

Test of the equivalence principle using the odd multipoles of the NPCF

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Outline

- ▶ A short video introduction to equivalence principle
- ▶ History of free-fall experiments for particles of different masses
- ▶ Equivalence principle for fluids in general relativity
- ▶ Violations of the equivalence
- ▶ How to detect the violations using the large scale structure.

Free fall experiments

Year	Investigator	Sensitivity	Method
500?	Philoponus ^[14]	"small"	Drop tower
1585	Stevin ^[15]	5×10^{-2}	Drop tower
1590?	Galileo ^[16]	2×10^{-2}	Pendulum, drop tower
1686	Newton ^[17]	10^{-3}	Pendulum
1832	Bessel ^[18]	2×10^{-5}	Pendulum
1908 (1922)	Eötvös ^[19]	2×10^{-9}	Torsion balance
1910	Southerns ^[20]	5×10^{-6}	Pendulum
1918	Zeeman ^[21]	3×10^{-8}	Torsion balance
1923	Potter ^[22]	3×10^{-6}	Pendulum
1935	Renner ^[23]	2×10^{-9}	Torsion balance
1964	Dicke, Roll, Krotkov ^[10]	3×10^{-11}	Torsion balance
1972	Braginsky, Panov ^[24]	10^{-12}	Torsion balance
1976	Shapiro, et al. ^[25]	10^{-12}	Lunar laser ranging
1981	Keiser, Faller ^[26]	4×10^{-11}	Fluid support
1987	Niebauer, et al. ^[27]	10^{-10}	Drop tower
1989	Stubbs, et al. ^[28]	10^{-11}	Torsion balance
1990	Adelberger, Eric G.; et al. ^[29]	10^{-12}	Torsion balance
1999	Baessler, et al. ^{[30][31]}	5×10^{-14}	Torsion balance
2017	MICROSCOPE ^[32]	10^{-15}	Earth orbit

MICROSCOPE: Similarity of free-fall for two bodies

- ▶ MICROSCOPE:: 300 kg Satellite operated by National Centre for Space Studies
- ▶ Aim: To test the universality of free-fall with a precession to the order 10^{15} , 100 times more precession than what could be achieved on earth
- ▶ Two masses with different neutron-proton ratios
 - ▶ Platinum-Rhodium alloy
 - ▶ Titanium-Aluminium-Vanadium alloy

Equivalence principle

*"A little reflection will show that the law of the equality of the inertial and gravitational mass is equivalent to the assertion that the acceleration imparted to a body by a gravitational field is **independent of the nature of the body**. For Newton's equation of motion in a gravitational field, written out in full, it is:*

$$\begin{aligned} & (\text{Inertial mass}) \cdot (\text{Acceleration}) \\ &= (\text{Intensity of the gravitational field}) \cdot (\text{Gravitational mass}). \end{aligned}$$

It is only when there is numerical equality between the inertial and gravitational mass that the acceleration is independent of the nature of the body"

"How I created the theory of relativity "

by

Albert Einstein

Free-fall in curved spacetime

- ▶ Consider the action of particle free from all external, non-gravitational forces etc

$$S = \int_A^B ds = \int_A^B \sqrt{-g_{\mu\nu}(x) dx^\mu dx^\nu} = \int_A^B \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau$$

- ▶ The Euler-Lagrange equation leads to a geodesic equation

$$\frac{d^2 x^\beta}{d\tau^2} + \Gamma^\beta_{\alpha\nu} \frac{dx^\alpha}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

- ▶ Promote to fluids(fields): Imagine a family of curves and a define an average tangent vector to the family of curves

$$u^\mu = \frac{dx^\mu}{d\tau} \longrightarrow u^\nu \nabla_\nu u^\mu = 0$$

Equivalence principle: General relativity

- Consider a standard Einstein-Hilbert action with no funny fields or coupling

$$S = S_{\text{EH}}[g_{\mu\nu}] + S_{\text{SM}}[g_{\mu\nu}] + S_{\phi}[\phi, g_{\mu\nu}] + S_{\text{DM}}[\varphi, g_{\mu\nu}]$$

- For this action, you can write down

$$T_I^{\mu\nu} \equiv \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}(g_{\mu\nu}))}{\delta g^{\mu\nu}} = (\rho_I + P_I) u_I^\mu u_I^\nu + P_I g^{\mu\nu}.$$

- I = baryons, dark matter, etc

$$\nabla^\mu T_{\mu\nu} = (P_I + \rho_I) u_I^\mu \nabla_\mu u_I^\nu + g^{\mu\nu} \nabla_\nu P_I + u_I^\nu [\nabla_\mu (P_I + \rho_I) u_I^\mu] = 0$$

- Relativistic Euler equation

$$(P_I + \rho_I) \cancel{u_I^\mu \nabla_\mu u_I^\nu} + (g^{\mu\nu} + u_I^\mu u_I^\nu) \nabla_\nu P_I = 0$$

Inverse at late-time

- ▶ We consider metric perturbations

$$ds^2 = a^2 \left(- (1 + 2\Phi) d\eta^2 + (1 - 2\Psi) \delta_{ij} dx^i dx^j \right) ,$$

- ▶ The perturbation of the temporal and spatial components of its 4-velocity is given by

$$u_I^0 = 1 - \Phi^{(1)} + \frac{1}{2} \left[3[\Phi^{(1)}]^2 - \Phi^{(2)} + \partial_i v_I^{(1)} \partial^i v_I^{(1)} \right] ,$$

$$u_I^i = \partial^i v_I^{(1)} + \frac{1}{2} \partial^i v_I^{(2)} ,$$

- ▶ In the limit of vanishing pressure: $l = b, c, m$

$$\partial^i v_I^{(1)'} + \mathcal{H} \partial^i v_I^{(1)} + \partial^i \Phi^{(1)} = 0 ,$$

- ▶ Note that $v_{bc} = v_b - v_c = 0$.

- ▶ Zero relative velocity due to the equivalence principle

Late-time violations of the equivalence principle

- ▶ Recombination: tight coupling between baryons and photon
- ▶ Violation of the local Lorentz invariance (existence of a preferred frame, e.g Einstein-aether theory)
- ▶ Gravity contains degrees of freedom that couple differently to various matter fields (Scalar-Tensor theory)

Scalar-Tensor theory with interaction in the dark sector

- ▶ Consider the following action

$$S = S_{\text{EH}}[g_{\mu\nu}] + S_{\text{M}}[g_{\mu\nu}] + S_{\phi}[\phi, g_{\mu\nu}] + S_{\text{DM}}[\varphi, \tilde{g}_{\mu\nu}]$$

- ▶ The dark matter field φ sees the metric $\tilde{g}_{\mu\nu}$.
- ▶ The two metrics $\tilde{g}_{\mu\nu}$ and $g_{\mu\nu}$ are related according to

$$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_{\mu}\phi\partial_{\nu}\phi.$$

- ▶ Couples to the ϕ field through $C(\phi)$ and $D(\phi)$
- ▶ Energy-momentum conservation equations become

$$\begin{aligned}\nabla^{\mu} T_{\mu\nu}^{\text{sm}} &= 0 \\ \nabla^{\mu} T_{\mu\nu}^{\text{DM}} &= Q(\phi, u^{\mu}\nabla_{\mu}\phi, \rho_{\text{c}})\nabla_{\nu}\phi\end{aligned}$$

Velocities in the CDM/dark energy interacting universe

- ▶ Baryon peculiar velocity

$$\partial^i v_b^{(1)'} + \mathcal{H} \partial^i v_b^{(1)} + \partial^i \Phi^{(1)} = 0,$$

- ▶ CDM peculiar velocity

$$\partial^i v_c^{(1)'} + \mathcal{H} [1 + \Theta_1] \partial^i v_c^{(1)} + [1 + \Theta_2] \partial^i \Phi^{(1)} = 0,$$

- ▶ Baryon-CDM relative velocity $v_{bc} = v_b - v_c$

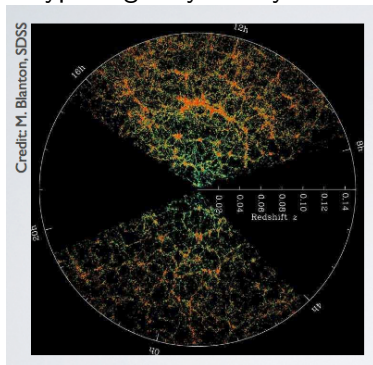
$$v_{bc}^{(1)'} = -\mathcal{H} (v_{bc}^{(1)} - \Theta_1 v_c^{(1)}) + \Theta_2 \Phi^{(1)}.$$

- ▶ Equivalence principle violation!

Can we test these in a model independent way in the universe?

Distribution of structures in the Universe

A typical galaxy survey

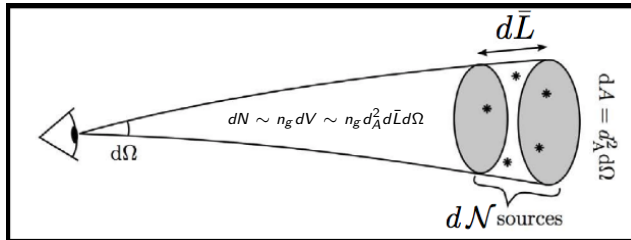


How structures are distributed in the Universe is determined by:

- ▶ The theory of gravity
- ▶ The matter content of the universe
- ▶ The initial conditions/origin

To extract these pieces of information we need to understand exactly what we are measuring.

Theory of what observation measures



Original proposals

- ▶ Kristian, J and Sachs, R. K, *Observations in Cosmology, Texas Symposium, (1969)*
- ▶ Ellis, G. F. R, *Relativistic cosmology, Varenna Lectures, (1969)*

Relativistic Number count fluctuations proposals

- ▶ Yoo, +, *Phys. Rev. D* **80**, 083514 (2009)
- ▶ Challinor, +, *Phys. Rev. D* **84**, 043516 (2011)
- ▶ Bonvin, +, *Phys. Rev. D* **84**, 063505 (2011)

Number count of sources

- ▶ Flux-limited number count of sources

$$\frac{dN^{\text{obs}}(z, \hat{\mathbf{n}}, F)}{dz d\Omega_o} = \mathcal{N}_g(z, \hat{\mathbf{n}}, F) d_A^2(z, \hat{\mathbf{n}}) [k_\mu u^\mu]_o \left| \frac{d\lambda}{dz} \right|,$$

- ▶ The observed number density is dependent on the flux-cut

$$\mathcal{N}_g(z, \hat{\mathbf{n}}, F) = \int_{\ln L(F)}^{\infty} d \ln L \, n_g(z, \hat{\mathbf{n}}, \ln L).$$

- ▶ The luminosity of the source is related to its flux by

$$L = 4\pi F d_L^2 = 4\pi F (1+z)^4 d_A^2$$

Evolution and flux constraint

- ▶ Flux-limited proper number density of galaxy

$$\mathcal{N}_g(z, \hat{\mathbf{n}}, \bar{L}) = \bar{\mathcal{N}}_g(z, \bar{L}) \left[1 + \delta_g + b_e \Delta_z + \mathcal{Q} \Delta_{d_L} + \frac{\partial \delta_g}{\partial \ln \bar{L}} \Delta_{d_L} \right],$$

- ▶ Parametrize these effects:

- ▶ The Evolution bias

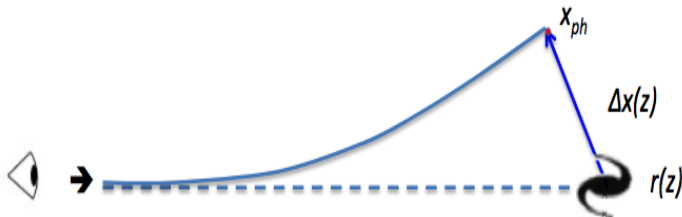
$$b_e(z, \bar{L}) = \frac{\partial \ln [\bar{\mathcal{N}}_g(z, > \bar{L})]}{\partial \ln a}$$

- ▶ The Magnification bias

$$\mathcal{Q}(z, \bar{L}) = - \frac{\partial \ln [\bar{\mathcal{N}}_g(z, > \bar{L})]}{\partial \ln \bar{L}} = \frac{2}{5} s(z, \bar{L})$$

Galaxy positions are distorted by inhomogeneities.

- ▶ We infer source position



- ▶ Radial and angular distortions

$$\mathbf{s}_{\text{obs}} = \mathbf{x}_{\text{phy}} + \Delta \mathbf{x}(z) = \mathbf{x}_{\text{phy}} + \Delta x_{\parallel} \hat{\mathbf{n}} + \Delta \mathbf{x}_{\perp},$$

- ▶ Radial distortions $\Delta x_{\parallel} \hat{\mathbf{n}} \sim \delta z / \mathcal{H} + \delta x_{\parallel}$
- ▶ Angular distortions $\Delta \mathbf{x}_{\perp}$

Radial distortions by inhomogeneities

- ▶ Radial displacement of the source position at leading order

$$\Delta x_{\parallel} = -\frac{1}{\mathcal{H}_s} \left[(\partial_{\parallel} v_s - \partial_{\parallel} v_o) - \Phi_s - \int_0^{\chi_s} (\Phi' + \Psi') d\chi \right] - \int_0^{\chi_s} (\Phi + \Psi) d\chi,$$

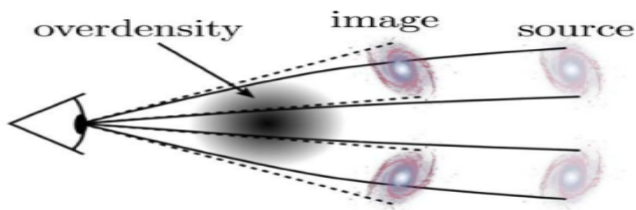
- ▶ Doppler effect
- ▶ Sachs-Wolfe effect (gravitational potential
- ▶ Integrated Sachs-Wolfe effects
- ▶ Time delay effect



Angular distortions by inhomogeneities

- ▶ Angular displacement of the image position AKA lensing

$$\Delta x_{\perp}^i = - \int_0^{\chi} d\chi' \left[(\chi - \chi') \frac{\chi'}{\chi} \partial_{\perp}^i (\Phi + \Psi) \right],$$



- ▶ Cosmic magnification

$$\Delta_{d_A} = \frac{1}{2} \left[\frac{1}{\chi_s} \Delta x_{\parallel} + \nabla_{\perp} \Delta x_{\perp}^I \right] \propto \frac{1}{\chi_s} \Delta x_{\parallel} - \kappa$$

Operational approximation

- ▶ We neglect all integrations terms
- ▶ Weak-field approximation: we neglect all terms that become important only on super-Horizon scales. i.e

$$\Phi, \quad v, \quad v\partial_{\parallel}\Phi, \quad \partial_i\Phi\partial^i\Phi$$

- ▶ We keep terms like $\delta_m \propto \partial^2\Phi$, $\partial_{\parallel}v \sim \partial_{\parallel}\Phi$, $\Phi\partial_{\parallel}\delta_m$

- ▶ First order:

$$\begin{aligned}\Delta_g^{(1)} &= \Delta_{\partial^2}^{(1)} + \Delta_{\partial^1}^{(1)} + \Delta_{\partial^0}^{(1)} \\ &\approx \Delta_{\partial^2}^{(1)} + \Delta_{\partial^1}^{(1)} = \Delta_N^{(1)} + \Delta_D^{(1)}\end{aligned}$$

- ▶ Second order

$$\begin{aligned}\Delta_g^{(2)} &= \Delta_{\partial^4}^{(2)} + \Delta_{\partial^3}^{(2)} + \Delta_{\partial^2}^{(2)} + \Delta_{\partial^1}^{(2)} + \Delta_{\partial^0}^{(2)} \\ &\approx \Delta_{\partial^4}^{(2)} + \Delta_{\partial^3}^{(2)} = \Delta_N^{(2)} + \Delta_D^{(2)}\end{aligned}$$

Gravity theory independent number count fluctuations

- ▶ Newtonian limit at linear order(Kaiser limit)

$$\Delta_{\text{N}}^{(1)} = \delta_g^{(1)} - \frac{1}{\mathcal{H}} \partial_{\parallel}^2 v_g^{(1)}$$

- ▶ General relativistic corrections at linear order

$$\begin{aligned} \Delta_{\text{D}}^{(1)} = & \partial_{\parallel} v_g^{(1)} + \frac{1}{\mathcal{H}} \left(\partial_{\parallel} v_g^{(1)'} + \partial_{\parallel} \Phi^{(1)} \right) \\ & + \left[b_e - 2\mathcal{Q} - \frac{2(1-\mathcal{Q})}{\chi\mathcal{H}} - \frac{\mathcal{H}'}{\mathcal{H}^2} \right] \partial_{\parallel} v_g^{(1)} \end{aligned}$$

- ▶ Newtonian limit at second order

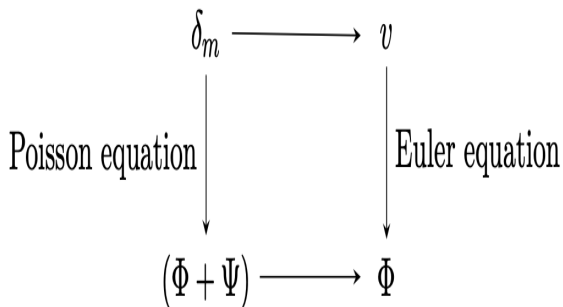
$$\begin{aligned} \Delta_{\text{N}}^{(2)} = & \delta_g^{(2)} - \frac{1}{\mathcal{H}} \partial_{\parallel}^2 v_g^{(2)} - \frac{2}{\mathcal{H}} \left[\delta_g^{(1)} \partial_{\parallel}^2 v_g^{(1)} + \partial_{\parallel} v_g^{(1)} \partial_{\parallel} \delta_g^{(1)} \right] \\ & + \frac{2}{\mathcal{H}^2} \left[\left(\partial_{\parallel}^2 v_g^{(1)} \right)^2 + \partial_{\parallel} v_g^{(1)} \partial_{\parallel}^3 v_g^{(1)} \right] \end{aligned}$$

Gravity theory independent number count fluctuations

$$\begin{aligned}
 \Delta_D^{(2)} = & \left[1 + b_e - 2\mathcal{Q} - \frac{2(1-\mathcal{Q})}{\chi\mathcal{H}} - \frac{\mathcal{H}'}{\mathcal{H}^2} \right] \partial_{\parallel} v_g^{(2)} \\
 & + \frac{1}{\mathcal{H}} \left(\partial_{\parallel} v_g^{(2)'} + \partial_{\parallel} \Phi^{(2)} \right) + \frac{2}{\mathcal{H}} \Phi^{(1)} \left[\partial_{\parallel} \delta_g^{(1)} - \frac{1}{\mathcal{H}} \partial_{\parallel}^3 v_g^{(1)} \right] \\
 & + \frac{2}{\mathcal{H}} \left[\delta_g^{(1)} - \frac{2}{\mathcal{H}} \partial_{\parallel}^2 v_g^{(1)} \right] \left[\partial_{\parallel} v_g^{(1)'} + \partial_{\parallel} \Phi^{(1)} \right] \\
 & + 4 \partial_{\parallel} v_g^{(1)} \left(1 - \frac{1}{\chi\mathcal{H}} \right) \frac{\partial \delta_g^{(1)}}{\partial \ln L} - \frac{2}{\mathcal{H}} \nabla_{\perp i} v_g^{(1)} \nabla_{\perp}^i \partial_{\parallel} v_g^{(1)} \\
 & + 2 \partial_{\parallel} v_g^{(1)} \delta_g^{(1)} \left[1 + b_e - 2\mathcal{Q} - \frac{2(1-\mathcal{Q})}{\chi\mathcal{H}} - \frac{\mathcal{H}'}{\mathcal{H}^2} \right] \\
 & + \frac{2}{\mathcal{H}} \partial_{\parallel} v_g^{(1)} \left[\delta_g^{(1)'} - \frac{2}{\mathcal{H}} \partial_{\parallel}^2 v_g^{(1)'} - \frac{1}{\mathcal{H}} \partial_{\parallel}^2 \Phi^{(1)} \right] \\
 & + \frac{2}{\mathcal{H}} \partial_{\parallel} v_g^{(1)} \partial_{\parallel}^2 v_g^{(1)} \left[-2 - 2b_e + 4\mathcal{Q} + \frac{4(1-\mathcal{Q})}{\chi\mathcal{H}} + 3 \frac{\mathcal{H}'}{\mathcal{H}^2} \right].
 \end{aligned}$$

Imprint of theory of gravity

Continuity equation



Anisotropic constraint

Prior from the solar system test

- Baryons obey the Equivalence principle

$$\partial^i v_b^{(1)'} + \mathcal{H} \partial^i v_b^{(1)} + \partial^i \Phi^{(1)} = 0,$$

- Leads to a velocity difference

$$\begin{aligned} \Delta_D^{(1)} = & \partial_{\parallel} v_g^{(1)} - \partial_{\parallel} v_b^{(1)} + \frac{1}{\mathcal{H}} \left(\partial_{\parallel} v_g^{(1)'} - \partial_{\parallel} v_b^{(1)'} \right) \\ & + \left[b_e - 2\mathcal{Q} - \frac{2(1-\mathcal{Q})}{\chi\mathcal{H}} - \frac{\mathcal{H}'}{\mathcal{H}^2} \right] \partial_{\parallel} v_g^{(1)} \end{aligned}$$

- Parametrise as

$$v_g^{(1)} - v_b^{(1)} = v_{gb}^{(1)} \equiv \Upsilon_1 v_g^{(1)},$$

- Conformal time derivative

$$v_g^{(1)'} - v_b^{(1)'} = v_{gb}^{(1)'} = \beta_1 v_g^{(1)'},$$

Prior from the solar system test

- ▶ Euler equation for baryons at second order

$$\partial_i v_b^{(2)'} + \mathcal{H} \partial_i v_b^{(2)} + \partial_i \Phi^{(2)} + 2 \partial_i \partial_j v_b^{(1)} \partial^j v_b^{(1)} = 0.$$

- ▶ Parametrise the velocity difference

$$v_g^{(2)} - v_b^{(2)} = v_{gb}^{(2)} \equiv \Upsilon_2 v_g^{(2)}.$$

- ▶ The conformal time derivative

$$v_g^{(2)'} - v_b^{(2)'} = v_{gb}^{(2)'} = \beta_2 v_g^{(2)'}$$

Odd multipoles of the galaxy power spectrum

- Multipoles of the cross-power spectrum

$$P_g^{AB}(k, \mu) = P_N^{AB}(k, \mu) + iP_D^{AB}(k, \mu)$$

$$P_\ell^{AB}(k) = \frac{(2\ell + 1)}{2} \int_{-1}^1 d\mu P_D^{AB}(k, \mu) \mathcal{L}_\ell(\mu),$$

- Dipole:

$$\begin{aligned} P_1^{AB}(k) = & (-i) \left[(b_1^B - b_1^A) \left[\frac{2}{\chi \mathcal{H}} + \frac{\mathcal{H}'}{\mathcal{H}^2} - \Upsilon_1 - \beta_1 X_{g1} \right] \right. \\ & + \left(1 - \frac{1}{\chi \mathcal{H}} \right) \left[3f_g(s^A - s^B) + 5(b_1^B s^A - b_1^A s^B) \right] \\ & \left. + \frac{3}{5} f_g (b_e^B - b_e^A) + (b_1^A b_e^B - b_1^B b_e^A) \right] f_g \frac{\mathcal{H}}{k} P_m(k) \end{aligned}$$

- Octupole:

$$P_3^{AB}(k) = 2 \left[\frac{1}{5} (b_e^A - b_e^B) - \left(1 - \frac{1}{\chi \mathcal{H}} \right) (s^A - s^B) \right] f_g^2 \frac{\mathcal{H}}{k} P_m(k)$$

Galaxy bispectrum

- ▶ Galaxy bispectrum in the Newtonian limit

$$B_{gN}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \mathcal{K}_N(\mathbf{k}) \mathcal{K}_N(\mathbf{k}_2) \mathcal{K}_N^{(2)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) P(k_1) P(k_2) + 2 \text{ c. p. },$$

- ▶ We expand in spherical harmonics

$$B_g(k_1, k_2, k_3, \mu_1, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} B_g^{\ell m}(k_1, k_2, k_3) Y_{\ell m}(\mu_1, \phi),$$

- ▶ Leads to even multipoles only $\ell = 0, 2, 4, 6, 8$.
- ▶ Adding relativistic Doppler corrections, generate $\ell = 1, 3, 5, 7$

Parametrization

- ▶ Redshift independent parametrization

$$\Upsilon_{1,2}(z) = \frac{1 - \Omega_m(z)}{1 - \Omega_m} \gamma_{1,2} ,$$

- ▶ Its derivative wrt redshift

$$\beta_{1,2}(z) = \left[\frac{1 - \Omega_m(z)}{1 - \Omega_m} \right]' \gamma_{1,2}$$

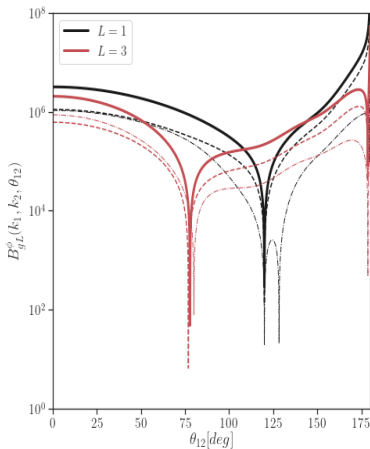
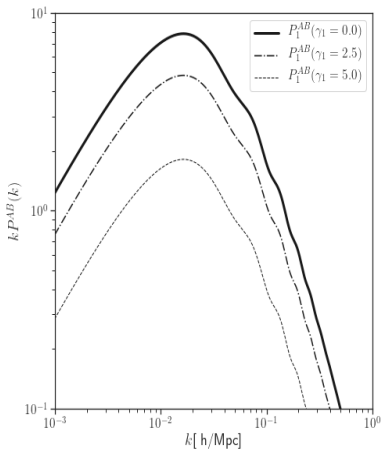
Equivalence principle, cosmological and Astrophysical parameters

- ▶ Equivalence principle parameters = $\{\gamma_1, \gamma_2\}$.
- ▶ Cosmological parameters = $\{D_m, f_g, \Omega_m, P_{\mathcal{O}}(k)\}$.
- ▶ Astrophysical parameters = $\left\{b_1, b_2, b_{s^2}, b_e, \mathcal{Q}, \frac{\partial b_1}{\partial \ln L}\right\}$.

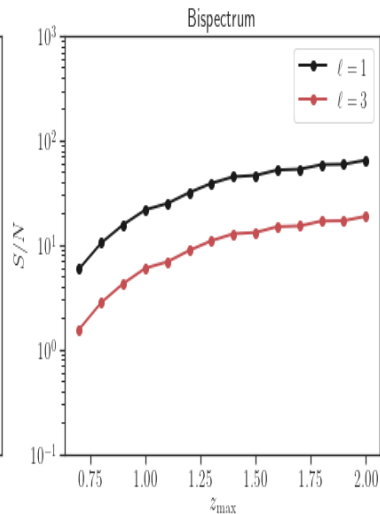
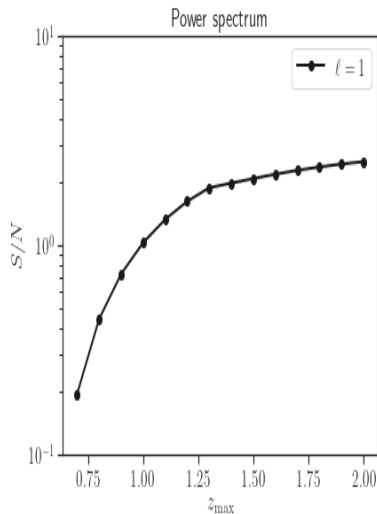
Future surveys

- ▶ $H\alpha$ emission line galaxies
- ▶ HI intensity mapping survey

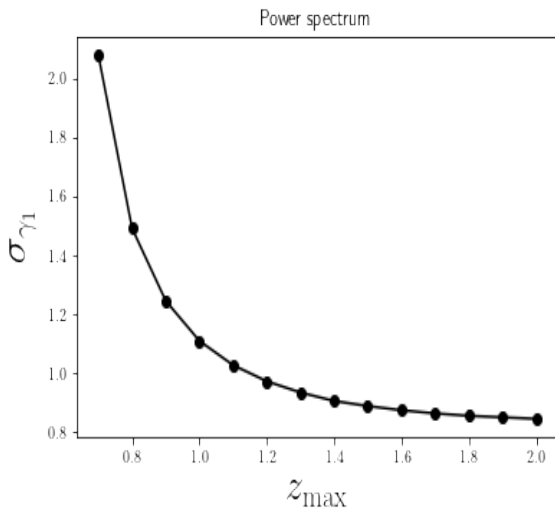
Multipoles of the cross-power spectrum and bispectrum



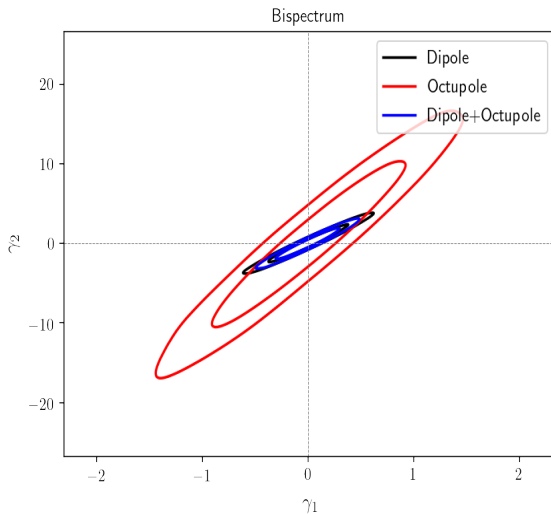
Detectability of the multipole moments



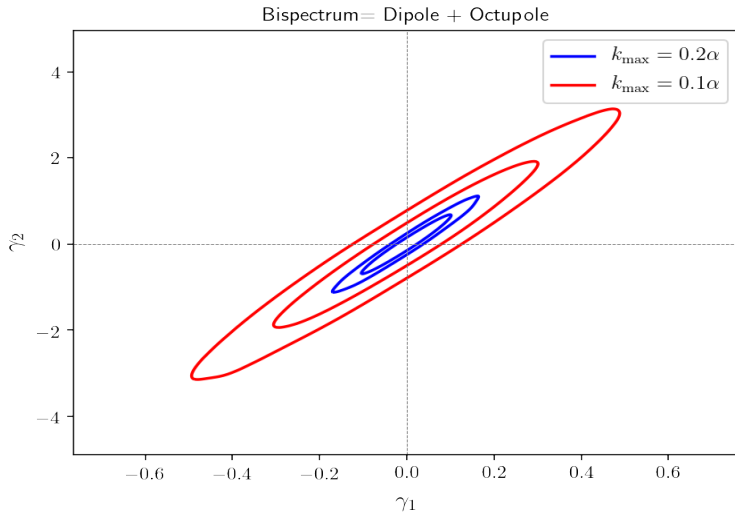
Power spectrum sensitive to γ_1



Bispectrum is sensitive to both $\gamma_{1,2}$



Importance of modelling small scales well



$$\alpha = (1+z)^{(2/(2+n_s))} [h/\text{Mpc}].$$

Conclusion

- ▶ Equivalence principle is the bedrock of general relativity, this is an opportunity to test it on cosmological scales
- ▶ This is a unique window to probe interacting dark sector
- ▶ Current surveys will be able to actually test this!

References

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2. **O. Umeh**, S. Jolicoeur, R. Maartens and C. Clarkson, “A general relativistic signature in the galaxy bispectrum: the local effects of observing on the lightcone,” JCAP **1703**, 034 (2017) [arXiv:1610.03351 [astro-ph.CO]].
3. S. Jolicoeur, R. Maartens, E. M. de Weerd, **O. Umeh**, C. Clarkson and S. Camera “Detecting the relativistic bispectrum in 21cm intensity maps,” arXiv: 2009.06197 [astro-ph.CO]
4. E. M. de Weerd, C. Clarkson, S. Jolicoeur, R. Maartens and **O. Umeh**, “Multipoles of the relativistic galaxy bispectrum,” JCAP **2005** 05 018 (2020) arXiv:1912.11016 [astro-ph.CO]