Stable, ghost-free solutions in UV non-local gravity

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based on arXiv:2005.01762 (PRD.102.024080) (K. Sravan Kumar, SM, Anupam Mazumdar, Jun Peng), arXiv:1905.03227 (PRD.100.064022) (K. Sravan Kumar, SM, Anupam Mazumdar)

Gravity in the UV

- Big Bang and black hole singularities in GR call for a UV completion
- GR is non-renormalizable
  - Counterterms are curvatures of higher order e.g. R^2
- In the spirit of EFT, we should include all possible higher dimensional operators consistent with symmetries to capture UV physics

Fourth order gravity: the good and the bad

- Quadratic curvature gravity[1]

$$S = \int d^4x\sqrt{-g}\ \left[\frac{M_P^2}{2}R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right]$$

- Expansion around flat space $\rightarrow$ Renormalizable
- 8 dofs
  - Massless, traceless, transverse graviton $\rightarrow$ 2
  - Massive scalar $\rightarrow$ 1
  - Massive spin-2 ghost $\rightarrow$ 5
- Higher derivative gravity: improved renormalizability comes at the cost of unitarity
- Is there a way to retain UV-stabilizing abilities of higher derivative terms without introducing new/ghost dofs?

[1] Stelle 76

Bouncing cosmology

- Bouncing cosmology: one way out of Big Bang singularity problem
  - A phase of contraction precedes expansion, through a finite value of scale factor at the bounce point
- In GR, bounce usually requires exotic matter which violates NEC
  - For spatially flat backgrounds, Friedmann equations give

$$\dot{H} = -\frac{1}{2}(\rho + p) \tag{1}$$

  - NEC violation typically implies ghost/gradient instabilities

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- Modify GR by adding higher derivative terms like R^2, $R_{\mu\nu}R^{\mu\nu}$, etc.
  - Typically lead to ghost instabilities
  - Eg. Bouncing solutions are possible in $R + R^2 + \Lambda$ theory, but they have either ghosts or $\rho_{radiation} < 0$
- How can we capture UV physics from higher derivative terms without introducing pathologies?

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- How can we capture UV physics from higher derivative terms without introducing pathologies? Non-localize the action

Non-locality

- Non-locality is a natural feature in many approaches towards quantum gravity like string theory, non-commutative geometry, loop quantum gravity, asymptotic safety and causal set theory
- Non-local field theories as UV-finite theories have a long history (Born, Pais and Uhlenbeck, Efimov, Krasnikov, Tomboulis, Modesto, Biswas, Mazumdar and Siegel, $\cdots$)

value, but identically). Then the field equations are

$$e^{-p^2} p^2 A_k = 0. \qquad \text{(VII.4)}$$

If one expands the exponential factor and retains only

(Max Born, 1949)

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- Non-local field theories as UV-finite theories have a long history (Born, Pais and Uhlenbeck, Efimov, Krasnikov, Tomboulis, Modesto, Biswas, Mazumdar and Siegel, $\cdots$)
- Instead of truncating terms to a finite higher order in derivatives, which typically introduces ghost instabilities, one may construct suitable terms which contain derivatives to all orders
- In string field theory (and p-adic string theory), one has non-local terms of the form $\sim \phi e^{\Box/M^2}\phi$

Non-local gravity - Action and equations of motion

Higher derivative, non-local gravity described by[2]

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + R\mathcal{F}(\Box)R - \Lambda\right] \quad (2)$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \left(\frac{\Box}{M_s}\right)^n$ and $M_s(< M_p)$ is a new UV scale

[2] Biswas, Mazumdar, Siegel 06; Biswas, Gerwick, Koivisto, Mazumdar 12

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$$\begin{aligned} &-\left[M_p^2 + 4\mathcal{F}(\Box)R\right] G^\mu{}_\nu - R\mathcal{F}(\Box)R\delta^\mu{}_\nu + 4\left(\nabla^\mu\partial_\nu - \delta^\mu{}_\nu\Box\right)\mathcal{F}(\Box)R \\ &\quad + 2\mathcal{K}^\mu{}_\nu - \delta^\mu{}_\nu(\mathcal{K}^\sigma{}_\sigma + \tilde{\mathcal{K}}) - \Lambda\delta^\mu{}_\nu = 0 \end{aligned} \quad (3)$$

where $\mathcal{K}^\mu{}_\nu = \frac{1}{M_s^2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1}(\partial^\mu\Box_s^l R)\ (\partial_\nu\Box_s^{n-l-1}R)$, $\qquad \Box_s = \Box/M_s^2$

$$\tilde{\mathcal{K}} = \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1}(\Box_s^l R)\ (\Box_s^{n-l}R)$$

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Non-local gravity - Finding stable solutions

- Search for background solutions of equations of motion
- Study perturbations around this background (at linearized EoM or quadratic action level)
- Deduce the structure of background-dependent form factor $\mathcal{F}(\bar{\Box})$ which makes perturbations free from any instabilities (ghost/gradient/tachyonic)

Background (in vacuum, $T_{\mu\nu} = 0$)

Background Ricci scalar ansatz for solving equations of motion:

$$\bar{\Box}\bar{R} = r_1 \bar{R} + r_2, \qquad r_{1,2} = \text{constants} \tag{4}$$

$$\bar{\Box}^n \bar{R} = r_1^n \left(\bar{R} + \frac{r_2}{r_1} \right) \implies \mathcal{F}(\bar{\Box})\bar{R} = \mathcal{F}_1 \bar{R} + \mathcal{F}_2 \tag{5}$$

$$\text{where} \quad \mathcal{F}_1 = \mathcal{F}(r_1) \qquad \text{and} \qquad \mathcal{F}_2 = \frac{r_2}{r_1}(\mathcal{F}_1 - f_0) \tag{6}$$

Substituting the ansatz (4) in the EoM (in vacuum) gives us the following unique conditions on the form factor $\mathcal{F}(\bar{\Box})$

$$\mathcal{F}_1 = \mathcal{F}'(r_1) = 0, \quad \mathcal{F}_2 = -\frac{M_p^2}{4}, \quad \Lambda = -\frac{M_p^2}{16 f_0} \tag{7}$$

From Eqs. (6) and (7), we get

$$f_0 < 0 \quad \text{for} \quad \Lambda > 0\,, \qquad f_0 > 0 \quad \text{for} \quad \Lambda < 0 \tag{8}$$

Quadratic action around the background $\bar{\Box}\bar{R} = r_1\bar{R} + r_2$

Second variation of the action around the ansatz $\bar{\Box}\bar{R} = r_1\bar{R} + r_2$, upon imposing the background conditions $\mathcal{F}_1 = 0$, $\mathcal{F}_2 = -\frac{M_p^2}{4}$:

$$\delta^2 S = \int d^4x\sqrt{-\bar{g}}\Bigg\{\frac{M_p^2}{4}(\delta_{GR} - \delta^{(2)}R) - \Lambda\left(\frac{h^2}{8} - \frac{1}{4}h_{\mu\nu}h^{\mu\nu}\right) + \frac{h}{2}\bar{R}\mathcal{F}(\bar{\Box})\delta R$$
$$+ \frac{h}{2}\bar{R}\delta\mathcal{F}(\Box)\bar{R} + \delta R\delta\mathcal{F}(\Box)\bar{R} + \bar{R}\delta\mathcal{F}(\Box)\delta R + \bar{R}\delta^{(2)}\mathcal{F}(\Box)\bar{R}\Bigg\} \tag{9}$$

where $\sqrt{-\bar{g}}\ \delta_{GR} \equiv \delta^{(2)}(\sqrt{-g}R)$

Upon imposing the unique background conditions, there remains no term in $\delta^2 S$ which could produce propagating vector or tensor modes.

Quadratic action around the background $\bar{\Box}\bar{R} = r_1\bar{R} + r_2$

Imposing all background conditions and after considerable manipulation of terms:

$$\delta^2 S = \int d^4x\sqrt{-\bar{g}}\;\zeta\mathcal{Z}(\bar{\Box})\zeta \tag{10}$$

$$\zeta = \delta(\Box)\bar{R} + (\bar{\Box} - r_1)\delta R \qquad\qquad \mathcal{Z}(\bar{\Box}) = \frac{\mathcal{F}(\bar{\Box})}{(\bar{\Box} - r_1)^2} \tag{11}$$

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- Fix the background $\bar{g}_{\mu\nu}$ as FLRW
- Full metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is the perturbation. In longitudinal gauge, $h_{\mu\nu}$ has $(2S + 2V + 2T)$ dofs:

$$h_{00} = a^2\,(-2\phi)\,,\;\; h_{0i} = a^2(\hat{B}_i),\;\; h_{ij} = a^2(-2\psi\delta_{ij} + 2\hat{h}_{ij}), \tag{12}$$

- ζ is a scalar dof, composed of only ϕ and ψ

Choosing $\mathcal{F}(\bar{\Box})$ which avoids instabilities

For the theory to have no extra degrees of freedom or ghosts at the quadratic level of the action, the kinetic operator $\mathcal{Z}(\bar{\Box})$ can have at most a single zero. Using the Weierstrass product theorem, we choose $\mathcal{F}(\bar{\Box})$ for $\Lambda > 0$ ($f_0 < 0$) as

$$\mathcal{F}(\bar{\Box}) = \frac{1}{M_s^6}(\bar{\Box} - m^2)(\bar{\Box} - r_1)^2 e^{\gamma(\bar{\Box})} \tag{13}$$

for some $m^2 \geq 0$, and γ is an arbitrary entire function of $\bar{\Box}/M_s^2$. This ensures that $\mathcal{Z}(\bar{\Box})$ has only one zero at $\bar{\Box} = m^2$. The kinetic term for ζ has the correct sign to avoid ghosts[3].

[3] Metric sign $(- + ++)$

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$$\mathcal{F}(\bar{\Box}) = \frac{1}{M_s^4}(\bar{\Box} - r_1)^2 e^{\gamma(\bar{\Box})} \quad (14)$$

In this case, $\mathcal{Z}(\bar{\Box})$ has no zeros and ζ acts like a p-adic scalar.

[3] Metric sign $(- + ++)$

Non-singular bouncing background solutions

We now specialize to known bouncing solutions[4]. They satisfy the ansatz $\bar{\Box}\bar{R} = r_1\bar{R} + r_2$, are vacuum solutions of the non-local theory, and possess only a single ghost-free scalar mode (and no vector or tensor modes) around the bouncing background at the linearized level when the non-local form factor $\mathcal{F}(\bar{\Box})$ is chosen appropriately as just discussed.

- Cosine hyperbolic bounce with $a(t) = a_0 \cosh(\sqrt{r_1/2}t)$. Here, $\Lambda > 0$. The background becomes dS at late times.
- Exponential bounce with $a(t) = a_0 e^{\frac{\lambda}{2}t^2}$. Here, $\Lambda > 0$ for $\lambda > 0$ and $\Lambda < 0$ for $\lambda < 0$.

[4] Biswas, Mazumdar, Siegel 06; Biswas, Koivisto, Mazumdar 11; Koshelev, Vernov 14

Physical spectrum around a cosh bouncing background

Let us restrict to the case of $\Lambda > 0$ and a cosine hyperbolic bounce, for which $\mathcal{F}(\bar{\Box})$ was derived earlier

$$\delta^2 S = \frac{1}{M_s^6} \int d^4x \sqrt{-\bar{g}}\, \zeta e^{\gamma(\bar{\Box})} (\bar{\Box} - m^2) \zeta \tag{15}$$

For a stable bounce, we would require the perturbation ζ to be well behaved in time. All solutions of the non-local EoM for ζ are captured by solutions of the local equation

$$(\bar{\Box} - m^2)\zeta = 0 \tag{16}$$

because the non-local factor $e^{\gamma(\bar{\Box})}$ does not introduce any new zeros in the kinetic operator.

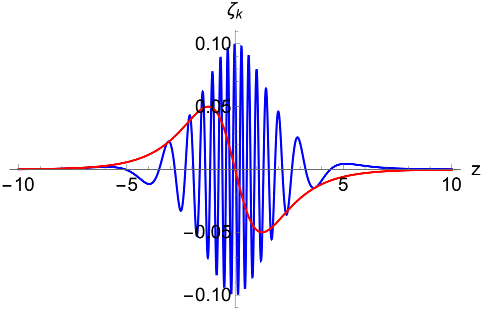

Figure: Evolution in cosmic time $z = M_s t$ of a Fourier component of the scalar mode ζ_k for cosine hyperbolic bounce. $k = 20M_s$ (blue curve) and $k = 0$ (red curve).

We can see that the solution ζ_k is oscillatory and bounded. This behavior persists to hold for any value of k with increasing number of bounded oscillations as $k \to \infty$.

Result and open questions

A higher derivative, non-local gravity model ($R + R\mathcal{F}R + \Lambda$) which admits non-singular bouncing solutions in the absence of matter. There exists a special vacuum (which is dS at late times for cosh bounce) with only a scalar propagating dof, and no vector or tensor modes (at the linearized level). By choosing a suitable function $\mathcal{F}$, the scalar can be made free from ghost instabilities around the dynamical background, and has oscillatory and bounded evolution across the bounce.

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Q. Is the isotropic and homogeneous solution in non-local gravity stable with respect to growing anisotropies in the contracting phase (BKL instability)?

Q. Can one have similar stable, non-singular bouncing scenarios in more general actions which include Ricci tensor and Weyl tensor terms, like $R + \Lambda + R\mathcal{F}_1 R + R_{\mu\nu}\mathcal{F}_2 R^{\mu\nu} + C_{\mu\nu\rho\sigma}\mathcal{F}_3 C^{\mu\nu\rho\sigma}$?

General analysis of non-(A)dS and special vacua in non-local gravity

Background quantities $(\bar{R}, \bar{g}_{\mu\nu}, \cdots)$ indicated by overbars.

$$\bar{R}_{\mu\nu\rho\sigma} = \underbrace{\frac{\bar{R}}{12}\left(\bar{g}_{\mu\rho}\bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\nu\rho}\right)}_{\text{dS/AdS}} + \underbrace{\frac{1}{2}\left(\bar{g}_{\nu\sigma}\bar{S}_{\mu\rho} + \bar{g}_{\mu\rho}\bar{S}_{\nu\sigma} - \bar{g}_{\nu\rho}\bar{S}_{\mu\sigma} - \bar{g}_{\mu\sigma}\bar{S}_{\nu\rho}\right) + \bar{C}_{\mu\nu\rho\sigma}}_{\text{deviation from dS/AdS}}$$

where traceless Ricci tensor $\bar{S}_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{1}{4}\bar{R}\bar{g}_{\mu\nu}$

- Turning on $\bar{S}_{\mu\nu}$ or/and $\bar{C}_{\mu\nu\rho\sigma} \implies$ non-(A)dS

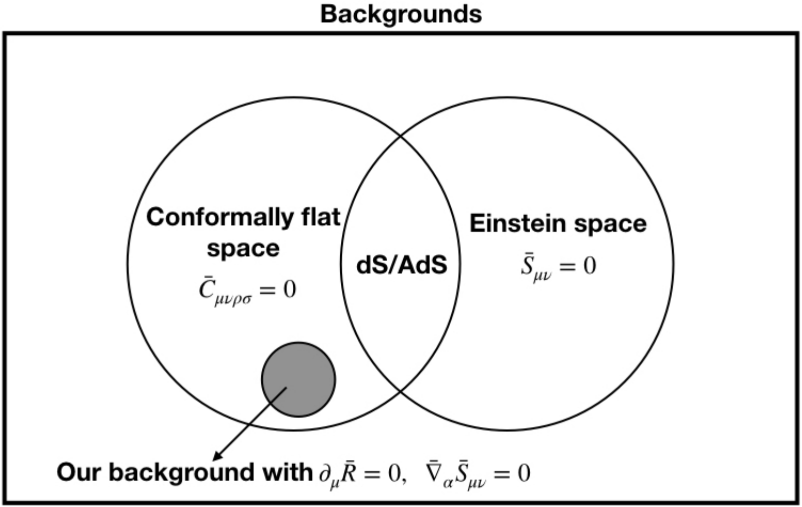
General analysis of non-(A)dS and special vacua in non-local gravity

- Background: a particular non-Maximally Symmetric Spacetime (MSS): Conformally-flat, covariantly constant curvature ($\bar{S}_{\mu\nu} =$ traceless Ricci tensor)

$$\bar{C}_{\mu\nu\rho\sigma} = 0, \quad \bar{S}_{\mu\nu} \neq 0, \quad \bar{\nabla}_\alpha \bar{S}_{\mu\nu} = 0, \qquad (\text{non-MSS} \xrightarrow{\bar{S}_{\mu\nu}=0} \text{MSS})$$

These conditions can be realized near the bounce point of a sufficiently slow, symmetric bouncing scenario.

Backgrounds



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- Covariant SVT decomposition (2S+1V+1T):

$$h_{\mu\nu} = \widehat{h}_{\mu\nu} + \bar{\nabla}_\mu A_\nu + \bar{\nabla}_\nu A_\mu + \bar{\nabla}_\mu \bar{\nabla}_\nu B - \frac{1}{4}\bar{g}_{\mu\nu}\phi$$

  - $\bar{\nabla}^\mu \widehat{h}_{\mu\nu} = 0, \quad \bar{g}^{\mu\nu}\widehat{h}_{\mu\nu} = 0, \quad \bar{\nabla}^\mu A_\mu = 0$

General action for non-local gravity

$$S = \frac{1}{2}\int d^4x \; \mathcal{L}_{EH+\Lambda} + \mathcal{L}_{R^2} + \mathcal{L}_{S^2} + \mathcal{L}_{C^2}$$

$$\mathcal{L}_{EH+\Lambda} = \sqrt{-g}\; M_P^2\,(R - 2\Lambda)$$

$$\mathcal{L}_{R^2} = \sqrt{-g}\; R\mathcal{F}_1(\Box)R$$

$$\mathcal{L}_{S^2} = \sqrt{-g}\; S^{\nu}{}_{\mu}\mathcal{F}_2(\Box)S^{\mu}{}_{\nu}$$

$$\mathcal{L}_{C^2} = \sqrt{-g}\; C^{\rho\sigma}{}_{\mu\nu}\mathcal{F}_3(\Box)C^{\mu\nu}{}_{\rho\sigma}$$

$$\mathcal{F}_i(\Box) = \sum_{n=0}^{\infty} f_{i,n}\left(\frac{\Box}{M_s^2}\right)^n, \qquad M_s(< M_P) = \text{UV scale of non-locality}$$

General structure of quadratic action $\delta^2 S$

$$S = \frac{1}{2}\int d^4x \; \mathcal{L}_{EH+\Lambda} + \mathcal{L}_{R^2} + \mathcal{L}_{S^2} + \mathcal{L}_{C^2}$$

$$\delta^2 S = \int d^4x \sqrt{-\bar{g}} \; \begin{bmatrix} B & \phi & A_\rho & \widehat{h}_{\mu\nu} \end{bmatrix} \begin{bmatrix} \mathcal{K}_{00} & \mathcal{K}_{01} & \mathcal{K}_{02} & \mathcal{K}_{03} \\ \mathcal{K}_{10} & \mathcal{K}_{11} & \mathcal{K}_{12} & \mathcal{K}_{13} \\ \mathcal{K}_{20} & \mathcal{K}_{21} & \mathcal{K}_{22} & \mathcal{K}_{23} \\ \mathcal{K}_{30} & \mathcal{K}_{31} & \mathcal{K}_{32} & \mathcal{K}_{33} \end{bmatrix} \begin{bmatrix} B \\ \phi \\ A_\sigma \\ \widehat{h}_{\alpha\beta} \end{bmatrix}$$

$\mathcal{K}$ around MSS[5]

$$\bar{g}_{\mu\nu} = \text{flat/dS/AdS}$$

$$\begin{bmatrix} B & \phi & A_\rho & \widehat{h}_{\mu\nu} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathcal{K}_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{K}_{33} \end{bmatrix} \begin{bmatrix} B \\ \phi \\ A_\sigma \\ \widehat{h}_{\alpha\beta} \end{bmatrix}$$

- Diagonal matrix $\implies$ no mode-mixing terms
- Kinetic (and mass) terms for ϕ and $\widehat{h}_{\mu\nu}$ modes
- No kinetic (and mass) terms for B and A_μ modes
- Can constrain $\mathcal{F}_i(\Box)$ to avoid new dofs/ghosts (example soon)

[5] Biswas, Koshelev, Mazumdar 16

$\mathcal{K}$ around non-MSS backgrounds

- Fix non-MSS background: Conformally-flat, covariantly constant curvature: $\bar{C}_{\mu\nu\rho\sigma} = 0$, $\bar{S}_{\mu\nu} \neq 0$, $\bar{\nabla}_\alpha \bar{S}_{\mu\nu} = 0$

$$\begin{bmatrix} B & \phi & A_\rho & \widehat{h}_{\mu\nu} \end{bmatrix} \begin{bmatrix} \mathcal{K}_{00} & \mathcal{K}_{01} & \mathcal{K}_{02} & \mathcal{K}_{03} \\ \mathcal{K}_{10} & \mathcal{K}_{11} & \mathcal{K}_{12} & \mathcal{K}_{13} \\ \mathcal{K}_{20} & \mathcal{K}_{21} & \mathcal{K}_{22} & \mathcal{K}_{23} \\ \mathcal{K}_{30} & \mathcal{K}_{31} & \mathcal{K}_{32} & \mathcal{K}_{33} \end{bmatrix} \begin{bmatrix} B \\ \phi \\ A_\sigma \\ \widehat{h}_{\alpha\beta} \end{bmatrix}$$

- Kinetic (and mass) terms for all SVT modes: $\phi, \widehat{h}_{\mu\nu}, B, A_\mu$
- Non-diagonal matrix $\rightarrow$ all 6 mode mixings present
- These new (non-MSS) terms in $\mathcal{K} \propto \bar{S}_{\mu\nu}$

Example 1: $R + \Lambda +$ non-local Weyl^2 around MSS

$$S = \frac{1}{2}\int d^4x\, \sqrt{-g}\left[M_P^2\left(R - 2\Lambda\right) + C^{\rho\sigma}{}_{\mu\nu}\mathcal{F}_3(\Box)C^{\mu\nu}{}_{\rho\sigma}\right]$$

$$\delta^2 S \sim \int \widetilde{h}_{\mu\nu}(\mathcal{K}_{33}^{\mu\nu\alpha\beta})\widetilde{h}_{\alpha\beta} - \widetilde{\phi}(\mathcal{K}_{11})\widetilde{\phi}$$

$$\mathcal{K}_{11} = \frac{3\bar{\Box} + \bar{R}}{6}$$

$$\mathcal{K}_{33}^{\mu\nu\alpha\beta} = \left(\bar{\Box} - \frac{\bar{R}}{6}\right)\left[1 + \frac{2}{M_P^2}\left(\bar{\Box} - \frac{\bar{R}}{3}\right)\mathcal{F}_3\left(\bar{\Box}_s + \frac{\bar{R}_s}{3}\right)\right]\bar{g}^{\alpha\mu}\bar{g}^{\beta\nu}$$

$\widetilde{h}_{\mu\nu}, \widetilde{\phi} \rightarrow$ canonically normalized fields

$\bar{\Box}_s = \bar{\Box}/M_s^2,\ \bar{R}_s = \bar{R}/M_s^2$

$R + \Lambda +$ non-local Weyl^2 around MSS

$$\mathcal{K}_{33}^{\mu\nu\alpha\beta} = \underbrace{\left(\bar{\Box} - \frac{\bar{R}}{6}\right)}_{\text{IR pole}} \underbrace{\left[1 + \frac{2}{M_P^2}\left(\bar{\Box} - \frac{\bar{R}}{3}\right)\mathcal{F}_3\left(\bar{\Box}_s + \frac{\bar{R}_s}{3}\right)\right]}_{\text{UV contribution}} \bar{g}^{\alpha\mu}\bar{g}^{\beta\nu}$$

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- $\mathcal{F}_3(\bar{\Box}_s) = \frac{M_P^2}{2}\left[\frac{e^{\alpha\left(\bar{\Box}_s - \frac{2\bar{R}_s}{3}\right)} - 1}{\left(\bar{\Box} - \frac{2\bar{R}}{3}\right)}\right] \implies$ No ghosts or new dofs other than spin-2 TT graviton

Example 2: $R + \Lambda +$ non-local R^2

$$S = \frac{1}{2} \int d^4x \, \sqrt{-g} \left[M_P^2 \left(R - 2\Lambda \right) + R\mathcal{F}(\Box) R \right]$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \left(\frac{\Box}{M_s} \right)^n$ and $M_s (< M_p)$ is a new UV scale

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- What are the degrees of freedom around the particular non-MSS background $(\bar{C}_{\mu\nu\rho\sigma} = 0, \bar{S}_{\mu\nu} \neq 0, \bar{\nabla}_\alpha \bar{S}_{\mu\nu} = 0)$?
- Background EoM: $\Omega \bar{S}^\mu_{\ \nu} = 0$ where $\Omega \equiv M_P^2 + 2f_0\bar{R}$
- 3 solutions
    - $\Omega = 0, \bar{S}_{\mu\nu} \neq 0$ (non-MSS)
    - $\Omega \neq 0, \bar{S}_{\mu\nu} = 0$ (MSS)
    - $\Omega = 0, \bar{S}_{\mu\nu} = 0$ (MSS)

$R + \Lambda +$ non-local R^2 around non-MSS

$$S = \frac{1}{2}\int d^4x\ \sqrt{-g}\left[M_P^2\left(R - 2\Lambda\right) + R\mathcal{F}(\Box)R\right]$$

$$\delta^2 S = \int d^4x\sqrt{-\bar{g}}\ \frac{1}{2}\psi\mathcal{F}(\bar{\Box})\psi, \quad \text{where } \psi \equiv (\bar{R} + 3\bar{\Box})\frac{\phi}{4} - \bar{S}_{\mu\nu}\widehat{h}^{\mu\nu}$$

Propagating, ghost-free dofs depend on the form of $\mathcal{F}(\bar{\Box})$ chosen:

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Propagating, ghost-free dofs depend on the form of $\mathcal{F}(\bar{\Box})$ chosen:

- $\mathcal{F}(\bar{\Box}) = \text{constant} \implies$ no kinetic term for ψ
- $\mathcal{F}(\bar{\Box}) = \left(1 - \frac{\bar{\Box}}{m^2}\right)^{\epsilon} e^{\alpha(\bar{\Box})}$, for some entire function $\alpha(\bar{\Box})$
    - $\epsilon = 0$: no pole
    - $\epsilon = 1$: one pole at $\bar{\Box} = m^2$ for some mass m

Example 2: $R + \Lambda$ + non-local R^2

$$S = \frac{1}{2}\int d^4x\, \sqrt{-g}\left[M_P^2\left(R - 2\Lambda\right) + R\mathcal{F}(\Box)R\right]$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \left(\frac{\Box}{M_s}\right)^n$ and $M_s(< M_p)$ is a new UV scale

- What are the degrees of freedom around the particular non-MSS background ($\bar{C}_{\mu\nu\rho\sigma} = 0, \bar{S}_{\mu\nu} \neq 0, \bar{\nabla}_\alpha \bar{S}_{\mu\nu} = 0$)?
- Background EoM: $\Omega \bar{S}^\mu_{\ \nu} = 0$ where $\Omega \equiv M_P^2 + 2f_0\bar{R}$
- 3 solutions
  - $\Omega = 0, \bar{S}_{\mu\nu} \neq 0$ (non-MSS)
  - $\Omega \neq 0, \bar{S}_{\mu\nu} = 0$ (MSS)
  - $\Omega = 0, \bar{S}_{\mu\nu} = 0$ (MSS)

$R + \Lambda +$ non-local R^2 around MSS

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[M_P^2 (R - 2\Lambda) + R\mathcal{F}(\Box)R \right]$$

What are the dofs in $\delta^2 S$ around the two MSS backgrounds?

($\Omega \equiv M_P^2 + 2f_0\bar{R}$, $\mathcal{K} =$ kinetic matrix)

(i) $\Omega \neq 0, \bar{S}_{\mu\nu} = 0$: Diagonal $\mathcal{K}$ with ϕ and $\widehat{h}_{\mu\nu}$ modes

(ii) $\Omega = 0, \bar{S}_{\mu\nu} = 0$: Diagonal $\mathcal{K}$ with only ϕ mode

- Tuned parameters obeying $\bar{R} = -\dfrac{M_P^2}{2f_0} = 4\Lambda$

- For both MSS backgrounds (i) and (ii), one can constrain $\mathcal{F}(\Box)$ so that there are no new dofs/ghosts
- (i) and (ii): distinct dS/AdS vacua because of distinct spectra[6]

[6] In local gravity, $R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \Lambda$, this was done by Pope, Lu 11 (Critical Gravity)

Result and open questions

- A higher derivative, non-local gravity model $(R + R\mathcal{F}R + \Lambda)$ can be made free from ghost instabilities around non-maximally symmetric backgrounds. This involves choosing a suitable $\mathcal{F}$.
- Upon a critical tuning of parameters in the theory, one can have special (A)dS vacua which have physical spectra different from usual expectations.
- More general actions like $R + \Lambda + R\mathcal{F}_1 R + R_{\mu\nu}\mathcal{F}_2 R^{\mu\nu} + C_{\mu\nu\rho\sigma}\mathcal{F}_3 C^{\mu\nu\rho\sigma}$ have SVT mode mixing around non-MSS backgrounds. It is challenging to find stable, ghost-free solutions in such cases.

Result and open questions

- A higher derivative, non-local gravity model $(R + R\mathcal{F}R + \Lambda)$ can be made free from ghost instabilities around non-maximally symmetric backgrounds. This involves choosing a suitable $\mathcal{F}$.
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Q. Are black holes (or black hole-like objects) in higher derivative, non-local gravity stable with respect to generic perturbations?
Q. The form factor $\mathcal{F}$ needed to produce ghost-free perturbations is background dependent. How can one construct a background independent theory of UV non-local gravity?

Thank you