The Boundary of the SMEFT

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Motivation

Motivation

Unique era of particle physics

- Higgs discovery
- No SUSY seen at LHC, post-naturalness?
- Model-building wide open: new physics soon, or desert?

New physics may come from precision measurement, rather than new on-shell states

- HL-LHC
- Higgs factories
- nEDM, eEDM
- Muon g-2
- Flavor violation

$\Longrightarrow$Standard Model effective field theory

The space of EFTs

How do we build a quantum field theory?

- Write down a Lagrangian, built out of operators $\mathcal{O}_i$, with couplings c_i:

$$\mathcal{L} = \overline{\mathcal{L}} + \sum_i c_i \mathcal{O}_i$$

and then just quantize it.

Standard Model Lagrangian

Higher-dimension operators

The space of EFTs

How do we build a quantum field theory?

- Write down a Lagrangian, built out of operators $\mathcal{O}_i$, with couplings c_i:

$$\mathcal{L} = \overline{\mathcal{L}} + \sum_i c_i \mathcal{O}_i$$

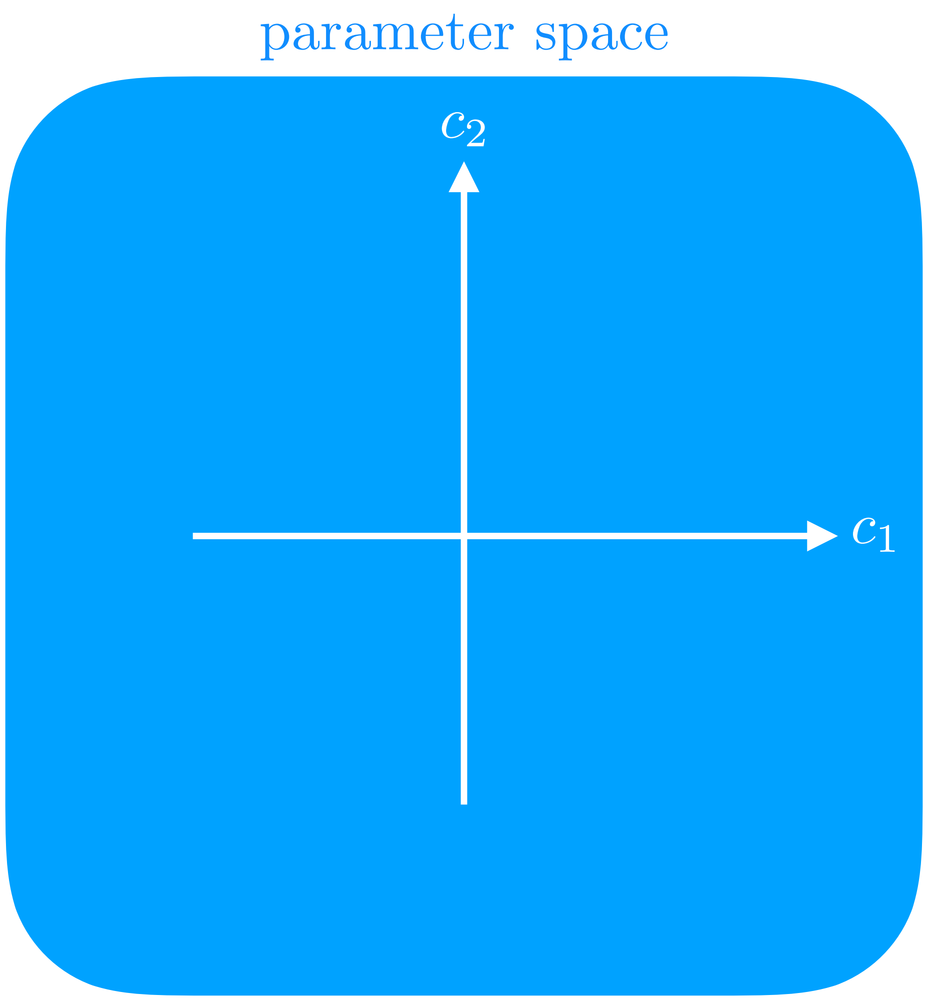
  and then just quantize it.

- Is this guaranteed to create a consistent EFT? No! Not all couplings c_i are allowed
- Certain signs of couplings violate infrared physics principles:
    - Unitarity
    - Causality
    - Analyticity
    - Examples:
        - Einstein-Maxwell theory Cheung, GR [1407.7865]; Cheung, Liu, GR [1801.08546, 1903.09156]; Bellazzini, Lewandowski, Serra [1902.03250]
        - Higher-curvature gravity (R^2, R^4 terms) Bellazzini, Cheung, GR [1509.00851]; Cheung, GR [1608.02942]
        - Massive gravity Cheung, GR [1601.04068]
        - $(\partial\phi)^4$ and F^4 couplings Adams et al. [hep-th/0602178]
        - Higher-point couplings Chandrasekaran, GR, Shahbazi-Moghaddam [1804.03153]
        - Conformal galileon Nicolis, Rattazzi, Trincherini [0912.4258]
        - a-theorem in $D = 4$ Komargodski, Schwimmer [1107.3987]

The space of EFTs

Despite this progress, little has been previously done to apply IR consistency program to SMEFT itself.

Compelling experimental reasons for doing so:



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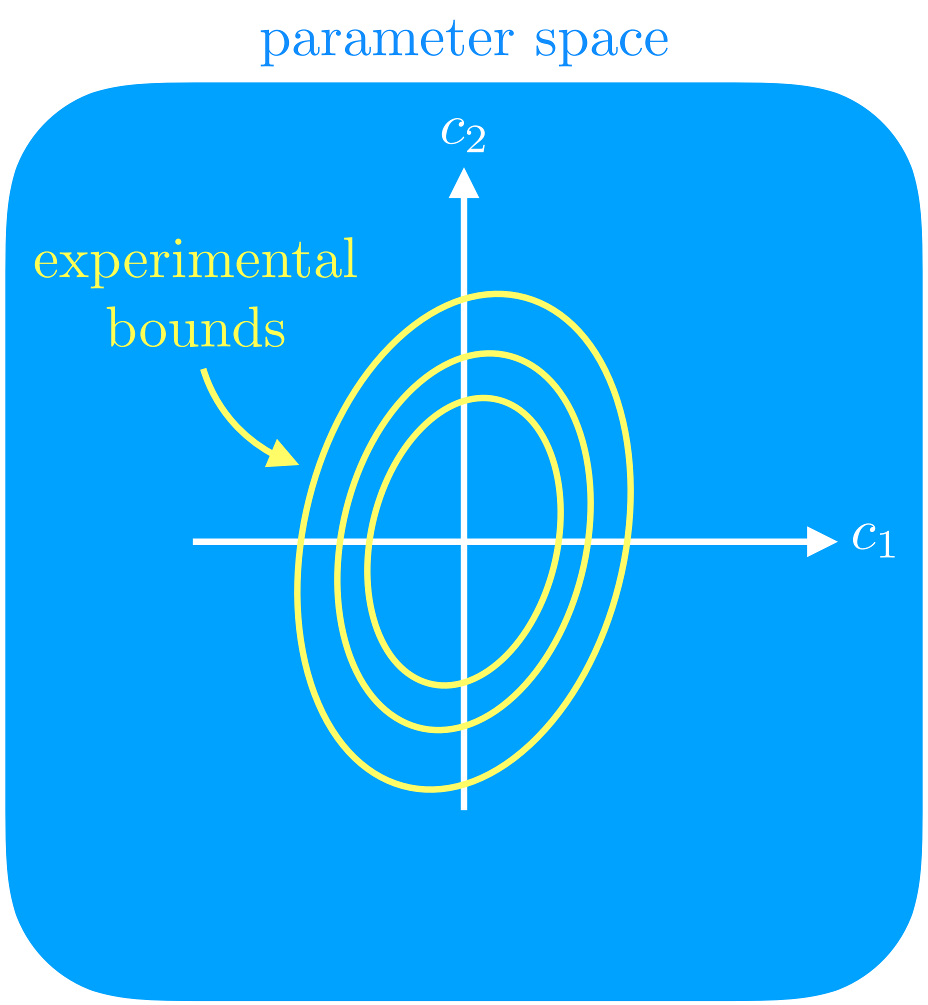

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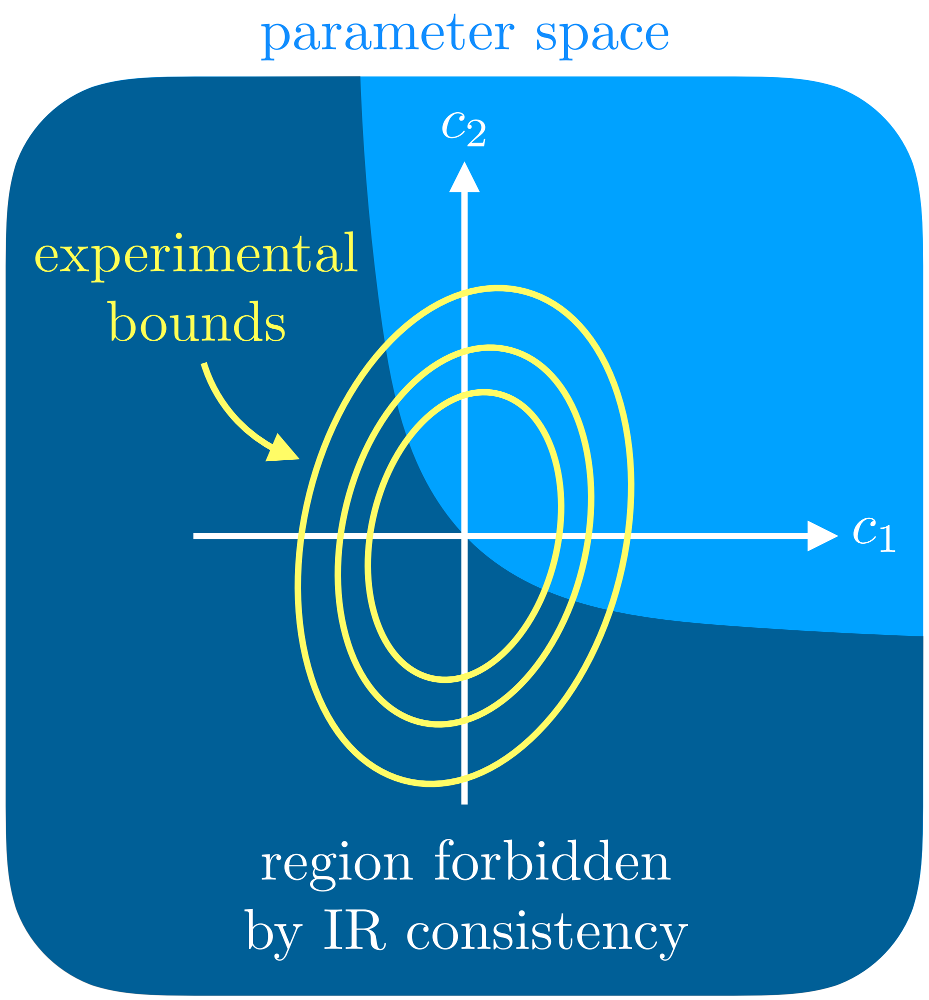

IR consistency conditions place powerful theoretical priors on parameter space.

The space of EFTs

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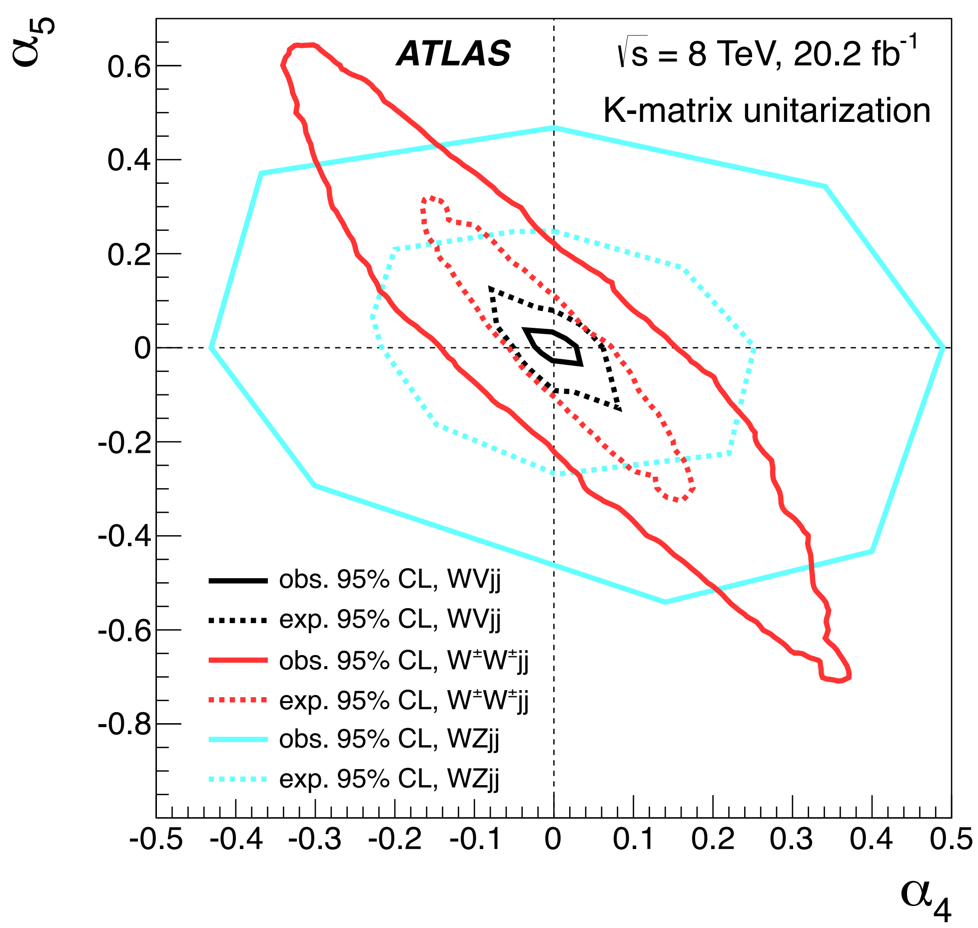
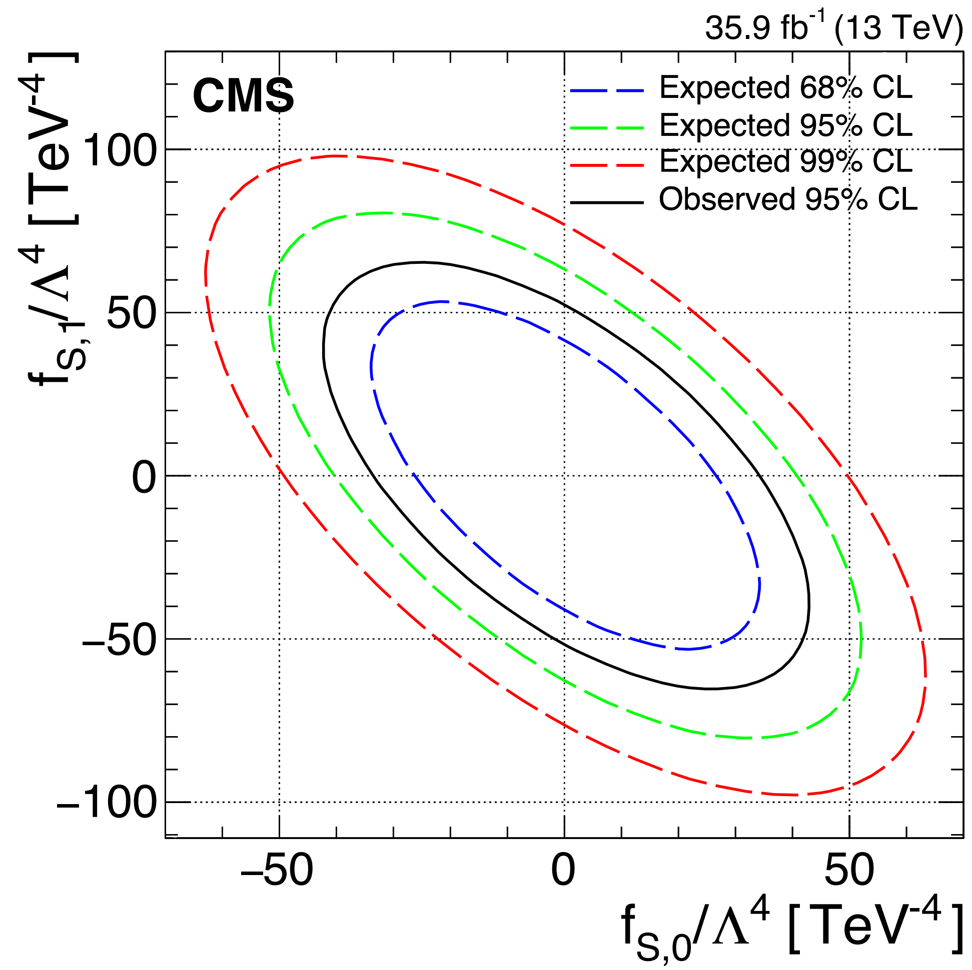
Compelling experimental reasons for doing so:



ATLAS [1609.05122]



CMS [1901.04060]

LHC is already bounding higher-dimension operators in the SMEFT.

Bounding SMEFT

This talk:

Apply IR consistency (analyticity & causality) bounds to constrain SMEFT.

- Four-point operators
- Need number of momenta divisible by 4
- Will consider either all-bosonic or all-fermionic operators and scatter states of fixed SM representation

$\Longrightarrow$ Mass dimension-eight operators $\sim 1/M^4$

In the SMEFT, there are:

- 64 bosonic operators
- 2763 fermionic operators

that we need to consider.

We will derive:

- 27 independent bounds
- 25 independent families of bounds

Connect LHC searches with other experiments (nEDM, Mu3e, etc.).

Powerful probe of fundamental physics (Lorentz invariance, causality, etc.): test of axioms of QFT and string theory.

Infrared consistency

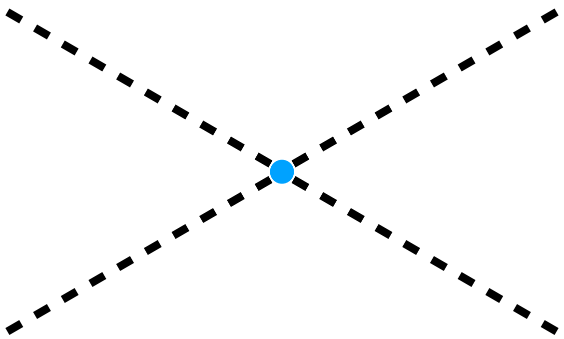
Example theory

We'll first briefly review how infrared consistency bounds the coefficients of an EFT, based on analyticity, unitarity, and causality. Adams et al. [hep-th/0602178]

Example EFT: massless scalar with shift symmetry

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$$

Two-to-two scattering amplitude:



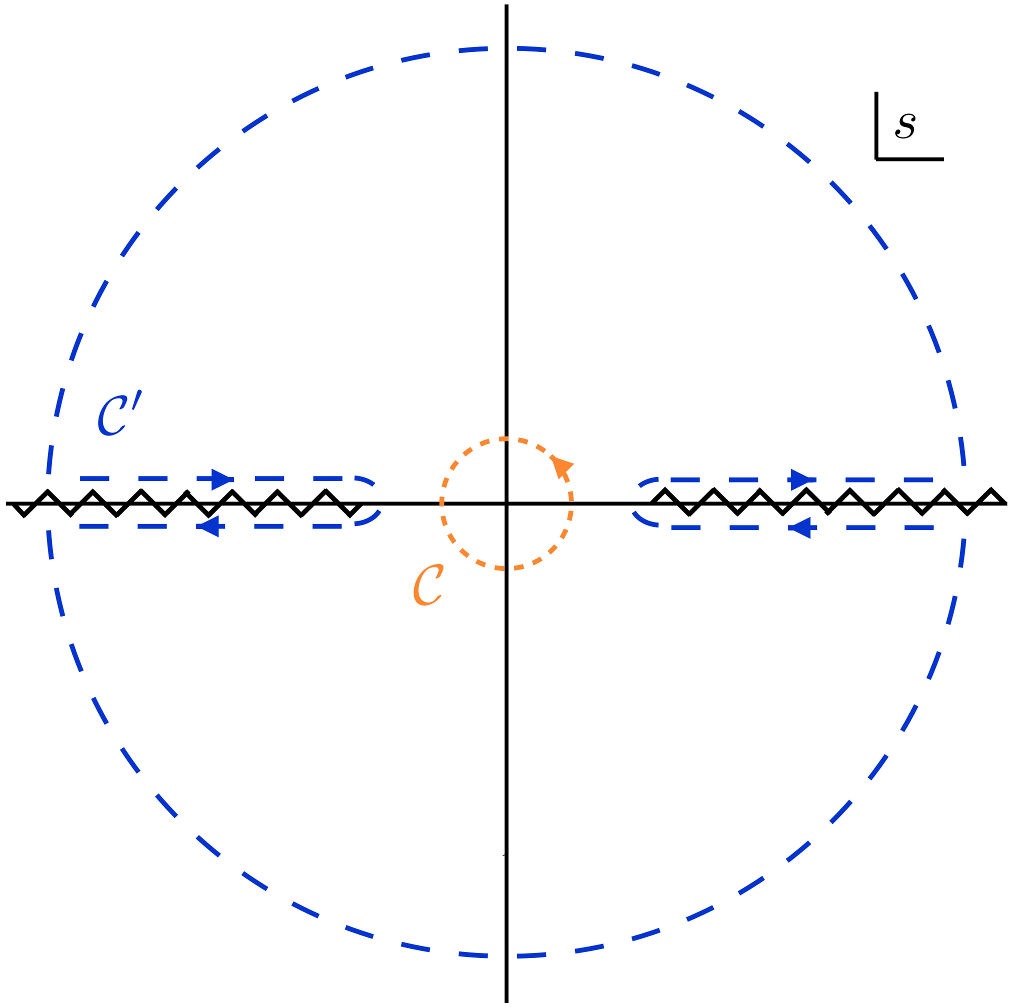
$$\mathcal{M}(s,t) = \frac{2c}{M^4}(s^2 + t^2 + u^2)$$

Forward amplitude (in state = out state):

$$\mathcal{A}(s) = \frac{4c}{M^4}s^2$$

Analyticity

The Wilson coefficient of interest can be extracted via a contour integral of the forward amplitude:

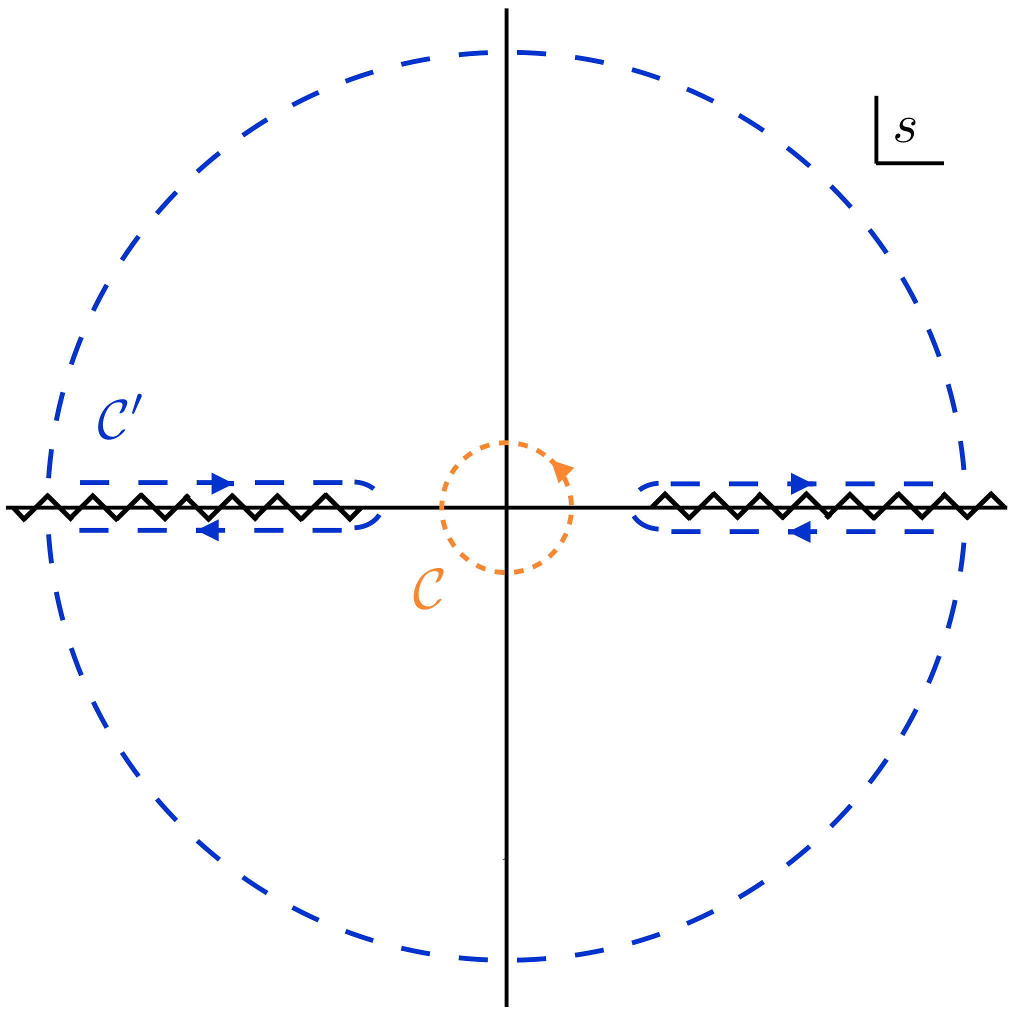


$$\frac{4c}{M^4} = \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{\mathrm{d}s}{s^3}\mathcal{A}(s)$$

residue theorem

Analyticity

The Wilson coefficient of interest can be extracted via a contour integral of the forward amplitude:



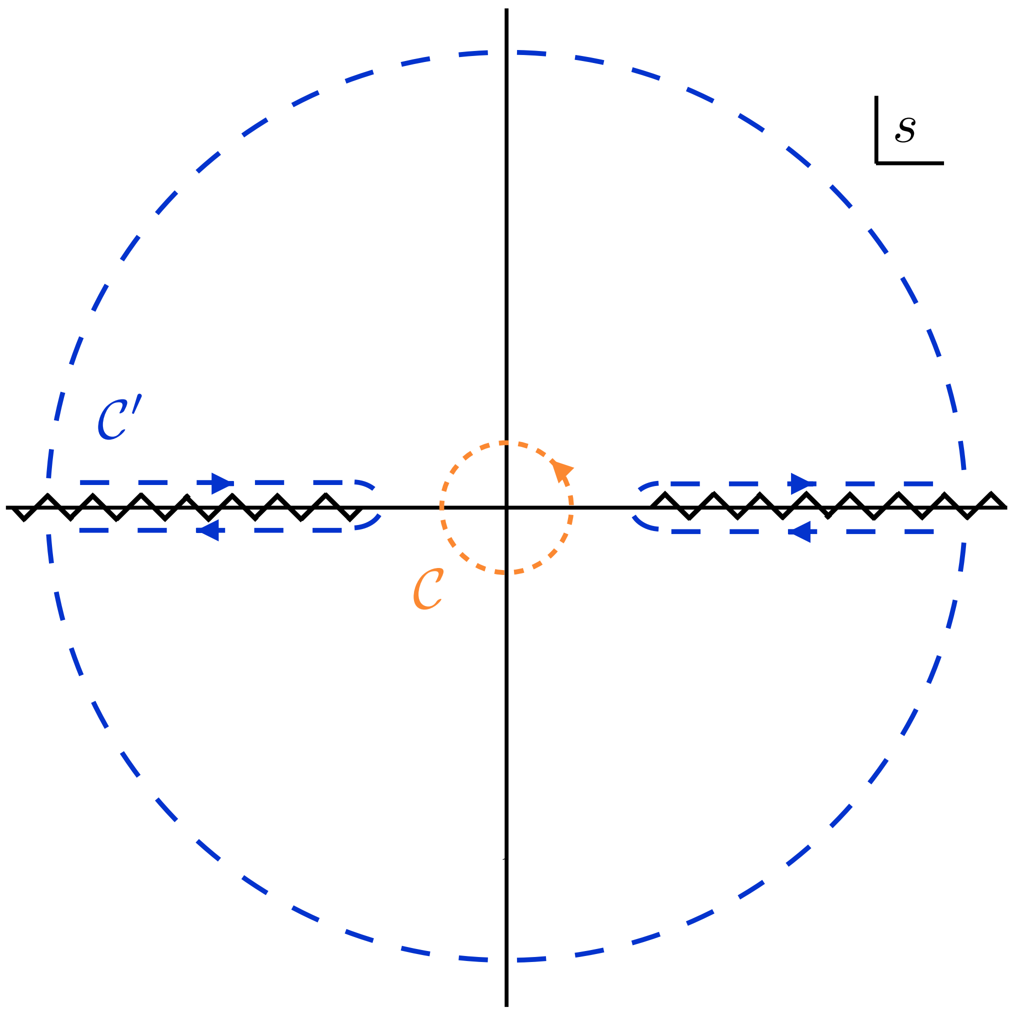
$$\frac{4c}{M^4} = \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{\mathrm{d}s}{s^3}\mathcal{A}(s)$$

$$= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{\mathrm{d}s}{s^3}\mathcal{A}(s)$$

use analyticity to deform the contour

Analyticity

The Wilson coefficient of interest can be extracted via a contour integral of the forward amplitude:

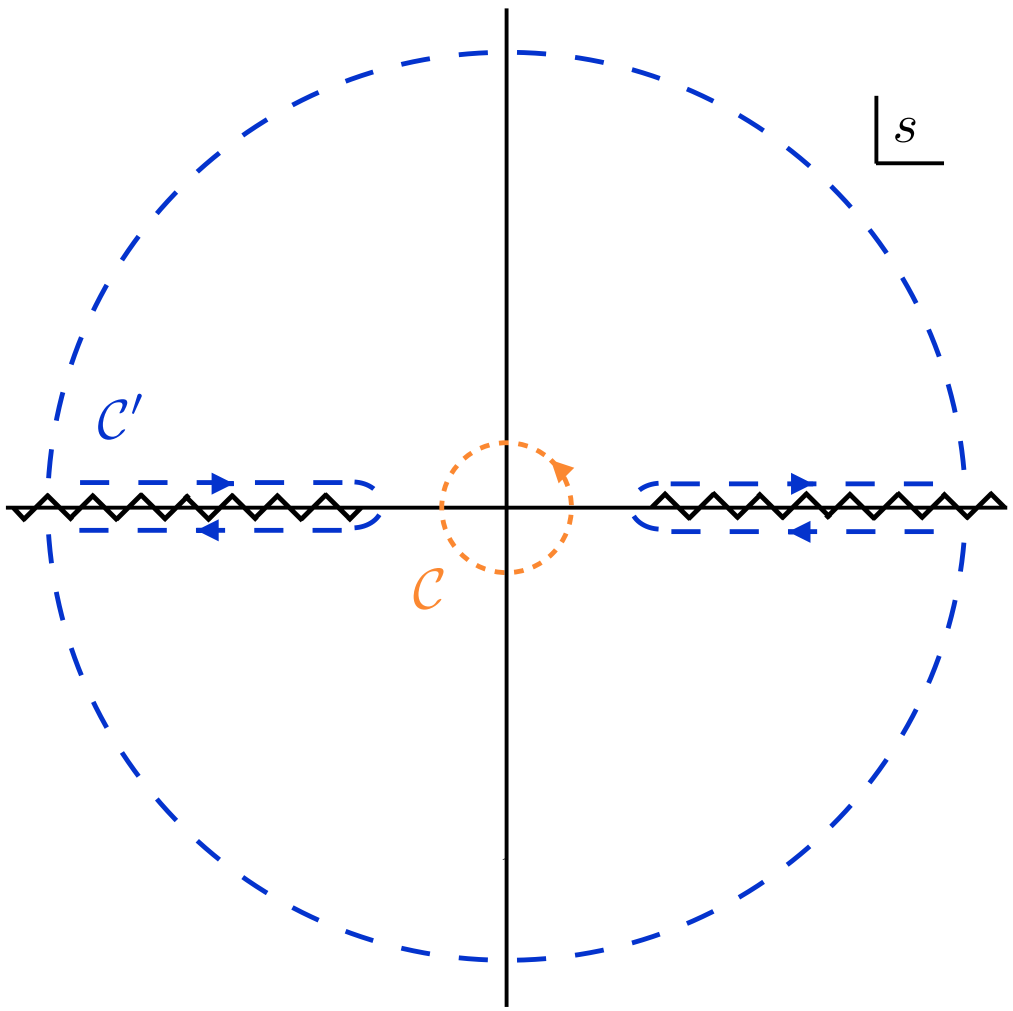


$$\begin{aligned}\frac{4c}{M^4} &= \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{\mathrm{d}s}{s^3}\mathcal{A}(s)\\ &= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{\mathrm{d}s}{s^3}\mathcal{A}(s)\\ &= \frac{1}{2\pi i}\left(\int_{-\infty}^{-s_{\mathrm{d}}} + \int_{s_{\mathrm{d}}}^{\infty}\right)\frac{\mathrm{d}s}{s^3}\mathrm{Disc}\,\mathcal{A}(s)\end{aligned}$$

boundary term at infinity vanishes

Analyticity

The Wilson coefficient of interest can be extracted via a contour integral of the forward amplitude:

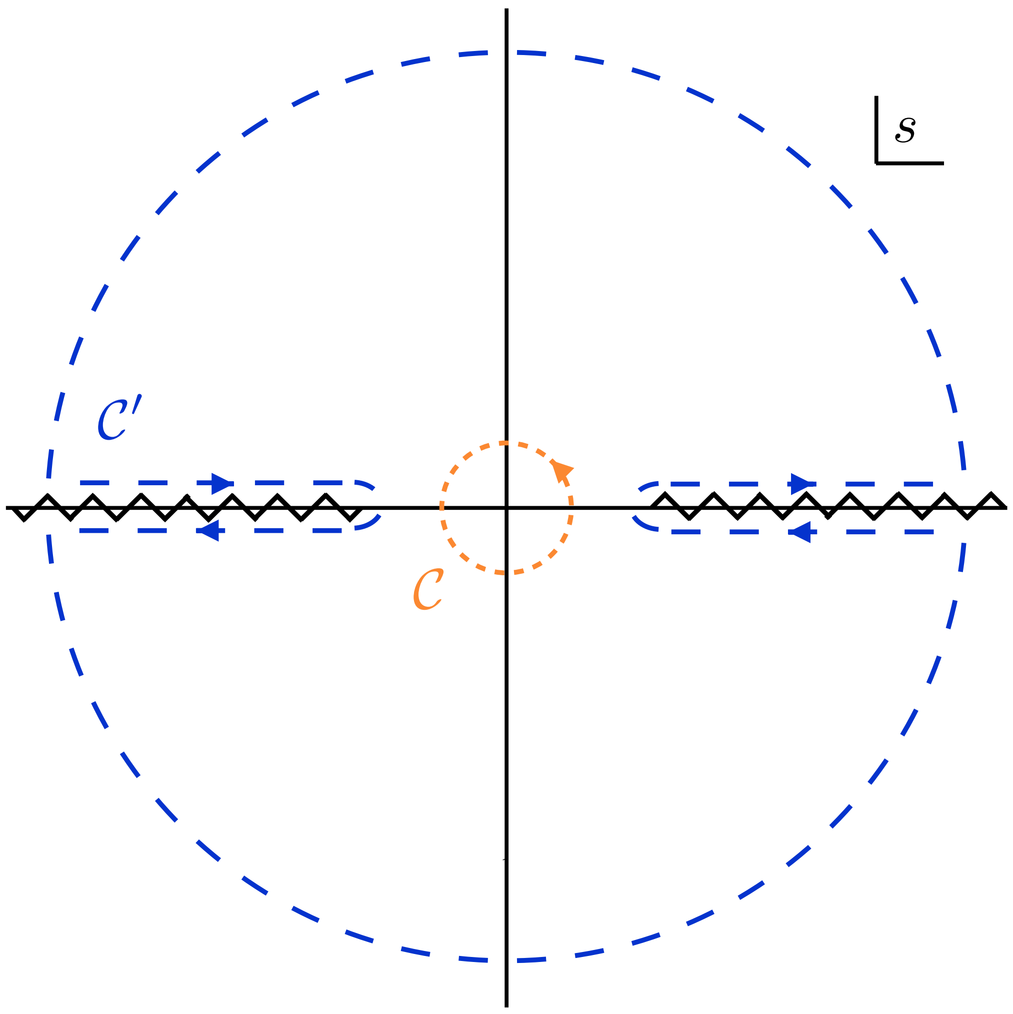


$$
\begin{aligned}
\frac{4c}{M^4} &= \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{\mathrm{d}s}{s^3}\mathcal{A}(s) \\
&= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{\mathrm{d}s}{s^3}\mathcal{A}(s) \\
&= \frac{1}{2\pi i}\left(\int_{-\infty}^{-s_{\mathrm{d}}} + \int_{s_{\mathrm{d}}}^{\infty}\right)\frac{\mathrm{d}s}{s^3}\mathrm{Disc}\,\mathcal{A}(s) \\
&= \frac{1}{i\pi}\int_{s_{\mathrm{d}}}^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{Disc}\,\mathcal{A}(s)
\end{aligned}
$$

crossing symmetry: $\mathcal{A}(-s) = \mathcal{A}(s)$

Analyticity

The Wilson coefficient of interest can be extracted via a contour integral of the forward amplitude:



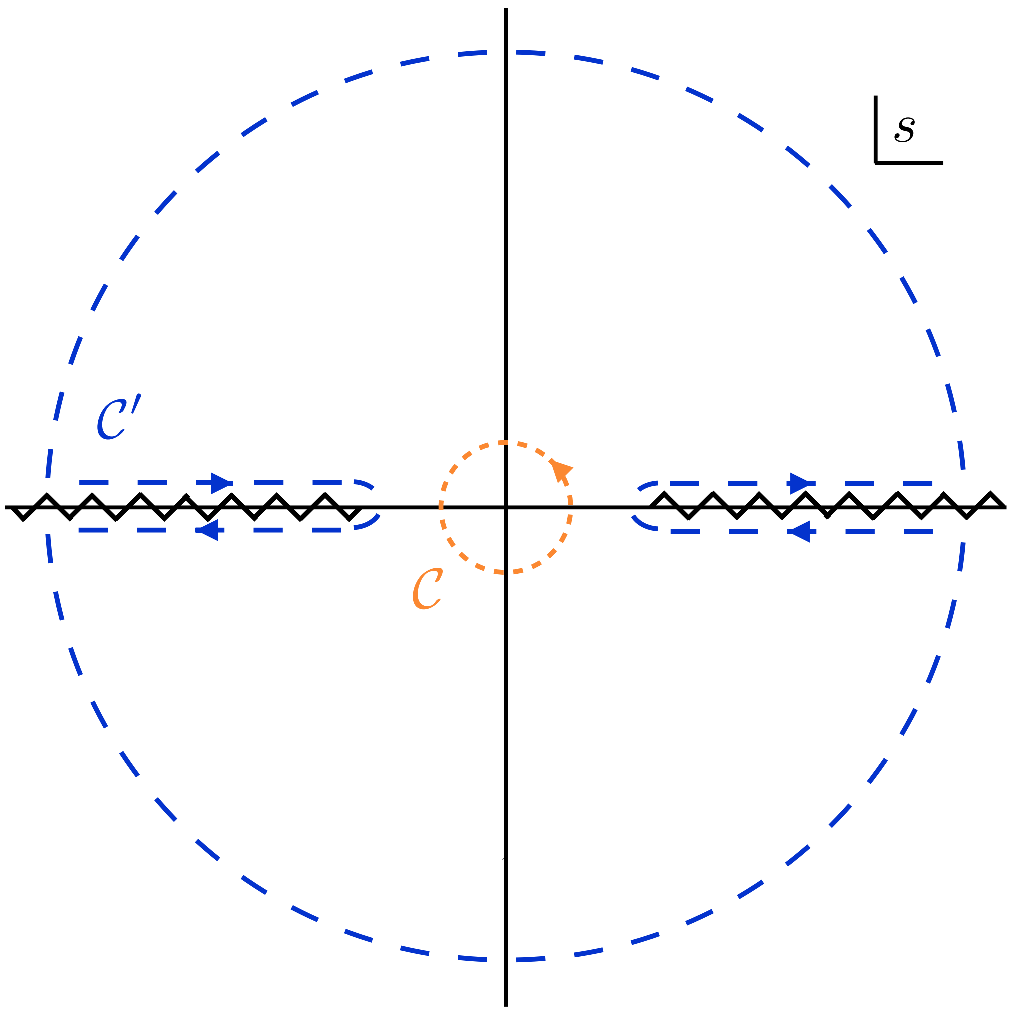
$$\frac{4c}{M^4} = \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{Disc}\,\mathcal{A}(s)$$

$$= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\lim_{\epsilon\to 0}\left[\mathcal{A}(s+i\epsilon)-\mathcal{A}(s-i\epsilon)\right]$$

by definition

Analyticity

The Wilson coefficient of interest can be extracted via a contour integral of the forward amplitude:



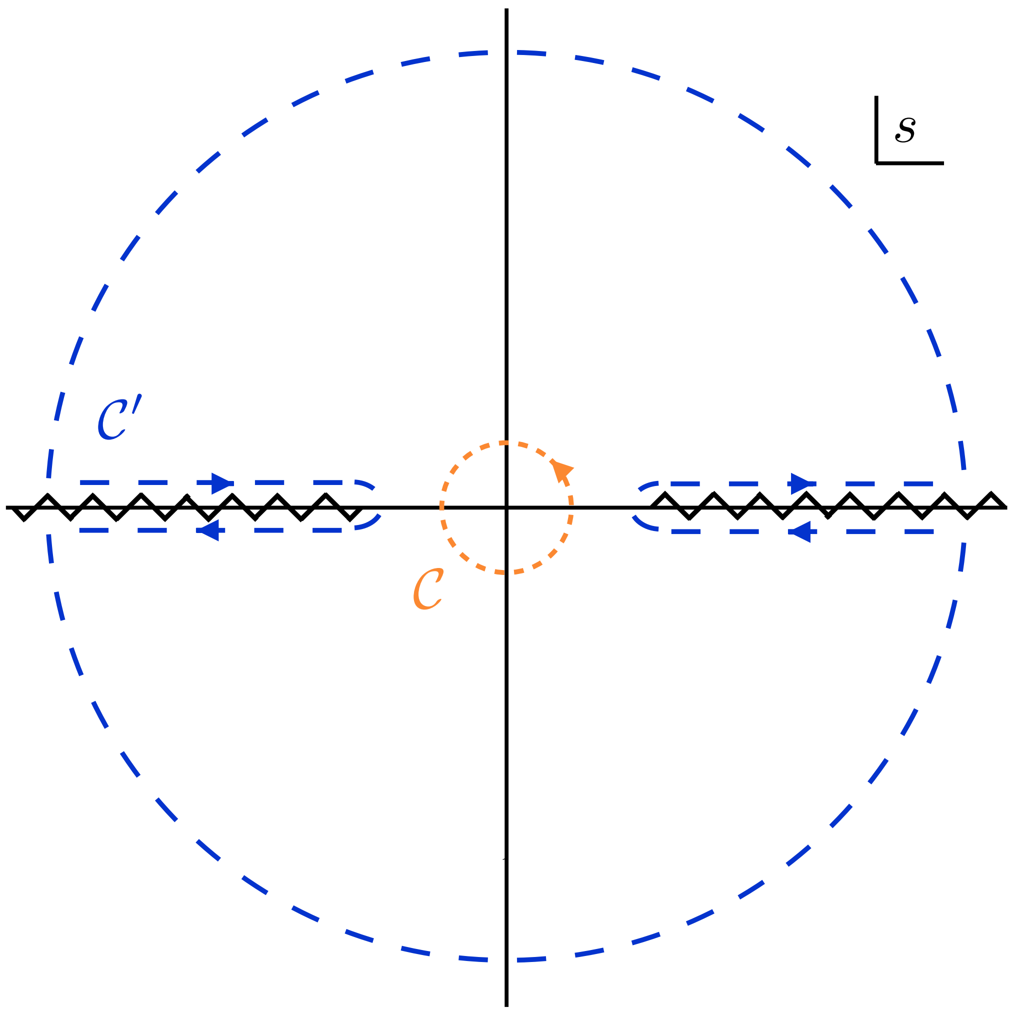
$$
\begin{aligned}
\frac{4c}{M^4} &= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\,\mathrm{Disc}\,\mathcal{A}(s)\\
&= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\lim_{\epsilon\to 0}\left[\mathcal{A}(s+i\epsilon)-\mathcal{A}(s-i\epsilon)\right]\\
&= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\lim_{\epsilon\to 0}\left[\mathcal{A}(s+i\epsilon)-(\mathcal{A}(s+i\epsilon))^*\right]
\end{aligned}
$$

Schwarz reflection principle:

$$\mathcal{A}(s^*) = [\mathcal{A}(s)]^*$$

Analyticity

The Wilson coefficient of interest can be extracted via a contour integral of the forward amplitude:

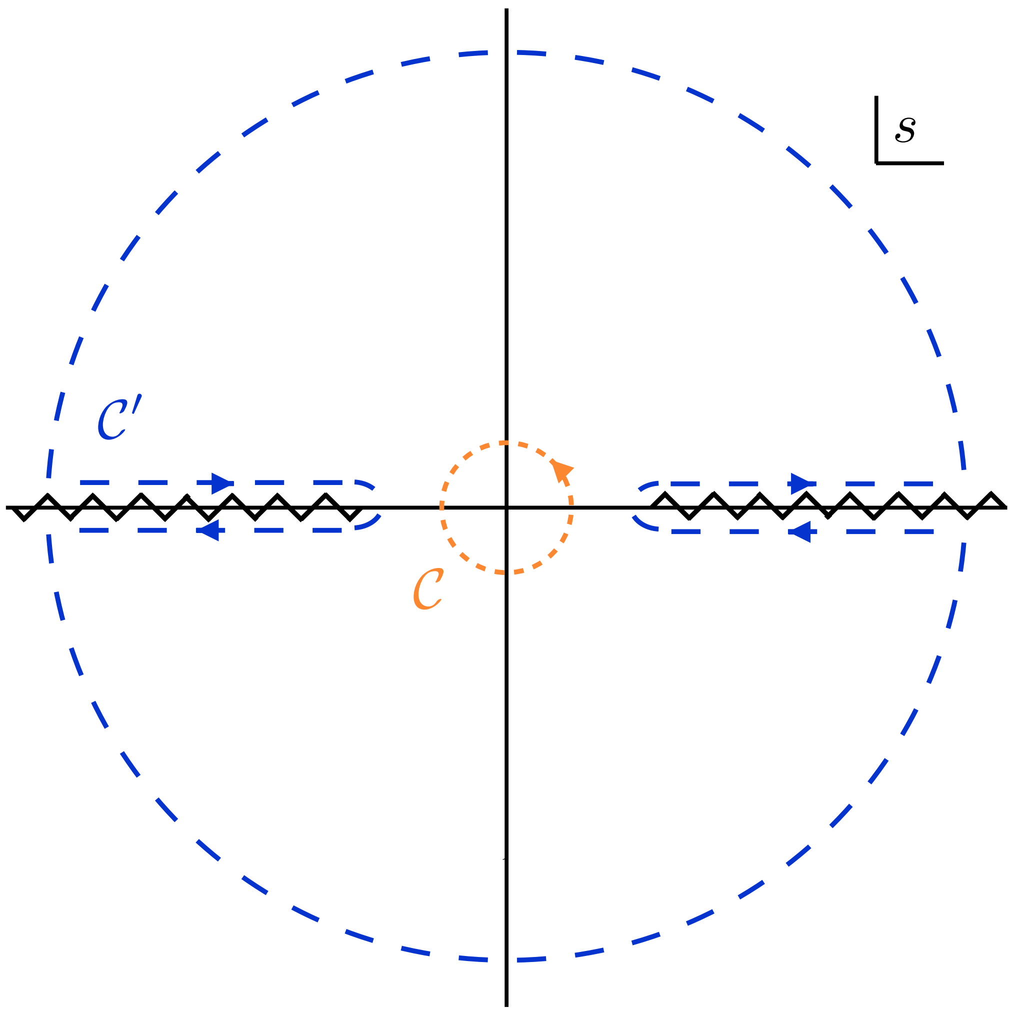


$$
\begin{aligned}
\frac{4c}{M^4} &= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{Disc}\,\mathcal{A}(s) \\
&= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\lim_{\epsilon\to 0}\left[\mathcal{A}(s+i\epsilon)-\mathcal{A}(s-i\epsilon)\right] \\
&= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\lim_{\epsilon\to 0}\left[\mathcal{A}(s+i\epsilon)-(\mathcal{A}(s+i\epsilon))^*\right] \\
&= \frac{2}{\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{Im}\,\mathcal{A}(s)
\end{aligned}
$$

by definition

Analyticity

The Wilson coefficient of interest can be extracted via a contour integral of the forward amplitude:



$$
\begin{aligned}
\frac{4c}{M^4} &= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{Disc}\,\mathcal{A}(s) \\
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&= \frac{1}{i\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\lim_{\epsilon\to 0}\left[\mathcal{A}(s+i\epsilon)-(\mathcal{A}(s+i\epsilon))^*\right] \\
&= \frac{2}{\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^3}\mathrm{Im}\,\mathcal{A}(s) \\
&= \frac{2}{\pi}\int_{s_\mathrm{d}}^{\infty}\frac{\mathrm{d}s}{s^2}\sigma(s)
\end{aligned}
$$

using the optical theorem (unitarity):

$$\mathrm{Im}\,\mathcal{A}(s) = s\,\sigma(s)$$

$$\implies c > 0$$

Adams et al. [hep-th/0602178]

Causality

Equation of motion for $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$:

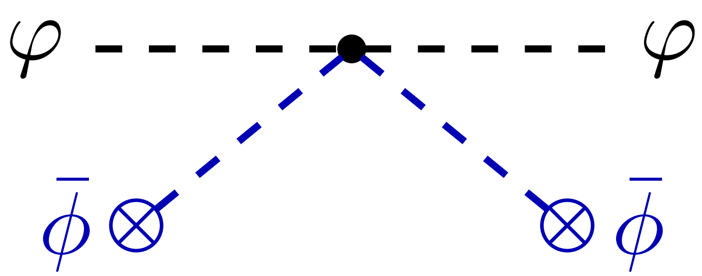
$$\Box\phi - \frac{4c}{M^4}\left[\Box\phi(\partial\phi)^2 + 2(\partial^\mu\phi)(\partial^\nu\phi)(\partial_\mu\partial_\nu\phi)\right] = 0$$

Let's expand about a condensate background:

$$\phi = \overline{\phi} + \varphi, \quad \overline{\partial_\mu\phi} = q_\mu = \text{constant}$$

so

$$\left(-1 + \frac{4cq^2}{M^4}\right)\Box\varphi + \frac{8c}{M^4}q^\mu q^\nu \partial_\mu\partial_\nu\varphi = 0$$



GR, Rodd [1908.09845]

Causality

Expanding our perturbation in plane waves, $\varphi \propto e^{ik\cdot x}$, we have a dispersion relation:

$$\left(-1 + \frac{4cq^2}{M^4}\right) k^2 + \frac{8c}{M^4}(q \cdot k)^2 = 0$$

Writing the wave vector as $k_\mu = (k_0, \mathbf{k})$, the speed of the perturbation is:

$$v = \frac{k_0}{|\mathbf{k}|} = \sqrt{1 - \frac{8c(q\cdot k)^2}{|\mathbf{k}|^2(M^4 - 4cq^2)}} \simeq 1 - \frac{4c(q\cdot k)^2}{M^4 k_0^2}$$

If $c < 0$, then $v > 1$, and a causal paradox can be engineered by giving two bubbles of condensate a large relative boost. Adams et al. [hep-th/0602178]

Bosonic operator basis

GR, Rodd [1908.09845]

Building blocks

We want all bosonic four-point operators that have four derivatives and/or field strengths. Morozov (1984); Hays et al. [1808.00442]

- Ingredients:
  - Gauge field strengths:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g_2 \epsilon^{IJK} W^J_\mu W^K_\nu$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_3 f^{abc} G^b_\mu G^c_\nu$$

  - Higgs:

$$H_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad H^\dagger_i = \frac{1}{\sqrt{2}} \begin{pmatrix} -\phi_3 + i\phi_4 \\ \phi_1 - i\phi_2 \end{pmatrix}$$

$$D_\mu H = (\partial_\mu + \tfrac{1}{2} i g_1 B_\mu + i g_2 \tau^I W^I_\mu) H$$

Building blocks

We want all bosonic four-point operators that have four derivatives and/or field strengths. Morozov (1984); Hays et al. [1808.00442]

- Must be Lorentz and gauge invariant. Contract indices using:

$$g^{\mu\nu}, \epsilon^{\mu\nu\rho\sigma}, \delta^{IJ}, \epsilon^{IJK}, \delta^{ab}, f^{abc}, d^{abc}$$

- Not all contractions are independent, due to identities.
    - Levi-Civita: $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta} = -24\,\delta^{[\mu}_{\alpha}\,\delta^{\nu}_{\beta}\,\delta^{\rho}_{\gamma}\,\delta^{\sigma]}_{\delta}$
    - Schouten: $\epsilon^{[\alpha\beta\gamma\delta}g^{\mu]\nu} = 0$
    - SU(N) identities
    - Define

$$\begin{aligned}
\widetilde{B}^{\mu\nu} &= \epsilon^{\mu\nu\rho\sigma}B_{\rho\sigma}/2 \\
\widetilde{W}^{I\mu\nu} &= \epsilon^{\mu\nu\rho\sigma}W^{I}_{\rho\sigma}/2 \\
\widetilde{G}^{a\mu\nu} &= \epsilon^{\mu\nu\rho\sigma}G^{a}_{\rho\sigma}/2
\end{aligned}$$

Field strength self-quartics

Basis of independent operators:

$\mathcal{O}_1^{B^4}$	$(BB)(BB)$
$\mathcal{O}_2^{B^4}$	$(B\widetilde{B})(B\widetilde{B})$
$\widetilde{\mathcal{O}}_1^{B^4}$	$(BB)(B\widetilde{B})$

$\mathcal{O}_1^{W^4}$	$(W^I W^I)(W^J W^J)$
$\mathcal{O}_2^{W^4}$	$(W^I \widetilde{W}^I)(W^J \widetilde{W}^J)$
$\mathcal{O}_3^{W^4}$	$(W^I W^J)(W^I W^J)$
$\mathcal{O}_4^{W^4}$	$(W^I \widetilde{W}^J)(W^I \widetilde{W}^J)$
$\widetilde{\mathcal{O}}_1^{W^4}$	$(W^I W^I)(W^J \widetilde{W}^J)$
$\widetilde{\mathcal{O}}_2^{W^4}$	$(W^I W^J)(W^I \widetilde{W}^J)$

$\mathcal{O}_1^{G^4}$	$(G^a G^a)(G^b G^b)$
$\mathcal{O}_2^{G^4}$	$(G^a \widetilde{G}^a)(G^b \widetilde{G}^b)$
$\mathcal{O}_3^{G^4}$	$(G^a G^b)(G^a G^b)$
$\mathcal{O}_4^{G^4}$	$(G^a \widetilde{G}^b)(G^a \widetilde{G}^b)$
$\mathcal{O}_5^{G^4}$	$d^{abe} d^{cde} (G^a G^b)(G^c G^d)$
$\mathcal{O}_6^{G^4}$	$d^{abe} d^{cde} (G^a \widetilde{G}^b)(G^c \widetilde{G}^d)$
$\widetilde{\mathcal{O}}_1^{G^4}$	$(G^a G^a)(G^b \widetilde{G}^b)$
$\widetilde{\mathcal{O}}_2^{G^4}$	$(G^a G^b)(G^a \widetilde{G}^b)$
$\widetilde{\mathcal{O}}_3^{G^4}$	$d^{abe} d^{cde} (G^a G^b)(G^c \widetilde{G}^d)$

writing $(AB) = A_{\mu\nu} B^{\mu\nu}$

Field strength cross-quartics

Basis of independent operators:

$\mathcal{O}_1^{B^2W^2}$	$(BB)(W^IW^I)$
$\mathcal{O}_2^{B^2W^2}$	$(B\widetilde{B})(W^I\widetilde{W}^I)$
$\mathcal{O}_3^{B^2W^2}$	$(BW^I)(BW^I)$
$\mathcal{O}_4^{B^2W^2}$	$(B\widetilde{W}^I)(B\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_1^{B^2W^2}$	$(B\widetilde{B})(W^IW^I)$
$\widetilde{\mathcal{O}}_2^{B^2W^2}$	$(BB)(W^I\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_3^{B^2W^2}$	$(BW^I)(B\widetilde{W}^I)$

$\mathcal{O}_1^{B^2G^2}$	$(BB)(G^aG^a)$
$\mathcal{O}_2^{B^2G^2}$	$(B\widetilde{B})(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{B^2G^2}$	$(BG^a)(BG^a)$
$\mathcal{O}_4^{B^2G^2}$	$(B\widetilde{G}^a)(B\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_1^{B^2G^2}$	$(B\widetilde{B})(G^aG^a)$
$\widetilde{\mathcal{O}}_2^{B^2G^2}$	$(BB)(G^a\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_3^{B^2G^2}$	$(BG^a)(B\widetilde{G}^a)$

$\mathcal{O}_1^{W^2G^2}$	$(W^IW^I)(G^aG^a)$
$\mathcal{O}_2^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{W^2G^2}$	$(W^IG^a)(W^IG^a)$
$\mathcal{O}_4^{W^2G^2}$	$(W^I\widetilde{G}^a)(W^I\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_1^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^aG^a)$
$\widetilde{\mathcal{O}}_2^{W^2G^2}$	$(W^IW^I)(G^a\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_3^{W^2G^2}$	$(W^IG^a)(W^I\widetilde{G}^a)$

$\mathcal{O}_1^{BG^3}$	$d^{abc}(BG^a)(G^bG^c)$
$\mathcal{O}_2^{BG^3}$	$d^{abc}(B\widetilde{G}^a)(G^b\widetilde{G}^c)$
$\widetilde{\mathcal{O}}_1^{BG^3}$	$d^{abc}(B\widetilde{G}^a)(G^bG^c)$
$\widetilde{\mathcal{O}}_2^{BG^3}$	$d^{abc}(BG^a)(G^b\widetilde{G}^c)$

Higgs self-quartics

Basis of independent operators:

$\mathcal{O}_1^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_2^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_3^{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

Higgs and field strength cross-quartics

Basis of independent operators:

$$\mathcal{O}_1^{H^2B^2} \qquad (D^\mu H^\dagger D^\nu H) B_{\mu\rho} B_\nu{}^\rho$$

$$\mathcal{O}_2^{H^2B^2} \qquad (D^\mu H^\dagger D_\mu H) B_{\rho\sigma} B^{\rho\sigma}$$

$$\widetilde{\mathcal{O}}_1^{H^2B^2} \qquad (D^\mu H^\dagger D_\mu H) B_{\rho\sigma} \widetilde{B}^{\rho\sigma}$$

$$\mathcal{O}_1^{H^2BW} \qquad (D^\mu H^\dagger \tau^I D_\mu H) B_{\rho\sigma} W^{I\rho\sigma}$$

$$\mathcal{O}_2^{H^2BW} \qquad I(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho} W_\nu^{I\rho} - B_{\nu\rho} W_\mu^{I\rho})$$

$$\mathcal{O}_3^{H^2BW} \qquad (D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho} W_\nu^{I\rho} B_{\nu\rho} W_\mu^{I\rho})$$

$$\widetilde{\mathcal{O}}_1^{H^2BW} \qquad (D^\mu H^\dagger \tau^I D_\mu H) B_{\rho\sigma} \widetilde{W}^{I\rho\sigma}$$

$$\widetilde{\mathcal{O}}_2^{H^2BW} \qquad i(D^\mu H^\dagger \tau^I D^\nu H)(B_{\rho[\mu} \widetilde{W}_{\nu]}^{I\rho} - \widetilde{B}_{\rho[\mu} W_{\nu]}^{I\rho})$$

$$\widetilde{\mathcal{O}}_3^{H^2BW} \qquad (D^\mu H^\dagger \tau^I D^\nu H)(B_{\rho(\mu} \widetilde{W}_{\nu)}^{I\rho} + \widetilde{B}_{\rho(\mu} W_{\nu)}^{I\rho})$$

$$\mathcal{O}_1^{H^2W^2} \qquad (D^\mu H^\dagger D^\nu H) W_{\mu\rho}^I W_\nu^{I\rho}$$

$$\mathcal{O}_2^{H^2W^2} \qquad (D^\mu H^\dagger D_\mu H) W_{\rho\sigma}^I W^{I\rho\sigma}$$

$$\mathcal{O}_3^{H^2W^2} \qquad i\,\epsilon^{IJK} (D^\mu H^\dagger \tau^I D^\nu H) W_{\mu\rho}^J W_\nu^{K\rho}$$

$$\widetilde{\mathcal{O}}_1^{H^2W^2} \qquad (D^\mu H^\dagger D_\mu H) W_{\rho\sigma}^I \widetilde{W}^{I\rho\sigma}$$

$$\widetilde{\mathcal{O}}_2^{H^2W^2} \qquad \epsilon^{IJK} (D^\mu H^\dagger \tau^I D^\nu H)(W_{\mu\rho}^J \widetilde{W}\nu^{K\rho} - \widetilde{W}_{\mu\rho}^J W\nu^{K\rho})$$

$$\widetilde{\mathcal{O}}_3^{H^2W^2} \qquad i\,\epsilon^{IJK} (D^\mu H^\dagger \tau^I D^\nu H)(W_{\mu\rho}^J \widetilde{W}_\nu^{K\rho} + W_{\mu\rho}^J W_\nu^{K\rho})$$

$$\mathcal{O}_1^{H^2G^2} \qquad (D^\mu H^\dagger D^\nu H) G_{\mu\rho}^a G_\nu^{a\rho}$$

$$\mathcal{O}_2^{H^2G^2} \qquad (D^\mu H^\dagger D_\mu H) G_{\rho\sigma}^a G^{a\rho\sigma}$$

$$\widetilde{\mathcal{O}}_1^{H^2G^2} \qquad (D^\mu H^\dagger D_\mu H) G_{\rho\sigma}^a \widetilde{G}^{a\rho\sigma}$$

Lower-dimension operators?

- Dimension-5 and dimension-7 operators always contain fermions.
  Only contribute to four-point boson scattering at loop level.
  $\Longrightarrow$ Can ignore at leading order.

- Contributions from SM scattering?
  SM obeys perturbative unitarity, so amplitudes scale more weakly than s^2.
  $\Longrightarrow$ SM amplitude does not contribute to our contour integral

- We assume UV scale M is above the weak scale, so treat bosons as massless to good approximation.

- One loophole left to consider: dimension-6 operators

Dimension-6 operators?

There are 15 bosonic operators at dimension six in the SMEFT: Grzadkowski et al. [1008.4884]

$\mathcal{O}^{H^2B^2}_{\text{dim-6}}$	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}^{W^3}_{\text{dim-6}}$	$\epsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$
$\widetilde{\mathcal{O}}^{H^2B^2}_{\text{dim-6}}$	$H^\dagger H B_{\mu\nu} \widetilde{B}^{\mu\nu}$	$\widetilde{\mathcal{O}}^{W^3}_{\text{dim-6}}$	$\epsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} \widetilde{W}^{K\mu}_{\rho}$
$\mathcal{O}^{H^2W^2}_{\text{dim-6}}$	$H^\dagger H W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}^{G^3}_{\text{dim-6}}$	$f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} G^{c\mu}_{\rho}$
$\widetilde{\mathcal{O}}^{H^2W^2}_{\text{dim-6}}$	$H^\dagger H W^I_{\mu\nu} \widetilde{W}^{I\mu\nu}$	$\widetilde{\mathcal{O}}^{G^3}_{\text{dim-6}}$	$f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} \widetilde{G}^{c\mu}_{\rho}$
$\mathcal{O}^{H^2G^2}_{\text{dim-6}}$	$H^\dagger H G^a_{\mu\nu} G^{a\mu\nu}$	$\mathcal{O}^{H^4}_{1,\text{dim-6}}$	$(H^\dagger H)\Box(H^\dagger H)$
$\widetilde{\mathcal{O}}^{H^2G^2}_{\text{dim-6}}$	$H^\dagger H G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$	$\mathcal{O}^{H^4}_{2,\text{dim-6}}$	$(H^\dagger D^\mu H)^\star (H^\dagger D_\mu H)$
$\mathcal{O}^{H^2BW}_{\text{dim-6}}$	$H^\dagger \tau^I H B_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}^{H^6}_{\text{dim-6}}$	$(H^\dagger H)^3$
$\widetilde{\mathcal{O}}^{H^2BW}_{\text{dim-6}}$	$H^\dagger \tau^I H B_{\mu\nu} \widetilde{W}^{I\mu\nu}$		

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$\mathcal{O}^{H^2B^2}_{\text{dim-6}}$	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}^{W^3}_{\text{dim-6}}$	$\epsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$
$\widetilde{\mathcal{O}}^{H^2B^2}_{\text{dim-6}}$	$H^\dagger H B_{\mu\nu} \widetilde{B}^{\mu\nu}$	$\widetilde{\mathcal{O}}^{W^3}_{\text{dim-6}}$	$\epsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} \widetilde{W}^{K\mu}_{\rho}$
$\mathcal{O}^{H^2W^2}_{\text{dim-6}}$	$H^\dagger H W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}^{G^3}_{\text{dim-6}}$	$f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} G^{c\mu}_{\rho}$
$\widetilde{\mathcal{O}}^{H^2W^2}_{\text{dim-6}}$	$H^\dagger H W^I_{\mu\nu} \widetilde{W}^{I\mu\nu}$	$\widetilde{\mathcal{O}}^{G^3}_{\text{dim-6}}$	$f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} \widetilde{G}^{c\mu}_{\rho}$
$\mathcal{O}^{H^2G^2}_{\text{dim-6}}$	$H^\dagger H G^a_{\mu\nu} G^{a\mu\nu}$	$\mathcal{O}^{H^4}_{1,\text{dim-6}}$	$(H^\dagger H)\Box(H^\dagger H)$
$\widetilde{\mathcal{O}}^{H^2G^2}_{\text{dim-6}}$	$H^\dagger H G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$	$\mathcal{O}^{H^4}_{2,\text{dim-6}}$	$(H^\dagger D^\mu H)^\star (H^\dagger D_\mu H)$
$\mathcal{O}^{H^2BW}_{\text{dim-6}}$	$H^\dagger \tau^I H B_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}^{H^6}_{\text{dim-6}}$	$(H^\dagger H)^3$
$\widetilde{\mathcal{O}}^{H^2BW}_{\text{dim-6}}$	$H^\dagger \tau^I H B_{\mu\nu} \widetilde{W}^{I\mu\nu}$		

Single insertion gives amplitude lower-order in derivatives and zero contribution to speed calculation.

We are working at leading order in the EFT, so need not consider multiple insertions of quartic operators (loops).

Dimension-6 operators?

There are 15 bosonic operators at dimension six in the SMEFT: Grzadkowski et al. [1008.4884]

[redacted]

$\mathcal{O}^{W^3}_{\text{dim-6}}$	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$\widetilde{\mathcal{O}}^{W^3}_{\text{dim-6}}$	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} \widetilde{W}_\rho^{K\mu}$
$\mathcal{O}^{G^3}_{\text{dim-6}}$	$f^{abc} G_\mu^{a\nu} G_\nu^{b\rho} G_\rho^{c\mu}$
$\widetilde{\mathcal{O}}^{G^3}_{\text{dim-6}}$	$f^{abc} G_\mu^{a\nu} G_\nu^{b\rho} \widetilde{G}_\rho^{c\mu}$
[redacted]	
$\mathcal{O}^{H^6}_{\text{dim-6}}$	$(H^\dagger H)^3$

Six-point: does not contribute to four-point scattering at leading order.

Dimension-6 operators?

There are 15 bosonic operators at dimension six in the SMEFT: Grzadkowski et al. [1008.4884]

[redacted]

$\mathcal{O}^{W^3}_{\text{dim-6}}$	$\epsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$
$\widetilde{\mathcal{O}}^{W^3}_{\text{dim-6}}$	$\epsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} \widetilde{W}^{K\mu}_{\rho}$
$\mathcal{O}^{G^3}_{\text{dim-6}}$	$f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} G^{c\mu}_{\rho}$
$\widetilde{\mathcal{O}}^{G^3}_{\text{dim-6}}$	$f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} \widetilde{G}^{c\mu}_{\rho}$

[redacted]

Give both 3- and 4-point vertices. Two insertions of 3-point vertex contributes as $\sim s^2/M^4$, going like the *square* of the Wilson coefficient.
A priori obstructs the placement of positivity bounds.

Can remove this obstacle by scattering gluons with commuting colors:

$$\boxed{f^{abc} u_1^b u_2^c = 0}$$

and similarly for the Ws.

Bosonic bounds

GR, Rodd [1908.09845]

Single field strength quartics

We now will compute IR consistency bounds for the $\mathcal{O}_i$. Let's start with operators of the form F^4. Generalizing to SU(N), we have:

$$
\begin{array}{ll}
\mathcal{O}_1^{F^4} & (F^a F^a)(F^b F^b) \\
\mathcal{O}_2^{F^4} & (F^a \widetilde{F}^a)(F^b \widetilde{F}^b) \\
\mathcal{O}_3^{F^4} & (F^a F^b)(F^a F^b) \\
\mathcal{O}_4^{F^4} & (F^a \widetilde{F}^b)(F^a \widetilde{F}^b) \\
\mathcal{O}_5^{F^4} & d^{abe} d^{cde} (F^a F^b)(F^c F^d) \\
\mathcal{O}_6^{F^4} & d^{abe} d^{cde} (F^a \widetilde{F}^b)(F^c \widetilde{F}^d) \\
\widetilde{\mathcal{O}}_1^{F^4} & (F^a F^a)(F^b \widetilde{F}^b) \\
\widetilde{\mathcal{O}}_2^{F^4} & (F^a F^b)(F^a \widetilde{F}^b) \\
\widetilde{\mathcal{O}}_3^{F^4} & d^{abe} d^{cde} (F^a F^b)(F^c \widetilde{F}^d) \\
\widetilde{\mathcal{O}}_4^{F^4} & d^{ace} d^{bde} (F^a F^b)(F^c \widetilde{F}^d)
\end{array}
$$

$$
\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{M^4} \sum_i c_i \mathcal{O}_i
$$

Single field strength quartics

To zeroth order in the c_i, the Yang-Mills equation of motion is

$$D^\mu F^a_{\mu\nu} = \partial^\mu F^a_{\mu\nu} + g f^{abc} A^{\mu b} F^c_{\mu\nu} = 0$$

This is satisfied by the SM background solution:

$$\overline{A}^a_\mu = u^a_1 \epsilon_{1\mu} w$$

constant vector in color space

arbitrary four-vector

coordinate, $\partial_\mu w = \ell_\mu$

Single field strength quartics

To zeroth order in the c_i, the Yang-Mills equation of motion is

$$D^\mu F^a_{\mu\nu} = \partial^\mu F^a_{\mu\nu} + g f^{abc} A^{\mu b} F^c_{\mu\nu} = 0$$

This is satisfied by the SM background solution:

$$\overline{A}^a_\mu = u^a_1 \epsilon_{1\mu} w$$

We wish to consider plane-wave perturbation around this solution:

$$A^a_\mu = \overline{A}^a_\mu + \delta A^a_\mu$$
$$\delta A^a_\mu = u^a_2 \epsilon_{2\mu} e^{ik\cdot x}$$

where $k^2 = k \cdot \epsilon_2 = 0$. Still solves the equations of motion if $f^{abc} u^b_1 u^c_2 = 0$.

Single field strength quartics

We now compute the modified dispersion relation for the plane wave, to first order in the c_i. The resulting speed is:

$$v = 1 - \frac{8}{M^4 \epsilon_2^2 u_2^2 k_0^2 N} \{[(\epsilon_1 \cdot k)(\epsilon_2 \cdot \ell) - (k \cdot \ell)(\epsilon_1 \cdot \epsilon_2)]^2 A + (\epsilon_1^\mu \epsilon_2^\nu k^\rho \ell^\sigma \epsilon_{\mu\nu\rho\sigma})^2 B$$
$$- [(\epsilon_1 \cdot k)(\epsilon_2 \cdot \ell) - (k \cdot \ell)(\epsilon_1 \cdot \epsilon_2)] \epsilon_1^\mu \epsilon_2^\nu k^\rho \ell^\sigma \epsilon_{\mu\nu\rho\sigma} C\}$$

where

$$A = N\left[(2c_1 + c_3)(u_1 u_2)^2 + c_3 u_1^2 u_2^2 + 2(c_5 + c_7)U^2\right] + 2c_7\left[(u_1 u_2)^2 - u_1^2 u_2^2\right]$$
$$B = N\left[(2c_2 + c_4)(u_1 u_2)^2 + c_4 u_1^2 u_2^2 + 2(c_6 + c_8)U^2\right] + 2c_8\left[(u_1 u_2)^2 - u_1^2 u_2^2\right]$$
$$C = N\left[(2\widetilde{c}_1 + \widetilde{c}_2)(u_1 u_2)^2 + \widetilde{c}_2 u_1^2 u_2^2 + 2(\widetilde{c}_3 + \widetilde{c}_4)U^2\right] + 2\widetilde{c}_4\left[(u_1 u_2)^2 - u_1^2 u_2^2\right]$$
$$U^a = d^{abc} u_1^b u_2^c$$

Single field strength quartics

We now compute the modified dispersion relation for the plane wave, to first order in the c_i. The resulting speed is:

$$v = 1 - \frac{8E^2}{M^4 N}\left(AX^2 + BY^2 + CXY\right)$$

where

$$\begin{aligned}
k_\mu &= (k_0, 0, 0, |\mathbf{k}|) \\
\epsilon_1^\mu &= (0, 1, 0, 0) \\
\ell_\mu &= (\cos\theta_1, \sin\theta_1\cos\theta_2, \sin\theta_1\sin\theta_2\cos\theta_3, \sin\theta_1\sin\theta_2\sin\theta_3) \\
\epsilon_2^\mu &= E(\cos\phi_1, \sin\phi_1\cos\phi_2, \sin\phi_1\sin\phi_2\cos\phi_3, \sin\phi_1\sin\phi_2\sin\phi_3)
\end{aligned}$$

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \sin\theta_1\cos\theta_2 & \sin\phi_1\cos\phi_2 \\ \sin\theta_1\sin\theta_2\cos\theta_3 & \sin\phi_1\sin\phi_2\cos\phi_3 \end{pmatrix} \begin{pmatrix} \cos\phi_1 + \sin\phi_1\sin\phi_2\sin\phi_3 \\ \cos\theta_1 - \sin\theta_1\sin\theta_2\sin\theta_3 \end{pmatrix}$$

Single field strength quartics

We now compute the modified dispersion relation for the plane wave, to first order in the c_i. The resulting speed is:

$$v = 1 - \frac{8E^2}{M^4 N}\left(AX^2 + BY^2 + CXY\right)$$

Writing $X = Z\cos\psi,\ Y = Z\sin\psi$, the causality bound becomes:

$$A\cos^2\psi + B\sin^2\psi + C\cos\psi\sin\psi > 0$$

for all ψ.

Single field strength quartics

Alternatively, we can obtain this bound from the forward four-point scattering amplitude:

$$\mathcal{A}_{F^4}(s) = \frac{8}{M^4 N}\left\{A(\epsilon_1\cdot\epsilon_2)^2 s^2 + B[\epsilon_1^2\epsilon_2^2 - (\epsilon_1\cdot\epsilon_2)^2]s^2 + 2C(\epsilon_1\cdot\epsilon_2)\epsilon_1^\mu\epsilon_2^\nu k_1^\rho k_2^\sigma \epsilon_{\mu\nu\rho\sigma}s\right\}$$
$$= \frac{8s^2}{M^4 N}\left[A\cos^2\psi + B\sin^2\psi + C\cos\psi\sin\psi\right]$$

Marginalizing over ψ gives the minimal set of independent conditions:

$$A > 0$$
$$B > 0$$
$$C^2 < 4AB$$

The third condition can be obtained by setting $\psi = \pm\arctan\sqrt{A/B}$. Cannot be found from bounding fixed-helicity amplitudes.

SU(3)

For SU(3),

$$A = 3c_3^{G^4} + 2c_5^{G^4} + 3(2c_1^{G^4} + c_3^{G^4})\cos^2\zeta$$
$$B = 3c_4^{G^4} + 2c_6^{G^4} + 3(2c_2^{G^4} + c_4^{G^4})\cos^2\zeta$$
$$C = 3\tilde{c}_2^{G^4} + 2\tilde{c}_3^{G^4} + 3(2\tilde{c}_1^{G^4} + \tilde{c}_2^{G^4})\cos^2\zeta$$

where $u_1^a u_2^a = \cos\zeta$. Marginalizing over ζ, we obtain the basis of bounds:

$$\begin{aligned}
3c_1^{G^4} + 3c_3^{G^4} + c_5^{G^4} &> 0 \\
3c_3^{G^4} + 2c_5^{G^4} &> 0 \\
3c_2^{G^4} + 3c_4^{G^4} + c_6^{G^4} &> 0 \\
3c_4^{G^4} + 2c_6^{G^4} &> 0 \\
(3\tilde{c}_1^{G^4} + 3\tilde{c}_2^{G^4} + \tilde{c}_3^{G^4})^2 &< 4(3c_1^{G^4} + 3c_3^{G^4} + c_5^{G^4})(3c_2^{G^4} + 3c_4^{G^4} + c_6^{G^4}) \\
(3\tilde{c}_2^{G^4} + 2\tilde{c}_3^{G^4})^2 &< 4(3c_3^{G^4} + 2c_5^{G^4})(3c_4^{G^4} + 2c_6^{G^4})
\end{aligned}$$

SU(2)

For SU(2), $d^{abc} \to 0,\ f^{abc} \to \epsilon^{IJK}$, and we require $u_1 = \pm u_2$. Then

$$A = 4(c_1^{W^4} + c_3^{W^4})$$
$$B = 4(c_2^{W^4} + c_4^{W^4})$$
$$C = 4(\widetilde{c}_1^{W^4} + \widetilde{c}_2^{W^4})$$

Independent bounds:

$$c_1^{W^4} + c_3^{W^4} > 0$$
$$c_2^{W^4} + c_4^{W^4} > 0$$
$$(\widetilde{c}_1^{W^4} + \widetilde{c}_2^{W^4})^2 < 4(c_1^{W^4} + c_3^{W^4})(c_2^{W^4} + c_4^{W^4})$$

U(1)

For U(1), $f^{abc} \to 0$ and we have:

$$A = 2c_1^{B^4}$$

$$B = 2c_2^{B^4}$$

$$C = 2\widetilde{c}_1^{B^4}$$

Independent bounds:

$$c_1^{B^4} > 0$$

$$c_2^{B^4} > 0$$

$$(\widetilde{c}_1^{B^4})^2 < 4c_1^{B^4} c_2^{B^4}$$

Field strength cross-quartics

Let's now consider operators with more than one kind of gauge field strength. In particular, let's generalize to $\mathrm{SU}(N) \otimes \mathrm{SU}(n)$ and define

$$
\begin{aligned}
\mathcal{O}_1 &= F^a_{\mu\nu} F^{a\mu\nu} f^A_{\rho\sigma} f^{A\rho\sigma} \\
\mathcal{O}_2 &= F^a_{\mu\nu} \widetilde{F}^{a\mu\nu} f^A_{\rho\sigma} \widetilde{f}^{A\rho\sigma} \\
\mathcal{O}_3 &= F^a_{\mu\nu} f^{A\mu\nu} F^a_{\rho\sigma} f^{A\rho\sigma} \\
\mathcal{O}_4 &= F^a_{\mu\nu} \widetilde{f}^{A\mu\nu} \widetilde{F}^a_{\rho\sigma} f^{A\rho\sigma} \\
\widetilde{\mathcal{O}}_1 &= F^a_{\mu\nu} \widetilde{F}^{a\mu\nu} f^A_{\rho\sigma} f^{A\rho\sigma} \\
\widetilde{\mathcal{O}}_2 &= F^a_{\mu\nu} F^{a\mu\nu} f^A_{\rho\sigma} \widetilde{f}^{A\rho\sigma} \\
\widetilde{\mathcal{O}}_3 &= F^a_{\mu\nu} f^{A\mu\nu} \widetilde{F}^a_{\rho\sigma} f^{A\rho\sigma}
\end{aligned}
$$

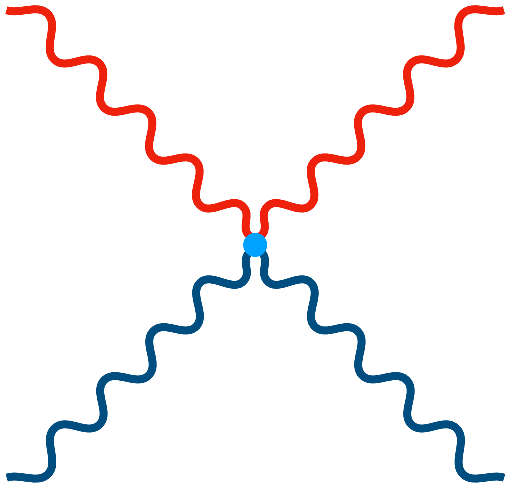
Field strength cross-quartics

In a background of nonzero $\overline{A}_{\mu}^{a}$ as before, the speed of propagation for a fluctuation δa_{μ}^{A} of the $\mathrm{SU}(n)$ field is:

$$v = 1 - \frac{4E^2}{M^4}(c_3 X^2 + c_4 Y^2 + \widetilde{c}_3 XY)$$

Four-point forward scattering amplitude:

$$\mathcal{A}_{F^2 f^2}(s) = \frac{4s^2}{M^4}\left[c_3 \cos^2\psi + c_4 \sin^2\psi + \widetilde{c}_3 \cos\psi \sin\psi\right]$$



Bounds:

$$\boxed{\begin{aligned} c_3 &> 0 \\ c_4 &> 0 \\ \widetilde{c}_3^2 &< 4c_3c_4 \end{aligned}}$$

Field strength cross-quartics

Applying this to the basis in the SMEFT, we have:

$$
\begin{aligned}
c_3^{B^2W^2} &> 0\\
c_4^{B^2W^2} &> 0\\
(\widetilde{c}_3^{B^2W^2})^2 &< 4c_3^{B^2W^2}c_4^{B^2W^2}\\
c_3^{B^2G^2} &> 0\\
c_4^{B^2G^2} &> 0\\
(\widetilde{c}_3^{B^2G^2})^2 &< 4c_3^{B^2G^2}c_4^{B^2G^2}\\
c_3^{W^2G^2} &> 0\\
c_4^{W^2G^2} &> 0\\
(\widetilde{c}_3^{W^2G^2})^2 &< 4c_3^{W^2G^2}c_4^{W^2G^2}
\end{aligned}
$$

Higgs quartics

Take the 3 $(DH)^4$ operators and expand in the real scalar fields $\phi_{1,2,3,4}$:

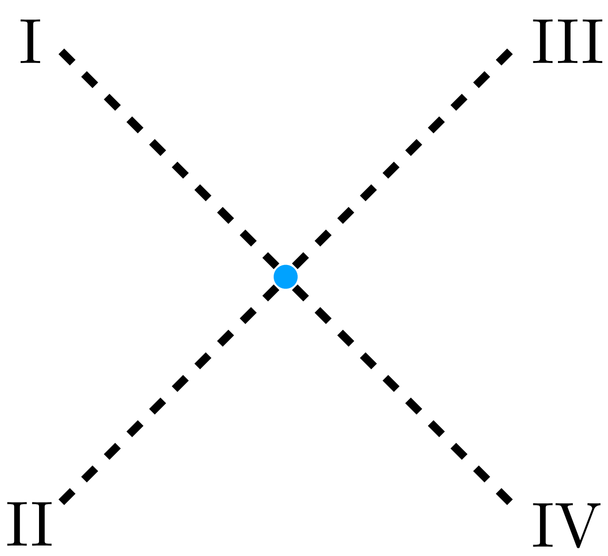
$$\mathcal{O}_1^{H^4} \to \frac{1}{4}\sum_{i=1}^{4}(\partial\phi_i)^4 + \frac{1}{2}\left[(\partial_\mu\phi_1\partial^\mu\phi_3)^2 + (\partial_\mu\phi_1\partial^\mu\phi_4)^2 + (\partial_\mu\phi_2\partial^\mu\phi_3)^2 + (\partial_\mu\phi_2\partial^\mu\phi_4)^2\right]$$
$$+ \frac{1}{2}\left[(\partial\phi_1)^2(\partial\phi_2)^2 + (\partial\phi_3)^2(\partial\phi_4)^2\right] - \partial_\mu\phi_1\partial^\mu\phi_4\partial_\nu\phi_2\partial^\nu\phi_3 + \partial_\mu\phi_1\partial^\mu\phi_3\partial_\nu\phi_2\partial^\nu\phi_4$$

$$\mathcal{O}_2^{H^4} \to \frac{1}{4}\sum_{i=1}^{4}(\partial\phi_i)^4 + \frac{1}{2}\left[(\partial_\mu\phi_1\partial^\mu\phi_3)^2 + (\partial_\mu\phi_1\partial^\mu\phi_4)^2 + (\partial_\mu\phi_2\partial^\mu\phi_3)^2 + (\partial_\mu\phi_2\partial^\mu\phi_4)^2\right]$$
$$- \frac{1}{2}\left[(\partial\phi_1)^2(\partial\phi_2)^2 + (\partial\phi_3)^2(\partial\phi_4)^2\right] + (\partial_\mu\phi_1\partial^\mu\phi_2)^2 + (\partial_\mu\phi_3\partial^\mu\phi_4)^2$$
$$+ \partial_\mu\phi_1\partial^\mu\phi_4\partial_\nu\phi_2\partial^\nu\phi_3 - \partial_\mu\phi_1\partial^\mu\phi_3\partial_\nu\phi_2\partial^\nu\phi_4$$

$$\mathcal{O}_3^{H^4} \to \frac{1}{4}\sum_{i=1}^{4}(\partial\phi_i)^4 + \frac{1}{2}\left[(\partial\phi_1)^2(\partial\phi_2)^2 + (\partial\phi_1)^2(\partial\phi_3)^2 + (\partial\phi_2)^2(\partial\phi_3)^2\right]$$
$$+ \frac{1}{2}\left[(\partial\phi_1)^2(\partial\phi_4)^2 + (\partial\phi_2)^2(\partial\phi_4)^2 + (\partial\phi_3)^2(\partial\phi_4)^2\right]$$

Higgs quartics

Two-to-two Higgs scattering: $|\mathrm{I}\rangle|\mathrm{II}\rangle \to |\mathrm{III}\rangle|\mathrm{IV}\rangle$



Arbitrary superposition of ϕ_i:

$$|\mathrm{I}\rangle = \sum_{i=1}^{4} \alpha_i |\phi_i\rangle \qquad |\mathrm{III}\rangle = \sum_{i=1}^{4} \gamma_i |\phi_i\rangle$$

$$|\mathrm{II}\rangle = \sum_{i=1}^{4} \beta_i |\phi_i\rangle \qquad |\mathrm{IV}\rangle = \sum_{i=1}^{4} \delta_i |\phi_i\rangle$$

$$\textstyle\sum_i |\alpha_i|^2 = \sum_i |\beta_i|^2 = \sum_i |\gamma_i|^2 = \sum_i |\delta_i|^2 = 1$$

Forward: $\alpha_i = \gamma_i^*, \;\; \beta_i = \delta_i^*$

s^2 part of forward amplitude: $\mathcal{A}_{H^4}(s) = \sum_{ijkl} K_{ijkl} \alpha_i \beta_j \alpha_k^* \beta_l^* \dfrac{s^2}{M^4}$

$$\textstyle\sum_{ijkl} K_{ijkl} \alpha_i \beta_j \alpha_k \beta_l > 0 \;\; \forall \alpha_i, \beta_i \quad \Longrightarrow \quad \begin{aligned} c_1^{H^4} + c_2^{H^4} + c_3^{H^4} &> 0 \\ c_1^{H^4} + c_2^{H^4} &> 0 \\ c_2^{H^4} &> 0 \end{aligned}$$

Higgs/field strength cross-quartics

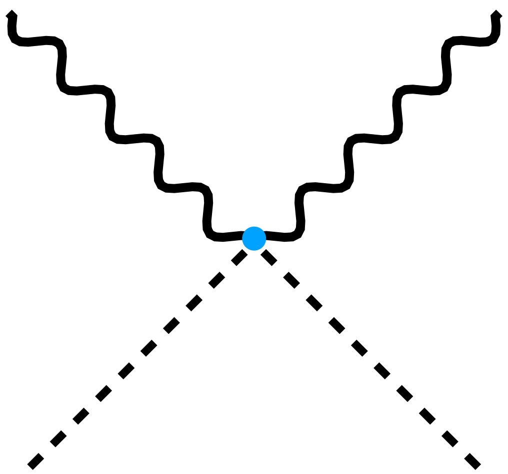
Four-point scattering of Higgs and B, W, G:

At $\mathcal{O}(s^2)$,

$$\mathcal{A}(s)_{H^2B^2} = c_1^{H^2B^2}\frac{s^2}{2M^4}$$

$$\mathcal{A}(s)_{H^2W^2} = c_1^{H^2W^2}\frac{s^2}{2M^4}$$

or $$\mathcal{A}(s)_{H^2G^2} = c_1^{H^2G^2}\frac{s^2}{2M^4}$$



Bounds:

$$\boxed{\begin{aligned} c_1^{H^2B^2} &> 0 \\ c_1^{H^2W^2} &> 0 \\ c_1^{H^2G^2} &> 0 \end{aligned}}$$

UV completions: bosonic operators

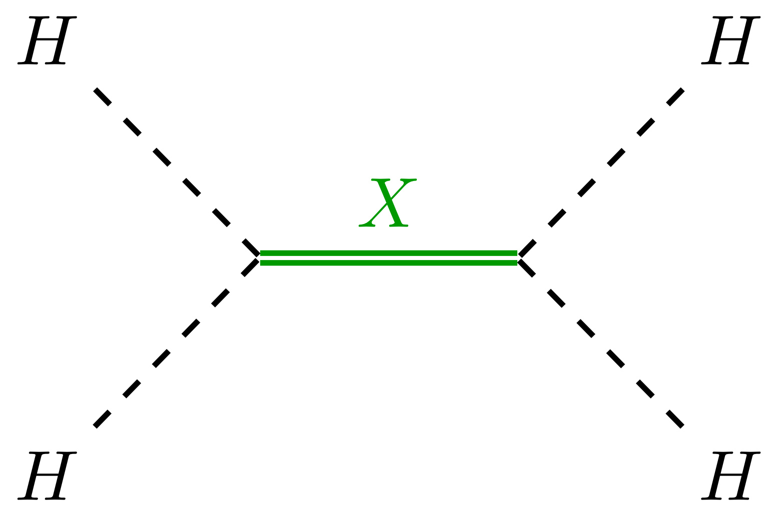
GR, Rodd [1908.09845]

Tree completions of $(DH)^4$ operators

Let's look at some example completions of the operators:

$$\mathcal{O}_1^{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H) \qquad c_1^{H^4} + c_2^{H^4} + c_3^{H^4} > 0$$
$$\mathcal{O}_2^{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H) \qquad c_1^{H^4} + c_2^{H^4} > 0$$
$$\mathcal{O}_3^{H^4} = (D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H) \qquad c_2^{H^4} > 0$$

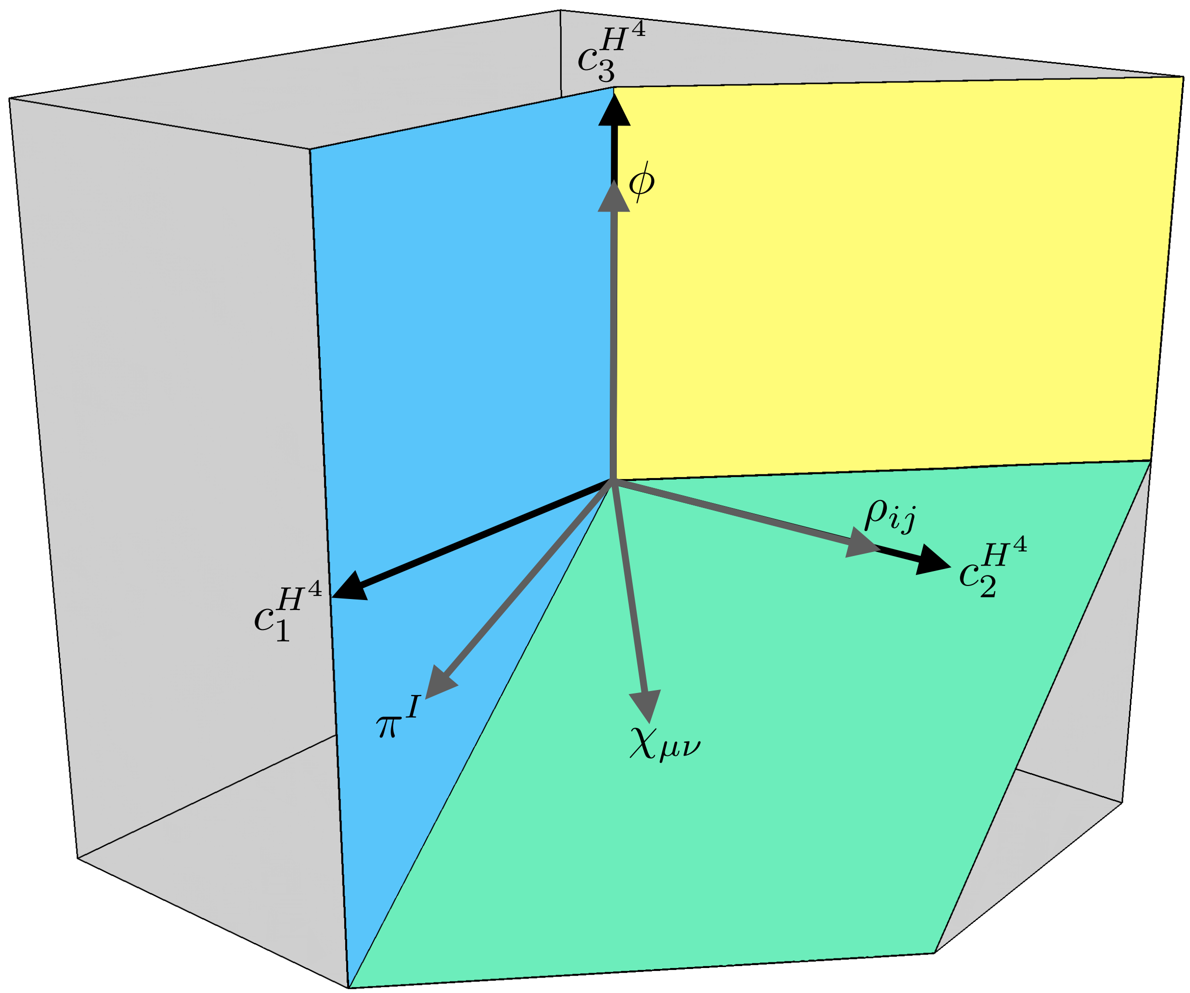
Tree-level completion: Massive state exchanged between $(DH)^2$



Tree completions of $(DH)^4$ operators

Recall our bounds:

$$
\begin{aligned}
c_1^{H^4} + c_2^{H^4} + c_3^{H^4} &> 0 \\
c_1^{H^4} + c_2^{H^4} &> 0 \\
c_2^{H^4} &> 0
\end{aligned}
$$

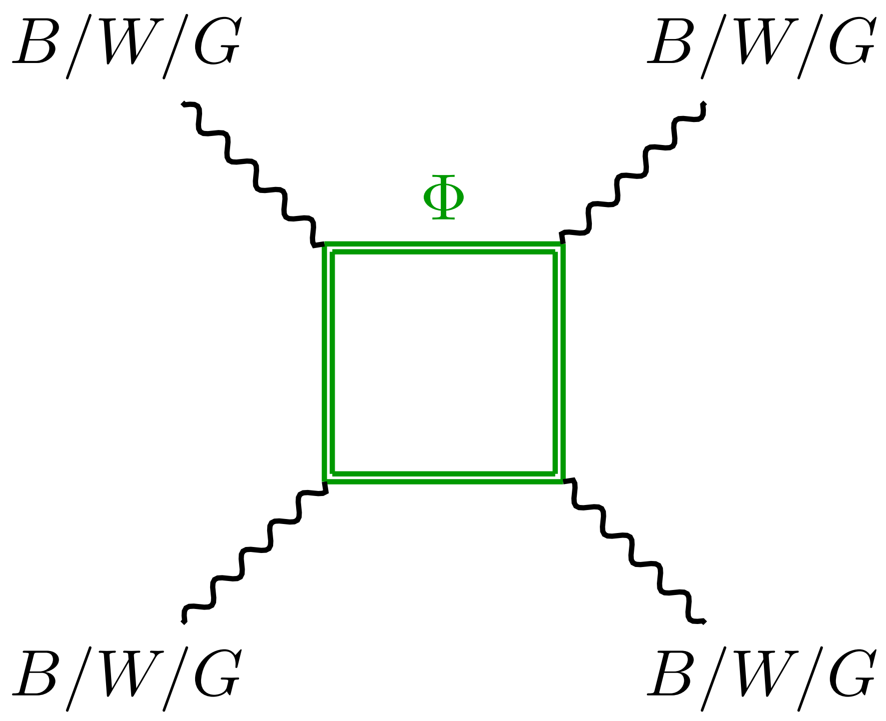


All of our example tree-level completions automatically satisfy our bounds.

One-loop completions of gauge operators

Now let's look at a large class of one-loop completions:

Consider a massive state Φ coupled to the gauge bosons.



Generalization of how the electron couples to the photon in QED, and integrating out the electron gives F^4 terms (Euler-Heisenberg Lagrangian)

One-loop completions of gauge operators

Wilson coefficients: Quevillon, Smith, & Touati [1810.06994]

	scalar	fermion	vector
$c_1^{B^4}$	$\frac{7}{32}g_1^4Q^4$	$\frac{1}{2}g_1^4Q^4$	$\frac{261}{32}g_1^4Q^4$
$c_2^{B^4}$	$\frac{1}{32}g_1^4Q^4$	$\frac{7}{8}g_1^4Q^4$	$\frac{243}{32}g_1^4Q^4$
$c_1^{W^4}$	$g_2^4\left[\frac{7}{32}\Lambda(\mathbf{R}_2)+\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{1}{2}\Lambda(\mathbf{R}_2)+\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{261}{32}\Lambda(\mathbf{R}_2)-\frac{3}{16}I_2(\mathbf{R}_2)\right]$
$c_2^{W^4}$	$g_2^4\left[\frac{1}{32}\Lambda(\mathbf{R}_2)+\frac{1}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{7}{8}\Lambda(\mathbf{R}_2)+\frac{19}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{243}{32}\Lambda(\mathbf{R}_2)-\frac{27}{112}I_2(\mathbf{R}_2)\right]$
$c_3^{W^4}$	$g_2^4\left[\frac{7}{16}\Lambda(\mathbf{R}_2)-\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\Lambda(\mathbf{R}_2)-\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{261}{16}\Lambda(\mathbf{R}_2)+\frac{3}{16}I_2(\mathbf{R}_2)\right]$
$c_4^{W^4}$	$g_2^4\left[\frac{1}{16}\Lambda(\mathbf{R}_2)-\frac{1}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{7}{4}\Lambda(\mathbf{R}_2)-\frac{19}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{243}{16}\Lambda(\mathbf{R}_2)+\frac{27}{112}I_2(\mathbf{R}_2)\right]$
$c_1^{G^4}$	$g_3^4\left[\frac{7}{32}\Lambda(\mathbf{R}_3)+\frac{1}{96}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{1}{2}\Lambda(\mathbf{R}_3)+\frac{1}{96}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{261}{32}\Lambda(\mathbf{R}_3)-\frac{3}{32}I_2(\mathbf{R}_3)\right]$
$c_2^{G^4}$	$g_3^4\left[\frac{1}{32}\Lambda(\mathbf{R}_3)+\frac{1}{672}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{7}{8}\Lambda(\mathbf{R}_3)+\frac{19}{672}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{243}{32}\Lambda(\mathbf{R}_3)-\frac{27}{224}I_2(\mathbf{R}_3)\right]$
$c_3^{G^4}$	$g_3^4\left[\frac{7}{16}\Lambda(\mathbf{R}_3)-\frac{1}{48}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\Lambda(\mathbf{R}_3)-\frac{1}{48}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{261}{16}\Lambda(\mathbf{R}_3)+\frac{3}{16}I_2(\mathbf{R}_3)\right]$
$c_4^{G^4}$	$g_3^4\left[\frac{1}{16}\Lambda(\mathbf{R}_3)-\frac{1}{336}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{7}{4}\Lambda(\mathbf{R}_3)-\frac{19}{336}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{243}{16}\Lambda(\mathbf{R}_3)+\frac{27}{112}I_2(\mathbf{R}_3)\right]$
$c_5^{G^4}$	$\frac{1}{32}g_3^4I_2(\mathbf{R}_3)$	$\frac{1}{32}g_3^4I_2(\mathbf{R}_3)$	$-\frac{9}{32}g_3^4I_2(\mathbf{R}_3)$
$c_6^{G^4}$	$\frac{1}{224}g_3^4I_2(\mathbf{R}_3)$	$\frac{19}{224}g_3^4I_2(\mathbf{R}_3)$	$-\frac{81}{224}g_3^4I_2(\mathbf{R}_3)$
$c_1^{B^2W^2}$	$\frac{7}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{261}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
$c_2^{B^2W^2}$	$\frac{1}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{7}{4}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{243}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
$c_3^{B^2W^2}$	$\frac{7}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$2g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{261}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
$c_4^{B^2W^2}$	$\frac{1}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{7}{2}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{243}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
$c_1^{B^2G^2}$	$\frac{7}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{261}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_2^{B^2G^2}$	$\frac{1}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{7}{4}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{243}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_3^{B^2G^2}$	$\frac{7}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$2g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{261}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_4^{B^2G^2}$	$\frac{1}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{7}{2}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{243}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_1^{W^2G^2}$	$\frac{7}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{261}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_2^{W^2G^2}$	$\frac{1}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{7}{4}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{243}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_3^{W^2G^2}$	$\frac{7}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$2g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{261}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_4^{W^2G^2}$	$\frac{1}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{7}{2}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{243}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_1^{BG^3}$	$\frac{7}{32}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{1}{2}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{261}{32}g_1g_3^3QI_3(\mathbf{R}_3)$
$c_2^{BG^3}$	$\frac{1}{32}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{7}{8}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{243}{32}g_1g_3^3QI_3(\mathbf{R}_3)$

One-loop completions of gauge operators

Relevant bounds:

$$\begin{aligned}
c_1^{B^4} &> 0 \\
c_2^{B^4} &> 0 \\
c_1^{W^4} + c_3^{W^4} &> 0 \\
c_2^{W^4} + c_4^{W^4} &> 0 \\
3c_1^{G^4} + 3c_3^{G^4} + c_5^{G^4} &> 0 \\
3c_3^{G^4} + 2c_5^{G^4} &> 0 \\
3c_2^{G^4} + 3c_4^{G^4} + c_6^{G^4} &> 0 \\
3c_4^{G^4} + 2c_6^{G^4} &> 0 \\
c_3^{B^2W^2} &> 0 \\
c_4^{B^2W^2} &> 0 \\
c_3^{B^2G^2} &> 0 \\
c_4^{B^2G^2} &> 0 \\
c_3^{W^2G^2} &> 0 \\
c_4^{W^2G^2} &> 0
\end{aligned}$$

One-loop completions of gauge operators

Relevant bounds:

scalar case:	fermion case:	vector case:
$\frac{7}{32} g_1^4 Q^4 > 0$	$\frac{1}{2} g_1^4 Q^4 > 0$	$\frac{261}{32} g_1^4 Q^4 > 0$
$\frac{1}{32} g_1^4 Q^4 > 0$	$\frac{7}{8} g_1^4 Q^4 > 0$	$\frac{243}{32} g_1^4 Q^4 > 0$
$\frac{21}{32} g_2^4 \Lambda(\mathbf{R}_2) > 0$	$\frac{3}{2} g_2^4 \Lambda(\mathbf{R}_2) > 0$	$\frac{783}{32} g_2^4 \Lambda(\mathbf{R}_2) > 0$
$\frac{3}{16} g_2^4 \Lambda(\mathbf{R}_2) > 0$	$\frac{21}{8} g_2^4 \Lambda(\mathbf{R}_2) > 0$	$\frac{729}{32} g_2^4 \Lambda(\mathbf{R}_2) > 0$
$\frac{63}{32} g_3^4 \Lambda(\mathbf{R}_3) > 0$	$\frac{9}{2} g_3^4 \Lambda(\mathbf{R}_3) > 0$	$\frac{2349}{32} g_3^4 \Lambda(\mathbf{R}_3) > 0$
$\frac{21}{16} g_3^4 \Lambda(\mathbf{R}_3) > 0$	$3 g_3^4 \Lambda(\mathbf{R}_3) > 0$	$\frac{783}{16} g_3^4 \Lambda(\mathbf{R}_3) > 0$
$\frac{9}{32} g_3^4 \Lambda(\mathbf{R}_3) > 0$	$\frac{63}{8} g_3^4 \Lambda(\mathbf{R}_3) > 0$	$\frac{2187}{32} g_3^4 \Lambda(\mathbf{R}_3) > 0$
$\frac{3}{16} g_3^4 \Lambda(\mathbf{R}_3) > 0$	$\frac{21}{4} g_3^4 \Lambda(\mathbf{R}_3) > 0$	$\frac{729}{16} g_3^4 \Lambda(\mathbf{R}_3) > 0$
$\frac{7}{8} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2) > 0$	$2 g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2) > 0$	$\frac{261}{8} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2) > 0$
$\frac{1}{8} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2) > 0$	$\frac{7}{2} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2) > 0$	$\frac{243}{8} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2) > 0$
$\frac{7}{8} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3) > 0$	$2 g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3) > 0$	$\frac{261}{8} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3) > 0$
$\frac{1}{8} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3) > 0$	$\frac{7}{2} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3) > 0$	$\frac{243}{8} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3) > 0$
$\frac{7}{8} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3) > 0$	$2 g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3) > 0$	$\frac{261}{8} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3) > 0$
$\frac{1}{8} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3) > 0$	$\frac{7}{2} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3) > 0$	$\frac{243}{8} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3) > 0$

One-loop completions of gauge operators

Relevant bounds:

Since $I_2(\mathbf{R}_2),\ I_2(\mathbf{R}_3),\ \Lambda(\mathbf{R}_2),\ \Lambda(\mathbf{R}_3)$ are all nonnegative by definition, the bounds are manifestly satisfied for arbitrary one-loop completions.

Born-Infeld

A string-inspired UV extension of the SMEFT: Born-Infeld Lagrangian

$$\mathcal{L} = M^4 \left[1 - \sqrt{1 + \frac{1}{2M^4} F^A_{\mu\nu} F^{A\mu\nu} - \frac{1}{16M^8} (F^A_{\mu\nu} \widetilde{F}^{A\mu\nu})^2} \right]$$

where $F^A_{\mu\nu} = (B_{\mu\nu}, W^I_{\mu\nu}, G^a_{\mu\nu})$

EFT at low energies:

$$\begin{aligned} \mathcal{L} =& -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \\ &+ \frac{1}{32M^4} \left(\mathcal{O}_1^{B^4} + \mathcal{O}_2^{B^4} + \mathcal{O}_1^{W^4} + \mathcal{O}_2^{W^4} + \mathcal{O}_1^{G^4} + \mathcal{O}_2^{G^4} \right) \\ &+ \frac{1}{16M^4} \left(\mathcal{O}_1^{B^2W^2} + \mathcal{O}_2^{B^2W^2} + \mathcal{O}_1^{B^2G^2} + \mathcal{O}_2^{B^2G^2} + \mathcal{O}_1^{W^2G^2} + \mathcal{O}_2^{W^2G^2} \right) \end{aligned}$$

Manifestly satisfies positivity bounds.

CP violation and completing the square

We notice a pattern in the SMEFT positivity bounds—there are two distinct forms:

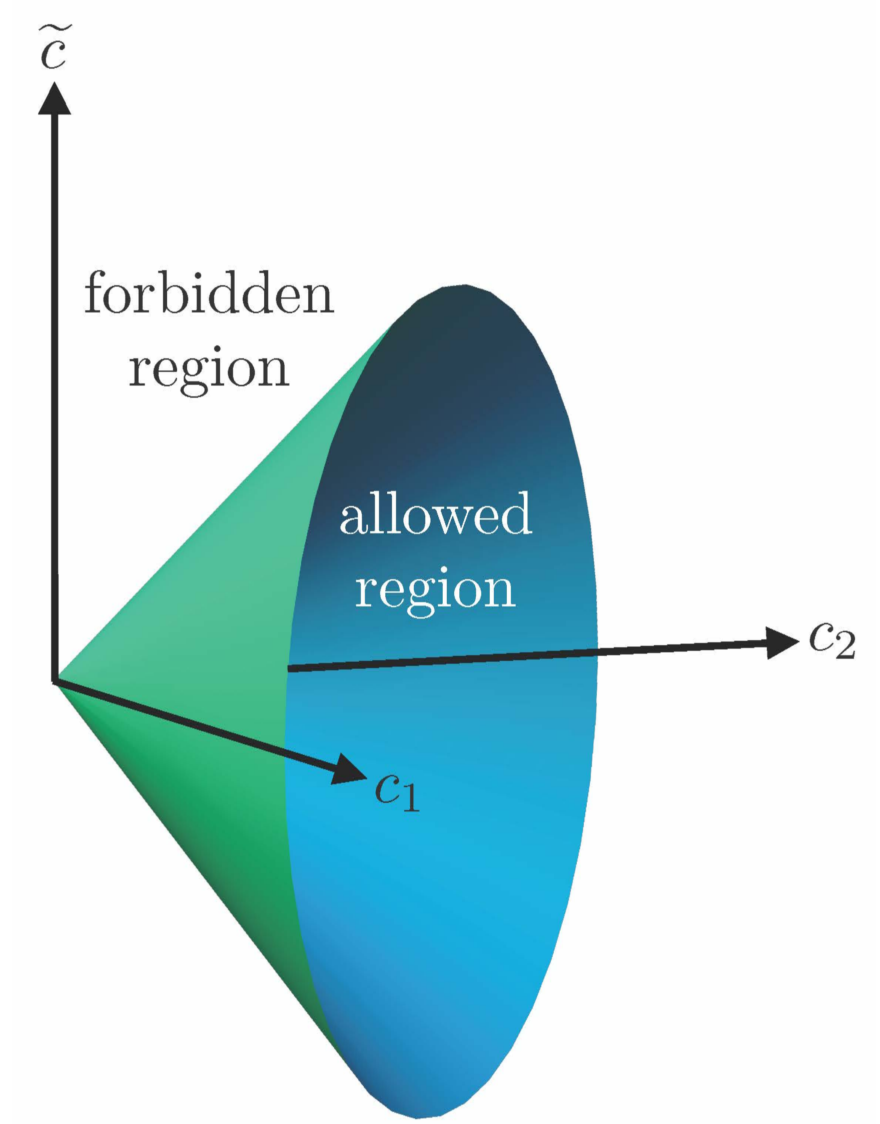
1. $c_1 > 0,\ c_2 > 0$ for $c_{1,2}$ coefficients of CP-conserving terms

2. $\widetilde{c}^2 < 4c_1c_2$ for $\widetilde{c}$ the coefficient of a CP-violating term

Bounds form a cone:

$$\widetilde{c}^2 + c_-^2 < c_+^2,\ c_+ > 0$$

where $c_\pm = c_1 \pm c_2$



CP violation and completing the square

Let's take the U(1) $\mathcal{O}_i^{B^4}$ terms as an example:

$$\Delta\mathcal{L} = \frac{1}{M^4}\left[c_1^{B^4}(B_{\mu\nu}B^{\mu\nu})^2 + c_2^{B^4}(B_{\mu\nu}\widetilde{B}^{\mu\nu})^2 + \widetilde{c}_1^{B^4}B_{\mu\nu}B^{\mu\nu}B_{\rho\sigma}\widetilde{B}^{\rho\sigma}\right]$$

The positivity bounds $c_{1,2}^{B^4} > 0,\ (\widetilde{c}_1^{B^4})^2 < 4c_1^{B^4}c_2^{B^4}$ imply that $\Delta\mathcal{L}$ can be written as a sum of perfect squares:

$$\Delta\mathcal{L} = \frac{\alpha^2}{2M^4}\left[(B_{\mu\nu}B^{\mu\nu} + \beta B_{\mu\nu}\widetilde{B}^{\mu\nu})^2 + \gamma^2(B_{\mu\nu}B^{\mu\nu} - \beta B_{\mu\nu}\widetilde{B}^{\mu\nu})^2\right]$$

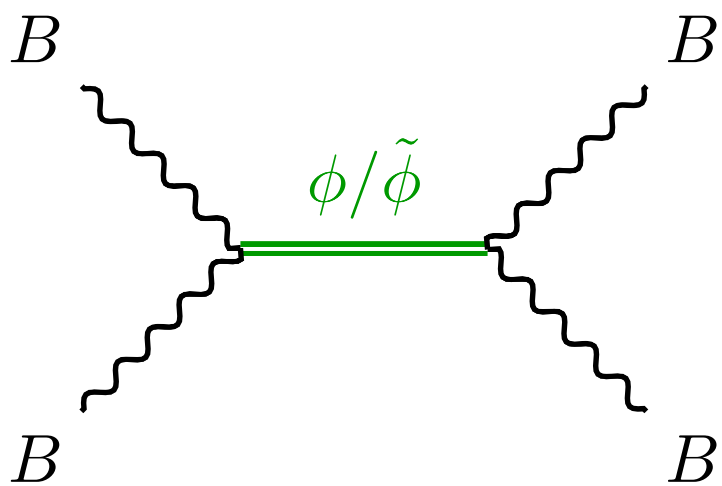
where $\alpha, \beta, \gamma \in \mathbb{R}$ are chosen such that

$$\alpha^2(1+\gamma^2) = 2c_1^{B^4}$$
$$\alpha^2\beta^2(1+\gamma^2) = 2c_2^{B^4}$$
$$\alpha^2\beta(1-\gamma^2) = \widetilde{c}_1^{B^4}$$

i.e., $(\widetilde{c}_1^{B^4})^2/4c_1^{B^4}c_2^{B^4} = [(1-\gamma^2)/(1+\gamma^2)]^2$

CP violation and completing the square

UV completion of these terms via a dilaton ϕ or axion $\widetilde{\phi}$ with mass mixing:



$$\mathcal{L} \supset -\frac{M^2}{2}(\phi + \widetilde{\phi})^2 - \frac{M^2}{2\gamma^2}(\phi - \widetilde{\phi})^2 + \frac{2\alpha}{M}\phi B_{\mu\nu}B^{\mu\nu} + \frac{2\alpha\beta}{M}\widetilde{\phi}B_{\mu\nu}\widetilde{B}^{\mu\nu}$$

Mass eigenstates: $\phi + \widetilde{\phi}$ (mass M) and $\phi - \widetilde{\phi}$ (mass $M/|\gamma|$)

Integrating out $\phi, \widetilde{\phi}$ gives

$$\Delta\mathcal{L} = \frac{\alpha^2}{2M^4}\left[(B_{\mu\nu}B^{\mu\nu} + \beta B_{\mu\nu}\widetilde{B}^{\mu\nu})^2 + \gamma^2(B_{\mu\nu}B^{\mu\nu} - \beta B_{\mu\nu}\widetilde{B}^{\mu\nu})^2\right]$$

Fermionic operator basis

GR, Rodd [2004.02885]

Building blocks

We want all bosonic four-point fermionic operators that have two extra derivatives. Counting can be found in Henning et al. [1512.03433], but not operators themselves.

- Ingredients: SM fermions

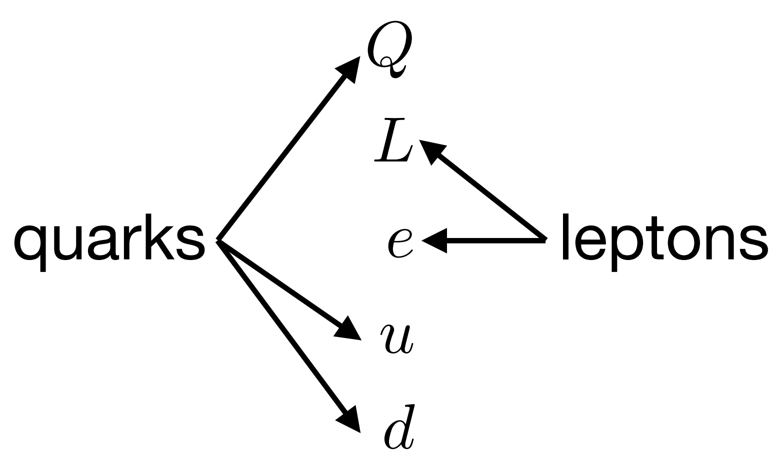
$$\text{left-handed} \begin{cases} Q \\ L \end{cases}$$

$$\left. \begin{matrix} e \\ u \\ d \end{matrix} \right\} \text{right-handed}$$

Building blocks

We want all bosonic four-point fermionic operators that have two extra derivatives. Counting can be found in Henning et al. [1512.03433], but not operators themselves.

- Ingredients: SM fermions



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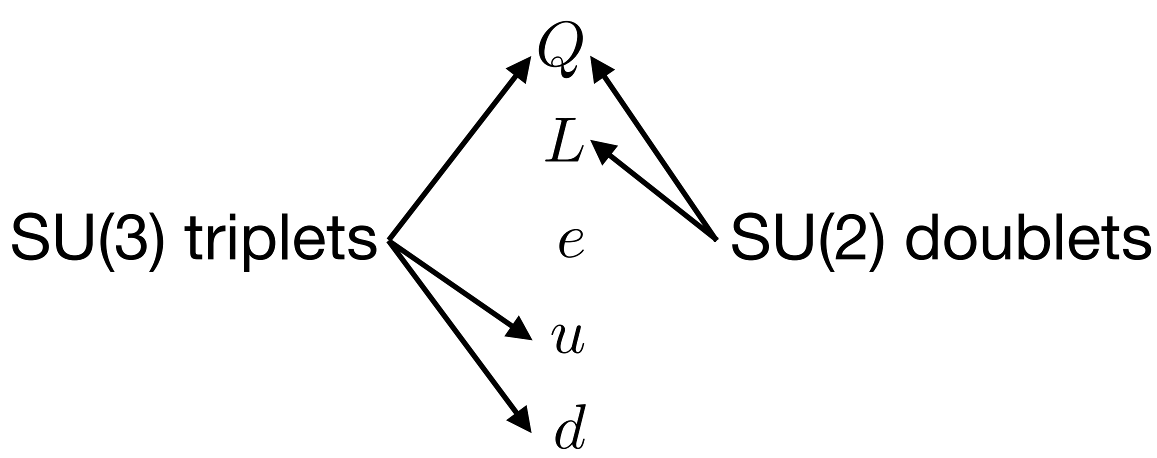


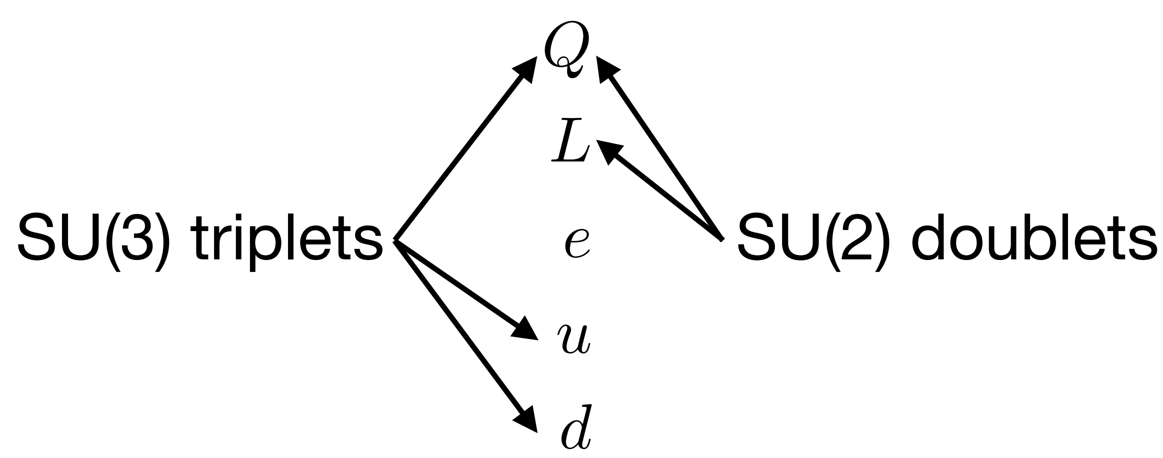
- Each fermion field has a **generation index** m, n, p, q (e.g., e_m) running over $1, \ldots, N_f$, where in the SM, $N_f = 3$

- This defines **flavor** quantum numbers:
    - Lepton flavor number (electron, muon, tau)
    - Strong isospin
    - Strangeness, bottomness, charm, topness

Building blocks

We want all bosonic four-point fermionic operators that have two extra derivatives. Counting can be found in Henning et al. [1512.03433], but not operators themselves.

- Ingredients: SM fermions



- Each fermion field has a **generation index** m, n, p, q (e.g., e_m) running over $1, \ldots, N_f$, where in the SM, $N_f = 3$
- Since we will scatter states of fixed SM representation, we require operators containing an even number of each fermionic field, modulo flavor.

Building blocks

We want all bosonic four-point fermionic operators that have two extra derivatives. Counting can be found in Henning et al. [1512.03433], but not operators themselves.

- As for the bosons, to construct the minimal basis, we must mod out by:
    - Fierz/Schouten identities
    - SU(N) identities
    - Integration by parts
    - Equation of motion (work in effectively massless limit, so $\partial\!\!\!/\psi = 0$)

Self-hermitian self-quartics

Define currents:

$$J^\mu[\psi]_{mn} = \bar{\psi}_m \gamma_\mu \psi_n \qquad J^\mu[\psi]^a_{mn} = \bar{\psi}_m T^a \gamma_\mu \psi_n$$

$$J^\mu[\psi]^I_{mn} = \bar{\psi}_m \tau^I \gamma_\mu \psi_n \qquad J^\mu[\psi]^{Ia}_{mn} = \bar{\psi}_m \tau^I T^a \gamma_\mu \psi_n$$

where $\psi = Q, L, e, d, u$, τ^I= SU(2) generator, T^a = SU(3) generator

Operators:

$$\begin{aligned}
\mathcal{O}_1[\psi] &= c^{\psi,1}_{mnpq} \partial_\mu J_\nu[\psi]_{mn} \partial^\mu J^\nu[\psi]_{pq}, & \psi &= \text{any} \\
\mathcal{O}_2[\psi] &= c^{\psi,2}_{mnpq} \partial_\mu J_\nu[\psi]^I_{mn} \partial^\mu J^\nu[\psi]^I_{pq}, & \psi &= L,Q \\
\mathcal{O}_3[\psi] &= c^{\psi,3}_{mnpq} \partial_\mu J_\nu[\psi]^a_{mn} \partial^\mu J^\nu[\psi]^a_{pq}, & \psi &= d,u,Q \\
\mathcal{O}_4[Q] &= c^{Q,4}_{mnpq} \partial_\mu J_\nu[Q]^{Ia}_{mn} \partial^\mu J^\nu[Q]^{Ia}_{pq} & &
\end{aligned}$$

$c_{mnpq} \in \mathbb{C}$

$N_f^2(N_f^2+1)/2$ independent real operators per line for each choice of ψ

Symmetry: $c_{mnpq} = c_{pqmn}$
Hermitian condition: $c_{mnpq} = c^*_{nmqp}$

Self-hermitian cross-quartics 1

Define currents:

$$J^\mu[\psi]_{mn} = \bar{\psi}_m \gamma_\mu \psi_n \qquad J^\mu[\psi]^a_{mn} = \bar{\psi}_m T^a \gamma_\mu \psi_n$$
$$J^\mu[\psi]^I_{mn} = \bar{\psi}_m \tau^I \gamma_\mu \psi_n \qquad J^\mu[\psi]^{Ia}_{mn} = \bar{\psi}_m \tau^I T^a \gamma_\mu \psi_n$$

where $\psi = Q, L, e, d, u$, τ^I= SU(2) generator, T^a = SU(3) generator

Operators:

$$\mathcal{O}_{J1}[\psi,\chi] = b^{\psi\chi,1}_{mnpq} \partial_\mu J_\nu[\psi]_{mq} \partial^\mu J^\nu[\chi]_{np}, \qquad \psi,\chi = \text{any}$$
$$\mathcal{O}_{J2}[Q,L] = b^{QL,2}_{mnpq} \partial_\mu J_\nu[Q]^I_{mq} \partial^\mu J^\nu[L]^I_{np}$$
$$\mathcal{O}_{J3}[\psi,\chi] = b^{\psi\chi,3}_{mnpq} \partial_\mu J_\nu[\psi]^a_{mq} \partial^\mu J^\nu[\chi]^a_{np}, \qquad \psi,\chi \in \{d,u,Q\}$$

$b^{\psi\chi}_{mnpq} = b^{\chi\psi}_{nmqp} \in \mathbb{C}$ and $\psi \neq \chi$

N_f^4 independent real operators per line for each choice of ψ, χ

Hermitian condition: $b_{mnpq} = b^*_{qpnm}$

Self-hermitian cross-quartics 2

Define tensors:

$$K_{\mu\nu}[\psi]_{mn} = \bar{\psi}_m \gamma_\mu \partial_\nu \psi_n$$
$$K_{\mu\nu}[\psi]^I_{mn} = \bar{\psi}_m \tau^I \gamma_\mu \partial_\nu \psi_n$$
$$K_{\mu\nu}[\psi]^a_{mn} = \bar{\psi}_m T^a \gamma_\mu \partial_\nu \psi_n$$

cf. Dirac stress-energy tensor:

$$T_{\mu\nu} = -i(K_{\mu\nu}[\psi]_{mm} + K_{\nu\mu}[\psi]_{mm} - g_{\mu\nu} K_\rho{}^\rho[\psi]_{mm})/2 + \text{h.c.}$$

Operators:

$$\mathcal{O}_{K1}[\psi,\chi] = -a^{\psi\chi,1}_{mnpq} K_{\mu\nu}[\psi]_{mq} K^{\nu\mu}[\chi]_{np}, \qquad \psi,\chi = \text{any}$$
$$\mathcal{O}_{K2}[Q,L] = -a^{QL,2}_{mnpq} K_{\mu\nu}[Q]^I_{mq} K^{\nu\mu}[L]^I_{np}$$
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N_f^4 independent real operators per line for each choice of ψ, χ

Hermitian condition: $a_{mnpq} = a^*_{qpnm}$

Lower-dimension operators?

- As before can ignore dimension-5 and dimension-7 operators, as well as SM scattering contribution.

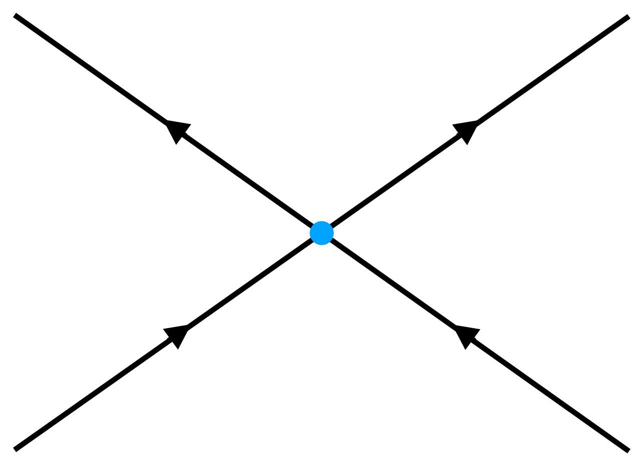
- Four-fermion dimension-6 operators can be present. Grzadkowski et al. [1008.4884]
    - Generate s^2 amplitudes via loop processes.
    - We will assume a sufficiently weakly-coupled UV completion that we can ignore these at leading order.

Fermionic bounds

GR, Rodd [2004.02885]

Fermion scattering

Let's first scatter right-handed leptons e_m in an arbitrary superposition of flavors:



Forward scattering, $\begin{pmatrix} |\psi_1\rangle \leftrightarrow \langle\psi_4| \\ |\psi_2\rangle \leftrightarrow \langle\psi_3| \end{pmatrix}$, requires: $\begin{aligned} \alpha_m &= \delta_m^* \\ \beta_m &= \gamma_m^* \end{aligned}$

Forward amplitudes:

$$\mathcal{A}(\bar{e}^- e^+ \bar{e}^- e^+) = \mathcal{A}(\bar{e}^- \bar{e}^- e^+ e^+) = 4c^{e,1}_{mnpq}\alpha_m \beta_n \beta_p^* \alpha_q^* s^2$$

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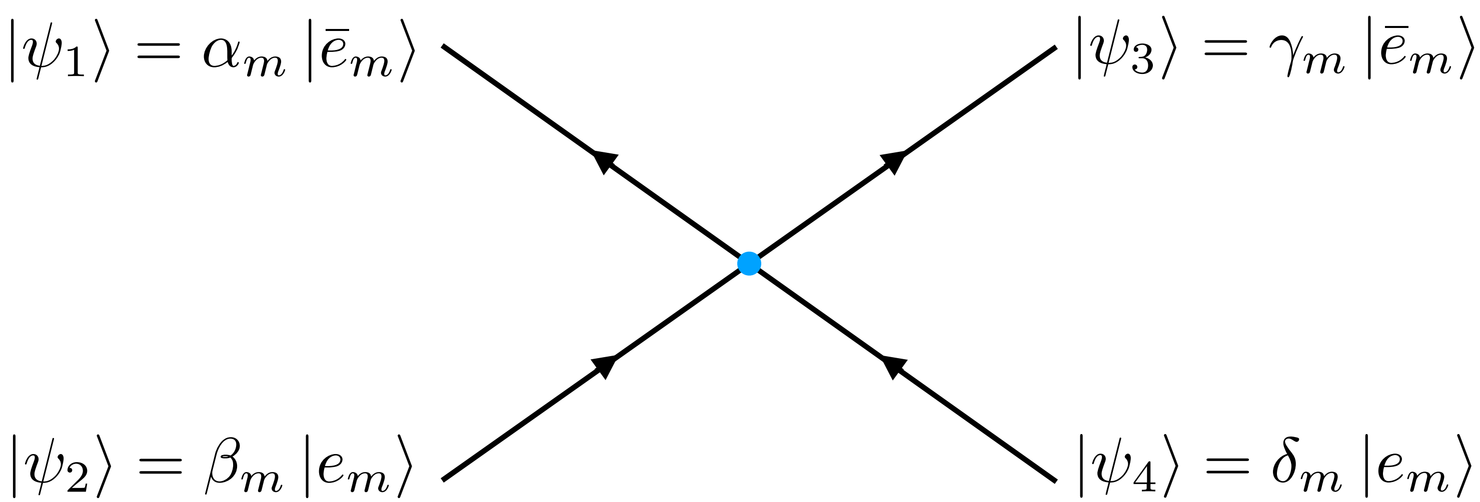


Unitarity and analyticity thus require that

$$c^{e,1}_{mnpq}\alpha_m\beta_n\beta^*_p\alpha^*_q > 0$$

for all vectors α and β.

Density matrix bound

We can think of the outer products for α and β as matrices:

$$\rho^{\alpha}_{mn} = \alpha_m \alpha^*_n$$
$$\rho^{\beta}_{mn} = \beta_m \beta^*_n$$

These matrices are automatically idempotent ($\rho^2 = \rho$), hermitian, and $\mathrm{Tr}(\rho) = 1$

$\implies$ The ρ's are *density matrices* for a pure state on a Hilbert space of dimension N_f.

$\rho^{\alpha}_{mn} = \alpha_m \alpha^*_n$ is the Schmidt decomposition.

Defining $c^{e,1}_{\alpha\beta} = c^{e,1}_{mnpq} \rho^{\alpha}_{mn} \rho^{\beta}_{pq}$, the positivity bound is then the requirement that

$$\boxed{c^{e,1}_{\alpha\beta} > 0 \quad \forall \;\; \alpha,\, \beta}$$

Structure of the flavohedron

Before we consider other operators, let's explore the nontrivial structure of the bound $c_{\alpha\beta} > 0$ in an example toy theory:

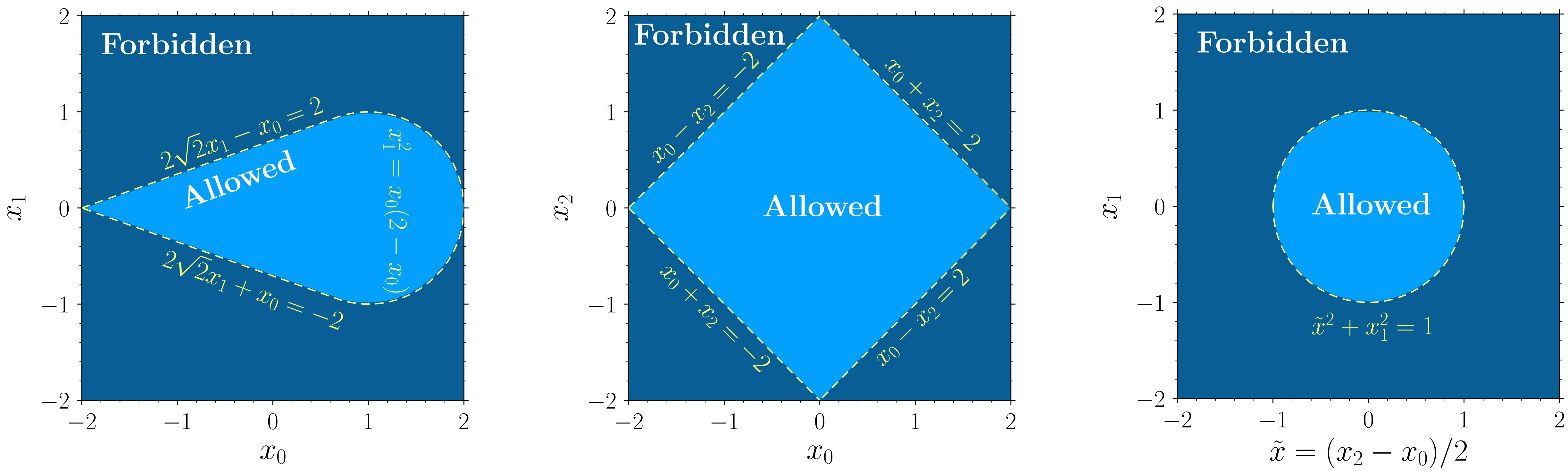
- Two flavors
- CP-conserving, so $c_{mnpq} \in \mathbb{R}$
- Assume/define:

$$\begin{aligned} c_{1111} = c_{2222} = c_{1221} &= c \\ c_{1122} &= c_0 \\ c_{1112} = c_{1222} &= c_1 \\ c_{1212} &= c_2 \\ x_{0,1,2} &= c_{0,1,2}/c \end{aligned}$$

Then the bound becomes:

$$\begin{aligned} c &> 0 \\ -2 + 4|x_1| < x_0 + x_2 &< 2 \\ 2|x_0 - x_2| &< 2 - x_0 - x_2 + \sqrt{(x_0 + x_2 + 2 - 4x_1)(x_0 + x_2 + 2 + 4x_1)} \end{aligned}$$

Structure of the flavohedron



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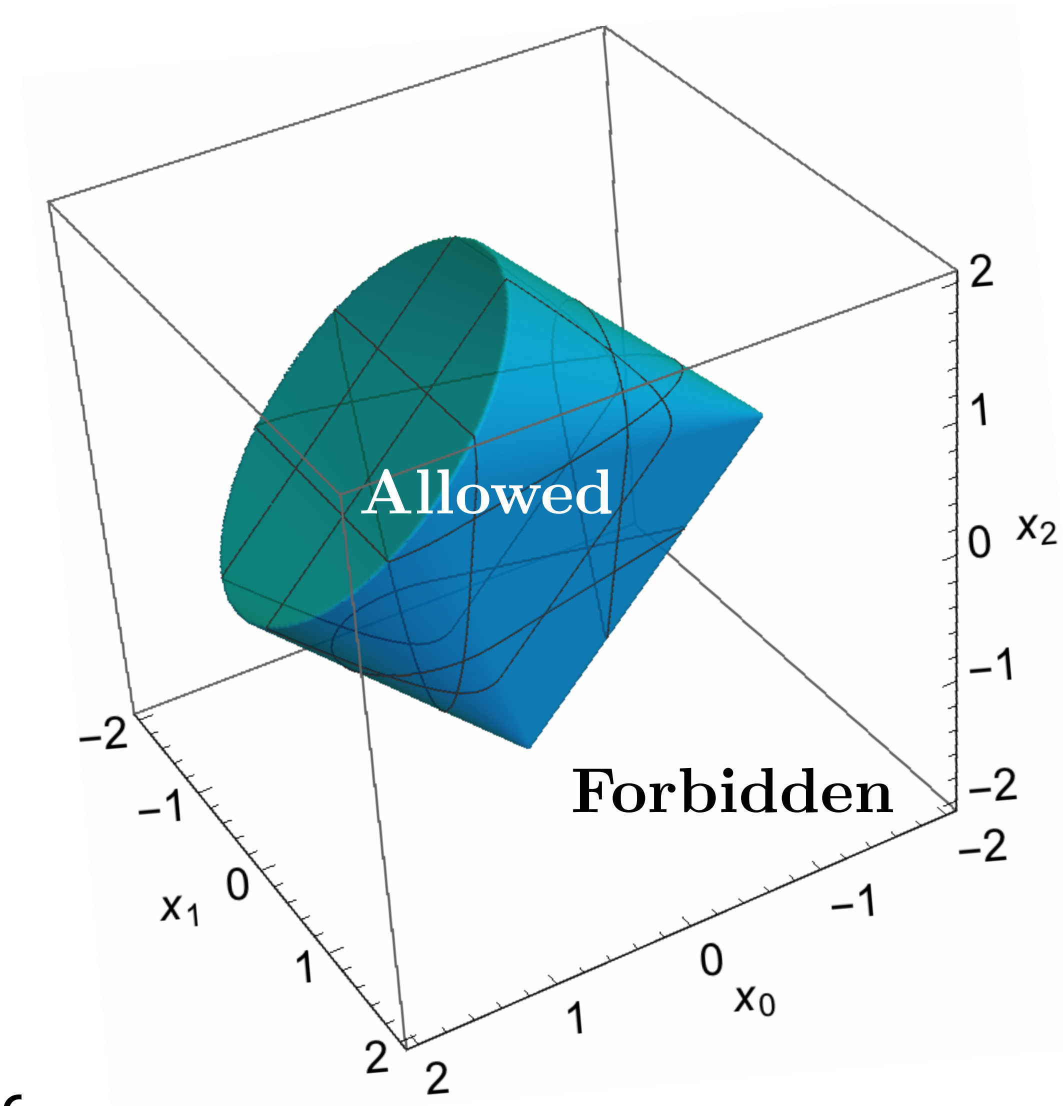

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SU(2) charges

Next, let's consider the L^4 operators:

$$\mathcal{O}_1[L] = c^{L,1}_{mnpq}\partial_\nu(\bar{L}_m\gamma_\mu L_n)\partial^\nu(\bar{L}_p\gamma^\mu L_q)$$
$$\mathcal{O}_2[L] = c^{L,2}_{mnpq}\partial_\nu(\bar{L}_m\tau^I\gamma_\mu L_n)\partial^\nu(\bar{L}_p\tau^I\gamma^\mu L_q)$$

We can now scatter superpositions of generations *and* SU(2) charges:

$$|\psi_1\rangle = \alpha_{mi}L_{mi}$$
$$|\psi_2\rangle = \beta_{mi}L_{mi}$$

Forward amplitudes:

$$\begin{aligned}\mathcal{A}(\bar{L}^+L^-\bar{L}^+L^-) &= \mathcal{A}(\bar{L}^+\bar{L}^+L^-L^-)\\ &= 4s^2\left[\left(c^{L,1}_{mnpq} - \frac{1}{4}c^{L,2}_{mnpq}\right)\alpha^*_{mi}\beta_{ni}\beta^*_{pj}\alpha_{qj} + \frac{1}{2}c^{L,2}_{mnpq}\alpha^*_{mi}\beta_{nj}\beta^*_{pj}\alpha_{qi}\right]\end{aligned}$$

SU(2) charges

Each operator is an SU(2) singlet, so without loss of generality, take:

$$\alpha_{mi} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \alpha_m \qquad \text{and} \qquad \beta_{mi} = \begin{pmatrix} \cos\theta \\ e^{i\phi}\sin\theta \end{pmatrix} \beta_m$$

$$\Longrightarrow \left[\left(c^{L^4 1}_{mnpq} - \frac{1}{4} c^{L^4 2}_{mnpq} \right) \cos^2\theta + \frac{1}{2} c^{L^4 2}_{mnpq} \right] \alpha^*_m \beta_n \beta^*_p \alpha_q > 0$$

Marginalize over θ to obtain the bounds:

$$c^{L,1}_{\alpha\beta} + \frac{1}{4} c^{L,2}_{\alpha\beta} > 0$$

$$c^{L,2}_{\alpha\beta} > 0$$

Quartic quark bounds

Analogously, for the Q^4, e^4, u^4 operators, we find:

$$c^{u,1}_{\alpha\beta} + \frac{1}{3}c^{u,3}_{\alpha\beta},$$
$$c^{u,3}_{\alpha\beta},$$
$$c^{d,1}_{\alpha\beta} + \frac{1}{3}c^{d,3}_{\alpha\beta},$$
$$c^{d,3}_{\alpha\beta},$$

$$c^{Q,1}_{\alpha\beta} + \frac{1}{4}c^{Q,2}_{\alpha\beta} + \frac{1}{3}c^{Q,3}_{\alpha\beta} + \frac{1}{12}c^{Q,4}_{\alpha\beta},$$
$$c^{Q,2}_{\alpha\beta} + \frac{1}{3}c^{Q,4}_{\alpha\beta},$$
$$c^{Q,3}_{\alpha\beta} + \frac{1}{4}c^{Q,4}_{\alpha\beta},$$
$$c^{Q,4}_{\alpha\beta}$$

are all > 0

Cross-quartic fermion bounds

Cross-quartic scattering (e.g., $de \to de$) analogously allows us to derive the bounds:

$$a^{de,1},\ a^{ue,1}_{\alpha\beta},\ a^{eL,1}_{\alpha\beta}, \qquad a^{dL,1}_{\alpha\beta},\ a^{uL,1}_{\alpha\beta},\ a^{eQ,1}_{\alpha\beta},$$

$$a^{QL,1}_{\alpha\beta} \pm \tfrac{1}{4} a^{QL,2}_{\alpha\beta}, \qquad a^{du,1}_{\alpha\beta} + \tfrac{1\pm 3}{12} a^{du,3}_{\alpha\beta},$$

$$a^{dQ,1}_{\alpha\beta} + \tfrac{1\pm 3}{12} a^{dQ,3}_{\alpha\beta}, \qquad a^{uQ,1}_{\alpha\beta} + \tfrac{1\pm 3}{12} a^{uQ,3}_{\alpha\beta},$$

$$\text{are all} > 0$$

Flavor violation

- Some of our operators conserve flavor, while others are flavor-violating
    - e.g., $\partial_\mu(\bar{e}_1\gamma_\nu e_3)\partial^\mu(\bar{e}_3\gamma^\nu e_1)$ and $\partial_\mu(\bar{e}_1\gamma_\nu e_2)\partial^\mu(\bar{e}_2\gamma^\nu e_1)$ have $(\Delta L_e, \Delta L_\mu, \Delta L_\tau) = (0, 0, 0)$
    - $\partial_\mu(\bar{e}_1\gamma_\nu e_2)\partial^\mu(\bar{e}_3\gamma^\nu e_1)$ has $(\Delta L_e, \Delta L_\mu, \Delta L_\tau) = (0, +1, -1)$
- Our bounds imply positivity of various flavor-conserving coefficients, e.g., $c_{1111}, c_{1221} > 0$.
  Correspond to diagonal density matrices ρ^α, ρ^β
- More general matrices: *magnitude* constraints on flavor-violating operators

Flavor violation

- Consider a scattering with $\alpha_m = (0, 0, 1),\ \beta_m = (0, \cos\theta, e^{i\phi}\sin\theta)$

  Marginalizing over all (θ, ϕ) gives us a quadratic bound:

$$c_{1221}c_{1331} > |c_{1231}|^2$$

- Flavor-violating operators are forbidden by unitarity from being too large:

Flavor-violating couplings are upper bounded by their flavor-conserving cousins.

- Reminiscent of our completing-the-square bounds we found for CP violation for bosons.

- In fact, one can show that the *only* possible CP-violating (i.e., imaginary coupling) four-fermion operators are *also* flavor-violating, so the flavor bound also constrains CP violation, just like for bosons. Generalizes to arbitrary U(1) symmetry.

UV completion: fermionic operators

- The completing-the-squares pattern in the bounds again suggests the structure of a UV completion.

- Suppose we have some collection of complex, two-Lorentz-index, SM singlet fields $\phi_{\mu\nu mn}$:

$$\mathcal{L} \supset -m^2 \phi_{\mu\nu mn} \phi^{*\mu\nu}_{mn} + (m y \phi^{\mu\nu}_{mn} \partial_\mu J_\nu[e]_{mn} + \text{h.c.})$$

Integrating out ϕ generates $|y|^2 \delta_{mq} \delta_{np} \partial_\mu J_\nu[e]_{mn} \partial^\mu J^\nu[e]_{pq}$

$$\implies \quad c^{e,1}_{\alpha\beta} = |y|^2 > 0 \quad \forall \quad \alpha, \beta \qquad \checkmark$$

- Well-defined UV completion: Kaluza-Klein graviton $\phi_{\mu\nu}$ coupling as $\kappa \phi^{\mu\nu} T_{\mu\nu}$ generates $c^{e,1}_{mnpq} = \kappa^2 (4\delta_{mq}\delta_{np} - \delta_{mn}\delta_{pq})/2m^2$

$$\implies \quad c^{e,1}_{\alpha\beta} = \kappa^2 (4|\alpha|^2 |\beta|^2 - |\alpha \cdot \beta|^2)/2m^2 > 0 \qquad \checkmark$$

Phenomenological consequences

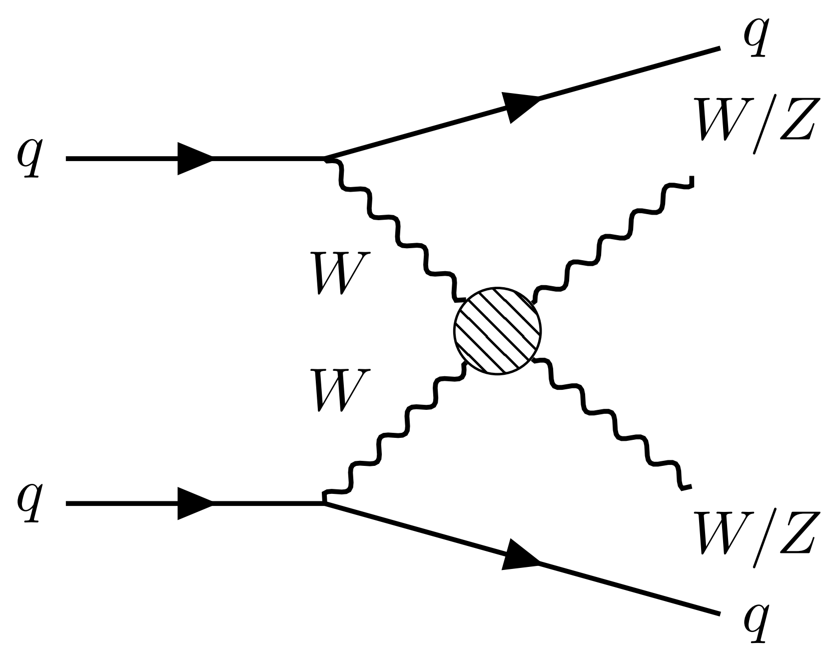
GR, Rodd [1908.09845, 2004.02885]

aQGCs

- Previously, focus on experimental signals of dimension-eight operators has focused on the electroweak sector in anomalous quartic gauge-boson couplings (aQGCs).

- Induce corrections to SM couplings, e.g., $WWWW$, $WWZZ$, $WWZ\gamma$ or induce non-SM couplings like $ZZZZ$

- Can be probed using channels like $qq \to qqWW$, $qq \to qqZZ$



- Can be distinguished from dimension-six operators, since these generate triple boson couplings heavily constrained by LEP and LHC

aQGCs

Traditional set of operators used for studies of aQGCs:

- 3 scalar "S-type" operators of the form $(DH)^4$
- 7 mixed "M-type" operators of the form $(DH)^2 F^2$
- 8 tensor "T-type" operators of the form F^4

Eboli, Gonzalez-Garcia, Mizukoshi [hep-ph/0606118]; Rauch [1610.08420]; Zhang, Zhou [1808.00010]; Bi, Zhang, Zhou [1902.08977]

Our set of CP-even operators in the electroweak sector has:

- 3 S-type
- 8 M-type
- 10 T-type

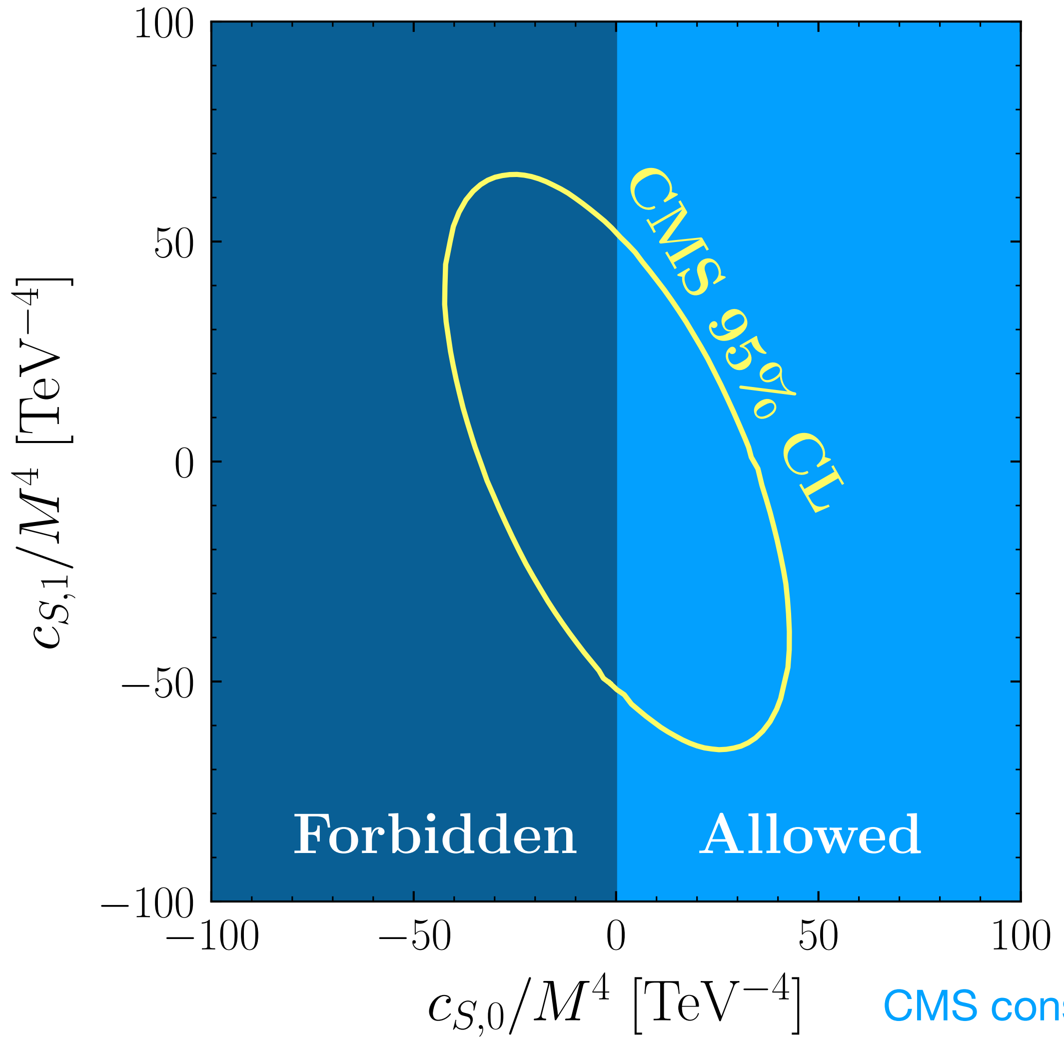
Set of aQGCs used in the literature is missing $\mathcal{O}_2^{W^4}$, $\mathcal{O}_2^{B^2W^2}$, and a linear combination of $\mathcal{O}_2^{H^2BW}$ and $\mathcal{O}_3^{H^2BW}$.

Can show that these are indeed independent operators, so experiments have been bounding an *incomplete basis* of aQGCs.

Moreover, experimental constraints (e.g., from the LHC) assume that all but one, or all but two, of the aQGC couplings vanish: highly non-generic EFT. Better: place bounds after *marginalizing* over other couplings.

aQGCs

LHC has already placed constraints on aQGCs:

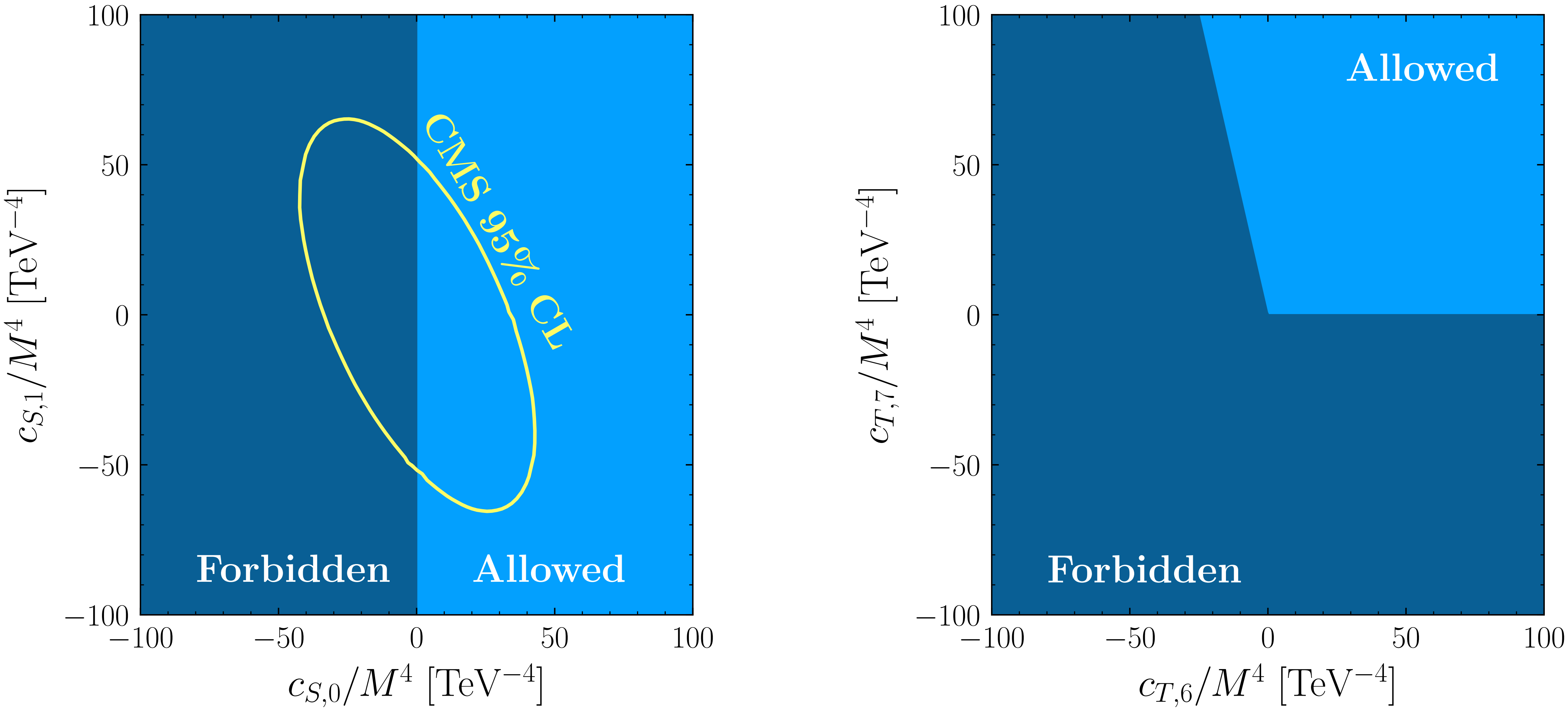


CMS constraint from [1901.04060]

$$\mathcal{O}_{S,0} = \mathcal{O}_2^{H^4},\ \mathcal{O}_{S,1} = \mathcal{O}_3^{H^4}$$

aQGCs

LHC has already placed constraints on aQGCs:



$$\mathcal{O}_{T,6} = \frac{g_1^2 g_2^2}{8}\mathcal{O}_3^{B^2W^2}, \quad \mathcal{O}_{T,7} = \frac{g_1^2 g_2^2}{32}\left(\mathcal{O}_1^{B^2W^2} + \mathcal{O}_3^{B^2W^2} + \mathcal{O}_4^{B^2W^2}\right)$$

aQGCs

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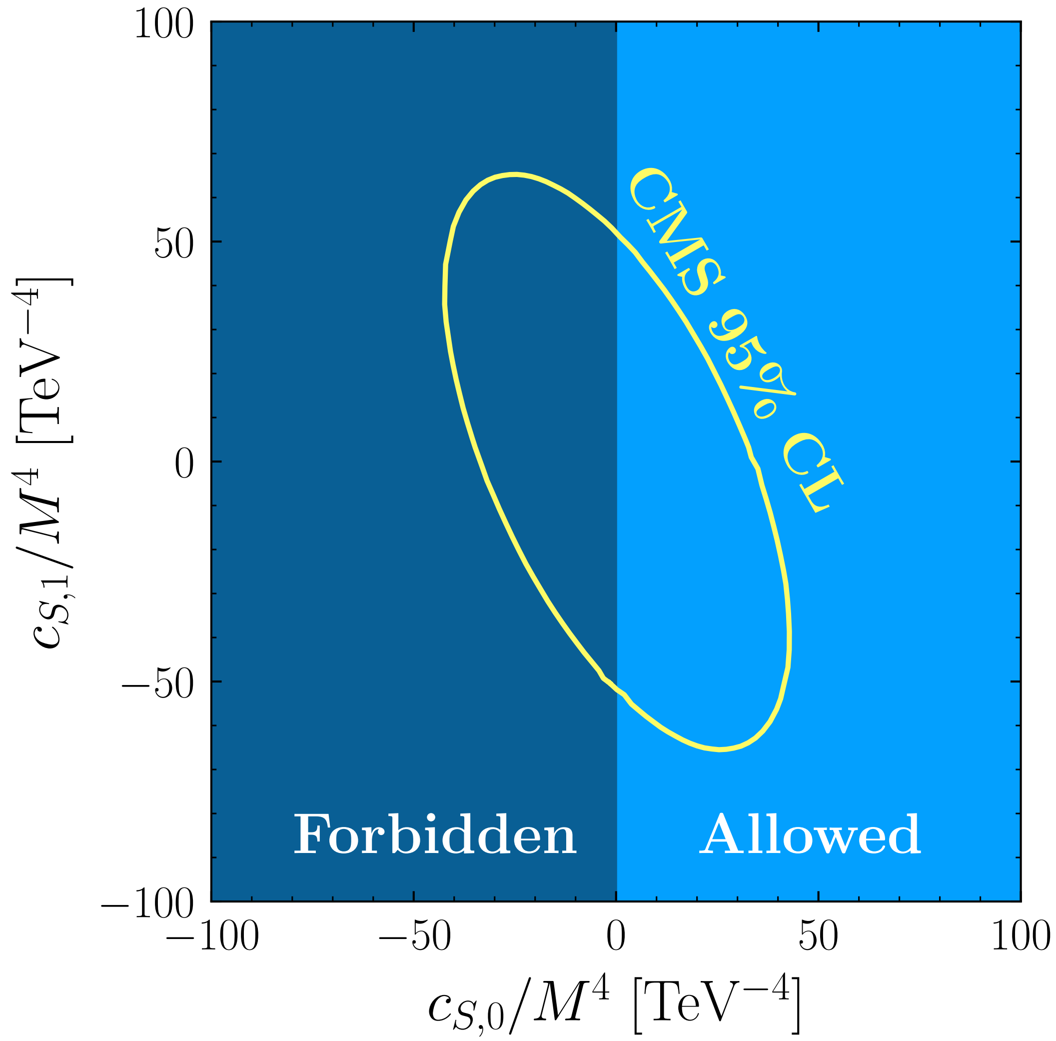
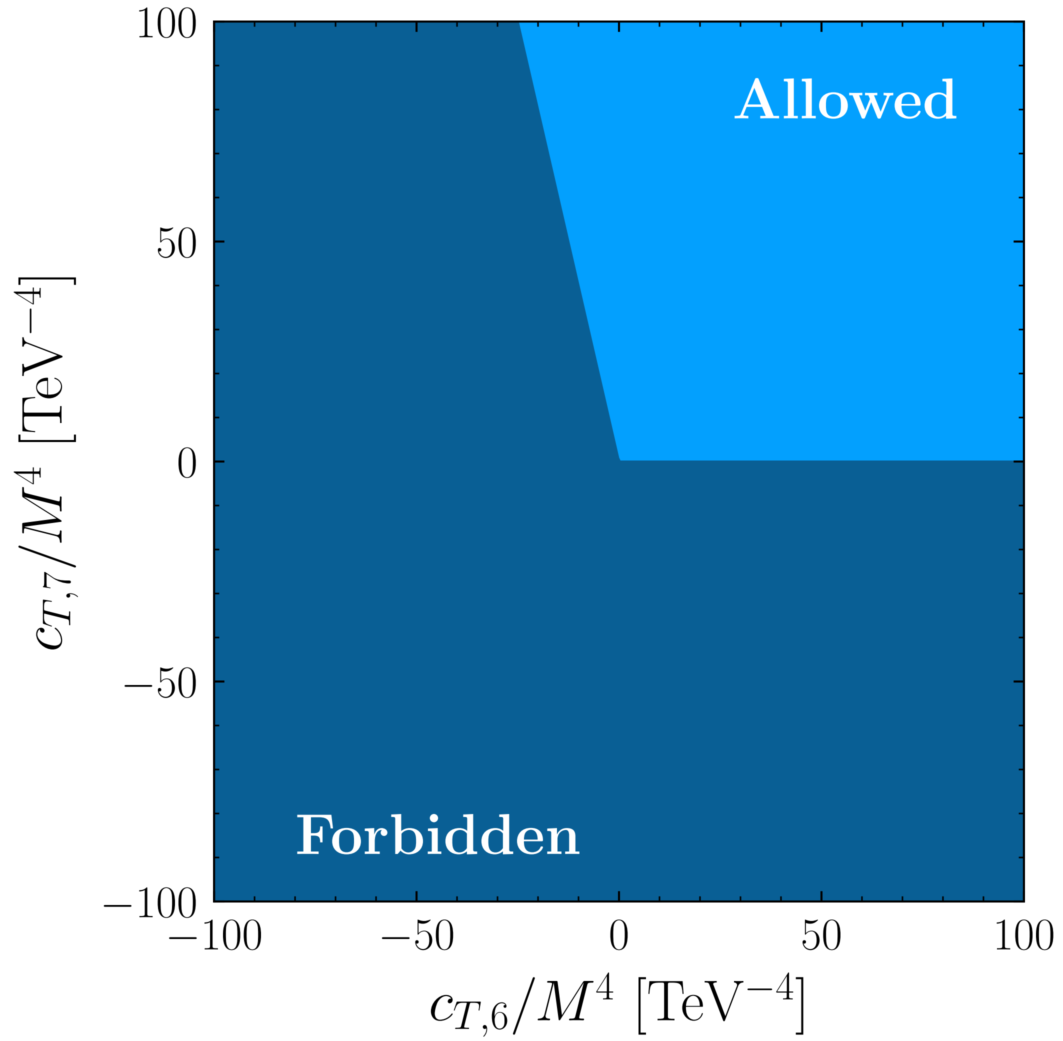




IR consistency bounds can sharpen bounds and motivate new places to look.

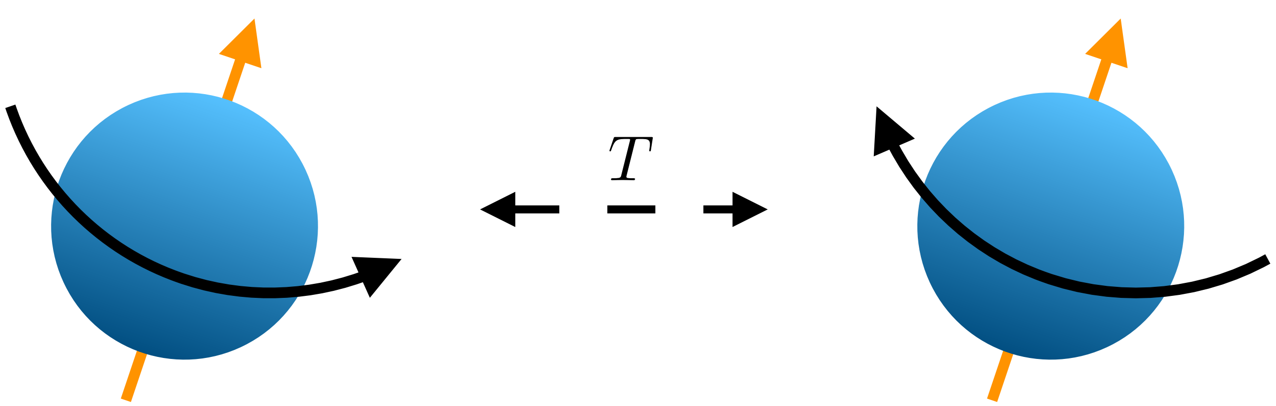
Neutron EDM

Our IR consistency bounds on CP-odd operators are always connected with CP-even bounds.

$\Longrightarrow$ Connect different experimental measurements?

Example: neutron electric dipole moment

- $G\widetilde{G}$ operator has famously small coupling (strong CP problem)
- Dominant nEDM arises from higher-dimension operators?
- Dimension-6 operator $f^{abc}G^{a\nu}_{\mu}G^{b\rho}_{\nu}\widetilde{G}^{c\mu}_{\rho}$ can generate nEDM in multi-Higgs models. Weinberg (1989) Depending on details of model, dimension-8 operators can provide dominant contribution. e.g., Chemtob (1993)



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For an operator $\frac{c}{M^4}G^3\widetilde{G}$, dimensional analysis tells us how to estimate the nEDM: Manohar, Georgi (1984); Georgi, Randall (1986)

$$|d_n| \sim \frac{e\,c}{M^4}\frac{\Lambda_{\chi\mathrm{SB}}^3}{(4\pi)^2} \approx c\left(\frac{1\ \mathrm{TeV}}{M}\right)^4 \times 10^{-28}\,e\,\mathrm{cm}$$

Experimental bounds require $|d_n| \lesssim 10^{-26}\,e\,\mathrm{cm}$, so the scale of M is being probed to 100s of GeV

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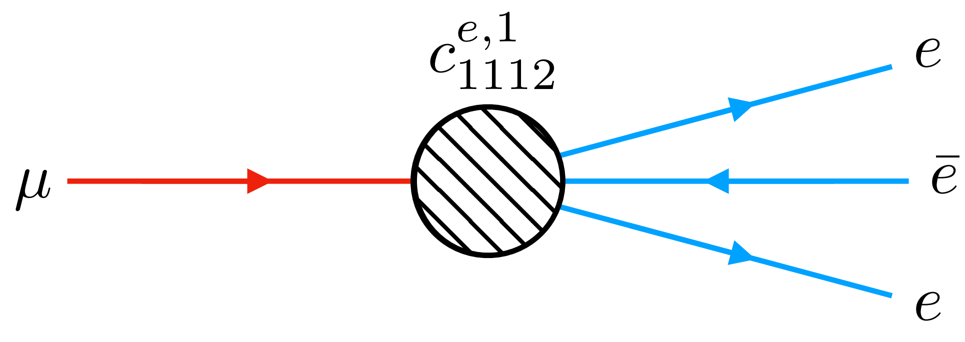
Discovery of nonzero nEDM from dimension-8 operators would lead directly to IR-consistency predictions for CP-even operators observable at the LHC.

Flavor change

Precision experiments are currently searching for flavor violation.

Our bounds could allow collider measurements to be connected to these low-energy, precision experiments, analogous with CP.

- Muon-to-electron conversion:



- If the Mu3e experiment Blondel et al. [1301.6113] finds nonzero $\mathrm{Br}(\mu \to 3e)$ via $c^{e,1}_{1112}$, our bound:

$$c^{e,1}_{1111} c^{e,1}_{2112} > |c^{e,1}_{1112}|^2$$

implies nonzero $c^{e,1}_{1111}$, $c^{e,1}_{2112}$, which could be tested at colliders in the dilepton distribution.

Dimension 6 versus 8

- There are dimension-6 analogues of all of our operators Grzadkowski et al. [1008.4884]
- By power counting, dimension-8 amplitudes are suppressed relative to dimension-6 by $(\Lambda_{\rm IR}/\Lambda_{\rm UV})^2$
- For muon decay, $\Lambda_{\rm IR} \sim m_\mu$, so how to distinguish dimension-8?
    - In colliders, hard contribution gives $\Lambda_{\rm IR} \sim \sqrt{s}$
    - Higher-ℓ angular distribution Alioli et al. [2003.11615]
    - High-p_T tail Greljo, Marzocca [1704.09015]
- Scaling means that higher mass scale can be more important than tighter branching ratio bound in constraining dimension-8 operators:

  $\mathrm{Br}(\mu \to 3e) \lesssim 10^{-12}$ vs. $\mathrm{Br}(\tau \to 3e) \lesssim 10^{-8}$ but $(m_\tau/m_\mu)^4 \sim 10^5$

  $(m_t/m_\mu)^4 \sim 10^{13}$, FCNCs?

Other probes

Flavor-violating:

- Neutral meson mixing

See GR, Rodd [2004.02885] for other refs.

Flavor-conserving:

- Nonresonant dilepton, dijet events at colliders

Dimension-6 bounds into several TeV scale:

- Dilepton, diphoton, dijet, and top production at LHC
- Neutrino scattering
- Electron and atomic parity violation

Fawlikowski et al. [1706.03783]

UV motivations:

- KK graviton
- Leptoquarks
- Fermion compositeness

Minimal flavor violation: Our bounds are orthogonal to the MFV hypothesis (neither forbid nor mandate).

Similary, lepton universality is unprotected by our bounds:
$a_{3112}^{QL,i} \neq a_{3222}^{QL,i}$ would generate $\Delta\Gamma(B \to K^{(*)} e^+ e^-) \neq \Delta\Gamma(B \to K^{(*)} \mu^+ \mu^-)$

Future directions

Future directions

Multiple avenues for future work:

- Finding more connections between CP-even and CP-odd measurement
- Superpositions of SM representations, B-violating operators
- Deriving SMEFT bounds for operators containing both fermions and bosons
- Other EFTs? Connections to swampland program?
- More bounds beyond the forward limit: beyond-positivity techniques, EFThedron, etc.?

Connecting the IR consistency program with the SMEFT and accessible experimental signals represents an important new bridge between phenomenology and formal theory, connecting physics at different energy scales and offering both a test of fundamental properties of QFT up to very high energies and a sharpening of our search for new physics.