Quasinormal modes of a blackhole with quadrupole moment

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Outline

- Blackholes
- Axisymmetric metrics
  - δ −metric
  - Hartle-Thorne metric
- Light ring method
- Quasinormal modes
  - SQ-metric
  - Hartle-Thorne metric
- Future work
- Conclusion

Blackholes

- 1905 special relativity was postulated.

- 1907 Minkowski wrote the theory using a spacetime metric.
- 1907-1915
- Equivalence principle: locally gravitational field can be removed

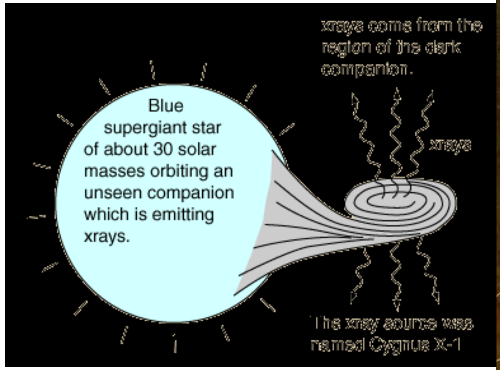
- Locally special relativity applies.

- Singularities appear in the solutions.

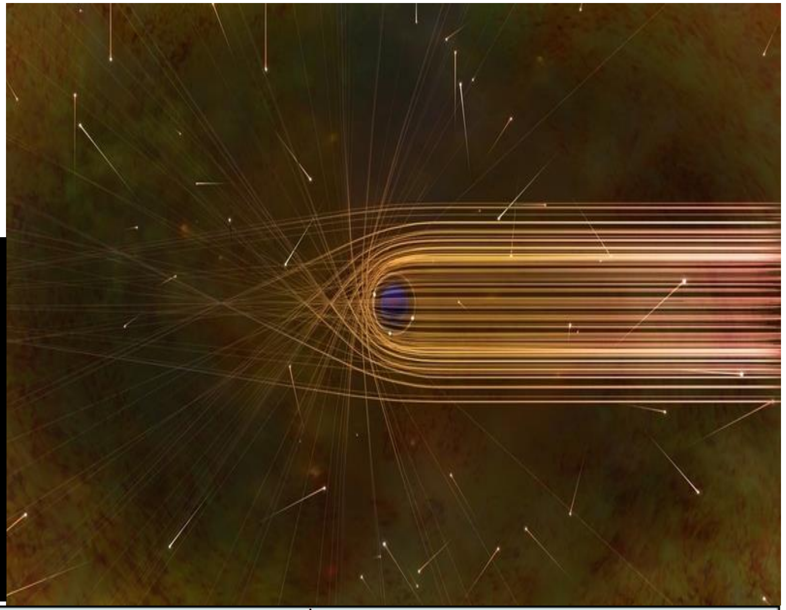
❖ Hawking- Penrose singularity theorem showed singularities are unavoidable if some form of energy condition holds

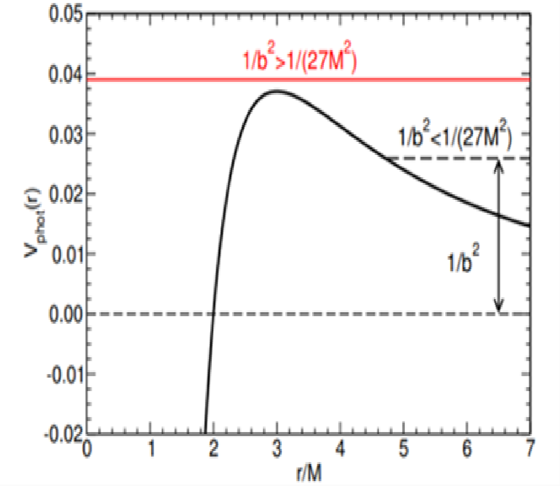
Blackholes

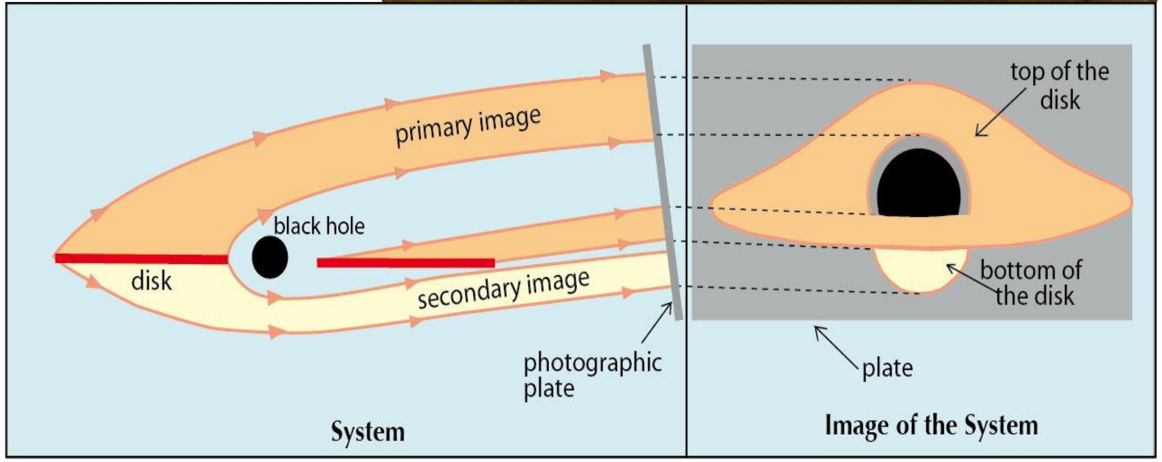
- Observational signature of black holes
  - detection of gravitational waves from merging black holes
  - motions of stars orbiting a blackhole
  - emission of vast amounts of electromagnetic radiation (X-rays)
  - lensing
  - shadow of a blackhole



Shadow of a blackhole



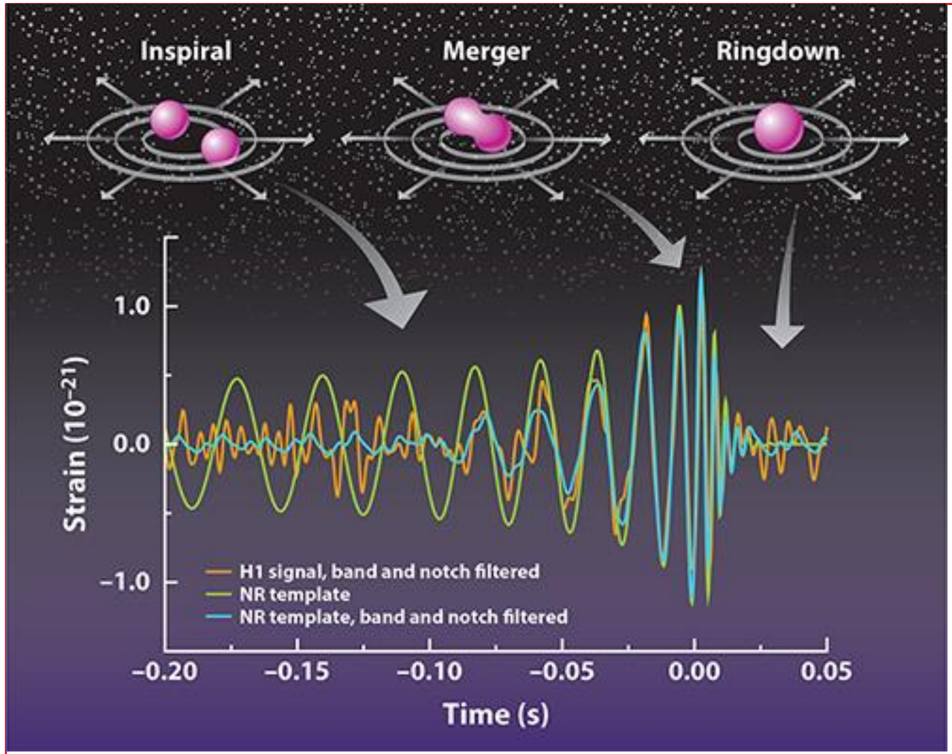




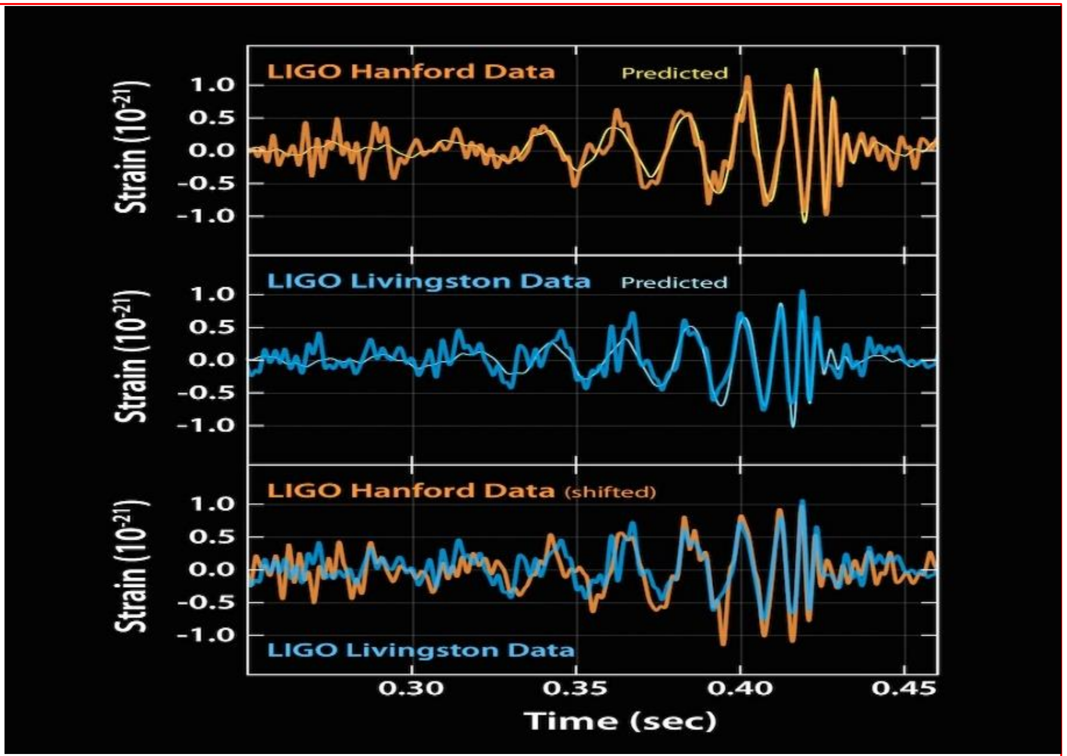
www.uanews.arizona.edu

Blackholes

Merging blackholes undergo different phases of evolution



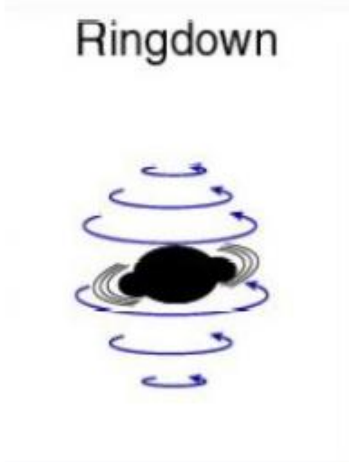
physics.aps.org



www.ligo.org/

Blackholes

- No hair theorem: mass, charge, angular momentum.
- Cosmic censorship: singularities should be covered by event horizons.
- It is believed that other moments are radiated away (Price).



Blackholes

- **Our motivation** is why the mass estimation by LIGO-VIRGO observations confirm large mass blackholes.

- It could be that the observed state is not a blackhole in the usual sense.

- We consider **generalized blackholes**.

Axisymmetric metrics

- Axisymmetric metrics can be written in Weyl canonical line element as

$$ds^2 = -e^{2\psi}\, dt^2 + e^{-2\psi}\, [e^{2\gamma}(d\rho^2 + dz^2) + \rho^2 d\phi^2] \qquad \psi,_{\rho\rho} + \frac{1}{\rho}\,\psi,_{\rho} + \psi,_{zz} = 0$$

- Axis is regular if $\gamma(\rho, z)$ vanishes as $\rho \to 0$.

- Prolate spheroidal coordinates

$$ds^2 = -e^{2\psi}\, dt^2 + \sigma^2\, e^{2\gamma - 2\psi}\, (x^2 - y^2) \left(\frac{dx^2}{x^2 - 1} + \frac{dy^2}{1 - y^2} \right) + \sigma^2 e^{-2\psi}(x^2 - 1)(1 - y^2)\, d\phi^2$$

- The solutions are not unique.

Axisymmetric metrics\ δ -metric

- The solution can be given by

$$\psi(x,y) = \sum_{n=0}^{\infty} (-1)^{n+1} q_n \, \mathcal{Q}_n(x) P_n(y)$$

- In spherical coordinates we have

$$ds^2 = -e^{2\psi}\, dt^2 + e^{2\gamma - 2\psi}\, \mathbb{B} \left(\frac{dr^2}{\mathbb{A}} + r^2\, d\theta^2 \right) + e^{-2\psi}\, \mathbb{A}\, r^2 \, \sin^2\theta\, d\phi^2$$

$$\mathbb{A} = 1 - \frac{2m}{r}, \qquad \mathbb{B} = 1 - \frac{2m}{r} + \frac{m^2}{r^2} \, \sin^2\theta$$

- δ -metric is (Bach and Weyl (1922))

$$ds^2 = -\mathbb{A}^{\delta}\, dt^2 + \mathbb{A}^{-\delta} \left(\frac{\mathbb{A}}{\mathbb{B}} \right)^{\delta^2 - 1} dr^2 + \mathbb{A}^{1-\delta} \left(\frac{\mathbb{A}}{\mathbb{B}} \right)^{\delta^2 - 1} r^2\, d\theta^2 + \mathbb{A}^{1-\delta}\, r^2 \, \sin^2\theta\, d\phi^2$$

❖ D. Papadopoulos, B. Stewart and L. Witten, “Some Properties of a Particular Static, Axially Symmetric Space-time,”

- Schwarzschild

$$\psi = \frac{1}{2} \ln \left(\frac{x-1}{x+1} \right) = \frac{1}{2} \ln \mathbb{A}, \qquad \gamma = \frac{1}{2} \ln \left(\frac{x^2 - 1}{x^2 - y^2} \right) = \frac{1}{2} \ln \left(\frac{\mathbb{A}}{\mathbb{B}} \right)$$

Axisymmetric metrics\ δ -metric

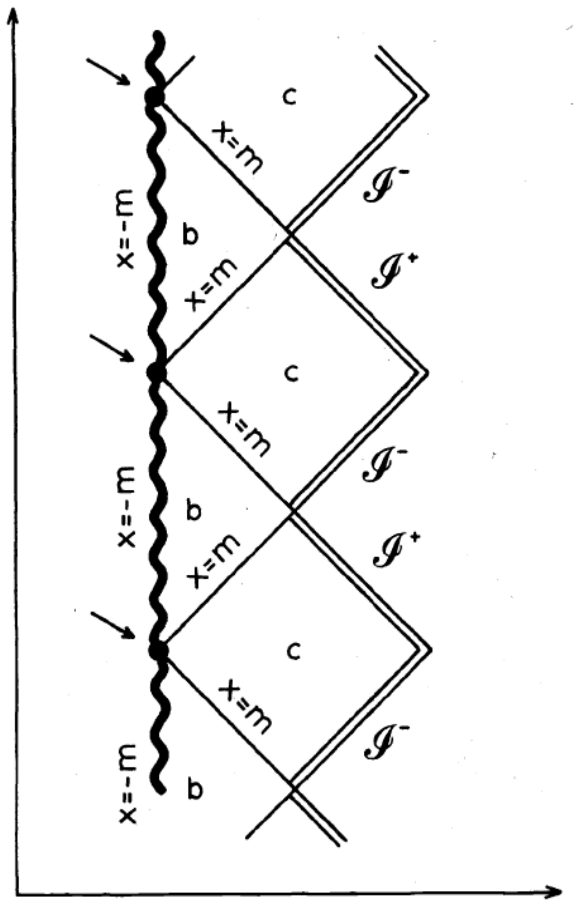


FIG. 3. The conformal representation of the space-time (x, t) plane when $\gamma \geq 2$ and even, $y = +1$. The wavy line represents the physical singularity. The arrows refer to the exceptional points. These points are not covered by the coordinate patch but are at infinite affine distance.

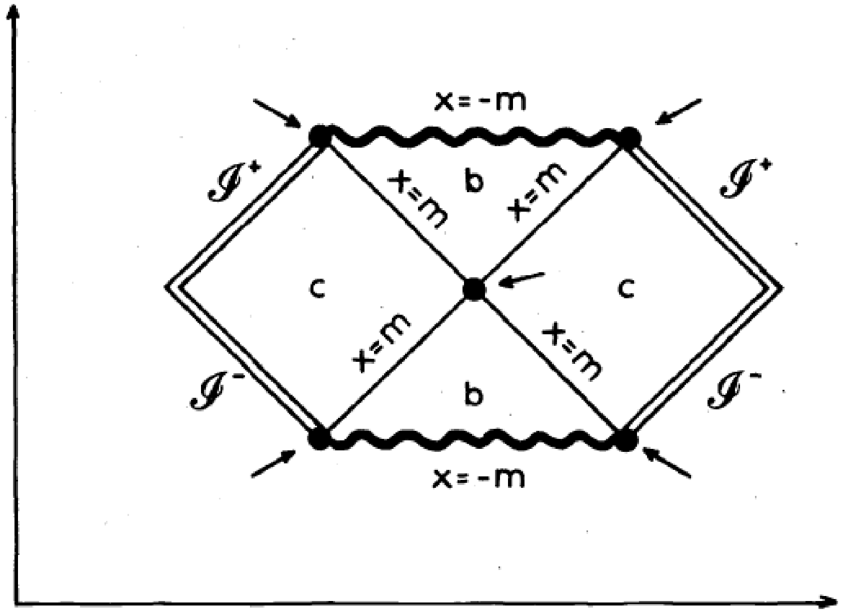


FIG. 4. The conformal representation of the space-time (x, t) plane when $\gamma > 2$ and odd, $y = +1$. The wavy line represents the physical singularity. The arrows refer to the exceptional points. These points are not covered by the coordinate patch but are at an infinite affine distance.

Axisymmetric metrics\ δ -metric

- Consider r=2m hypersurface, its normal N satisfies

$$N \cdot N = g^{rr} = \mathbb{A}^{-\delta^2+\delta+1}\,\mathbb{B}^{\delta^2-1}$$

- We find that this surface is null for

$$-(\sqrt{5}+1)/2 < q < (\sqrt{5}-1)/2$$

- Also r=2m is an infinite redshift surface

Axisymmetric metrics\ δ -metric

- To interpret δ -metric, we expand the metric in the weak field limit and bring it to the Newtonian form

$$ds^2 = -c^2\,(1+2\,\Phi_N/c^2)\,dt^2 + \frac{d\rho^2}{1+2\,\Phi_N/c^2} + U(\rho,\vartheta)\rho^2(d\vartheta^2 + \sin^2\vartheta\,d\phi^2)$$

- where we have defined

$$\Phi_N = -\frac{GM}{\rho} + \frac{GQ}{\rho^3}\,P_2(\cos\vartheta)\,, \qquad U(\rho,\vartheta) = 1 - 2\,\frac{GQ}{c^2\rho^3}\,P_2(\cos\vartheta)$$

- This metric describes a source with mass and quadrupole

$$M = (1+q)\,m\,, \qquad Q = \frac{2}{3}m^3\,q$$

- δ metric expanded to first order in q for the physical mass M is the SQ metric.

$$ds^2_{SQ} = -\left[1 + q\left(\frac{2M}{r\mathbb{A}} + \ln\mathbb{A}\right)\right]\mathbb{A}\,dt^2 + \left[1 - q\left(\frac{2M}{r\mathbb{A}} + \ln\frac{\mathbb{B}^2}{\mathbb{A}}\right)\right]\frac{dr^2}{\mathbb{A}} + \left(1 - q\ln\frac{\mathbb{B}^2}{\mathbb{A}}\right)r^2\,d\theta^2 + (1 - q\ln\mathbb{A})\,r^2\sin^2\theta\,d\phi^2\,.$$

Axisymmetric \Hartle -Thorne metric

- Erez_Rosen metric is given by

$$\psi = \frac{1}{2}\ln\left(\frac{x-1}{x+1}\right) + q_2\,\frac{1}{2}(3y^2-1)\left[\frac{3x^2-1}{4}\,\ln\left(\frac{x-1}{x+1}\right) + \frac{3}{2}\,x\right]$$

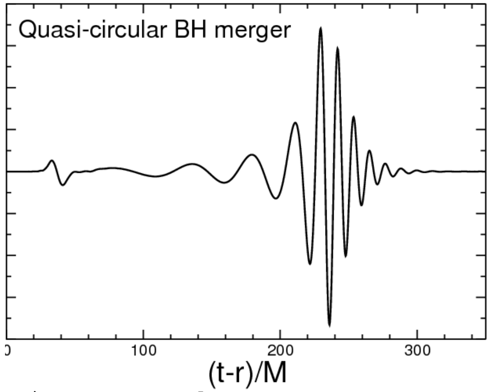
- The Hartle-Thorne metric is

$$ds^2_{HT} = -\mathcal{F}\,dt^2 + \frac{1}{\mathcal{F}}\,dr^2 + \mathcal{G}\,r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

❖ (J. B. Hartle and K. S. Thorne, “Slowly Rotating Relativistic Stars”)

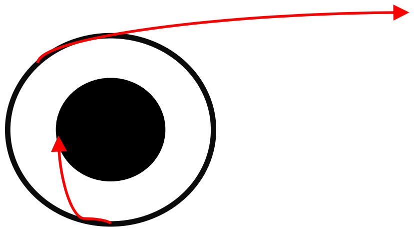
Light ring method

- Quasinormal modes are damped oscillations of a BH which are ingoing at the horizon and out going at infinity

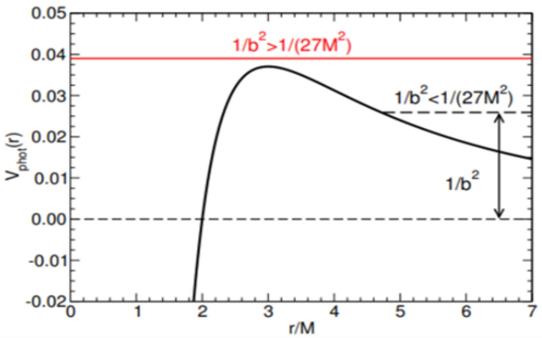


- After merger, we may consider a blackhole with slight perturbations.

- The QNMs are related to the decay of the photon congruence in the unstable orbit.
  - ❖ V. Ferrari et.al



❖ E. Berti et.al



- we consider the bundle of null rays in the unstable circular equatorial orbit at r = r0

$$K^\mu = \frac{dx^\mu}{d\lambda} = [1 - \epsilon h', \epsilon r_0 f', 0, \omega_\pm (1 - \epsilon h')]$$

- For a congruence of null rays, the law of conservation of rays is given by $(\rho_n K^\mu)_{;\mu} = 0$

Light ring method

- Null geodesics

$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\mu}{d\lambda}=0\,, \qquad \frac{d^2x^\alpha}{d\lambda^2}+\Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\mu}{d\lambda}=0$$

- Slight perturbations of unstable photon orbits results in

$$f(t)=\beta_{SQ}\sinh(\gamma_{SQ}\,t)\,, \qquad g(t)=0\,,$$
$$h(t)=2\frac{(1-q)\beta_{SQ}}{\gamma_{SQ}}\left[\cosh(\gamma_{SQ}\,t)-1\right]+C_{SQ}\,t\,,$$

$$\gamma_{SQ}^2=\frac{1+2q\left[1+2\ln(2/3)\right]}{27M^2}$$

Quasinormal modes

- SQ metric QNMs

$$\omega_{QNM} = \omega^0_{SQ} + i\Gamma_{SQ} = \pm j\omega_{\pm} + i\gamma_{SQ}\left(n + \frac{1}{2}\right), \qquad n = 0, 1, 2, 3, \cdots,$$

where

$$\omega_{\pm} = \pm\frac{1 - q(\ln 3 - 1)}{3\sqrt{3}M}, \qquad \gamma_{SQ} = \frac{1 + q\,[1 + 2\ln(2/3)]}{3\sqrt{3}M}, \qquad q = \frac{3Q}{2M^3}.$$

❖ **A. Allahyari, H. Firouzjahi and B. Mashhoon, ``Quasinormal Modes of a Black Hole with Quadrupole Moment,''**

Quasinormal modes

- HT metric QNMs

$$\omega_{QNM} = \omega^0_{HT} + i\Gamma_{HT} = \pm\, j\omega_\pm + i\gamma_{HT}\left(n + \frac{1}{2}\right)\,, \qquad n = 0, 1, 2, 3, \cdots\,,$$

where

$$\omega_\pm = \pm\frac{1 - \frac{1}{2}\hat{q}\,(-16 + 15\ln 3)}{3\sqrt{3}\,M}\,, \qquad \gamma_{HT} = \frac{1 + \hat{q}\,(-16 + 15\ln 3)}{3\sqrt{3}M}\,, \qquad \hat{q} := \frac{5Q}{8M^3}\,.$$

❖ **A.Allahyari, H.Firouzjahi and B.Mashhoon, ``Quasinormal Modes of a Black Hole with Quadrupole Moment,''**

Ongoing

- Rotating δ-metric

$$ds^2 = -F\,dt^2 + 2\,F\,\omega\,dt\,d\phi + \frac{e^{2\gamma}}{F}\frac{\mathbb{B}}{\mathbb{A}}\,dr^2 + r^2\,\frac{e^{2\gamma}}{F}\,\mathbb{B}\,d\theta^2 + \left[\frac{r^2}{F}\,\mathbb{A}\,\sin^2\theta - F\,\omega^2\right]d\phi^2\,,$$

- Rotation contribute to the quadrupole

$$M = m(1+q)\,, \qquad Q = \frac{2}{3}M^3 q + \frac{J^2}{M}\,, \qquad J = M\,a$$

- The mere fact that the source rotates increases the corresponding range of quadrupole.

❖ **A. Allahyari, H. Firouzjahi and B. Mashhoon, “Quasinormal Modes of Generalized Black Holes: delta-Kerr Spacetime”, Classical and Quantum Gravity, 10.1088/1361-6382/ab6860**

Ongoing

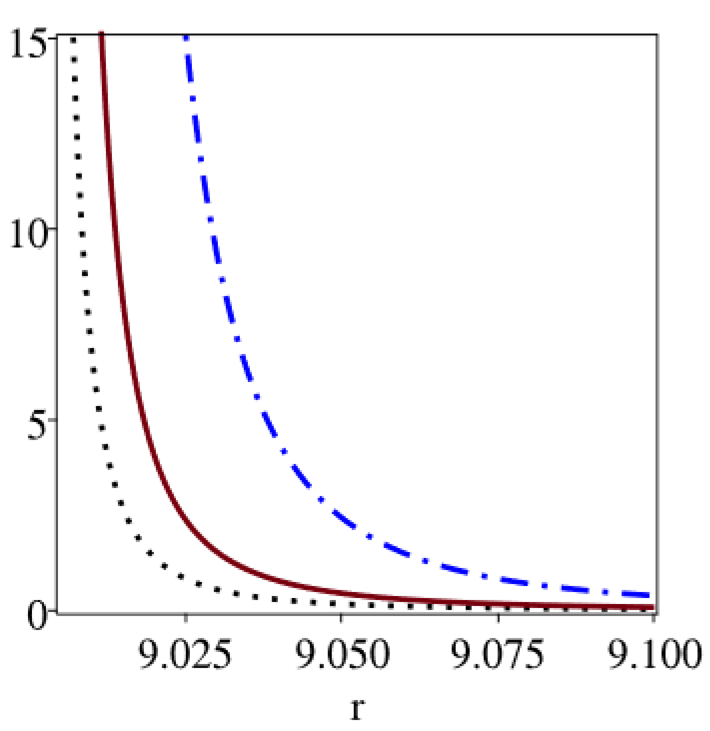


FIG. 1: The behavior of the Kretschmann invariant $K(r,\theta)$ for $m = 5$, $a = 3$ and $q = 1/8$ (black dot), $q = 1/6$ (solid red) and $q = 1/4$ (blue dot-dash). Left panel: K as a function of $r : 9 \to \infty$ for $\theta = \pi/2$. The singularity of K occurs at the outer Kerr horizon $r_S = 9$. Right panel: K as a function of $\theta : 0 \to \pi$ for $r = 10$.

Ongoing

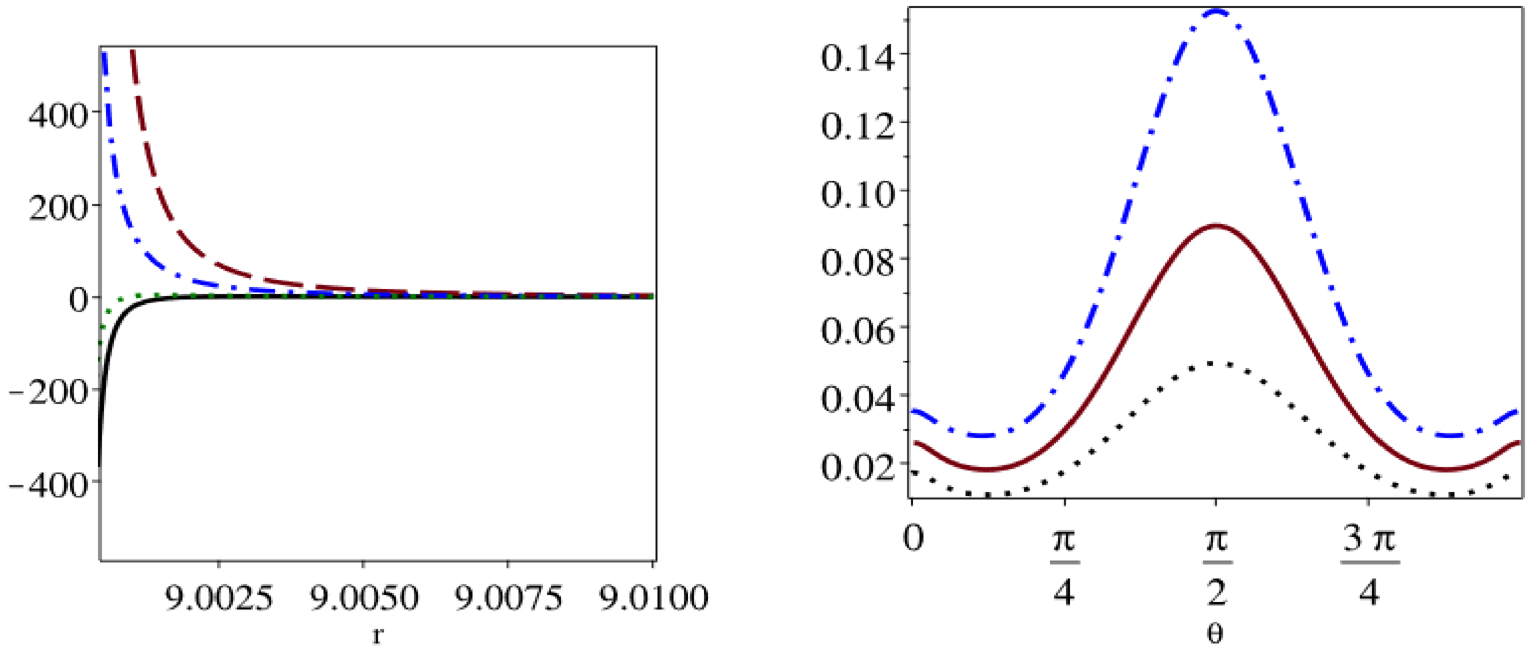


FIG. 2: The behavior of the Kretschmann invariant $K(r,\theta)$ for $m = 5$ and $a = 3$ near the outer singularity $r_S = 9$. Left panel: K as a function of $r : 9 \to \infty$ for $q = 0.1$ and $\theta = \pi/10$ (green dot), $\theta = \pi/6$ (solid black), $\theta = \pi/3$ (blue dot-dash) and $\theta = \pi/2$ (red dash). We have checked numerically that these results remain essentially the same for various values of q. Right panel: K as a function of $\theta : 0 \to \pi$ for $r = 9.05$ and $q = 0.08$ (black dot), $q = 0.1$ (solid red) and $q = 0.12$ (blue dot-dash).

Ongoing

- The quasinormal modes are

$$\Omega^0_{KQ} + i\,\Gamma_{KQ} = \frac{j}{3\sqrt{3}M}\left[1 - q\,(-1+\ln 3) \pm \frac{2a}{3\sqrt{3}M} + \frac{11}{54}\frac{a^2}{M^2}\right], \\ + i\,\frac{n+\frac{1}{2}}{3\sqrt{3}M}\left\{1 + q\left[1 + 2\ln\left(\frac{2}{3}\right)\right] - \frac{2}{27}\frac{a^2}{M^2}\right\}, \qquad n = 0, 1, 2, 3, \cdots,$$

- QNM for H-T metric

$$\Omega^0_{RHT} + i\,\Gamma_{RHT} = \frac{j}{3\sqrt{3}M}\left[1 - \frac{1}{2}\tilde{q}\,(-16+15\ln 3) \pm \frac{2J}{3\sqrt{3}M^2} + \frac{11}{54}\frac{J^2}{M^4}\right] \\ + i\,\frac{n+\frac{1}{2}}{3\sqrt{3}M}\left[1 + \tilde{q}\,(-16+15\ln 3) - \frac{2}{27}\frac{J^2}{M^4}\right], \qquad n = 0, 1, 2, 3, \cdots,$$

$$\tilde{q} := \frac{5}{8}\frac{Q - J^2/M}{M^3}$$

- Numerically equal to the quasinormal modes of rotating H-T metric.

Conclusion

- Axisymmetric configuration can undergo quasinormal oscillations.

- We could determine mass and quadrupole of two solutions.

- Essentially the QNMs for both SQ and static HT spacetimes to linear order in the quadrupole moment are the same.

- The quadrupole increases the mass estimate in the real part of QNM spectrum.

Thanks…