Dark Matter During Supercooled Confinement

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`2007.08440, 2ymm.nnnnn`

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IPMU Astro Particle Experimental Cosmo Seminar
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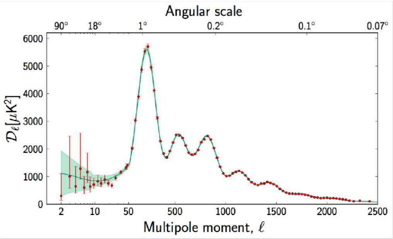
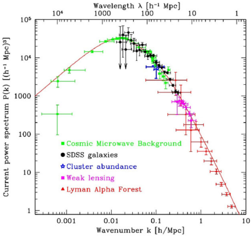
Overview

Overview

1. Motivation
2. DM from supercooling
3. Application to composite DM
4. Dilaton portal pheno

Motivation...





Observed DM density

$$Y_{\rm DM} \equiv \frac{n_{\rm DM}}{s} \approx \frac{0.4{\rm eV}}{M_{\rm DM}}$$

As has been known for a long time:

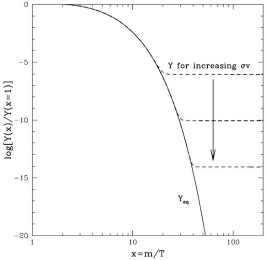
- UV thermal value $Y^{\rm eq}_{\rm DM} = 45\zeta(3) g_{\rm DM}/2\pi^4 g_{*s} \sim 0.01 - 0.001$.
- But standard hot DM doesn't work.
- Need a non-thermal production (QCD axion), heavy DM (WIMP), other complications (secluded sectors...)

Heavy DM density - Usual picture

Observed DM density

$$Y_{\rm DM} \equiv \frac{n_{\rm DM}}{s} \approx 0.4 \times 10^{-12} \left(\frac{\rm TeV}{M_{\rm DM}}\right)$$

Need to suppress from UV thermal value $Y_{\rm DM}^{\rm eq} \sim 0.001$.



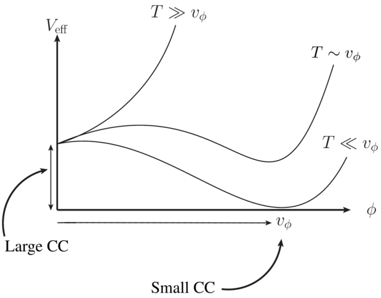
Usual way: $\langle \sigma v \rangle \sim 10^{-26} {\rm cm}^3/{\rm s}$.

Limited by unitarity $\sigma v < 4\pi(2J+1)/(M_{\rm DM}^2 v)$ to $M_{\rm DM} \lesssim 100$ TeV

One may instead consider some modified expansion due to

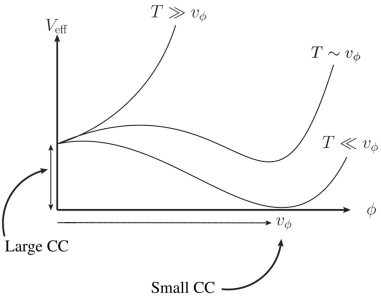
- Early matter (well studied)
- Or vacuum domination.

Alternative: Some late time inflation



- Field becomes stuck behind the barrier.
- Inflation starts when $\rho_{\rm vac} \sim v_\phi^4 \sim g_* T^4$.
- Phase transition takes place at some $T_n \ll v_\phi$.
- Bubbles accelerate and collide, reheating universe: $\rho_{\rm vac} \to$ Bubble walls $\to$ Oscillations $\to$ Radiation.
- Chemically decoupled species are diluted.

Alternative: Some late time inflation



- Temperature evolution avoids graceful exit problem
- DM should not come back into equilibrium follow the PT
- PT may also be followed by matter domination

DM is diluted by a volume factor

$$Y_{\rm DM} = Y_{\rm DM}^{\rm eq} \left(\frac{T_{\rm nuc}}{T_{\rm infl}} \right)^3 \frac{T_{\rm RH}}{T_{\rm infl}}$$

Supercooling

For this to work, we require $T_n \ll T_c$, which is possible in close-to-conformal potentials.

Some examples

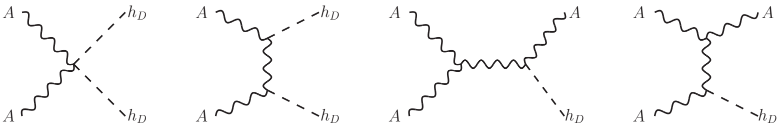
1. Coleman-Weinberg potential from radiative symmetry breaking
2. Confining theores with a light dilaton $\to$ focus here
3. RS with the Goldberger-Wise mechanism.

The first was studied in the paper Supercool DM - Hambye, Teresi, Strumia 1805.01473
Let me say a few words on this as an introduction as it serves as a template for the composite case.

Example 1: Elementary relic

Supercool DM - Hambye et al. 1805.01473

$SU(2)_D$ Vector Dark Matter with a Classically Scale Invariant Potential.

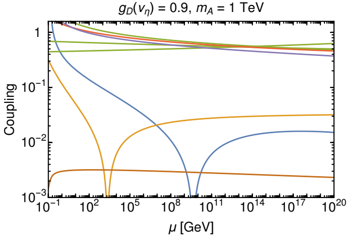


The Model: $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_D$

$$\mathcal{L} \supset -\frac{1}{4} F_D \cdot F_D + (\mathcal{D} H_D)^\dagger (\mathcal{D} H_D) - \cancel{\frac{\mu^2}{2} H_D^\dagger H_D} - \lambda_\eta \, (H_D^\dagger H_D)^2 - \lambda_{h\eta} \, H_D^\dagger H_D \, H^\dagger H$$

The dark gauge bosons A make up the DM. They communicate to the SM via the higgs portal.

Radiative Symmetry Breaking



We start with a classically scale invariant theory

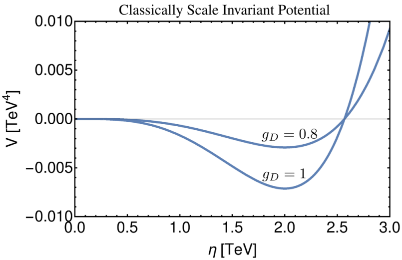
- The dark gauge coupling drives the exotic quartic negative in the IR

$$\beta_{\lambda_\eta} = \frac{1}{(4\pi)^2}\left(\frac{9}{8}g_D^4 - 9g_D^2\lambda_\eta + 2\lambda_{h\eta}^2 + 24\lambda_\eta^2\right)$$

- This signals radiative symmetry breaking - Coleman, E. Weinberg '73
- The potential is approximated in the flat direction in field space - Gildener, S. Weinberg '76

Classically Scale Invariant Potential

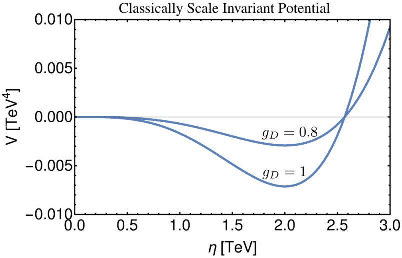
- Hambye, Strumia 1306.2329



Potential at $T = 0$

$$V_1^0(\eta) \simeq \frac{9 g_D^4 \eta^4}{512\pi^2} \left(\mathrm{Ln}\left[\frac{\eta}{v_\eta}\right] - \frac{1}{4} \right)$$

Nucleation Temperature



Thermal Contribution

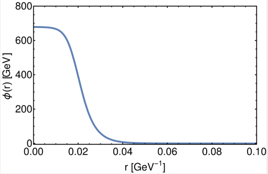
$$\frac{2\pi^2}{T^4} V_1^T(\eta, T) = \int_0^\infty y^2 \mathrm{Log}\left(1 - e^{-\sqrt{y^2 + m^2(\eta)/T^2}}\right) \mathrm{d}y$$
$$\approx -\frac{\pi^4}{45} + \frac{\pi^2 m^2}{12 T^2} - \frac{\pi m^3}{6 T^3} - \frac{m^4}{32 T^4} \mathrm{Ln}\left(\frac{m^2}{220 T^2}\right)$$

$$V_{\mathrm{eff}} = V_1^0(\eta) + V_1^T(\eta, T) + (V_{\mathrm{daisy}} + V_{\mathrm{QCD}})$$

Nucleation Temperature

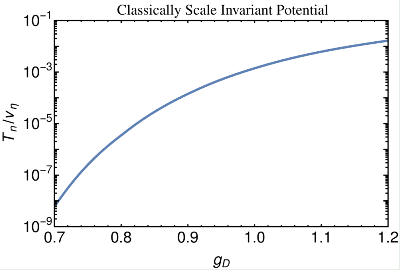
Euclidean Action

$$S_3 = 4\pi \int r^2 \left(\frac{1}{2} \left(\frac{d\phi_i}{dr} \right)^2 + \Delta V(\phi, \eta, T) \right) dr$$

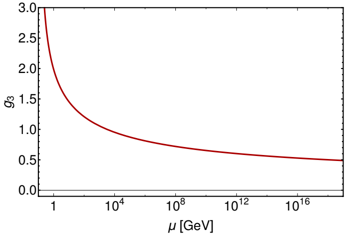


Nucleation when $\Gamma/V \sim T^4 e^{-S_3/T} \sim H^4$.

Universe generically becomes vacuum dominated before PT



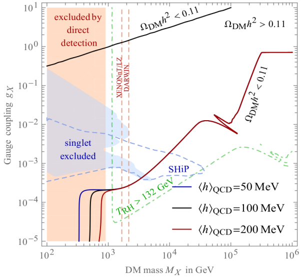
Taking into account QCD



If $T_n \lesssim \Lambda_{\rm QCD}$, QCD confinement must be taken into account.

- When QCD confines a mass scale enters the potential.
- EW Symmetry is broken by the quark condensate.
- The Higgs gets a VEV $\langle h \rangle \sim \Lambda_{\rm QCD}$ induced by $y_t h \langle \overline{t_L} t_R \rangle$.
  - Witten '81
- This gives a mass term $V_{\rm eff} \supset -\lambda_{h\eta} \Lambda_{QCD}^2 \eta^2$.
- The thermal barrier disappears at $T \sim m_h \Lambda_{QCD} / m_A$.
  - Iso, Serpico, Shimada 1704.04955

DM relic density

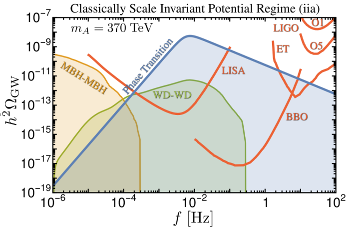


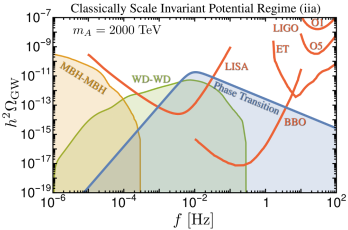
Super-cool DM - Hambye, Strumia, Teresi 1805.01473

$$Y_{\rm DM}|_{\rm super-cool} = Y_{\rm DM}^{\rm eq} \frac{T_{\rm RH}}{T_{\rm infl}} \left(\frac{T_{\rm n}}{T_{\rm infl}} \right)^3$$

$$Y_{\rm DM}|_{\rm sub-thermal} = M_{\rm Pl} M_{\rm DM} \langle \sigma_{\rm ann} v_{\rm rel} \rangle \sqrt{\frac{\pi g_*}{45}} \int_{z_{\rm RH}}^{\infty} \frac{dz}{z^2} Y_{\rm eq}^2$$

GW test





- Sufficient supercooling possible to set the relic density.
- GWs can test the scenario for $m_A \gtrsim 370$ TeV. IB, Garcia-Cely 1809.01198
- Signal is supressed as m_A increases due to inefficient reheating through $-\lambda_{h\eta}\, H_D^\dagger H_D\, H^\dagger H$
- Lower DM masses possible but GW signal suppressed by QCD effects.

More on GWs later...

Composite models

Composite models

- We now want to extend this picture to composite DM models.
- For a low compositeness scale the EW hierarchy problem may also be addressed.
- We will also, however, consider high scales for which tuning is necessary.

IB, Y. Gouttenoire, F. Sala 2007.08440

+ G. Servant 2ymm.nnnnn

Example 2: Composite relic

Supercooled Composite DM from a Confining Phase Transition.

Assume for now

- There is a strong sector confining at a scale f.
- Along with the gluons there are a number of $\sim$ massless quarks.
- The DM is a hadron stable due to some underlying global symmetry of one of its constituent quarks.*
- The DM is a composite state with $m_{\rm DM} \sim f$.
- Strong sector communicates to the SM via the dilaton portal (pNGB of the scale symmetry).

* In analogy with proton/antiproton stability or K mesons which would be stable in the absence of strangeness violating weak interactions.

The dilaton potential

We assume:

- In the UV: The theory is described by a strongly-interacting conformal field theory
- Plus a small explicit breaking of scale invariance, $\epsilon\,\mathcal{O}_\epsilon$, with scaling dimension $d = 4 + \gamma_\epsilon \lesssim 4$
- We assume $\mathcal{O}_\epsilon$ stays close-to-marginal until confinement
- So $|\gamma_\epsilon| \ll 1$, and $\partial\epsilon/\partial \log\mu \simeq \gamma_\epsilon\,\epsilon$ until scale invariance gets spontaneously broken.
- This lead to spontaneous symmetry breaking $\langle\chi'\rangle \neq 0$.
- And a potential for the dilaton, $\epsilon(\chi')\chi'^{4+\gamma_\epsilon}$.

We parameterise the non-canonical dilaton field as:

$$\chi'(x) = f e^{\frac{\sigma'(x)}{f}}$$

where $\sigma'(x) \to \sigma'(\lambda x) + f \log\lambda$ under dilatations.

The dilaton potential

The dilaton potential at $T = 0$ is

$$V(\chi) = \frac{c_\chi}{Z^4} g_\chi^2 \chi^4 \left[1 - \frac{1}{1+\gamma_\epsilon/4}\left(\frac{\chi}{Zf}\right)^{\gamma_\epsilon}\right],$$

with a minimum at $\langle\chi\rangle = Z\,f$. (In RS dual $Z = \sqrt{24}$.)

The dilaton mass:

$$m_\sigma^2 = \frac{\partial^2}{\partial\sigma^2} V\left(\chi(\sigma)\right)\Big|_{\sigma=0} = -4\,\gamma_\epsilon\, c_\chi g_\chi^2\, \frac{f^2}{Z^2}.$$

- The CFT is spontaneously broken and a confinement scale f is generated.
- Tower of resonances at $\sim f$.
- We assume the dilaton is lighter than these resonances
- It is then possible to integrate them out and describe the phase transition in terms of the dilaton field only

Dilaton potential at finite temperature

The dilaton potential at $T = 0$ is

$$V(\chi) = \frac{c_\chi}{Z^4} g_\chi^2 \chi^4 \left[1 - \frac{1}{1 + \gamma_\epsilon/4} \left(\frac{\chi}{Zf} \right)^{\gamma_\epsilon} \right],$$

with a minimum at $\langle \chi \rangle = Z\, f$.

We approximate the deconfined phase by an $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills - Witten '98

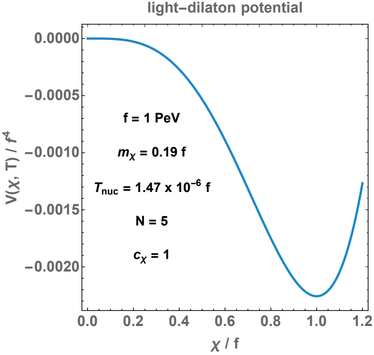
$$F_{\rm dec} = -\frac{\pi^2}{8} N^2 T^4.$$

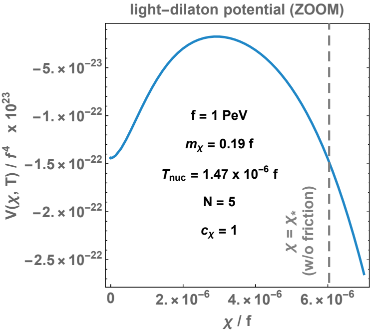
To match onto this: - Creminelli, Nicolis, Rattazzi hep-ph/0607158

$$V_{\rm T}(\chi, T) = \sum_{\text{CFT bosons}} \frac{n T^4}{2\pi^2} J_B \left(\frac{m_{\rm CFT}^2}{T^2} \right)$$

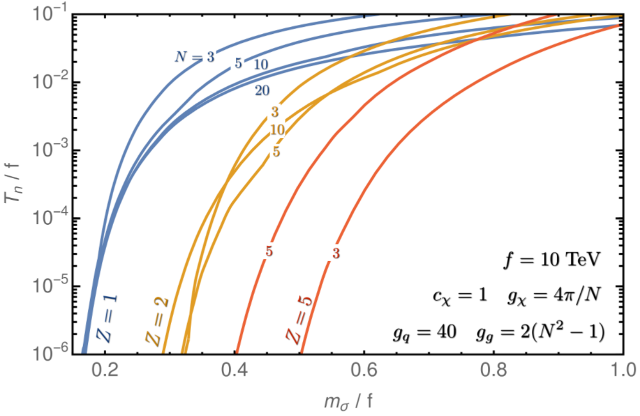
with $m_{\rm CFT} = g_\chi \chi / Z$ and $\sum n = \frac{45 N^2}{4}$.
(This is a guess for $0 < \chi < T/g_\chi$.)

Example potential





Composite Sector with a Light Dilaton



A light dilaton arises if the beta function of the strong sector is small when the theory confines.

This implies strong supercooling.

Dilaton portal DM

Assuming the SM is part of the CFT: The dilaton couplings to the SM are known (with some model dependence for $c_{\rm EM}$, $c_{\rm G}$).

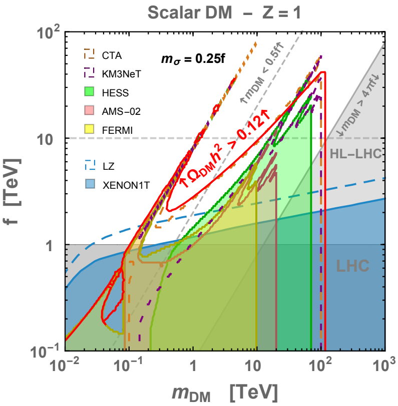
$$\begin{aligned}\mathcal{L} \supset & -\left(\frac{\sigma}{Z\,f}\right)\sum_q (1+\gamma_q) m_q \bar{q} q \\ & +\left(\frac{2\sigma}{Z\,f}+\frac{\sigma^2}{Z^2\,f^2}\right)\left[m_W^2 W^{+\mu}W^-_\mu + \frac{1}{2}m_Z^2 Z_\mu Z^\mu - \frac{1}{2}m_h^2 h^2\right] \\ & +\frac{\alpha_{\rm EM}}{8\pi\,Z\,f}\,c_{\rm EM}\,\sigma\,F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_{\rm s}}{8\pi\,Z\,f}\,c_{\rm G}\,\sigma\,G_{a\mu\nu}G^{a\mu\nu} \\ & +\frac{\sigma}{Zf}\partial_\mu h \partial^\mu h - \frac{\sigma}{Zf} m_h^2 h^2\end{aligned}$$

The dilaton also couples to DM:

$$\mathcal{L}_{\rm DM} \supset -\left(2\frac{\sigma}{Zf}+\frac{\sigma^2}{Z^2 f^2}\right)\frac{1}{2}m_{\rm DM}^2\,\eta^2.$$

Dilaton portal DM! - Bai et al. '09, Blum et al. '14, Efrati et al. '14

Dilaton portal DM

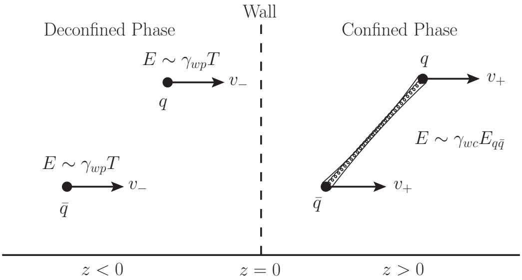


Standard (no supercooling)

- For fixed m_σ, relic abundance sets relation between m_{DM} and f
- Limits from direct detection (t-channel dilaton exchange)
- And indirect detection ($\eta\eta \to \sigma* \to WW,\ ZZ,\ \sigma\sigma$).

Quark Confinement

Now consider if the strong sector phase transition is supercooled.



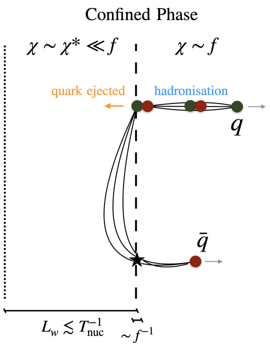
With strong supercooling the elementary techniquanta are seperated by $d \gg f^{-1}$ when entering the bubbles.

Cornell potential at large distances

$$E_{q\bar{q}} \approx f^2 \langle d_c \rangle \approx \frac{f^2}{T_{\rm nuc}} \left(\frac{\gamma_{\rm cw}}{\gamma_{\rm wp}} \right)^{1/3} \approx \left(\frac{f^3}{T_{\rm nuc}} \right)^{1/2}$$

A lower energy configuration, however, is also possible

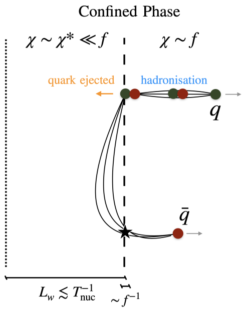
Lower energy configuration possible



Complication:

- String can form oriented towards bubble wall
- Need to estimate dynamics of string formation/breaking
- Final state hadrons are boosted in the plasma frame → additional production via DIS

String formation/breaking



Model this:

- Quark enters with $E \sim 3\gamma_{\rm wp} T$
- Quark at end ejected with $E \sim f$.
- Energy in the string COM frame: $E_{\rm CM} \sim \sqrt{3\gamma_{\rm wp} T f}$.

Yield of composite states

$$Y_\psi^{\rm Str.} \approx Y_{\rm TC}^{\rm eq} \left(\frac{T_n}{T_{\rm start}} \right)^3 \frac{T_{\rm RH}}{T_{\rm start}} \mathcal{P} \left(\log \left[\frac{E_{\rm cm}}{m_*} \right] \right)$$

For QCD $\mathcal{P}(x) \approx 1 + 0.4x + ...$

One can check the shell of ejected particles does not significantly change the incoming particle energy

Boosted composite particles and DIS

Close to collision (runaway wall)

$$\gamma_{\rm wp} \sim \frac{T_n M_{\rm pl}}{f^2}$$

Change of frame (runaway wall)

$$\gamma_{\rm cp} \approx \frac{1}{2}\sqrt{\frac{\gamma_{\rm wp}}{3}\frac{f}{T_n}} \sim \sqrt{\frac{M_{\rm pl}}{f}}$$

Resonances are highly boosted in plasma frame.

In our estimates, for $m_{\rm sigma}^4 \ll \Delta V$, energy is dissipated via DIS with preheated plasma at $E_{\rm PRH} \sim m_\sigma$

$$Y_{\rm DM}^{\rm DIS} \approx \frac{0.43\,{\rm eV}}{m_*}\frac{\rm BR}{10^{-6}}\frac{1}{Z^{5/2}}\frac{g_{\rm TC}}{130}\frac{4\pi}{g_*}\left(\frac{0.2}{m_\sigma/f}\right)^{3/2}\left(\frac{T_n/f}{10^{-5.9}}\right)^4$$

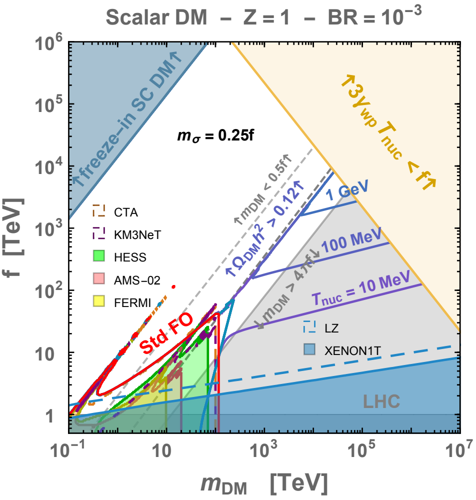
Annihilations after DIS

After the return to kinetic equilibrium (Important if $T_{\rm RH}$ is high)

$$\frac{dY_{\rm DM}}{dx} = -\sqrt{\frac{8\pi^2 g_{*s}}{45}} \frac{M_{\rm pl} m_{\rm DM} \langle \sigma v_{\rm rel} \rangle}{x^2} \left(Y_{\rm DM}^2 - Y_{\rm DM}^{{\rm eq}\,2} \right)$$

This is just the standard Boltzmann equation for DM freezeout, solved with appropriate initial conditions.

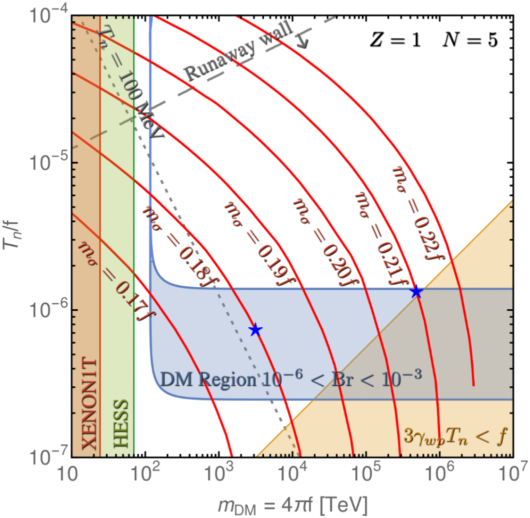
Combining Everything



Combining everything

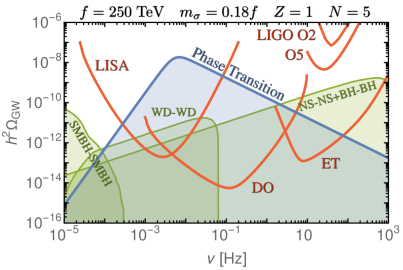
Effects important for $m_{\rm DM} > f$.

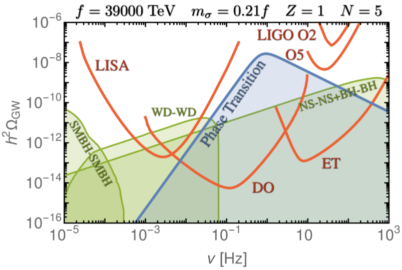
Summary 1



Fixing the DM to be heavy compared to f

GW signal



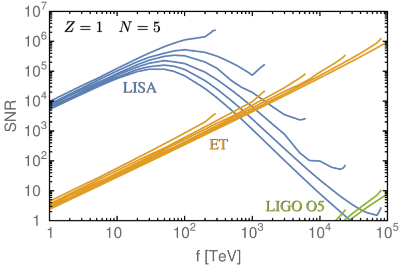


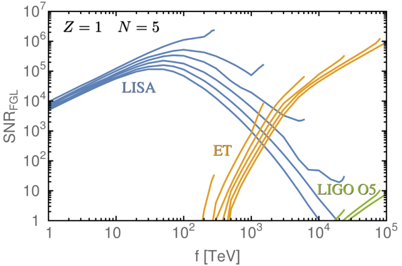
$$\alpha \equiv \frac{\Delta V}{\rho_{\rm rad}(T_n)}, \qquad \beta \equiv \frac{dS}{dt}\Big|_{\rm nuc} = -H_* T \frac{dS}{dT}\Big|_{\rm nuc}, \qquad \kappa_\phi \equiv \frac{\rho_\phi}{\Delta V} \simeq 1.$$

The dilaton decays rapidly leading to an unsupressed signal.

- In Composite scenario: $(\sigma/f)(D_\mu H)^2$, implicitly assuming Higgs is a pNGB of the same sector that breaks scale invariance.
- In Coleman-Weinberg: reheating through suppressed $\lambda_{h\eta} h^2 \eta^2$, where $\lambda_{h\eta} \sim (v_{\rm EW}/v_\eta)^2$.

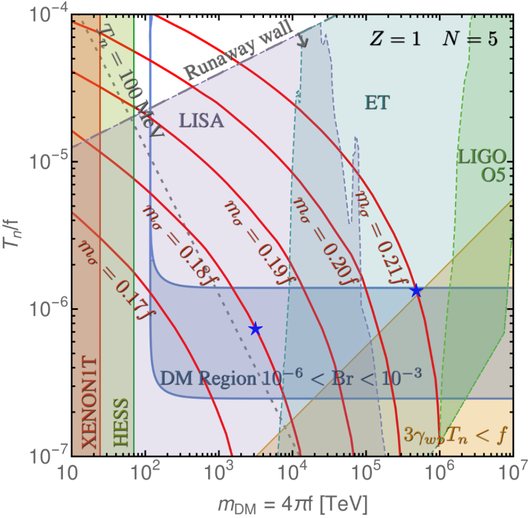
GW signal





$$\mathrm{SNR} = \sqrt{t_{\mathrm{obs}} \int \left(\frac{\Omega_{\mathrm{GW}}(\nu)}{\Omega_{\mathrm{sens}}(\nu)} \right)^2 d\nu}$$

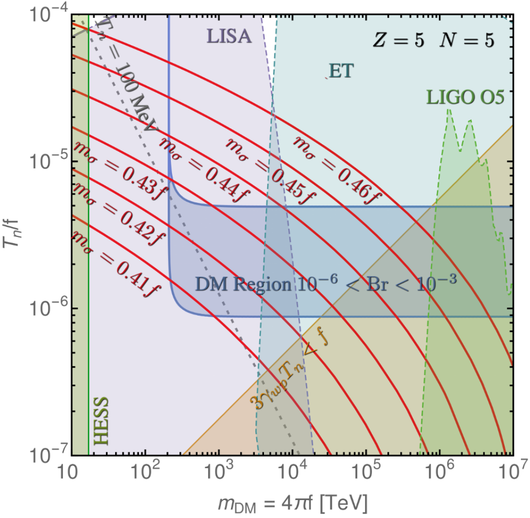
$$\mathrm{SNR}_{\mathrm{FGL}} = \sqrt{t_{\mathrm{obs}} \int \left(\frac{\mathrm{Max}[0, \Omega_{\mathrm{GW}}(\nu) - \Omega_{\mathrm{FG}}(\nu)]}{\Omega_{\mathrm{sens}}(\nu)} \right)^2 d\nu}$$



$\mathrm{SNR_{FGL}} > 100$ for LISA/ET

$\mathrm{SNR_{FGL}} > 10$ for LIGO

Summary



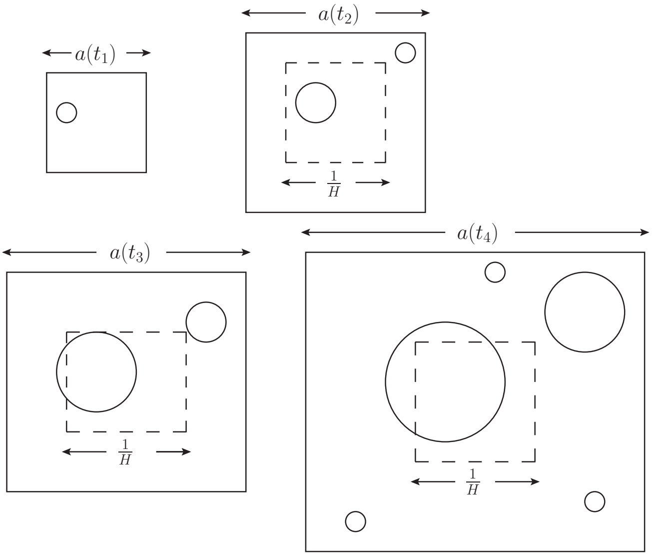
$\mathrm{SNR_{FGL}} > 100$ for LISA/ET
$\mathrm{SNR_{FGL}} > 10$ for LIGO

Conclusions

Conclusions

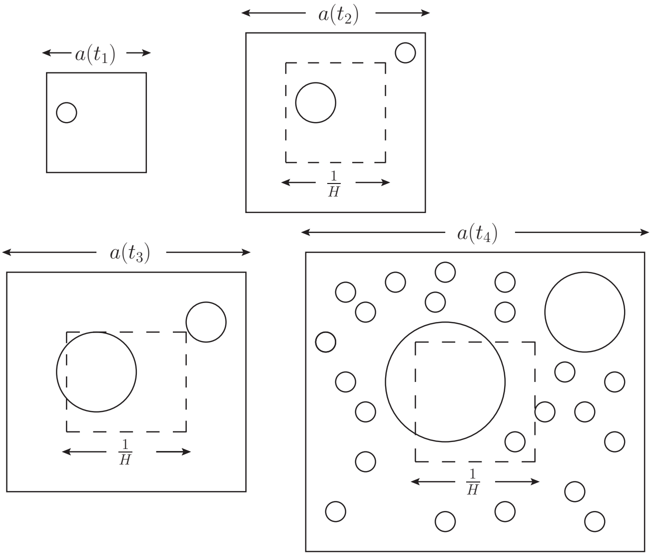
- The DM density could be set by a period of late time inflation terminated by a first order PT.
- The composite DM scenario results in a number of complications.
- The dilaton portal offers an attractive scenario for testing via GWs.
- LISA, which will launch in 2034, will test scenarios with significant supercooling, together with ET.

Completion of the Phase Transtion



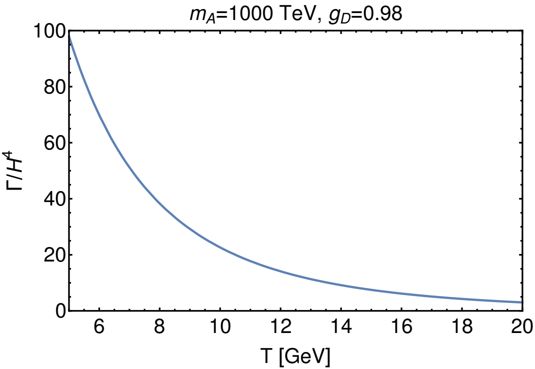
If nucleation rate is low, we can form bubbles which never meet.

Completion of the Phase Transtion



If nucleation grows enough, sufficient bubbles to meet will nucleate.

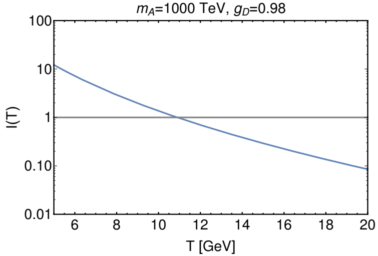
Completion of the Phase Transtion

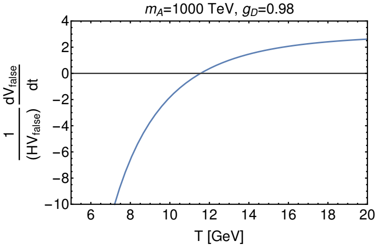


In the classically scale invariant potential we have a slow transition but an exponentially growing nucleation rate.

Completion of the Phase Transtion

We can explicitly check the volume of false vacuum decreases and the bubbles will percolate.



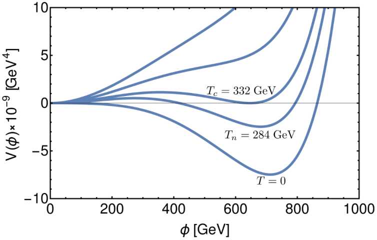


$$P(T) \equiv e^{-I(T)} \lesssim 1/e \implies I(T) = \frac{4\pi}{3}\int_{t_c}^{t} dt'\Gamma(t')a(t')^3 r(t,t')^3 \gtrsim 1$$

$$\frac{1}{H\mathcal{V}_{\rm false}}\frac{d\mathcal{V}_{\rm false}}{dt} = 3 + T\frac{dI}{dT} \lesssim -1.$$

Also see Ellis, Lewicki, No 1809.08242 Ellis, Lewicki, No, Vaskonen 1903.09642

Intro to PTs

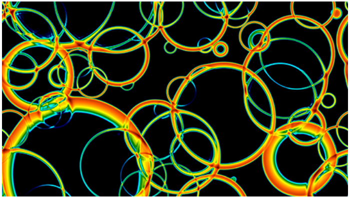


1. Determine $V_{\text{eff}}(\phi, T)$.
2. Use this to calculate the bubble nucleation rate.
3. Nucleation occurs when $\Gamma/V \sim T^4 e^{-S} \sim H^4$.

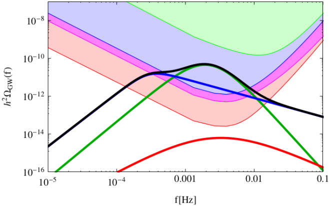
The GW spectrum depends on the bulk properties of the PT

- The Hubble scale
- Latent heat $\alpha \simeq \rho_{\text{vac}}/\rho_{\text{rad}}$
- Inverse timescale of the transition $\beta = -\frac{dS}{dt}$
- The wall velocity v_w

Predicted GW spectra - Recap



From a simulation by Weir et al.



LISA working group 1512.06239

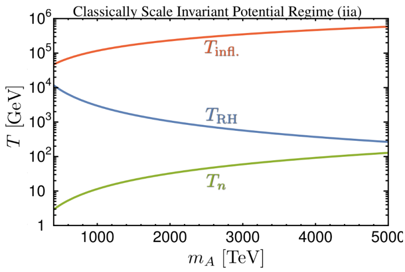
Update: 1910.13125

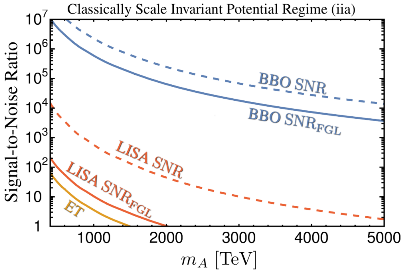
$$h^2\Omega_{\rm GW}(f) \equiv h^2 \frac{f}{\rho_c}\frac{d\rho_{\rm GW}}{df}$$

Three contributions

1. Scalar field contribution
2. Sound waves in the plasma
3. Magnetohydrodynamic Turbulence.

Example 1: Elementary relic



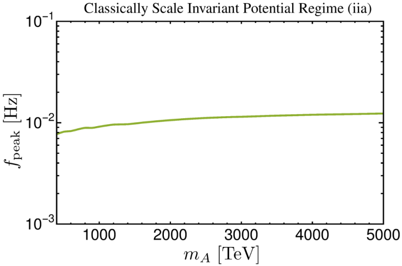


We correct for the period of matter domination after the PT.

$$f_{\rm peak} \to \left(\frac{T_{\rm RH}}{T_{\rm infl}}\right)^{1/3} f_{\rm peak} \qquad \Omega_{\rm GW} \to \left(\frac{T_{\rm RH}}{T_{\rm infl}}\right)^{4/3} \Omega_{\rm GW}$$

Reheating becomes inefficient due to the small width of the h_D into the SM.

Example 1: Elementary relic

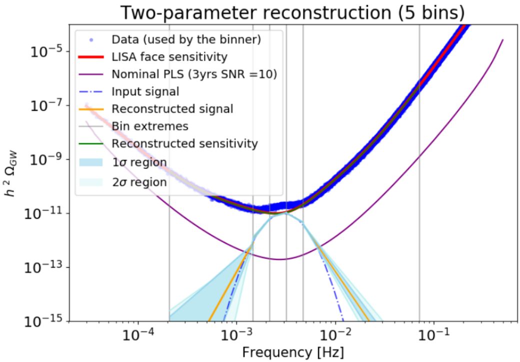


Key prediction of the model

We find the peak frequency here is $\sim 10^{-2}$ Hz almost independent of m_A.

Caveat: calculation can be improved as the running of g_D potentially important.

The SNR above is for power-law backgrounds.

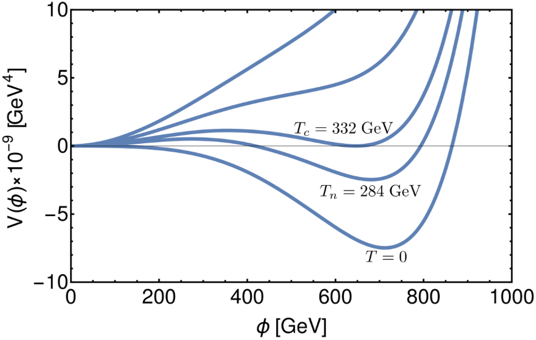


With stronger signals also the shape can be reconstructed.

Caprini et al. 1906.09244, Flauger et al. 2009.11845

Calculation of the GW spectrum - Step 1: V_{eff}

Find the finite temperature effective potential which depends on underlying particle physics parameters.

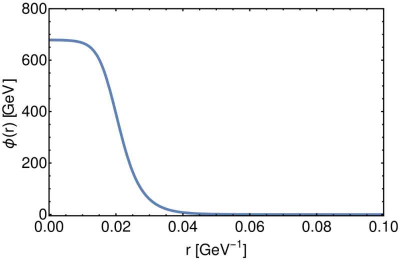


$$V_{\mathrm{eff}}(\phi, T) = V_{\mathrm{tree}}(\phi) + V_1^0(\phi) + V_1^T(\phi, T) + ...$$

This will allow us to predict T_n and other important quantities.
Further techniques: dimensional reduction, lattice calculations.

Calculation of the GW spectrum - Step 2: T_n

Nucleation when $\Gamma/V \sim T^4 e^{-S} \sim H^4$.



Euclidean Action

$$S_4 = 2\pi^2 \int r^3 \left(\frac{1}{2} \left(\frac{d\phi_i}{dr} \right)^2 + \Delta V(\phi, \eta, T) \right) dr \quad \text{Quantum tunneling}$$

$$\frac{S_3}{T} = \frac{4\pi}{T} \int r^2 \left(\frac{1}{2} \left(\frac{d\phi_i}{dr} \right)^2 + \Delta V(\phi, \eta, T) \right) dr \quad \text{Thermal transition}$$

Calculation of the GW spectrum Step 3:

The spectra depend on the macroscopic properties

- Latent heat $\alpha \simeq \rho_{\rm vac}/\rho_{\rm rad}$
- Inverse timescale of the transition $\beta = -\frac{dS}{dt}$
- The Hubble scale (or almost equivalently T_n)
- The wall velocity v_w

These are all calculable from microphysics.
All but v_w are easily found from calculation of T_n.
We can calculate these quantities and then match onto results from simulations/semi-analytic studies.

Calculation of the GW spectrum Step 3:

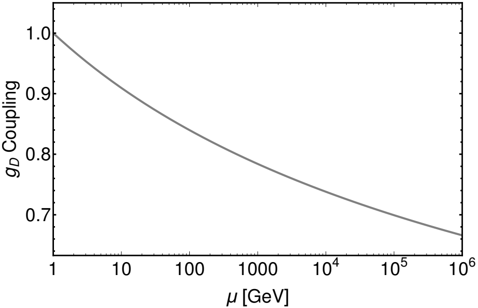
Find the latent heat and timescale of the PT

$$\alpha = \frac{1}{\rho_{\rm rad}}\left(1 - T\frac{\partial}{\partial T}\right)\left(V[\phi_0,\eta_0] - V[\phi_n,\eta_n]\right)\Big|_{T_n} \simeq \frac{\rho_{\rm vac}}{\rho_{\rm rad}}$$

$$\beta = -\frac{d}{dt}(S) = H\,T_n\frac{d}{dT}(S)\Big|_{T_n}$$

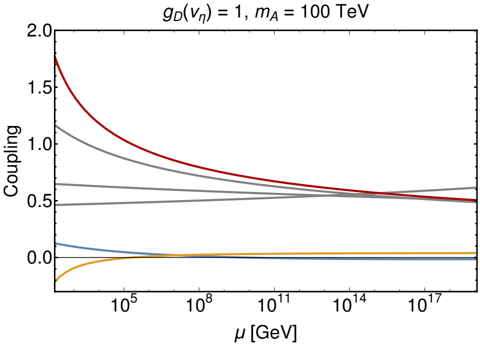
Now we have T_n, α, β, and assume $v_w \simeq 1$.
We now match onto results from numerical simulations.
The peak frequency is related to the bubble size at collision together with a redshifting.

Dark Running



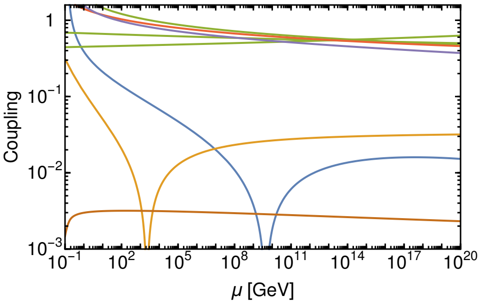
$$\frac{dg_D}{d\ ln(\mu)} = \frac{g_D^3}{(4\pi)^2}\left(-\frac{22}{3} + \frac{1}{6}\right)$$

Dark Running - Including All Coupligns

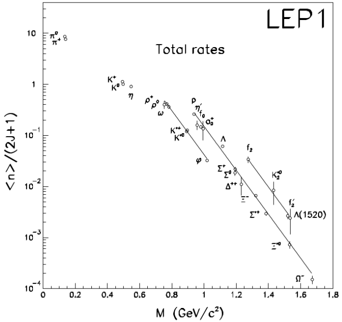


$$\frac{dg_D}{d\,ln(\mu)} = \frac{g_D^3}{(4\pi)^2}\left(-\frac{22}{3} + \frac{1}{6}\right)$$

$g_D(v_\eta) = 0.9,\ m_A = 1$ TeV



- If DM is a heavy resonance - additional suppression.



Thermal Model - Chliapnikov, Phys. Lett. B462, 341 (1999)

$$\langle r \rangle \propto (2J+1)\mathrm{Exp}\left[-\frac{M}{f}\right]$$

Indirect detection

Constraints coming from

- Gamma-ray measurements of the Galactic Centre from HESS
- Measurements of gamma-rays from dwarf spheroidal galaxies (dSphs) from FERMI
- easurements of the anti-proton flux from AMS-02.

We exclude regions of parameter space which satisfy

$$\mathrm{BR}(\sigma \to WW, ZZ, bb)\, \sigma v_{\mathrm{rel}}|_{\eta\eta\to\sigma\sigma} + \sigma v_{\mathrm{rel}}|_{\eta\eta\to \mathrm{WW,ZZ,bb}} \;>\; 2\, \langle \sigma v_{\mathrm{rel}} \rangle^{\mathrm{ID}}_{\mathrm{limit}}$$