Compact gravitational systems with negative pressure

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Gravitational structures with negative pressure

Work with student Philip Beltracchi on

formation of dark energy stars

Beltracchi, Gondolo PRD 99 (2019) 044037

collapse of a constant-density star into a dark energy star

Beltracchi, Gondolo PRD 99 (2019) 084021

a curious dark energy object resembling a singular isothermal sphere

Beltracchi, Gondolo, arXiv:1910.08166

Gravitational structures with negative pressure

False/true vacuum bubble (non-static, has past/future singularities)

Coleman, De Luccia 1980, Sato, Sasaki, Kodama, Maeda 1981+, Blau, Guendelman, Guth 1987, Farhi, Guth 1987; …

Interior-deSitter/exterior-Schwarzschild geometry (static)

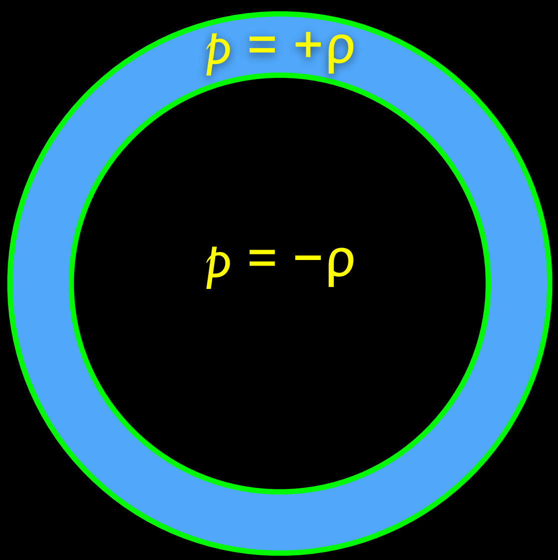
A spherically symmetric geometry that is asymptotically de Sitter as $r \to 0$ and asymptotically Schwarzschild as $r \to \infty$. For example, regular black holes with $p = -\rho$ at the center, G-lumps, gravastars, …

Sakharov 1966, Gliner 1966, Dymnikova 1992+, Mazur, Mottola 2002+, Catoen, Faber, Visser 2005, Ansoldi, Sindoni 2008, …

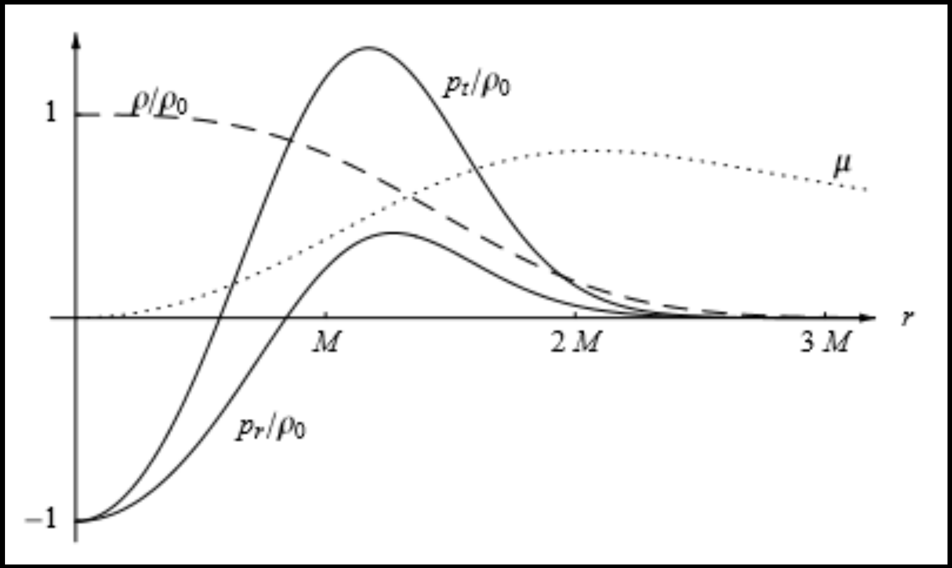
Gravastars

Originally, a star with a $p = -\rho$ interior of volume V matched to a Schwarzschild exterior of mass $M = \rho\, V$ and having no horizons or spacetime singularities.

Mazur, Mottola 2002+, Catoen, Faber, Visser 2005, …



- A center with negative pressure
- Radius > Schwarzschild radius
- Initial model by Mazur and Mottola: a pure dark energy core, stiff matter shell, and vacuum exterior, with infinitesimal boundary layers
- Cattoen, Faber, and Visser replaced boundary layers with anisotropic stress



DeBenedictis, Horvat, Ilijic, Kloster, Viswanathan (2007)

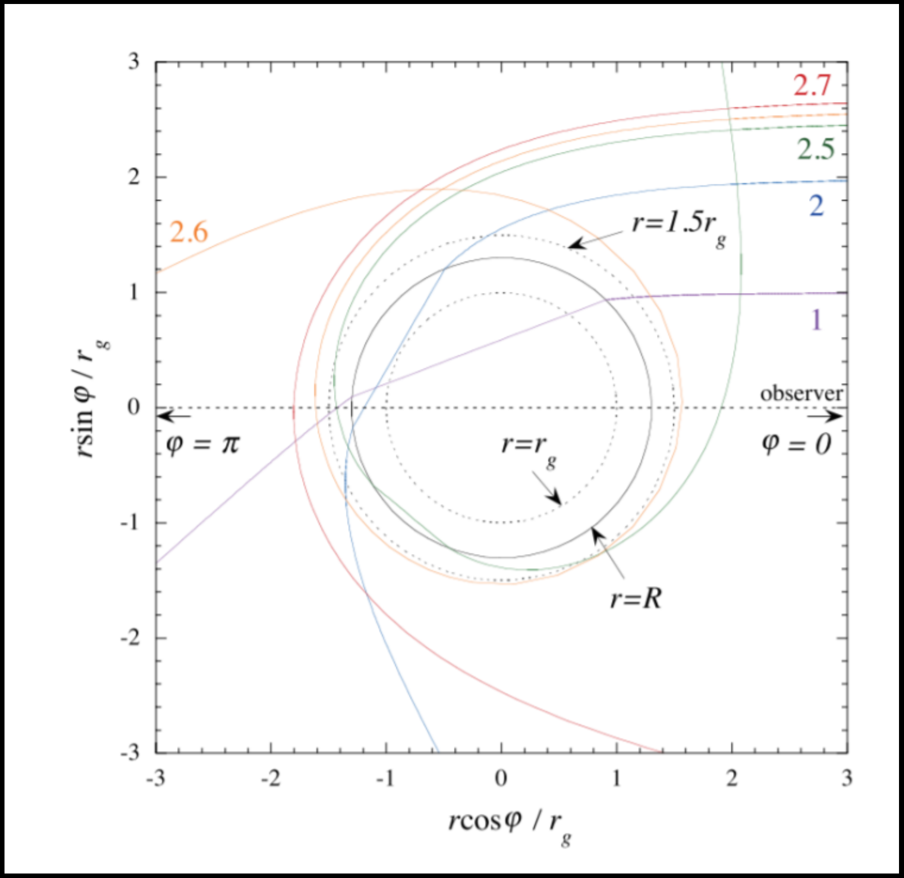
Gravastars

Gravitational lensing

- gravastars do not require event horizons
- it may be possible to have light pass through
- interesting lensing trajectories

Gravitational waves

- matter on surface can possibly give a "seismic" signature in gravitational waves



Sakai, Saida, Tamaki 2014

Gravastars

- "Gravitational condensates": temperature/entropy term must be zero

  Mazur, Mottola 2002

- Numerical simulations indicate (slow) rigid rotation and angular momentum are possible, Schwarzschild interior dark energy star nearly matches Kerr source

  Chirenti, Rezzolla 2008, Posada 2016

- Gravastars can be electrically charged

  Horvat, Ilijic, Marunovic 2008

- Gravastar stability has been studied. Possible to oscillate between radii rather than settle (bounded excursion)

  Chirenti, Rezzolla 2007; Rocha et al 2008

- Formation from normal matter configurations

  Beltracchi, Gondolo 2019

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Formation of dark energy stars

Formation of dark energy stars

Beltracchi, Gondolo 2019a

A dark energy star is a gravitationally-bound object with a finite-volume dark energy ($p = -\rho$) core.

Time-dependent spherically-symmetric anisotropic solution

$$ds^2 = -e^{2\Phi(t,r)}\, dt^2 + \frac{dr^2}{1 - \frac{2Gm(t,r)}{r}} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

$$T_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \rho & -S_r & 0 & 0 \\ -S_r & p_r & 0 & 0 \\ 0 & 0 & p_T & 0 \\ 0 & 0 & 0 & p_T \end{pmatrix}$$

pressure anisotropy $\Delta = p_T - p_r$

radial momentum flow S_r

Formation of dark energy stars

Beltracchi, Gondolo 2019a

Einstein's equations

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho$$

$$\frac{\partial \Phi}{\partial r} = \frac{G\left(m + 4\pi r^3 p_r\right)}{r^2\left(1 - \frac{2Gm}{r}\right)}$$

$$\frac{\partial \rho}{\partial \tau} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\sqrt{1 - \frac{2Gm}{r}}\, S_r\right)$$

$$-\frac{\partial p_r}{\partial r} - \frac{G\left(m + 4\pi r^3 p_r\right)(\rho + p_r)}{r^2\left(1 - \frac{2Gm}{r}\right)} + \frac{2\Delta}{r} = \sqrt{1 - \frac{2Gm}{r}}\,\frac{\partial}{\partial \tau}\left(\frac{S_r}{1 - \frac{2Gm}{r}}\right)$$

Force equation $F = \mathrm{d}p/\mathrm{d}t$

In the static case, it reduces to the Tolman-Oppenheimer-Volkoff equation plus an anisotropy term

τ is proper time at fixed r, θ, ϕ $\quad (d\tau = e^{\Phi(t,r)}\, dt)$

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Einstein's equations

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho$$

$$\frac{\partial \Phi}{\partial r} = \frac{G\left(m + 4\pi r^3 p_r\right)}{r^2\left(1 - \frac{2Gm}{r}\right)}$$

$$\frac{\partial \rho}{\partial \tau} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\sqrt{1-\frac{2Gm}{r}}\,S_r\right)$$

$$-\frac{\partial p_r}{\partial r} - \frac{G\left(m+4\pi r^3 p_r\right)\left(\rho + p_r\right)}{r^2\left(1-\frac{2Gm}{r}\right)} + \frac{2\Delta}{r} = \sqrt{1-\frac{2Gm}{r}}\,\frac{\partial}{\partial \tau}\left(\frac{S_r}{1-\frac{2Gm}{r}}\right)$$

Give $\rho(t,r)$ and $p_r(t,r)$

Find $S_r(t,r)$ and $\Delta(t,r)$

Check energy conditions

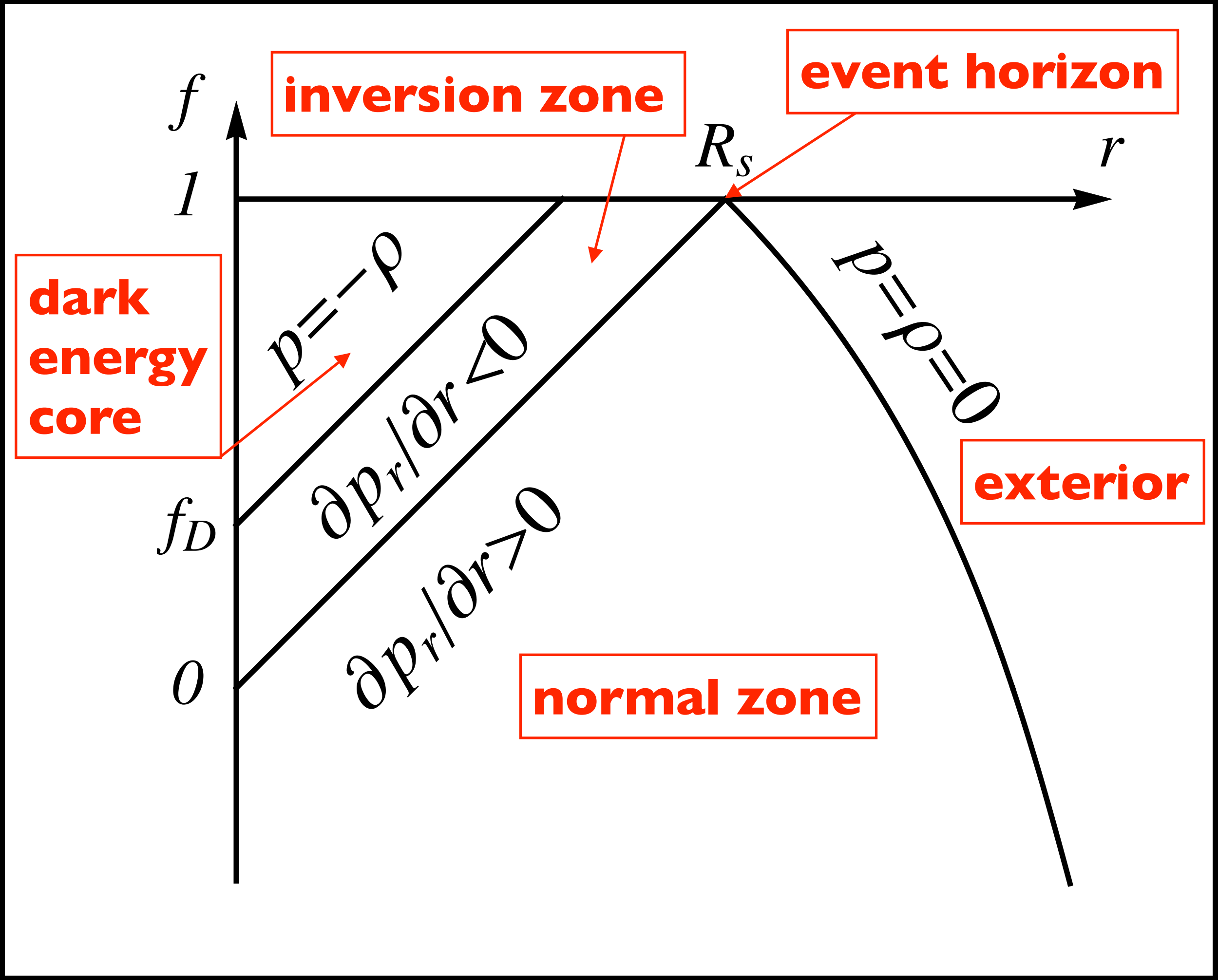
τ is proper time at fixed r, θ, ϕ $\quad (d\tau = e^{\Phi(t,r)}\,dt)$

Pile-up model

Beltracchi, Gondolo 2019a

Form a $p_r = p_T = -\rho = \text{const}$ core of increasing radius

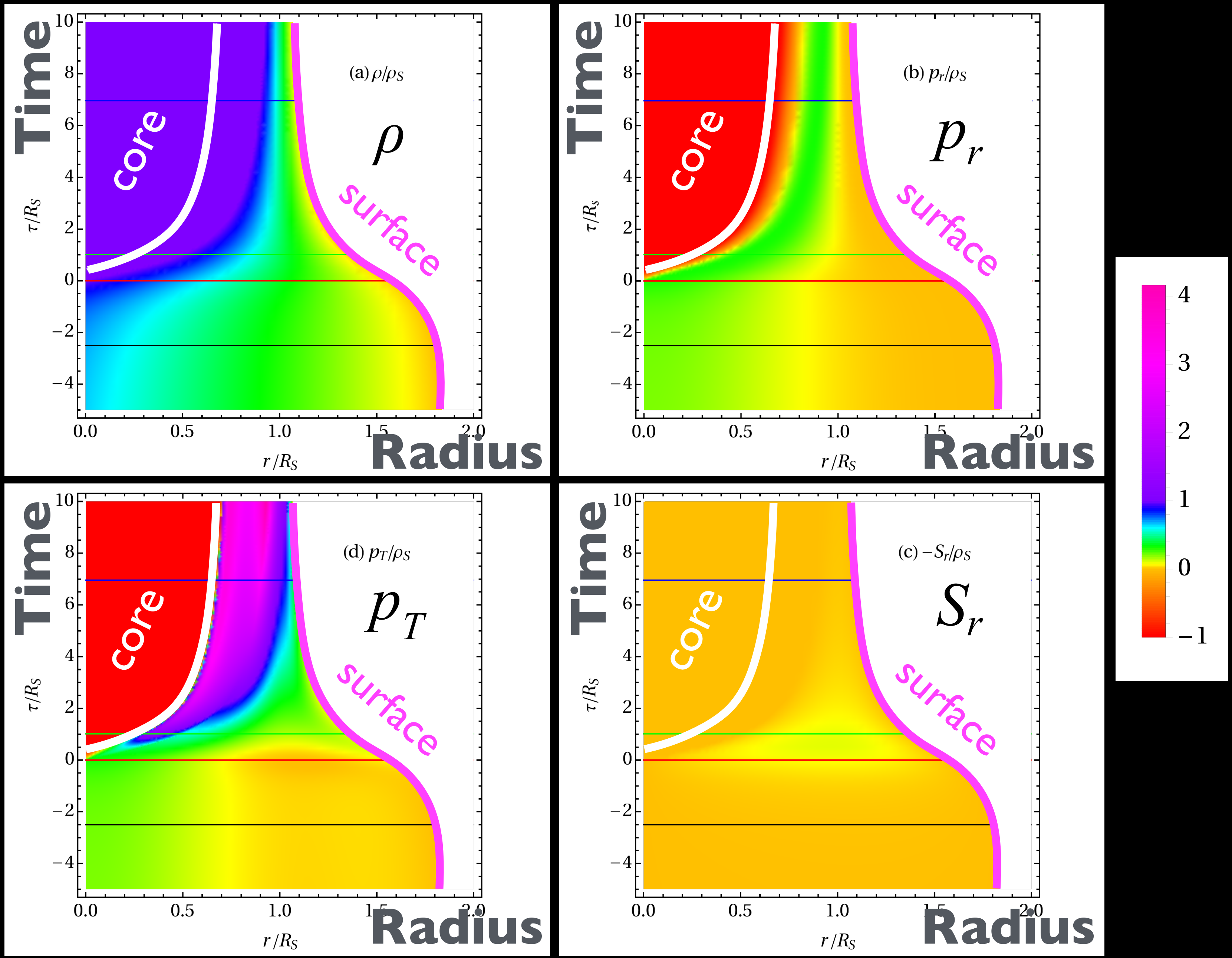
Evolution parameter $f = f(t)$ controls formation of singularities and horizons



Pile-up model

Beltracchi, Gondolo 2019a

Evolution of density, pressure, and energy flow



Pile-up model

Beltracchi, Gondolo 2019a

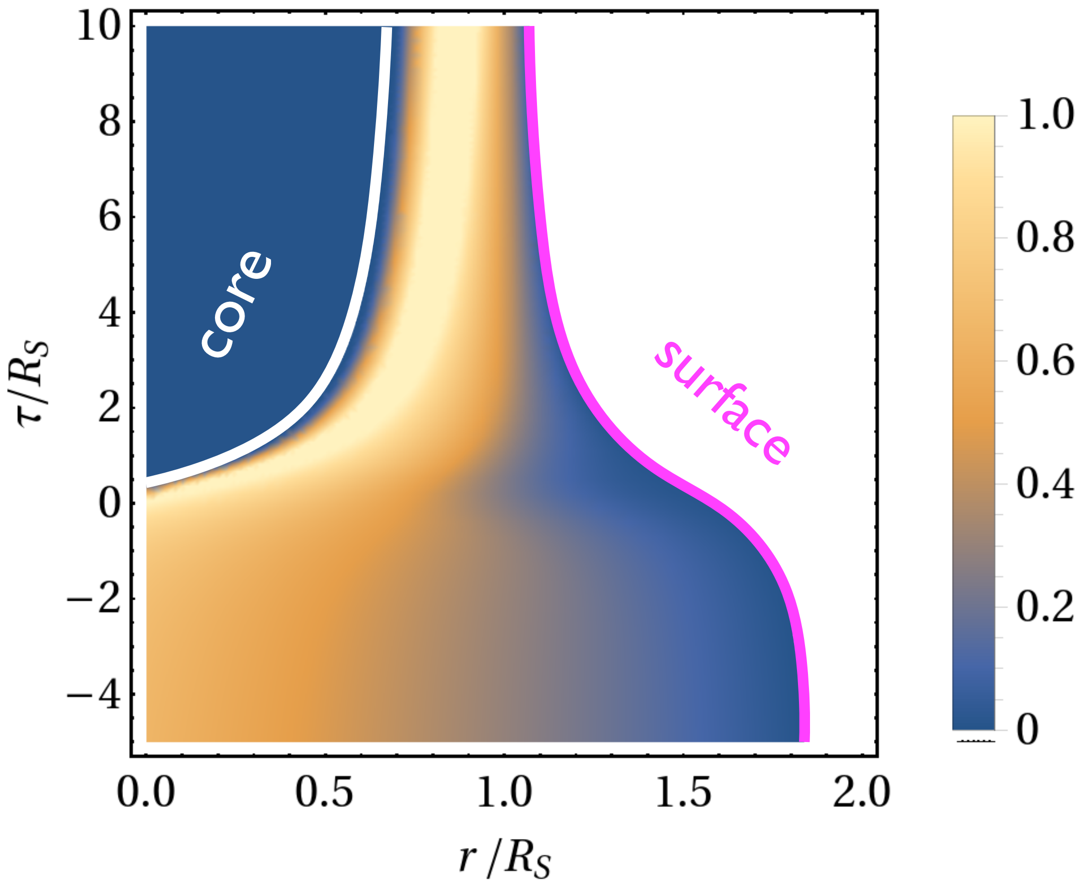
Weak energy condition

$T_{\mu\nu} k^\mu k^\nu \geq 0$ for all time-like vectors k^μ

Null energy condition

$T_{\mu\nu} k^\mu k^\nu \geq 0$ for all light-like vectors k^μ

The weak and the null energy conditions are satisfied at any position and time.



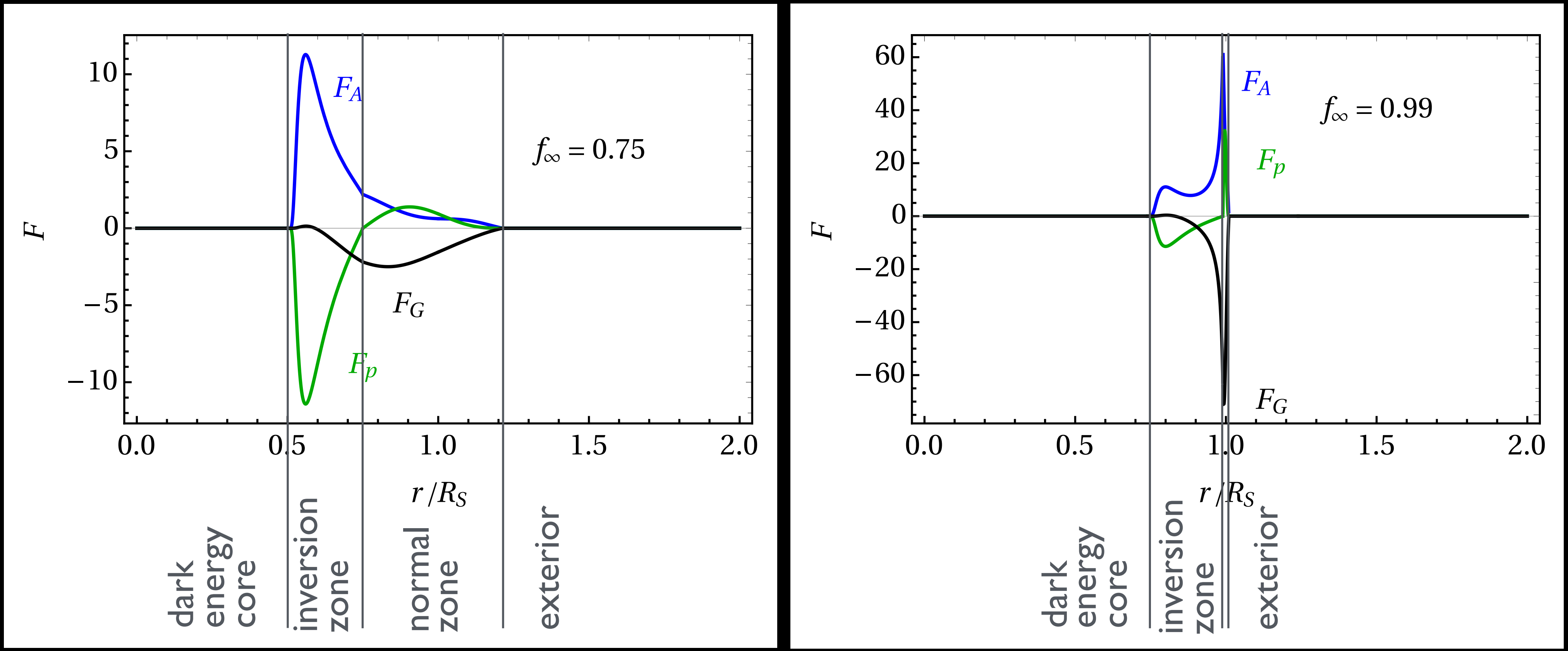
minimum $T_{\mu\nu}k^\mu k^\nu$ over time-like k^μ

Pile-up model

Beltracchi, Gondolo 2019a

Form a $p_r = p_T = -\rho = \text{const}$ core of increasing radius

Force balance in equilibrium configuration



In the inversion zone, the pressure gradient force and the gravitational force point inwards and are balanced by the anisotropy force.

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An exact time-dependent interior Schwarzschild solution

Schwarzschild stars

A spherically-symmetric, static, constant density star *Schwarzschild 1916*

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)$$

with

$$f(r) = \begin{cases} \frac{1}{4}(3a-b)^2, & r \le R, \\ 1 - \frac{R_s}{r}, & r \ge R, \end{cases}$$

$$h(r) = \begin{cases} b^2, & r \le R, \\ 1 - \frac{R_s}{r}, & r \ge R. \end{cases}$$

where

$$a = \sqrt{1 - \frac{R_s}{R}}, \qquad b = \sqrt{1 - \frac{R_s r^2}{R}}$$

Schwarzschild stars

- The pressure is everywhere finite if $R/R_s > 9/8 = 1.125$ (Buchdahl bound).

- For $R/R_s < 9/8$, the pressure diverges at finite radius $R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}$

- The singularity is integrable in the sense that

$$M_{\rm grav}(V) = \int_V (\rho + p_x + p_y + p_z)\,\sqrt{-g_{tt}}\,dV$$

is finite in any volume V *Mazur, Mottola 2015*

Formation of a Schwarzschild star

Beltracchi, Gondolo 2019b

Exact time-dependent solution of Einstein's equations

Ansatz: the radius of the Schwarzschild star depends on time, $R = R(t)$

Then $T_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \rho & S_r & 0 & 0 \\ S_r & p_r & 0 & 0 \\ 0 & 0 & p_T & 0 \\ 0 & 0 & 0 & p_T \end{pmatrix}$

anisotropic pressure $p_r \neq p_T$
momentum flow S_r

Continuity of p_r and p_T at the surface of the star gives

$$2R\ddot{R}(R_s - R) + \dot{R}^2(R_s + 8R) = 0,$$

which can solved analytically to find

$$\frac{t-t_0}{t_s} = F\left(\frac{R_s}{R}\right)$$

where t_0 and t_s are integration constants and

$$F(x) = \frac{1}{2942}\left(\frac{8-28x+35x^2}{8(1-x)^{7/2}} - 1\right).$$

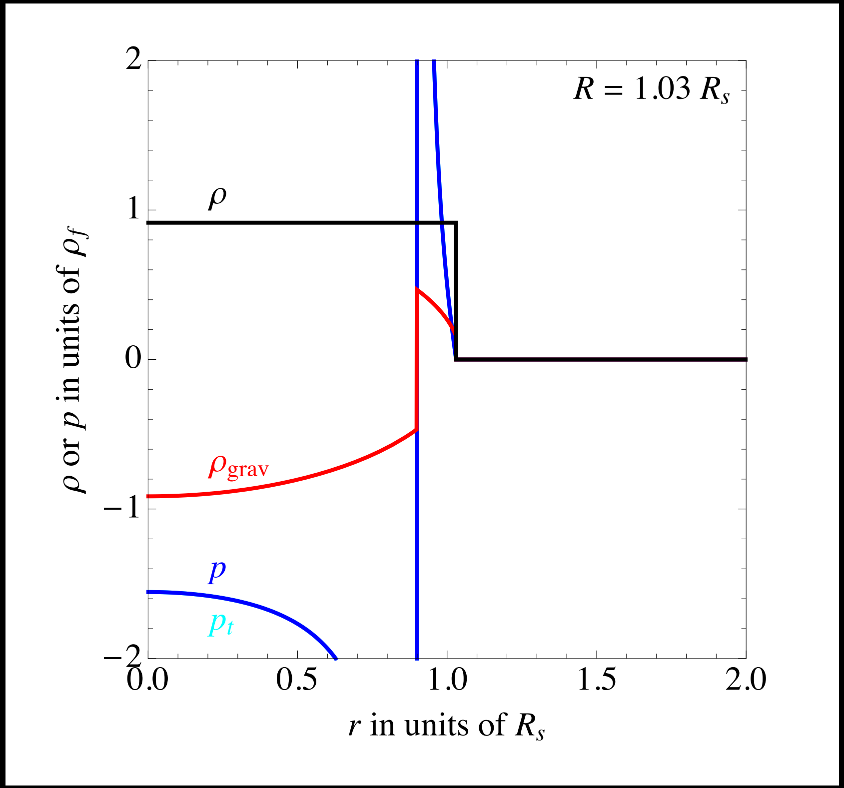
At t_0, R was infinite.
At $t_0 + t_s$, the pressure becomes singular.

Formation of a Schwarzschild star

Beltracchi, Gondolo 2019b

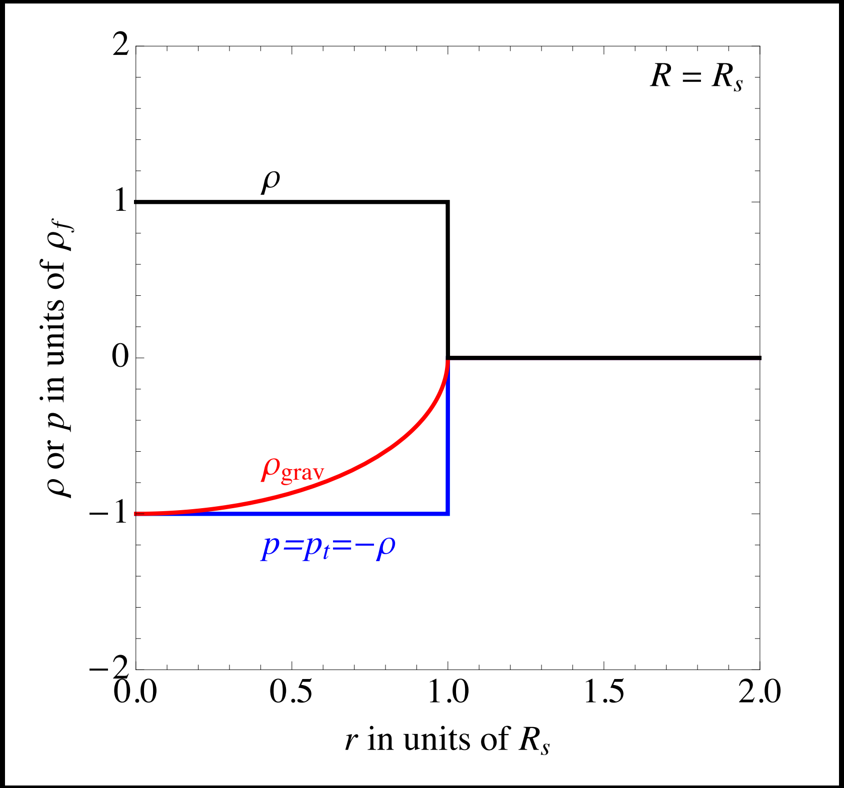
Energy density and pressure profiles

after the pressure diverges



Violates the weak and null energy conditions

at $t = \infty$



Ends in a gravastar

Formation of a Schwarzschild star

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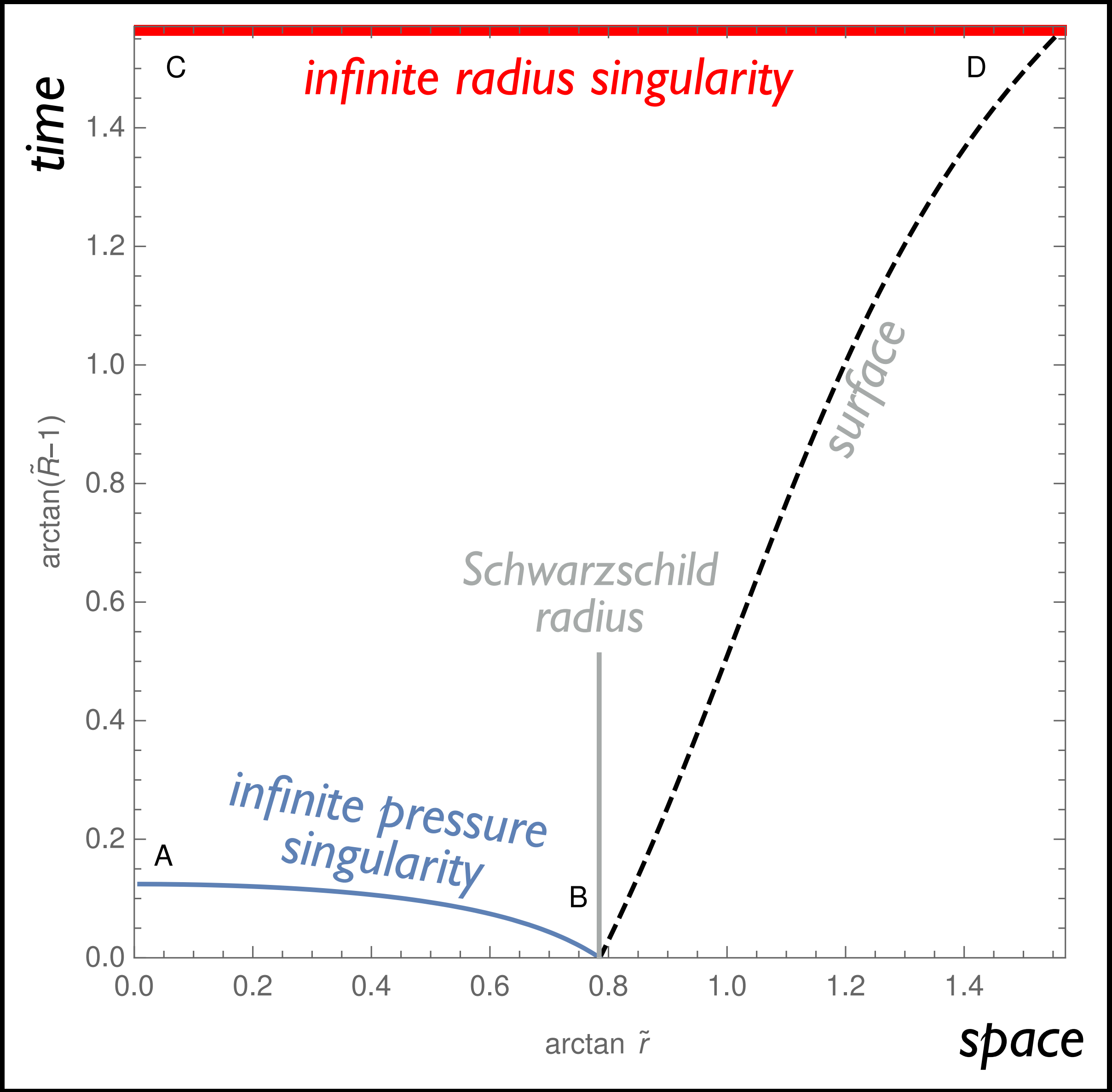
Location of curvature singularities in spacetime



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Uniaxial dark energy

Uniaxial continuum

A continuum with stress-energy tensor that can be diagonalized at every spacetime point into the diagonal form

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & & & \\ & -\rho & & \\ & & p_\perp & \\ & & & p_\perp \end{pmatrix}$$

Invariant under rotations about an axis and Lorentz boosts along that axis

- It may be characterized by an (effective) equation of state

$$p_\perp = p_\perp(\rho)$$

$$p \equiv \frac{-\rho + 2p_\perp}{3} = p(\rho)$$

- Examples:
  - cosmological constant ($p_\perp = -\rho$, $p = -\rho$)
  - Maxwell's electromagnetic theory ($p_\perp = \rho$, $p = \rho/3$)
  - Nonlinear electrodynamics, including Born-Infeld theory
- Not a scalar field that varies in space or time
- Segre type $[(11)(1,1)]$ and its degeneracy $[(111,1)]$

Static spherically-symmetric uniaxial continuum

- The boost-invariance axis is in the radial direction, $p_r = -\rho,\ p_T = p_\perp$.
- The metric is of Kerr-Schild type

$$ds^2 = -\left(1 - \frac{2Gm(r)}{c^2 r}\right) dt^2 + \frac{dr^2}{1 - \frac{2Gm(r)}{c^2 r}} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

with $\quad m(r) = \int_0^r \rho(r')\, 4\pi r'^2 dr' \quad$ and $\quad \frac{r}{2}\frac{d\rho}{dr} + \rho = -p_T(\rho)$

> TOV equation but no gravitational force because $p_r = -\rho$.
> Same density profile as in the absence of gravity.
> (Coulomb field gives M=0 Reissner-Nordstrom metric)

- Solutions obey a superposition principle

$$T_{\mu\nu} = T^{(1)}_{\mu\nu} + T^{(2)}_{\mu\nu} \qquad m(r) = m^{(1)}(r) + m^{(2)}(r)$$

The Reissner-Nordstrom-de Sitter metric has $-g_{tt} = 1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{4\pi\epsilon_0 c^4 r^2} - H^2 r^2$

Static spherically-symmetric uniaxial continuum

The special case $p_\perp = 0$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & & & \\ & -\rho & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$$

For $\rho=\mu\,\delta(z)$, this is the stress-energy tensor of a cosmic string. Thus this system can be thought of as a collection of very many cosmic strings through a single point, like a koosh.

The metric is $ds^2 = -\kappa^2\,dt^2 + \frac{dr^2}{\kappa^2} + r^2\,d\theta^2 + r^2 \sin^2\theta\,d\phi^2$

$$\kappa = \sqrt{1 - \frac{2G\lambda}{c^2}} = \text{constant}$$

"hyperconical"

Letelier 1979; Barriola, Vilenkin 1989

Static spherically-symmetric uniaxial continuum

The special case $p_\perp = 0$

It is the magnetic monopole solution of a nonlinear electrodynamic theory with Lagrangian density $\mathcal{L} = a\,(F^{\mu\nu} F_{\mu\nu})^{1/2}$

It is the metric of an O(3) global monopole at the large distances

It is the metric of a Born-Infeld magnetic monopole at the small distances

The density profile is the same as a singular isothermal sphere

$$\rho = \frac{\lambda}{4\pi r^2} \qquad\qquad m = \lambda\, r$$

Gravitational lensing looks the same as for a singular isothermal sphere

deflection angle $\alpha = \pi \dfrac{1-\kappa}{\kappa}$ *independent of distance from center*

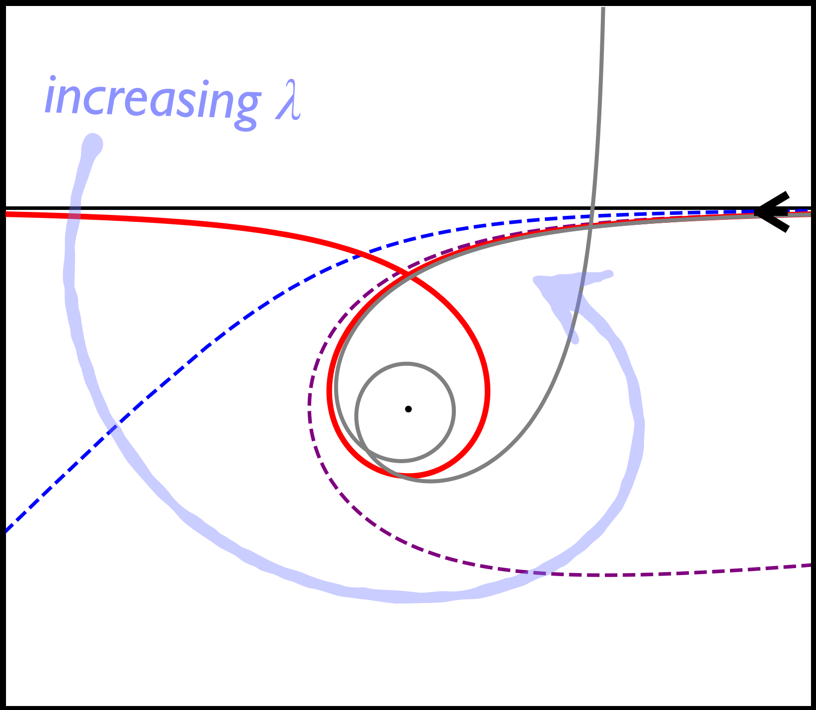
Static spherically-symmetric uniaxial continuum

The special case $p_\perp = 0$

Massive and massless particles follow the same trajectories

$$r\cos(\kappa\phi) = \kappa b \qquad\qquad \kappa = \sqrt{1 - \frac{2G\lambda}{c^2}}$$

There are no bound orbits



Looping trajectories at

$G\lambda > 3c^2/8 \simeq (0.6c)^2$

Conclusions

Gravitationally-bound dark energy structures

We have been examining theoretical possibilities to form gravitationally bound dark energy objects.

We found an exact time-dependent solution of Einstein's equations describing the collapse of a constant-density star into a gravastar (it violates the weak energy condition).

We found explicit time-dependent semi-analytic solutions of Einstein's equations giving the collapse of a spherical object to a dark energy star (they have no horizons/singularities and they obey the weak energy condition).

We are exploring dark energy with anisotropic stress, and found a curious dark energy object that resembles the singular isothermal sphere in some aspects but with no bound orbits (it may be thought of as infinitely-many strings through a center: a koosh).