Axion assisted Schwinger effect

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DESY

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Based on **2101.05192** with V. Domcke (CERN) and K. Mukaida (CERN)

See also **1910.01205**

Main result

- Rate of standard Schwinger effect:

$$\text{(rate)} \propto \exp\left[-\frac{\pi\left(m^2+p_T^2\right)}{gE}\right] \text{ with } p_T^2 = p_x^2 + p_y^2 \text{ for } \overrightarrow{E} \propto \hat{z}\,.$$

- Axion velocity assists the production if coupled, resulting in

$$\text{(rate)} \propto \exp\left[-\frac{\pi m^2}{gE}\right] \text{ for } \dot{\phi}/f_a \gg m,\ p_T\,.$$

- Can be interpreted as axion induced spin-momentum interaction.

Motivation

- Solving Dirac equation is fun.

- Axion inflation:

  $\phi F\tilde{F}$ induces tachyonic gauge field production. [Anber, Peloso 09; ...]

  → Interesting phenos: non-gaussianity, gravitational wave, etc..

  ➡ Fermion production stops gauge field production, can be a game changer.

- Friction term to relaxion:

  Tachyonic gauge field production used for friction. [Hook, Marques-Tavares 16]

  ➡ Fermion production can again change the story.

Outline

1. Introduction
2. Review of Schwinger effect
3. Axion assisted Schwinger effect
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Schwinger effect

Schwinger effect: pair production in a strong electric field.

- Assume constant electric field in z-direction → $A_\mu = (0, 0, 0, Et)$.

$$\Omega^2(t) = (p_z + gEt)^2 + p_T^2 + m^2, \quad p_T^2 = p_x^2 + p_y^2 .$$

- Frequency depends on time → positive and negative frequency modes mixed.

$$ue^{-i\Theta} \to \alpha\, ue^{-i\Theta} + \beta\, ve^{i\Theta}, \quad \Theta = \int \Omega dt .$$

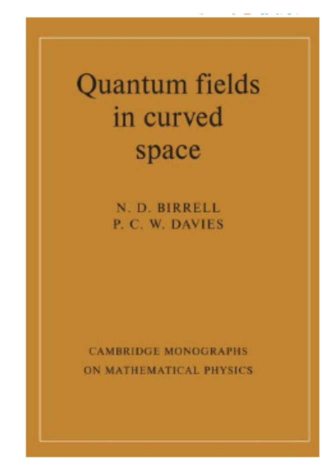
- Time evolution of Bogoliubov coefficients derived from Dirac equation:

$$\dot{\alpha} = -\frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{2i\Theta}\beta, \quad \dot{\beta} = \frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{-2i\Theta}\alpha, \quad \Pi_z = p_z + gEt .$$

- Interpreted as production from time-dependent background.

occupation number: $f_p = 2\left|\beta\right|^2 .$

chirality



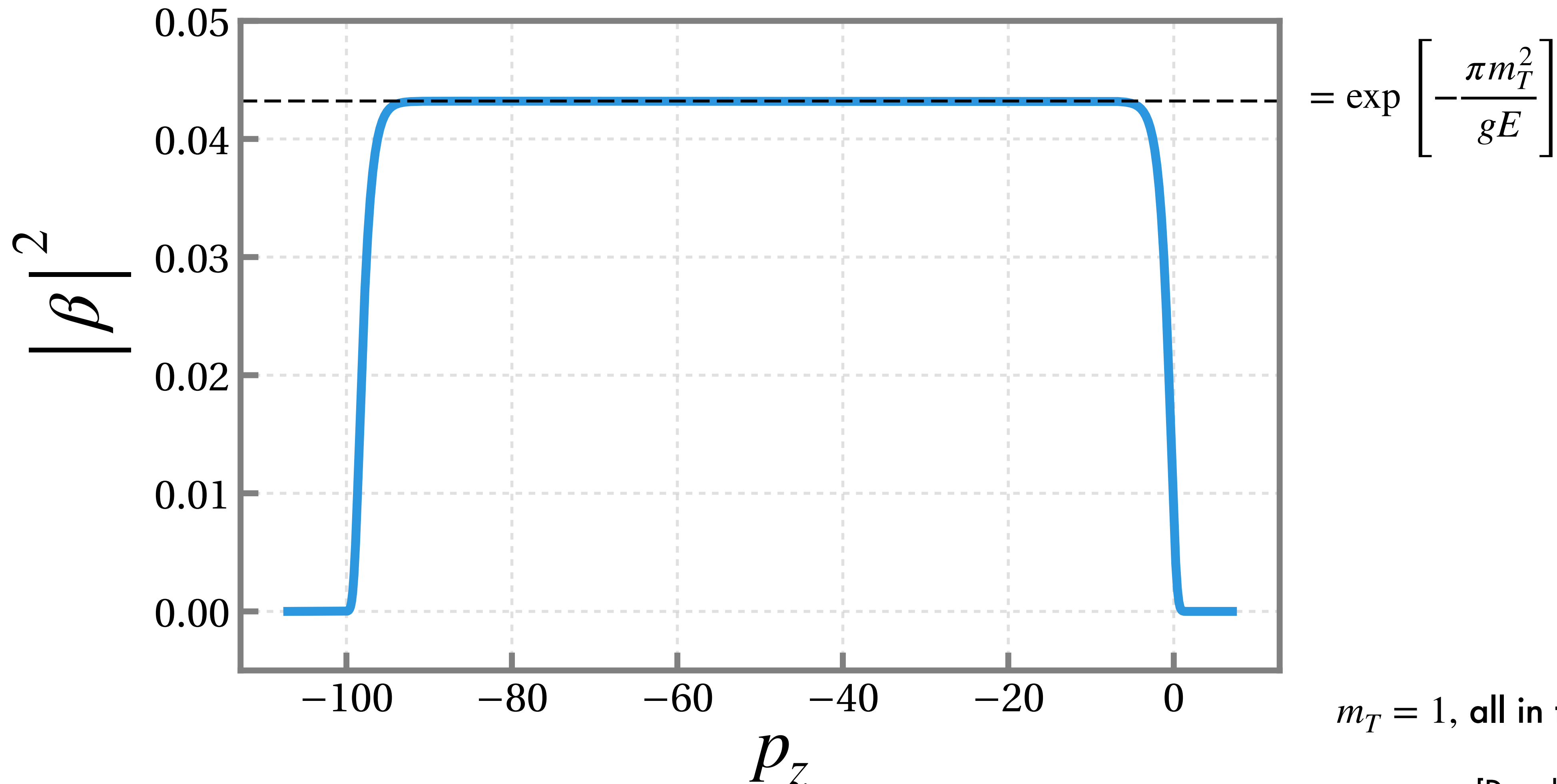
Numerical results

- Impose electric field for $0 < t < 100$ (turned on and off adiabatically).
- Develop a plateau for $-100 < p_z < 0$, modes that pass $\Pi_z = 0$.

* $\Pi_z = p_z + gEt\,.$

- Height of the plateau:

$$\left|\beta\right|^2 \simeq \exp\left[-\frac{\pi m_T^2}{gE}\right], \quad m_T^2 = p_T^2 + m^2\,.$$

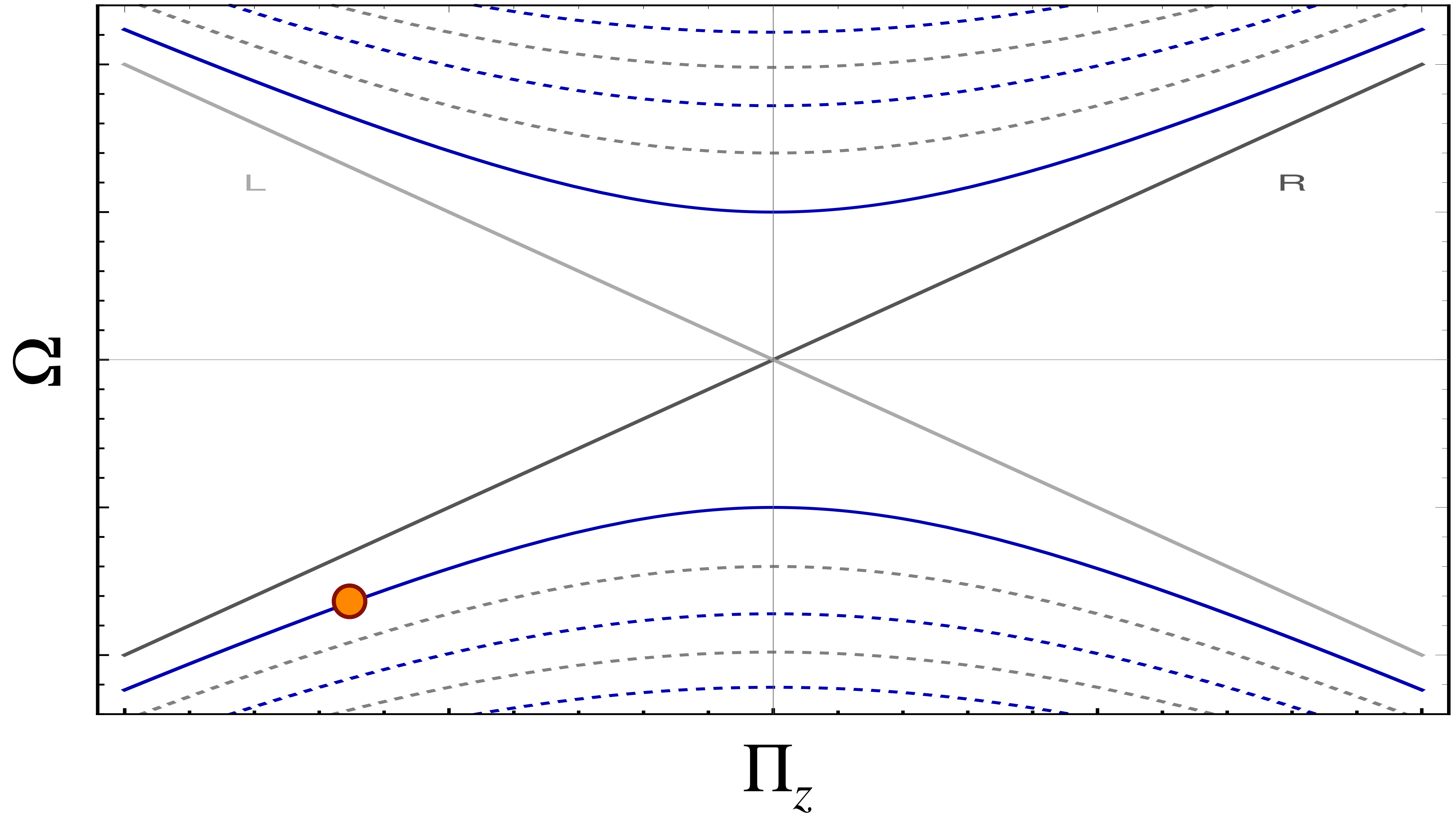


$m_T = 1$, all in the unit $gE = 1$.

[Domcke, YE, Mukaida 19]

Intuitive picture

- Particles in Dirac sea accelerated by electric field.
- Tunneling most probable for $\Pi_z = 0$ due to the smallest gap.
- Gap size $= 2m_T$ → suppression controlled by this parameter.



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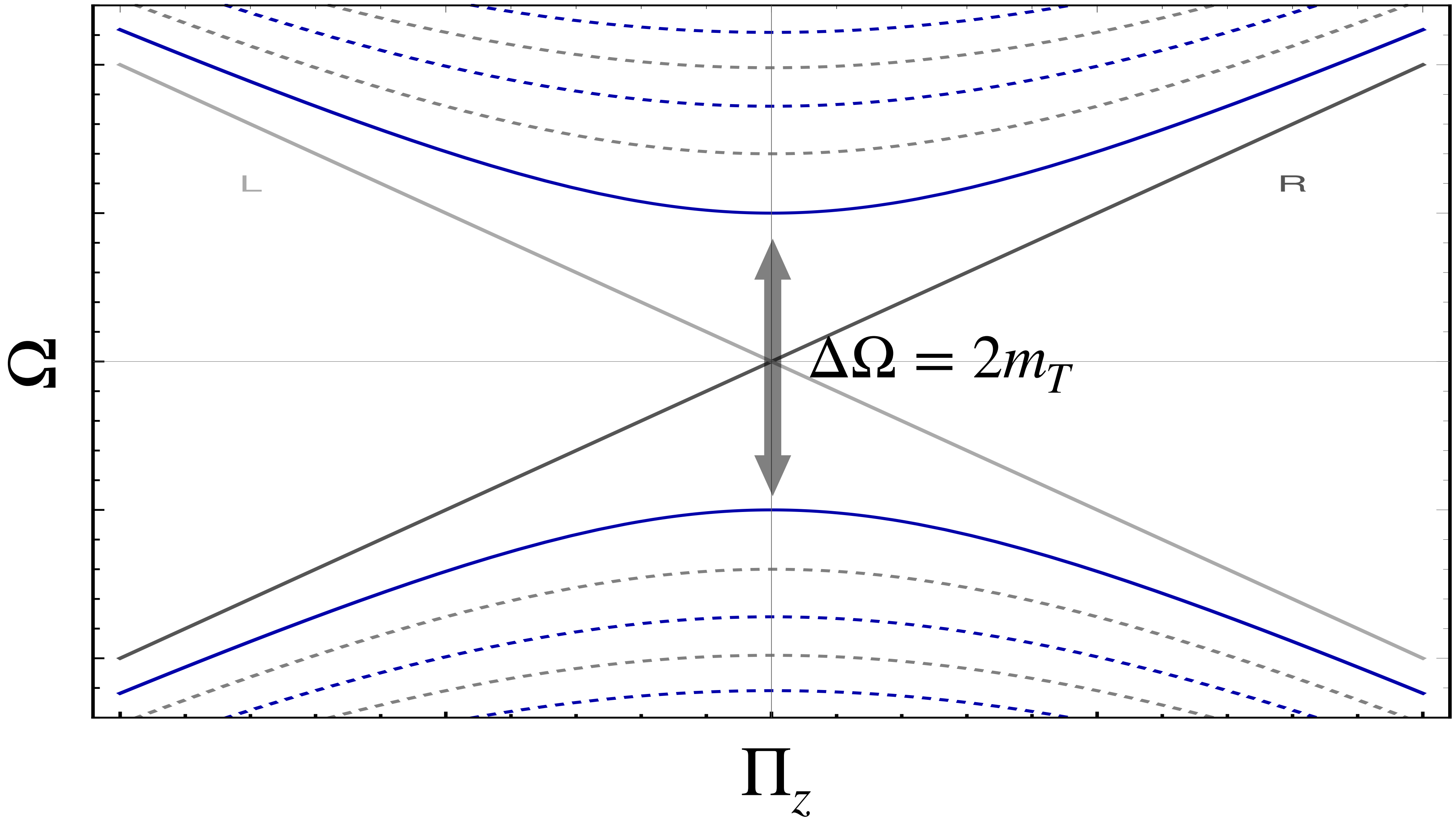


Phase integral method

- One can quantify this intuitive picture by a semiclassical method. [Dumlu, Dunne 11; ...]

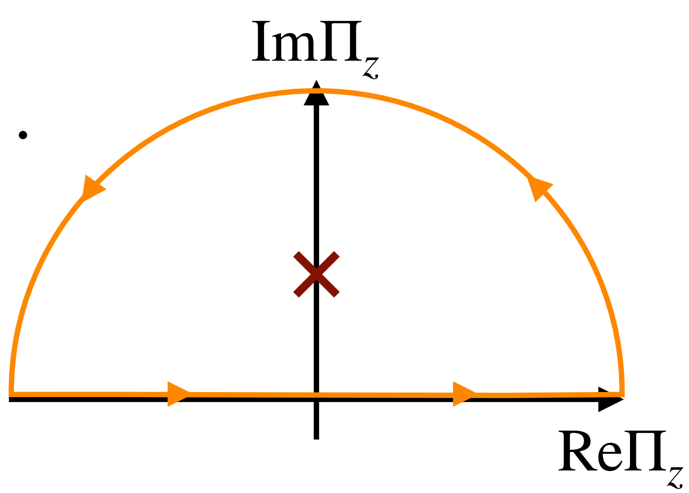
- Production exponentially suppressed → Born approximation: $\alpha \simeq 1$.

$$\dot{\beta} \simeq \frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{-2i\Theta}$$, integrated as $$\beta \simeq \int_{-\infty}^{\infty} dt \, \frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{-2i\Theta} .$$

- Close the integral contour and apply residue theorem:

$$\beta \simeq \frac{2\pi i}{gE} \sum_{\Pi_z = \Pi_\otimes} \mathrm{Res}_{\Pi_\otimes} \left[\frac{m_T \dot{\Pi}_z}{2\Omega^2} e^{-2i \int^{\Pi_z} d\Pi_z \frac{\Omega}{gE}} \right] .$$

poles: $\Omega = 0$ (turning points) → $\Pi_\otimes = \pm i m_T$.



- Suppression = "distance" of the pole from the real axis (~ gap size).

$$|\beta|^2 \sim \exp\left[-4 \int_0^{m_T} \sqrt{m_T^2 - \Pi_z^2} \, \frac{d\Pi_z}{gE} \right] = \exp\left[-\frac{\pi m_T^2}{gE} \right] .$$

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Preliminary

- Include all dim-5 operators of axion-fermion-gauge field system.

Dirac eqn: $$\left[i\gamma^\mu D_\mu - m e^{2ic_m\gamma_5\phi/f_a} + c_5 \frac{\partial_\mu \phi}{f_a}\gamma^\mu\gamma_5 \right]\psi = 0.$$

- Assume constant electric field + constant axion velocity.

➡ Again positive and negative frequency modes mixed.

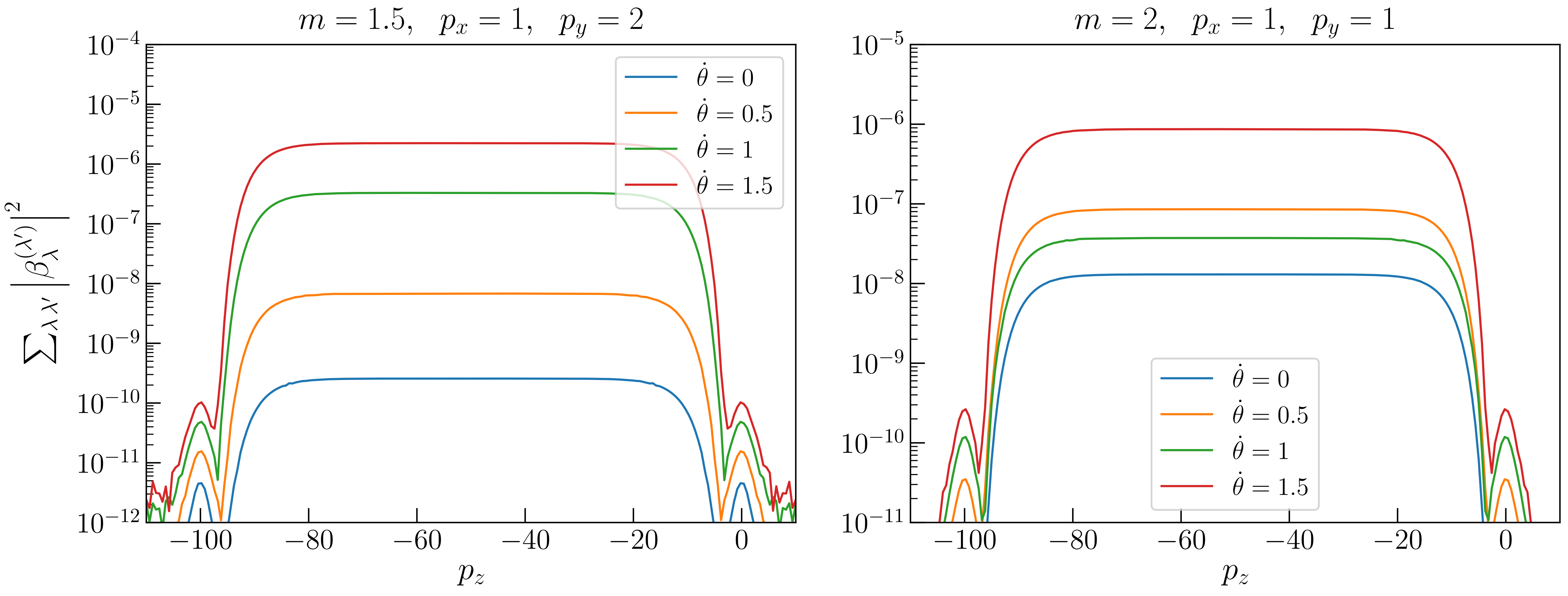
- Time evolution of Bogoliubov coefficients again from Dirac eqn:

$$\begin{pmatrix} \dot\alpha_1 \\ \dot\alpha_2 \\ \dot\beta_1 \\ \dot\beta_2 \end{pmatrix} = \left[i\dot\theta_{5+m} \begin{pmatrix} -\frac{m}{m_T}\frac{\Pi_z}{\Omega} & \frac{p_T}{m_T} & \frac{m}{\Omega}e^{2i\Theta} & 0 \\ \frac{p_T}{m_T} & \frac{m}{m_T}\frac{\Pi_z}{\Omega} & 0 & -\frac{m}{\Omega}e^{2i\Theta} \\ \frac{m}{\Omega}e^{-2i\Theta} & 0 & \frac{m}{m_T}\frac{\Pi_z}{\Omega} & \frac{p_T}{m_T} \\ 0 & -\frac{m}{\Omega}e^{-2i\Theta} & \frac{p_T}{m_T} & -\frac{m}{m_T}\frac{\Pi_z}{\Omega} \end{pmatrix} + \frac{m_T\dot\Pi_z}{2\Omega^2} \begin{pmatrix} 0 & 0 & -e^{2i\Theta} & 0 \\ 0 & 0 & 0 & -e^{2i\Theta} \\ e^{-2i\Theta} & 0 & 0 & 0 \\ 0 & e^{-2i\Theta} & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{pmatrix},$$

where $\theta_{5+m} = (c_5 + c_m)\dfrac{\phi}{f_a}$ and $1,2$: remnants of helicities.

Numerical results: spectrum

- Impose electric field and axion velocity for $0 < t < 100$ (turned on and off adiabatically).
- Again develop a plateau for $-100 < p_z < 0$.
- Height of the plateau: enhanced and strongly dependent on $\dot{\theta}_{5+m}$ (!)

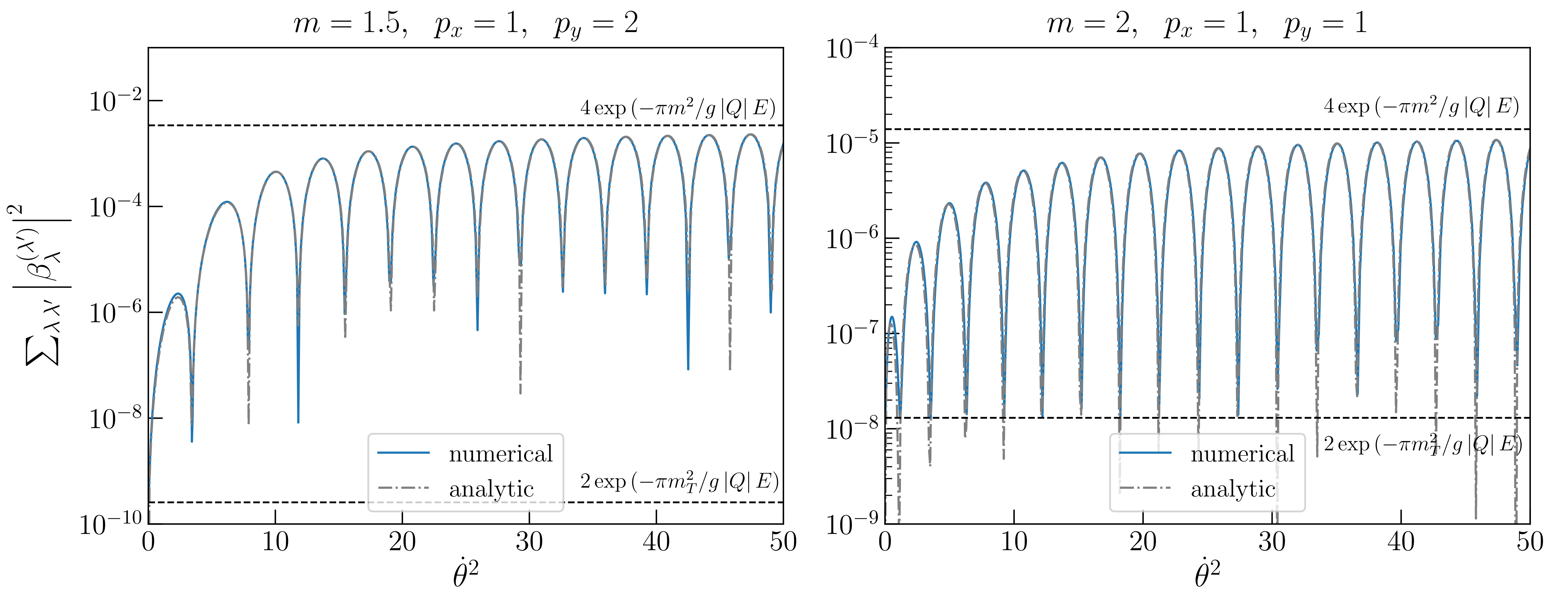


Numerical results: height

- Plot the occupation number versus $\dot{\theta}_{5+m}$ for $p_z = -50$ (middle of the plateau).
- Occupation number for large enough $\dot{\theta}_{5+m}$:

$$\left|\beta\right|^2 \sim \exp\left[-\frac{\pi m^2}{gE}\right]$$, or suppression from p_T is gone!

- On top of the enhancement, the height oscillates with $\dot{\theta}_{5+m}$.



Interpret: NR limit

- Take the non-relativistic limit $m^2 \gg gE,\ \dot\theta^2_{5+m},\ p_T^2$:

$$\mathscr{L}_\eta = \eta^\dagger i\partial_0\eta + \frac{1}{2m}\eta^\dagger\left(\vec\Pi^2 - 2\dot\theta_{5+m}\vec\Pi\cdot\vec\sigma + \dot\theta^2_{5+m}\right)\eta + \mathcal{O}\left(\frac{1}{m^2}\right), \quad \vec\Pi = \vec p + g\vec A .$$

axion induces spin-momentum interaction.

- Diagonalizing the positive frequency two component spinor η gives

$$\tilde\Omega^\pm_{\rm NR} = \frac{1}{2m}\left(\sqrt{\Pi_z^2 + p_T^2} \pm \dot\theta_{5+m}\right)^2 .$$

"gap" is smaller for $\dot\theta_{5+m} \neq 0$ and eventually gone for $\dot\theta_{5+m} > p_T$.

- We may then expect enhancement by $\exp\left[\pi p_T^2/gE\right]$, exactly what we saw.

Interpret: empirical formula

- Empirical formula that well reproduces numerical results for $m, p_T \gtrsim gE$:

$$\left|\beta\right|^2 \simeq 2\left|\exp\left(2i\int_0^{\Pi_+}\frac{d\Pi_z}{gE}\tilde{\Omega}^-\right)\right|^2 - 2\mathrm{Re}\left[\exp\left(2i\int_{\Pi_-}^{\Pi_+}\frac{d\Pi_z}{gE}\tilde{\Omega}^-\right)\right],$$

where $\tilde{\Omega}^- = \sqrt{\left(\sqrt{\Pi_z^2 + p_T^2} - \dot{\theta}_{5+m}\right)^2 + m^2}$, $\Pi_\pm = \pm\sqrt{\left(\dot{\theta}_{5+m} + im\right)^2 - p_T^2}$, or $\tilde{\Omega}^-\Big|_{\Pi_z=\Pi_\pm} = 0.$

- Oscillatory feature from second term: interference between Π_+ and Π_- .

- Empirical formula indicates that

$$\beta \sim \int_{-\infty}^{\infty} d\Pi_z f(\Pi_z)\exp\left[-2i\int\frac{d\Pi_z}{gE}\tilde{\Omega}^-\right]$$ with some function $f(\Pi_z)$.

  ➡ $\tilde{\Omega}^-$ should play the central role in particle production.

- $\tilde{\Omega}^-$ reduces to $\tilde{\Omega}^-_{\mathrm{NR}}$ in the NR limit.

  ➡ Origin would be relativistic version of spin-momentum interaction.

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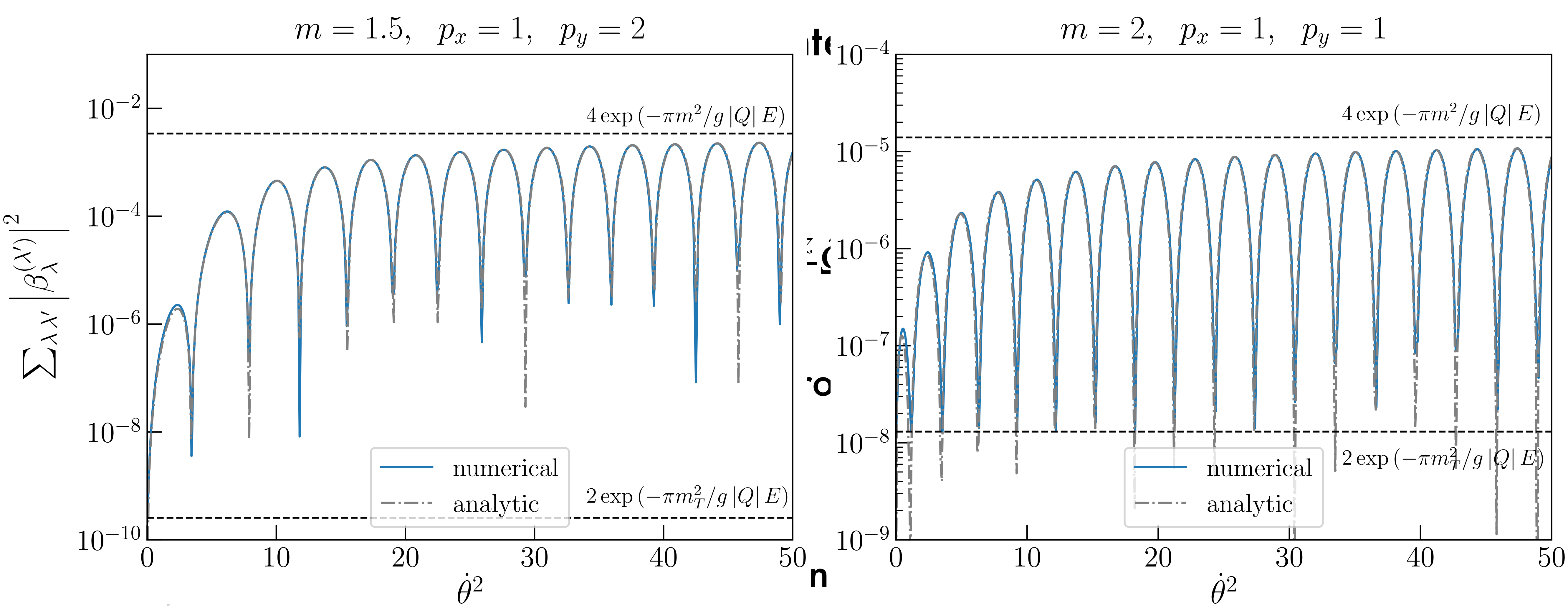

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Theoretical aspects

- Chiral rotation invariance:

✓ c_5 and c_m always appears in the combination $c_5 + c_m$ in our computation.

- Anomaly equation:

$$\partial_\mu J_5^\mu = 2im\bar{\psi}e^{2i\theta_m\gamma_5}\gamma_5\psi - \frac{g^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

➡ non-trivial check of computation for $\vec{E} \parallel \vec{B}$ ($F\tilde{F} = -4\vec{E}\cdot\vec{B}$).

✓ Follows from EoM of the Bogoliubov coefficients.

- Euler-Heisenberg Lagrangian: [Domcke, YE, Mukaida 19]

α and β contain terms suppressed by power, not exp, of m for $\dot{E} \neq 0$.

✓ Identified as higher dimensional operators, i.e., Euler-Heisenberg terms.

(Gone after turning off time-dependence, not to be confused with particle production.)

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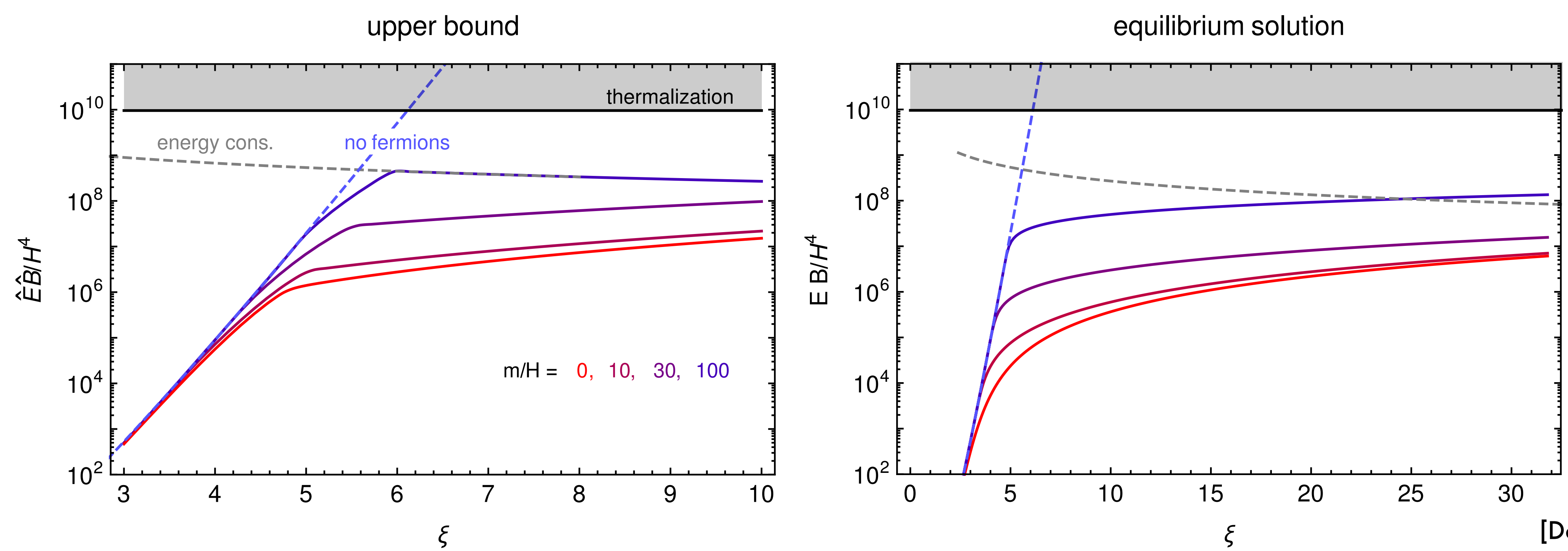
Back up

Phenomenology

- Axion-gauge field coupling induces tachyonic gauge field production.

$$\langle \vec{E}^2 \rangle \simeq 10^{-4} \frac{e^{2\pi\xi}}{\xi^3} H^4, \quad \langle \vec{B}^2 \rangle \simeq 10^{-4} \frac{e^{2\pi\xi}}{\xi^5} H^4, \quad \xi = c_A \frac{\dot{\phi}}{H f_a}.$$

- Fermion can suppress gauge boson production, even without axion coupling.



[Domcke, YE, Mukaida 19]

- Axion coupling enhances induced current, can be more effective:

$$g\langle J_z \rangle \sim \tau \times \frac{g^3 E^2}{2\pi^2} e^{-\frac{\pi m^2}{gE}} \times \max\left[\frac{B}{E}\coth\left(\frac{\pi B}{E}\right), \frac{\dot{\theta}_{5+m}^2}{\pi m^2}\right],$$ τ : duration of electric field.

Gauge boson production

- Axion velocity induces tachyonic production of gauge boson: [Anber, Sorbo 09; ...]

$$0 = \left[\frac{d^2}{d\eta^2} + k\left(k \pm 2\lambda aH\xi\right)\right] A_\pm(\eta, \vec{k}), \quad \xi = \frac{\alpha|\dot{\phi}|}{2\pi f_a H}, \quad \lambda = \mathrm{sgn}(\dot{\phi})\,.$$

➡ one helicity exponentially enhanced: $A_{-\lambda} \simeq \frac{1}{\sqrt{2k}}\left(\frac{k}{2\xi aH}\right)^{1/4} e^{\pi\xi - 2\sqrt{2k\xi/aH}}$.

- Helical gauge boson production at the end of inflation:

$$\langle \vec{E}^2 \rangle \simeq 2.6 \times 10^{-4} \frac{e^{2\pi\xi}}{\xi^3} H^4, \quad \langle \vec{B}^2 \rangle \simeq 3.0 \times 10^{-4} \frac{e^{2\pi\xi}}{\xi^5} H^4,$$

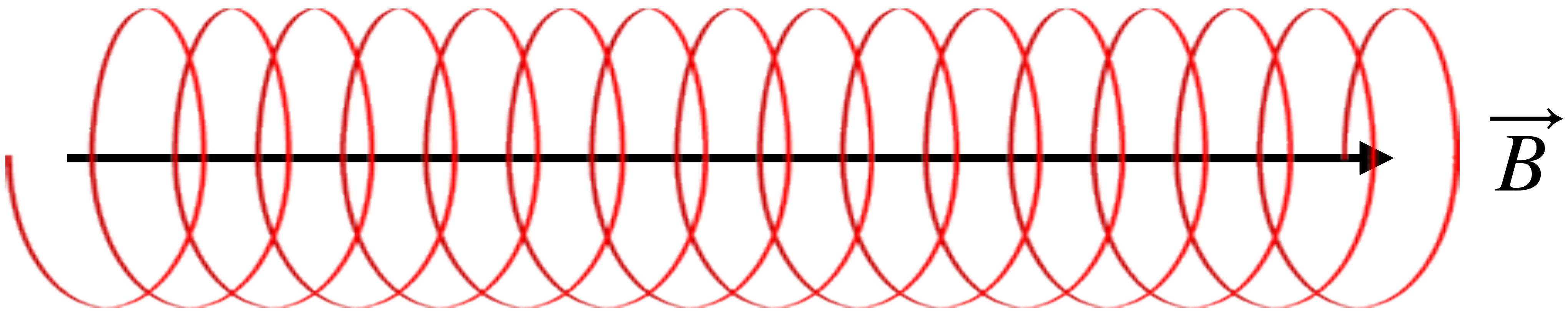
$$\langle \vec{E} \cdot \vec{B} \rangle \simeq 2.6 \times 10 - 4\lambda \frac{e^{2\pi\xi}}{\xi^4} H^4\,.$$

Homogenous within each Hubble patch and $\vec{E} \parallel \vec{B}$.

➡ $F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\vec{E} \cdot \vec{B} \neq 0$, nontrivial Chern-Simons term.

Magnetic field

- Charged particles classically spiral under magnetic field.



- Transverse motion quantum-mechanically discretized: Landau levels.

transverse momentum p_T^2 replaced by $m_B^2 = 2ng\,|\,QB\,|$ with $n = 0,\ 1,\ 2,...$

$\begin{cases} n = 0 : \text{Lowest Landau level (moving parallel to } B\text{).} \\ n \geq 1 : \text{higher Landau level (moving with transverse momentum).} \end{cases}$

Equation of motion

- Lowest Landau level: only one component appears.

$$\dot{\alpha}_0 = i\dot{\theta}_{5+m}\frac{\Pi_z}{\Omega_0}\alpha_0 - \left(\frac{m\dot{\Pi}_z}{2\Omega_0^2} + i\dot{\theta}_{5+m}\frac{m}{\Omega_0}\right)e^{2i\Theta_0}\beta_0,$$

$$\dot{\beta}_0 = -i\dot{\theta}_{5+m}\frac{\Pi_z}{\Omega_0}\alpha_0 + \left(\frac{m\dot{\Pi}_z}{2\Omega_0^2} - i\dot{\theta}_{5+m}\frac{m}{\Omega_0}\right)e^{2i\Theta_0}\beta_0,$$

where $\Omega_0 = \sqrt{\Pi_z^2 + m^2}, \quad \Theta_0 = \int dt\, \Omega_0\,.$

- Higher Landau levels: the same as without B after $p_T \to m_B$.

$$\begin{pmatrix}\dot{\alpha}_1\\ \dot{\alpha}_2\\ \dot{\beta}_1\\ \dot{\beta}_2\end{pmatrix} = \left[i\dot{\theta}_{5+m}\begin{pmatrix} -\frac{m}{m_T}\frac{\Pi_z}{\Omega} & \frac{m_B}{m_T} & \frac{m}{\Omega}e^{2i\Theta} & 0 \\ \frac{m_B}{m_T} & \frac{m}{m_T}\frac{\Pi_z}{\Omega} & 0 & -\frac{m}{\Omega}e^{2i\Theta} \\ \frac{m}{\Omega}e^{-2i\Theta} & 0 & \frac{m}{m_T}\frac{\Pi_z}{\Omega} & \frac{m_B}{m_T} \\ 0 & -\frac{m}{\Omega}e^{-2i\Theta} & \frac{m_B}{m_T} & -\frac{m}{m_T}\frac{\Pi_z}{\Omega} \end{pmatrix} + \frac{m_T\dot{\Pi}_z}{2\Omega^2}\begin{pmatrix} 0 & 0 & -e^{2i\Theta} & 0 \\ 0 & 0 & 0 & -e^{2i\Theta} \\ e^{-2i\Theta} & 0 & 0 & 0 \\ 0 & e^{-2i\Theta} & 0 & 0 \end{pmatrix}\right]\begin{pmatrix}\alpha_1\\ \alpha_2\\ \beta_1\\ \beta_2\end{pmatrix}.$$

Anomaly equation

- Anomaly equation should hold in any background spacetime.

$$\partial_\mu J_5^\mu = -\frac{g^2Q^2}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + 2im\bar{\psi}e^{2i\theta_m\gamma_5}\gamma_5\psi, \quad J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi\,.$$

➡ spatially averaged version: $\dot{q}_5 = \dfrac{g^2Q^2EB}{2\pi^2} + 2im\left\langle\bar{\psi}e^{2i\theta_m\gamma_5}\gamma_5\psi\right\rangle.$

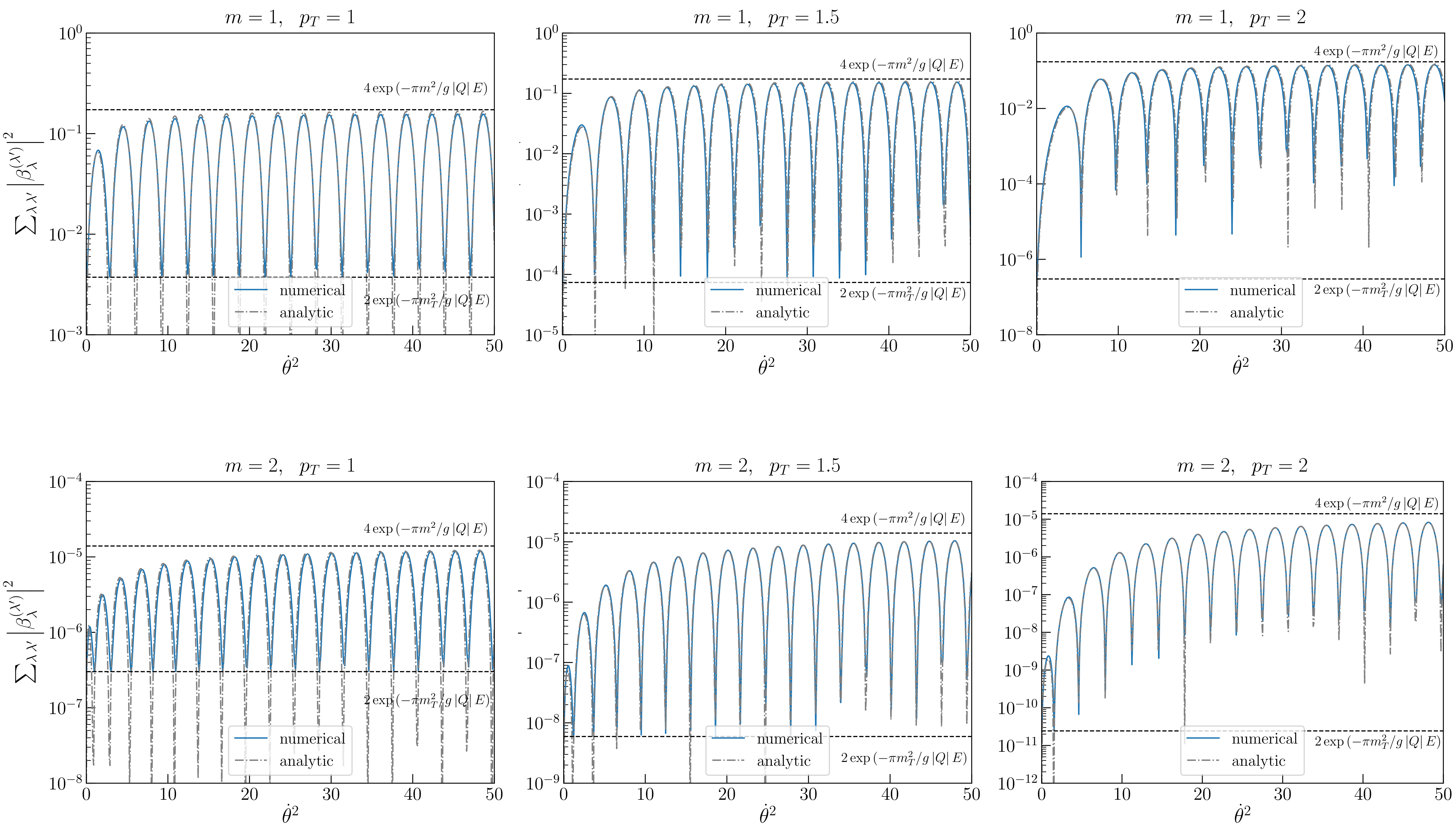
- Follows directly from equations of motion of α and β in the current case.

$$\begin{cases} \text{lowest Landau level: } \dot{q}_{5,0} = \dfrac{g^2Q^2EB}{2\pi^2} + 2im\left\langle\bar{\psi}e^{2i\theta_m\gamma_5}\gamma_5\psi\right\rangle\Big|_{\mathrm{LLL}}. \\ \text{higher Landau level: } \dot{q}_{5,n} = 2im\left\langle\bar{\psi}e^{2i\theta_m\gamma_5}\gamma_5\psi\right\rangle\Big|_{\mathrm{HLL},n}. \end{cases}$$

➡ Chern-Simons term served entirely by the lowest Landau level.

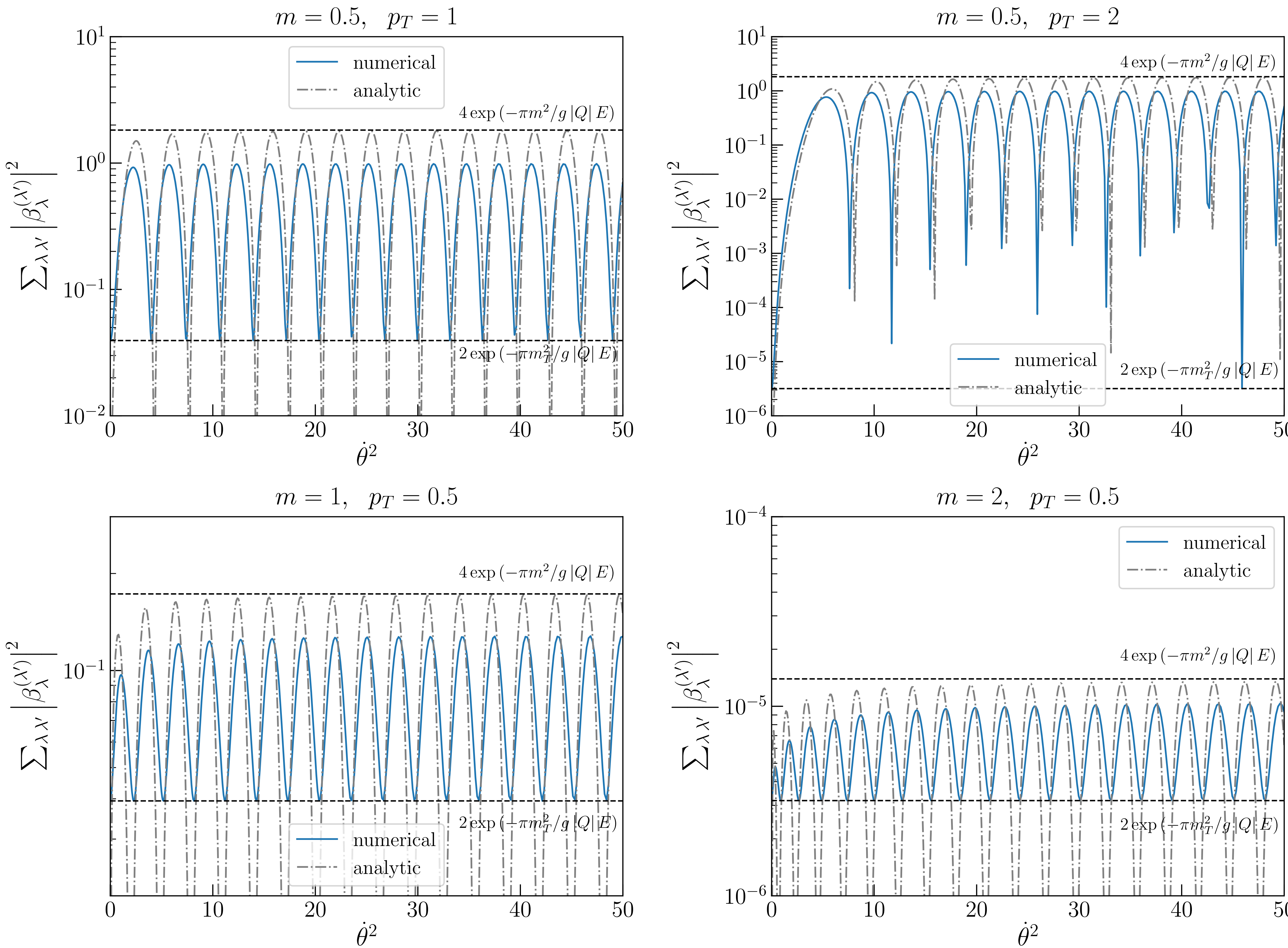
Analytic formula

Well reproduces numerical results for $m, p_T \gtrsim gE$.



Limitation

Does not work well for $m, p_T \lesssim gE$.

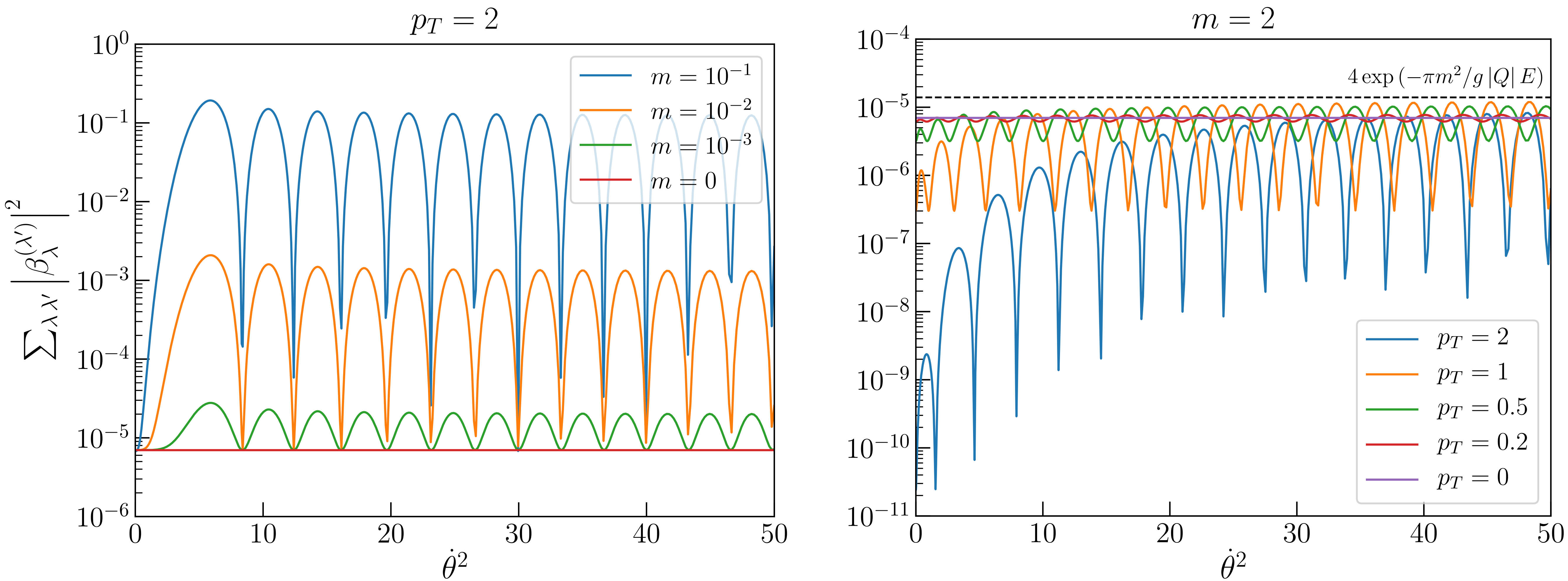


Small m or p_T limit

- Enhancement is gone in the limit $m \to 0$.

  Should be the case since $c_5 + c_m$ is unphysical in this limit.

- Enhancement is more important for larger p_T.



Larger axion velocity

- Gap of $\tilde{\Omega}^-$: takes minimum at $\Pi_z = \pm\sqrt{\dot{\theta}^2_{5+m} - p_T^2}$, not at $\Pi_z = 0$.

Remember that $\tilde{\Omega}^- = \sqrt{\left(\sqrt{\Pi_z^2 + p_T^2} - \dot{\theta}_{5+m}\right)^2 + m^2}$.

- Can be seen also in the numerical results.