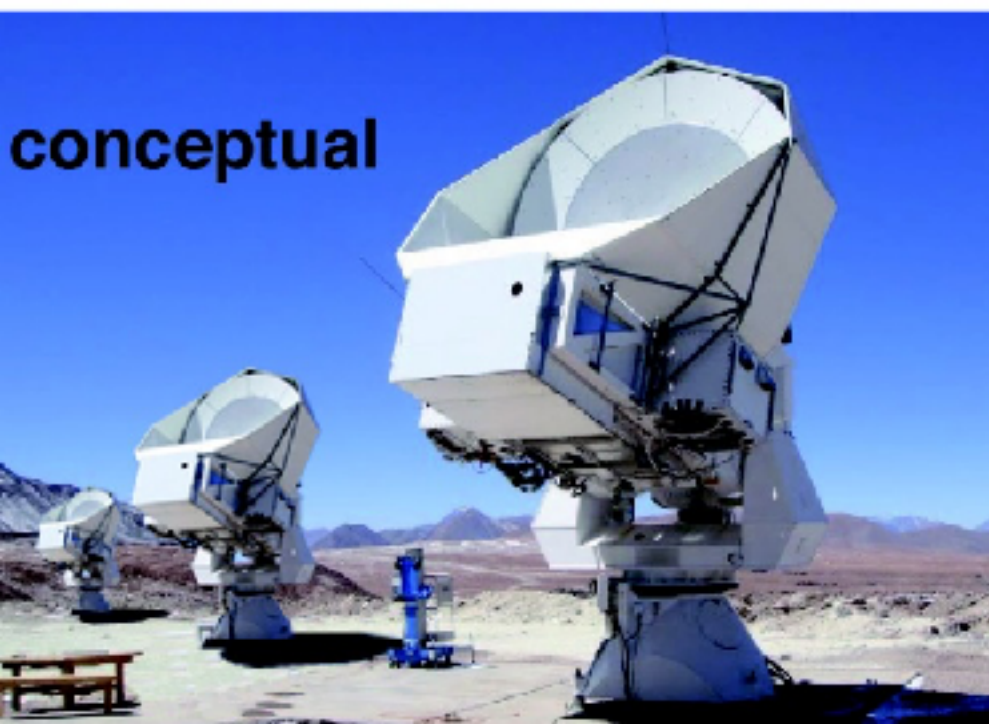


# South Pole Telescope “3G”



# What comes next?

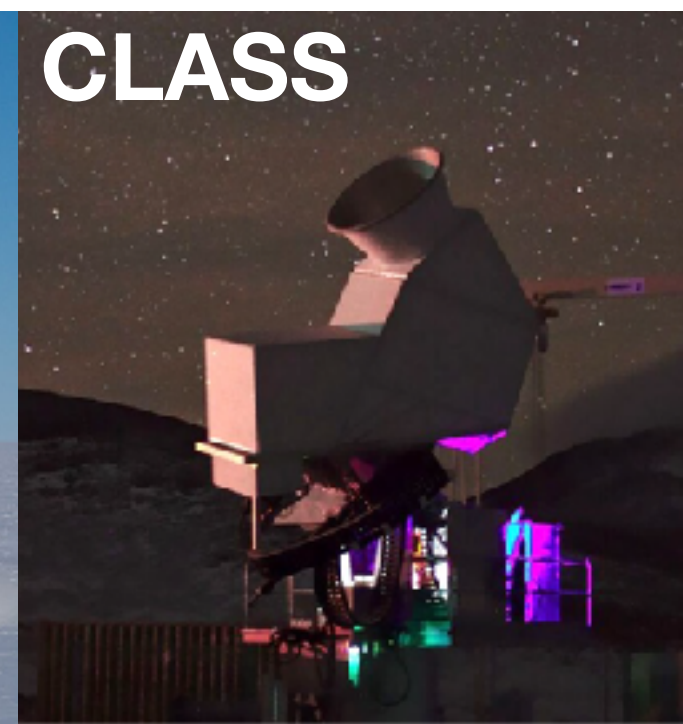
## The Simons Array



## BICEP/Keck Array



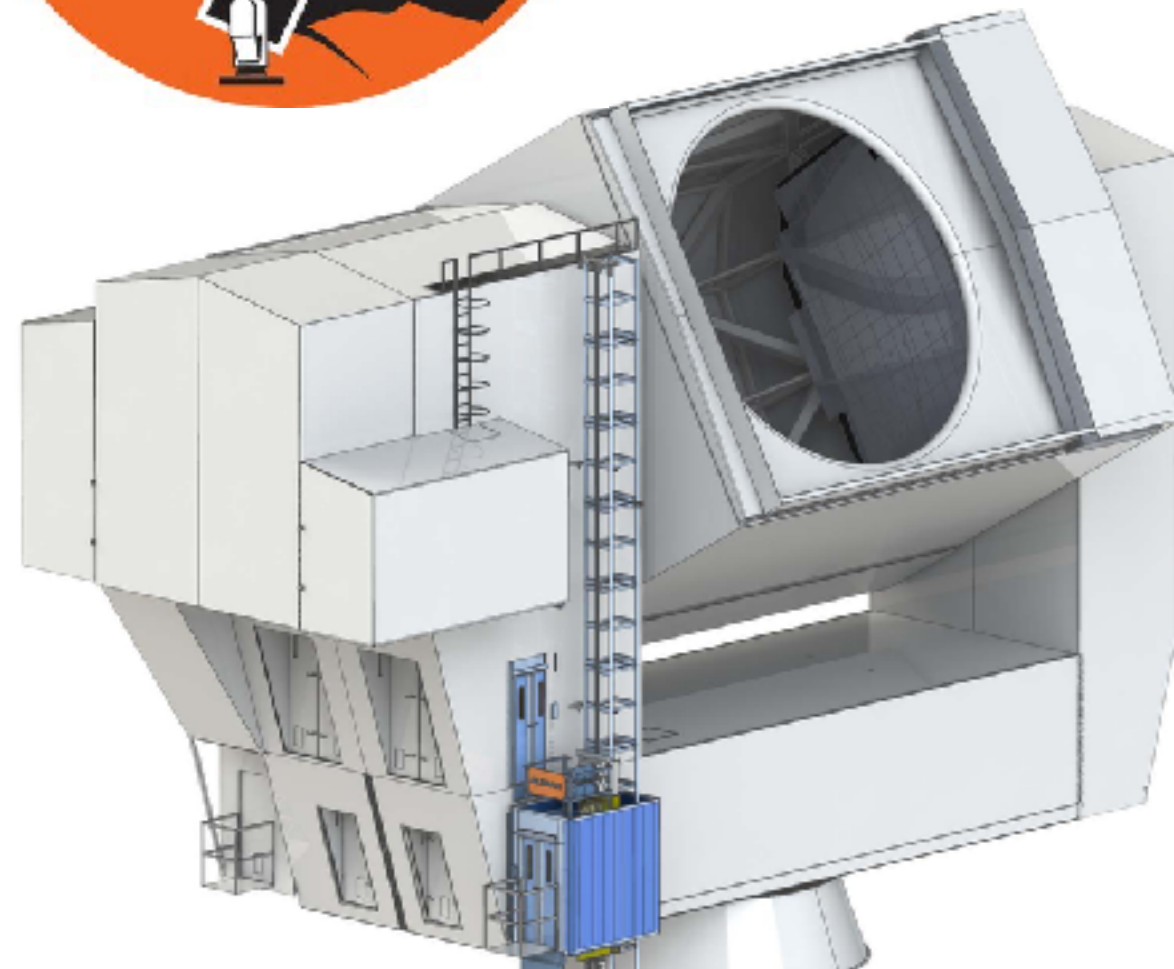
## CLASS

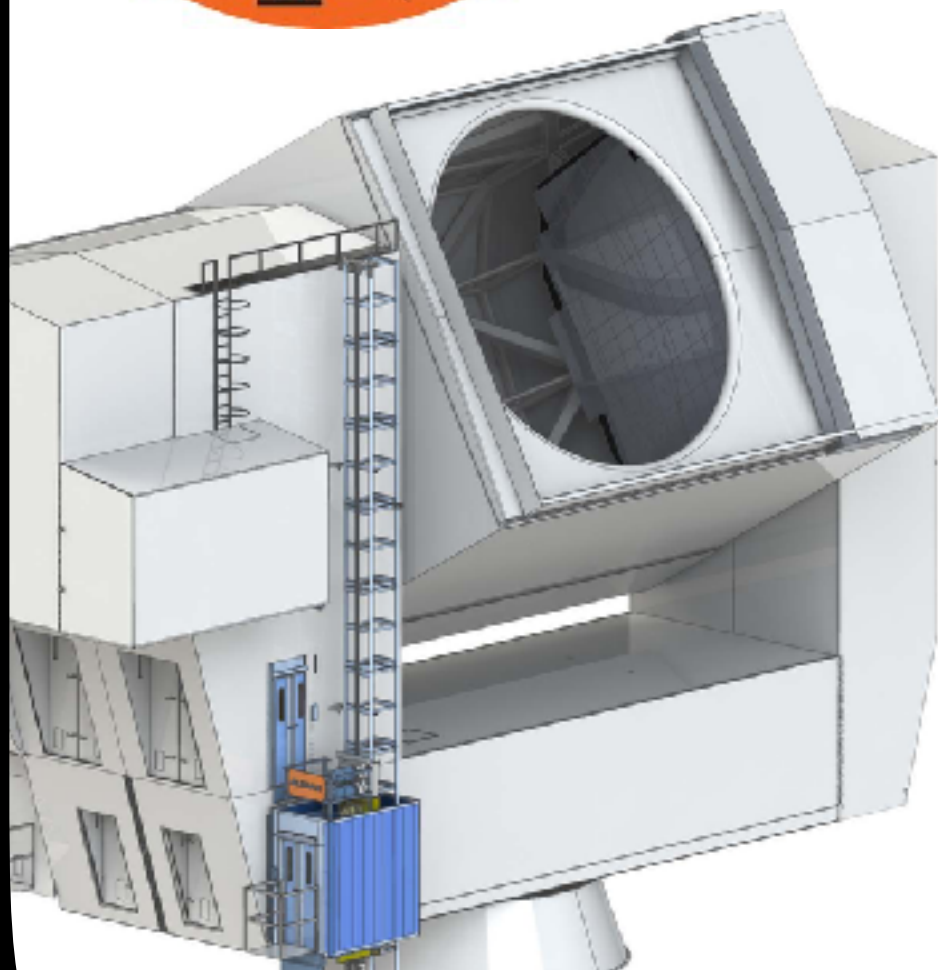
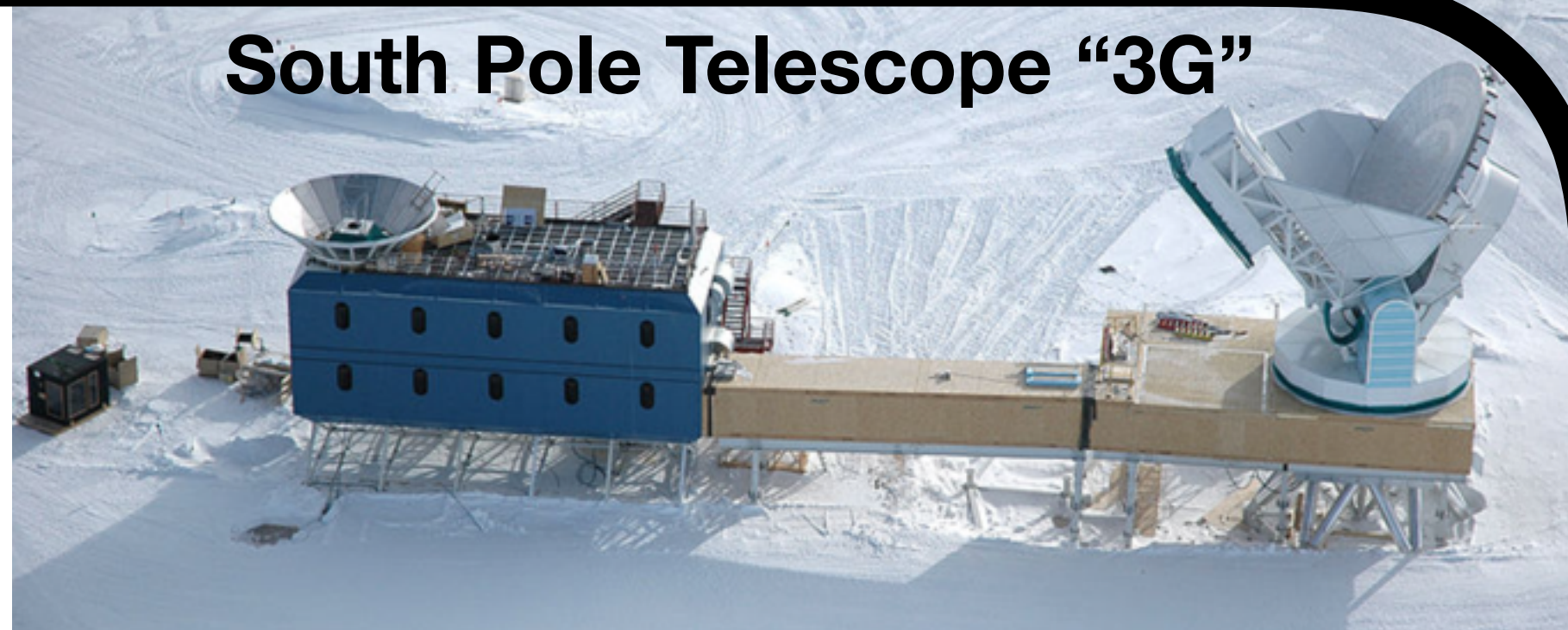


# Advanced Atacama Cosmology Telescope

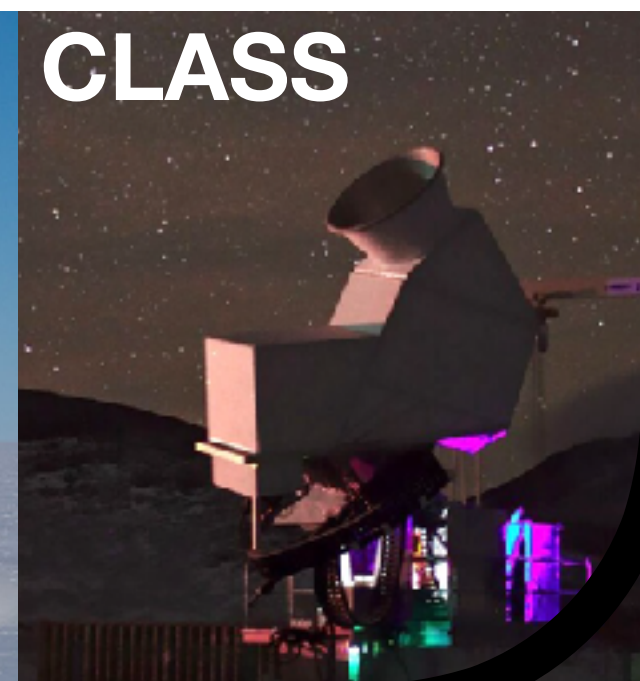


**The Simons Array**





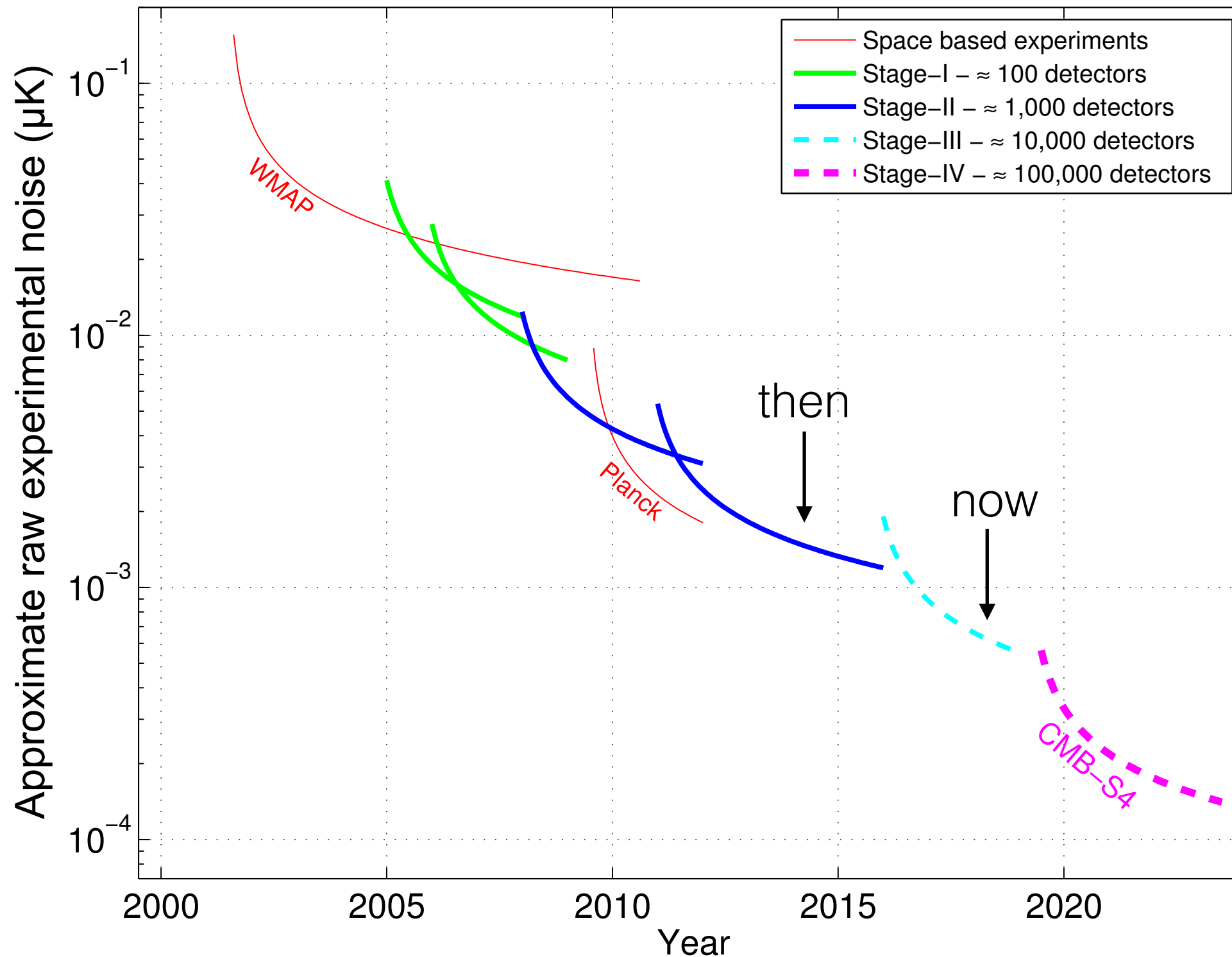
# CMB-S4(?)



# CMB-S4

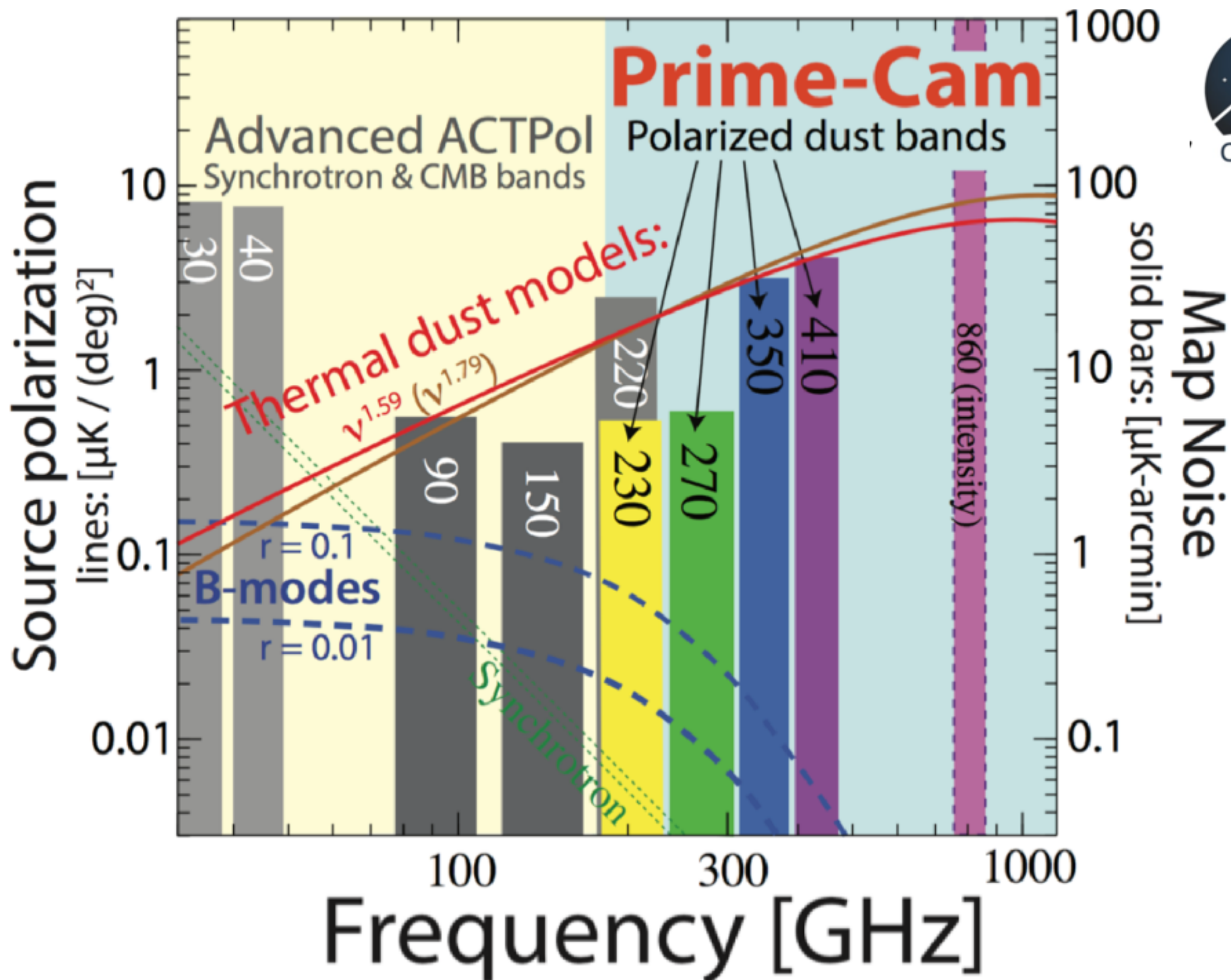
Next Generation CMB Experiment

## CMB Stages



# The Biggest Enemy: Polarised Dust Emission

- The upcoming data will **NOT** be limited by statistics, but by systematic effects such as the Galactic contamination
- **Solution**: Observe the sky at multiple frequencies, especially at high frequencies ( $>300$  GHz)
- This is challenging, unless we have a superb, high-altitude site with low water vapour
- CCAT-p!



# Where is CCAT-p?

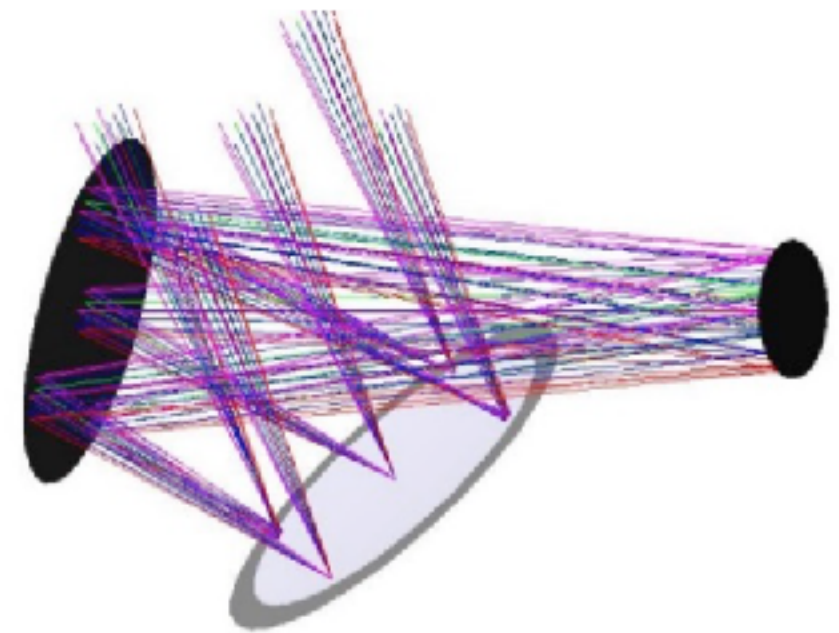
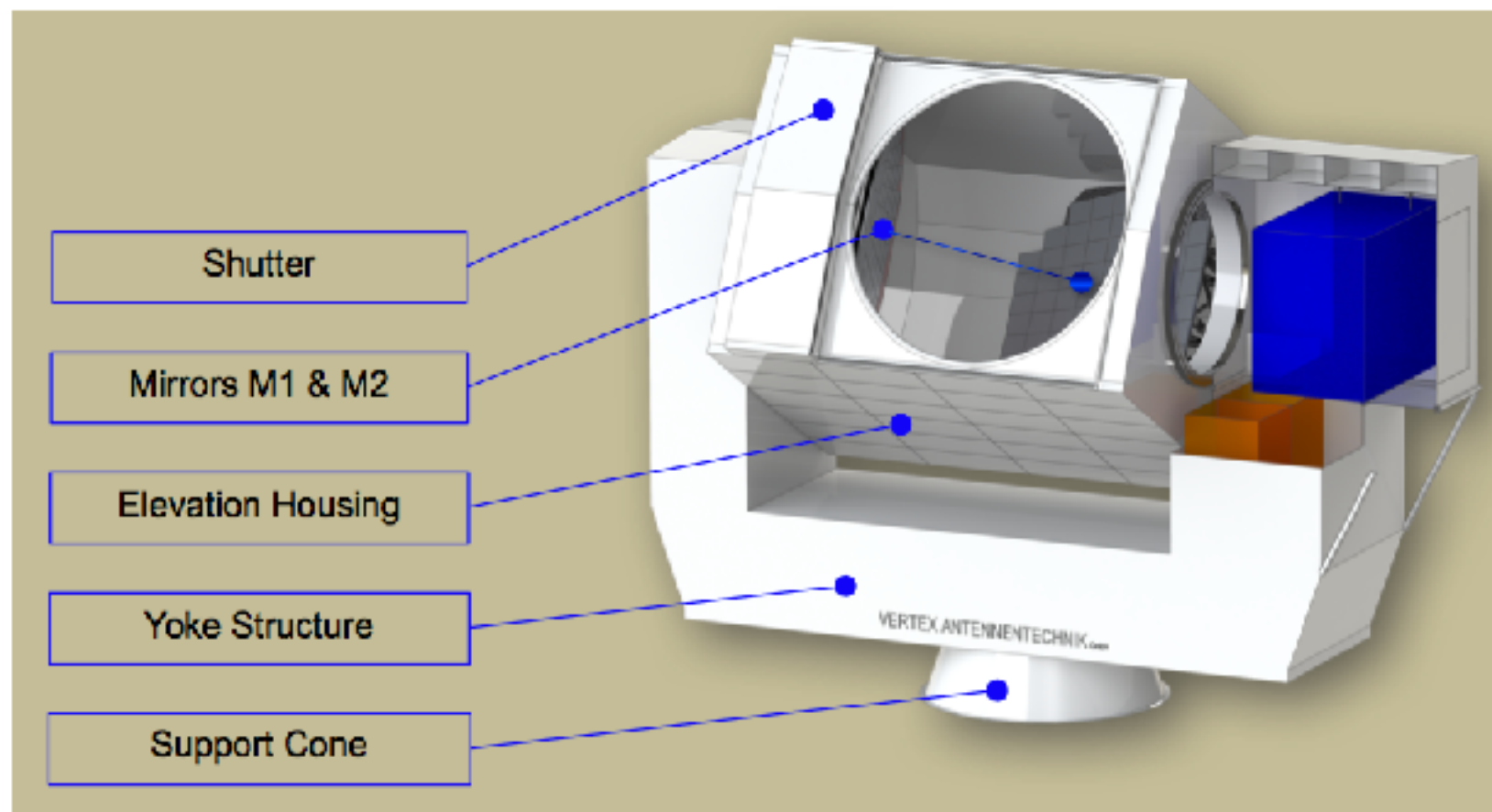
**Cerro Chajnantor at 5600 m w/ TAO**





# What is CCAT-p?

CCAT-prime is a high surface accuracy /  
throughput 6 m submm (0.3-3mm) telescope



Cornell U. + German consortium + Canadian consortium + ...

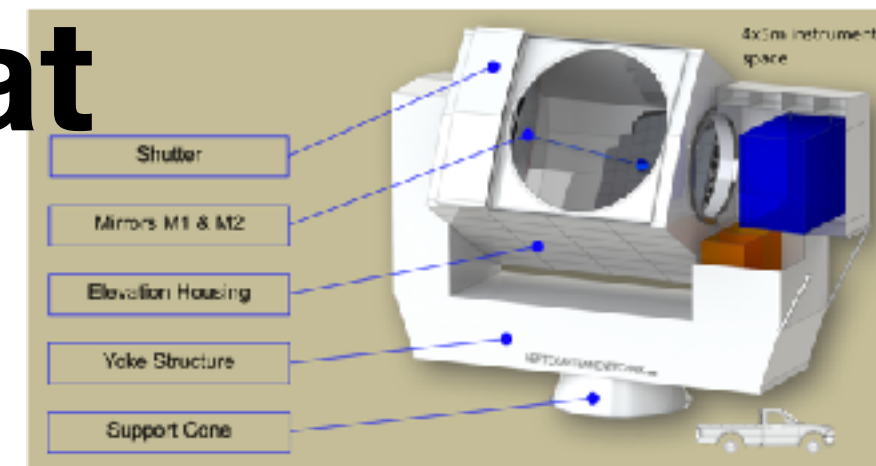
# A Game Changer

- **CCAT-p**: 6-m, **Cross-dragone** design, on Cerro Chajnantor (5600 m)

- **Germany makes great telescopes!**

CCAT-prime

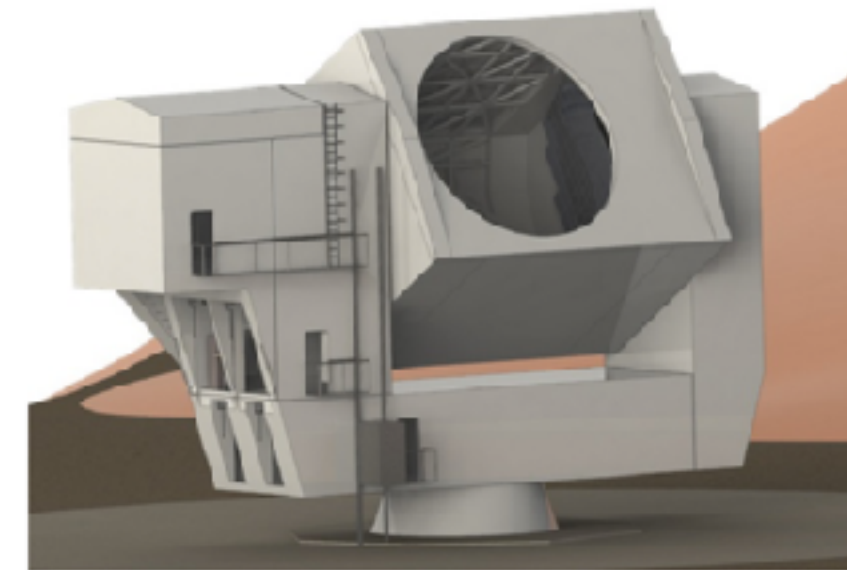
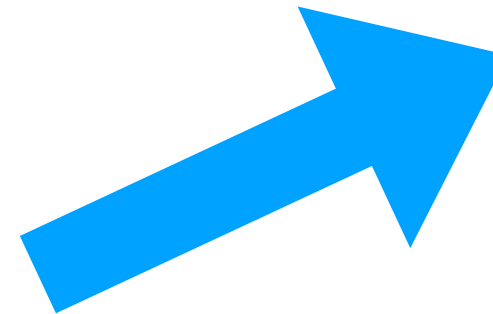
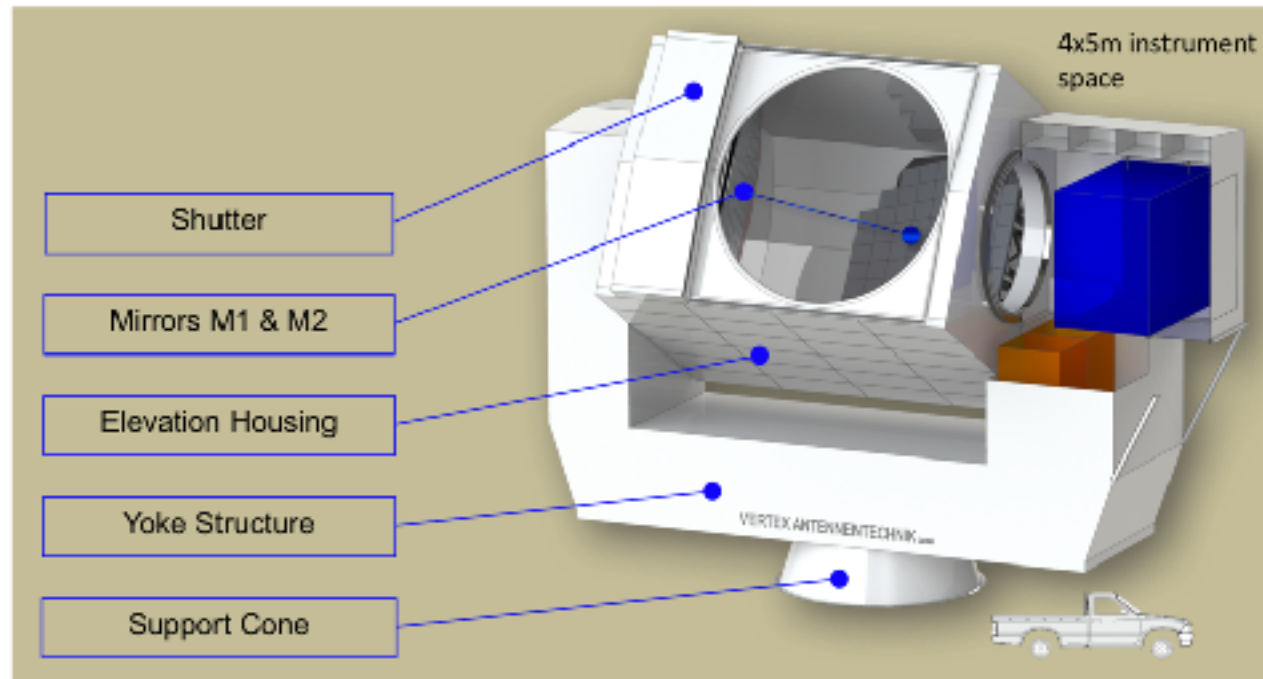
designed and built by Vertex Antennentechnik GmbH, Duisburg



- Design study completed, and the contract has been signed by “VERTEX Antennentechnik GmbH”
  - CCAT-p is a great opportunity for Germany to make significant contributions towards the CMB S-4 landscape (both US and Europe) by providing telescope designs and the “lessons learned” with prototypes.

## CCAT-prime

designed and built by Vertex Antennentechnik GmbH, Duisburg



*A rendering of the unique and powerful radio telescope. Image courtesy of VERTEX ANTENNENTECHNIK.*

## Simons Observatory (USA)

in collaboration

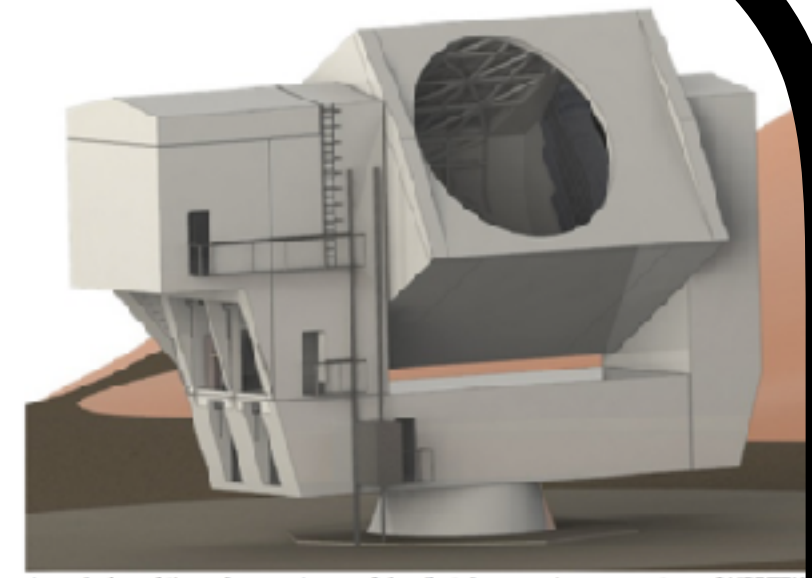
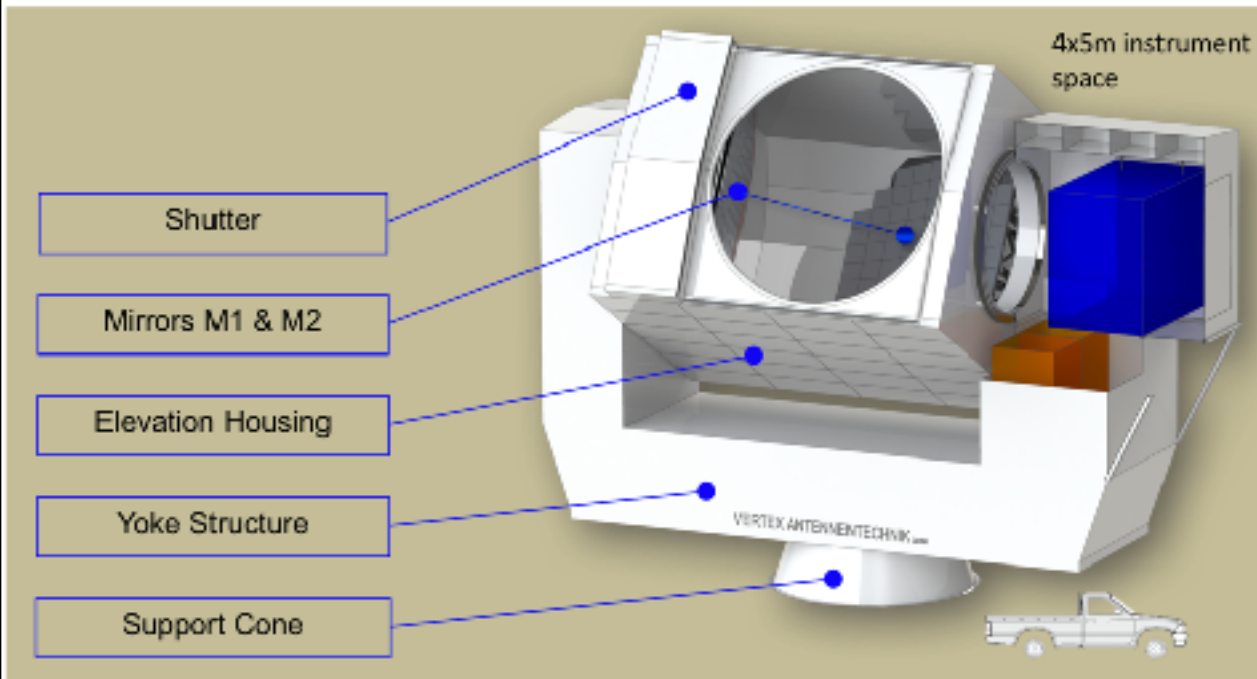


South Pole?

# This could be “CMB-S4”

## CCAT-prime

designed and built by Vertex Antennentechnik GmbH, Duisburg



A rendering of the unique and powerful radio telescope. Image courtesy of VERTEX ANTENNENTECHNIK.

**Simons Observatory  
(USA)**

in collaboration



**South Pole?**

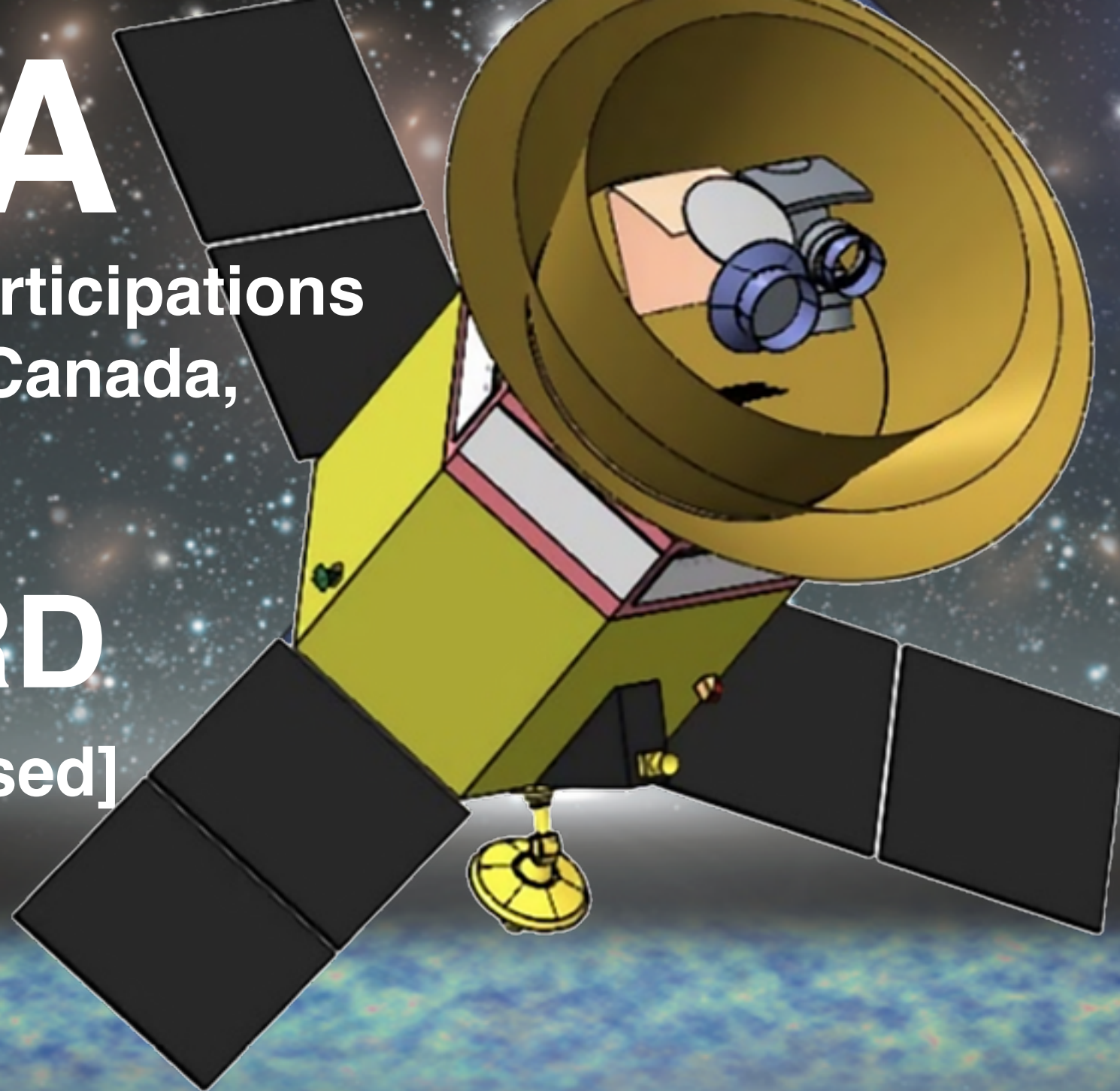
**To have even more  
frequency coverage...**

# JAXA

+ possible participations  
from USA, Canada,  
Europe

## LiteBIRD

2027– [proposed]



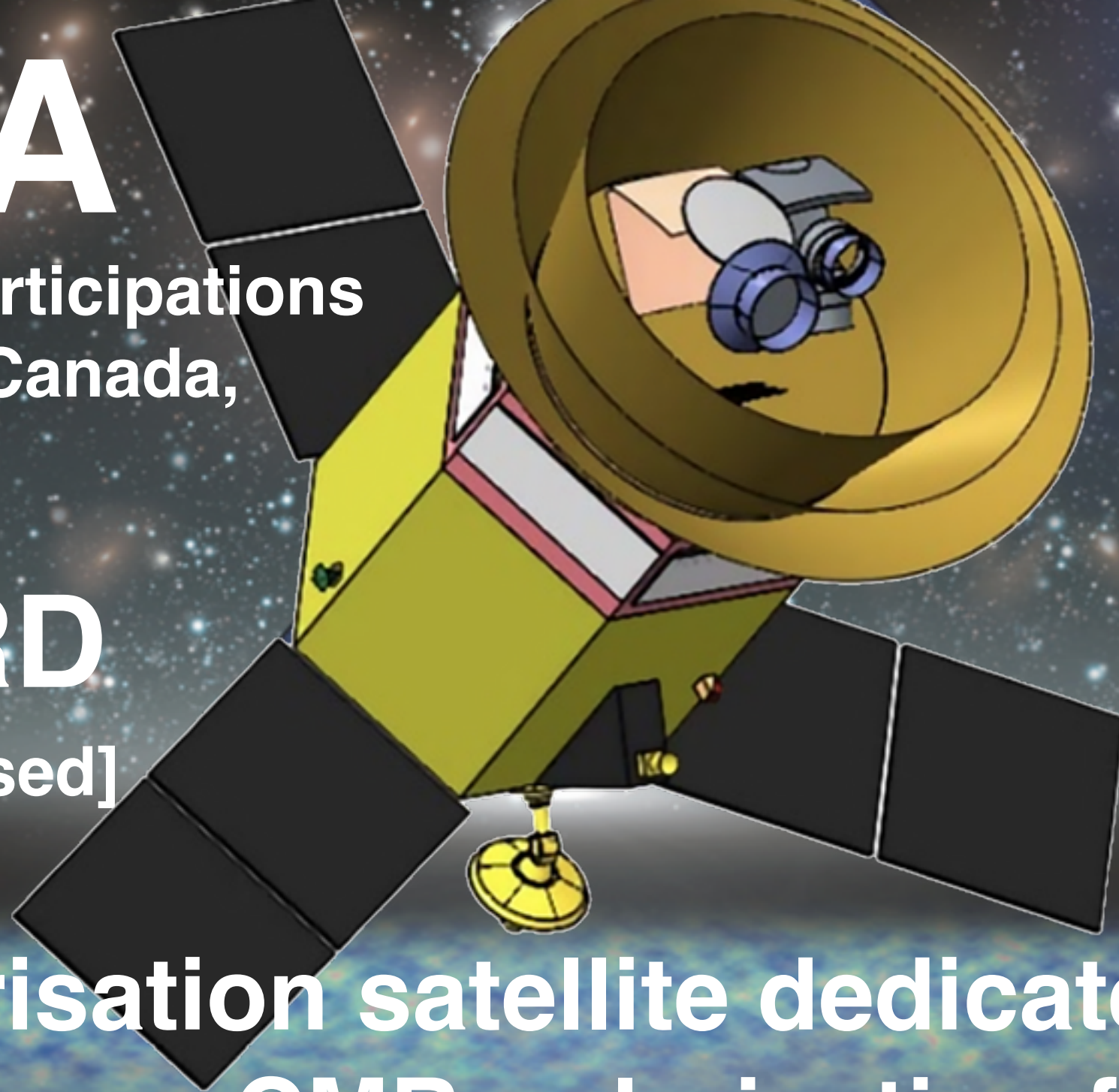
Target:  $\delta r < 0.001$  (68%CL)

# JAXA

+ possible participations  
from USA, Canada,  
Europe

## LiteBIRD

2027– [proposed]



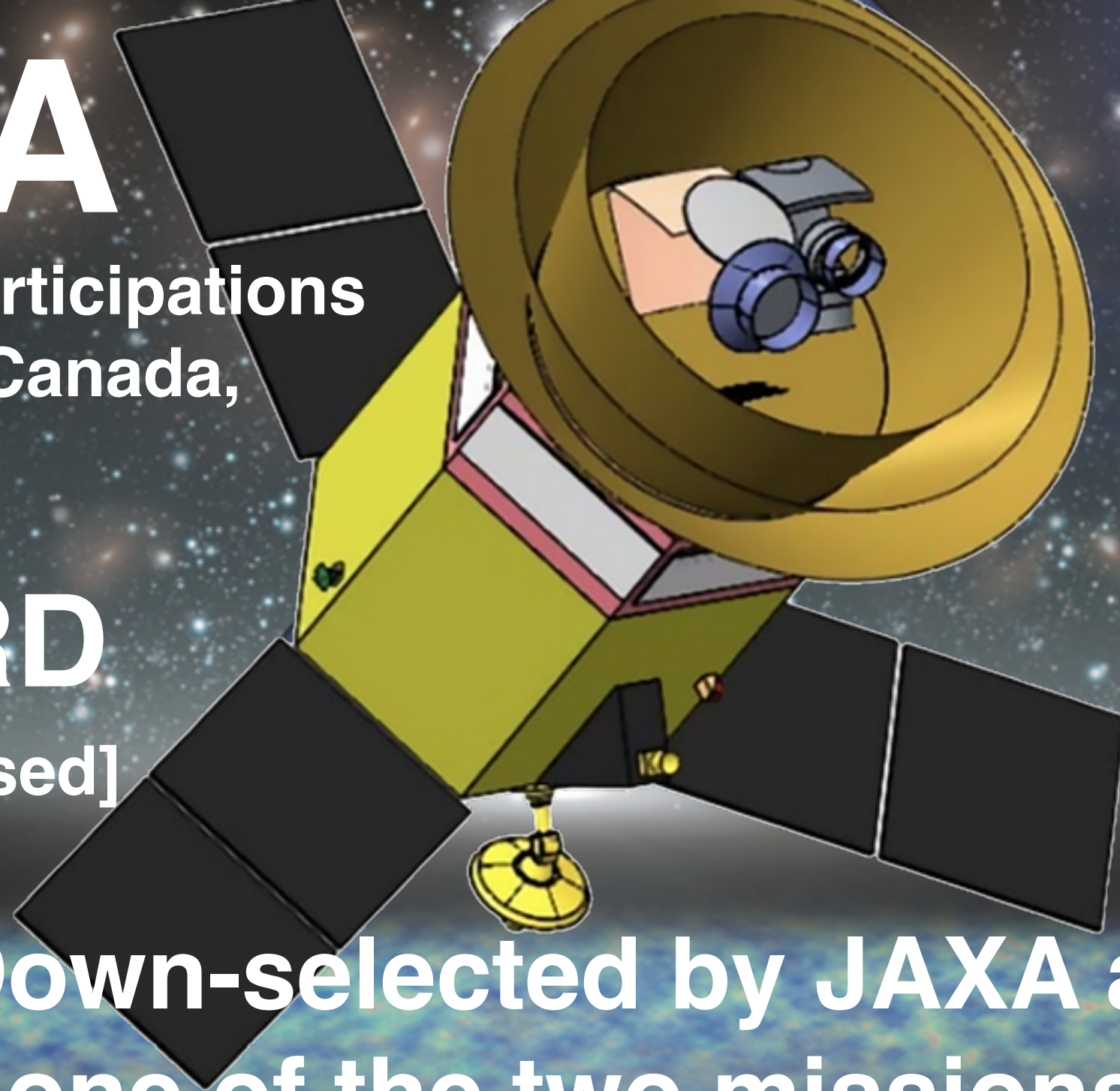
**Polarisation satellite dedicated to  
measure CMB polarisation from  
primordial GW, with a few thousand  
super-conducting detectors in space**

# JAXA

+ possible participations  
from USA, Canada,  
Europe

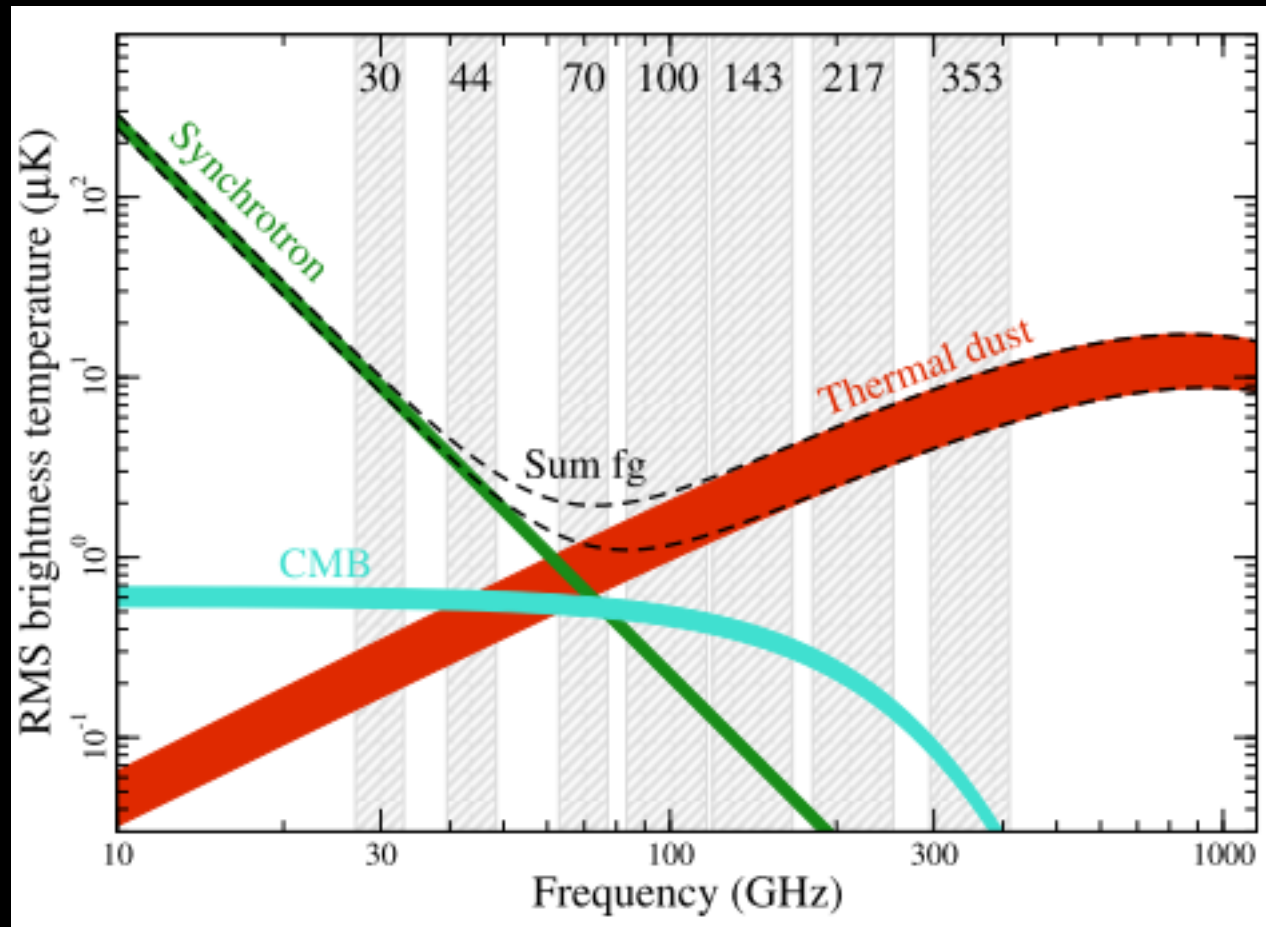
## LiteBIRD

2027– [proposed]

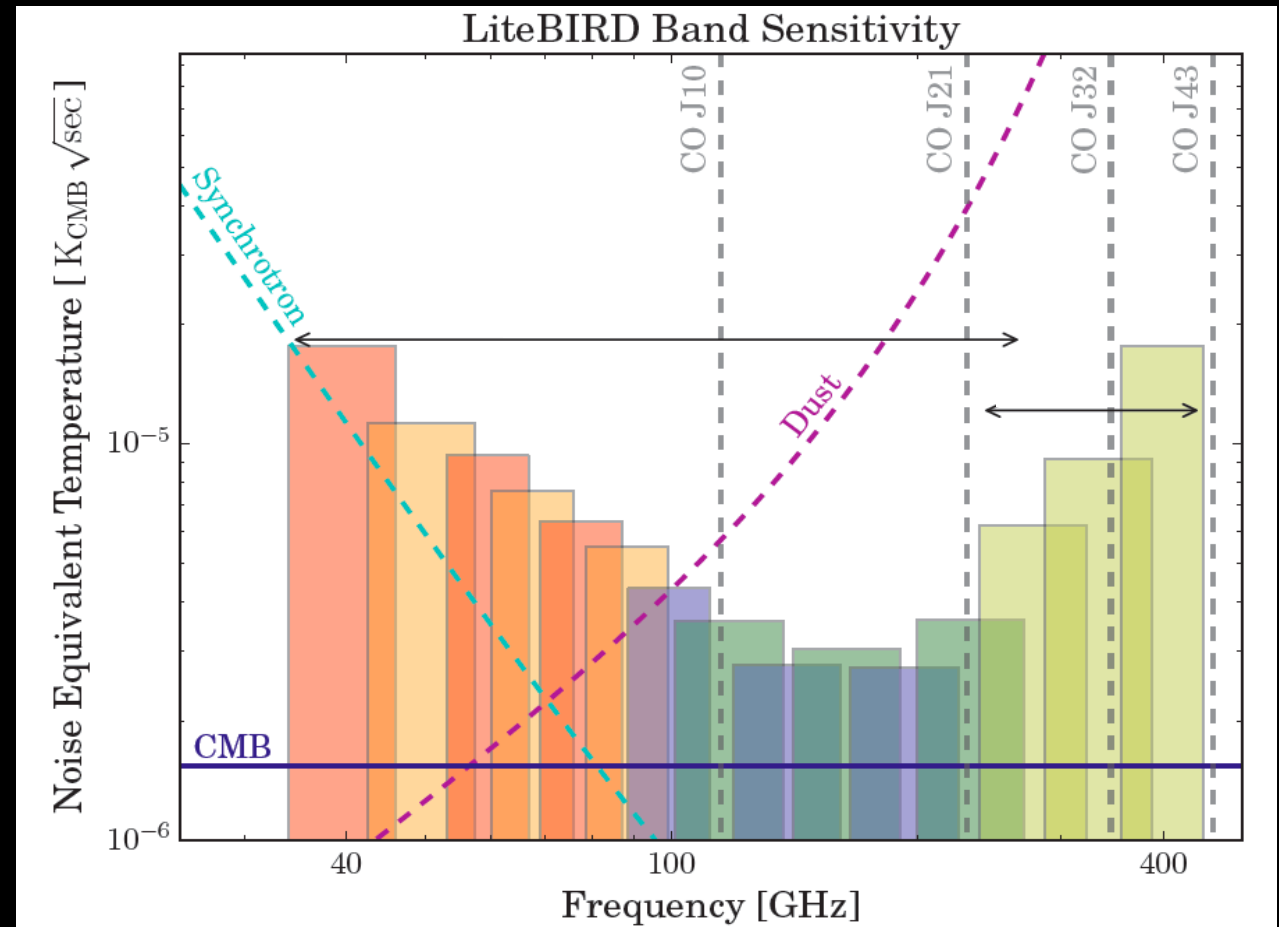


**Down-selected by JAXA as  
one of the two missions  
competing for a launch in mid 2020's**

# Foreground Removal



Polarized galactic emission (Planck X)

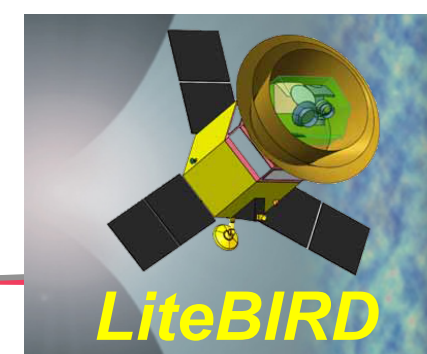


LiteBIRD: 15 frequency bands

- Polarized foregrounds
  - Synchrotron radiation and thermal emission from inter-galactic dust
  - Characterize and remove foregrounds
- 15 frequency bands between 40 GHz - 400 GHz
  - Split between Low Frequency Telescope (LFT) and High Frequency Telescope (HFT)
  - LFT: 40 GHz – 235 GHz
  - HFT: 280 GHz – 400 GHz

*Slide courtesy Toki Suzuki (Berkeley)*

# LiteBIRD Spacecraft



JAXA  
H3

4.5 m

LFT (Low frequency telescope) 34 – 161 GHz : Synchrotron + CMB

HFT (high frequency telescope) 89 – 448 GHz : CMB + Dust

**European Contribution**

LFT (5K)

**HFT (5K)**

Focal plane 0.1K

V-groove

30K

100K

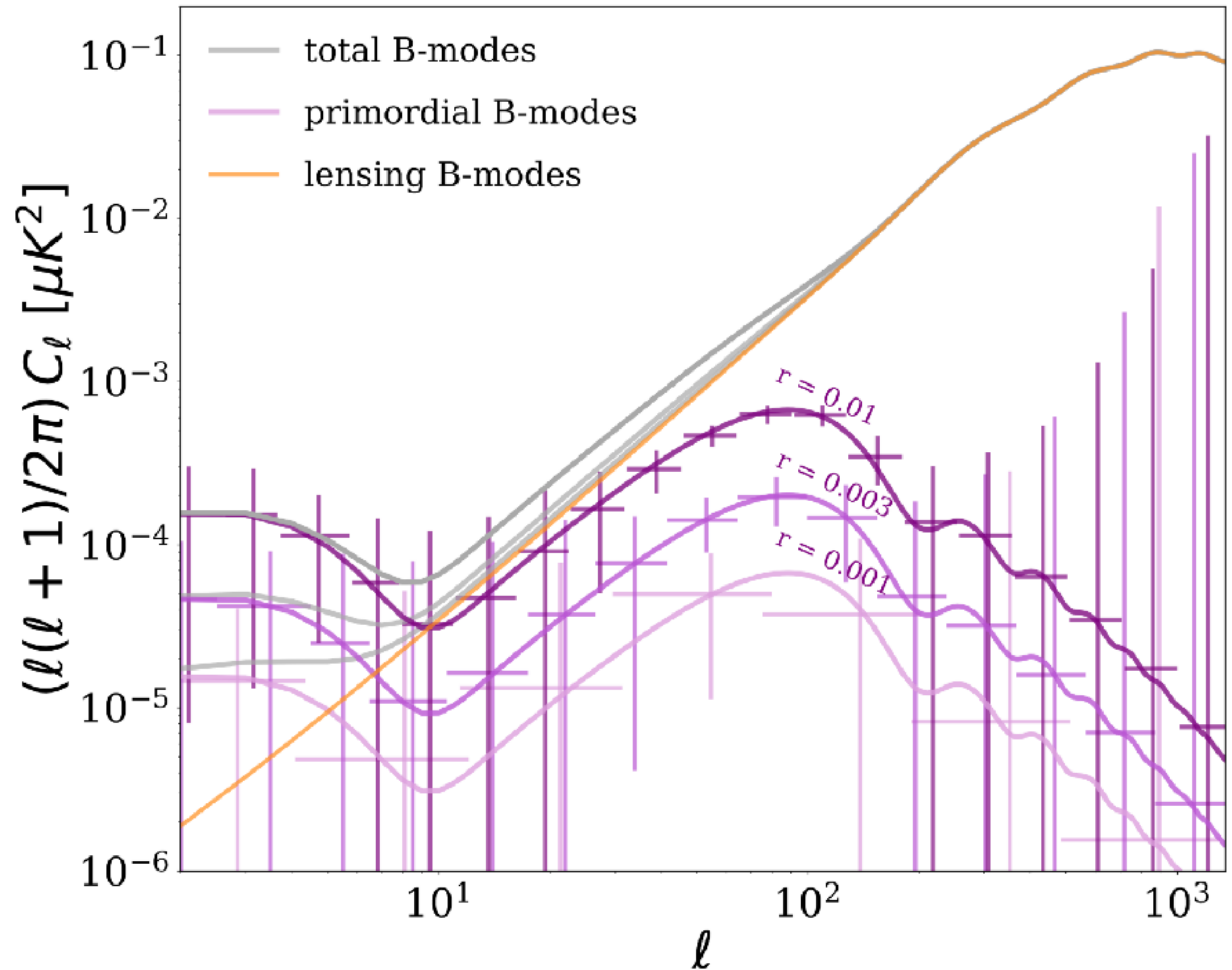
200K

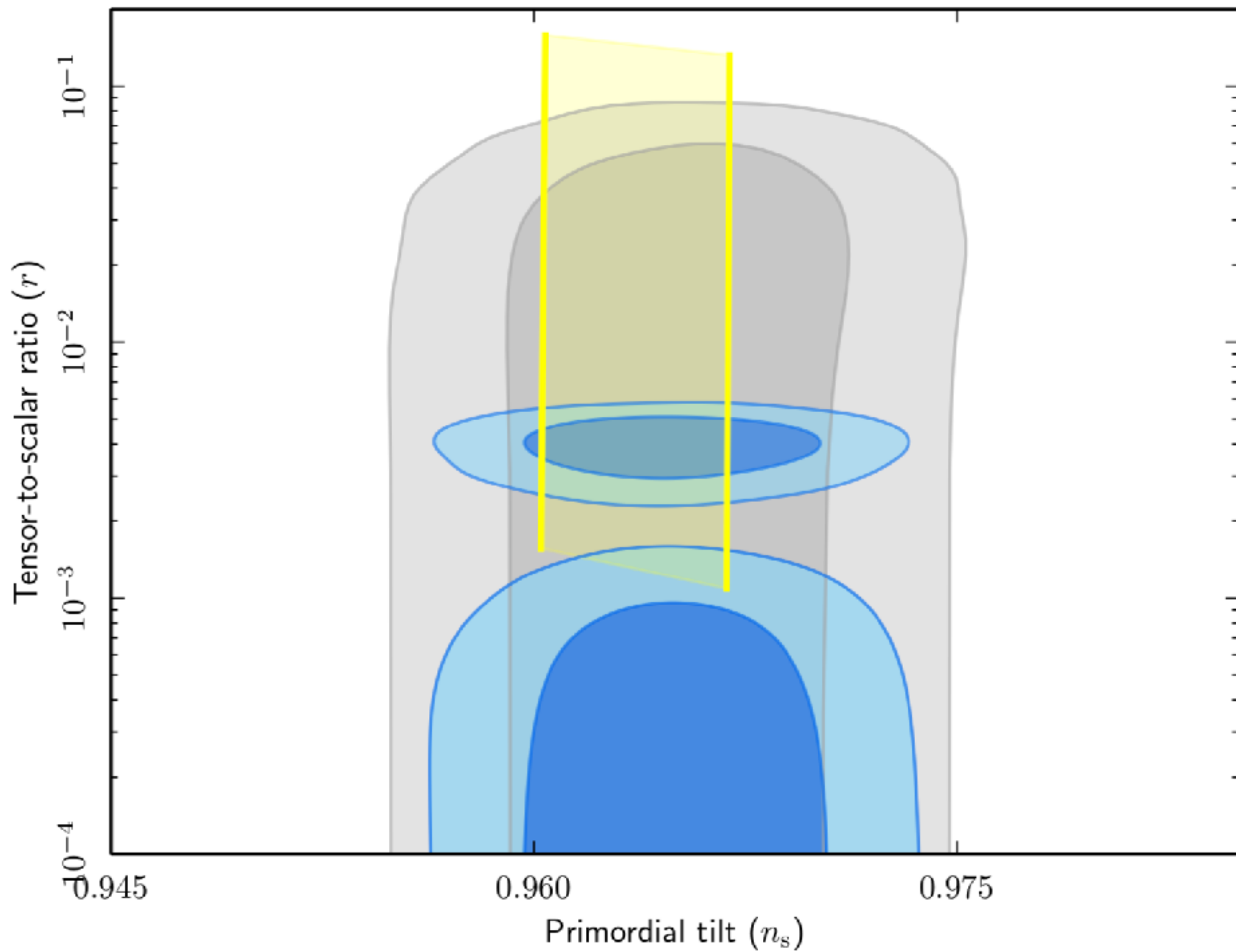
SVM/BUS

HG-antenna

PLM

radiators





# Summary

- **Next frontier**: Using CMB polarisation to find GWs from inflation.  
**Critical test of the physics of inflation!**
- With *CCAT-p*, we can remove the dust polarisation to reach  $r \sim 10^{-2}$  **reliably**, i.e., 10 times better than the current bound
- With *LiteBIRD* we plan to reach  $r \sim 10^{-3}$ , i.e., 100 times better than the current bound
- GW from vacuum or sources? An exciting window to new physics
  - Check not only for scale invariant, but also for chirality and **non-Gaussianity**

# SU(2), confronted

- The SU(2) model of Dimastrogiovanni et al. predicts:

$$f_{\text{NL}}^{\text{tens}} \approx \frac{125}{18\sqrt{2}} \frac{r^2}{\epsilon_B} \approx 2.5 \frac{r^2}{\Omega_A}$$

- The current 68%CL constraint is  $f_{\text{NL}}^{\text{tens}} = 400 \pm 1500$ 
  - This is already constraining!