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The **semiclassical approximation** to quantum gravity and its relation to **cosmological observations**

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Phys. Rev. D **93**, 104035 (2016), `arXiv:1511.05545`
Phys. Rev. D **94**, 123527 (2016), `arXiv:1611.02932`
Phys. Rev. D **99**, 104007 (2019), `arXiv:1903.01234`

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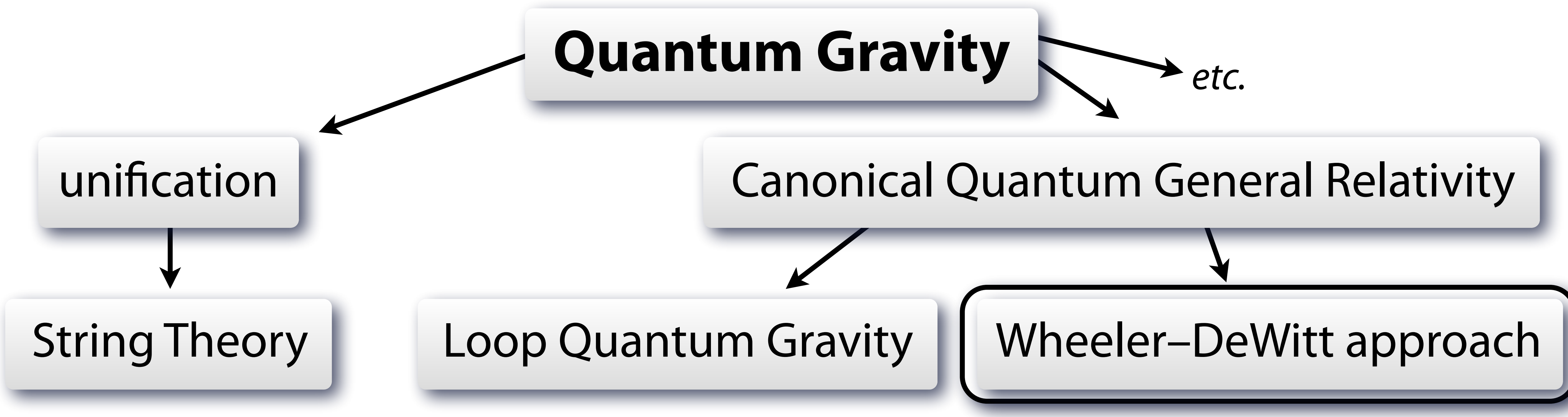
Outline

1. Introduction and motivation
2. The Wheeler–DeWitt approach to Quantum Gravity
3. Inflation and perturbations within quantum cosmology
4. The semiclassical approximation
5. Calculation of quantum-gravitational effects
   - ➔ de Sitter
   - ➔ slow-roll approximation
   - ➔ excited initial states
6. Comparison with other approaches
7. Summary and Outlook

Tests of theories of Quantum Gravity



- we have several approaches to Quantum Gravity at hand
- in order to decide which is the correct one, we need testable predictions
- *problem:* quantum-gravitational effects are suppressed by $\propto \dfrac{1}{m_{\mathrm{P}}^2}$

$$m_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}} \simeq 1.22 \times 10^{19}\,\mathrm{GeV}/c^2$$

- best chances to find sizeable QG effects ➔ inflationary universe

➡ ***Can QG effects be observed in the Cosmic Microwave Background?***

A conservative approach to Quantum Gravity

- Schrödinger equation is the quantum wave equation that leads to the classical Hamilton–Jacobi equation in the semiclassical limit

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\Psi = \hat{H}\Psi \qquad \longrightarrow \qquad H + \frac{\partial S}{\partial t} = 0$$

➡ What is the quantum wave equation that immediately gives Einstein's equations (in their Hamiltonian form) in the semiclassical limit?

➡ **Wheeler–DeWitt equation:**

$$\hat{\mathcal{H}}\,\Psi[h_{ij}(\mathbf{x}), \phi(\mathbf{x})] = 0$$

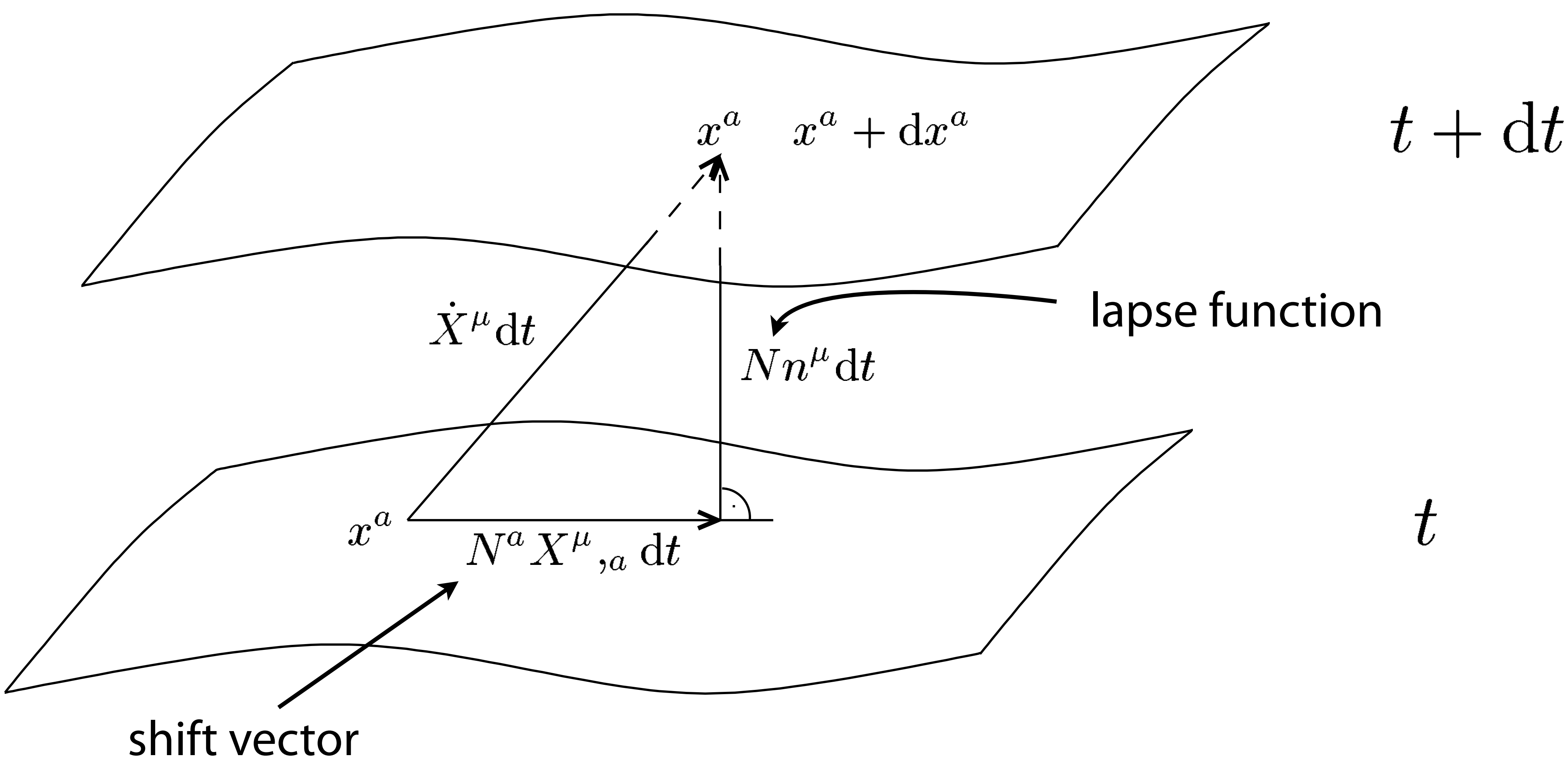
wave functional 3-metric matter field

$$\frac{1}{\sqrt{h}}\left(\frac{1}{2}h_{ij}h_{kl} - h_{ik}h_{jl}\right)\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}} + \sqrt{h}\,{}^{(3)}R = 0$$

B. S. DeWitt, Phys. Rev. **160**, 113 (1967); J. A. Wheeler, in: Battelle rencontres, 242 (1968).

Derivation of the Wheeler–DeWitt equation

- reformulate General Relativity as a Hamiltonian theory by means of the **ADM formalism**
  - 3+1 decomposition by foliating spacetime $(\mathcal{M}, g)$ into a set of three-dimensional space-like hypersurfaces Σ_t with an induced spatial metric h_{ij}



C. Kiefer, *Quantum geometrodynamics: whence, whither?*, arXiv: 0812.0295

Derivation of the Wheeler–DeWitt equation

- canonical variables: h_{ij} and its conjugate momentum p^{ij}
- Hamiltonian: $H^{\mathrm{g}} = \int \mathrm{d}^3x\,\left(N\,\mathcal{H}^{\mathrm{g}}_{\perp} + N^i\,\mathcal{H}^{\mathrm{g}}_{i}\right)$
- dynamics classically given by constraints: $\mathcal{H}^{\mathrm{g}}_{\perp} \approx 0 \qquad \mathcal{H}^{\mathrm{g}}_{i} \approx 0$

➡ quantization: $\left[\hat{h}_{ij}(\mathbf{x}), \hat{p}^{kl}(\mathbf{y})\right] = \mathrm{i}\,\hbar\,\delta^{k}_{(i}\delta^{l}_{j)}\,\delta(\mathbf{x},\mathbf{y})$

$$\hat{h}_{kl}\,\Psi[h_{ij}] = h_{kl}\,\Psi[h_{ij}] \qquad\qquad \hat{p}^{kl}\,\Psi[h_{ij}] = -\,\mathrm{i}\,\hbar\,\frac{\delta}{\delta h_{kl}}\Psi[h_{ij}]$$

- ▸ Hamiltonian constraint: $\mathcal{H}^{\mathrm{g}}_{\perp}\,\Psi[h_{ij}(\mathbf{x})] = 0$
- ▸ diffeomorphism constraint: $\mathcal{H}^{\mathrm{g}}_{i}\Psi[h_{ij}(\mathbf{x})] = 0$

➡ Wheeler–DeWitt equation follows from Hamiltonian constraint:

$$\left[-\,16\pi G\,\hbar^2\,G_{ijkl}\,\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \frac{\sqrt{h}}{16\pi G}\left({}^{(3)}R - 2\,\Lambda\right) + \mathcal{H}_{\mathrm{mat}}[h_{ij},\phi]\right]\Psi[h_{ij},\phi] = 0$$

DeWitt metric $\qquad$ $\det(h_{ij})$ $\qquad$ 3-dim. Ricci scalar $\qquad$ cosmol. constant $\qquad$ matter field

Wheeler–DeWitt equation

$$\left[-16\pi G\,\hbar^2\,G_{ijkl}\,\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}-\frac{\sqrt{h}}{16\pi G}\left({}^{(3)}R-2\,\Lambda\right)+\mathcal{H}_{\rm mat}[h_{ij},\phi]\right]\Psi[h_{ij},\phi]=0$$

DeWitt metric $\det(h_{ij})$ 3-dim. Ricci scalar cosmol. constant matter field

- timeless (GR: *dynamical time* **vs.** QM: *absolute time* ➔ QG: *no time*)

- intrinsic time can be recovered in the semiclassical limit
  ➔ Born–Oppenheimer approximation with respect to $m_{\rm P}^2 \propto G^{-1}$

$\mathcal{O}(m_{\rm P}^2)$: Hamilton–Jacobi equation of General Relativity

$\mathcal{O}(m_{\rm P}^0)$: functional Schrödinger equation for matter field; WKB time
➔ recovery of QFT in curved spacetime

$\mathcal{O}(m_{\rm P}^{-2})$: quantum-gravitational correction terms to Schrödinger eq.

details: Kiefer and Singh, Phys. Rev. D **44**, 1067 (1991).

➡ *WDW equation might not hold at the most fundamental level, but can be used as an effective equation to study conceptual questions in QG*

The road to Quantum Cosmology

- full Wheeler–DeWitt equation is mathematically difficult to handle

➡ quantization of a symmetry-reduced model of the universe

- consider a spatially flat homogeneous and isotropic universe

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\Omega_3^2$$

with a minimally coupled scalar field ϕ with potential $\mathcal{V}(\phi)$

- infinitely many degrees of freedom of *"superspace"* are reduced to two:

➔ scale factor a and scalar field ϕ ➔ ***<u>mini</u>superspace***

➡ Wheeler–DeWitt equation:

$$\frac{\hbar^2}{2}\left(\frac{4\pi G}{3a^2}\frac{\partial}{\partial a}\left(a\,\frac{\partial}{\partial a}\right) - \frac{1}{a^3}\,\frac{\partial^2}{\partial \phi^2}\right)\Psi(a,\phi) + a^3\,\mathcal{V}(\phi)\,\Psi(a,\phi) = 0$$

➡ ***How can one calculate QG effects in the CMB anisotropies from this?***

Overview: The WDW eq. and its semiclassical approx.

- de Sitter universe with scale factor a, $\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right)$, constant scalar field leading to constant Hubble parameter H_0, with perturbations v_k

$\hbar = c = 1$

$$\Bigg[\underbrace{\frac{1}{2a}\boxed{\frac{1}{m_\mathrm{P}^2}}\frac{\partial}{\partial a}\left(a\,\frac{\partial}{\partial a}\right) + \frac{a^4}{2}\boxed{m_\mathrm{P}^2}H_0^2}_{} + \sum_k \mathcal{H}_k^{\mathrm{pert}}(v_k)\Bigg]\,\Psi(a, v_k) = 0$$

$$\boxed{m_\mathrm{P}^2 := \frac{3}{4\pi G}}$$

semiclassical approximation

$\mathcal{O}(m_\mathrm{P}^2)$: classical dynamics

$$\frac{\partial}{\partial\eta} = a^2 H_0^2\,\frac{\partial}{\partial a}$$

$\mathcal{O}(m_\mathrm{P}^0)$: QM/QFT on curved spacetime

$$\mathrm{i}\,\frac{\partial}{\partial\eta}\,\psi_k^{(0)} = \mathcal{H}_k\psi_k^{(0)}$$

Kiefer and Singh, Phys. Rev. D **44**, 1067 (1991).

$\mathcal{O}(m_\mathrm{P}^{-2})$: QG corrections

$$\mathrm{i}\,\frac{\partial}{\partial\eta}\,\psi_k^{(1)} = \mathcal{H}_k\psi_k^{(1)} - \frac{\psi_k^{(1)}}{2\,m_\mathrm{P}^2\,\psi_k^{(0)}}\left[\frac{\left(\mathcal{H}_k\right)^2}{V}\,\psi_k^{(0)} + \mathrm{i}\,\frac{\partial}{\partial\eta}\left(\frac{\mathcal{H}_k}{V}\right)\psi_k^{(0)}\right]$$

$V(\eta) = H_0^{-2}\eta^{-4}$

Background: The WDW eq. for an inflationary universe

- inflation modelled using a scalar field ϕ with potential $\mathcal{V}(\phi)$

  ➔ slow roll: $\dot{\phi}^2 \ll |\mathcal{V}(\phi)|$ ➔ slow-roll parameters:

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \delta = \epsilon - \frac{\dot{\epsilon}}{2H\epsilon} = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

- for a flat Friedmann–Lemaître universe
  with minimally coupled scalar field

$$\alpha := \ln(a/a_0)$$

➡ Wheeler–DeWitt equation:

$$\mathcal{H}_0\Psi(\alpha,\phi) = \frac{1}{2}\,\mathrm{e}^{-2\alpha}\left[\frac{1}{m_\mathrm{P}^2}\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} + 2\,\mathrm{e}^{6\alpha}\,\mathcal{V}(\phi)\right]\Psi(\alpha,\phi) = 0$$

- de Sitter background: neglect ϕ-kinetic term and set $\mathcal{V} = \frac{1}{2}\,m_\mathrm{P}^2\,H_0^2$

- slow-roll background: rescale $\tilde{\phi} = m_\mathrm{P}^{-2}\,\phi$

$$\mathcal{V} = \frac{1}{2}\,m_\mathrm{P}^2 H^2\left(1 - \frac{\epsilon}{3}\right) \quad ➔ \quad V = \frac{1}{H_k^2\eta^4(k\eta)^{2\epsilon}}\left(1 + \frac{11\epsilon}{3}\right) + \mathcal{O}(2)$$

at horizon crossing

Adding scalar and tensor perturbations

- origin of CMB anisotropies: quantum fluctuations “amplified” by inflation
  - gauge-invariant ***scalar*** perturbations to the metric

$$\mathrm{d}s^2 = a^2(\eta)\left\{-\left(1-2\boxed{A}\right)\mathrm{d}\eta^2 + 2\left(\boxed{\partial_i B}\right)\mathrm{d}x^i\mathrm{d}\eta + \left[\left(1-2\boxed{\psi}\right)\delta_{ij} + 2\partial_i\partial_j \boxed{E}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

combined with perturbations of the scalar field ϕ

$$\delta\phi^{(\mathrm{gi})}(\eta,\mathbf{x}) = \delta\phi + \phi'\left(B - E'\right)$$

- *additionally:* ***tensor*** perturbations ➔ primordial gravitational waves

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \left(\delta_{ij} + \boxed{h_{ij}}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

➡ gauge-invariant *Mukhanov–Sasaki* variable $\boxed{v_k \propto a\,\delta x^{(\mathrm{gi})}_{\mathrm{S,T}}}$

➡ for <u>each</u> mode (for <u>both</u> scalars and tensors), we get a WDW equation

$$\boxed{\left[\mathcal{H}_0 + \cancel{\sum_{\mathrm{S,T};k}}\,{}^{\mathrm{S,T}}\mathcal{H}_k\right]\Psi_k(\alpha, v_k) = 0}$$

$${}^{\mathrm{S,T}}\mathcal{H}_k = \frac{1}{2}\left[-\frac{\partial^2}{\partial v_k^2} + {}^{\mathrm{S,T}}\omega_k^2(\eta)\,v_k^2\right]$$

Semiclassical approximation

- Born–Oppenheimer approximation, WKB ansatz: $\Psi_k(\alpha, v_k) = \mathrm{e}^{\mathrm{i}\, S(\alpha, v_k)}$
- expansion of $S(\alpha, f_k)$: $S = m_{\mathrm{P}}^2\, S_0 + m_{\mathrm{P}}^0\, S_1 + m_{\mathrm{P}}^{-2}\, S_2 + \dots$
- insert WKB ansatz into WDW eq. and equate terms of equal power of m_{P}
  - $\mathcal{O}(m_{\mathrm{P}}^2)$: Hamilton–Jacobi equation ➔ Friedmann equation
  - $\mathcal{O}(m_{\mathrm{P}}^0)$: define $\psi_k^{(0)}(\alpha, v_k) := \gamma(\alpha)\, \mathrm{e}^{\mathrm{i}\, S_1(\alpha, v_k)}$

➔ introduce WKB conf. time ➔ **Schrödinger equation**

$$\frac{\partial}{\partial \eta} := -\,\mathrm{e}^{-2\alpha}\, \frac{\partial S_0}{\partial \alpha}\, \frac{\partial}{\partial \alpha}$$

$$\boxed{\mathrm{i}\, \frac{\partial}{\partial \eta}\, \psi_k^{(0)} = \mathcal{H}_k \psi_k^{(0)}}$$

  - $\mathcal{O}(m_{\mathrm{P}}^{-2})$: **quantum-gravitationally corrected Schrödinger equation**

$$\boxed{\mathrm{i}\, \frac{\partial}{\partial \eta}\, \psi_k^{(1)} = \mathcal{H}_k \psi_k^{(1)} - \frac{\psi_k^{(1)}}{2\, m_{\mathrm{P}}^2\, \psi_k^{(0)}} \left[\frac{\left(\mathcal{H}_k\right)^2}{V}\, \psi_k^{(0)} + \mathrm{i}\, \frac{\partial}{\partial \eta} \left(\frac{\mathcal{H}_k}{V} \right) \psi_k^{(0)} \right]}$$

Derivation of the power spectra in the *de Sitter* case

- Gaussian ansatz $\psi_k^{(0)}(\eta, v_k) = \mathcal{N}_k^{(0)}(\eta)\,\mathrm{e}^{-\frac{1}{2}\,\Omega_k^{(0)}(\eta)\,v_k^2}$ ➔ Schrödinger eq.
- we have to solve: $\mathrm{i}\,\Omega_k'^{(0)}(\eta) = \left(\Omega_k^{(0)}(\eta)\right)^2 - {}^{\mathrm{S,T}}\omega_k^2(\eta)$

$$= k^2 - \frac{2}{\eta^2}$$

➡ solution: $\Omega_k^{(0)}(\eta) = \dfrac{k^2\eta}{\mathrm{i} + k\eta} + \dfrac{\mathrm{i}}{\eta}$

- power spectrum for scalar perturbations can be obtained via

$$\mathcal{P}_\mathrm{S}^{(0)}(k) = \frac{G\,H_0^2}{\pi\,\epsilon}\,\frac{k^3\eta^2}{\mathfrak{Re}\Omega_k^{(0)}}$$

- superhorizon limit $-k\eta \to 0$ ➔ $\mathfrak{Re}\Omega_k^{(0)}(\eta) = \dfrac{k^3\eta^2}{k^2\eta^2+1} \longrightarrow k^3\eta^2$

➡ **power spectrum for scalar pert.:** $\boxed{\mathcal{P}_\mathrm{S}^{(0)}(k) = \left.\dfrac{G\,H_0^2}{\pi\,\epsilon}\right|_{k=H_0 a}}$

$$r^{(0)} = \frac{\mathcal{P}_\mathrm{T}^{(0)}(k)}{\mathcal{P}_\mathrm{S}^{(0)}(k)} = 16\,\epsilon$$

➡ **tensor perturb.:** $\boxed{\mathcal{P}_\mathrm{T}^{(0)}(k) = \dfrac{16\,G\,H_0^2}{\pi}\,\dfrac{k^3\eta^2}{\mathfrak{Re}\Omega_k^{(0)}} = \dfrac{16\,G\,H_0^2}{\pi}}$

The uncorrected power spectra in the *slow-roll* case

- slow-roll parameters enter in all kinds of expressions
  - conformal time: $\eta = -\frac{1}{Ha}(1+\epsilon) + \mathcal{O}(2)$
  - "frequencies": ${}^{\mathrm{S}}\omega_k^2(\eta) = k^2 - \frac{2+6\epsilon-3\delta}{\eta^2}$ ${}^{\mathrm{T}}\omega_k^2(\eta) = k^2 - \frac{2+3\epsilon}{\eta^2}$

➡ **power spectrum for scalar perturbations:** $\gamma := 2\epsilon - \delta$

$$\boxed{\mathcal{P}_{\mathrm{S}}^{(0)}(k) = \frac{G\,H_k^2}{\pi\,\epsilon}\left[1 - 2\epsilon + \gamma(4 - 2\gamma_{\mathrm{E}} - 2\ln(2))\right]}$$

➡ **power spectrum for tensor perturbations:**

$$\boxed{\mathcal{P}_{\mathrm{T}}^{(0)}(k) = \frac{16\,G\,H_k^2}{\pi}\left[1 - 2\epsilon + \epsilon(4 - 2\gamma_{\mathrm{E}} - 2\ln(2))\right]}$$

➡ **tensor-to scalar ratio:** $r^{(0)} = 16\,\epsilon + \mathcal{O}(2)$

Calculation of the quantum-gravitational corrections

$$\mathrm{i}\,\frac{\partial}{\partial\eta}\,\psi_k^{(1)} = \mathcal{H}_k\psi_k^{(1)} - \frac{\psi_k^{(1)}}{2\,m_\mathrm{P}^2\,\psi_k^{(0)}}\left[\frac{\left(\mathcal{H}_k\right)^2}{V}\,\psi_k^{(0)} + \mathrm{i}\,\frac{\partial}{\partial\eta}\left(\frac{\mathcal{H}_k}{V}\right)\psi_k^{(0)}\right]$$

- also assume Gaussianity for corrected Schrödinger equation:

$$\psi_k^{(1)}(\eta, v_k) = \mathcal{N}_k^{(1)}(\eta)\,\mathrm{e}^{-\frac{1}{2}\,\Omega_k^{(1)}(\eta)\,v_k^2}$$

- we have to solve: $\mathrm{i}\,\Omega_k'^{(1)}(\eta) = \left(\Omega_k^{(1)}(\eta)\right)^2 - \widetilde{\omega}_k^2(\eta)$

with $\widetilde{\omega}_k^2 := \omega_k^2 - \dfrac{1}{2m_\mathrm{P}^2 V}\left[\left(3\Omega_k^{(0)} - \mathrm{i}\,(\ln V)'\right)\left(\omega_k^2 - (\Omega_k^{(0)})^2\right) + 2\,\mathrm{i}\,\omega_k\,\omega_k'\right]$

- imaginary terms appear ➔ problem with unitarity

- additionally, numerical analysis of full equation with imaginary terms reveals that the solution oscillates heavily for early times

➡ no way to implement initial conditions ➔ neglect the imaginary terms

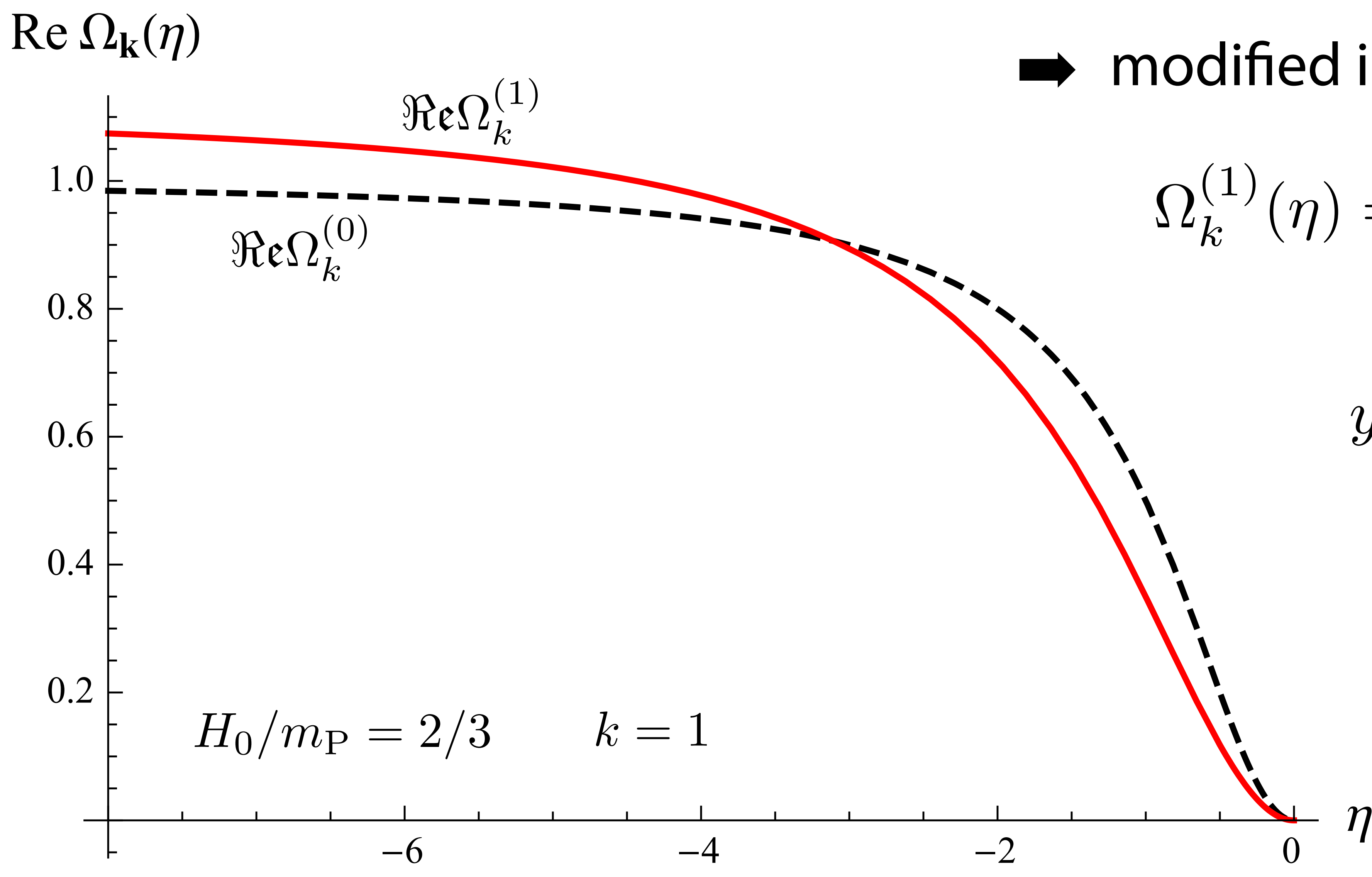
➡ justification: C. Kiefer and D. Wichmann, Gen. Rel. Grav. **50**, 66 (2018).

The *de Sitter* case: QG corrections – numerics

- equation we have to solve after removal of imaginary terms:

$$\mathrm{i}\,\Omega_k'^{(1)} = \left(\Omega_k^{(1)}\right)^2 - \omega_k^2 + \frac{H_0^2\eta^4}{2m_\mathrm{P}^2}\,\frac{k^3(11-k^2\eta^2)}{(1+k^2\eta^2)^3}$$

- numerical solution with Bunch–Davies initial conditions
  ➔ oscillation with constant amplitude around mean value $k + \dfrac{H_0^2}{4k\,m_\mathrm{P}^2}$



➡ modified initial conditions:

$$\Omega_k^{(1)}(\eta) = -\,\mathrm{i}\,\frac{y_k^{(1)\prime}(\eta)}{y_k^{(1)}(\eta)}$$

$$y_k^{(1)} \propto \mathrm{e}^{\mathrm{i}\beta_k\eta}$$

$$\beta_k \approx k + \frac{H_0^2}{4k\,m_\mathrm{P}^2}$$

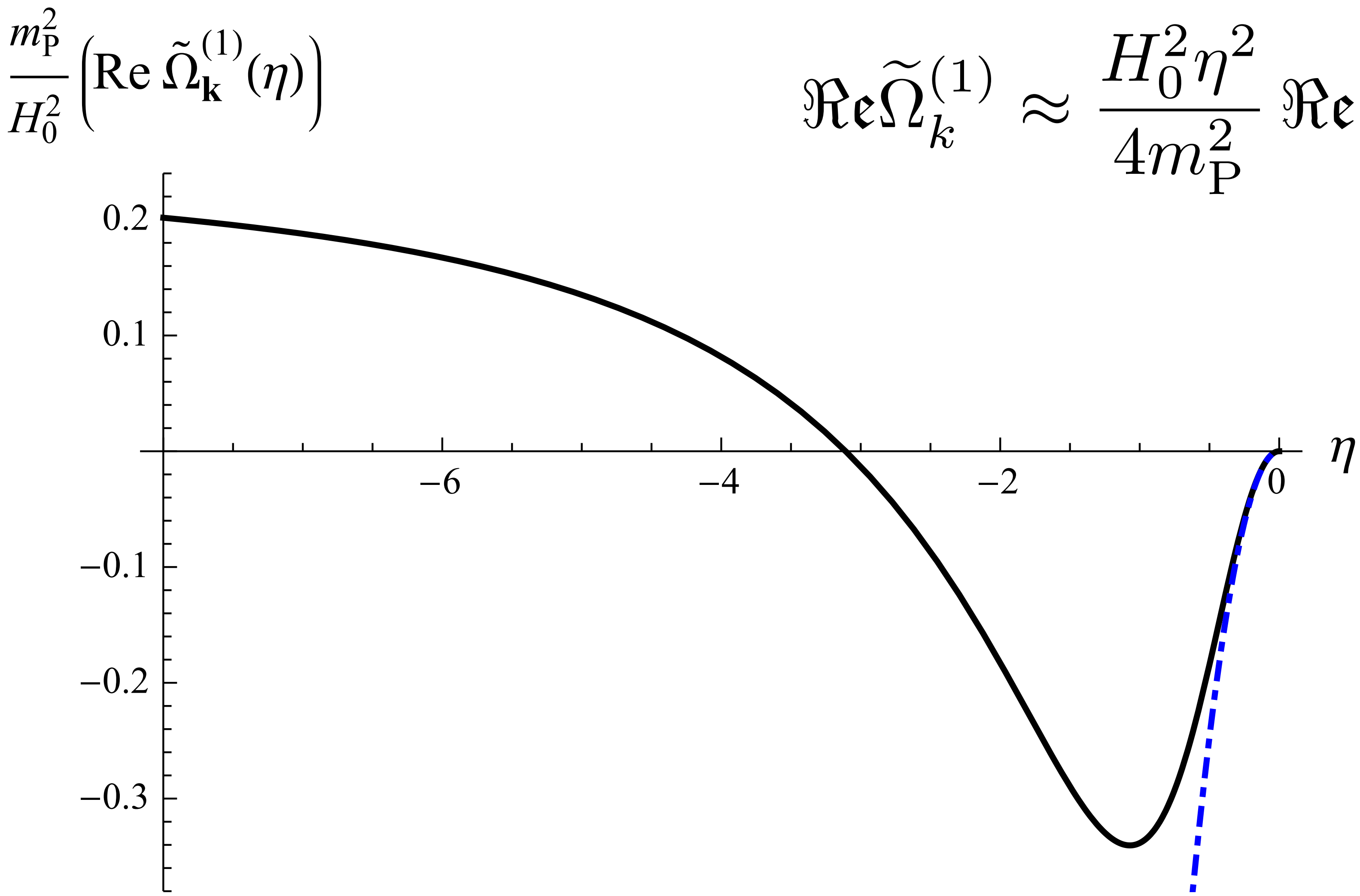
The *de Sitter* case: QG corrections – linearization

- find analytical solution at late times (superhorizon limit) $-k\eta \to 0$

➡ linearization around $\Omega_k^{(0)}$: $\quad \Omega_k^{(1)} = \Omega_k^{(0)} + \widetilde{\Omega}_k^{(1)}$

- we have to solve: $\mathrm{i}\,\widetilde{\Omega}_k'^{(1)} = 2\,\Omega_k^{(0)}\widetilde{\Omega}_k^{(1)} - \left(\widetilde{\omega}_k^2 - \omega_k^2\right)$

➡ behavior of the solution at $-k\eta \to 0$



$$\mathfrak{Re}\widetilde{\Omega}_k^{(1)} \approx \frac{H_0^2\eta^2}{4m_\mathrm{P}^2}\,\mathfrak{Re}\left(1 + 3\mathrm{e}^2\Gamma(0,2) + \frac{9\Gamma(0,-2)}{\mathrm{e}^2}\right)$$

$$\approx -0.988\,\frac{H_0^2\eta^2}{m_\mathrm{P}^2}$$

➔ agrees very well with numerical solution

The *de Sitter* case: QG corrections – power spectra

- QG corrected power spectrum:

$$\mathcal{P}^{(1)}_{\mathrm{S}}(k) = \frac{4\pi G}{a^2\,\epsilon}\,\frac{k^3}{4\pi^2}\,\left(\Re\mathfrak{e}\,\Omega^{(0)}_k + \Re\mathfrak{e}\,\widetilde{\Omega}^{(1)}_k\right)^{-1}$$

➡ **we get for both scalars and tensors:**

$$= \mathcal{P}^{(0)}_{\mathrm{S}}(k)\left[1 - \frac{\Re\mathfrak{e}\,\widetilde{\Omega}^{(1)}_k}{\Re\mathfrak{e}\,\Omega^{(0)}_k} + \mathcal{O}\left(\frac{H_0^4}{m_{\mathrm{P}}^4}\right)\right]$$

$$\boxed{\mathcal{P}^{(1)}_{\mathrm{S,T}}(k) = \mathcal{P}^{(0)}_{\mathrm{S,T}}(k)\left[1 + \frac{H_0^2}{m_{\mathrm{P}}^2}\,\frac{0.988}{k^3} + \mathcal{O}\left(\frac{H_0^4}{m_{\mathrm{P}}^4}\right)\right]}$$

- QG corrections lead to an ***enhancement*** of power on large scales
- upper bound on H_0^2/m_{P}^2 from tensor-to-scalar ratio $r \lesssim 0.11$

$$\frac{H_0^2}{m_{\mathrm{P}}^2} = \frac{2\,\mathcal{V}}{m_{\mathrm{P}}^4} \sim \frac{2\,r}{0.01}\left(\frac{10^{16}\,\mathrm{GeV}}{m_{\mathrm{P}}}\right)^4 \lesssim 1.74\times 10^{-10}$$

➡ upper limit:

$$\left|\frac{\mathcal{P}^{(1)}_{\mathrm{S,T}}(k) - \mathcal{P}^{(0)}_{\mathrm{S,T}}(k)}{\mathcal{P}^{(0)}_{\mathrm{S,T}}(k)}\right| \lesssim 1.72\times 10^{-10}\left(\frac{k_0}{k}\right)^3$$

The *slow-roll* case: Summary of the results

- **QG corrected power spectra:** $\mathcal{P}^{(1)}_{\mathrm{S,T}}(k) = \mathcal{P}^{(0)}_{\mathrm{S,T}}(k)\left\{1 + \Delta_{\mathrm{S,T}}\right\}$

$$\Delta_{\mathrm{S}} = \frac{H_k^2}{m_{\mathrm{P}}^2}\left(\frac{\bar{k}}{k}\right)^3 \left(0.988 + 3.14\,\epsilon - 2.56\,\delta\right)$$

$$\Delta_{\mathrm{T}} = \frac{H_k^2}{m_{\mathrm{P}}^2}\left(\frac{\bar{k}}{k}\right)^3 \left(0.988 + 0.58\,\epsilon\right)$$

reference scale

$$\bar{k} = k_* = 0.05\,\mathrm{Mpc}^{-1}$$

Planck pivot scale

- **QG corrected tensor-to-scalar ratio:**

$$r^{(1)} := \frac{\mathcal{P}^{(1)}_{\mathrm{T}}(k)}{\mathcal{P}^{(1)}_{\mathrm{S}}(k)} \approx 16\epsilon\left(1 + 2.56\,\frac{H_k^2}{m_{\mathrm{P}}^2}\left(\frac{\bar{k}}{k}\right)^3 (\delta - \epsilon)\right)$$

- upper bound on $H_{\mathrm{inf}}^2/m_{\mathrm{P}}^2$ from tensor-to-scalar ratio $r \lesssim 0.11$

$$\frac{H_{\mathrm{inf}}^2}{m_{\mathrm{P}}^2} = \frac{2\,\mathcal{V}}{m_{\mathrm{P}}^4} \sim \frac{2\,r}{0.01}\left(\frac{10^{16}\,\mathrm{GeV}}{m_{\mathrm{P}}}\right)^4 \lesssim 1.7\times 10^{-10}$$

- $n_{\mathrm{S}} \approx 0.968 \pm 0.006$ implies $\epsilon \lesssim 0.007$ and $\delta \approx -0.002$

➡ **upper limits:** $|\Delta_{\mathrm{S,T}}| \lesssim 2\times 10^{-10}$ $\quad \frac{\Delta_{\mathrm{S}}}{\Delta_{\mathrm{T}}} \approx 1.02$ $\quad \frac{\Delta r}{r^{(0)}} \approx -4\times 10^{-12}$

Excited initial states

- What if the perturbations start their evolution in an excited state?
- use number eigenstate for harm. oscillator with time-dep. frequency

$$\psi_N^{\mathrm{eig}}(y,t) \equiv \langle y|N\rangle = \frac{1}{\sqrt{\sigma(t)}} \exp\left(\frac{i\,m}{2}\,\frac{\sigma'(t)}{\sigma(t)}\,y^2\right) \varphi_N(y,t)$$

$$\varphi_N(y,t) = \frac{e^{-i(N+\frac{1}{2})\tau(t)}}{\sqrt{2^N N!}\,\pi^{\frac{1}{4}}}\, e^{-\frac{y^2}{2\sigma^2(t)}}\, \mathrm{H}_N\left(\frac{y}{\sigma(t)}\right)$$

- excitation number N_k

➡ **corrected power spectra for scalar and tensor perturbations:**

$${}^{\mathrm{S}}\mathcal{P}^{(1)}_{N_k}(k) = \frac{GH^2}{\pi\epsilon}\,(2N_k+1)\left(1+\frac{2N_k+1}{k^3}\,\frac{H^2}{m_{\mathrm{P}}^2}\,\beta_{N_k}\right)$$

$${}^{\mathrm{T}}\mathcal{P}^{(1)}_{N_k}(k) = \frac{16GH^2}{\pi}\,(2N_k+1)\left(1+\frac{2N_k+1}{k^3}\,\frac{H^2}{m_{\mathrm{P}}^2}\,\beta_{N_k}\right)$$

N_k even

$$\beta_{N_k} \approx \begin{cases} 0.9876 \\ 0.104 \end{cases}$$

N_k odd

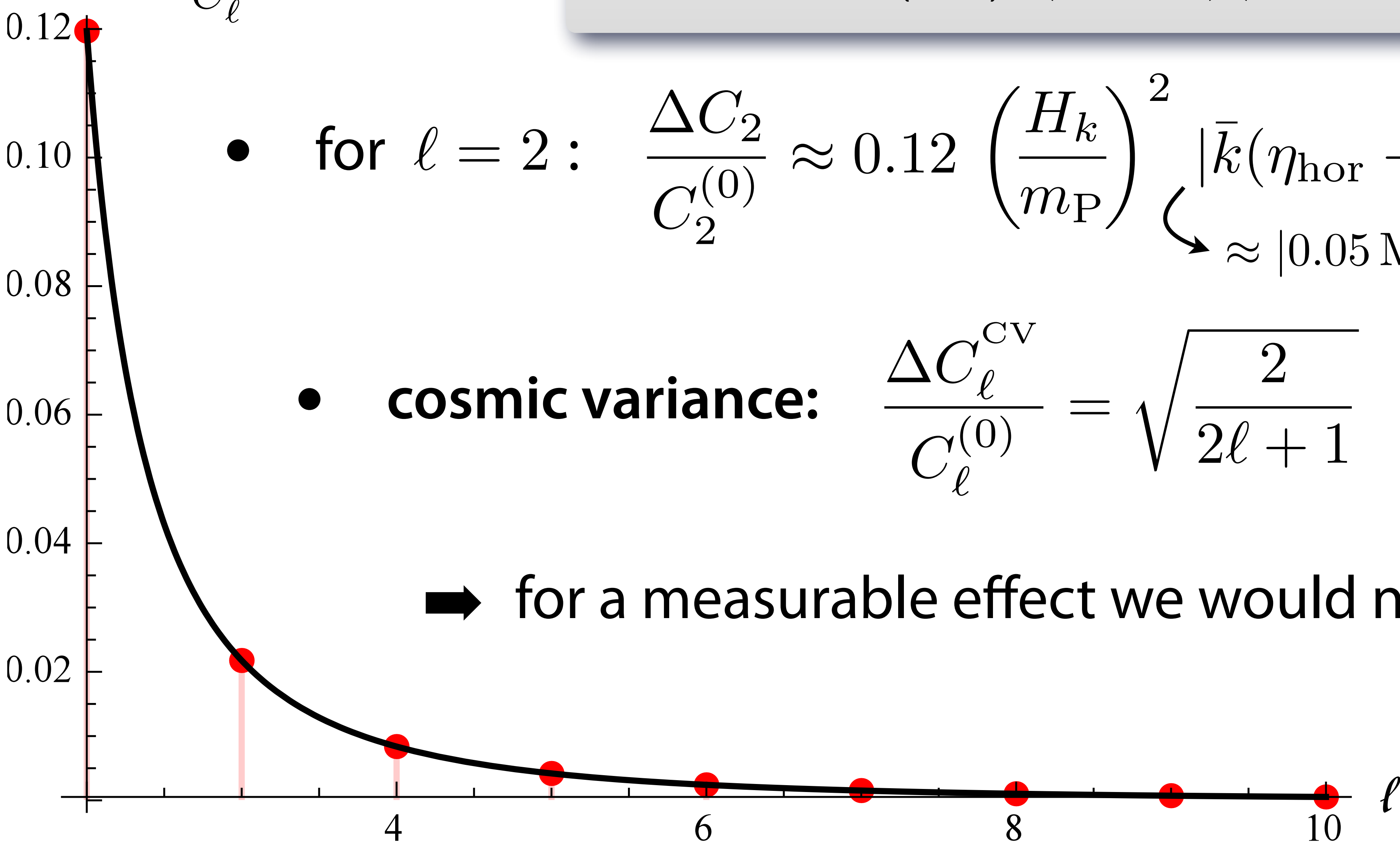
CMB temperature anisotropies

- $C_\ell^{(i)} = \int_0^\infty \frac{\mathrm{d}k}{k}\, \mathcal{P}_{\mathrm{S}}^{(i)}(k)\, \Theta_\ell^2(k)$ ➔ uncorr.: $C_\ell^{(0)} \approx \frac{1}{8\pi^2\epsilon}\left(\frac{H_k}{m_{\mathrm{P}}}\right)^2 \frac{1}{\ell(\ell+1)}$

- QG correction:

$$\Delta C_\ell \approx \frac{1}{4\pi^2}\int_0^\infty \frac{\mathrm{d}k}{k\,\epsilon}\left(\frac{H_k}{m_{\mathrm{P}}}\right)^4 \left(\frac{\bar{k}}{k}\right)^3 j_\ell^2(k[\eta_{\mathrm{hor}}-\eta_{\mathrm{rec}}])$$
$$\approx \frac{3}{4\pi\epsilon}\left(\frac{H_k}{m_{\mathrm{P}}}\right)^4 \frac{|\bar{k}(\eta_{\mathrm{hor}}-\eta_{\mathrm{rec}})|^3}{(2\ell-3)(2\ell-1)(2\ell+1)(2\ell+3)(2\ell+5)}$$

$$p_\ell := \frac{\Delta C_\ell}{C_\ell^{(0)}}$$



- for $\ell = 2$: $\frac{\Delta C_2}{C_2^{(0)}} \approx 0.12 \left(\frac{H_k}{m_{\mathrm{P}}}\right)^2 |\bar{k}(\eta_{\mathrm{hor}}-\eta_{\mathrm{rec}})|^3$

$\approx |0.05\,\mathrm{Mpc}^{-1}(700\,\mathrm{Mpc})|^3 \approx 5\times 10^4$

- **cosmic variance:** $\frac{\Delta C_\ell^{\mathrm{CV}}}{C_\ell^{(0)}} = \sqrt{\frac{2}{2\ell+1}}$ ➔ $\frac{\Delta C_2^{\mathrm{CV}}}{C_2^{(0)}} \approx 0.63$

➡ for a measurable effect we would need $\frac{H_k}{m_{\mathrm{P}}} \gtrsim 10^{-2}$

Comparison with other approaches

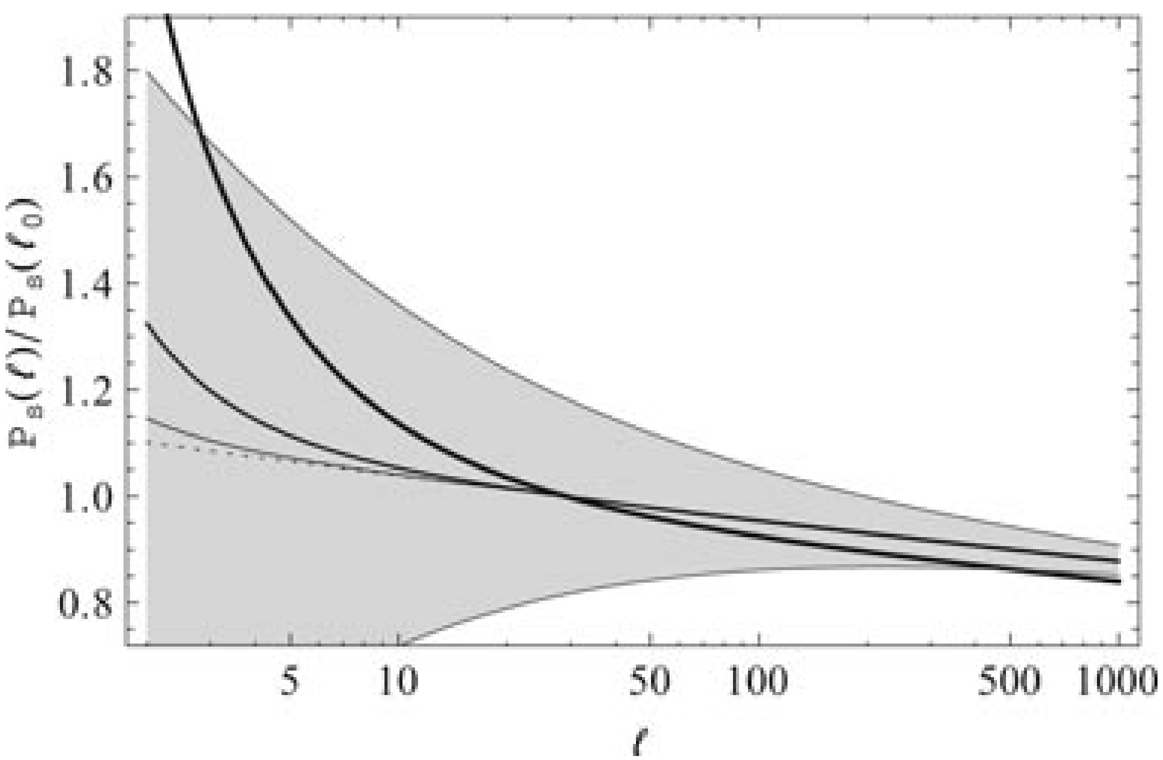
- different method to realize the semiclassical approx. to the WDW eq.
  A. Kamenshchik, A. Tronconi, G. Venturi: `1305.6138` (PLB), `1403.2961` (PLB), `1501.06404` (JCAP).

- decomposition of the wave function into an infinite set of moments
  D. Brizuela and U. Muniain, JCAP 04 (2019) 016

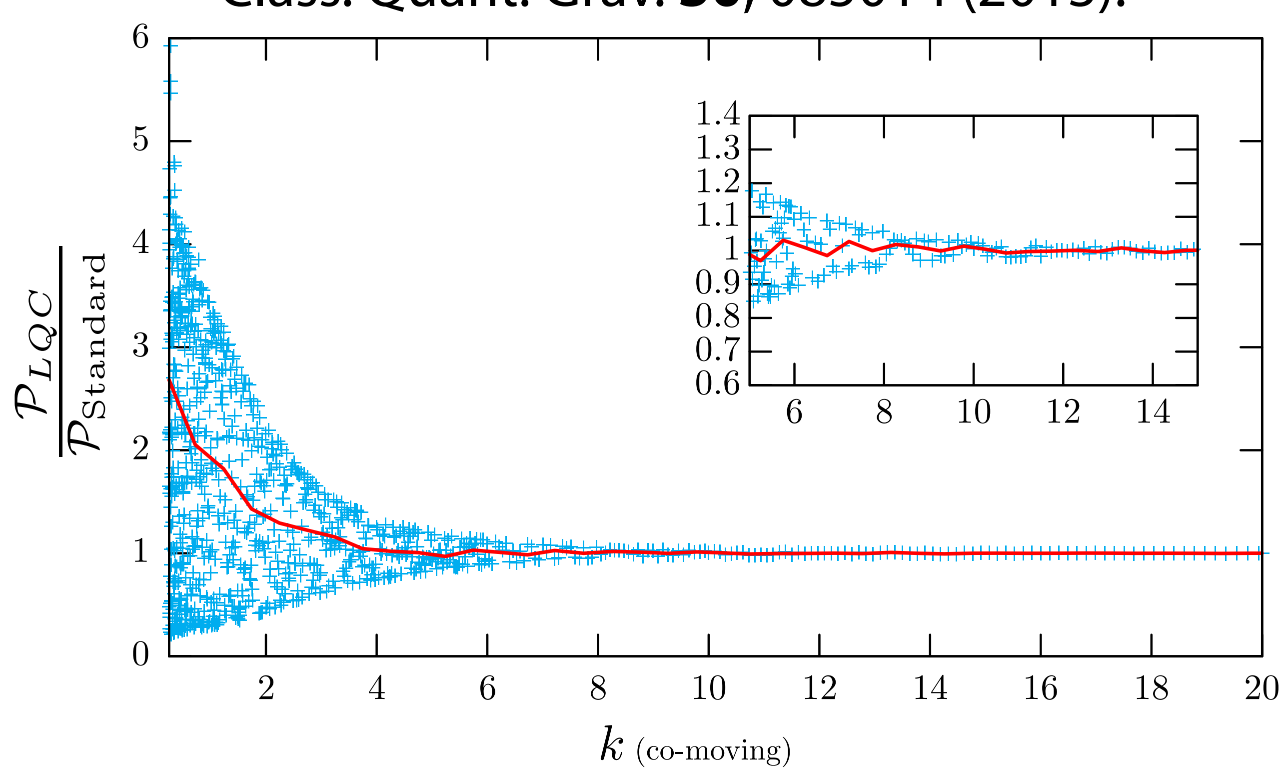
  ➔ *de Sitter:* same behavior $\propto k^{-3}$, same sign, slightly diff. prefactor

- **Loop Quantum Cosmology**

➡ *inverse-volume corrections:*
M. Bojowald, G. Calcagni, and S. Tsujikawa,
Phys. Rev. Lett. **107**, 211302 (2011),
JCAP **1111**, 046 (2011).



➡ *pre-inflationary dynamics:*
I. Agulló, A. Ashtekar, and W. Nelson,
Phys. Rev. Lett. **109**, 251301 (2012),
Phys. Rev. D **87**, 043507 (2013),
Class. Quant. Grav. **30**, 085014 (2013).

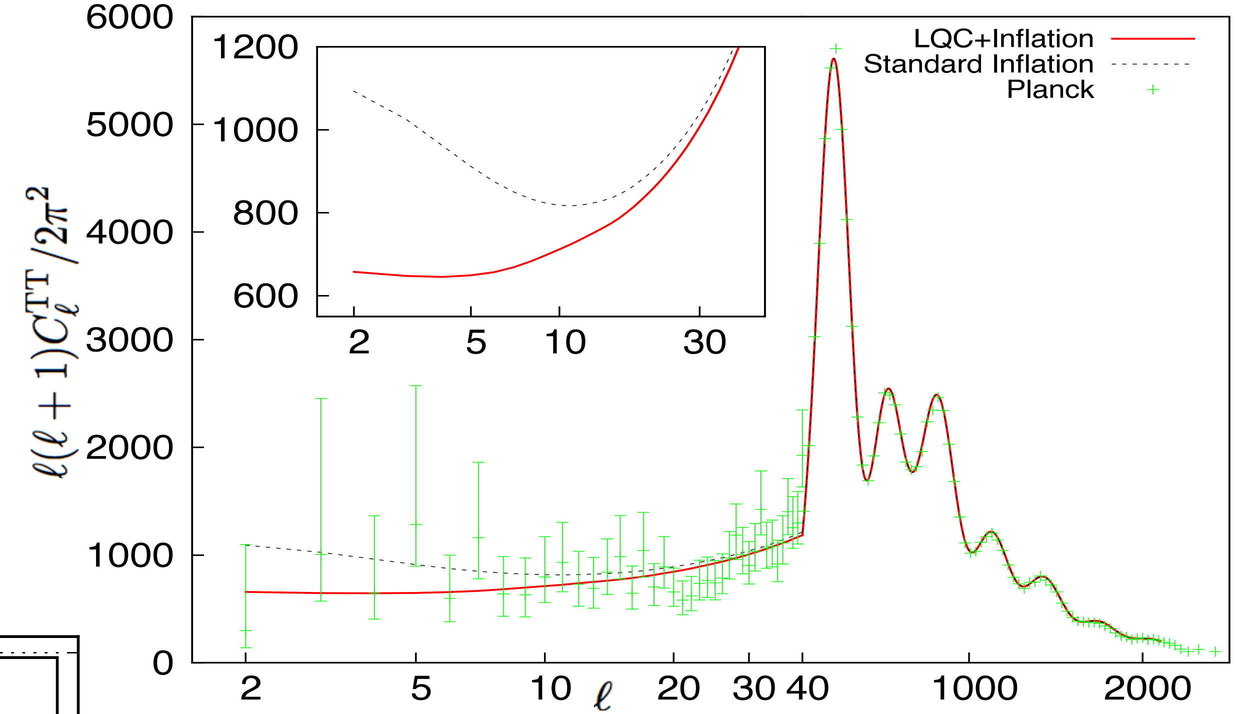


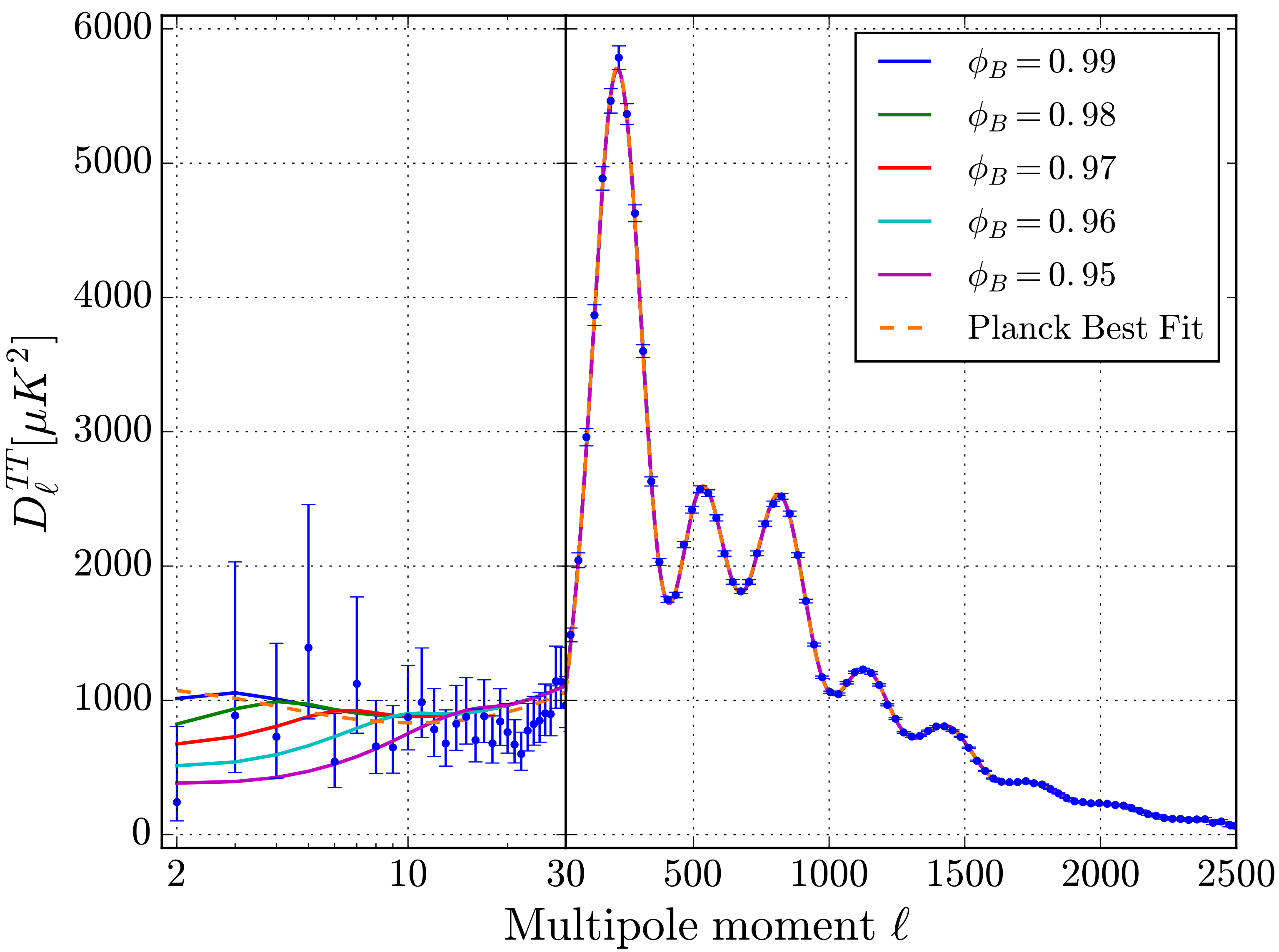
Comparison with other approaches II

- **Loop Quantum Cosmology** *(cont.)*

➡ *LQG-inspired initial conditions:*

A. Ashtekar and B. Gupt,
Class. Quant. Grav. **34**, 014002 (2017).





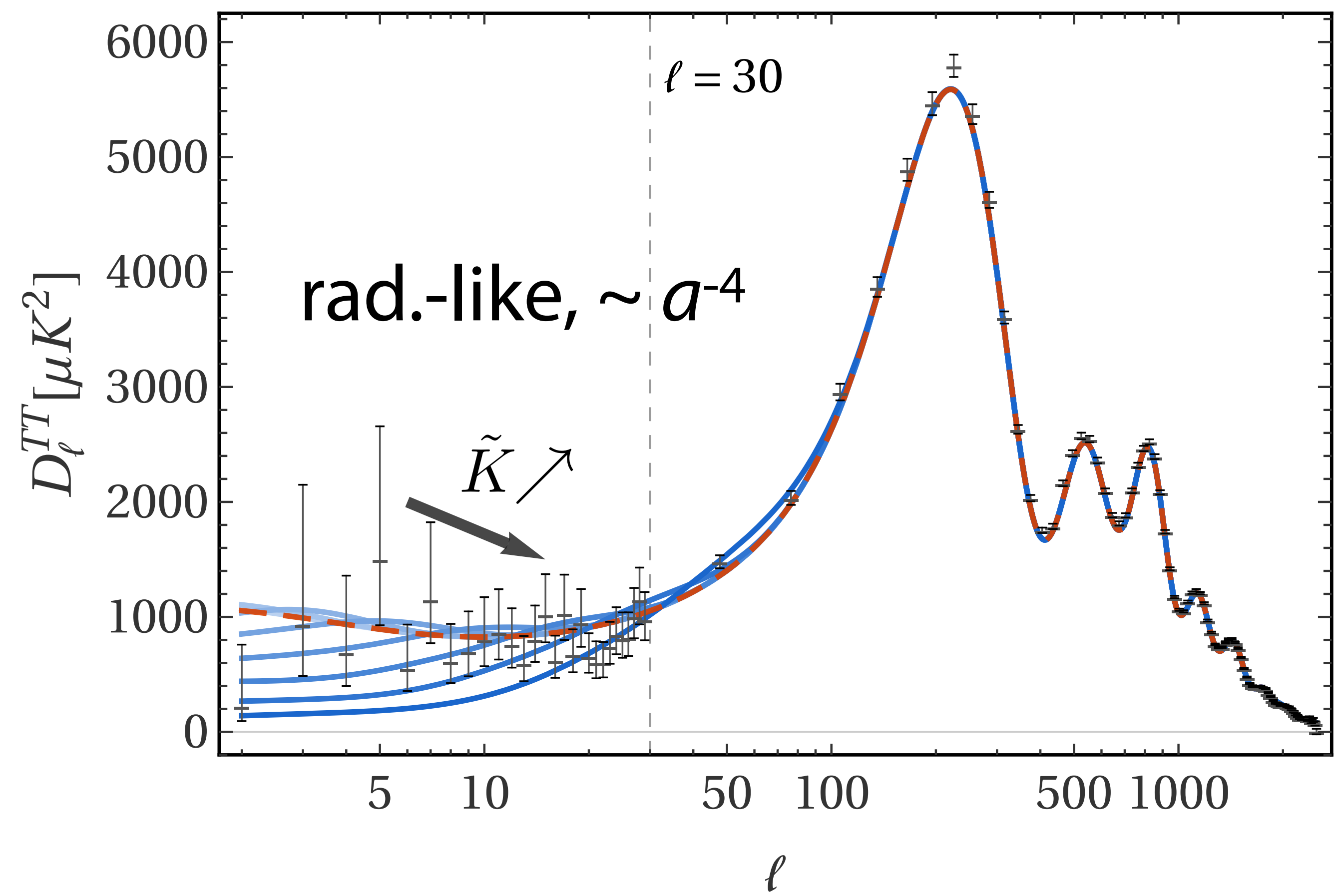
➡ *hybrid quantization:*

L. Castelló Gomar, G. A. Mena Marugán,
D. Martín de Blas, and J. Olmedo,
Phys. Rev. D **96**, 103528 (2017).

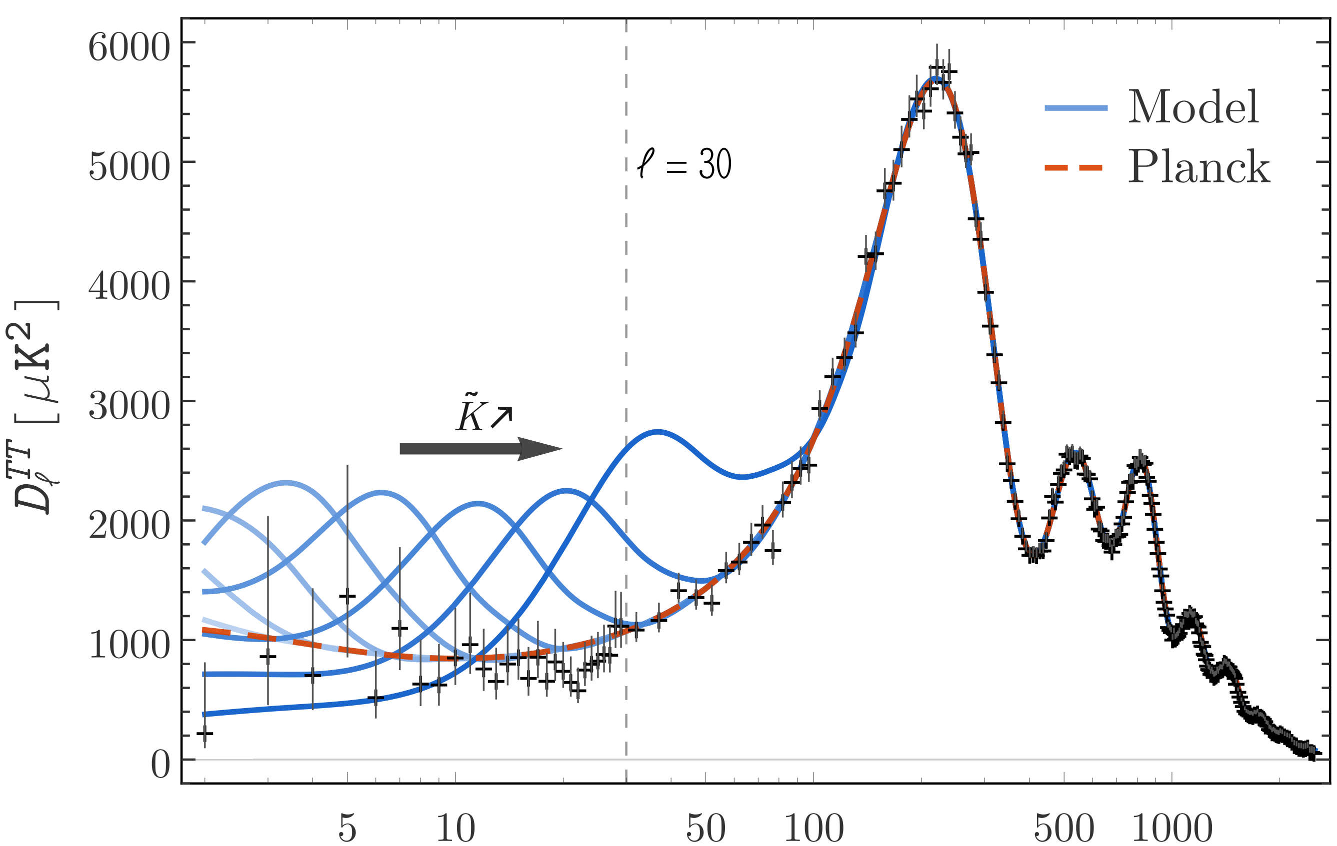
Comparison with other approaches III

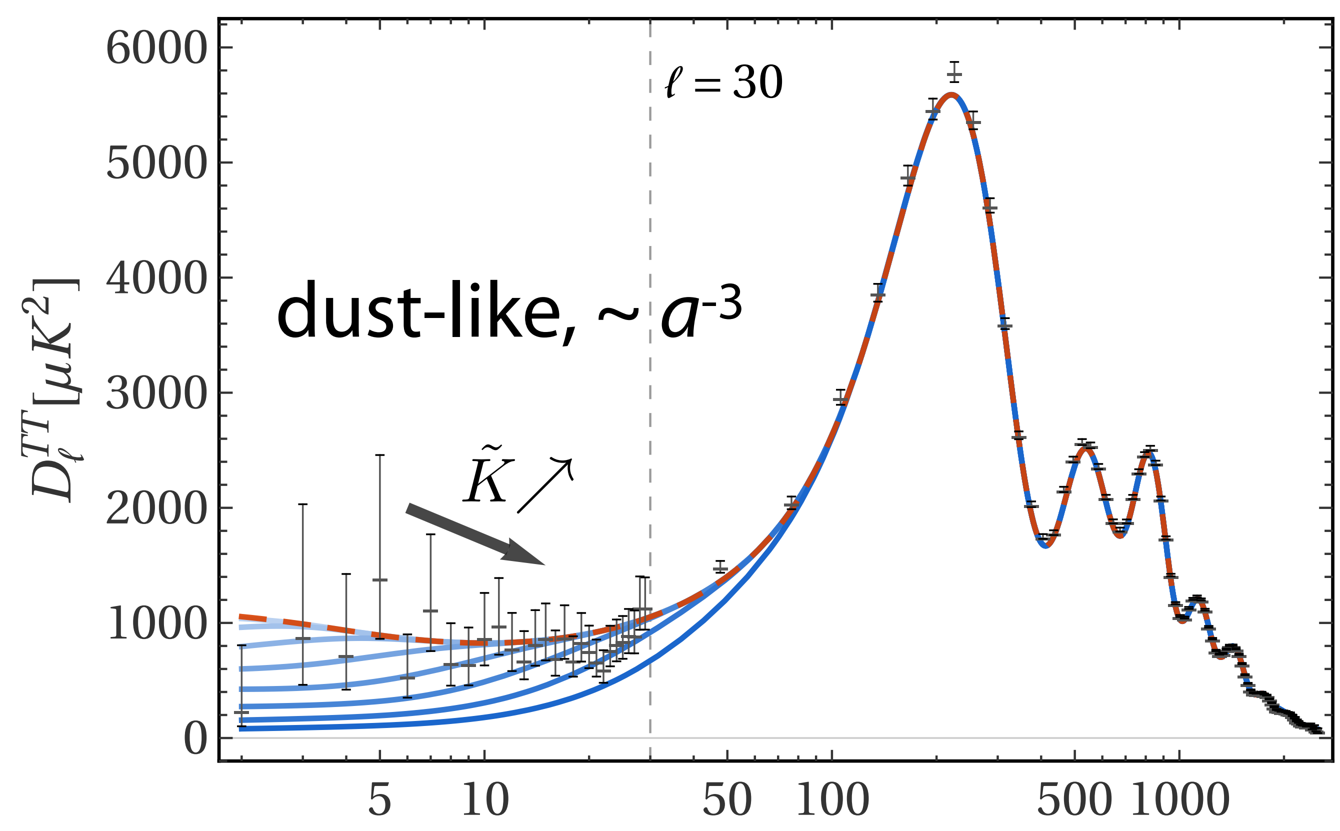
➡ pre-inflationary phase from "third quantization" models:

M. Bouhmadi-López, J. Morais, M.K. and Salvador Robles-Pérez,
Eur. Phys. J. C **77**, 718 (2017),
Eur. Phys. J. C **78**, 240 (2018),
JCAP 02 (2019) 057.



stiff-matter-like, ~ a^{-6}





Summary

- we calculated ***quantum-gravitational corrections*** to the power spectra of ***scalar*** and ***tensor*** perturbations during ***inflation*** by performing a ***semiclassical*** approximation to the ***Wheeler–DeWitt eq.***

➡ ***specific enhancement*** of power on large scales, too small to be measurable (with current bounds from observation)

- other QG approaches also lead to modification of power on large scales

➡ behavior like $\propto \dfrac{1}{k^3} \dfrac{H^2}{m_{\mathrm{P}}^2}$ universal feature for semiclassical approaches?

➡ LQC and other approaches can also incorporate a suppression

- ***Outlook:*** → non-Gaussianities
  → galaxy–galaxy correlations *(no cosmic variance)*

- D. Brizuela, C. Kiefer, M. K., `1511.05545` *(de Sitter)*, `1611.02932` *(slow-roll)*.
- D. Brizuela, C. Kiefer, M. K., S. Robles-Pérez, `1903.01234` *(excited states)*.