# Vector mesons on the wall

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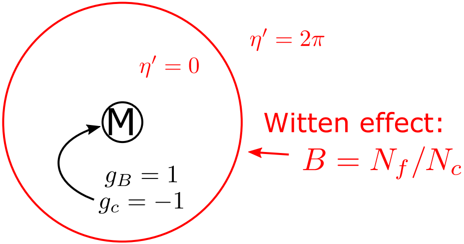
# Practice



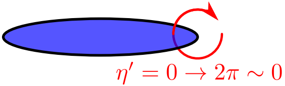

- At large $N_c$, $U(1)_A$ is restored and $\eta'$ is considered as the Goldstone boson.
- Consider a monopole for the baryon number symmetry $U(1)_B$ surrounded by a domain wall for $\eta'$.
- An 't Hooft anomaly for $U(1)_A \times U(1)_B$ induces the WZW term

$$\frac{N_f}{8\pi^2 N_c}\eta'(dA_B)^2.$$

- Witten effect implies the object has the baryon charge $B = N_f/N_c$, which is inconsistent with the Dirac quantization condition.
- The puzzle is originated from a mixed 't Hooft anomaly involving $\theta$ periodicity and the global symmetry. To solve the puzzle we need something on the wall.

# Overview

- We also consider the $\eta'$ string, that is, the domain wall with a boundary, which is called a pancake. We confirm that the object can be a baryon as proposed in Ref. [Z. Komargodski (2018)]:




= Baryon with spin $N_c/2$

- We find that the vector mesons play an important role on the pancake.
- A new understanding of the duality between the vector mesons and the gluons is obtained.
- The effective theory obtained gives a picture of the chiral phase transitions at finite temperature.

# Outline

1. The effective theory for $\eta'$
2. The effective theory on the pancake
3. Coupling of the vector mesons to the wall
4. Physical implications

# Outline

# The effective action for $\eta'$

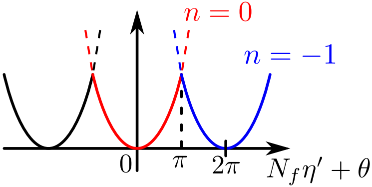
We consider large-$N_c$ QCD coupled with $N_f$ massless fermions,

$$\mathcal{L} = \frac{N_c}{4\lambda^2}|f|^2 + iN_c\bar{\psi}\not{D}\psi + i\theta\frac{1}{8\pi^2}\,\mathrm{tr}\,f^2.$$

The large-$N_c$ argument implies that the effective Lagrangian for $\eta'$ is

$$\mathcal{L}^{\text{eff}}_{\eta'} = \frac{N_f f_\pi^2}{8}|d\eta'|^2 + \frac{f_\pi^2}{8N_f}m_{\eta'}^2 \min_{n\in\mathbb{Z}}(N_f\eta' + \theta + 2\pi n)^2,$$

$$m_{\eta'}^2 = \mathcal{O}(1/N_c), \quad f_\pi^2 = \mathcal{O}(N_c),$$




- The potential has a multi-branch structure, which means that some heavier fields change when $\eta'$ crosses $N_f\eta' + \theta = \pi$.
- $\eta'$ is $2\pi$-periodic because under $U(1)_L$, $\psi \to e^{i\alpha P_L}\psi$, it changes as $\eta' \to \eta' + \alpha$.

# Coupling of the background fields

Let us consider how to couple the background fields to $\eta'$. We introduce the $SU(N_f)_V$ and $U(1)_V$ background fields $A_f$, $A_V$ through the covariant derivative

$$D\psi = (d - ia - iA_f - iA_V)\psi.$$

For $U(1)_A$ transformations to compensate constant shifts of the $\theta$, we add the counter term as

$$\mathcal{L} = \frac{1}{2g^2}|f|^2 + i\bar{\psi}\not{D}\psi + i\theta\frac{1}{8\pi^2 N_f}(N_f \operatorname{tr} f^2 + N_c \operatorname{tr} F_f^2 + N_c N_f F_V^2).$$

Because the $\theta$ dependence is compensated by the shift of $\eta'$, there should be the term

$$\mathcal{L}_{\text{topo}}^{\text{eff}} = i(N_f\eta' + \theta)\frac{1}{8\pi^2 N_f}(N_c \operatorname{tr} F_f^2 + N_c N_f F_V^2)$$

However, by considering carefully the effect of the division part of the global symmetry we find that this term has a problem.

# The division part of the global symmetry

The global symmetry is [Y. Tanizaki (2018)]

$$\frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_V}{\mathbb{Z}_{N_c} \times (\mathbb{Z}_{N_f})_V}$$

The division part means that the following elements of $SU(N_c) \times SU(N_f)_L \times SU(N_f)_R \times U(1)_V$ acts on the fermion in the same way:

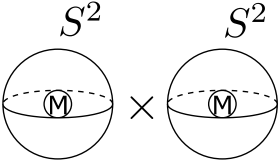
$$(U_c, U_L, U_R, U_V) \sim (e^{-i2\pi/N_c} U_c, e^{-i2\pi/N_f} U_L, e^{-i2\pi/N_f} U_R, e^{i(2\pi/N_c + 2\pi/N_f)} U_V).$$

Due to the division part, the background and dynamical gauge fields can have fractional instanton charges.

# Fractional instantons

The easiest way to see how the instanton numbers become fractional is as follows:

- Let the space time be $S^2 \times S^2$, and place the monopole "inside" each $S^2$.




- The instanton charge is roughly the product of the monopole charges.
- We focus on the subgroup $U(1)_V/\mathbb{Z}_c$. It is convenient to regard $a + A_V \mathbf{1}$ as the $U(N_c) = [SU(N_c) \times U(1)_V]/\mathbb{Z}_{N_c}$ gauge field.
- The minimal monopole with respect to the $U(1)$ subgroup $(e^{i\phi}, 1 \ldots, 1)$ of $U(N_c)$ is ($\theta, \varphi$: spherical coordinates of $S^2$)

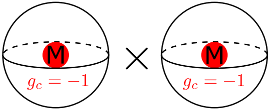
$$\frac{1}{2\pi}(f + F_V \mathbf{1}) = \mathrm{diag}(1, 0, \ldots, 0) \frac{\sin\theta d\theta d\varphi}{4\pi}.$$

It can be decomposed into $SU(N_c)$ part and $U(1)_V$ part as

$$\frac{1}{2\pi} f = \mathrm{diag}(\frac{N_c - 1}{N_c}, -\frac{1}{N_c}, \ldots, -\frac{1}{N_c}) \frac{\sin\theta d\theta d\varphi}{4\pi},$$
$$\frac{1}{2\pi} F_V = \frac{1}{N_c} \frac{\sin\theta d\theta d\varphi}{4\pi}.$$

- In addition, we place a monopole inside another $S^2$ in the same way. Then the fractional instanton is obtained:

$$\frac{1}{8\pi^2}\int \mathrm{tr}\, f^2 = 1 - \frac{1}{N_c}, \quad \frac{1}{8\pi^2}\int F_V^2 = \frac{1}{N_c^2}.$$




Note that $\int \mathrm{tr}(f + F_V \mathbf{1})^2/(8\pi^2)$ is an integer.

- Now we compare the $\theta$ terms in QCD and $\mathcal{L}^{\text{eff}}_{\text{topo}}$ in the effective theory:

$$\mathcal{L}_\theta = i\theta \frac{1}{8\pi^2 N_f}(N_f\, \mathrm{tr}\, f^2 + N_c\, \mathrm{tr}\, F_f^2 + N_c N_f F_V^2),$$

$$\mathcal{L}^{\text{eff}}_{\text{topo}} = i(N_f \eta' + \theta)\frac{1}{8\pi^2 N_f}(N_c\, \mathrm{tr}\, F_f^2 + N_c N_f F_V^2).$$

In QCD, $\theta$ is $2\pi$ periodic, however, in the effective theory $\theta$ is not $2\pi$ periodic, because there is not the gluon field.

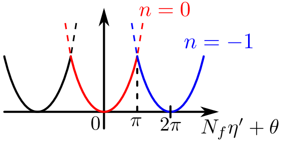
INCONSISTENT!

- Also, we see that the $2\pi$ periodicity of $\eta'$ is lost. Because $\eta'$ is introduced as $2\pi$-periodic scalar, any non-renormalized expectation value becomes zero in this case. In other words, we cannot place the monopole in the effective theory. This contradicts with QCD's feature.

# How to cure the pathology

We need to modify the effective action:

- The multi-branch structure of the $\eta'$ potential can be used.
- The $\eta'$ potential (including $\theta$) has several branches labeled by $n$. When $\theta$ changes from $0$ to $2\pi$, the branch changes from $n = 0$ to $n = -1$.

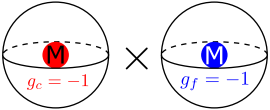



- For example, if we add the term to $\mathcal{L}^{\text{eff}}_{\text{topo}}$

$$i(N_f\eta' + \theta)\frac{1}{8\pi^2 N_f}(N_c \operatorname{tr} F_f^2 + N_c N_f F_V^2) + \underline{i2\pi n \frac{N_c}{8\pi^2} F_V^2},$$

  the periodicity seems to be recovered.
- However, there arises another problem for other background monopoles.

Consider other background monopoles:




$$\frac{1}{2\pi}F_V = \frac{1}{N_c}\frac{\sin\theta_1 d\theta_1 d\varphi_1}{4\pi} + \frac{1}{N_f}\frac{\sin\theta_2 d\theta_2 d\varphi_2}{4\pi}, \quad f = \ldots, \quad F_f = \ldots,$$

$$\Rightarrow \quad \frac{1}{8\pi^2}\int \mathrm{tr}\, f^2 = 0, \quad \frac{1}{8\pi^2}\int \mathrm{tr}\, F_f^2 = 0, \quad \frac{1}{8\pi^2}\int F_V^2 = \frac{1}{N_c N_f}.$$

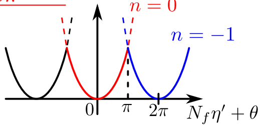
In QCD, the $2\pi$-periodicity of $\theta$ is violated,

$$i\theta\frac{1}{8\pi^2 N_f}(N_f\,\mathrm{tr}\, f^2 + N_c\,\mathrm{tr}\, F_f^2 + N_c N_f F_V^2).$$

However in the effective theory, if one adds the term, the periodicity is NOT violated:

$$i(N_f\eta' + \theta)\frac{1}{8\pi^2 N_f}(N_c\,\mathrm{tr}\, F_f^2 + N_c N_f F_V^2) + i2\pi n\frac{N_c}{8\pi^2}F_V^2$$

does not change under $(\theta, n) \to (\theta + 2\pi, n - 1)$.




INCONSISTENT!

- In order for the effective theory to be consistent with QCD for both cases by adding the term

$$\mathcal{L}_\theta = i\theta \frac{1}{8\pi^2 N_f}(N_f \operatorname{tr} f^2 + N_c \operatorname{tr} F_f^2 + N_c N_f F_V^2),$$

$$\mathcal{L}_{\text{topo}}^{\text{eff}+} = i(N_f \eta' + \theta)\frac{1}{8\pi^2 N_f}(N_c \operatorname{tr} F_f^2 + N_c N_f F_V^2) + 2\pi i n \frac{N_c l}{8\pi^2} F_V^2,$$

  it has to be satisfied that

$$1 - l/N_c = R, \quad l/N_f = J,$$
$$\Rightarrow N_c R - N_f J = 1 \quad \text{for } R, J \in \mathbb{Z}.$$

  It can be satisfied only if $\gcd(N_c, N_f) = 1$.

- For any choice of the counter $\theta$ terms, the $\theta$ periodicity is violated at least one of the two cases if $\gcd(N_c, N_f) \neq 1$ (an anomaly involving $\theta$-periodicity and $G_{\text{sub}}$ [C. Cordova, et al. (2020)]). The above result is a consequence of the anomaly.

For $\gcd(N_c, N_f) \neq 1$, we need to introduce two-form $\mathbb{Z}_{N_{c,f}}$ background gauge fields.

# Two-form $\mathbb{Z}_N$ gauge fields

The two-form $\mathbb{Z}_{N_{c,f}}$ background gauge fields are defined as a pair $(B^{(2)}_{c,f}, \widehat{A}_c)$ of $U(1)$ two-form and one-form gauge fields that satisfy constraints,

$$N_c B^{(2)}_{c,f} = d\widehat{A}_{c,f},$$

where the normalization is given as

$$\int d\widehat{A}_{c,f} \in 2\pi\mathbb{Z}.$$

A one-form gauge transformation acts as

$$B^{(2)}_{c,f} \to B^{(2)}_{c,f} + d\lambda^{(1)}_{c,f}, \quad \widehat{A}_{c,f} \to \widehat{A}_{c,f} + N_{c,f}\lambda^{(1)}_{c,f}.$$

We decompose $A_V$ using these as

$$A_V = \tilde{A}_V + \frac{1}{N_c}\widehat{A}_c + \frac{1}{N_f}\widehat{A}_f, \quad \int d\tilde{A}_V \in 2\pi\mathbb{Z}.$$

$\tilde{A}_V$ transforms under one-form gauge transformation as

$$\tilde{A}_V \to \tilde{A}_V - \lambda^{(1)}_c - \lambda^{(1)}_f$$

The fractional part is carried by $(B_c^{(2)}, \widehat{A}_c)$ and $(B_f^{(2)}, \widehat{A}_f)$. Thus

$$\frac{N_c}{8\pi^2}\int (B_c^{(2)})^2 = -\frac{1}{8\pi^2}\int \mathrm{tr}(f^2) \bmod 1.$$

Note that the left hand side is only determined up to addition by an integer due to the one-form gauge transformation. The effective theory becomes consistent with QCD in the both cases by adding the term:

$$\mathcal{L}^{\mathrm{eff}}_{\mathrm{topo}+} = i(N_f\eta' + \theta)\frac{1}{8\pi^2 N_f}(N_c \,\mathrm{tr}\, F_f^2 + N_c N_f F_V^2) - \underline{2\pi i n \frac{N_c}{8\pi^2}(B_c^{(2)})^2}.$$

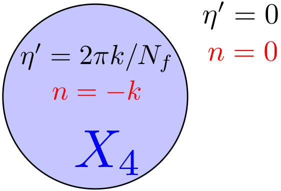
The term plays the role in the gluon contribution in the $\theta$ term of QCD:

$$\mathcal{L}_\theta = i\theta\frac{1}{8\pi^2 N_f}(N_f \,\mathrm{tr}\, f^2 + N_c \,\mathrm{tr}\, F_f^2 + N_c N_f F_V^2).$$

It seems OK, but...

# The necessity of a dynamical field on the wall

If one considers a domain wall, there appears a problem:




The term is not one-form-gauge invariant:

$$2\pi i \frac{kN_c}{8\pi^2} \int_{X_4} (B_c^{(2)})^2 \to \ldots + 2\pi i \frac{k}{8\pi^2} \int_{\partial X_4} (2\lambda_c^{(1)} d\widehat{A}_c + N_c \lambda_c^{(1)} d\lambda_c^{(1)}).$$

We need a theory with a $\mathbb{Z}_{N_c}$ one-form symmetry that has an anomaly on the domain wall. A candidate is a $U(k)_{-N_c}$ CS theory,

$$i\frac{1}{4\pi} \int_{\partial X_4} (-N_c \operatorname{tr}(cdc - i\frac{2}{3}c^3) + 2\operatorname{tr}(c)d\widehat{A}_c), \quad c \in \mathfrak{u}(k),$$

$$c \to c - \lambda^{(1)}\mathbf{1}, \quad \widehat{A}_c \to \widehat{A}_c + N_c \lambda^{(1)}.$$

# $c$ as a dual gluon

Final answer: Add the term

$$\mathcal{L}^{\text{eff}}_{\text{topo}+} = i(N_f\eta' + \theta)\frac{1}{8\pi^2 N_f}(N_c \operatorname{tr} F_f^2 + N_c N_f F_V^2) - \underline{2\pi i n \frac{N_c}{8\pi^2}(B_c^{(2)})^2},$$

and put the CS theory on the domain wall corresponding to $\eta' : 2\pi k/N_f \to 0$:

$$i\frac{1}{4\pi}\int_{\partial X_4}(-N_c \operatorname{tr}(cdc - i\frac{2}{3}c^3) + 2\operatorname{tr}(c)d\widehat{A}_c), \quad c \in \mathfrak{u}(k).$$

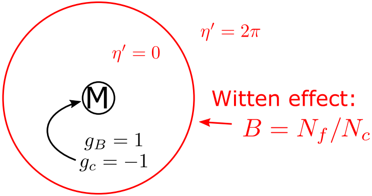
Roughly speaking, the situation can be explained as follows:

- The $\eta'$ domain wall corresponds to the interface of $\theta$ from $2\pi k$ to $0$.
- In the UV theory, there is a $SU(N_c)_k$ CS theory on the interface of $\theta$:

$$-i\frac{1}{8\pi^2}\int d\theta \operatorname{tr}(ada - i\frac{2}{3}a^3).$$

- If we assume that the confinement in the four-dimensional bulk does not affect the theory on the wall, the gluons remain on the wall forming the CS theory at low energy.
- $U(k)_{-N_c}$ is the level-rank dual of $SU(N_c)_k$, and thus $c$ can be regarded as a dual gluon field.

# Resolution of the puzzle




On the domain wall,

$$\frac{1}{4\pi}\int(-N_c\,\mathrm{tr}(cdc - i\frac{2}{3}c^3) - 2\,\mathrm{tr}(c)d\widehat{A}_c).$$

- The Gauss law constraint gives

$$dc - ic^2 = \frac{1}{N_c}d\widehat{A}_c\mathbf{1}$$

- Due to the monopole inside the wall, it is satisfied that $\int d\widehat{A}_c = 2\pi$.
- The constraint cannot be satisfied because $\int d\,\mathrm{tr}\,c \in 2\pi\mathbb{Z}$.
- Any non-normalized expectation value is zero.
- The object disappears (and the domain wall and the monopole exist separately.)

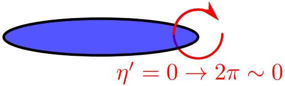
# Outline

# Domain walls bounded by strings

- We consider domain walls corresponding to a jump from $\eta' = 0$ to $2\pi$.
- This type of domain walls could be bounded by a string while the pion fields are kept fixed, because $\eta' = 0 \sim 2\pi$.
- $\eta'$ winds around the string, and $\eta'$ is ill-defined at the core of the string. In the full theory, it is expected that this singularity is regulated by restoring the chiral symmetry at the core.
- Note that the bulk term disappears for the domain wall with $k = N_f$,

$$2\pi i \frac{1}{8\pi^2} \int_{X_4} (-N_c (B_c^{(2)})^2 + N_c \,\mathrm{tr}\, F_f^2 + N_c N_f F_V^2) \in 2\pi i \mathbb{Z},$$

Thus it can have a boundary.




= Baryon with spin $N_c/2$

# Coupling between the backgrounds and the wall

The term

$$\mathcal{L}_{\mathrm{topo}}^{\mathrm{eff}} = i(N_f\eta' + \theta)\frac{1}{8\pi^2 N_f}(N_c \,\mathrm{tr}\, F_f^2 + N_c N_f F_V^2).$$

gives the CS term for the background on the wall as

$$\frac{1}{4\pi}\int_{M_3}\left(-N_c \,\mathrm{tr}(cdc - i\frac{2}{3}c^3) + N_c \,\mathrm{tr}(\mathcal{A}_f d\mathcal{A}_f - i\frac{2}{3}\mathcal{A}_f^3)\right),$$
$$c \in \mathfrak{u}(N_f), \quad \mathcal{A}_f = A_V\mathbf{1} + A_f \in \mathfrak{u}(N_f).$$

However, the second term violate the gauge symmetry for $\mathcal{A}_f$ because of the boundary. This is recovered by

1. adding the term at the boundary

$$\frac{N_c}{4\pi}\int_{\partial M_3} \mathrm{tr}(\mathcal{A}_f c), \text{ and,}$$

2. imposing the boundary condition for $c$ as ($t$: a specific component)

$$c_t - \mathcal{A}_t^f = 0.$$

To maintain this condition $c$ transforms as $U(N_f)_V$ gauge field on the boundary

# Chiral version of WZW model and baryons

- By integrating out $c_t$, we obtain the gauss law constraint, $\tilde{f}_c = 0$, the solution of which is $\tilde{c} = iW\tilde{d}W^{-1}$, $W \in U(N_f)$, where tilde denotes the other components.
- Then the action on the wall reduces to

$$\frac{N_c}{4\pi}\int_{\partial M_3} \mathrm{tr}\left(W\partial_y W^{-1} W\partial_t W^{-1} - 2iW\partial_y W^{-1}\mathcal{A}_t^f + \mathcal{A}_y^f \mathcal{A}_t^f\right) dydt$$
$$+ \frac{N_c}{4\pi}\int_{M_3} \mathrm{tr}\left(\frac{1}{3}(WdW^{-1})^3 - \mathcal{A}_f d\mathcal{A}_f + i\frac{2}{3}\mathcal{A}_f^3\right).$$

- $W$ transforms under $U(N_f)_V$ as $W \to g_f W$ at the boundary.
- The baryon charge is ($Y_2$: a time slice of $M_3$)

$$B = \frac{1}{2\pi}\int_{\partial Y_2} \mathrm{tr}(\mathcal{A}_f - iWdW^{-1}) - \frac{1}{2\pi}\int_{Y_2} \mathrm{tr}(\mathcal{F}_f),$$

  The contributions from the background cancels (in the gauge without Dirac strings).
  $\Rightarrow$ baryon number = winding number
- The baryon creation operator: $\rho_{\mathrm{Sym}^{N_c}(\square)}(W)$ [Z. Komargodski (2018)].
  The baryon charge: 1, The flavour representation: $\mathrm{Sym}^{N_c}(\square)$.

# Outline

# What happens when we introduce $A_L, A_R$?

Let us consider how the $SU(N_f)_{L,R}$ background fields $A_{L,R}$ couple to the domain wall. At large $N_c$, the global symmetry is $U(N_f)_L \times U(N_f)_R$. An 't Hooft anomaly implies the coupling between $\eta'$ and $A_{L,R}$ as

$$\frac{N_c}{8\pi^2}\eta'(F_L^2 + F_R^2)/2.$$

Thus on the wall there appears

$$\frac{N_c}{4\pi}\int_Y [\mathrm{tr}(A_L dA_L - i\frac{2}{3}A_L^3) + \mathrm{tr}(A_R dA_R - i\frac{2}{3}A_R^3)]/2$$

- When the domain wall has the boundary, how to restore the gauge invariance for $A_L$ and $A_R$ is not trivial.
- Naively, it seems that we can do it by using $U$. However, $U$ is not well-defined at the boundary, and we cannot use it.
- This problem can be solved by introducing the vector mesons as the gauge field for the hidden local symmetry.

# The hidden local symmetry

- Let $U$ be the $U(N_f)$-valued field that involves the pion field and $\eta'$ so that

$$\det U = \exp(i\eta').$$

- We divide $U$ into two parts,

$$U = \xi_L^\dagger \xi_R,$$

and regard $\xi_{L,R} \to h\xi_{L,R}$, $h \in U(N_f)$ as the gauge symmetry.

- We introduce the gauge field $v$ for this symmetry and its kinetic term. The covariant derivative is given as

$$D\xi_{L,R} := d\xi_{L,R} - iv\xi_{L,R} + i\xi_{L,R}\mathcal{A}_{L,R}.$$

The lowest derivative terms give the mass term for $v$, which is minimized when

$$v = \frac{1}{2}(\mathcal{A}_L^{\xi_L} + \mathcal{A}_R^{\xi_R}),$$

$$\mathcal{A}_{L,R}^{\xi_{L,R}} := \xi_{L,R}\mathcal{A}_{L,R}\xi_{L,R}^{-1} + i\xi_{L,R}d\xi_{L,R}^{-1}.$$

- Thanks to the mass term, $v$ can be treated as $\mathcal{A}_f$ roughly.

# Coupling to the vector mesons

Let $v \in \mathfrak{u}(N_f)$ be the vector meson field as the gauge field for the hidden local symmetry. We propose that the theory on the wall is

$$\frac{1}{4\pi}\int_{M_3}\left[N_c \operatorname{tr}\left(cdc - i\frac{2}{3}c^3\right) - N_c \operatorname{tr}\left(vdv - i\frac{2}{3}v^3\right)\right] + \frac{N_c}{4\pi}\int_{\partial M_3}\operatorname{tr}(vc),$$

which is gauge invariant in the same way.

- The coupling of $v$ to the wall comes from the topological term,

$$\frac{N_c}{8\pi^2}\int \eta' \operatorname{tr}(f_v^2).$$

- By minimizing the mass term for $v$, this reproduces the previous term if we neglect the irrelevant term to $U(1)_A$ breaking.
- To maintain the symmetry under $\eta' \to \eta' + 2\pi/N_f$, the instanton charge for the vector meson field has to be a multiple of $N_f$ when there are no background fields.

# Relation to the generalized WZW term

- The possible topological terms including $v$ is already discussed. (c.f. [M. Harada, K. Yamawaki (2003)])

$$\Gamma_v := -i\frac{N_c}{16\pi^2}\int_{M^4}\sum_{i=1}^{4} c_i\mathcal{L}_i,$$

$$\mathcal{L}_1 = \mathrm{tr}(\widehat{\alpha}_L^3\widehat{\alpha}_R - \widehat{\alpha}_R^3\widehat{\alpha}_L), \quad \mathcal{L}_2 = \mathrm{tr}(\widehat{\alpha}_L\widehat{\alpha}_R\widehat{\alpha}_L\widehat{\alpha}_R), \quad \mathcal{L}_3 = \mathrm{tr}(f_v(\widehat{\alpha}_L\widehat{\alpha}_R - \widehat{\alpha}_R\widehat{\alpha}_L)),$$

$$\mathcal{L}_4 = \frac{1}{2}\,\mathrm{tr}\left(\widehat{\mathcal{F}}_L(\widehat{\alpha}_L\widehat{\alpha}_R - \widehat{\alpha}_R\widehat{\alpha}_L) - \widehat{\mathcal{F}}_R(\widehat{\alpha}_R\widehat{\alpha}_L - \widehat{\alpha}_L\widehat{\alpha}_R)\right),$$

$$\widehat{\mathcal{F}}_{L,R} := \xi_{L,R}\mathcal{F}_{L,R}\xi_{L,R}^{-1}, \quad \widehat{\alpha}_{L,R} := \mathcal{A}_{L,R}^{\xi_{L,R}} - v$$

- The proposed term is reproduced for $c_1 = 2/3,\ c_2 = -1/3,\ c_3 = 1,\ c_4 = 1.$
- The values are consistent with the experiments. [A. Karasik (2020)]
- The phenomenology seems to prefer the vector meson dominance for $\pi^0 \to 2\gamma$ but not to prefer that for $\gamma \to 3\pi$:
  - $\pi^0 \to \rho^0 + \omega^0 \to 2\gamma$ when $c_4 = 1$, $c_3 = c_4$
  - $\gamma \to \omega \to 3\pi$ when $c_1 - c_2 + c_4 = 4/3$, $c_3 = c_4$
- $\omega \to \pi\rho \to 3\pi$ when $c_1 - c_2 - c_3 = 0.$
- Our consideration could explain the values of $c_i$.

# Outline

# Physical implication to the duality and the chiral phase transition

- At the edge of the pancake, $c$ transforms in the same way as $v$.
  $\Rightarrow$ This suggests that at the point where the chiral symmetry is restored, $v$ could be regarded as the dual of the gluons because $c$ is the level-rank dual of the gluon.
- At high temperature, $\eta'$ crosses the cusp of the potential more easily and frequently. In this situation, $c$ can propagate in the four dimensional space by creating the pancakes, and it mixed with $v$ on the $\eta'$ string. If this is correct, the pancakes can play an important role near the chiral phase transition.
- It is also interesting to consider $\theta$ winds around $S^1$ direction $N_f$ times. We expect as follows:
  For a small radius, $\theta$ rapidly changes and $\eta'$ cannot follow.
  $\Rightarrow$ The effective theory is a $U(N_f)_{-N_c}$ CS theory for $c$.

  For a large radius, $\eta'$ follow the change of $\theta$.
  $\Rightarrow$ The effective theory on the interface is a CS Higgs theory for $v$.

  The transition can be smooth due to the creation of the pancakes.
  (cf. [N. Kan et al. (2019)])

# Summary

- We consider large $N_c$ QCD with $N_f$ massless fermions.
- We show that we need a $U(N_f)_{-N_c}$ CS theory on the $\eta'$ domain wall to match the anomaly involving $\theta$ periodicity and the global symmetry.
- We confirm that the $\eta'$ domain wall bounded by the string can be regarded as a baryon with spin $N_c/2$.
- For the gauge invariance of the background fields at the boundary of the wall, it is concluded that the vector mesons should couple to the wall.
- At the boundary of the wall, the vector mesons should couple to the gauge field of $U(N_f)$ CS theory.

# Future Works

- The effective theory we obtained can be used at the finite density, in strong magnetic field, ...
  The chiral soliton lattice, the quark-hadron continuity, ...
- In general, what happens on a singular object of a Goldstone boson field when there is some anomaly?