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Relative geometric invariant theory

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Kummer's quartic

SURFER

The Kummer Quartic

In 1875, Eduard Kummer was the first person who explicitly asked the question of what the maximum number $\mu(d)$ of singularities on a surface of degree d are. In his case the degree was 4 and are called *quartics*.

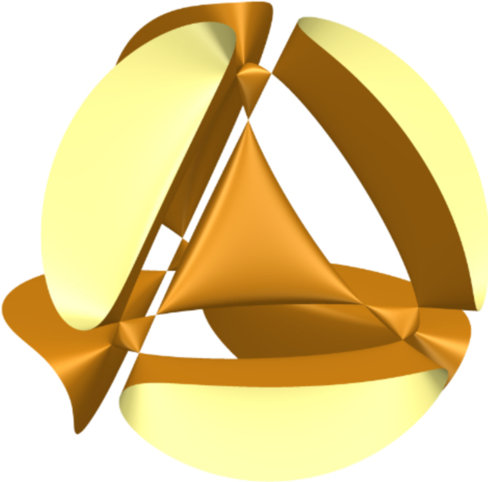
He showed that $\mu(4) = 16$. After that he studied quartics with 16 singularities in detail. A particularly beautiful family of such surfaces is given by:

$$(x^2 + y^2 + z^2 - \mu^2)^2 - \lambda\, y_0\, y_1\, y_2\, y_3,$$

where μ is a free parameter, and $\lambda = \frac{3\mu^2-1}{3-\mu^2}$; the y_i are the sides of a regular tetrahedron $y_0 = 1 - z - \sqrt{2}x$, $y_1 = 1 - z + \sqrt{2}x$, $y_2 = 1 + z + \sqrt{2}y$, $y_3 = 1 + z - \sqrt{2}y$ in order to make the surface symmetric. Not all members of this family have exactly 16 real singularities, although most of them do:

For some special values of the parameters, several of the singularities may coincide.

Start Info Colours

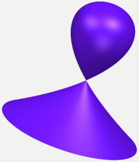
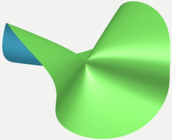
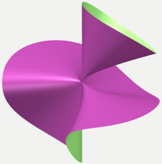


Semistable cubic surfaces

Theorem (Hilbert 1893)

A cubic form $f \in V_{3,3} \setminus \{0\}$ is semistable if and only if $V(f) \subset \mathbb{P}^3$ is smooth or has only isolated singuarities of type A_1 (conical nodes) and A_2 (binodes).

Double points of surfaces

Conical node	binode	uninode
A_1	$A_k,\ k \geq 2$	$D_k,\ k \geq 4,\ E_6,\ E_7,\ E_8$
		

The most singular semistable surface