Nonlinear Behaviour of Axions

Mark Hertzberg

Tufts University

IPMU APEC Seminar, December 16 2020

QCD-Axion Reminder

$$\Delta\mathcal{L}_{qcd} \sim \theta\,\mathbf{E}^a \cdot \mathbf{B}^a \qquad\qquad |\theta| \lesssim 10^{-10}$$

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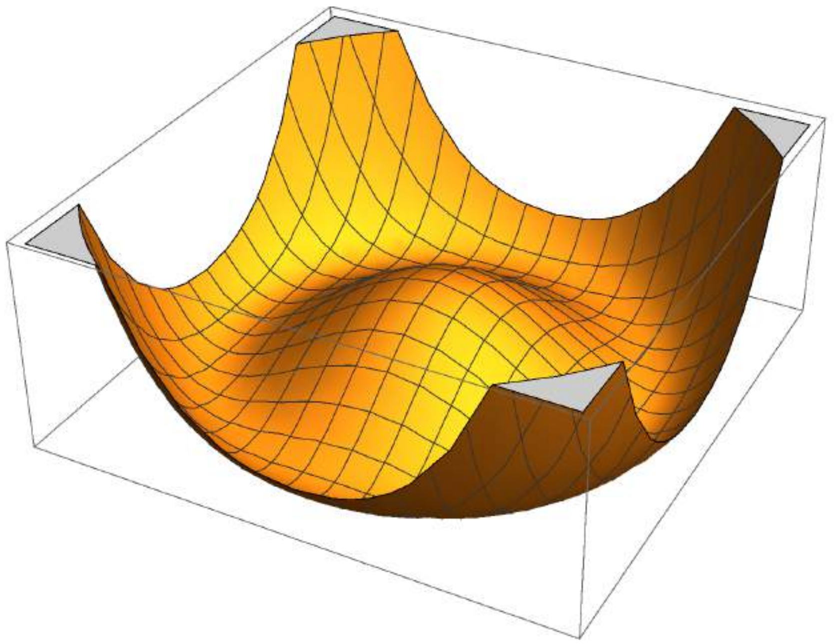
(Peccei-Quinn, Weinberg, Wilczek)

$$\theta \to \phi/f_a$$



$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\mathbf{E}^a \cdot \mathbf{B}^a + \frac{1}{2}(\partial\phi)^2$$

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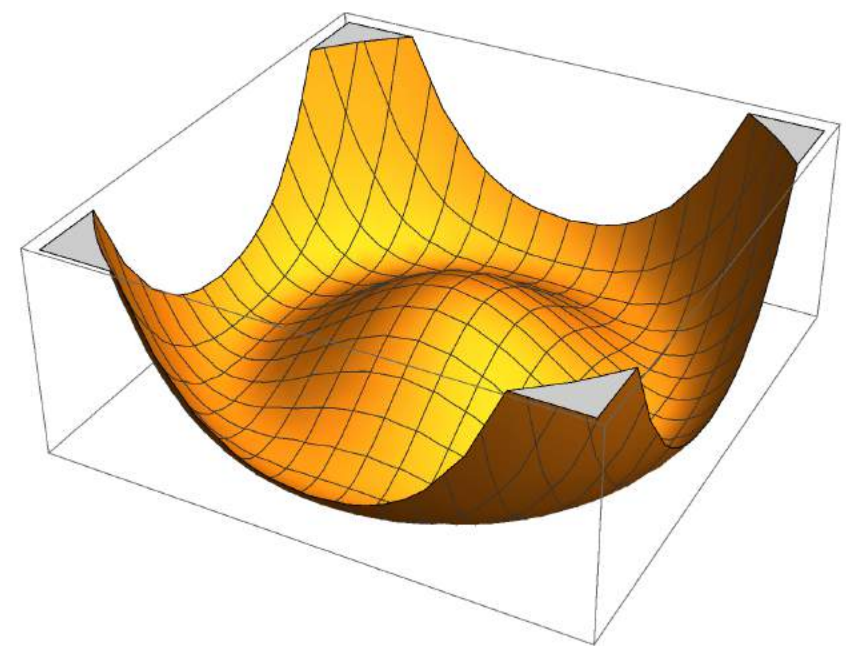

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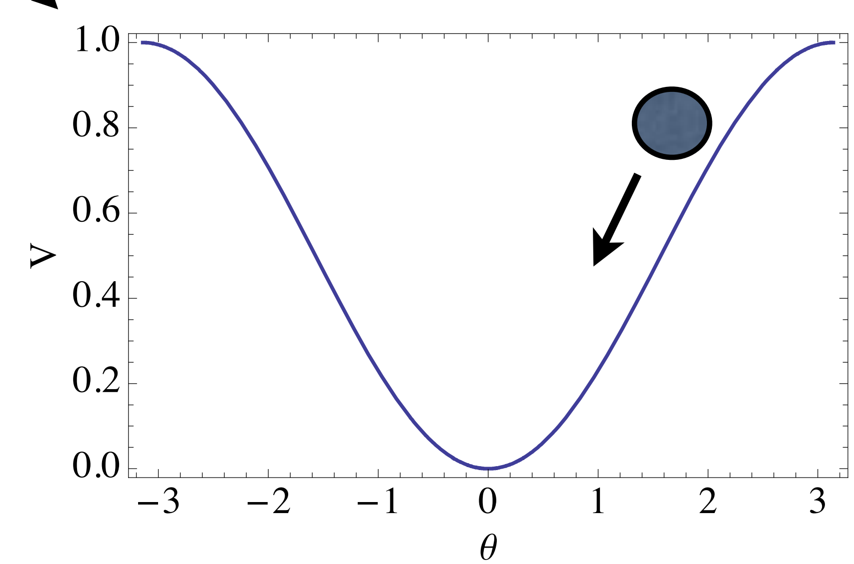

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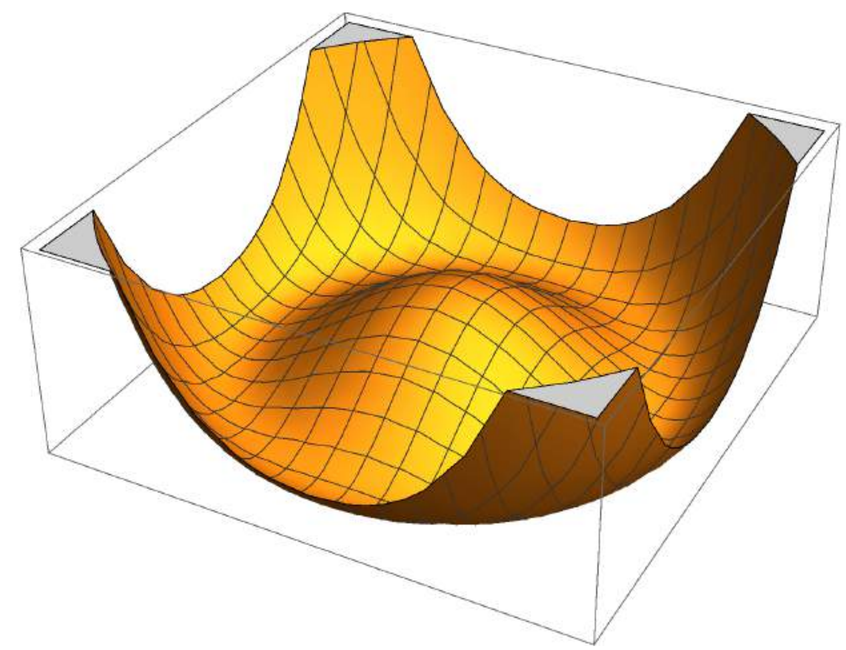
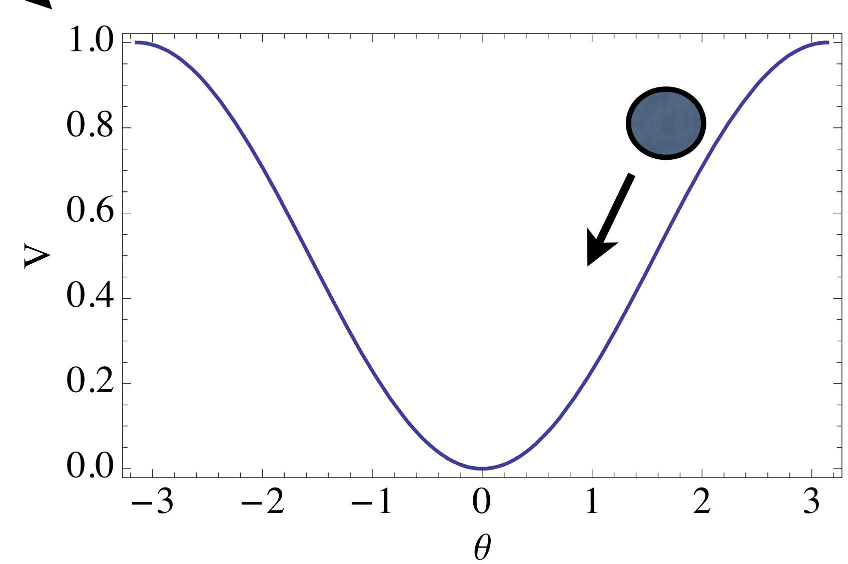


$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\mathbf{E}^a \cdot \mathbf{B}^a + \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V(\phi) = \frac{1}{2}m_a^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$$



Axion mass:

$$m_a \sim \frac{\Lambda_{qcd}^2}{f_a}$$

(Attractive) Self-Coupling:

$$\lambda \sim -\frac{\Lambda_{qcd}^4}{f_a^4}$$

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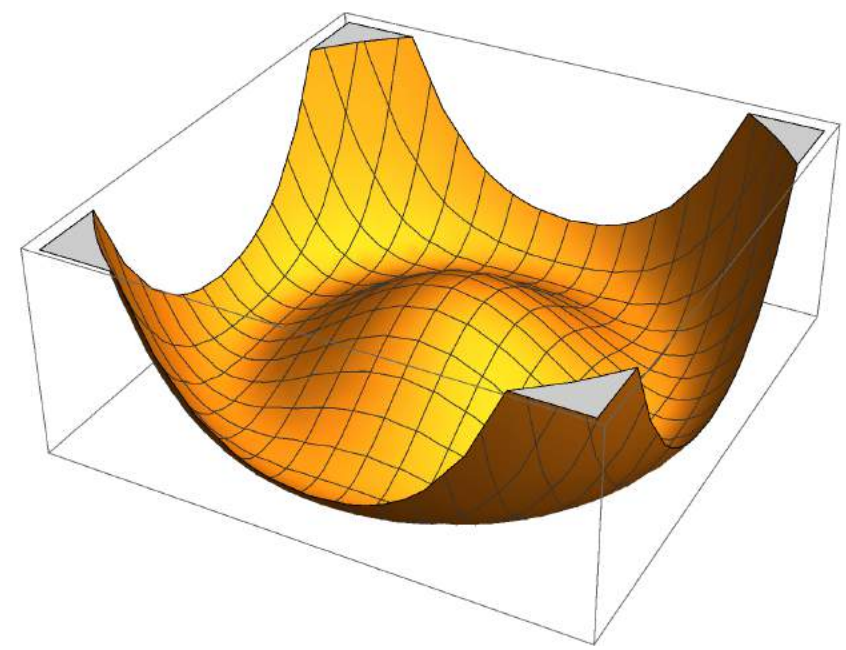

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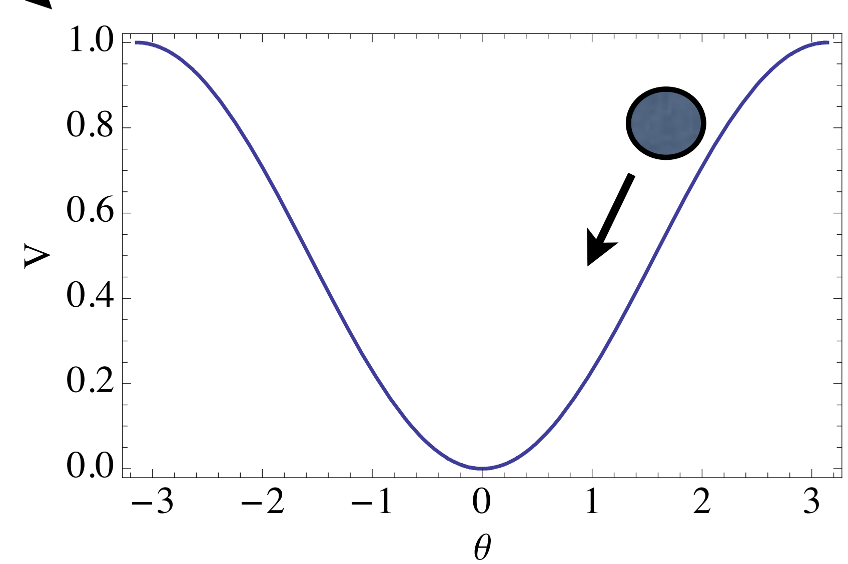
Abundance

$$\Omega_a \approx \langle\theta_i^2\rangle \left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{7/6} < 0.25$$



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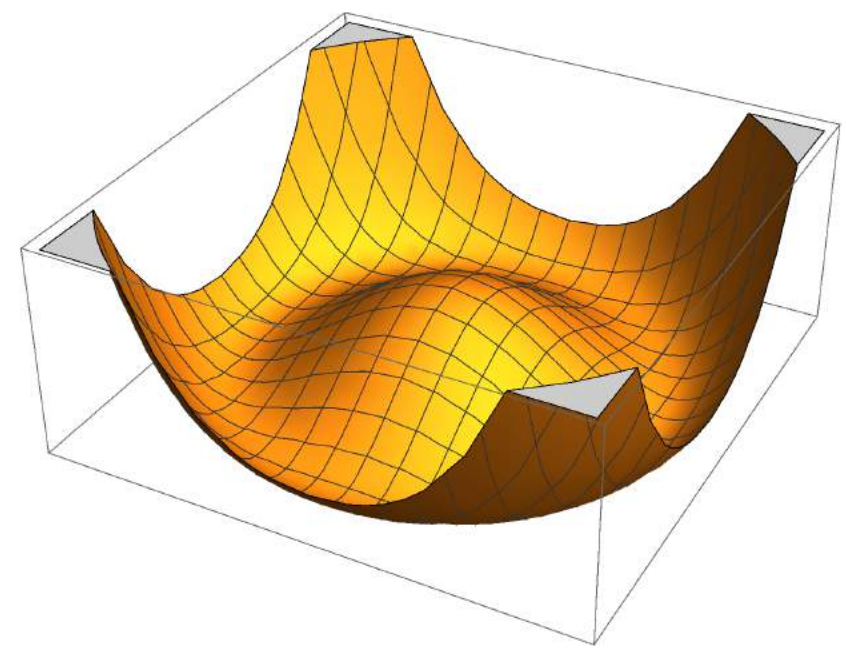

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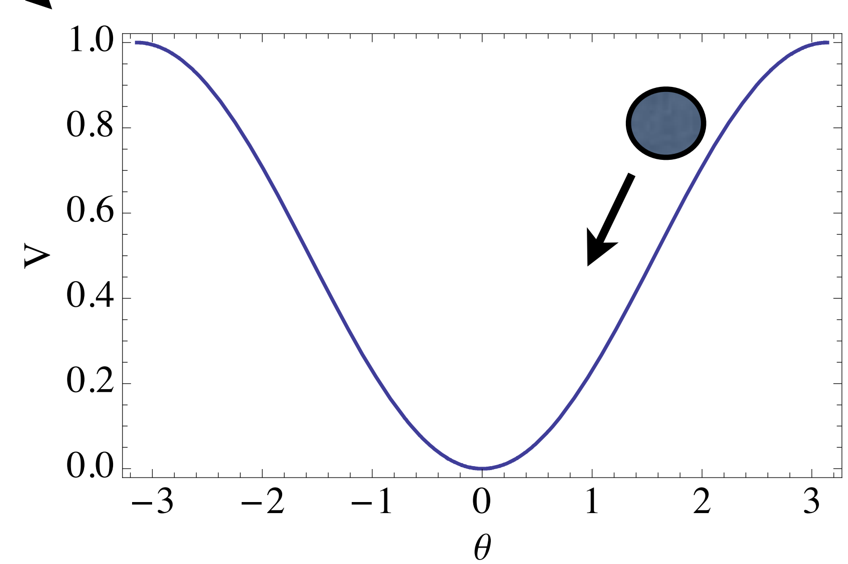
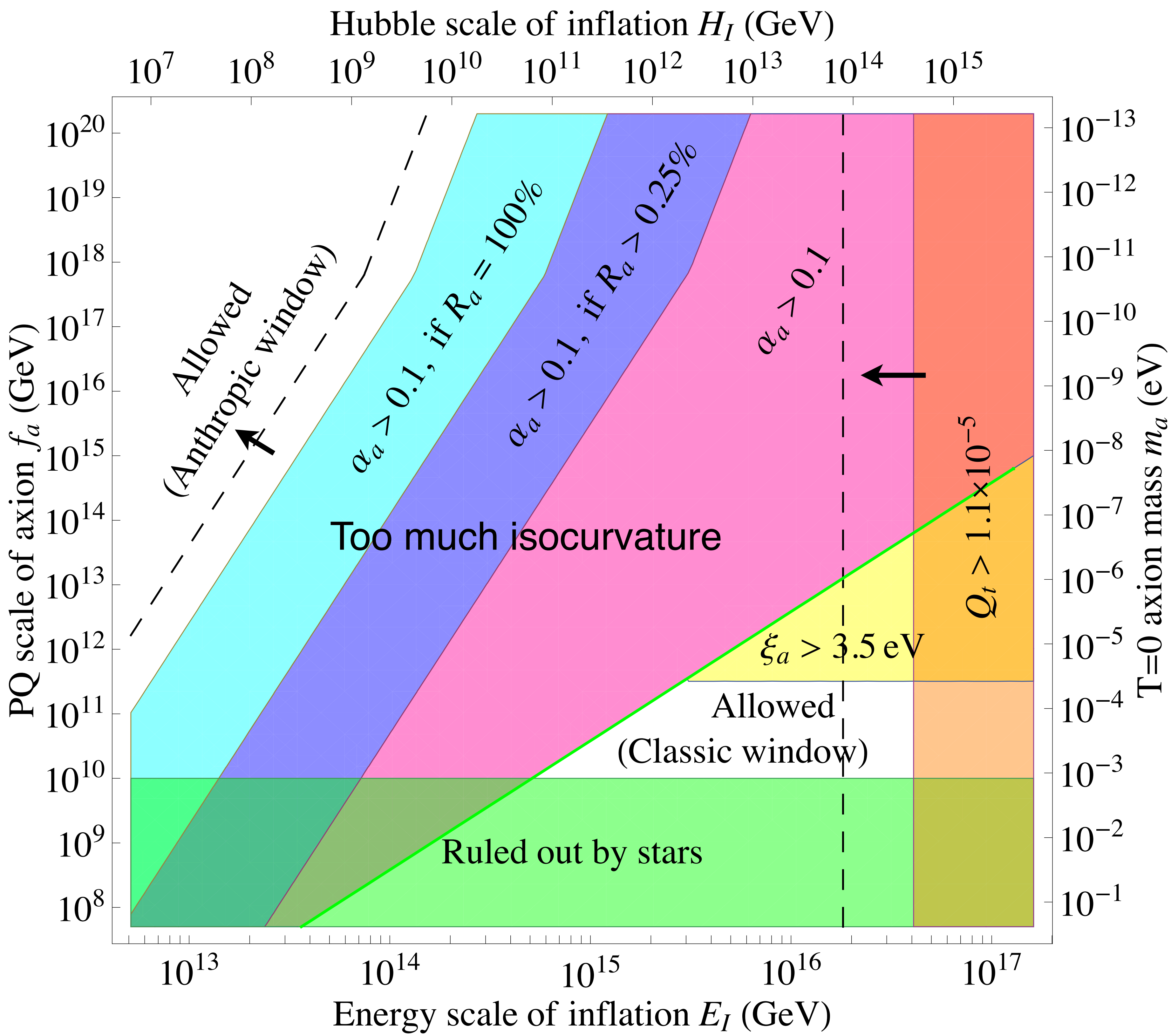


Related issues for string-axions, ALPs, light bosonic DM

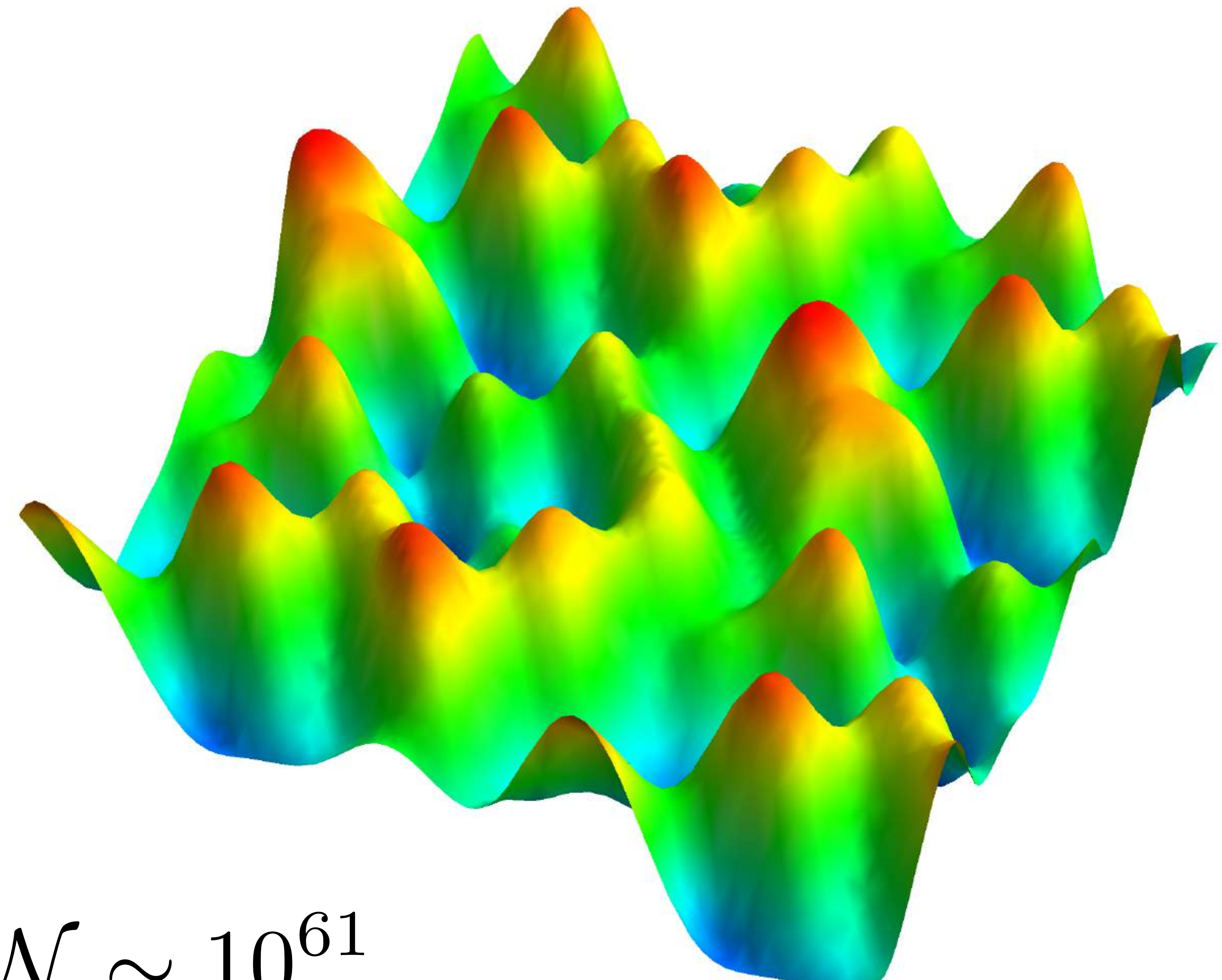
QCD-Axion Allowed Windows



Hertzberg, Tegmark, Wilczek 0807.1726

Focus on Classic Window

In Classic Window; Axion Initial Distribution



$\mathcal{N} \sim 10^{61}$

Consider Non-Relativistic Behavior

$$\phi(\mathbf{x},t) = \frac{1}{\sqrt{2\,m}}\left(e^{-imt}\psi(\mathbf{x},t) + e^{imt}\psi^*(\mathbf{x},t)\right)$$

(For rigorous treatment: Namjoo, Guth, Kaiser 2017,
Eby, Mukaida, Takimoto, Wijewardhana, Yamada 2018)

Consider Non-Relativistic Behavior

$$\phi(\mathbf{x},t) = \frac{1}{\sqrt{2\,m}}\left(e^{-imt}\psi(\mathbf{x},t) + e^{imt}\psi^*(\mathbf{x},t)\right)$$

Hamiltonian

$$\hat{H} = \hat{H}_{\mathrm{kin}} + \hat{H}_{\mathrm{int}} + \hat{H}_{\mathrm{grav}}$$

$$\hat{H}_{\mathrm{kin}} = \int d^3x \frac{1}{2m} \nabla\hat{\psi}^\dagger \cdot \nabla\hat{\psi}$$

$$\hat{H}_{\mathrm{int}} = \int d^3x \frac{\lambda}{16m^2} \hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}$$

$$\hat{H}_{\mathrm{grav}} = -\frac{Gm^2}{2}\int d^3x \int d^3x' \frac{\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}^\dagger(\mathbf{x}')\hat{\psi}(\mathbf{x})\hat{\psi}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

Number Density

$$\hat{n}(\mathbf{x}) = \hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x})$$

(For rigorous treatment: Namjoo, Guth, Kaiser 2017,
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Dynamical Time Scales

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

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Occupancy number change rate

$$\Gamma_k \equiv \frac{\dot{\mathcal{N}}_k}{\mathcal{N}_k} \sim \frac{G\,m^2\,n_{ave}}{k^2}$$

(See Sikivie, Yang 2009)

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Relaxation rate $$\Gamma_{rel} \sim \frac{G^2\,n^2\,m^5}{k^6} \qquad (\sim n\,\sigma\,v\,\mathcal{N})$$

(See Levkov, Panin, Tkachev 2018)

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(See Levkov, Panin, Tkachev 2018)

Equilibrium with high occupancy suggests BEC

Axion BEC Literature

- Sikivie, Yang (2009)
- Erken, Sikivie, Tam, Yang (2011)
- Chavanis (2012)
- Banik, Sikivie (2013)
- Davidson, Elmer (2013)
- Saikawa, Yamaguchi (2013)
- Noumi, Saikawa, Sato, Yamaguchi (2014)
- Vega, Sanchez (2014)
- Li, Rindler-Daller, Shapiro (2014)
- Berges, Haeckel (2014)
- Banik, Christopherson, Sikivie, Todarello (2015)
- Davidson (2015)
- Eby, Suranyi, Wijewardhana (2014, 2015, 2016, 2017, 2018, 2019, 2020) [w/Leembruggen, Ma, Street, Vaz]
- ……. many more……

Classical Description of BEC Phase Transition

Free Theory

$$F[\psi] = \int \frac{d^3k}{(2\pi)^3} \left[\frac{k^2}{2m} - \mu(T)\right] |\psi_k|^2$$

Number

$$\langle N \rangle = \frac{\int \mathcal{D}\psi \, N[\psi] \, \exp\left(-F[\psi]/T\right)}{\int \mathcal{D}\psi \, \exp\left(-F[\psi]/T\right)}$$

Density

$$n_{\rm th} = \int \frac{d^3k}{(2\pi)^3} \frac{T}{\frac{k^2}{2m} - \mu(T)}$$

Critical Temperature

$$T_{\rm crit} = \frac{\pi^2 \, n_{\rm tot}}{m \, k_{\rm UV}}$$

Guth, Hertzberg, Prescod-Weinstein 1412.5930

Classical vs Quantum with Interactions

What About Interactions?

Fundamental claim of Sikivie, Todarello, 1607.00949

On time scales $t > \tau = 1/\Gamma$ the classical description breaks down, requiring the full quantum theory, which is the only way to see thermalization

Toy Model

Second Quantized Language

$$\hat{H} = \sum_i \omega_i \, \hat{a}_i^\dagger \hat{a}_i + \frac{1}{4} \sum_{ijkl} \Lambda_{ij}^{kl} \, \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l,$$

Consider just 5 oscillators for simplicity

Initial quantum state $|\{N_i\}\rangle = |12, 25, 4, 12, 1\rangle$

Sikivie, Todarello, 1607.00949

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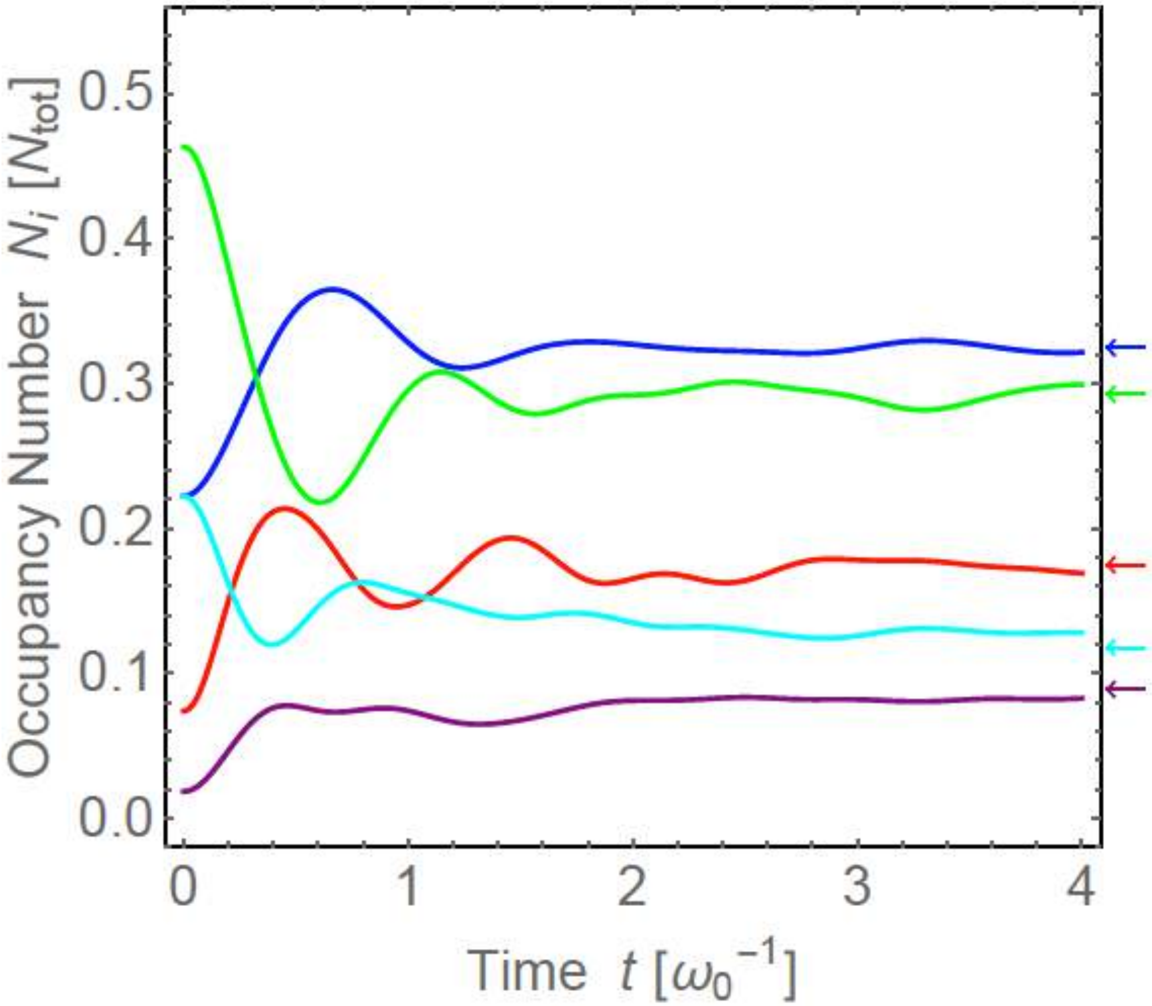
Initial classical state $\quad a_i = \sqrt{N_i}$

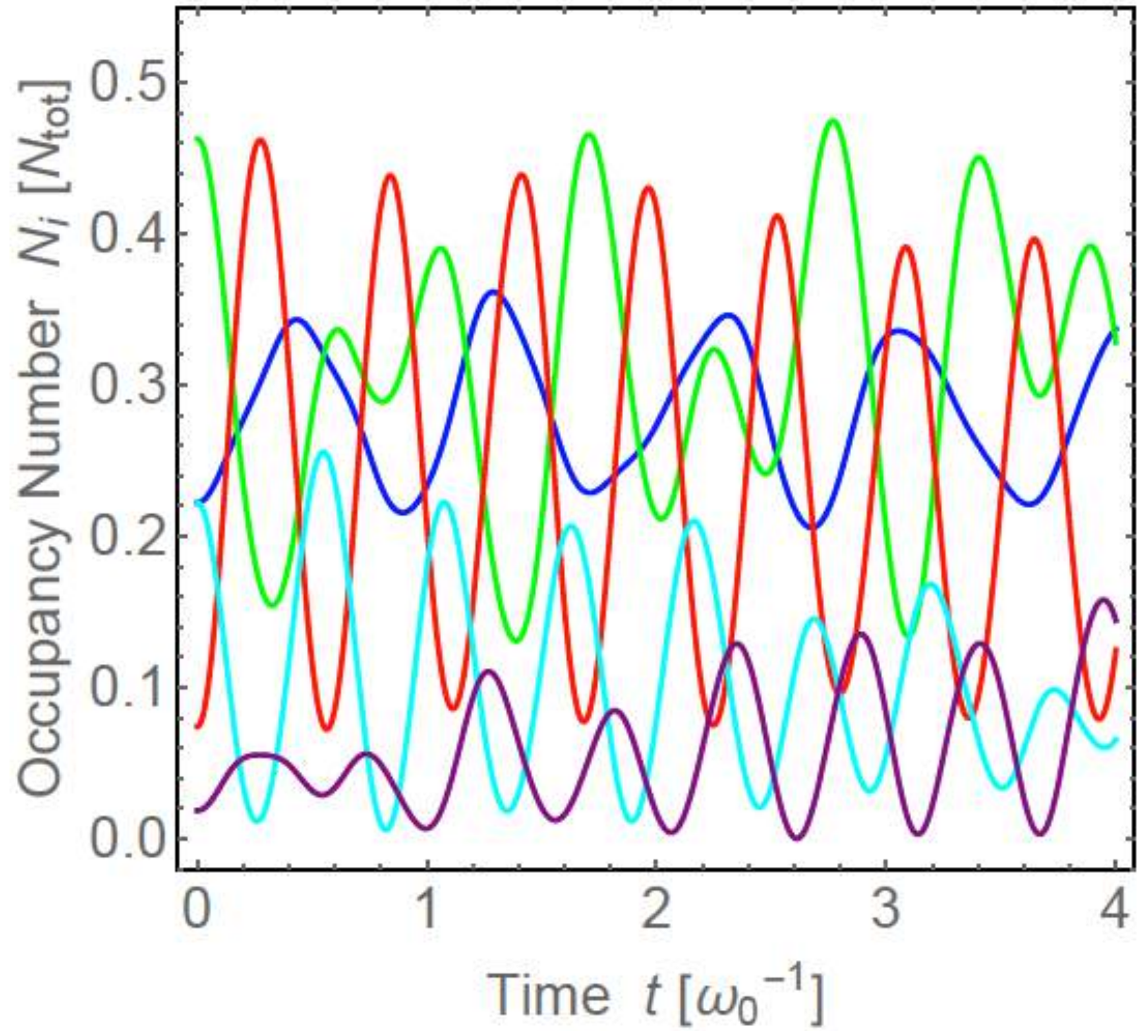
Sikivie, Todarello, 1607.00949

Quantum vs Classical??

Quantum



Classical



Sikivie, Todarello, 1607.00949

Correct Classical Treatment

Initial classical state $a_i = \sqrt{N_i}\, e^{I\theta_i}, \quad \theta_i \in [0, 2\pi)$

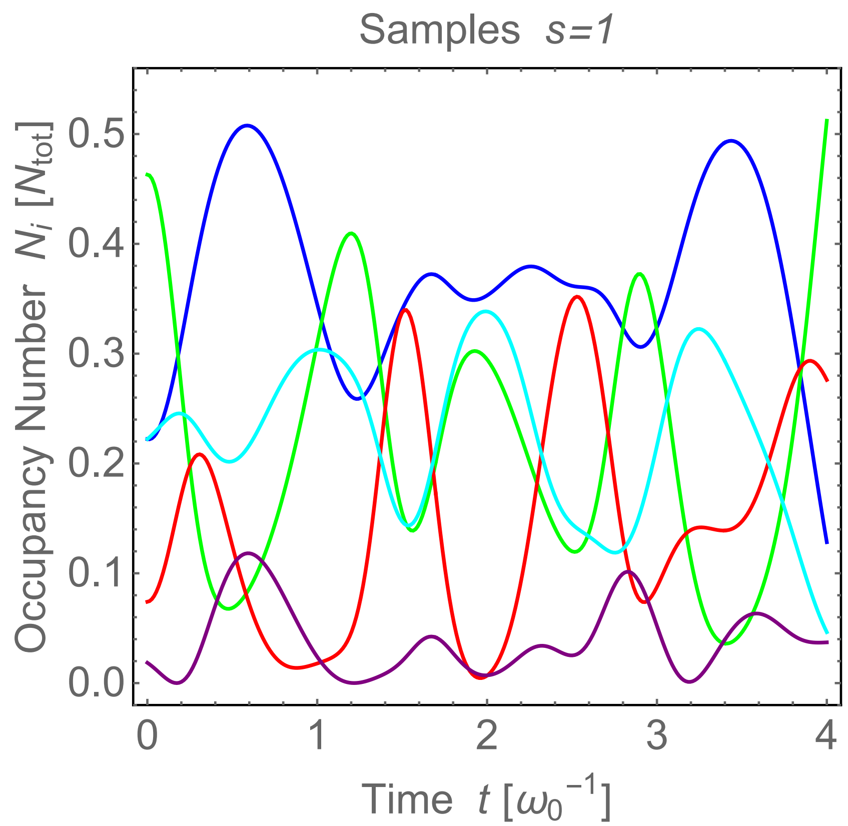
Ensemble average over random initial phases

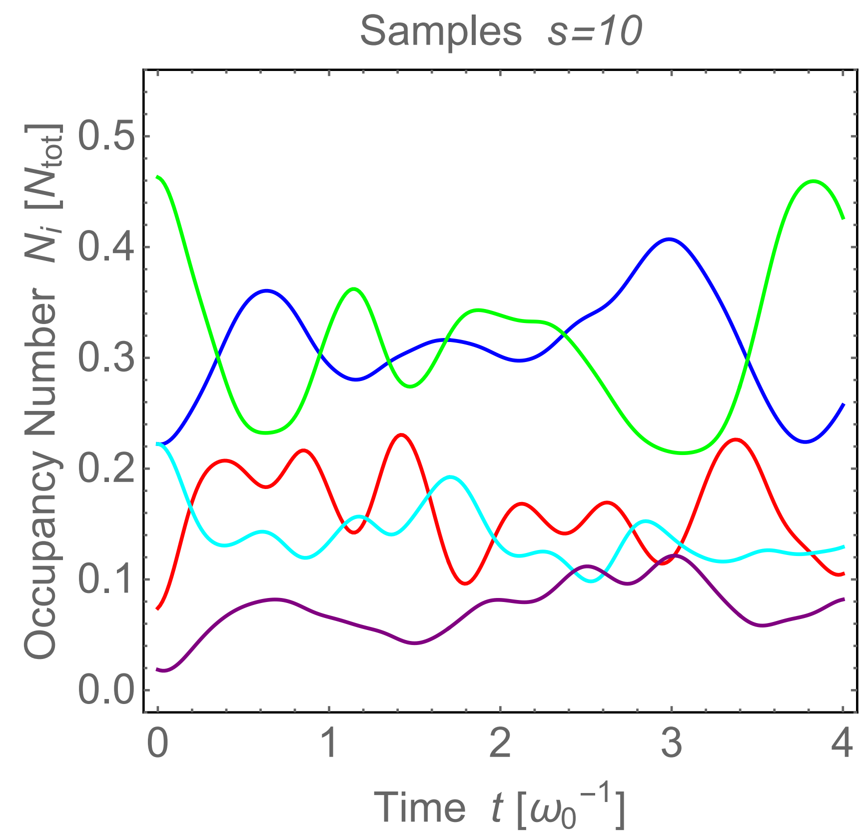
Meaningful comparison

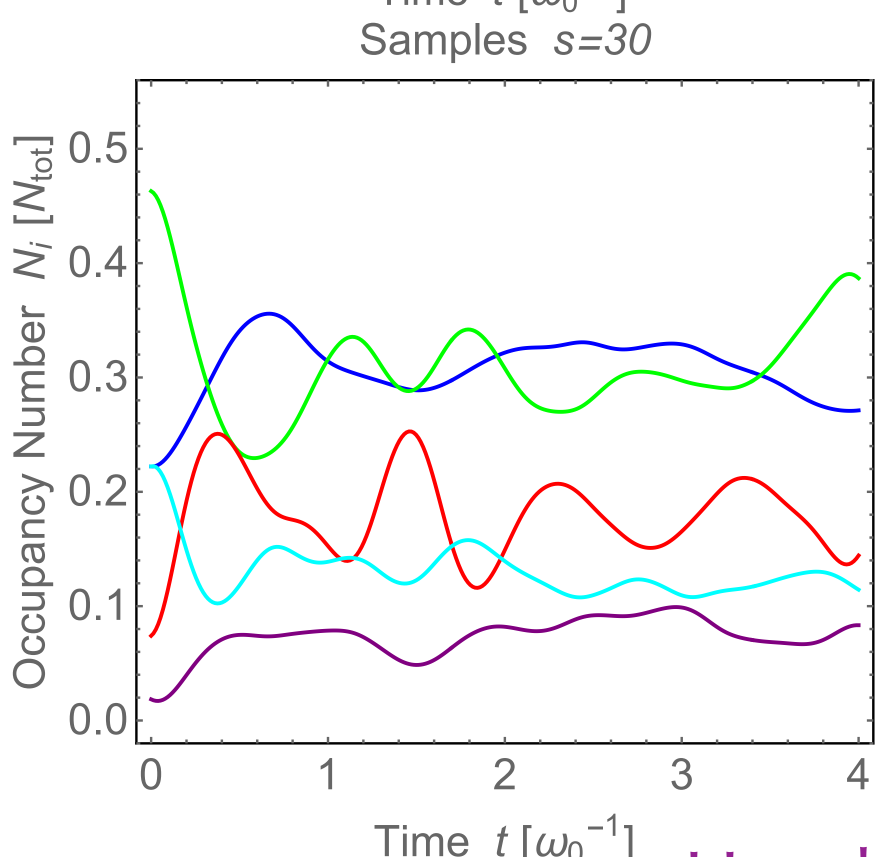
Connects to uncertainty in branch of wavefunction

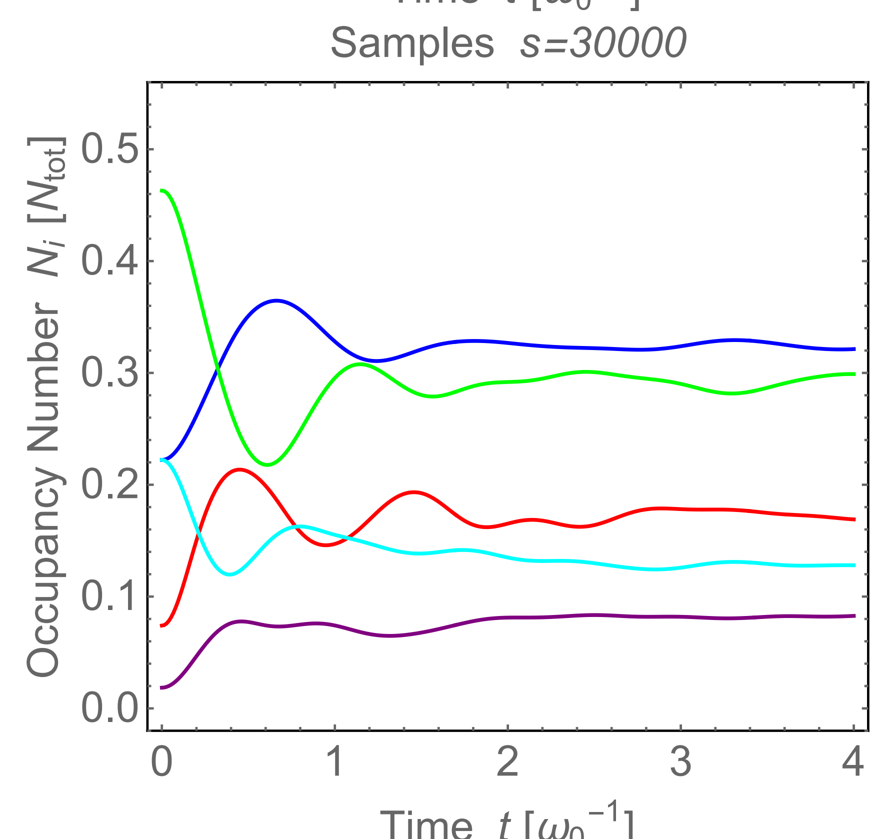
Hertzberg 1609.01342

Correct Classical Treatment









(matches quantum)

Hertzberg 1609.01342

Implication for Correlation Functions

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At high occupancy

$$\langle\{N_i\}|\hat{\psi}^\dagger(\mathbf{x},t)\,\hat{\psi}(\mathbf{y},t)|\{N_i\}\rangle \approx \langle\psi^*(\mathbf{x},t)\,\psi(\mathbf{y},t)\rangle_{ens}$$

Ergodic theorem

$$\langle\psi^*(\mathbf{x},t)\,\psi(\mathbf{y},t)\rangle_{ens} = \frac{1}{V}\int_V d^3z\,\psi_\mu^*(\mathbf{x}+\mathbf{z},t)\,\psi_\mu(\mathbf{y}+\mathbf{z},t)$$

Hertzberg 1609.01342

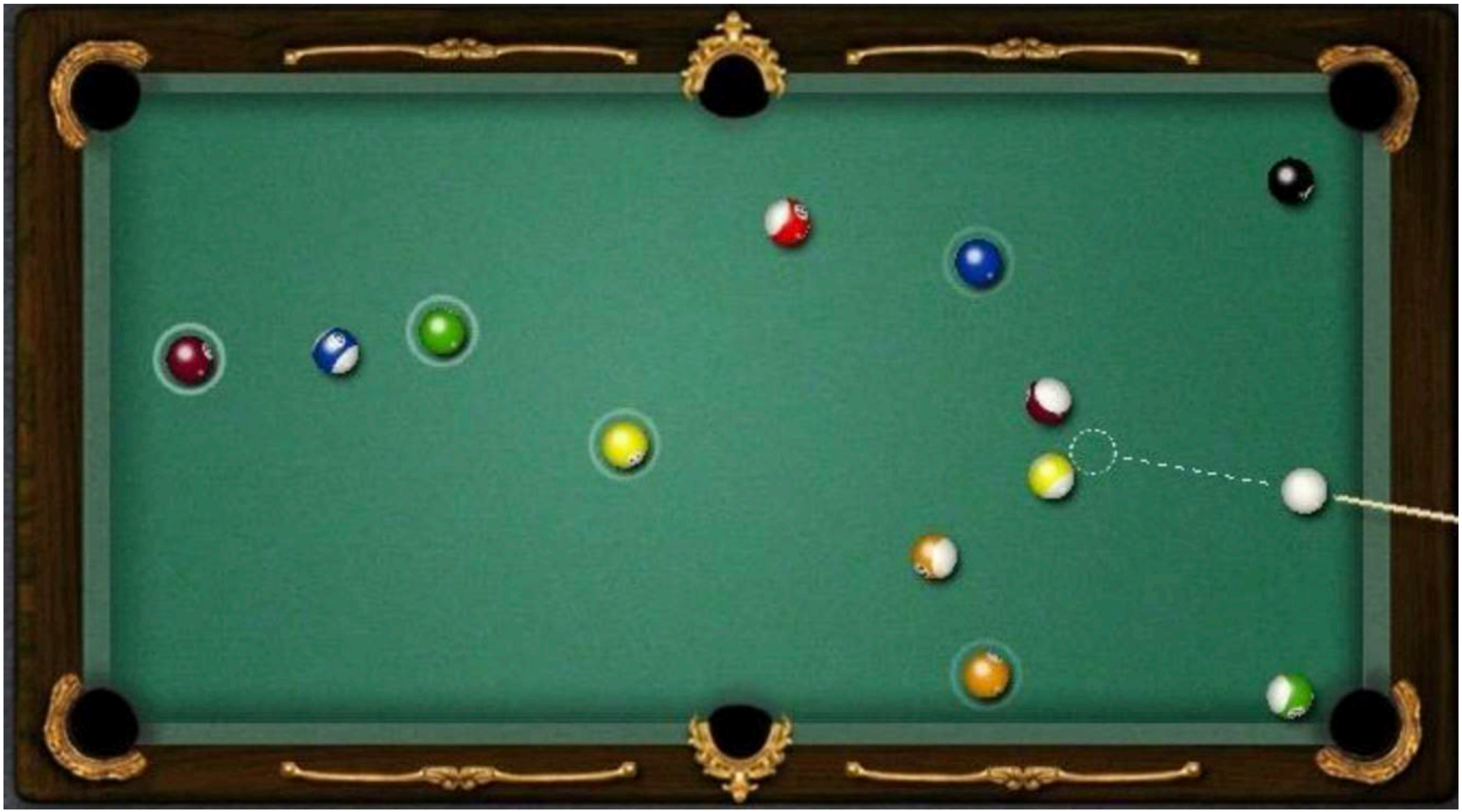
Implication for Axion Simulations

Correlation functions of quantum and classical micro-states agree at high occupancy, despite the macroscopic spreading of wave-functions in these chaotic systems

Note: this is not some trivial consequence of Ehrenfest theorem…

Hertzberg 1609.01342

Akin to billiard balls which exhibit chaos



Albrecht, Phillips 2012

(Return to this at end of talk)

Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

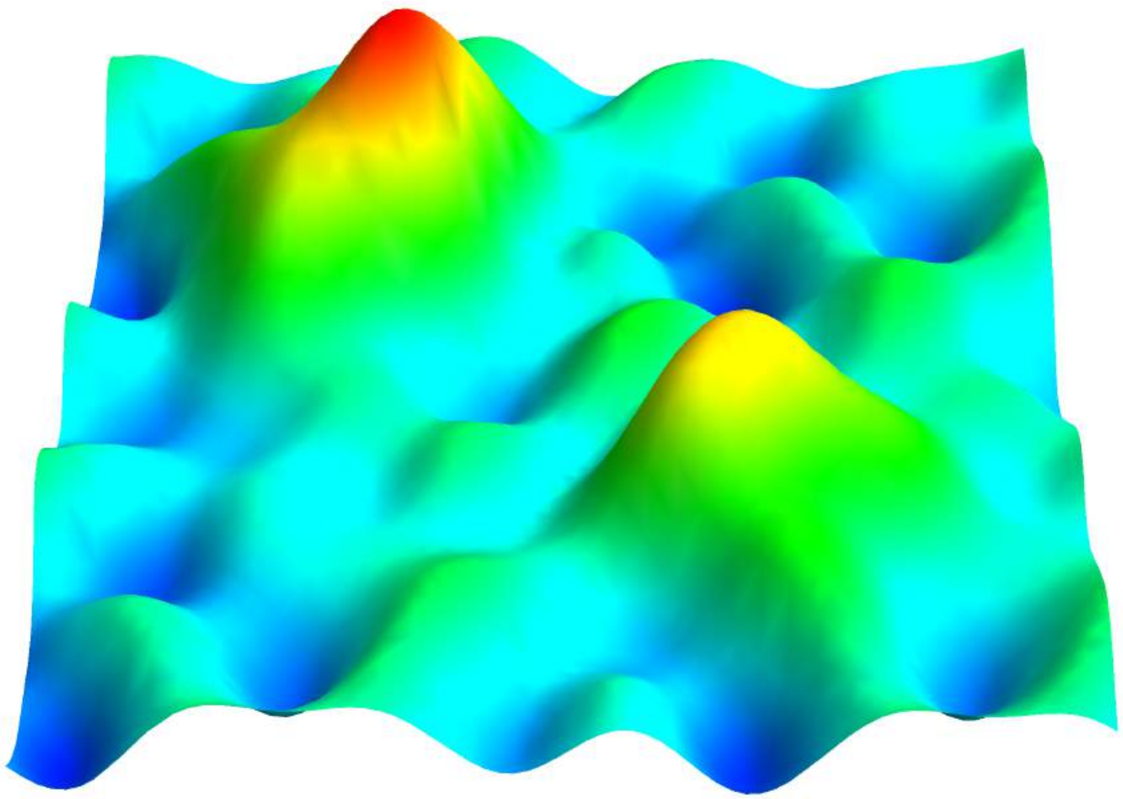
Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

Miniclusters
—> Axion stars

that may exist in galaxies



Hogan, Rees 1988; Kolb, Tkachev 1993, 1994, 1995; Barranco, Bernal 2001; Guth, Hertzberg, Prescod-Weinstein 2014; Fairbairn, Marsh, Quevillon, Rozier 2017; Kitajima, Soda, Urakawa 2018; Eby, Leembruggen, Ma, Street, Suranyi, Vaz, Wijewardhana 2014 — 2020

Axion Stars in Detail

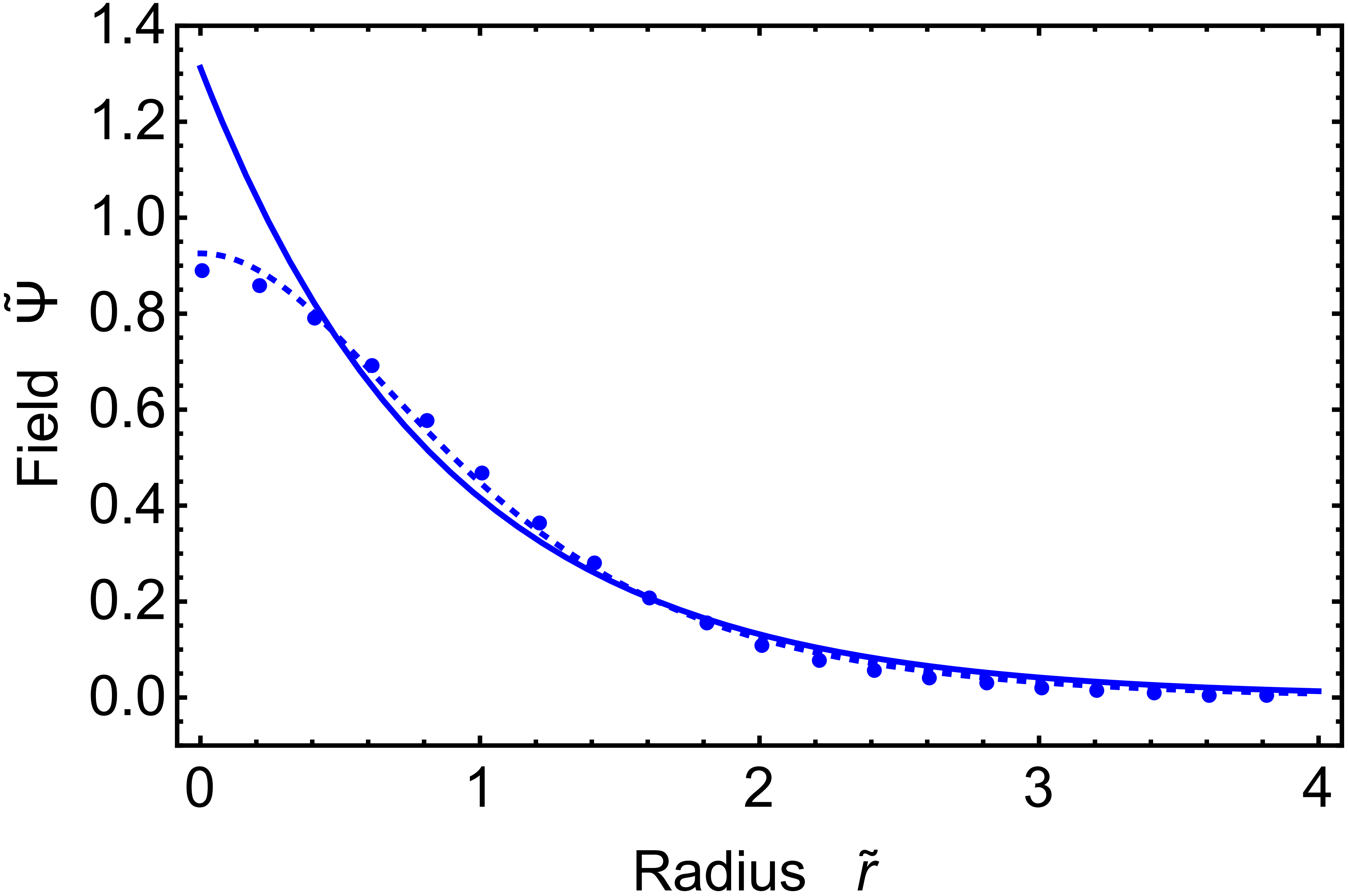
Return to Non-Relativistic Classical Field Theory

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

$$(\lambda < 0)$$

Star Solutions (BEC) at fixed N



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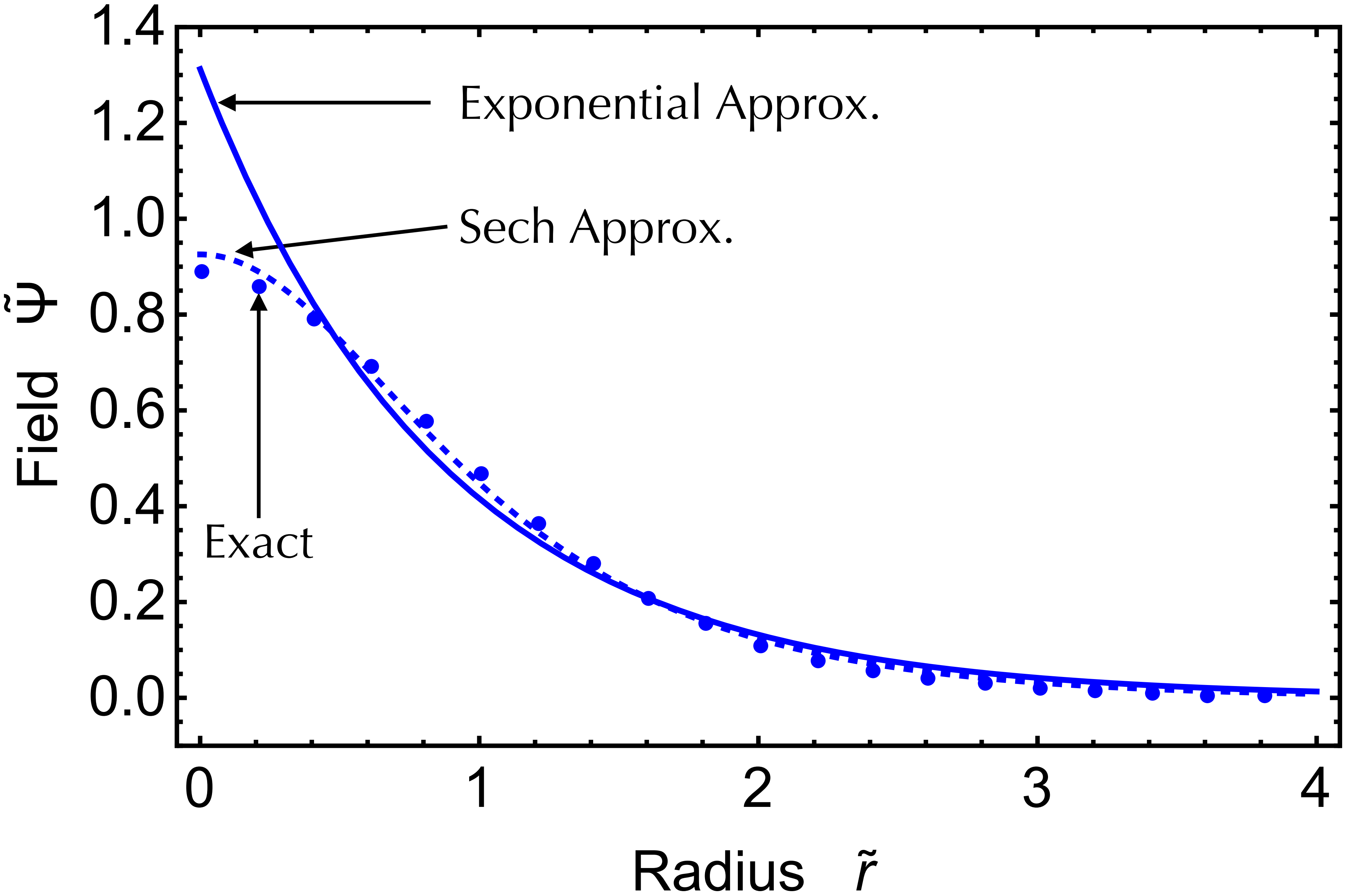
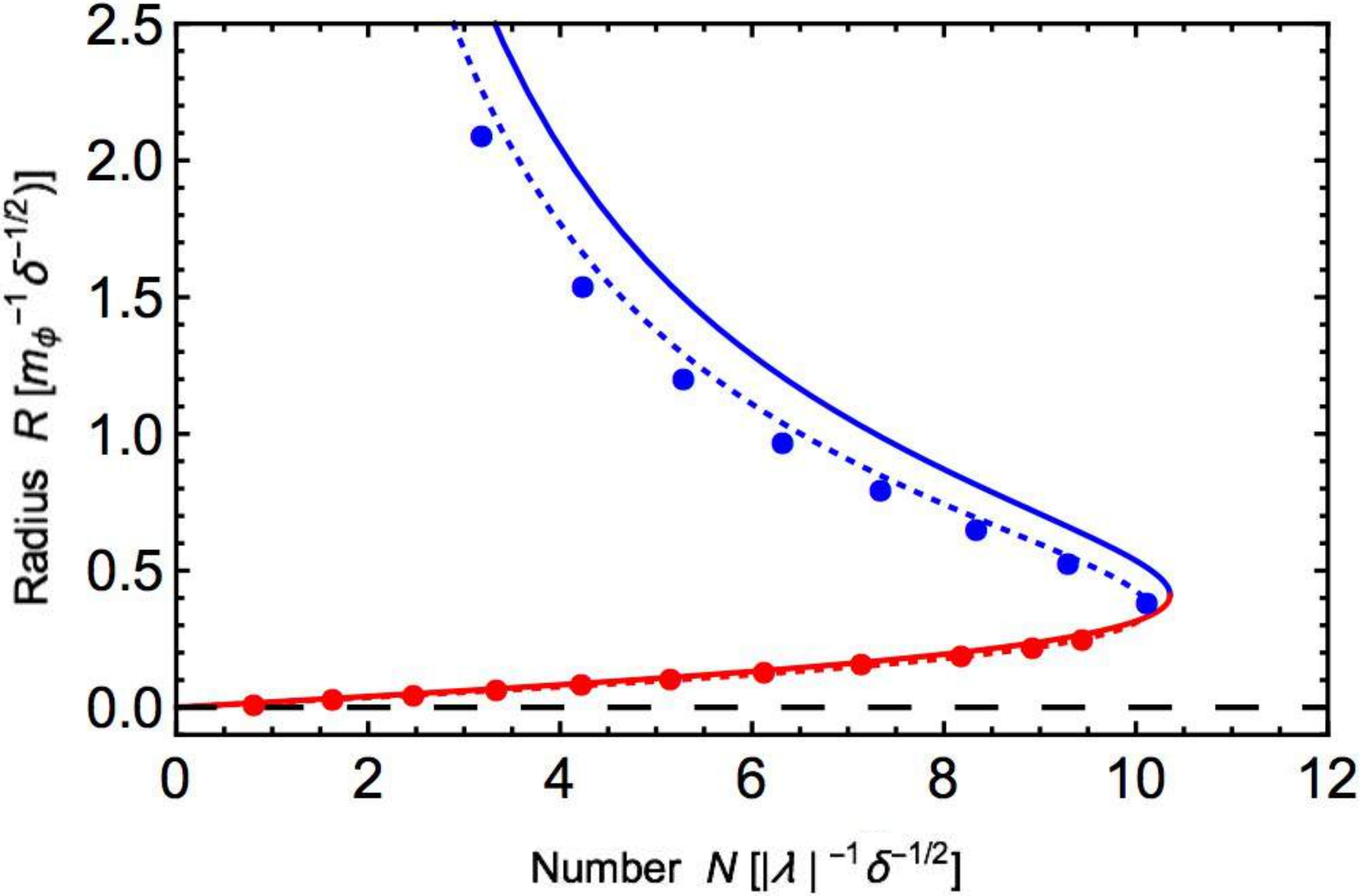


Schiappacasse, Hertzberg 1710.04729

Two Branches of Solutions



See Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729

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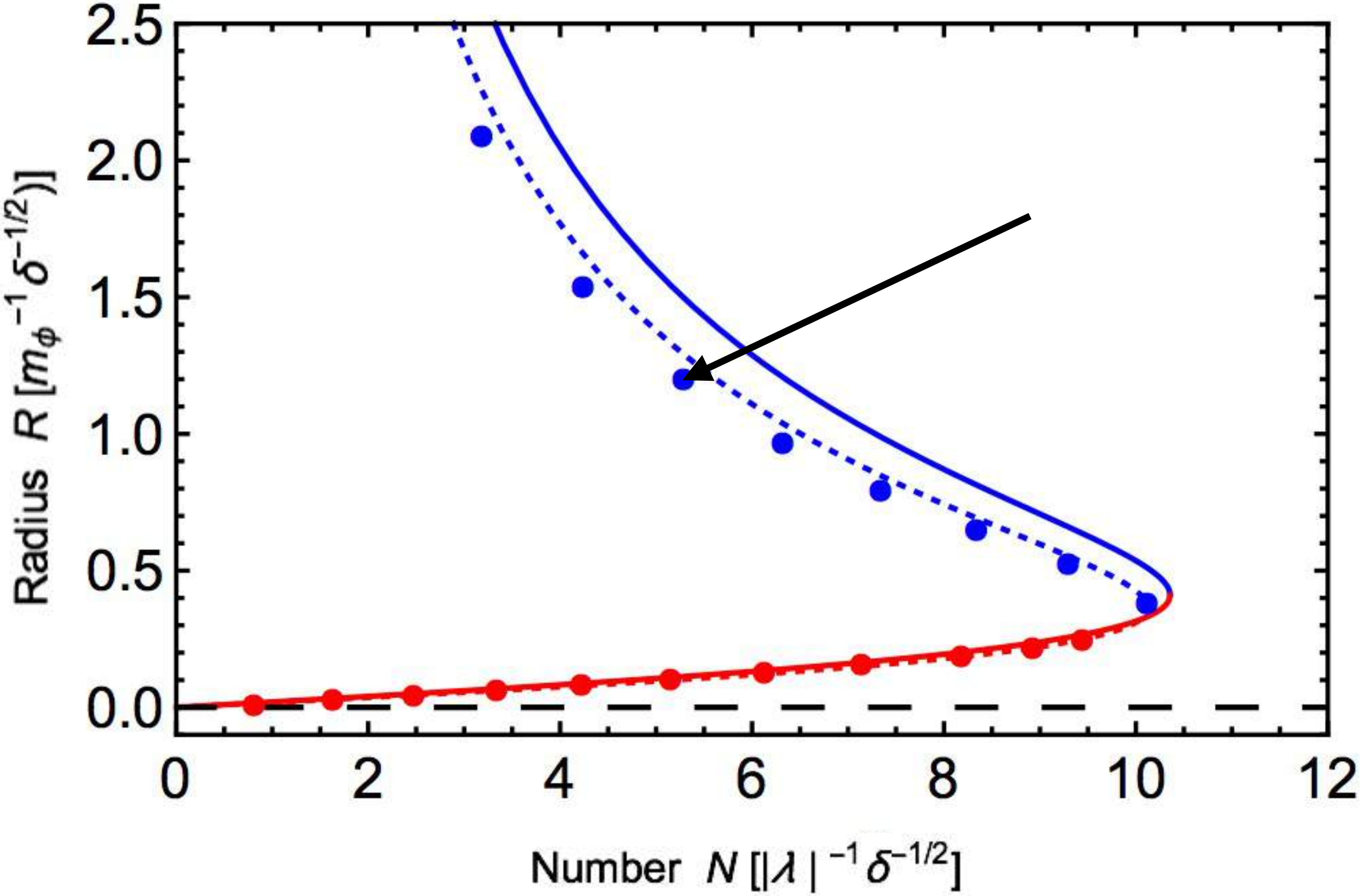

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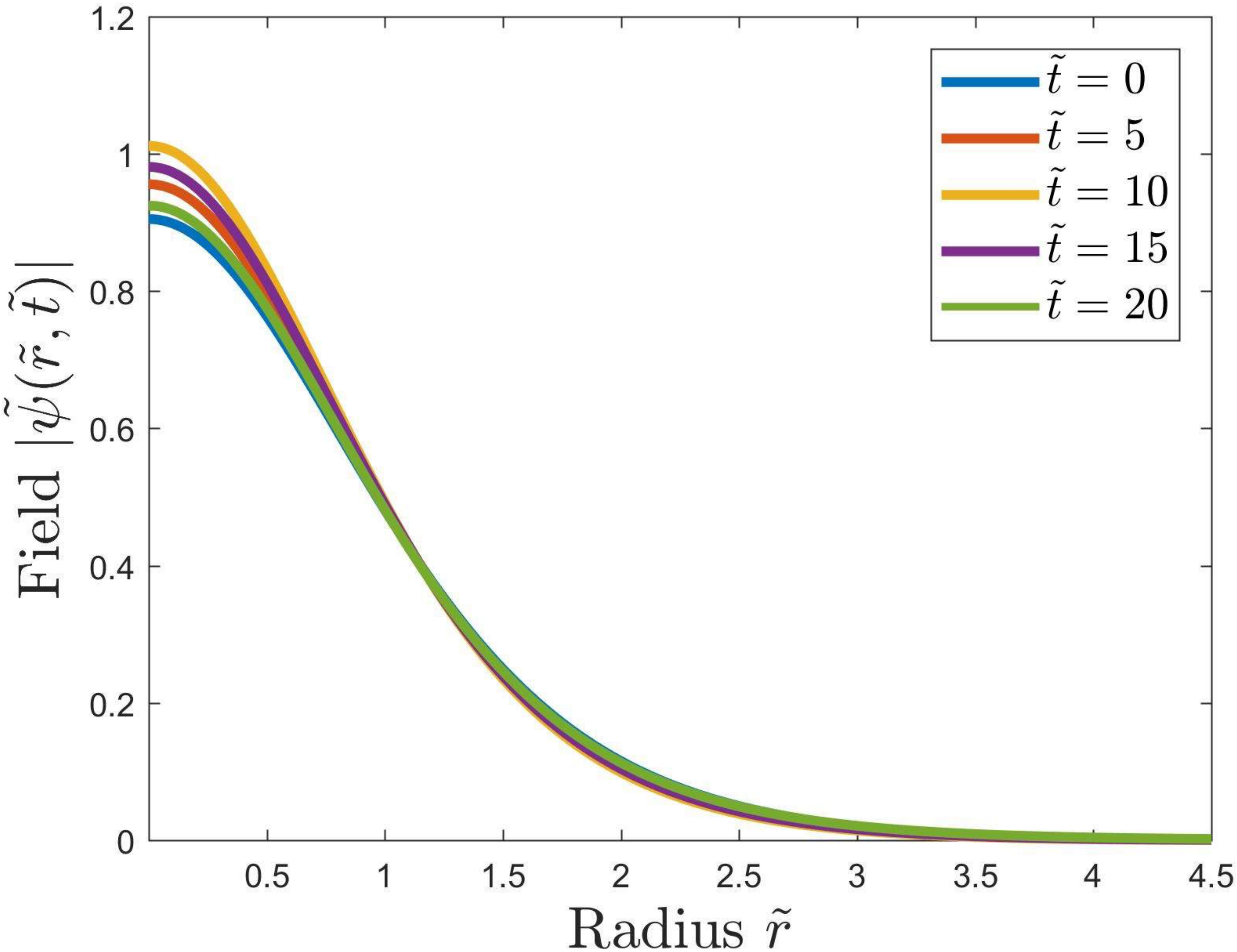
Schiappacasse, Hertzberg 1710.04729

Perturbing Upper Branch



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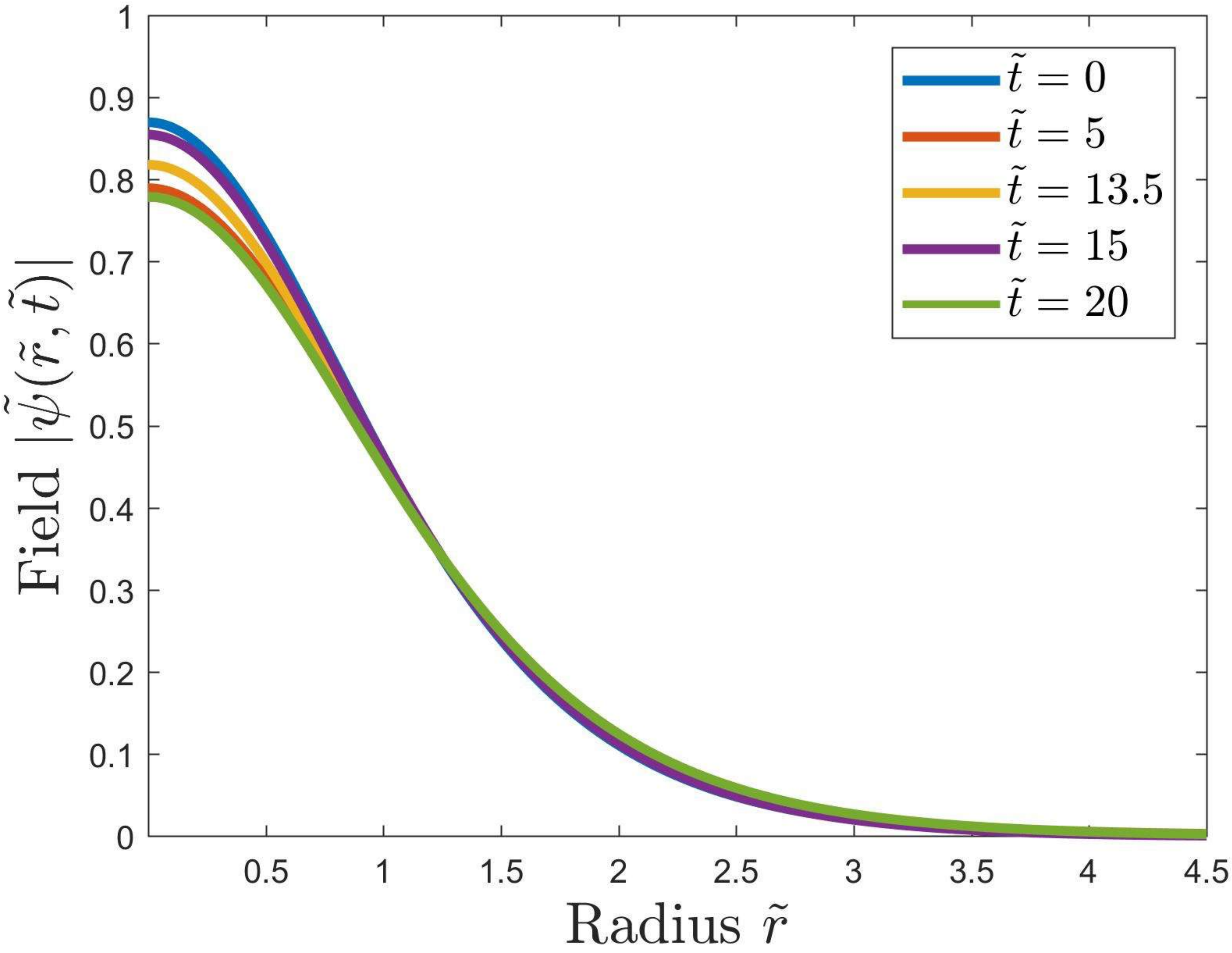

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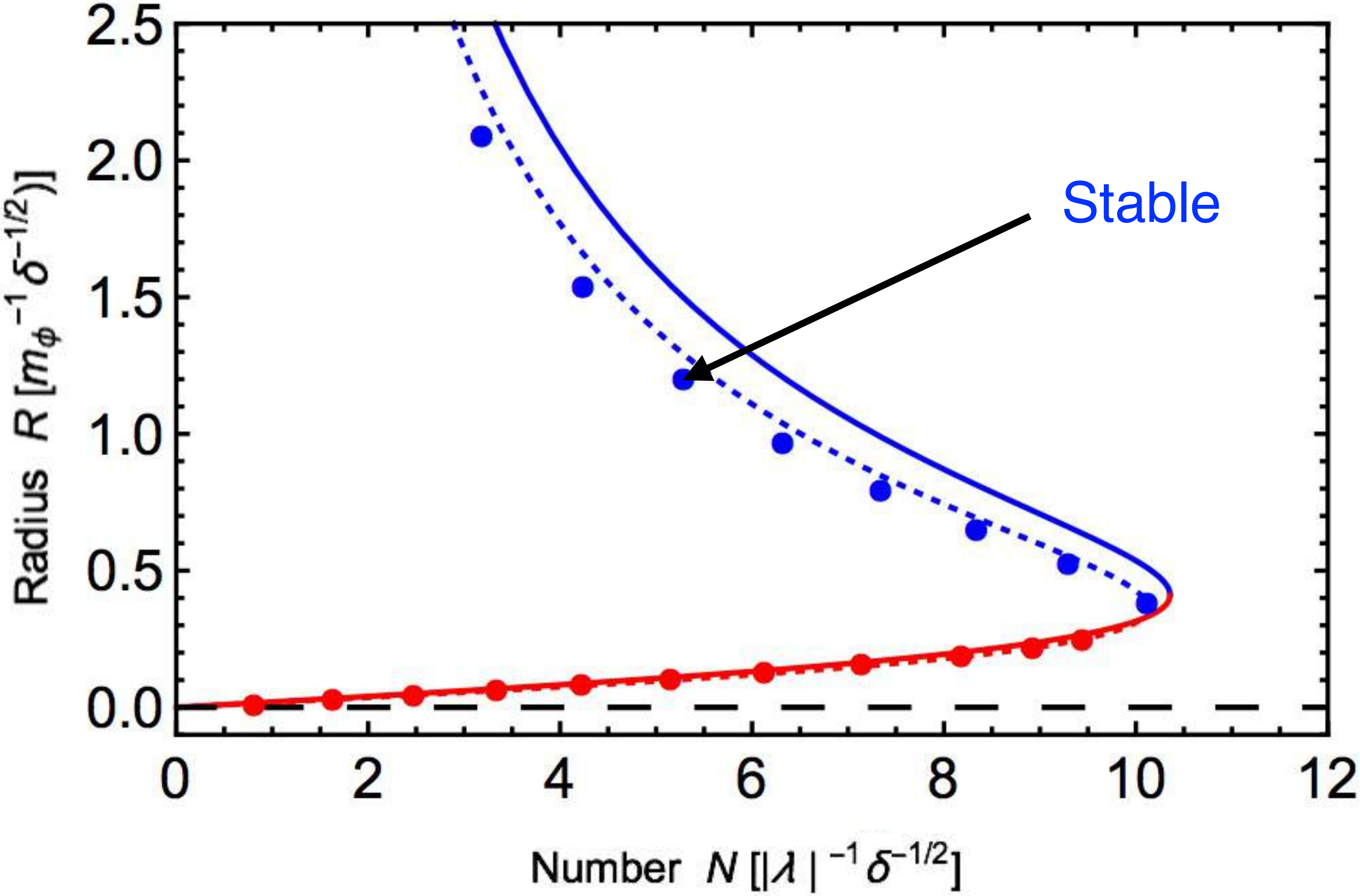
Two Branches of Solutions



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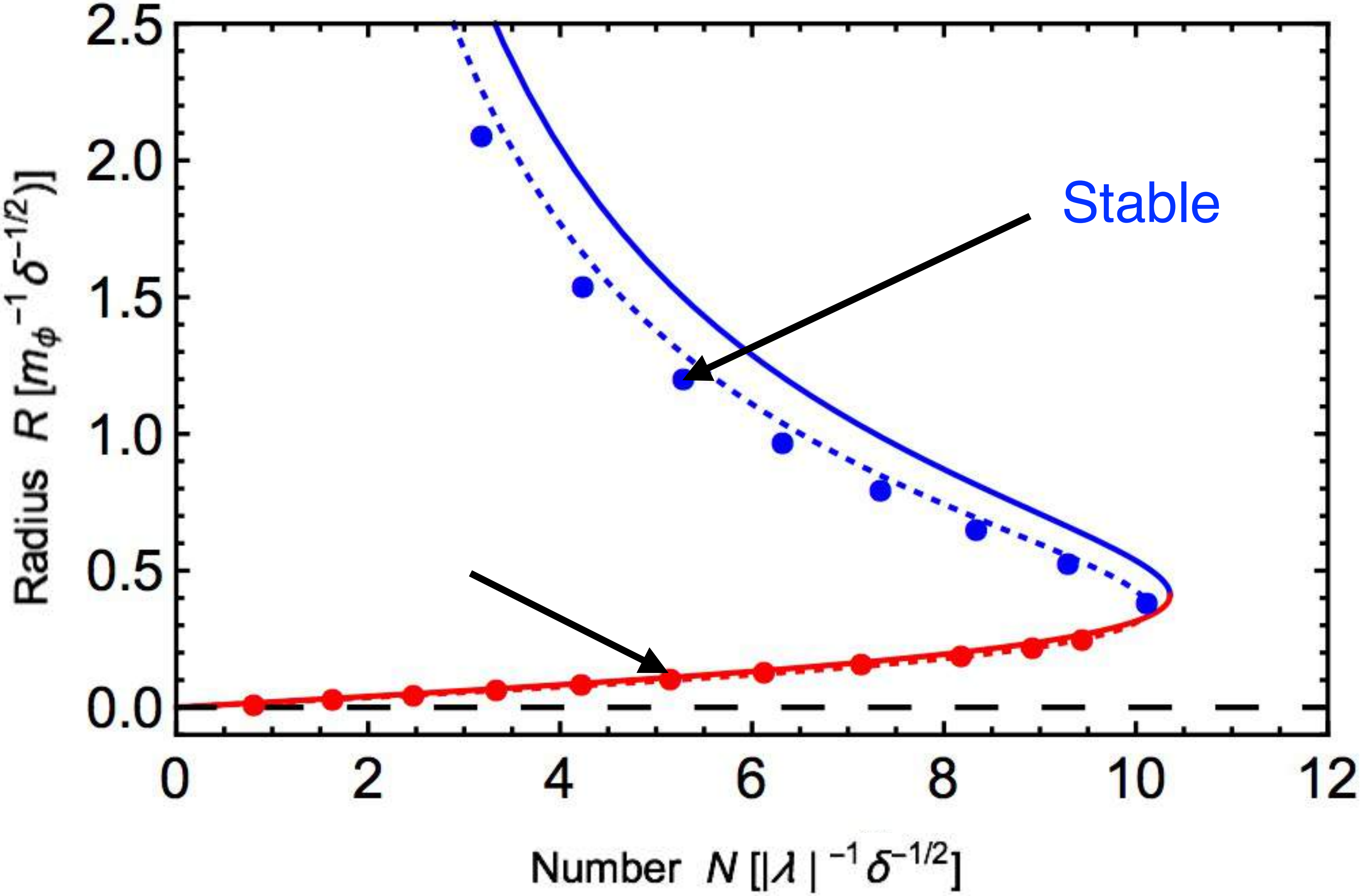
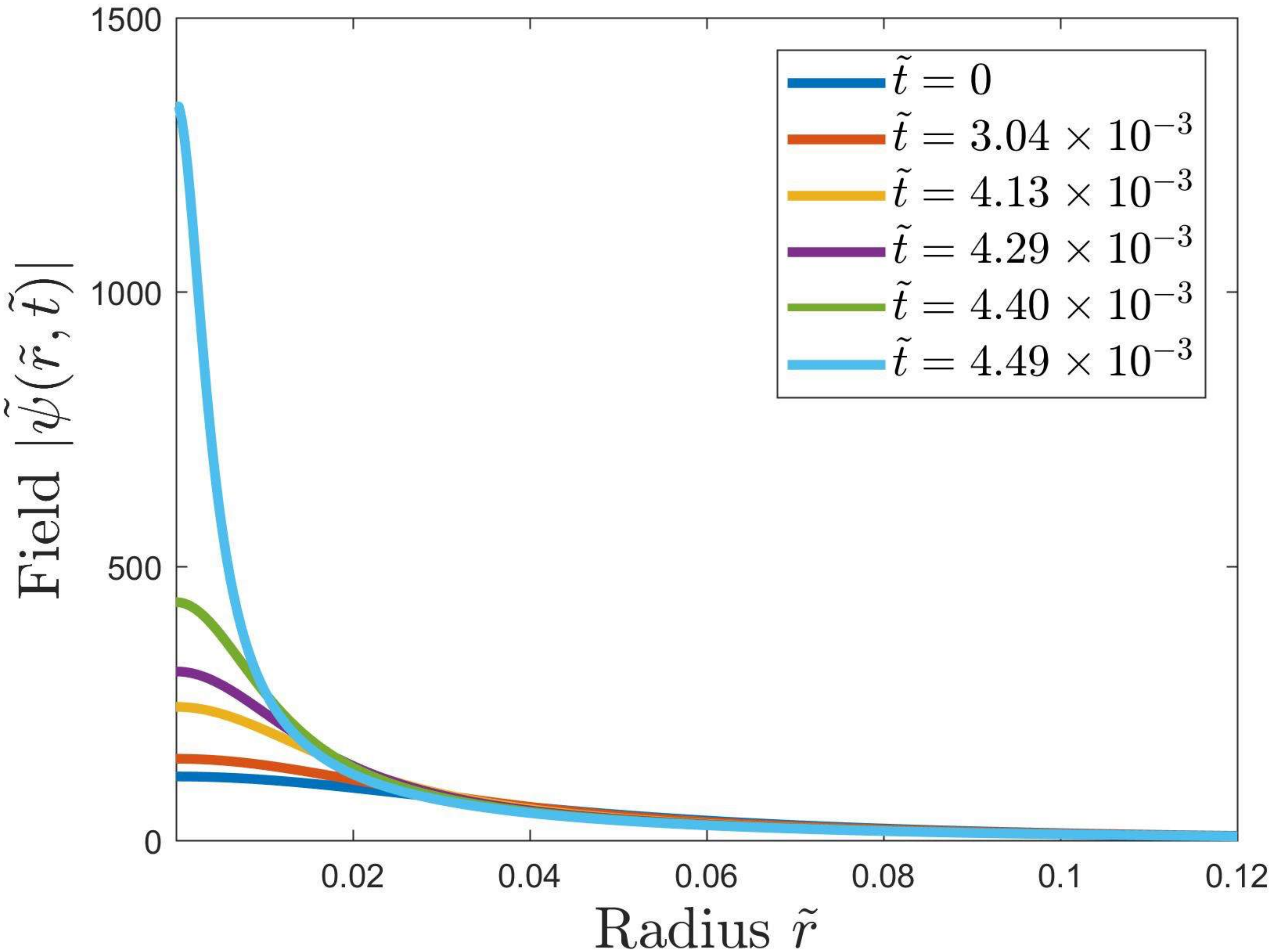


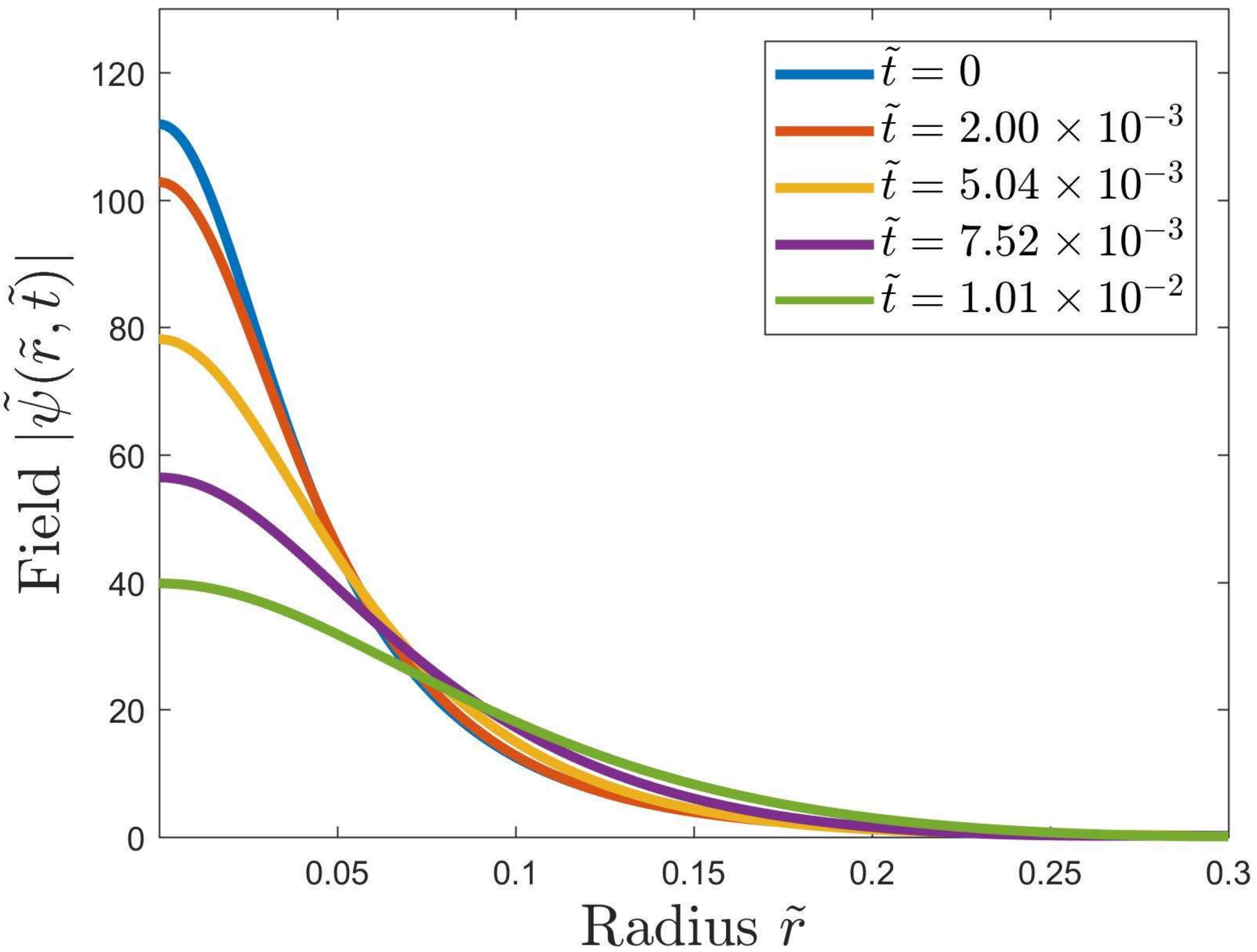
See Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729

Perturbing Lower Branch



Schiappacasse, Hertzberg 1710.04729

Perturbing Lower Branch



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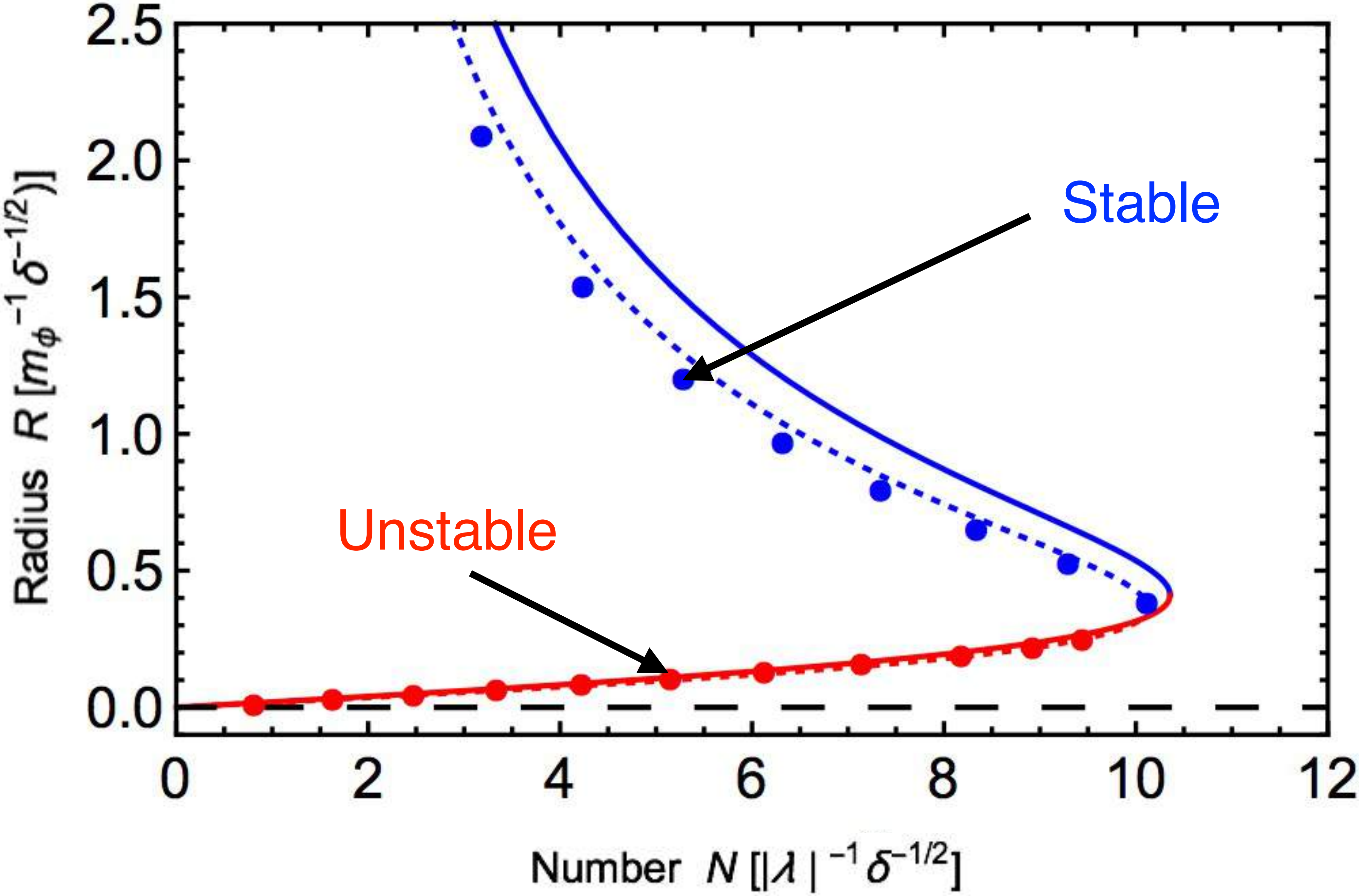
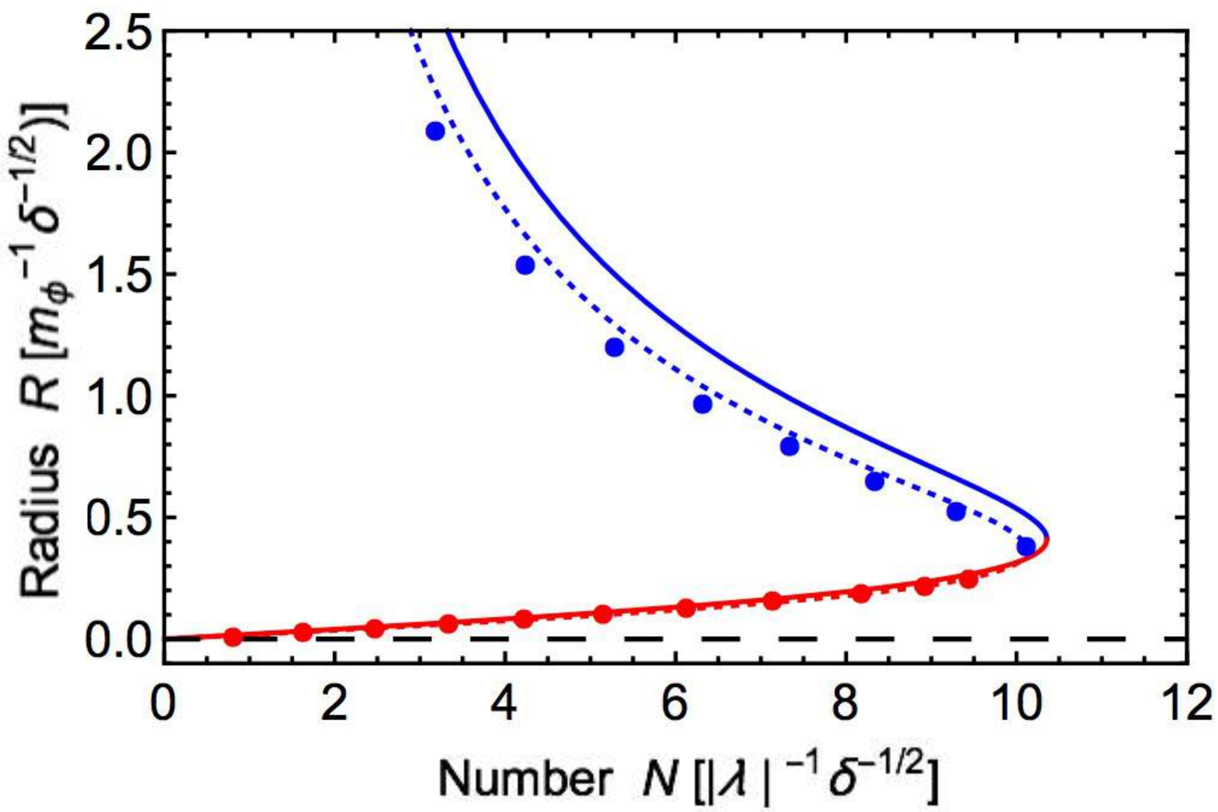
Two Branches of Solutions



See Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729

Two Branches of Solutions

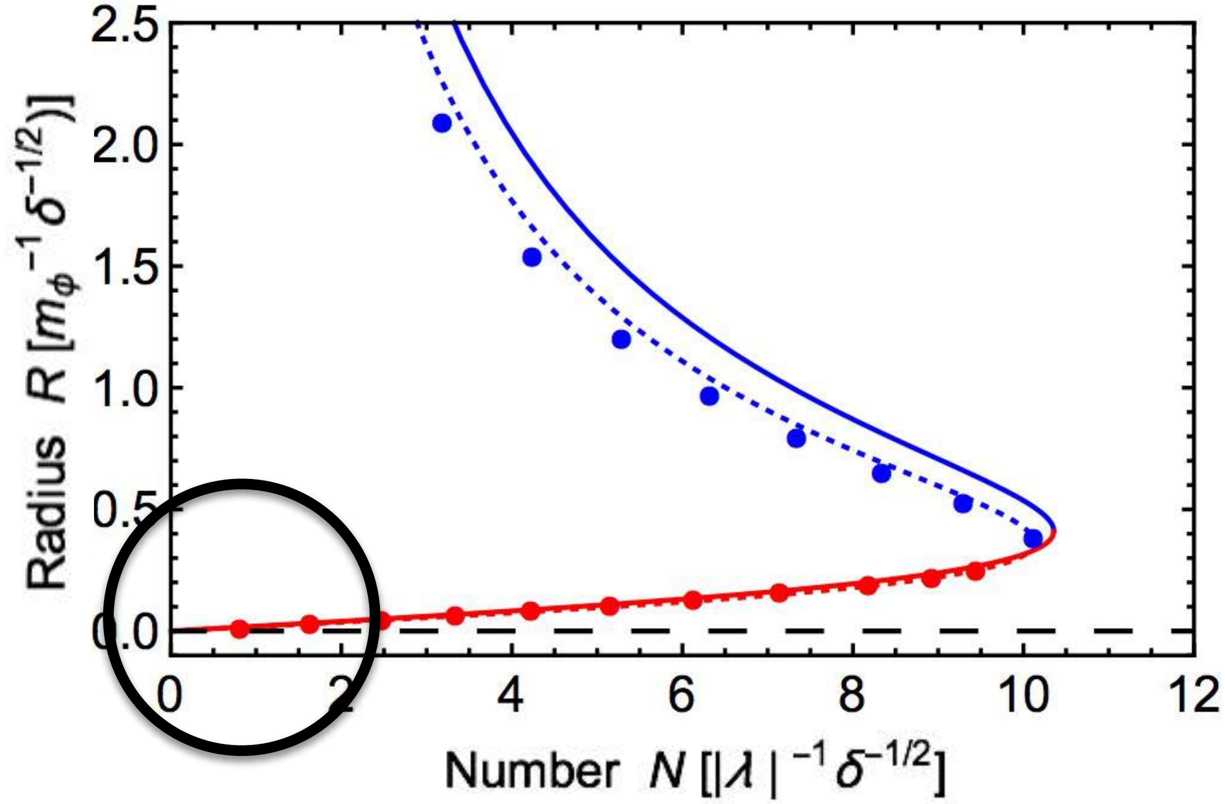
$$N_{max} = \frac{f_a}{m^2\sqrt{G}}\tilde{N}_{max} \sim 8\times 10^{59}\,(\tilde{m}^{-2}\tilde{f}_a)\,,$$
$$M_{max} = N_{max}\,m \qquad \sim 1.4\times 10^{19}\,\mathrm{kg}\,(\tilde{m}^{-1}\tilde{f}_a)\,,$$
$$R_{90,min} = \frac{a\,(\tilde{R}_{90}/\tilde{R})}{bN_{max}Gm^3} \sim 130\,\mathrm{km}\,(\tilde{m}^{-1}\tilde{f}_a^{-1})\,,$$



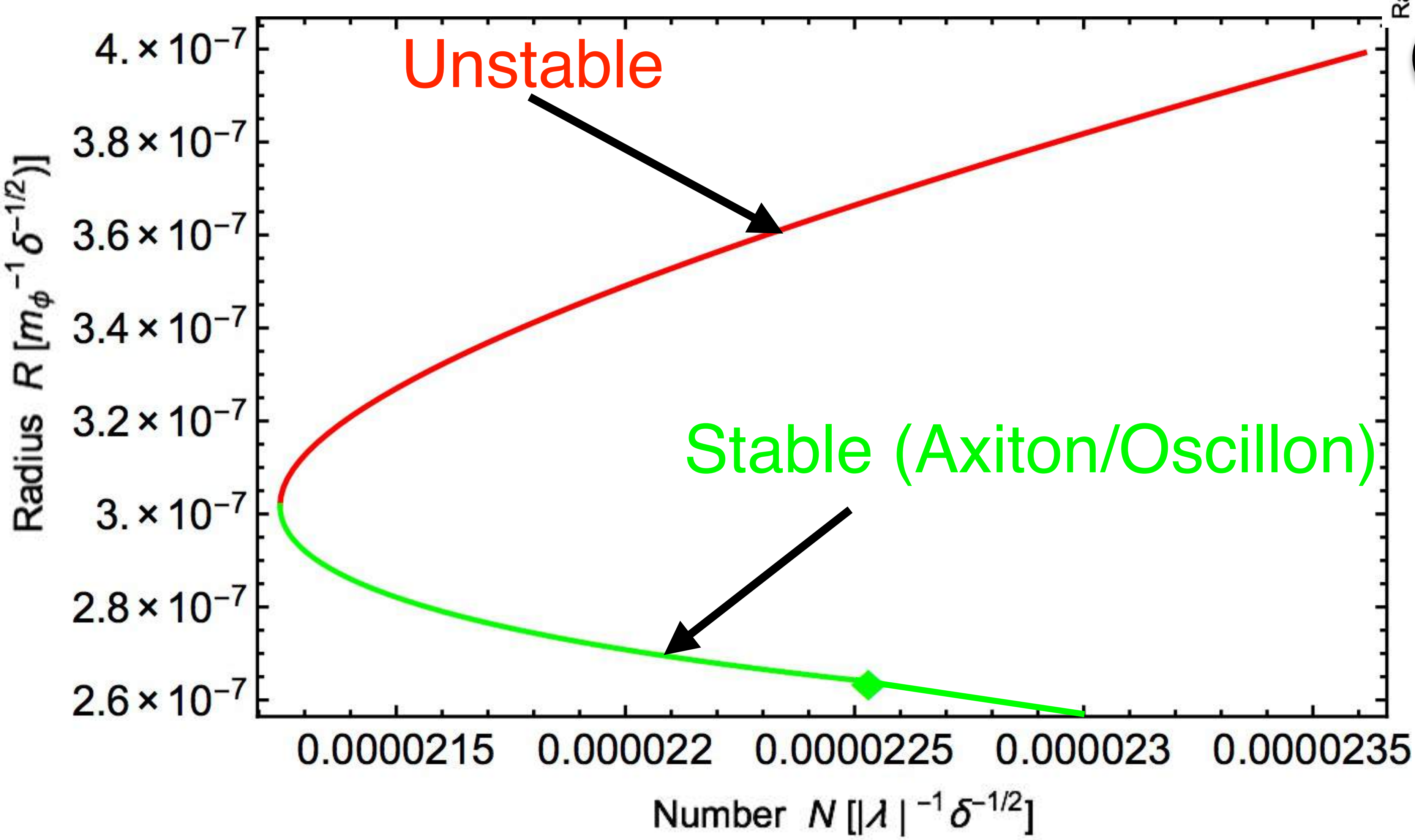
where $\tilde{f}_a \equiv f_a/(6\times 10^{11}\,\mathrm{GeV})$ and $\tilde{m} \equiv m/(10^{-5}\,\mathrm{eV})$.

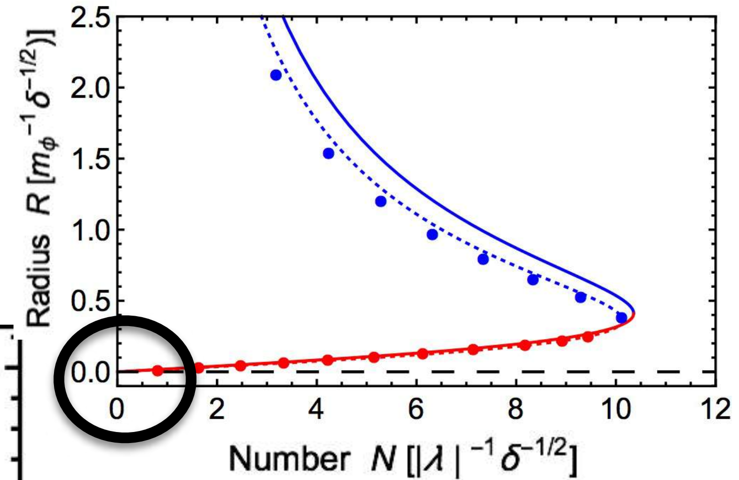
See Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729

Relativistic Branch (Axiton)



Relativistic Branch (Axiton/Oscillon)





Kolb, Tkachev astro-ph/9311037; Fodor, Fogacs, Horvath, Mezei 0903.0953; Hertzberg 1003.3459; Eby, Suranyi, Wijewardhana 1512.01709; Schiappacasse, Hertzberg 1710.04729; Visinelli, Baum, Redondo, Freese, Wilczek 1710.08910; …

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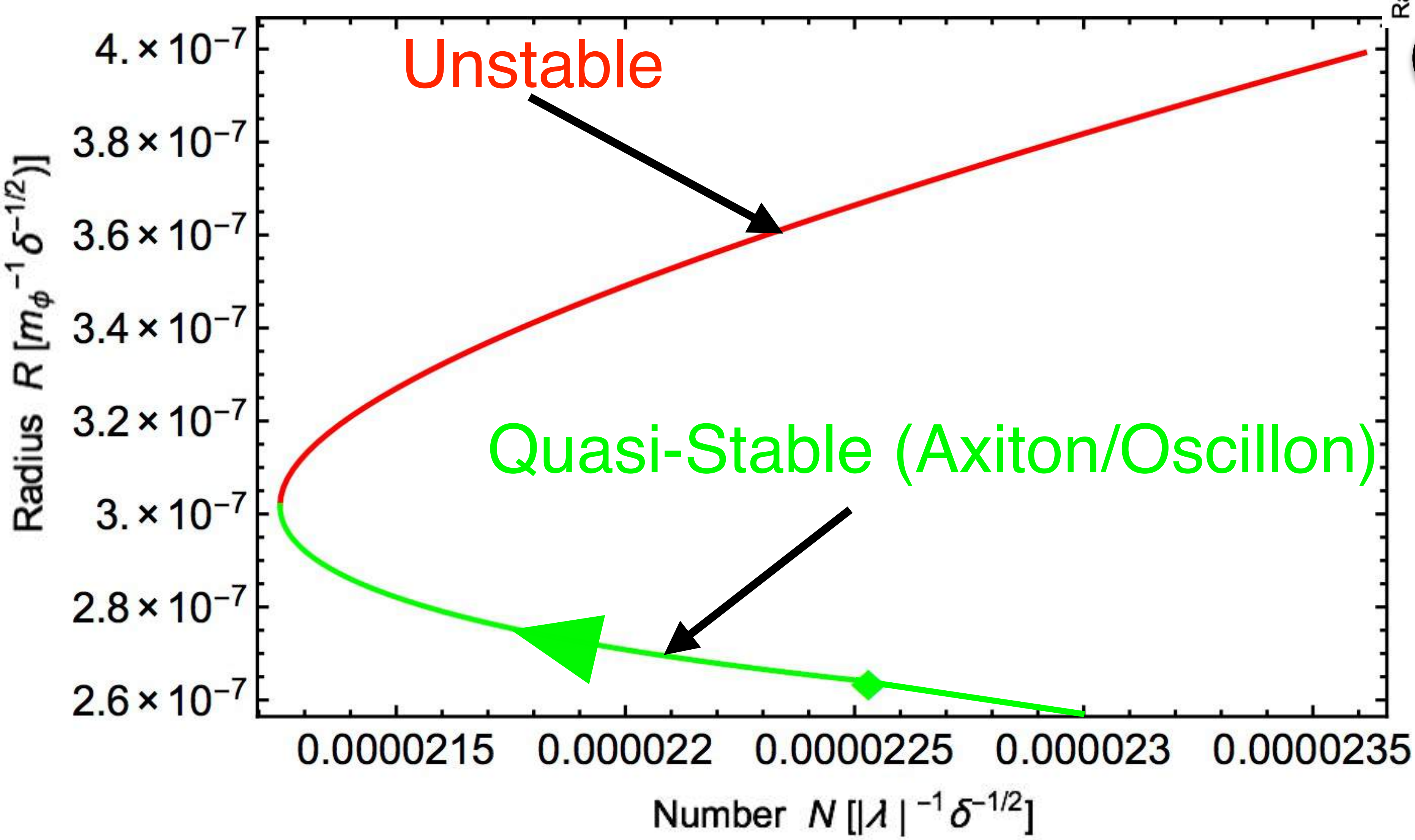
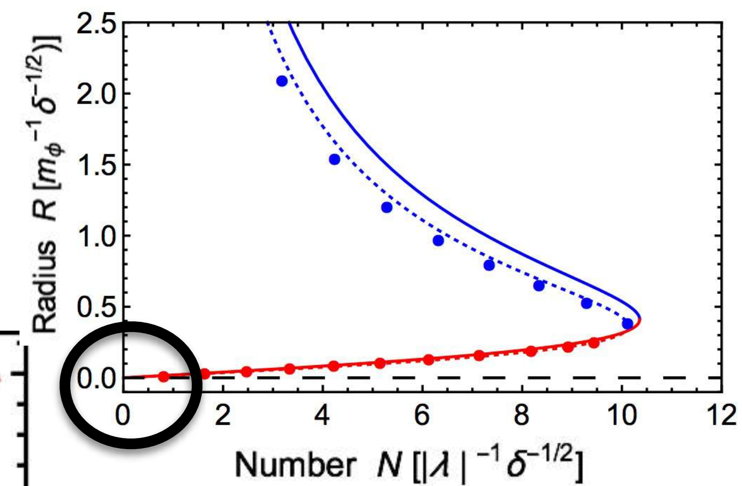
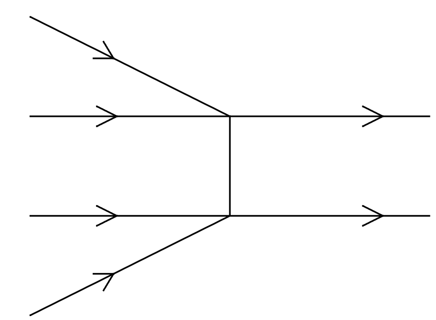
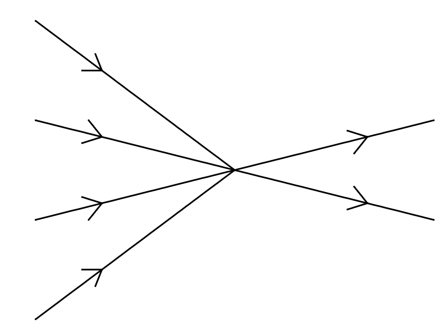






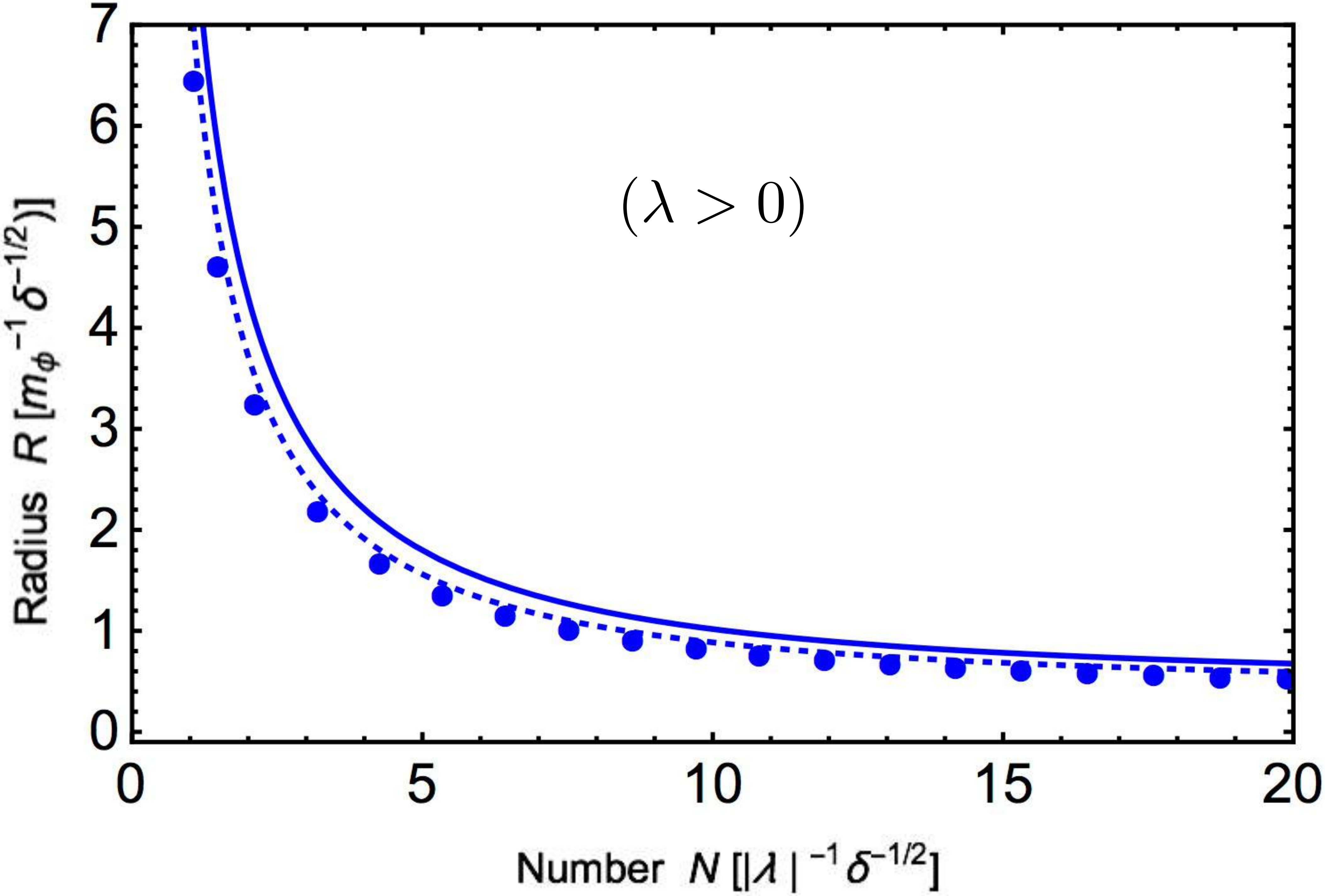


Kolb, Tkachev astro-ph/9311037; Fodor, Fogacs, Horvath, Mezei 0903.0953; Hertzberg 1003.3459; Eby, Suranyi, Wijewardhana 1512.01709; Schiappacasse, Hertzberg 1710.04729; Visinelli, Baum, Redondo, Freese, Wilczek 1710.08910; …

Repulsive Self Interactions

(see; Fan 2016)

Repulsive Self Interaction (Axion-Like Particle)



See Chavanis, Delfini 2011 and others…

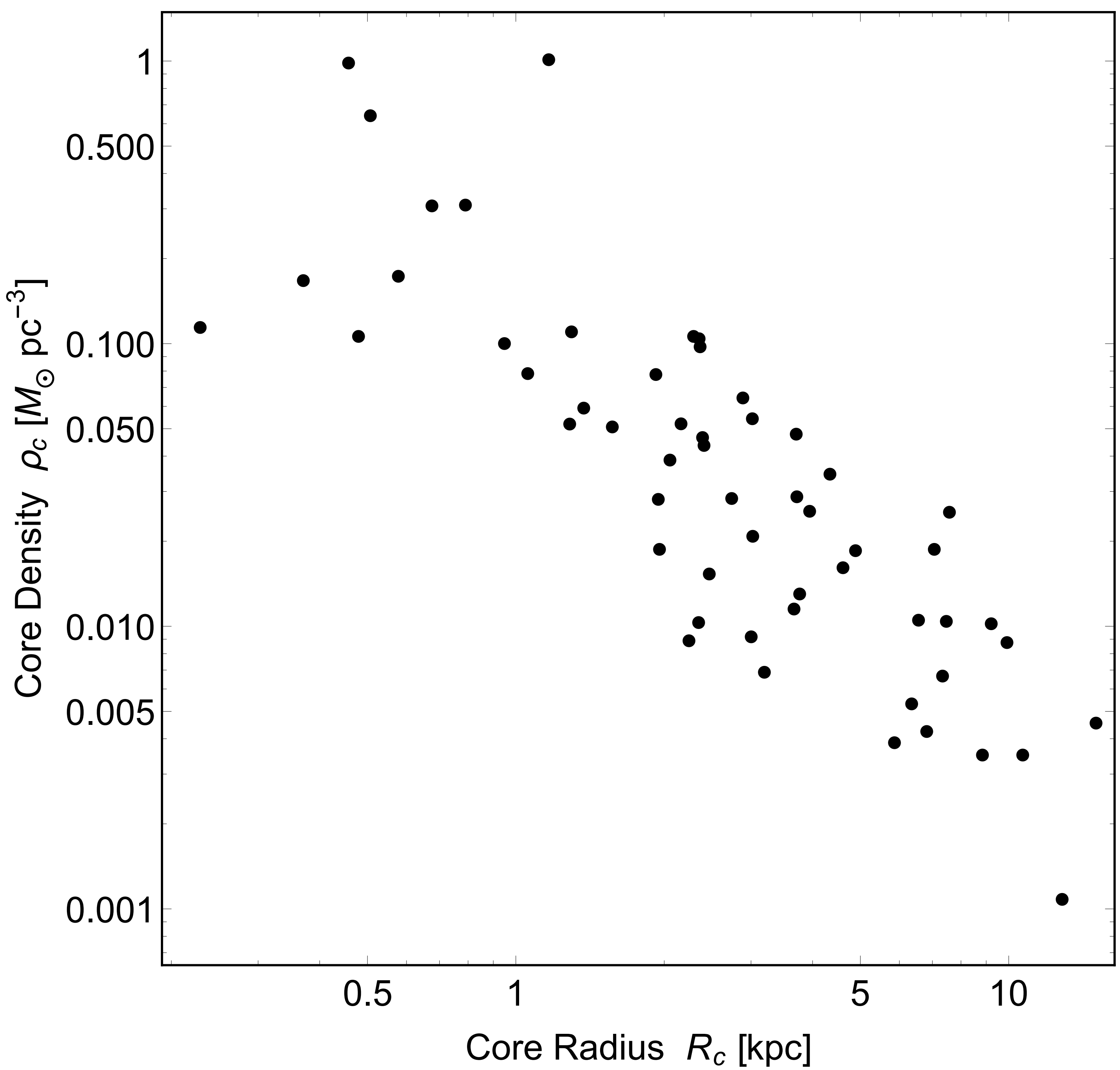
Schiappacasse, Hertzberg 1710.04729; Hertzberg, Rompineve, Yang 2010.07927

Implications for Fuzzy Dark Matter

Can it explain galactic cores?

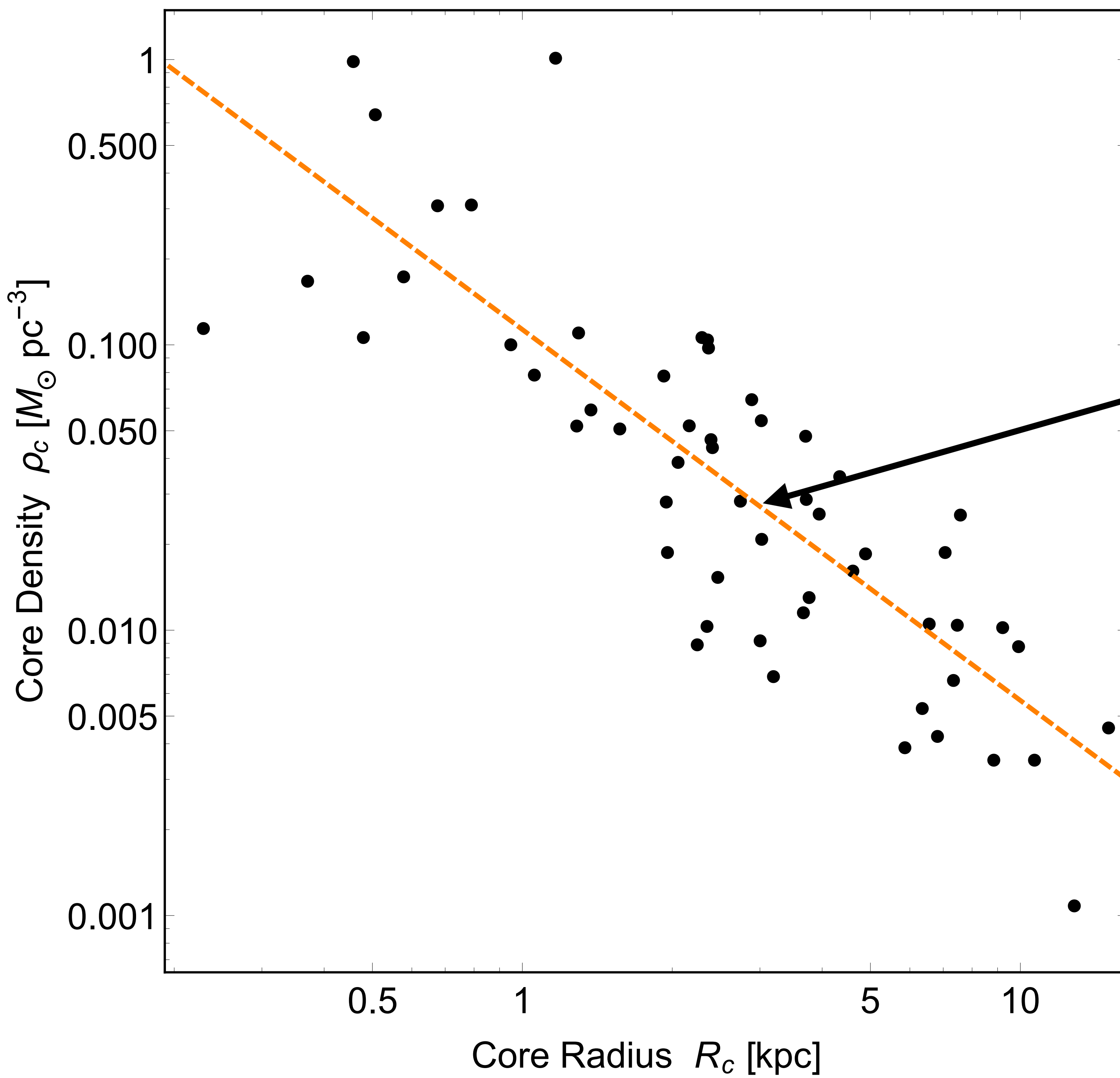
Hu, Barkana, Gruzinov 2000, ….

Core Density Vs Core Radius (Data)



Data from: Rodriguez, del Popolo, Marra, de Oliveira 2017

Core Density Vs Core Radius (Data)

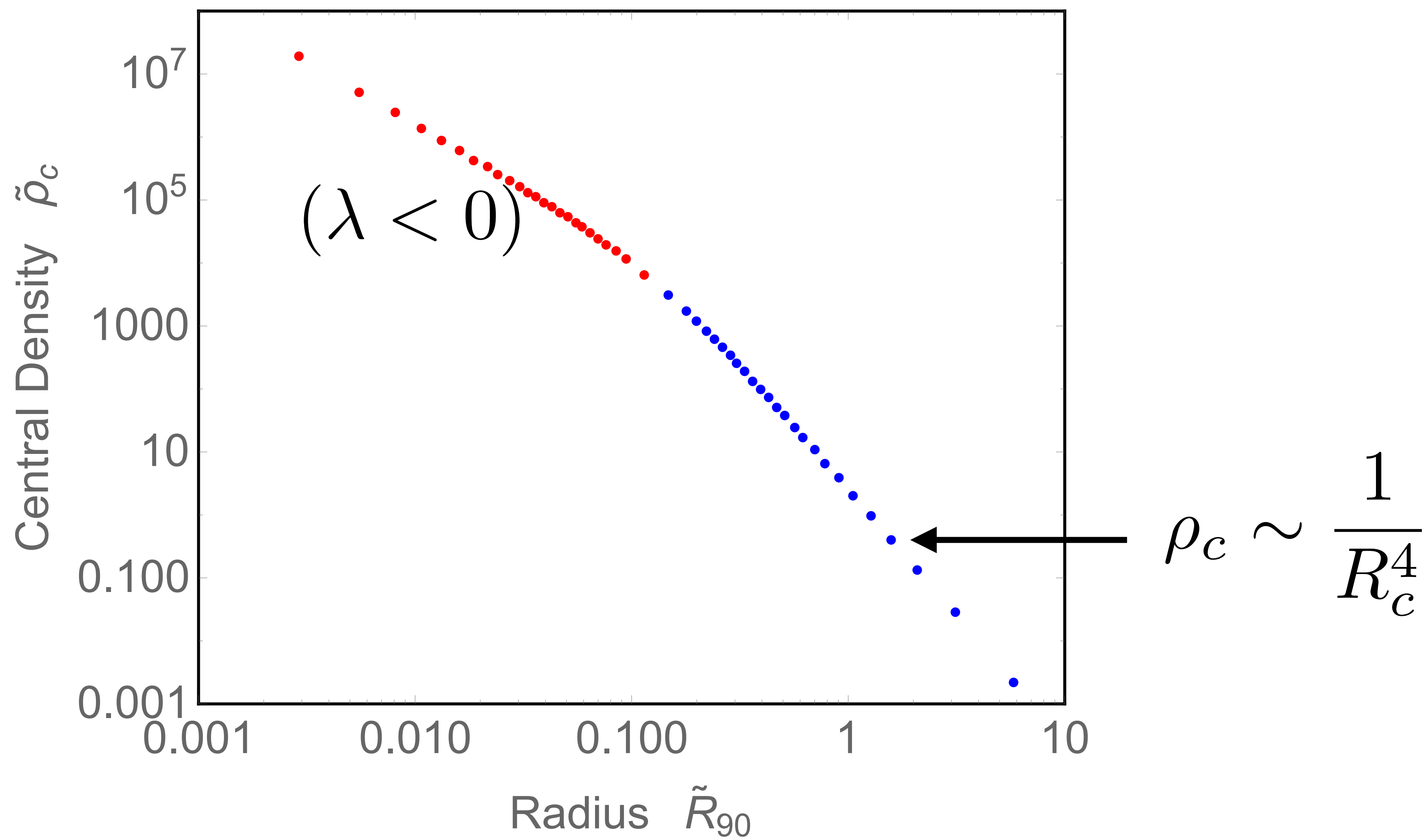


$$\rho_c \sim \frac{1}{R_c^\beta}$$

$$\beta \approx 1$$

Data from: Rodriguez, del Popolo, Marra, de Oliveira 2017

Core Density Vs Core Radius (Light Scalar in BEC)



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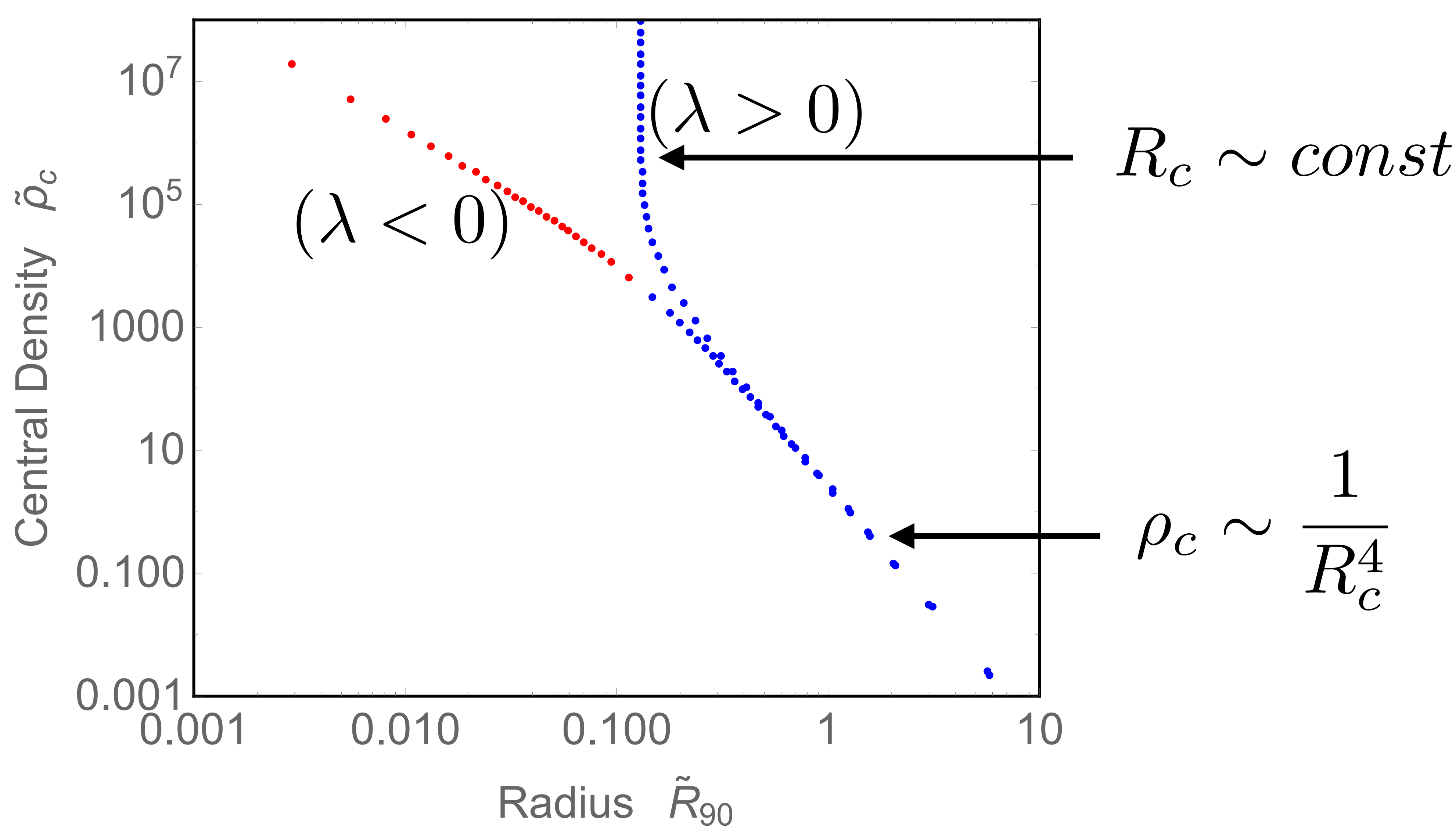
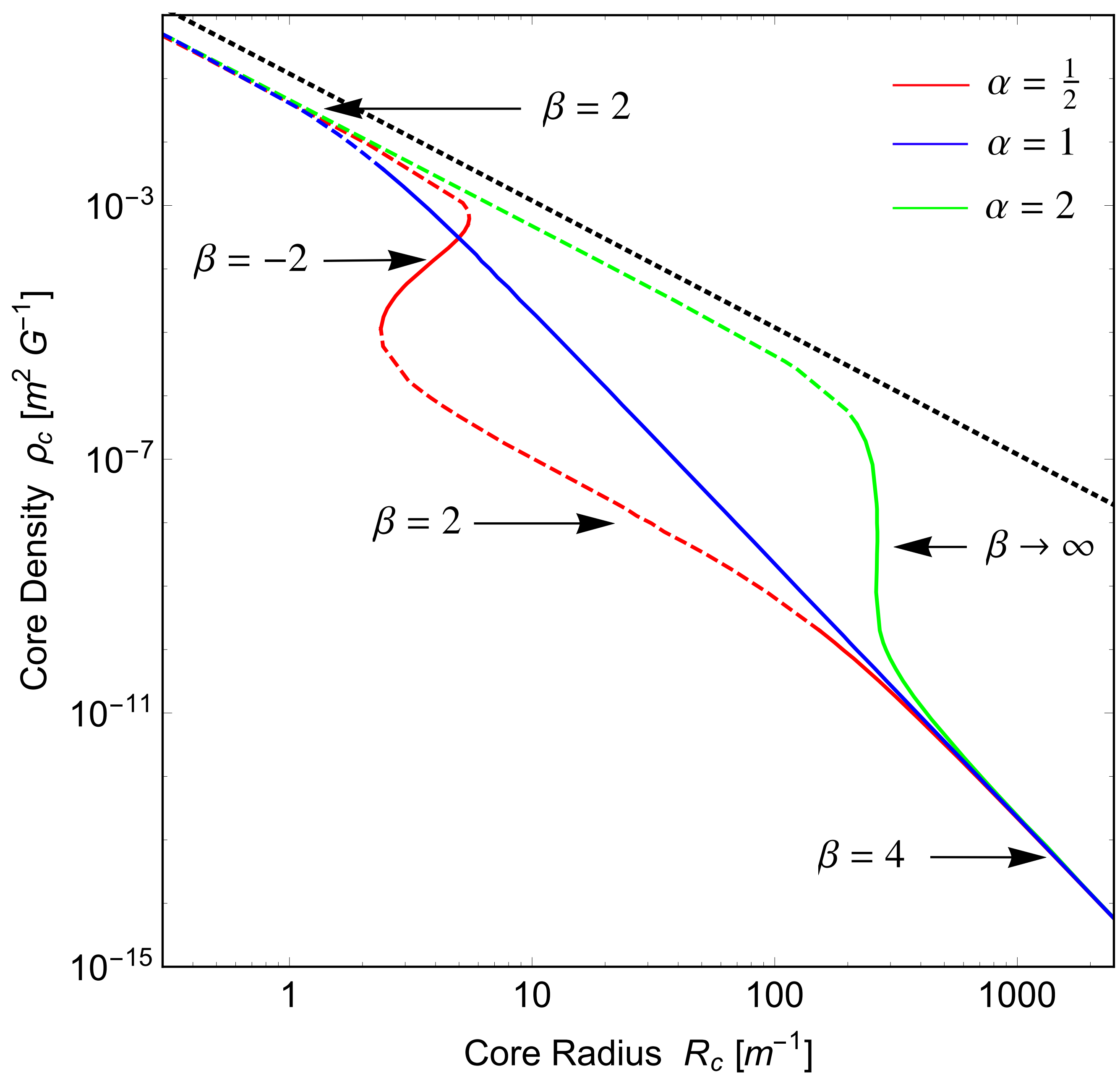


Core Density Vs Core Radius (Light Scalar in BEC)

Extension to general potentials, polytropes, full relativistic

$$V(\phi) \propto ((1 + \phi^2/F^2)^\alpha - 1)$$

Solid = Stable
Dashed = Unstable



$$\rho_c \sim \frac{1}{R_c^\beta}$$

Deng, Hertzberg, Namjoo, Masoumi 1804.05921

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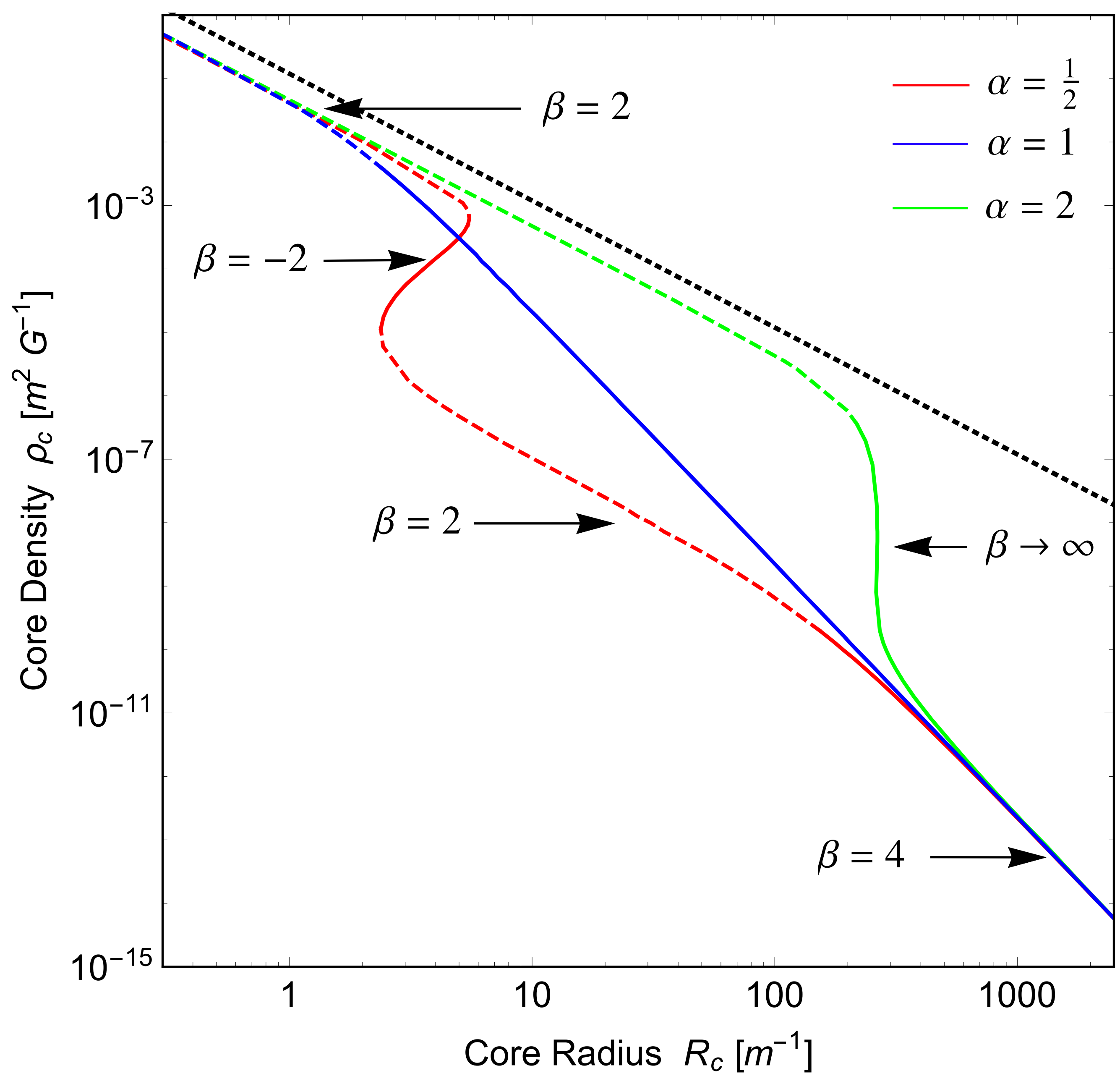

$$\rho_c \sim \frac{1}{R_c^\beta}$$

Never obtain $\beta \sim 1$ and stable

Deng, Hertzberg, Namjoo, Masoumi 1804.05921

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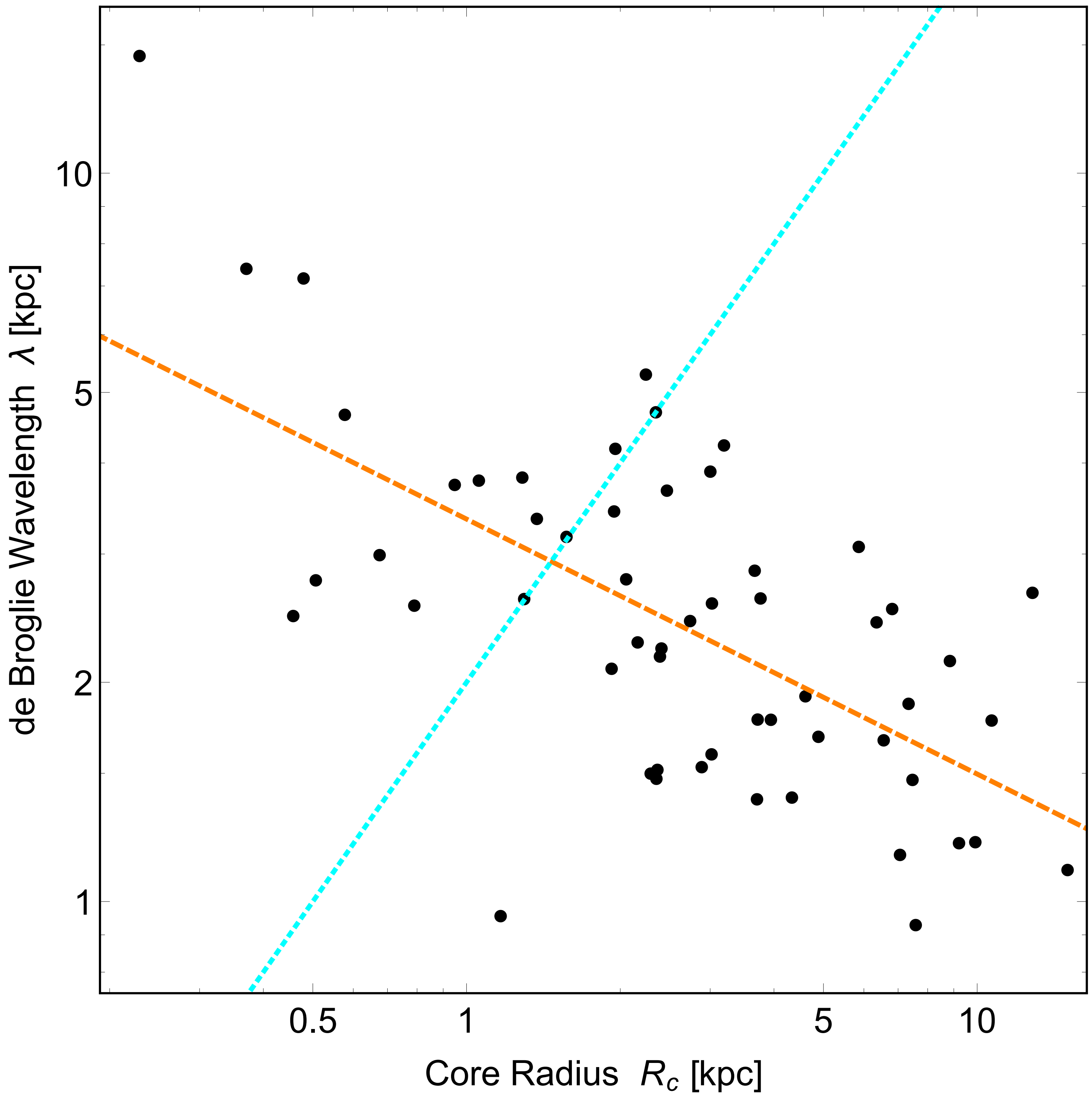

Deng, Hertzberg, Namjoo, Masoumi 1804.05921

Axion Star Resonance into Photons

Consider Axion to Photon Coupling

Photon Lagrangian $$\mathcal{L}_\gamma = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + g_{a\gamma}\,\phi\,\mathbf{E}\cdot\mathbf{B}$$

(Sikivie 1983; Adshead, Giblin, Scully, Sfakianakis 2015, 2016; Masaki, Aoki, Soda 2017)

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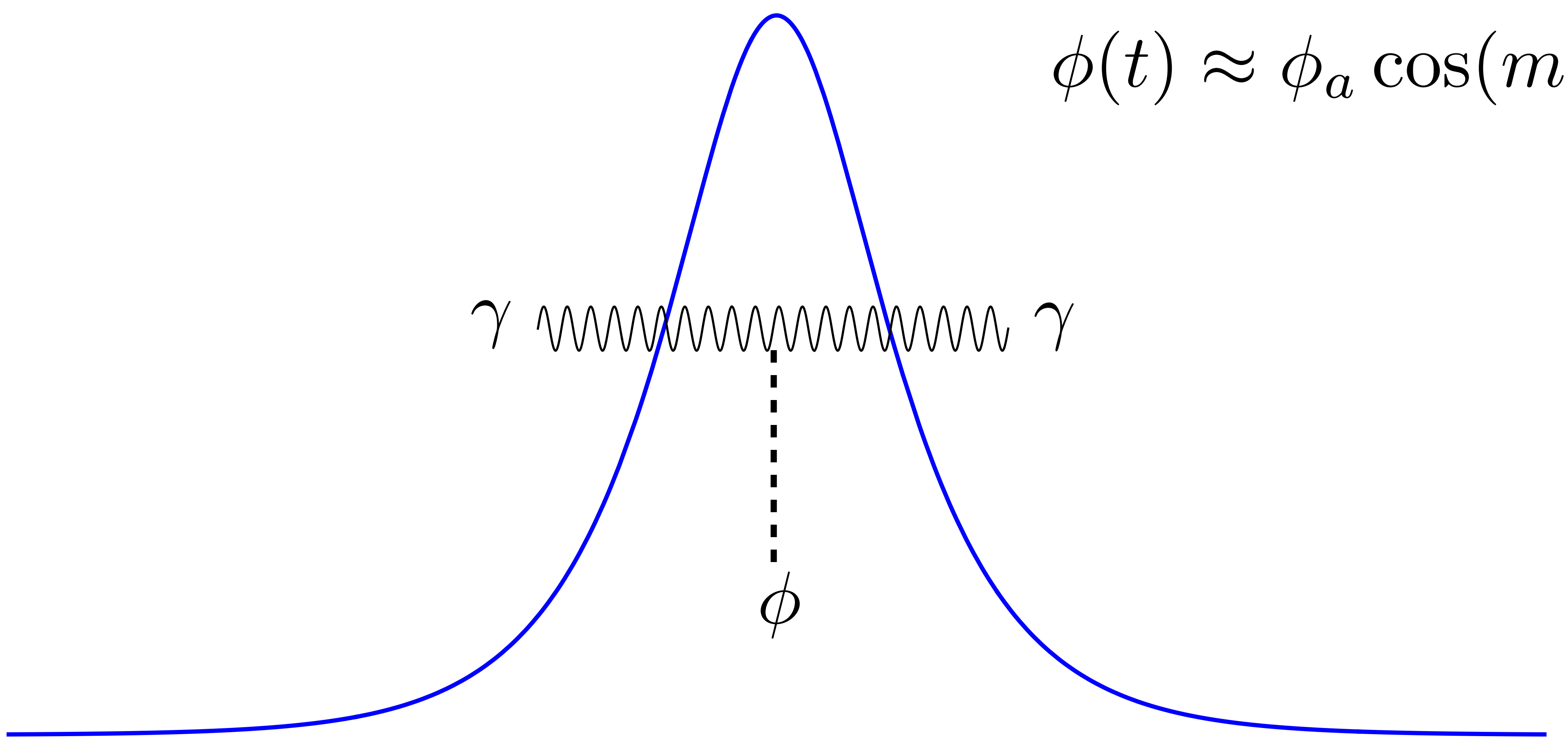
(Sikivie 1983; Adshead, Giblin, Scully, Sfakianakis 2015, 2016; Masaki, Aoki, Soda 2017)

Equation of motion $$\ddot{\mathbf{A}} - \nabla^2\mathbf{A} + g_{a\gamma}\,\partial_t\phi\,\nabla\times\mathbf{A} = 0$$

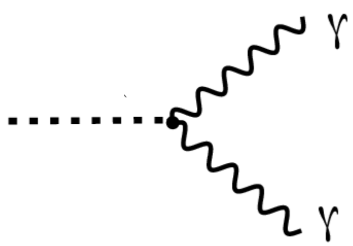
$$\phi(t) \approx \phi_a \cos(m_\phi t)$$



Homogeneous Axion Field

Mathieu Equation

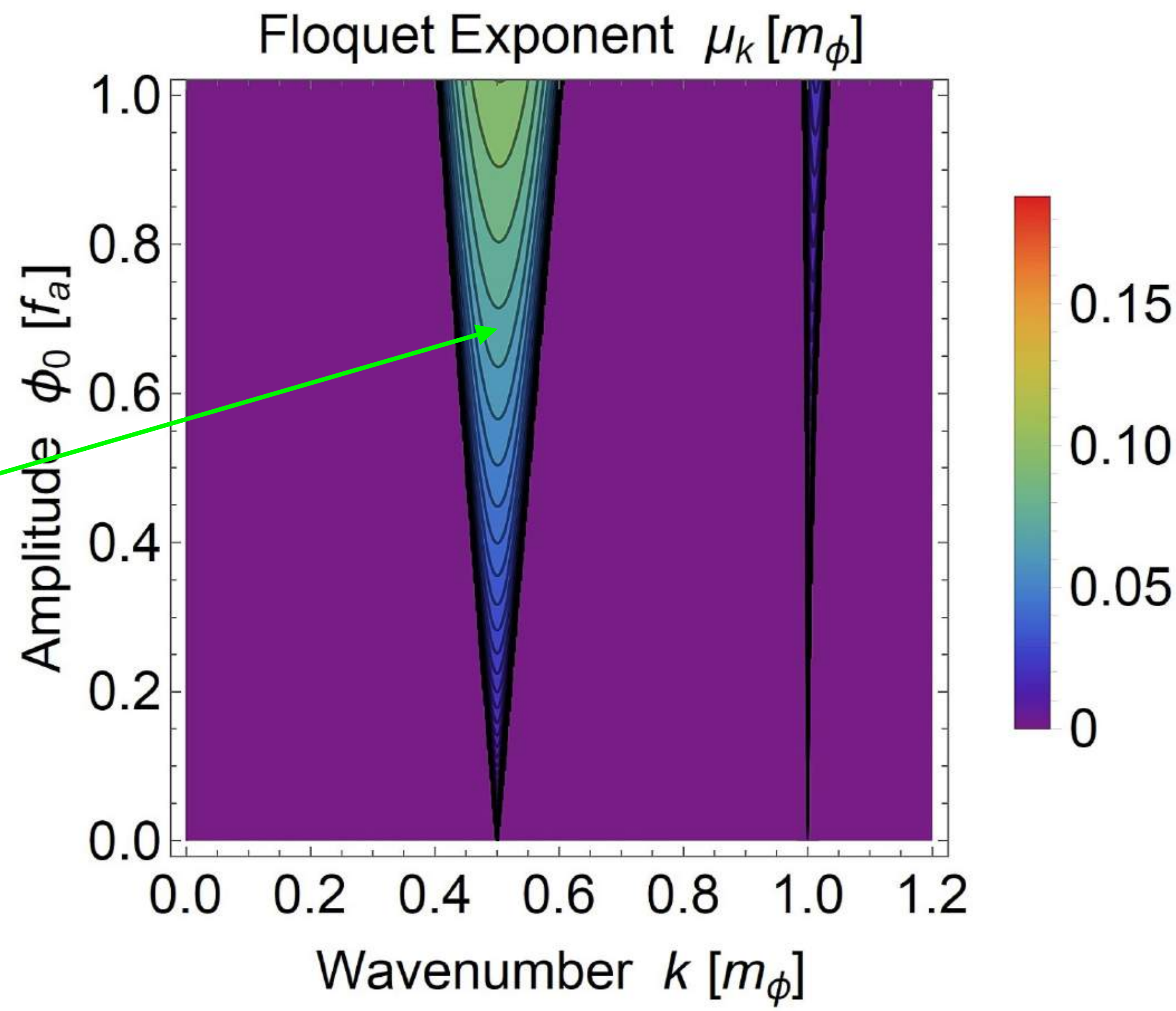
$$\ddot{\mathbf{A}}^T_{\mathbf{k}} + k^2 \mathbf{A}^T_{\mathbf{k}} + g_{a\gamma}\, k\, \partial_t \phi(t) \mathbf{A}^T_{\mathbf{k}} = 0$$



Parametric resonance
always present

$$k \approx \frac{m_a}{2}$$

$$\mu^*_H \approx \frac{1}{4} g_{a\gamma}\, m_\phi\, \phi_a$$



e.g., Yoshimura 1996

Comment on Photon Plasma Mass

In plasma, the photon acquires an effective mass $$\omega_p^2 = \frac{4\pi\alpha\, n_e}{m_e}$$

In VERY EARLY universe, this is huge; preventing resonance

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In plasma, the photon acquires an effective mass $$\omega_p^2 = \frac{4\pi\alpha\, n_e}{m_e}$$

In VERY EARLY universe, this is huge; preventing resonance

Clumps in halo: $$\omega_p^2 \approx \frac{n_e}{0.03\,\mathrm{cm}^{-3}} \left(6 \times 10^{-12}\,\mathrm{eV}\right)^2$$

Negligibly small; allowing for resonance

Inhomogeneous (Spherical) Axion Star

Decomposition into vector spherical harmonics

$$\mathbf{A}(\mathbf{r},t)=\sum_{lm}\int\frac{d^3k}{(2\pi)^3}\left[a_{lm}(k,t)\mathbf{N}_{lm}(k,\mathbf{r})+b_{lm}(k,t)\mathbf{M}_{lm}(k,\mathbf{r})\right]$$

where

$$\mathbf{M}_{lm}(k,\mathbf{r})=i\frac{j_l(kr)}{\sqrt{l(l+1)}}\nabla\times\left[Y_{lm}(\theta,\varphi)\mathbf{r}\right]$$

$$\mathbf{N}_{lm}(k,\mathbf{r})=\frac{i}{k}\nabla\times\mathbf{M}_{lm}$$

Hertzberg, Schiappacasse 1805.00430

Inhomogeneous (Spherical) Axion Star

Instability channel $\quad l = 1, m = 0, \quad b_{10} = -i\, a_{10}$

$$\ddot{a}_{10}(k,t) + k^2 a_{10}(k,t) + g_{a\gamma} k \int \frac{dk'}{(2\pi)} \partial_t \tilde{\phi}(k-k') a_{10}(k',t) = 0$$

Hertzberg, Schiappacasse 1805.00430

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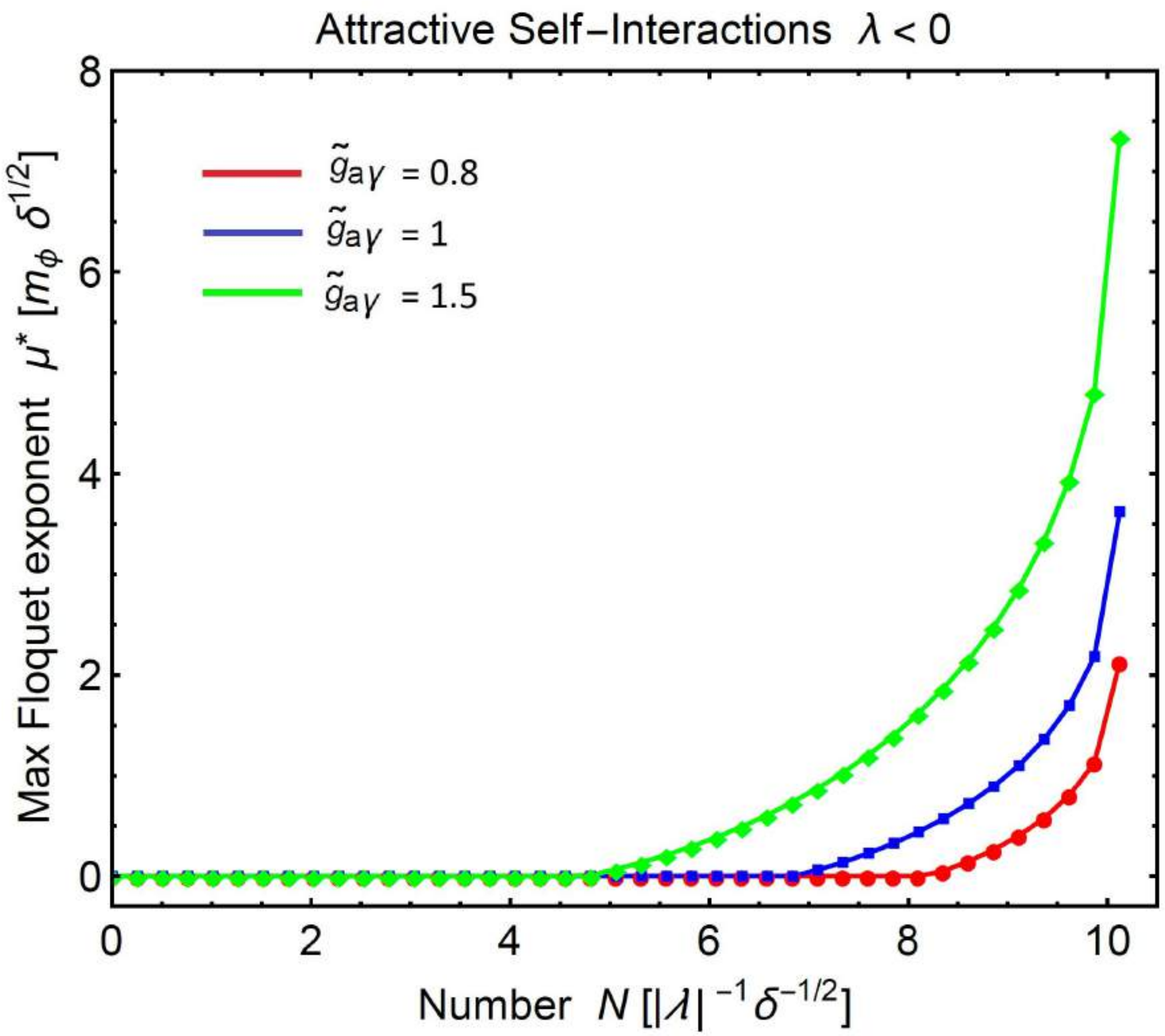

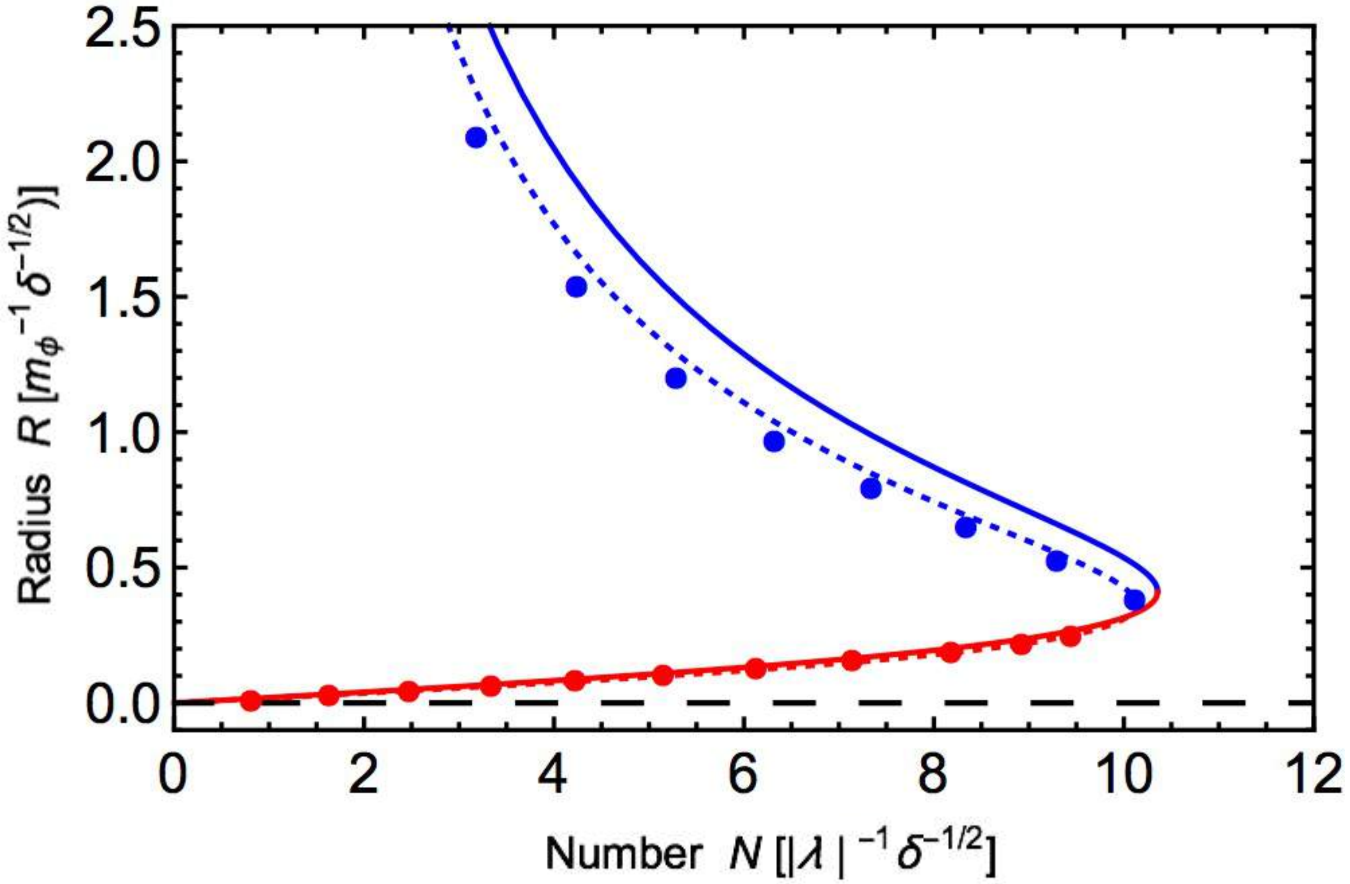

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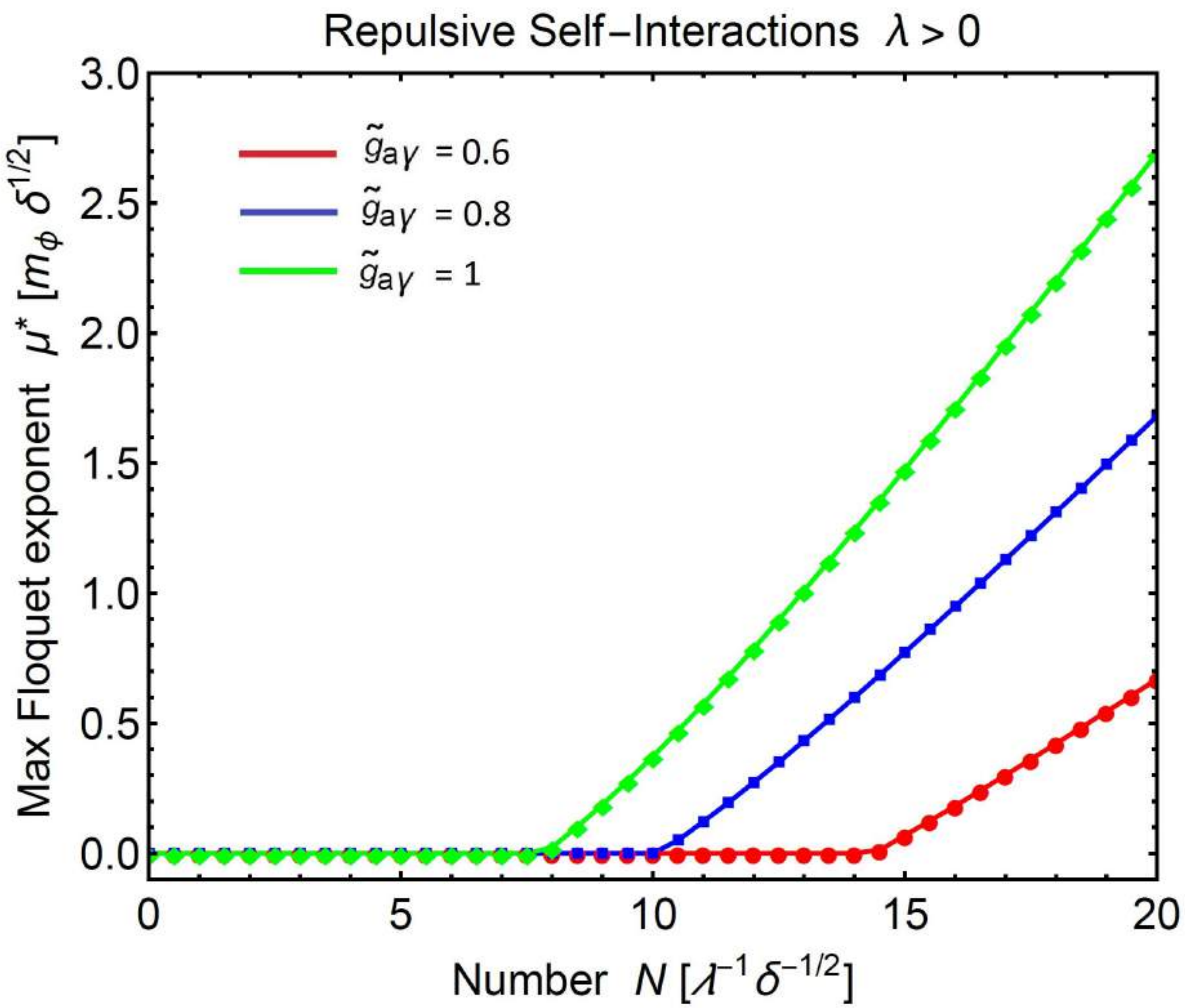

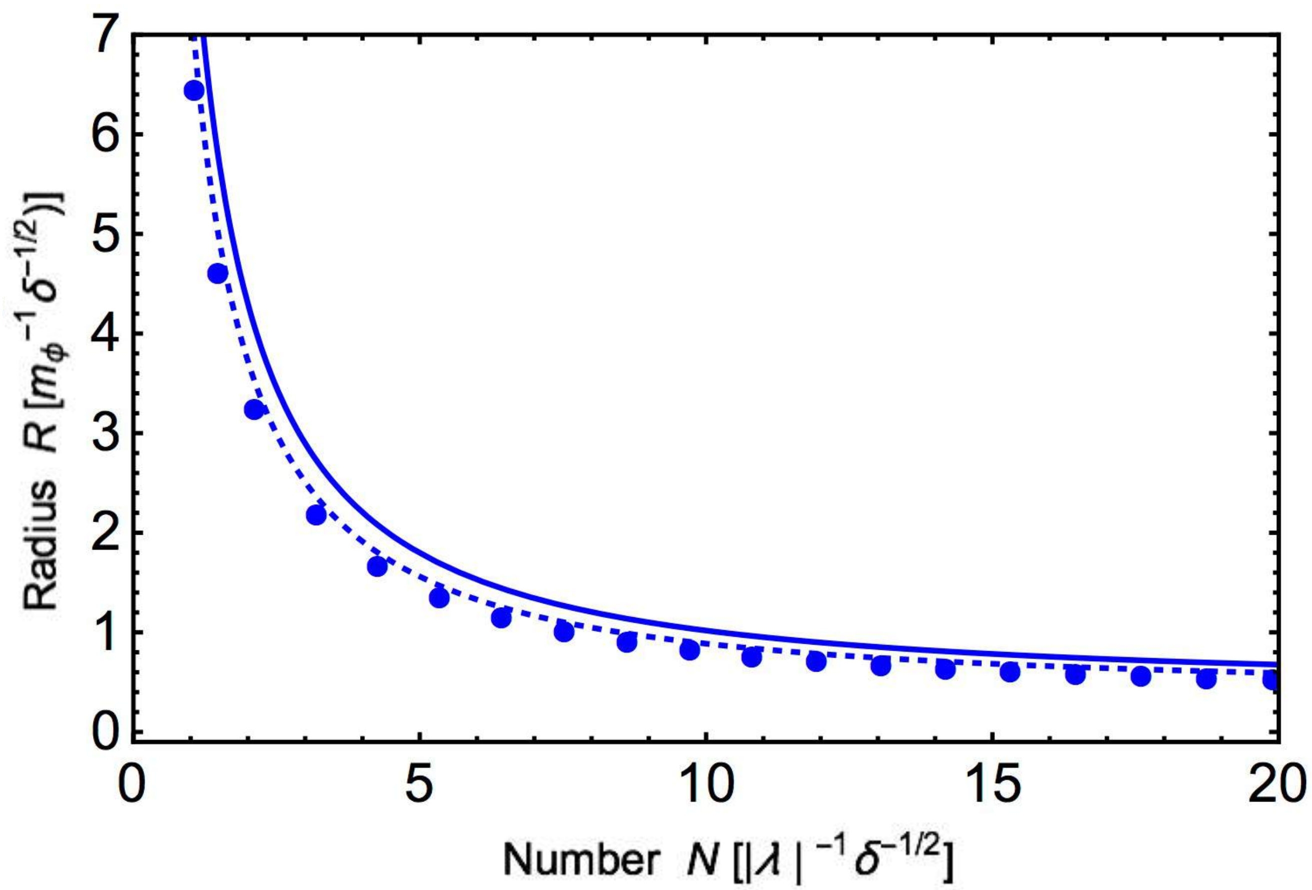

Hertzberg, Schiappacasse 1805.00430

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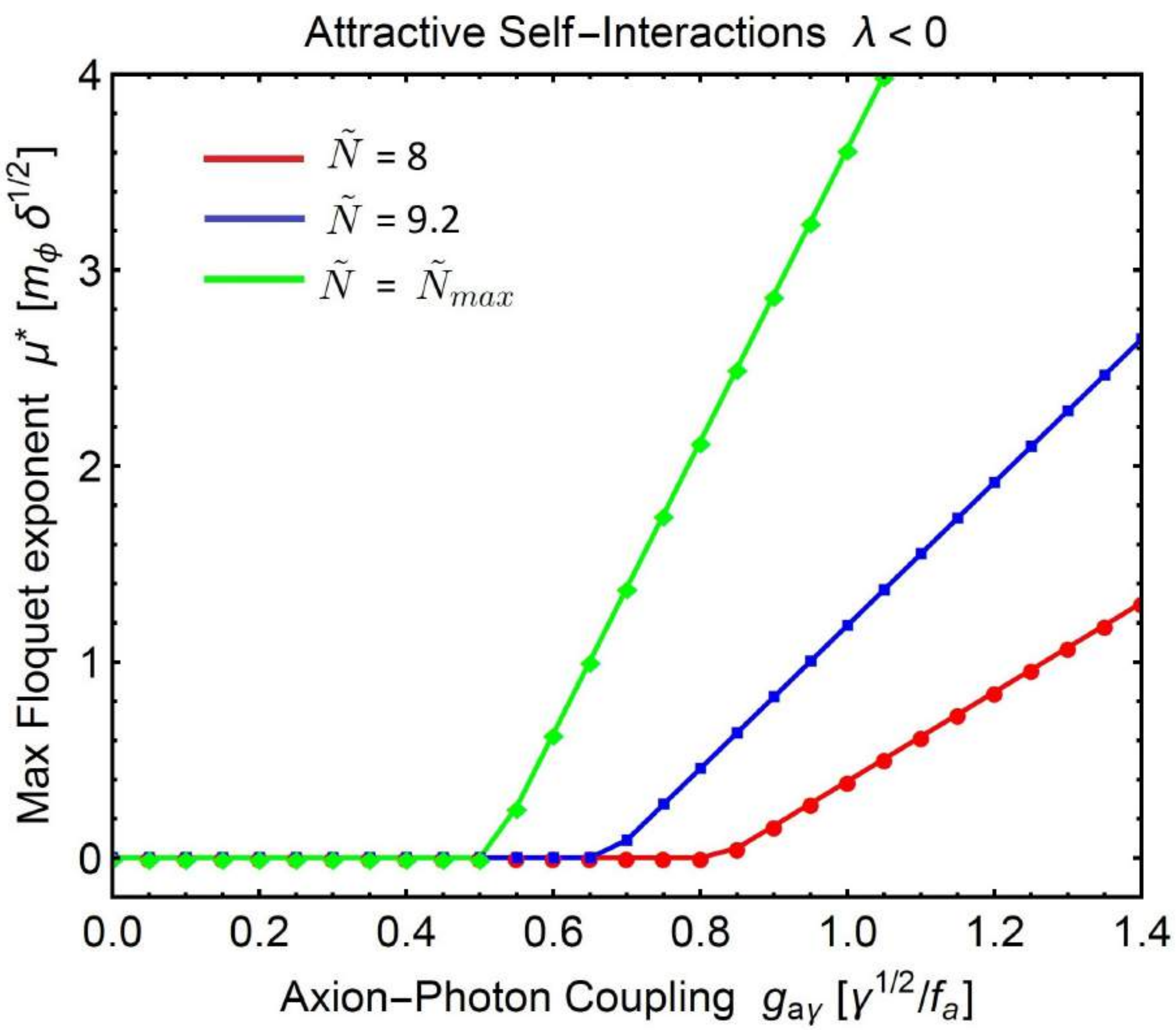

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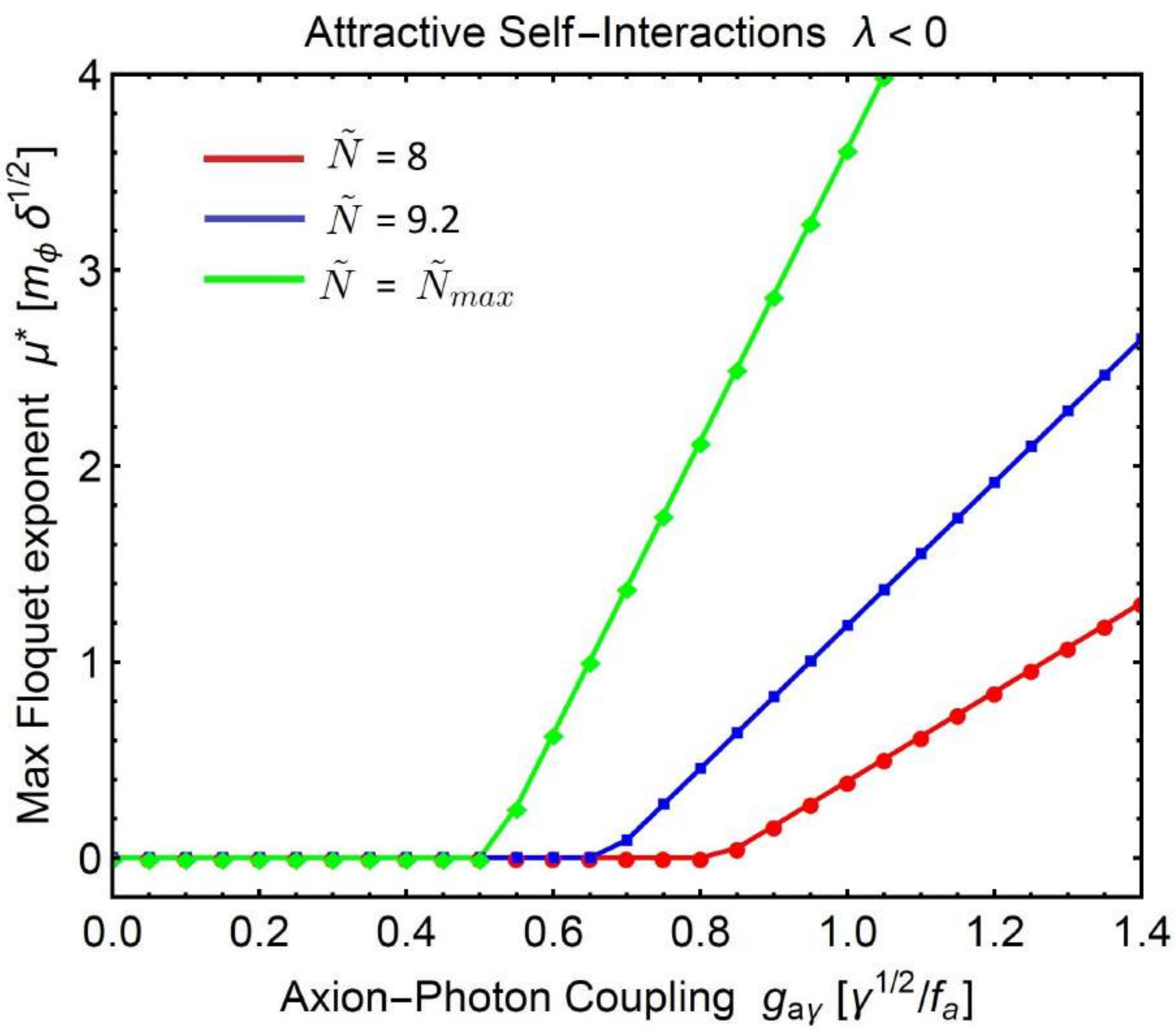
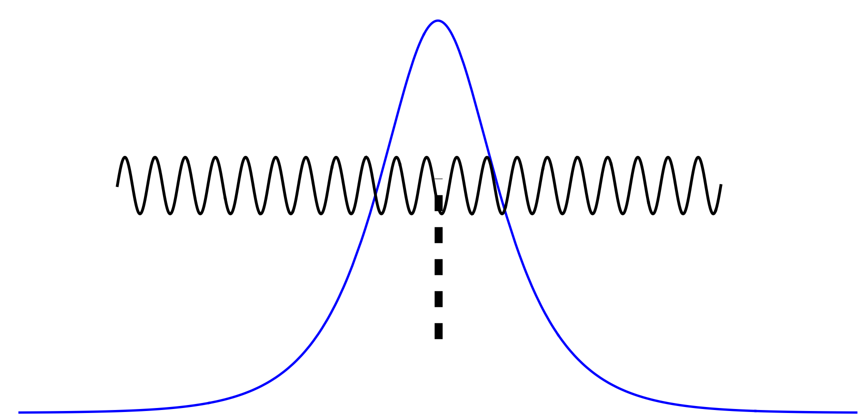


$$\mu_{hom} > \mu_{esc}$$



Hertzberg, Schiappacasse 1805.00430

Tkachev 1986, 1987, 2015;
Hertzberg 2010; Kawasaki, Yamada 2014

Resonance Condition (Spherical) Axion Star

$$\boxed{g_{a\gamma} > \frac{0.3}{f_a}}$$

$(\lambda < 0)$

No resonance for standard QCD axion-photon coupling

$$g_{a\gamma} \sim \frac{\alpha}{f_a}$$

Allowed for models with enhanced couplings, decay to hidden sectors, or repulsive interactions

(e.g., Daido, Takahashi, Yokozaki 2018)

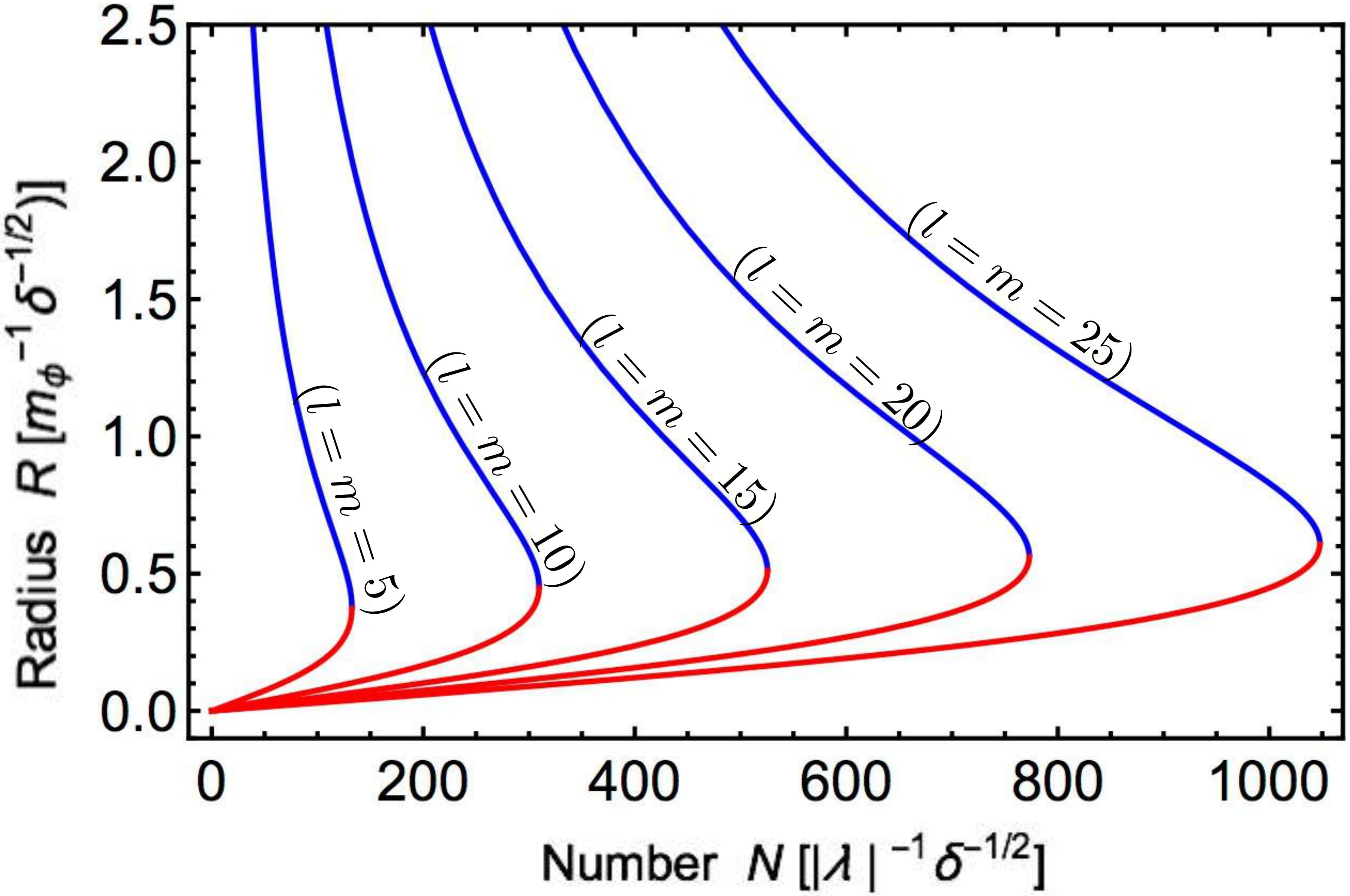
(e.g., Farina, Pappadopulo, Rompineve, Tesi 2016)

(e.g., Fan 2016)

Hertzberg, Schiappacasse 1805.00430

Including Angular Momentum

Two Branches of Solutions (with Angular Momentum)

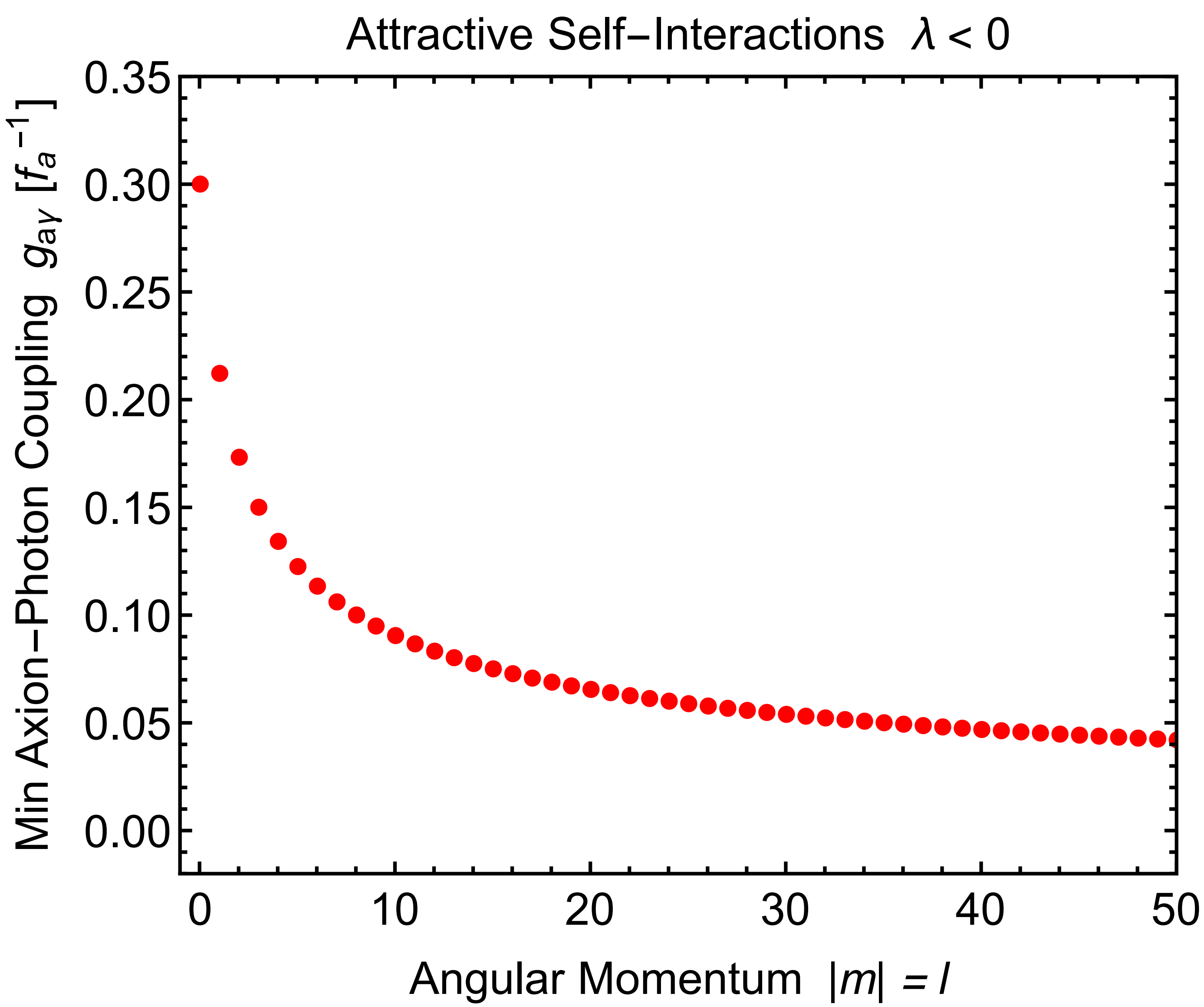


High angular momentum allows higher amplitude at core, which helps for resonance into photons

Hertzberg, Schiappacasse 1804.07255

Resonance Condition (Non-Spherical) Axion Star

$$\boxed{g_{a\gamma} > \frac{0.3}{f_a\sqrt{l+1}}} \qquad (\lambda < 0)$$

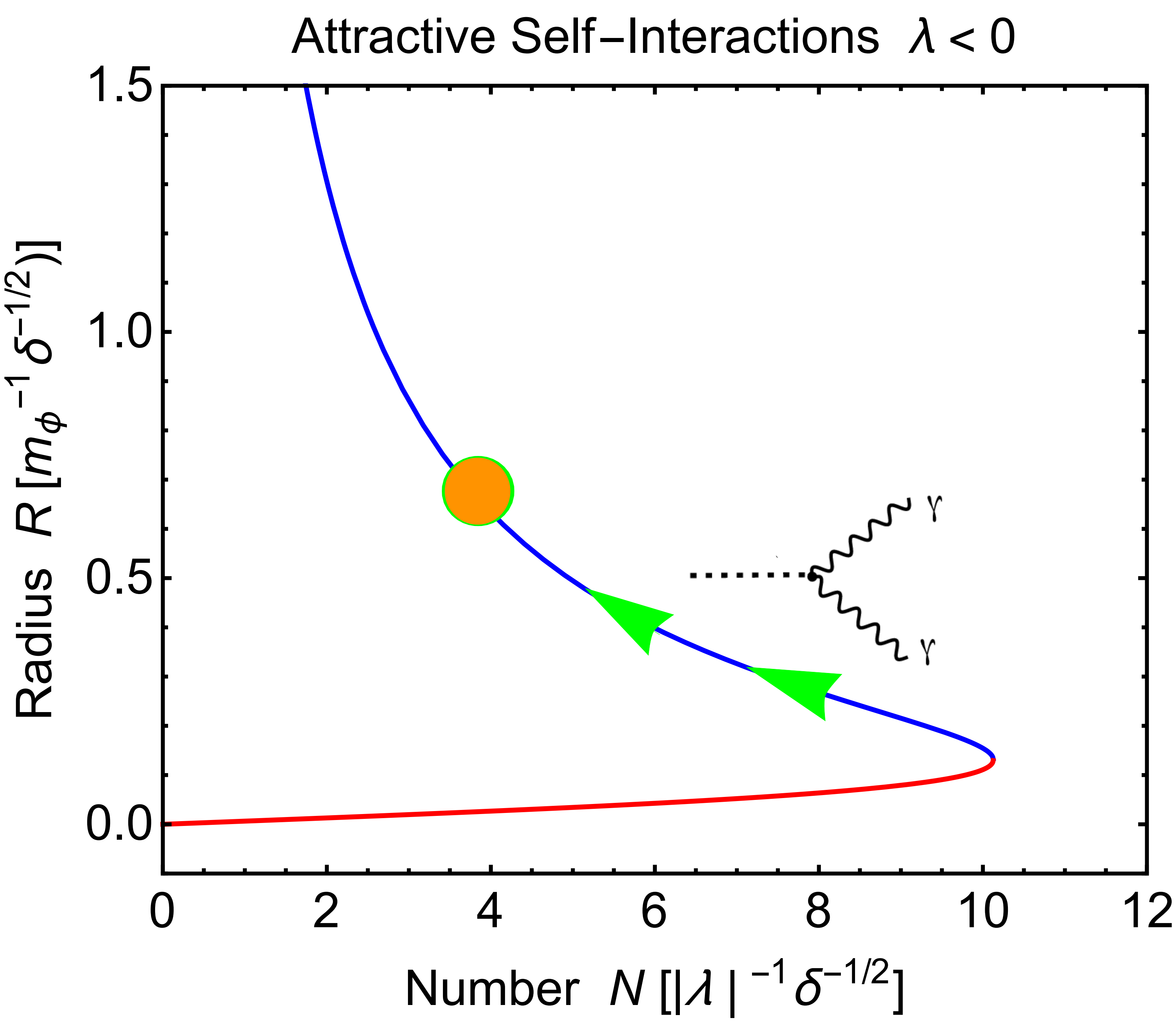


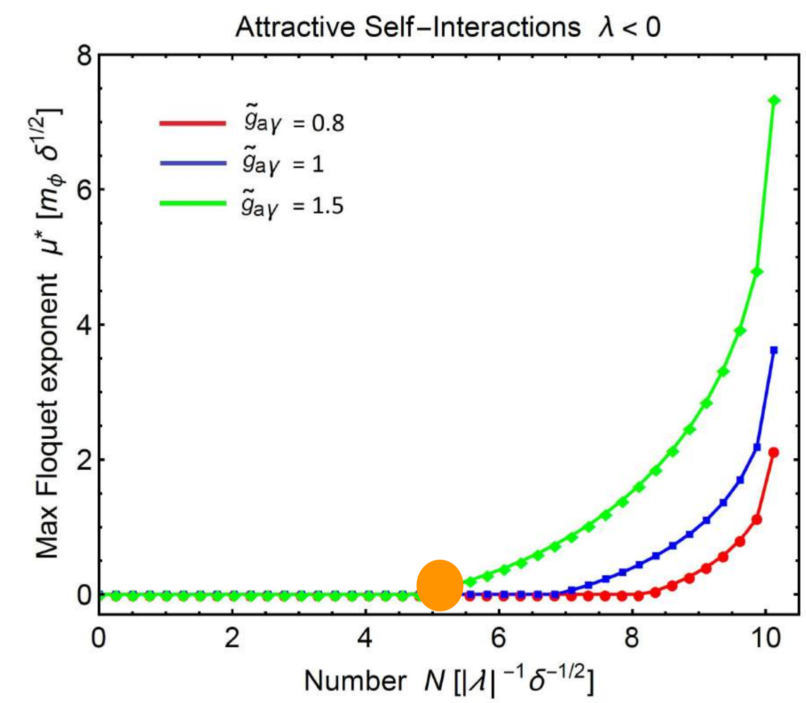
Resonance allowed for standard QCD axion-photon couplings, with high angular momentum

Hertzberg, Schiappacasse 1805.00430

Astrophysical Consequences

Lasing Stars Early Universe

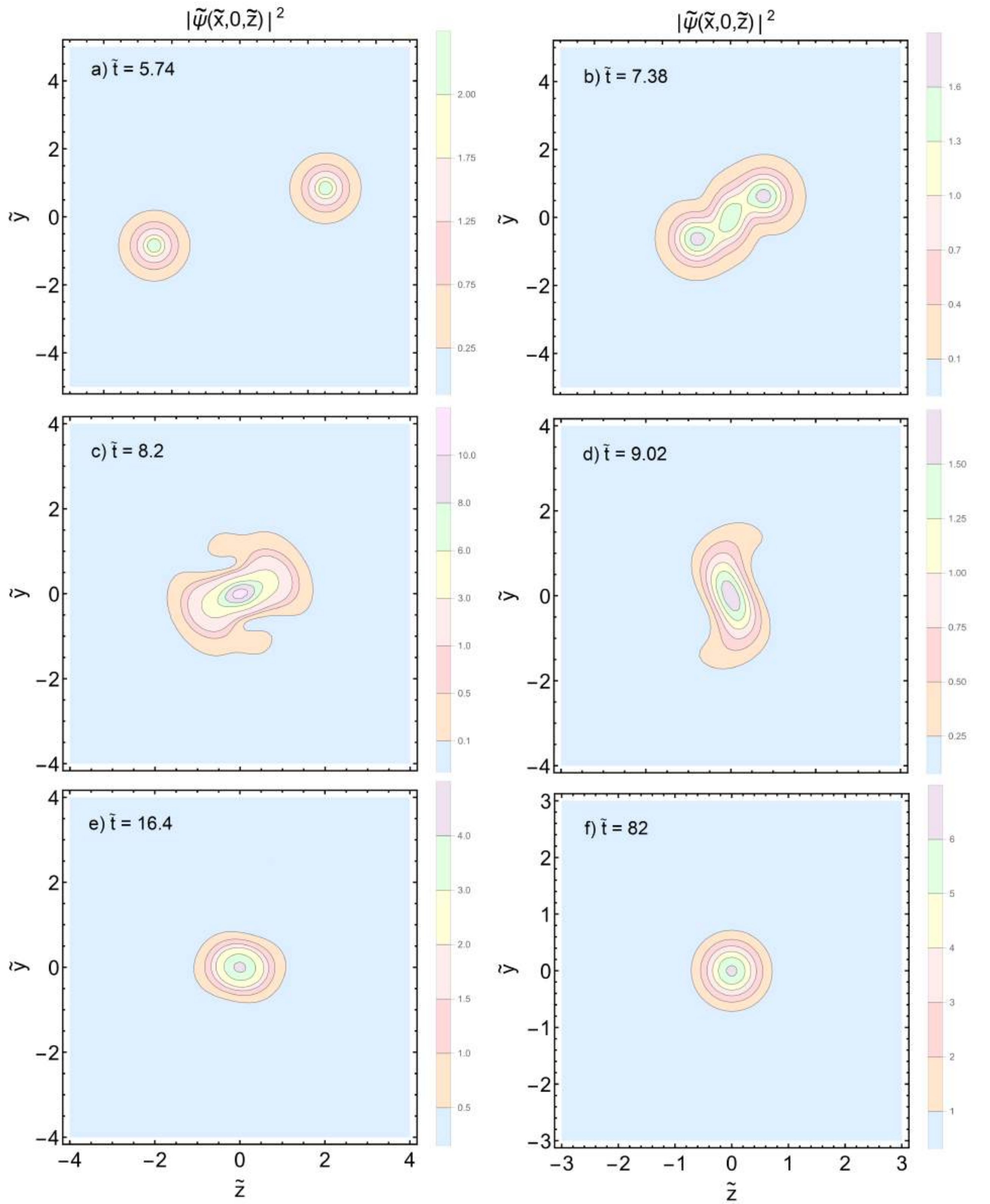




(i) Mass Pile Up

Hertzberg, Schiappacasse 1805.00430

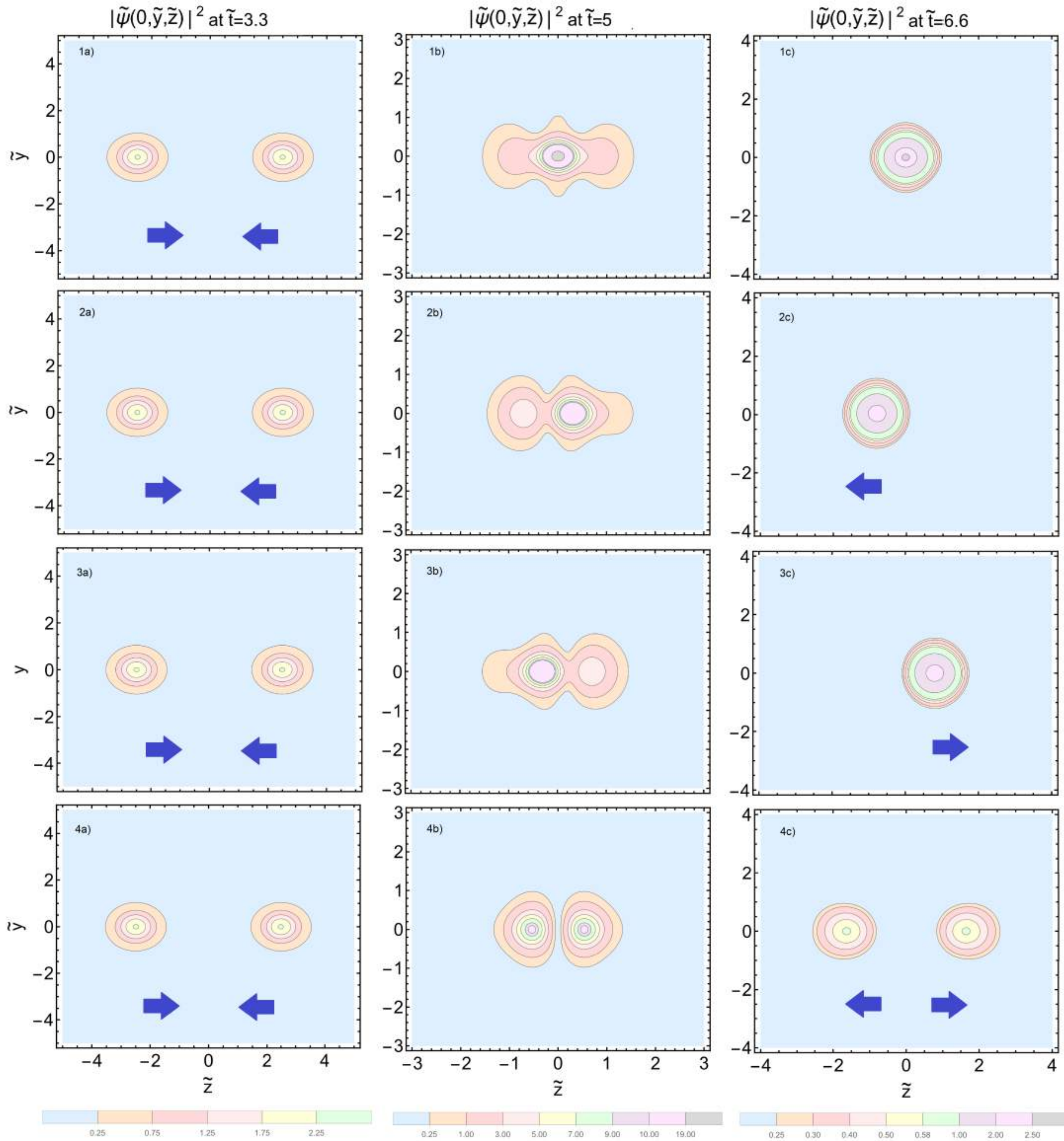
Axion Star Mergers



Hertzberg, Li, Schiappacasse 2005.02405

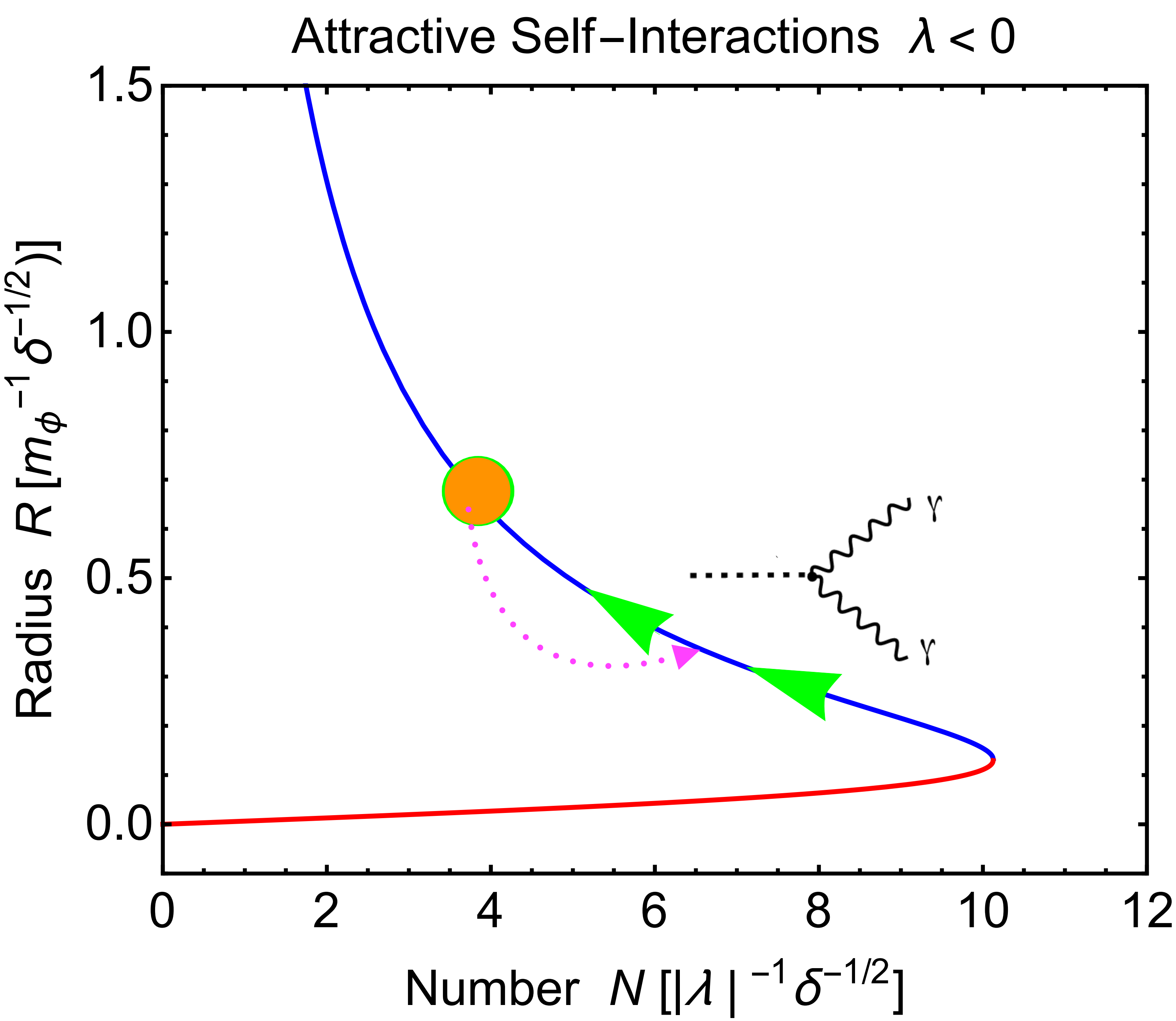
Axion Star Mergers

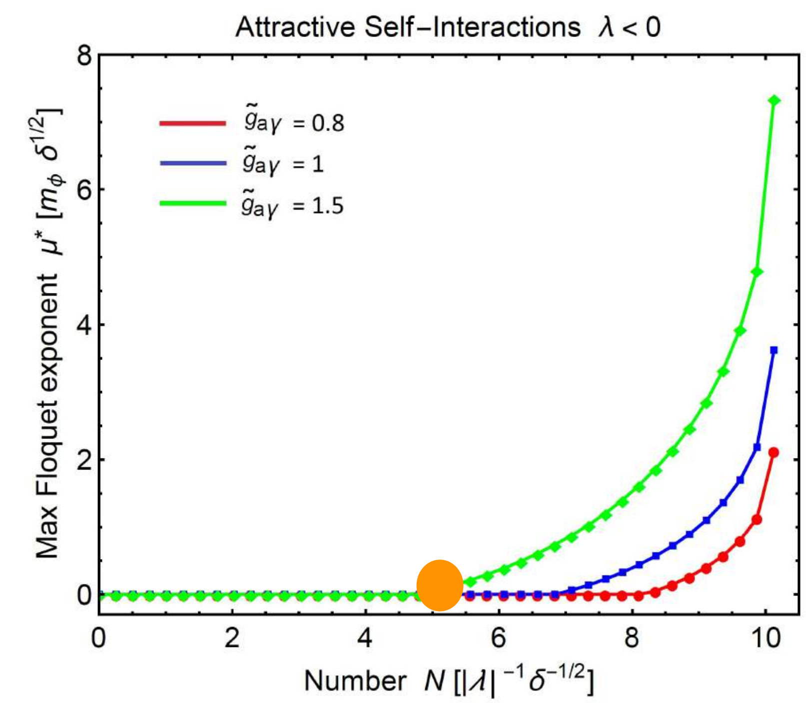
Phase dependence



Hertzberg, Li, Schiappacasse 2005.02405

Lasing Stars Late Universe





(i) Mass Pile Up

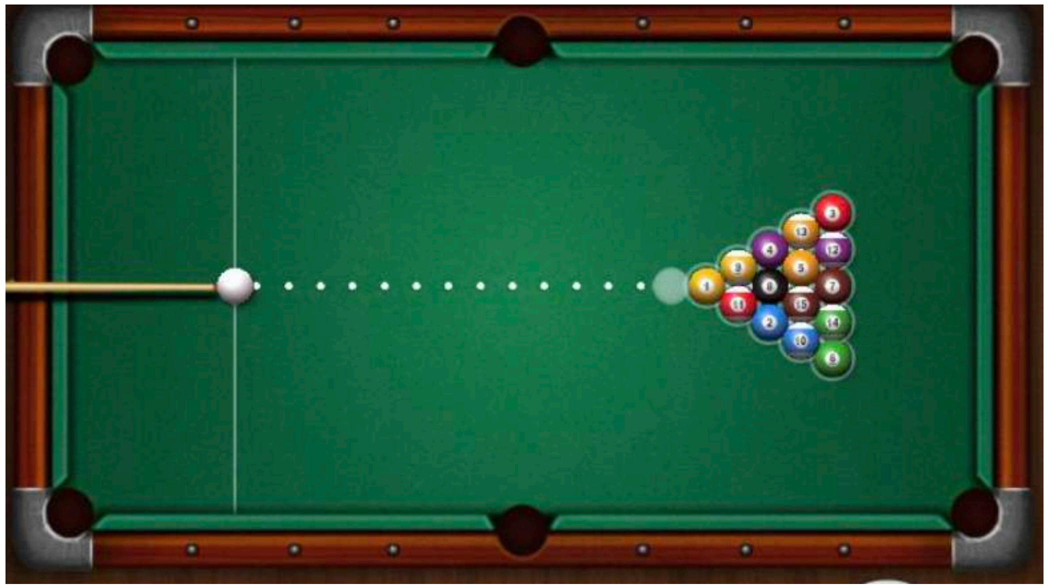
(ii) Late Time Mergers;
Radio-wave Bursts

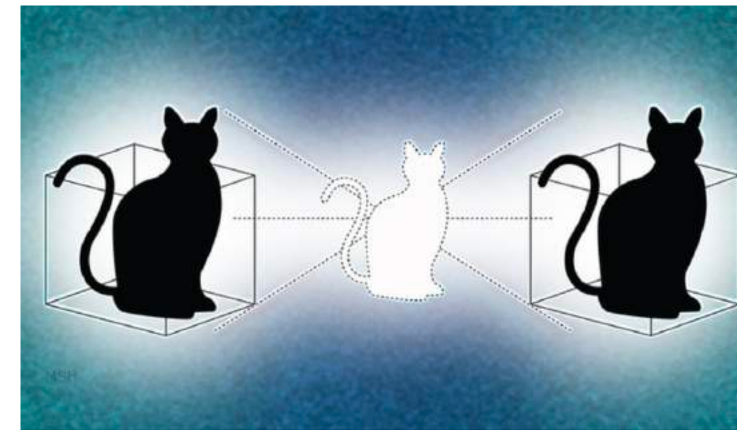
$$\lambda_{EM} = \frac{2\pi}{k} \approx \frac{4\pi}{m_a}$$
$$= \mathcal{O}(1)\, meters$$

Hertzberg, Li, Schiappacasse 2005.02405

(also Tkachev 2015)

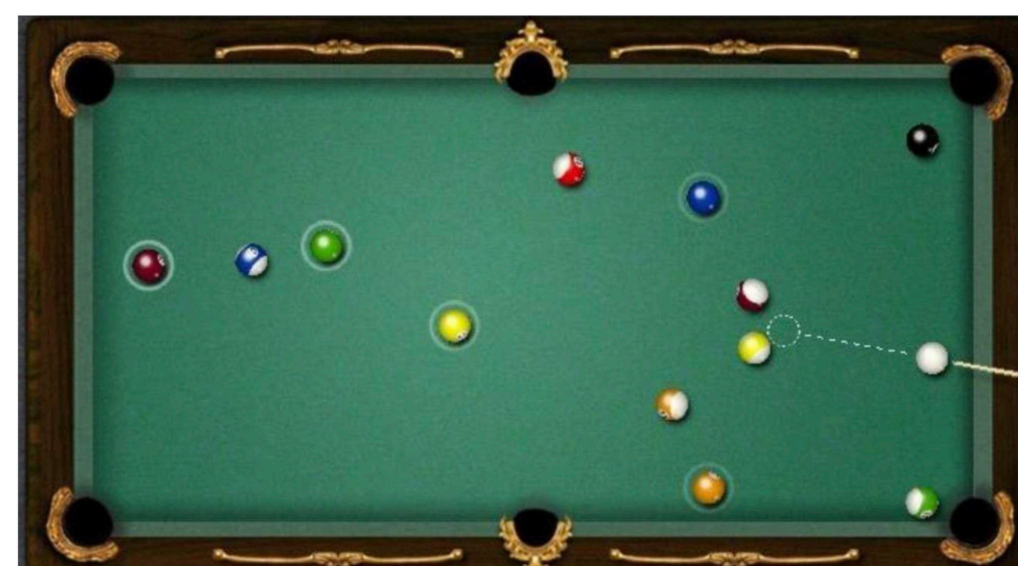
Recall that non-linear dynamics can launch states into Schrodinger cat-like states





Schrodinger Cat Billiards

+



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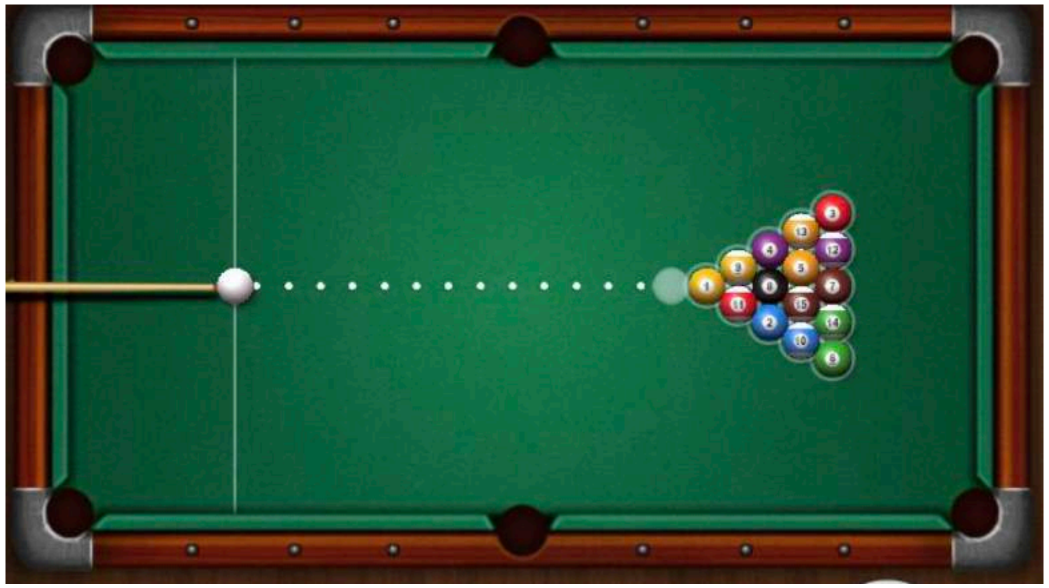
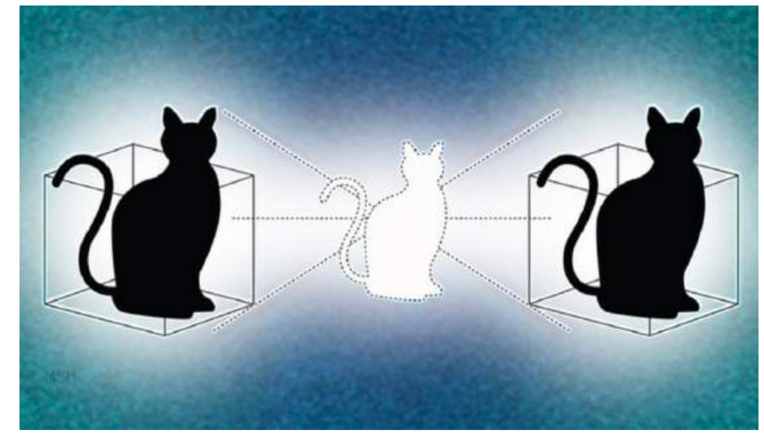
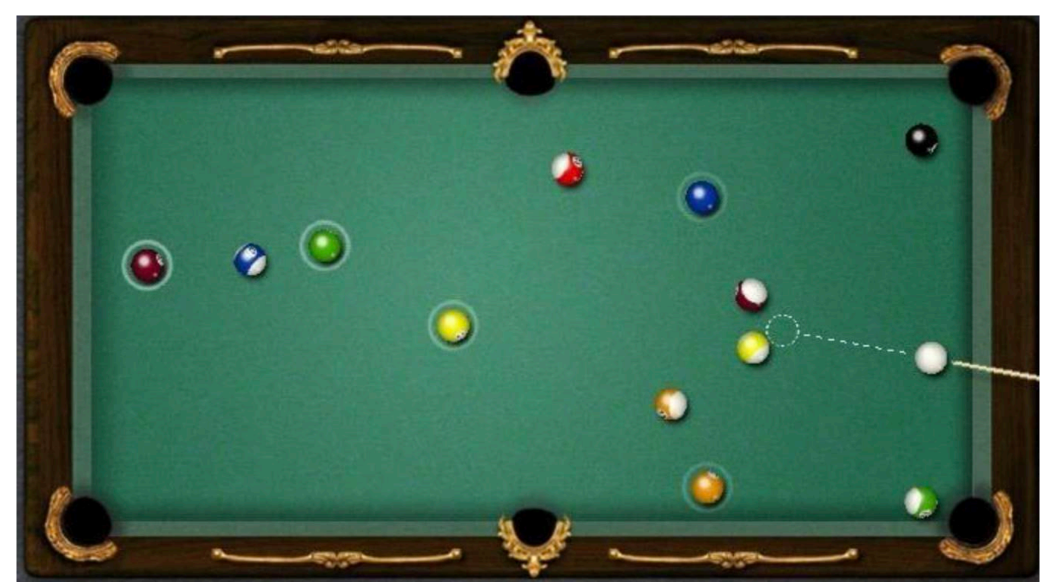




Quantumness destroyed due to DECOHERENCE

Schrodinger Cat Billiards

+



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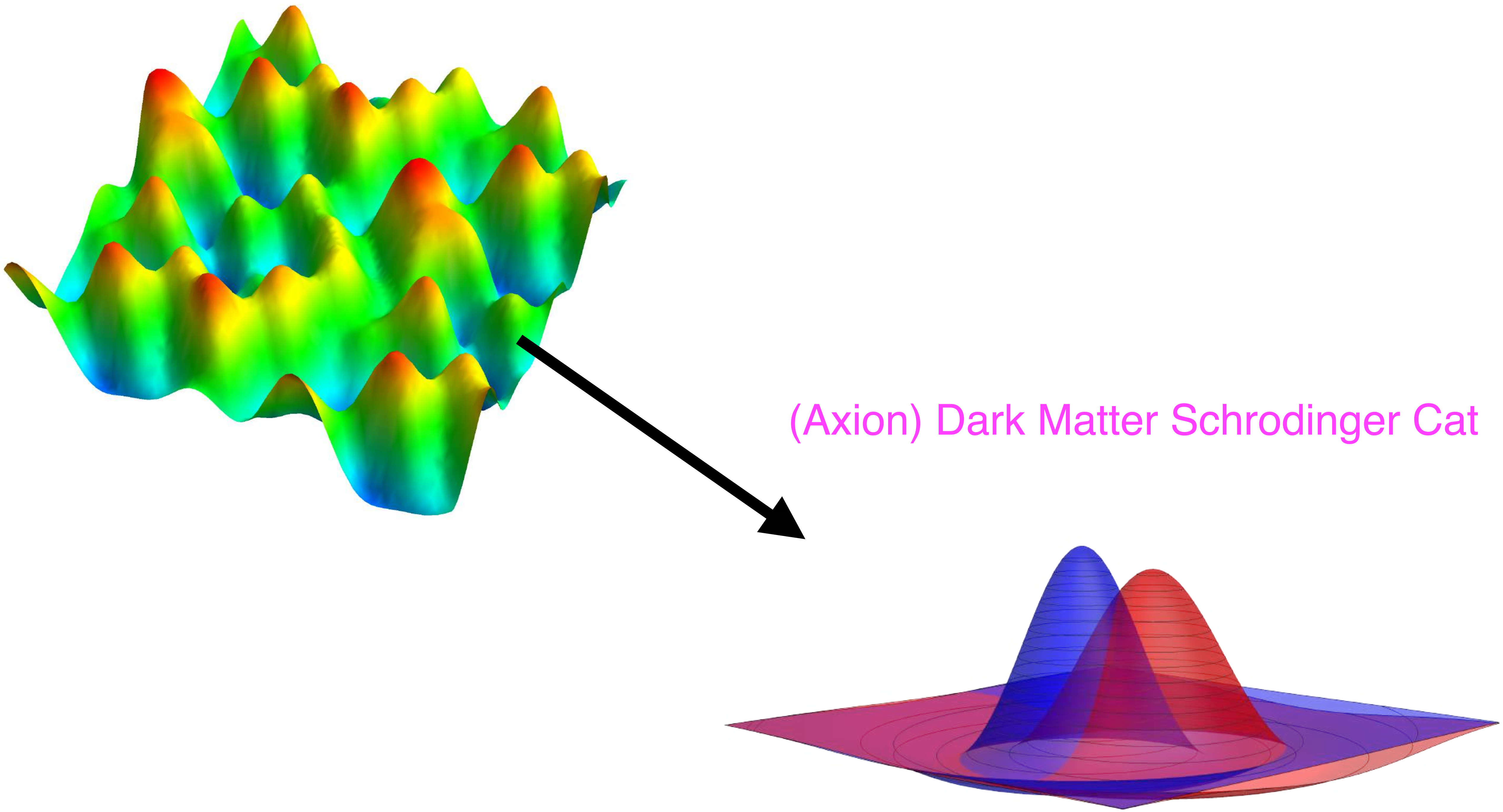

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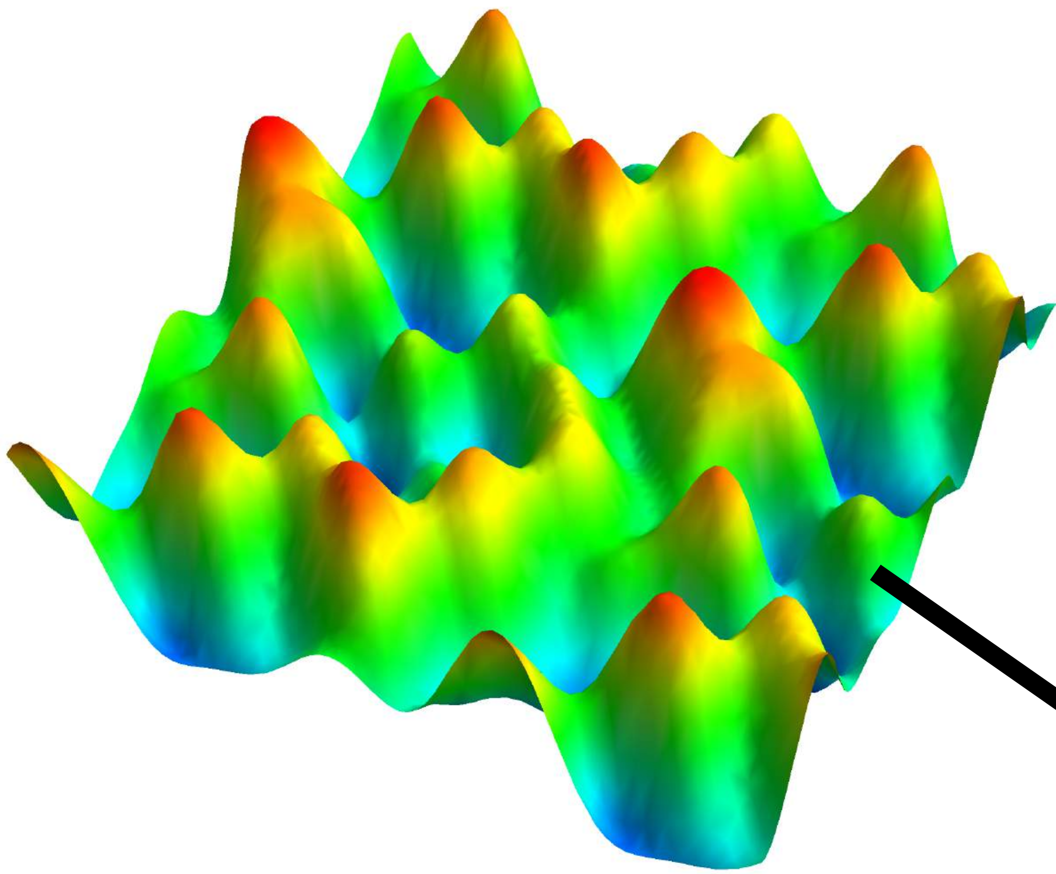
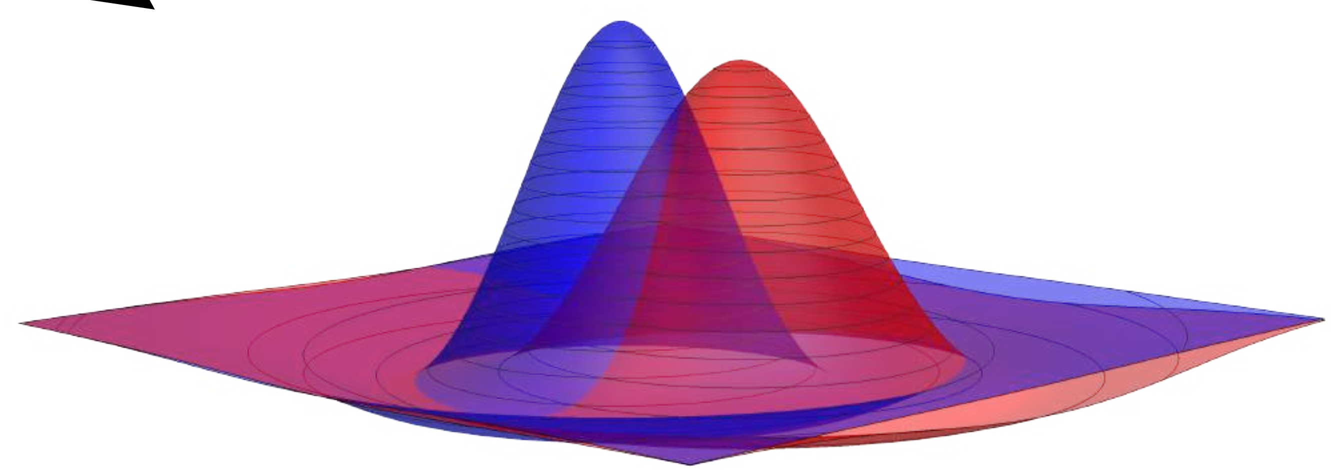


(Axion) Dark Matter Schrodinger Cat



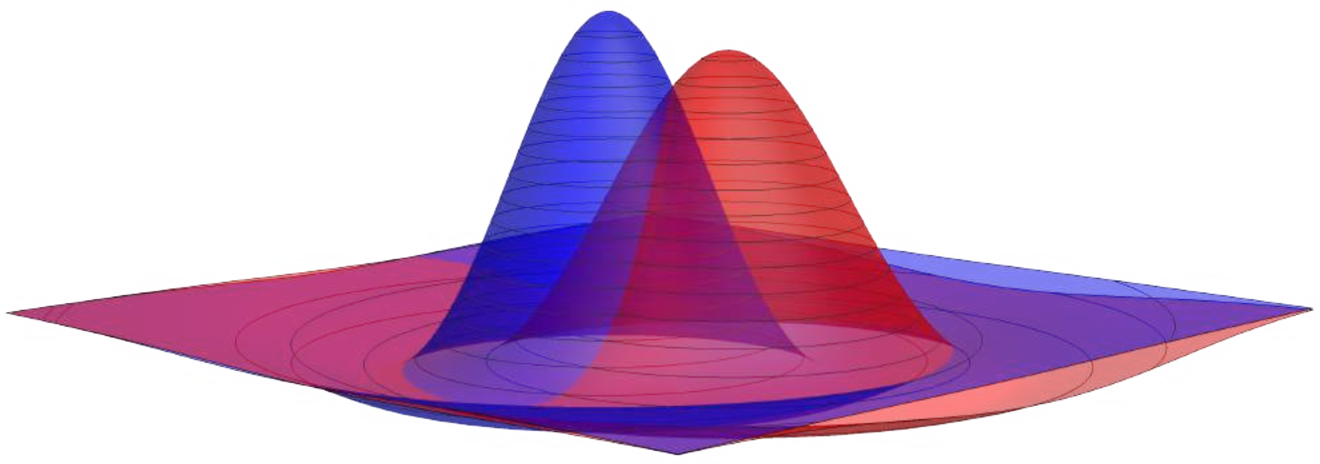
Quantumness destroyed due to DECOHERENCE???

Less clear because dark matter has tiny (non-gravitational) interactions

Could Dark Matter Schrodinger Cats Survive?

Entanglement from Gravitational Scattering

$|A_1\rangle + |A_2\rangle$



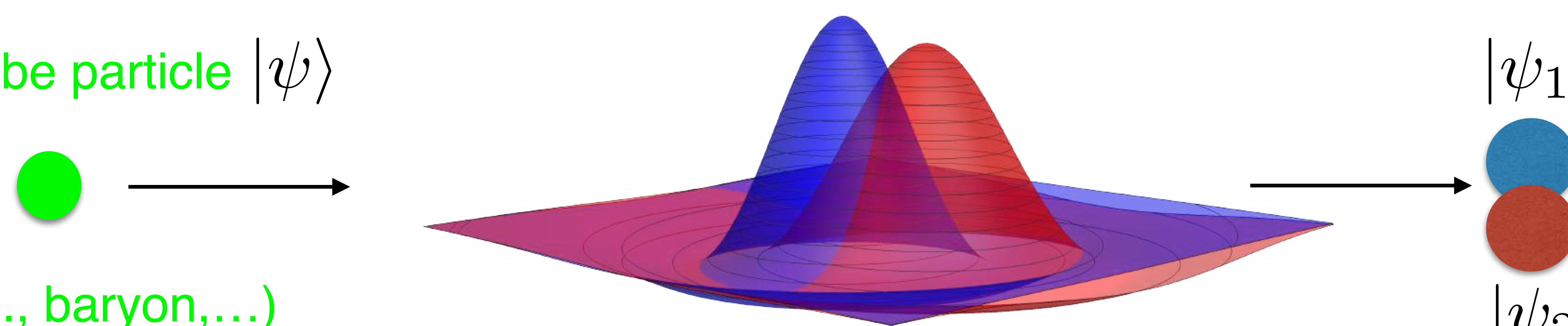
Probe particle $|\psi\rangle$

(e.g., baryon,…)

$$|\Psi_i\rangle = (|A_1\rangle + |A_2\rangle)|\psi\rangle$$

Product State

Entanglement from Gravitational Scattering



$$|\Psi_i\rangle = (|A_1\rangle + |A_2\rangle)|\psi\rangle$$

Product State

$$|\Psi_f\rangle = |A_1\rangle|\psi_1\rangle + |A_2\rangle|\psi_2\rangle$$

Entangled State

Trace Out Probe Particle

$$\rho = |\Psi_f\rangle\langle\Psi_f|$$ Full Density Matrix

$$\rho_{red} = \mathrm{Tr}_p[\rho]$$ Reduced Density Matrix

$$= |A_1\rangle\langle A_1| + |A_2\rangle\langle A_2| + \langle\psi_1|\psi_2\rangle|A_2\rangle\langle A_1| + \langle\psi_2|\psi_1\rangle|A_1\rangle\langle A_2|$$

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$$\rho_{red} = \mathrm{Tr}_p[\rho]$$ Reduced Density Matrix

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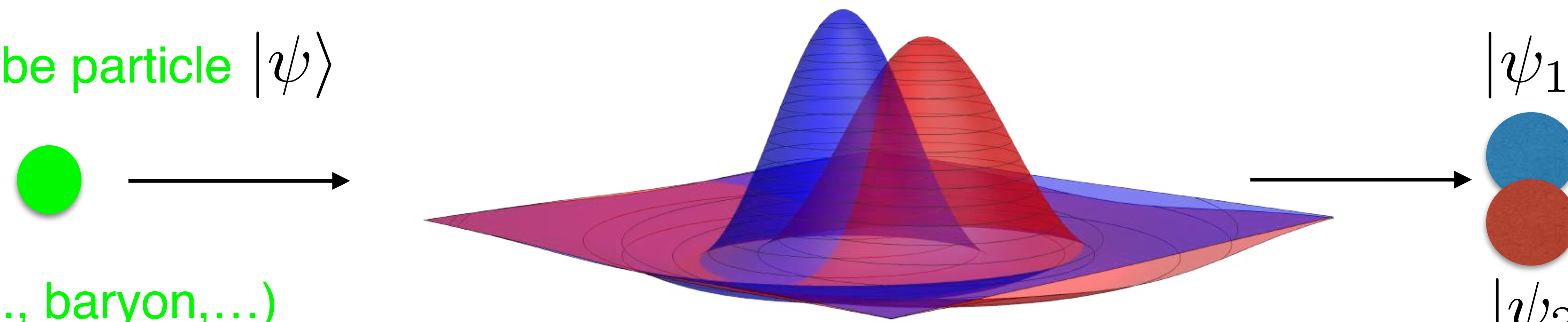
Off diagonal elements;
controlling true quantum effects

Overlap of Probe Particle States

$|A_1\rangle + |A_2\rangle$

Probe particle $|\psi\rangle$

$|\psi_1\rangle$



(e.g., baryon,…)

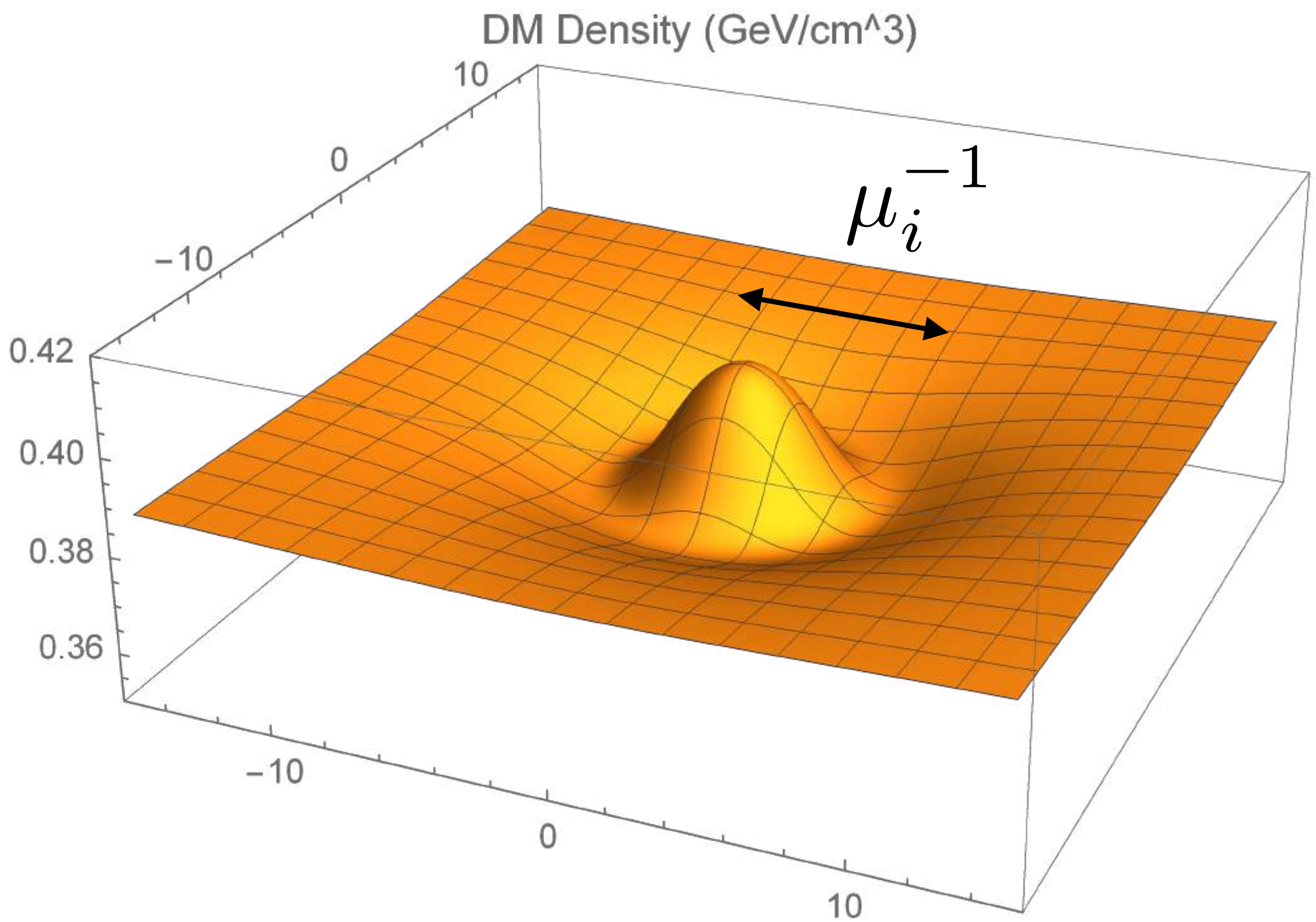
$|\psi_2\rangle$

$$|\psi_i\rangle = |\mathrm{uns}\rangle + |\mathrm{sc}_i\rangle$$

$$|\langle\psi_1|\psi_2\rangle|^2 = 1 + 2\Big(\langle sc_1|sc_2\rangle_{\mathrm{R}} - \frac{1}{2}\Big\{\langle sc_1|sc_1\rangle + \langle sc_2|sc_2\rangle\Big\}\Big) + \Big(\langle sc_1|uns\rangle_{\mathrm{I}} + \langle uns|sc_2\rangle_{\mathrm{I}}\Big)^2 + O(G^3)$$

Allali, Hertzberg 2005.12287

Choice of Axion State



$$\int d^3x\, \delta\rho_i(r) = 0$$

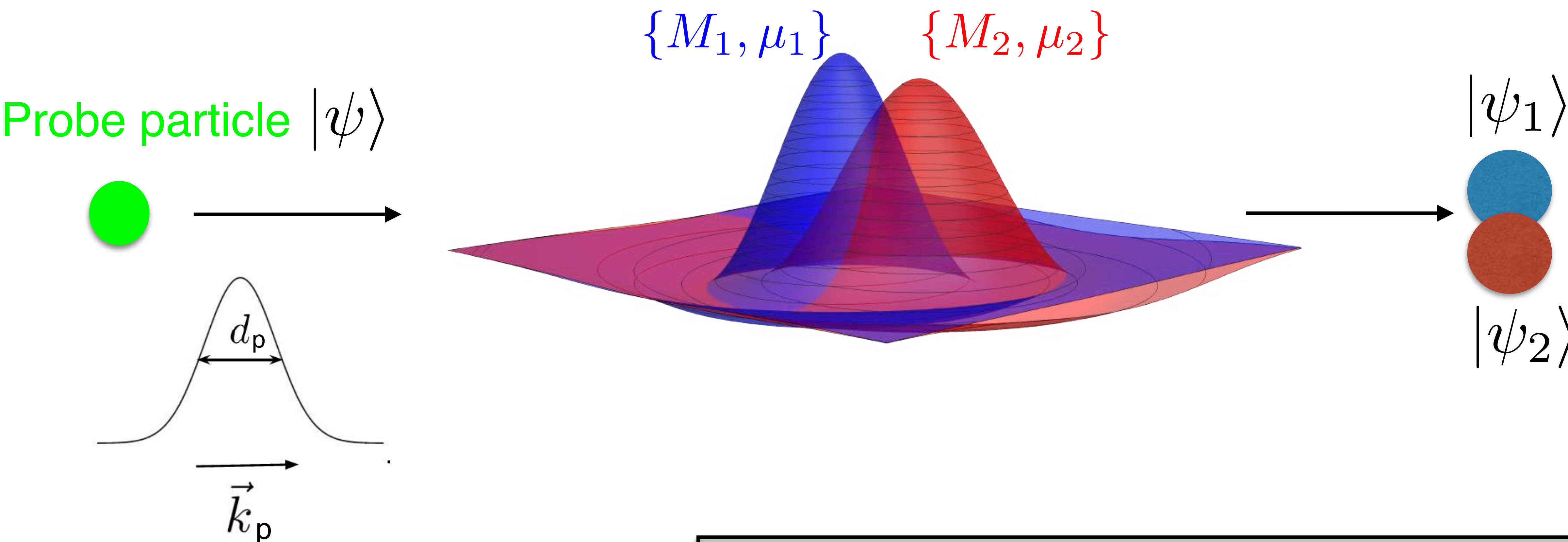
e.g., $$\delta\rho_i(r) = M_i \mu_i^3 (1 - 2r^2\mu_i^2/3) e^{-r^2\mu_i^2}$$

Zero integrated mass (otherwise even more rapid decoherence)

Allali, Hertzberg 2005.12287

Result for Overlap of Probe Particle States

We evolve a Gaussian wave packet using perturbation theory



$$|\langle\psi_1|\psi_2\rangle|^2 = 1 - 2\Delta$$

$$\boxed{\Delta_0 = \frac{2G^2m^4}{\hbar^4k^2d^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1M_2}{\mu_1\mu_2}\chi_{12}\right]}$$

Allali, Hertzberg 2005.12287

Decoherence Rate from N-Probe Particles

Off diagonal element of density matrix

$$\prod_{n=1}^{N} |\langle\psi_1|\psi_2\rangle|_n = \prod_{n=1}^{N}(1-\Delta_b) \sim e^{-\sum_{n=1}^{N}\Delta_b}$$

Decoherence rate

$$\Gamma_{\rm dec} = n\,v\int d^2b\,\Delta_b$$

$$\boxed{\Gamma_{\rm dec} = \frac{4\pi G^2 m^4 n\,v}{\hbar^4 k^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1 M_2}{\mu_1\mu_2}\chi_{12}\right]}$$

Allali, Hertzberg 2005.12287

Application to Diffuse Axions

Diffuse axions $\mu_i \sim \frac{1}{\lambda_{dB,a}} \sim \frac{2\pi}{m_a v_{vir}}$ $M_i \sim \frac{4\pi}{3} \rho_{DM} \lambda_{dB,a}^3$

Probe: Diffuse baryons $k_p \sim m_p \, v_{vir}$

Decoherence Rate $$\boxed{\Gamma_{dec} \sim \frac{G^2 \, m_p \, \rho_b \, \rho_{DM}^2}{m_a^8 \, v_{vir}^9}}$$

Decoherence Time $$t_{dec} = 1/\Gamma_{dec} \propto m_a^8$$

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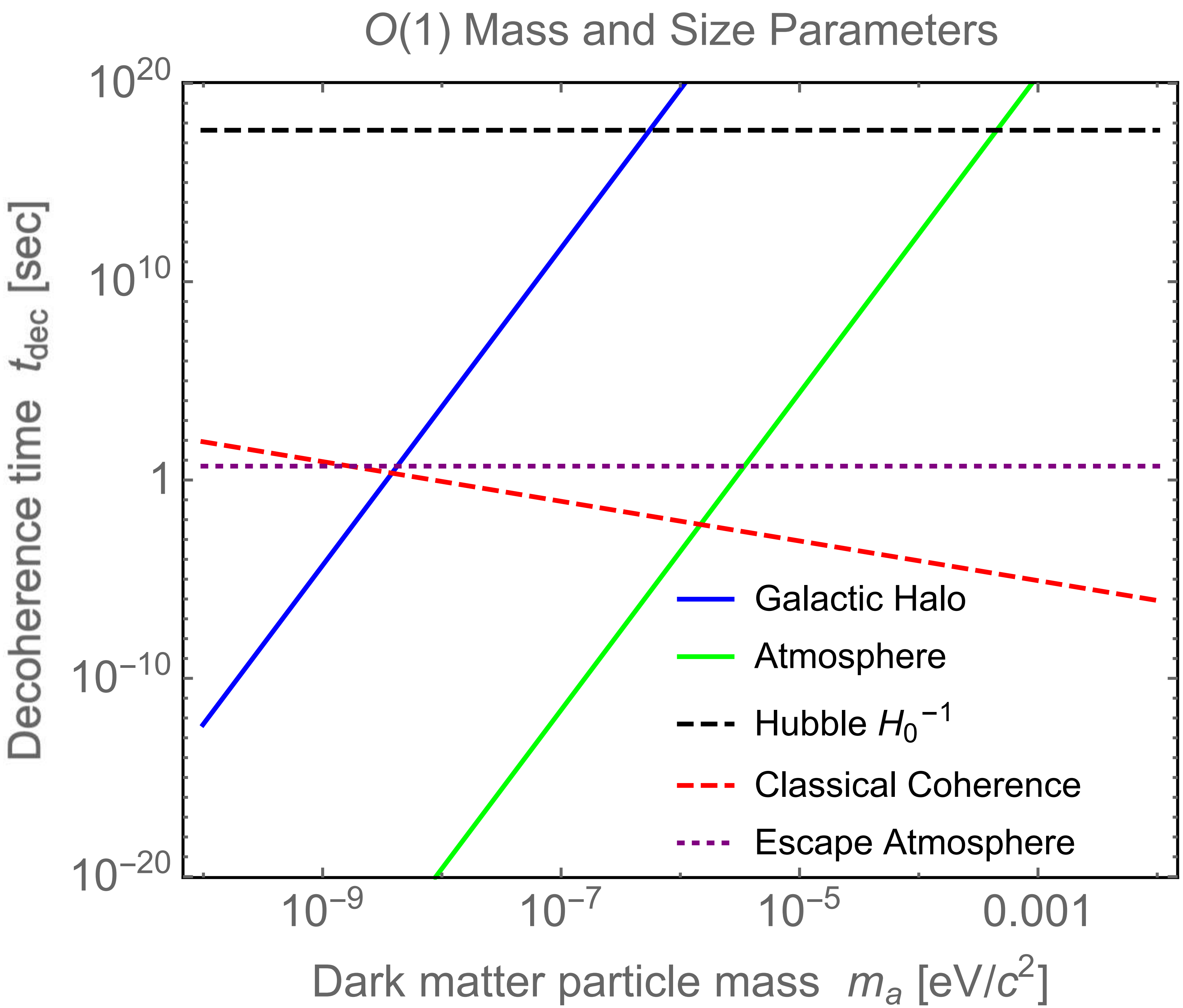
Application to Diffuse Axions



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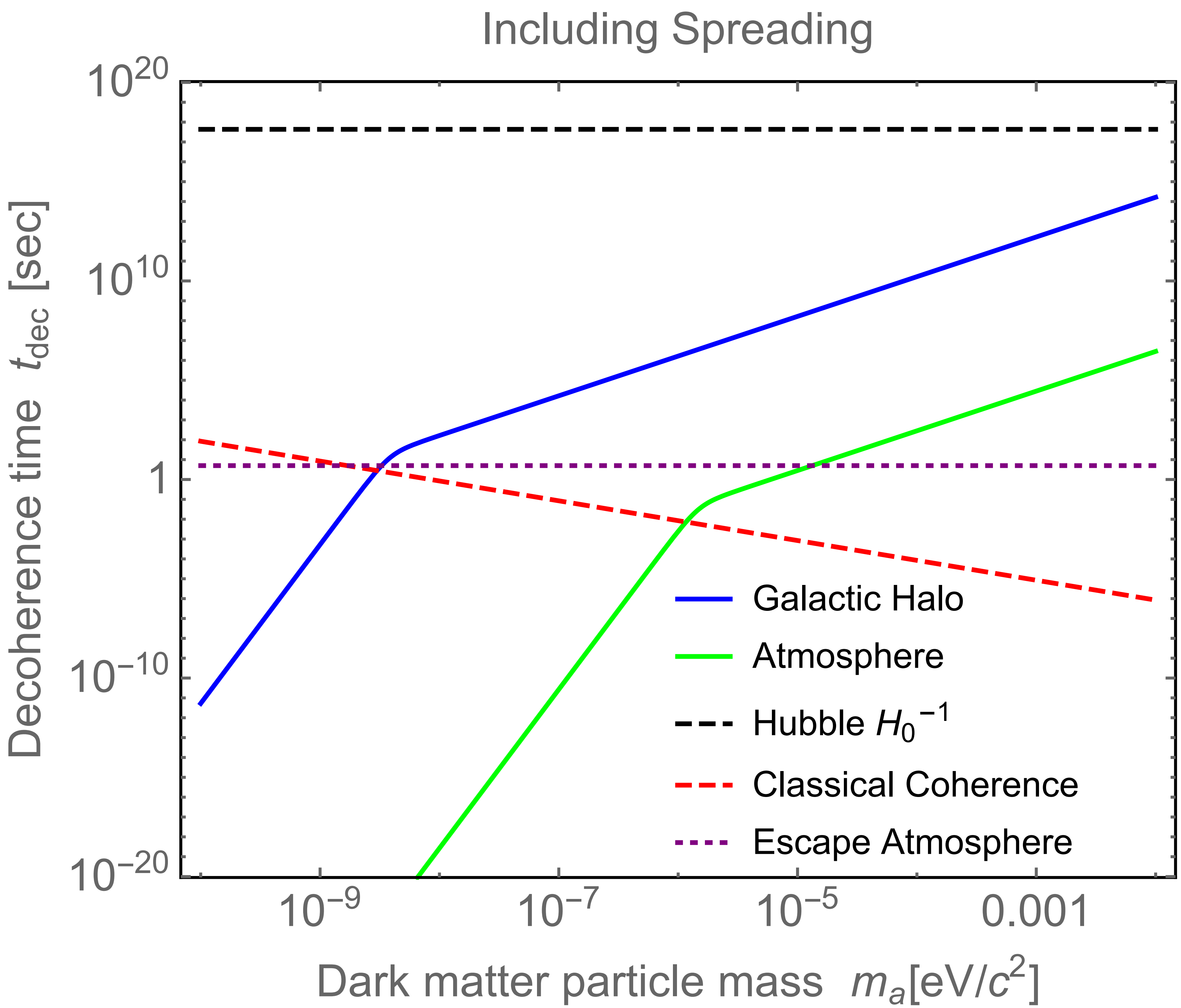
Application to Diffuse Axions



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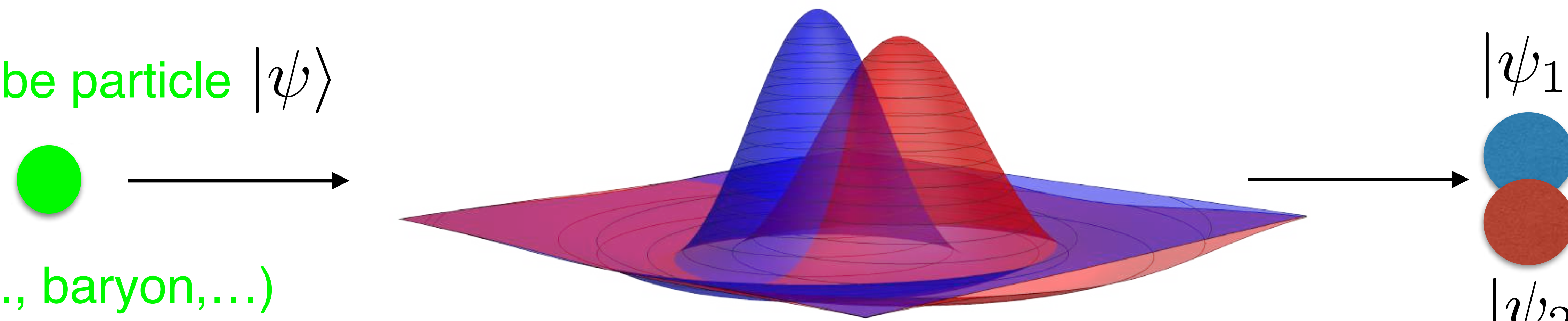
Application to Boson Stars



Boson stars are much denser

Decoherence Rate

$$\boxed{\Gamma_{\rm dec} \gtrsim \frac{\hbar^2 m_p \rho_p}{v_p m_a^4} \sim 10^{21}\,\text{sec}^{-1}\left(\frac{1\,\text{eV}}{m_a\,c^2}\right)^4}$$

Extremely rapid decoherence —> Very classical

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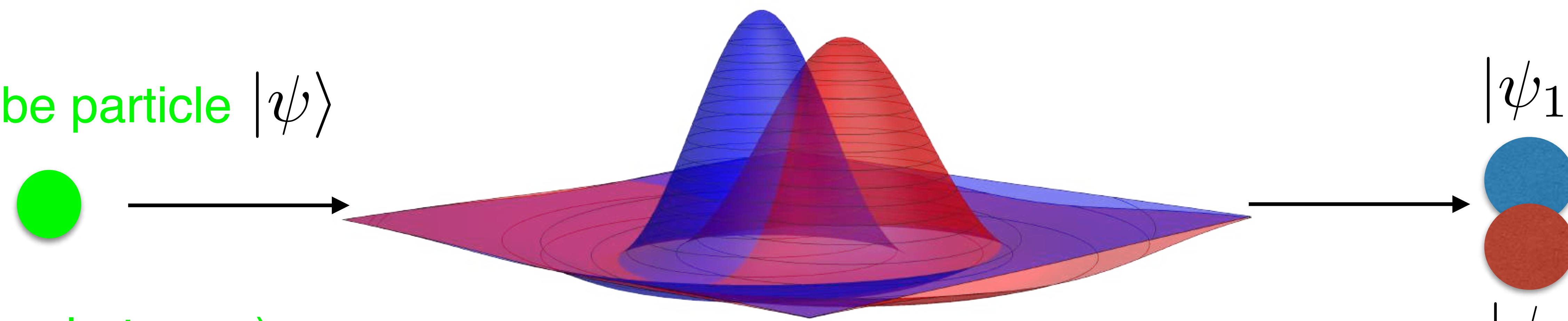
General Relativistic Extension

Probe particle $|\psi\rangle$

$|\psi_1\rangle$

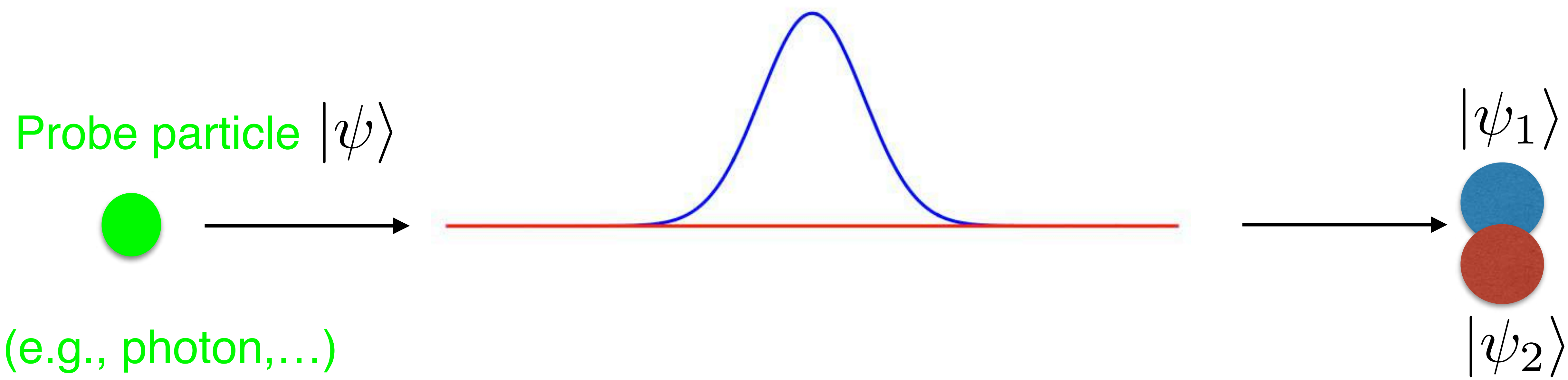


$|\psi_2\rangle$

(e.g., photon,…)

Allali, Hertzberg in preparation

General Relativistic Extension



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General Relativistic Extension

Starting from QFT, can derive in RSE for 1-particle sub-space: (ignoring spin)

$$(i\partial_t - \sqrt{-\nabla^2 + m^2})\psi(\mathbf{x},t) = \left(\Phi(\mathbf{x},t)\sqrt{-\nabla^2 + m^2} - \frac{\Psi(\mathbf{x},t)\nabla^2}{\sqrt{-\nabla^2 + m^2}}\right)\psi(\mathbf{x},t)$$

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Decoherence Rate
for superposition of
different phases

$$\boxed{\Gamma_{dec} \propto \exp\left[-\frac{m_a^2 E_p^2}{\mu^2 k^2}\right] \sim \exp\left[-\frac{1}{v_a^2 v_p^2}\right]}$$

Exponentially suppressed for non-relativistic axions or probes

So the phase is rather robust against decoherence

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Thank you