Spontaneously broken Boosts in CFT

Sridip Pal
Institute for Advanced Study

Based on 2102.12583 [hep-th]

Zohar Komargodski, Mark Mezei, **SP**, Avia Raviv-Moshe

Nambu Goldstone Theorems

Consider d dimensional QFT with continuous internal symmetry. Q be the generator.

$$\langle\Omega|[Q, O(0)]|\Omega\rangle \neq 0$$

- In finite volume it can not happen, one can diagonalize Q. Spontaneous symmetry breaking is an infinite volume phenomenon.
- Presence of Super-selection sectors.

- Algebraic decay of correlator: $\langle\Omega\,|j_0(x)O(0)|\,\Omega\rangle$ can not decay faster than $|x|^{-(d-1)}$

  $\langle\Omega\,|O^\dagger(x)O(0)|\,\Omega\rangle$ can not decay faster than $|x|^{-(d-2)}$

Nambu Goldstone Theorems: Space time symmetry

Consider d dimensional QFT with conserved Stress Energy tensor.

$$\langle\Omega|[Q^{(\xi)}, O(0)]|\Omega\rangle \neq 0$$

$\mathcal{J}^{(\xi)}_\mu = \xi^\nu T_{\mu\nu}$ For Killing vector ξ

From dimensional analysis, a nonzero commutator for the boost Killing vector (which we can take to be $\xi^1 = x^0, \xi^0 = x^1$, with the rest of the components vanishing) means that correlators of some components of the EM tensor and O decay not faster than $1/|x|^d$.

- Boost breaking is conceptually on different footing. There is no sense of Boost symmetry in compact space. No super selection sectors.

Nambu Goldstone Theorems: Space time symmetry

Consider d dimensional QFT with conserved Stress Energy tensor.

$$\langle\Omega|[Q^{(\xi)}, O(0)]|\Omega\rangle \neq 0$$

$\mathcal{J}_\mu^{(\xi)} = \xi^\nu T_{\mu\nu}$ For Killing vector ξ

- While there is no boost symmetry in compact space, if we focus on a small patch, there is an approximate boost symmetry, this can be broken spontaneously in infinite volume limit for some specially chosen states.

- The algebraic decay and gapless excitation around this state in the infinite volume theory implies constraint on the finite volume theory.

Nambu Goldstone Theorems: Bound on gap

- Consider a theory in a box of size L and a state with finite energy density ϵ .
- $\epsilon^{-1/d}$ is the length scale of the infinite volume theory. The algebraic decay in infinite volume means such decay in the range $\epsilon^{-1/d} \ll \Delta x \ll L$

- Algebraic decay of correlator in this range immediately implies the gap (in the finite volume theory) should be smaller than $\epsilon^{1/d}$

$\epsilon^{(1-\gamma)/d}$

$$E_{gap} \sim L^{-\gamma}$$

- Most natural value of the exponent is 1 (if we assume in the deep infrared, we reached a fixed point of RG flow and there is a scale invariant theory describing the physics)

Nambu Goldstone Theorems: CFT

- We want to play with these ideas in context of CFT with an aim to constrain the CFT data.
- We study the CFT on the cylinder $S^{d-1} \times \mathbb{R}$ with energy being $E = \Delta/R$

Existence of states below the following energy

$$E = s_{d-1}\epsilon R^{d-1} + c\epsilon^{(1-\gamma)/d} R^{-\gamma}$$

From previous consideration

Where c is some order one number.

$$\Delta = \Delta_\Omega + c\Delta_\Omega^{\frac{1-\gamma}{d}}$$

Nambu Goldstone Theorems: CFT

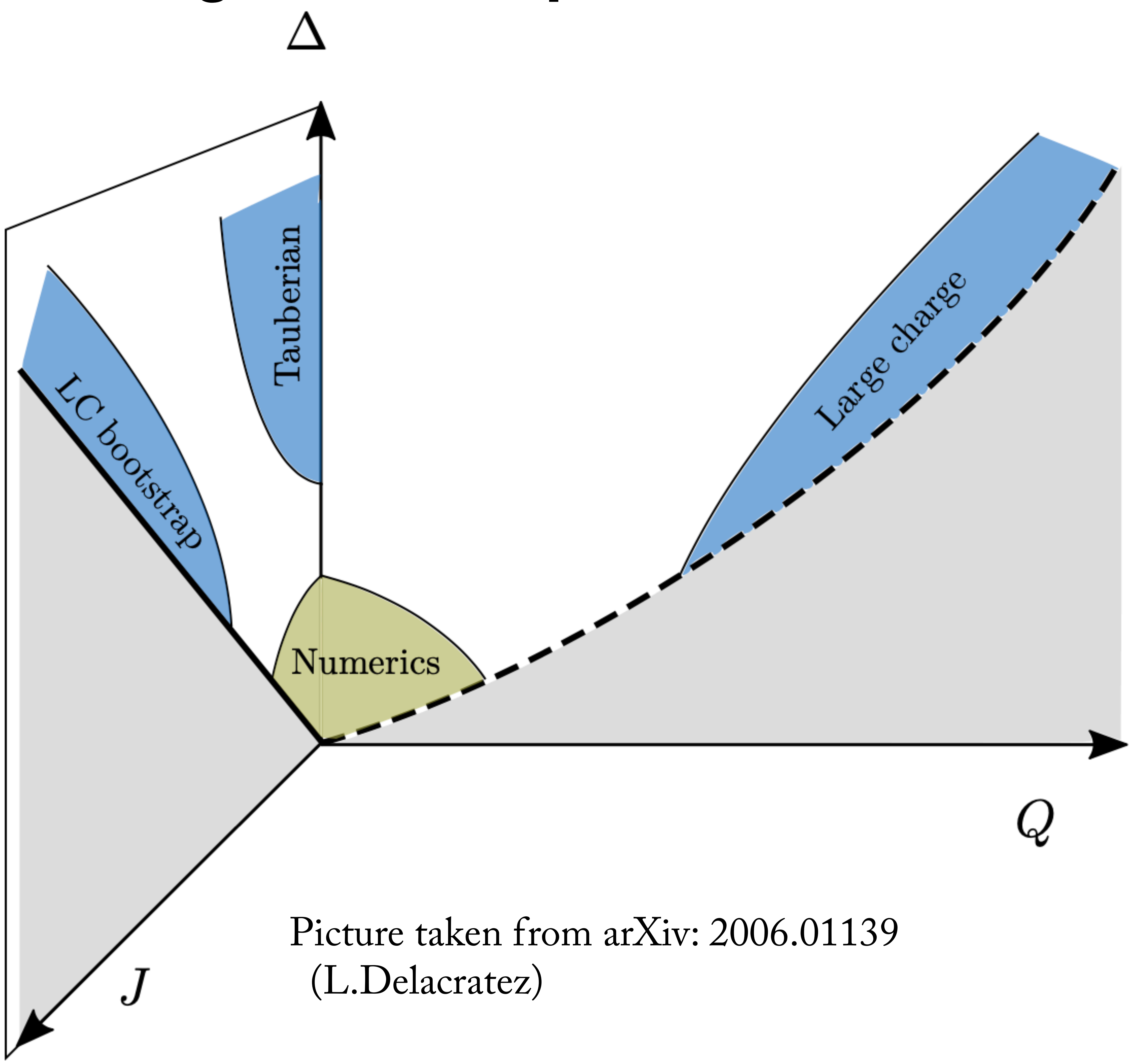
Existence of states below

$$\Delta = \Delta_\Omega + c\Delta_\Omega^{\frac{1-\gamma}{d}}$$

- Trivial for descendants, nonetheless the twist in the story comes from the fact that Descendants don't contribute to these NG theorems. Hence the bound constrains the Primary spectra, which we want.

- In what follows, we will make these physical insights precise and argue for $\gamma = 1$

Constraining the CFT spectra



Picture taken from arXiv: 2006.01139
(L.Delacratez)

Nambu Goldstone Theorems (Boost)

arXiv: 2001.06024; L.Albetre, A. Nicolis

One can consider Stress Energy tensor as order parameter if we have a finite energy density state.

$$\begin{aligned}\left\langle \Omega \left| \delta_{K^i} T^{0j}(t,x) \right| \Omega \right\rangle &= i \left\langle \Omega \left| \left[K^i, T^{0j}(t,x) \right] \right| \Omega \right\rangle \\ &= \left\langle \Omega \left| T^{00}(t,x)\delta^{ij} + T^{ij}(t,x) \right| \Omega \right\rangle \\ &= (\epsilon + P)\delta^{ij}\end{aligned}$$

$$i \left\langle \Omega \left| \left[K^i, T^{0j}(0) \right] \right| \Omega \right\rangle = -i \int d^{d-1}x x^i \left\langle \left[T^{00}(t,x), T^{0j}(0) \right] \right\rangle$$

$$2\pi\delta(\omega)(\epsilon + P)\delta_i^j = \lim_{k \to 0} \frac{\partial}{\partial k^i} G^{(comm)}_{T^{00},T^{0j}}(\omega, k)$$

Nambu Goldstone Theorems (Boost)

In terms of spectral density

$$\delta(\omega)(\epsilon + P)\delta_i^j = \lim_{k\to 0} \frac{\partial}{\partial k^i} \rho_{T^{00},T^{0j}}(\omega, k)$$

Similarly, one can consider current as order parameter if we have finite charge density.

$$\delta(\omega)\rho\delta_i^j = \lim_{k\to 0} \frac{\partial}{\partial k^i} \rho_{T^{00},J^j}(\omega, k)$$

Nambu Goldstone Theorems (Boost+Dilatation)

$$\delta(\omega)(\epsilon + P)\delta_i^j = \lim_{k\to 0} \frac{\partial}{\partial k^i} \rho_{T^{00},T^{0j}}(\omega, k)$$

$$\delta(\omega)\rho\delta_i^j = \lim_{k\to 0} \frac{\partial}{\partial k^i} \rho_{T^{00},J^j}(\omega, k)$$

$$\delta(\omega) d\epsilon = \lim_{k\to 0} \frac{\partial}{\partial k^i} \rho_{T^{i0},T^{00}}(\omega, k)$$

$$\delta(\omega)(d-1)\rho = \lim_{k\to 0} \frac{\partial}{\partial k^i} \rho_{T^{i0},J^0}(\omega, k)$$

One can also think of them appearing from the NG theorem for broken Dilatation

- SCT does give anything new!

Nambu Goldstone Theorems (T00T00)

$$\delta(\omega)(\epsilon + P)\delta_i^j = \lim_{k\to 0} \frac{\partial}{\partial k^i} \rho_{T^{00},T^{0j}}(\omega, k)$$

Conservation Equation

$$\rho_{T^{00},T^{00}}(\omega, k) \approx -(\epsilon + P)k^2\delta'(\omega)$$

Implication in CFT $\rho_{T^{00},T^{00}}(\omega, k) \approx -(\epsilon + P)k^2\delta'(\omega)$

- Consider a CFT on $R \times S^{d-1}$, pick a state with finite energy density and then take the macroscopic limit such that energy density is kept finite and fixed . The macroscopic limit should satisfy the sum rules.
- Aim is to identify the states that are responsible for saturating the sum rule.

- We want to understand the following correlator in S channel in the macroscopic limit

$$\langle \Omega \left| T_{00}\left(\tau, \vec{n}_1\right) T_{00}\left(0, \vec{n}_2\right) \right| \Omega \rangle$$

Descendants drop out in the macroscopic limit

$$\langle \Omega | \mathcal{O}(\tau, \vec{n}_1) \, \mathcal{O}(0, \vec{n}_2) | \Omega \rangle = \sum |C_{\Omega \mathcal{O} \Delta}|^2 \, g_{\Delta,\ell}(z, \bar{z})$$

$$\sqrt{z\bar{z}} = e^{\tau/R}, \qquad \frac{z+\bar{z}}{2\sqrt{z\bar{z}}} = \cos\theta$$

$$g_{\Delta,\ell}(z,\bar{z}) = \sum_{m,n} r_{m,n} (z\bar{z})^{(\Delta+m+n)/2} C_{\ell+m-n}$$

$$g_{\Delta,\ell}(z,\bar{z}) \equiv (z\bar{z})^{-\Delta_\Omega/2} \mathcal{G}_{\Delta_\Omega+\Delta,\ell}(z,\bar{z})$$

- In the macroscopic limit, m=n=0 term gives the dominant contribution.

$$z \equiv 1 + \frac{u}{R}$$

R becomes very large such that u is fixed and so in energy density.

Descendants drop out in the macroscopic limit

- Let us show this for the scalar blocks in 4D CFT

$$\Delta = O(1), \ell = O(1)$$

$$g_{\Delta,\ell}(z,\bar{z}) = (z\bar{z})^{-\Delta_\Omega/2}\mathcal{G}_{\Delta_\Omega+\Delta,\ell}(z,\bar{z}) = (\ell+1) + \frac{(\ell+1)\,(2\Delta-\Delta_{\mathcal{O}})\,(u+\bar{u})}{2R} + \ldots$$

$$z \equiv 1 + \frac{u}{R}$$

Should we retain the m=n=0 term we would have obtained the same result from Gegenbauer!

Descendants drop out in the macroscopic limit

- Let us show this for the scalar blocks in 4D CFT

$$\Delta = \mathcal{E}R, \quad \ell \equiv pR$$

$$g_{\Delta,\ell}(z,\bar{z}) = R\exp\left(\frac{\mathcal{E}}{2}(u+\bar{u})\right)\frac{2\sinh\left(\frac{p}{2}(u-\bar{u})\right)}{u-\bar{u}}$$

$$z \equiv 1 + \frac{u}{R}$$

↓

Should we retain the m=n=0 term we would have obtained the same result from Gegenbauer!

- General argument can be made using Dolan Osborn results.
- Similar arguments hold for Spinning correlator as well.

Nambu Goldstone Theorems (T00T00)

$$\rho_{T^{00},T^{00}}(\omega,k) \approx -(\epsilon+P)k^2\delta'(\omega)$$

- Related to Fourier transform of T00T00 correlator (to be precise commutator) in the heavy state.
- Write down the correlator on the Cylinder, take the macroscopic limit, rewrite it as Fourier transformation.

$$\begin{aligned}\langle\Omega\left|T_{00}\left(\tau,\vec{n}_1\right)T_{00}\left(0,\vec{n}_2\right)\right|\Omega\rangle =&\epsilon^2\left[1+\sum_{\ell}\frac{\#\ell!}{\Delta_\Omega^\ell}e^{-\ell|\tau|/R}C_\ell^{(d/2-1)}(\cos\theta)\right]\\ &+\sum_{\Delta}\frac{C^2_{\Omega T_{00}\Delta}}{R^{2d}}\left[e^{-\Delta|\tau|/R}C_{\ell(\Delta)}^{(d/2-1)}(\cos\theta)+O\left(1/\Delta_\Omega\right)\right]\end{aligned}$$

Nambu Goldstone Theorems (T00T00)

$$\langle\Omega\left|T_{00}\left(\tau,\vec{n}_1\right)T_{00}\left(0,\vec{n}_2\right)\right|\Omega\rangle =\epsilon^2\left[1+\sum_{\ell}\frac{\#\ell!}{\Delta_\Omega^\ell}e^{-\ell|\tau|/R}C_\ell^{(d/2-1)}(\cos\theta)\right]$$
$$+\sum_{\Delta}\frac{C^2_{\Omega T_{00}\Delta}}{R^{2d}}\left[e^{-\Delta|\tau|/R}C_{\ell(\Delta)}^{(d/2-1)}(\cos\theta)+O\left(1/\Delta_\Omega\right)\right]$$

$$\boxed{\lim_{R\to\infty}C_{pR}^{(d/2-1)}(\cos(x/R))=\frac{1}{2^{d-2}\pi^{(d-2)/2}\Gamma\left(\frac{d-2}{2}\right)}\frac{R^{d-3}}{p}\int d^{d-1}k\delta(|\vec{k}|-p)e^{i\vec{k}\cdot\vec{x}}}$$

Nambu Goldstone Theorems (T00T00)

$$\lim_{R\to\infty} C_{pR}^{(d/2-1)}(\cos(x/R)) = \frac{1}{2^{d-2}\pi^{(d-2)/2}\Gamma\left(\frac{d-2}{2}\right)}\frac{R^{d-3}}{p}\int d^{d-1}k\,\delta(|\vec{k}|-p)e^{i\vec{k}\cdot\vec{x}}$$

$$\begin{aligned}\langle\Omega\left|T_{00}\left(\tau,\vec{n}_1\right)T_{00}\left(0,\vec{n}_2\right)\right|\Omega\rangle =&\epsilon^2\left[1+\sum_{\ell}\frac{\#\ell!}{\Delta_\Omega^\ell}e^{-\ell|\tau|/R}C_\ell^{(d/2-1)}(\cos\theta)\right]\\ &+\sum_{\Delta}\frac{C^2_{\Omega T_{00}\Delta}}{R^{2d}}\left[e^{-\Delta|\tau|/R}C_{\ell(\Delta)}^{(d/2-1)}(\cos\theta)+O\left(1/\Delta_\Omega\right)\right]\end{aligned}$$

$$\boxed{\langle\Omega\left|T_{00}(\tau,x)T_{00}(0)\right|\Omega\rangle = \epsilon^2+\frac{1}{2^{d-2}\pi^{(d-2)/2}\Gamma\left(\frac{d-2}{2}\right)}\int d\omega d^{d-1}k K(\omega,k)e^{-\omega|\tau|}\frac{e^{i\vec{k}\cdot\vec{x}}}{k}}$$

$$K(\omega,p)\equiv\rho(\omega,p)C^2_{\Omega T_{00}\Delta}/R^{d+3}$$

Implication in CFT (T00T00)

$$\langle \Omega | T_{00}(\tau, x) T_{00}(0) | \Omega \rangle = \epsilon^2 + \frac{1}{2^{d-2}\pi^{(d-2)/2}\Gamma\left(\frac{d-2}{2}\right)} \int d\omega d^{d-1}k K(\omega, k) e^{-\omega|\tau|} \frac{e^{i\vec{k}\cdot\vec{x}}}{k}$$

$$\rho_{T^{00}T^{00}}(\omega, p) \sim \frac{1}{p}(K(\omega, p) - K(-\omega, p))$$

Compare this with the sum rule

$$\frac{\rho_{T^{00}T^{00}(\omega,p)}}{p^2} = -\frac{d\epsilon}{d-1}\delta'(\omega) + O(p)$$

$\Delta_{gap}/R \to 0$ OR $\Delta_{gap}/\Delta_{\Omega}^{1/d} \to 0$ AND $\ell/\Delta_{\Omega}^{1/d} \to 0$

Implication in CFT (T00T00)

$$\Delta_{\text{gap}} / \Delta_{\Omega}^{1/d} \to 0$$

AND

$$\ell / \Delta_{\Omega}^{1/d} \to 0$$

- This gap is little o of R, **not** quite order 1. This says that for every positive real number c, there exists a state below cR
- To argue for big O of 1 gap, one needs to further assume existence of scale invariant field theory describing the infinite volume physics.

Implication in CFT (T00T00)

- Not only can we bound the spectra, we can also bound the OPE coefficient.

$$\ell/\Delta_\Omega^{1/d} \to 0$$

$$\Delta_{\text{gap}} /\Delta_\Omega^{1/d} \to 0$$

$$\langle \Omega \,| T_{00}(\tau, x) T_{00}(0) |\, \Omega \rangle = \epsilon^2 + \frac{1}{2^{d-2}\pi^{(d-2)/2}\Gamma\left(\frac{d-2}{2}\right)} \int d\omega d^{d-1}k K(\omega, k) e^{-\omega|\tau|} \frac{e^{i\vec{k}\cdot\vec{x}}}{k}$$

$$K(\omega, p) \equiv \rho(\omega, p) C^2_{\Omega T_{00}\Delta}/R^{d+3}$$

$$C^2_{\Omega T_{00}\Delta} = O\left(\Delta_\Omega R^3\right)$$

Examples:

$$\ell / \Delta_\Omega^{1/d} \to 0$$

$$\Delta_{\text{gap}} / \Delta_\Omega^{1/d} \to 0$$

$$C^2_{\Omega T_{00} \Delta} = O\left(\Delta_\Omega R^3\right)$$

We can verify these in various set up:

$$\langle \Omega \left| T_{00}\left(\tau, \vec{n}_1\right) T_{00}\left(0, \vec{n}_2\right) \right| \Omega \rangle$$

1. Superfluid — Regge trajectory : Single particle states
2. Free Scalar Field (d>2) — Regge trajectory : particle-hole states, effectively single particle
3. Free Fermions — Particle hole pairs around Fermi surface
4. 2D CFT — Regge trajectory of Virasoro Descendants: Single particle states

Furthermore, one can also verify the sum rules for hydrodynamics.

Superfluid

S. Hellerman, D. Orlando, S. Reffert and M. Watanabe ;
A. Monin, D. Pirtskhalava, R. Rattazzi and F. K. Seibold

$$\langle \Omega \left| T_{00}\left(\tau, \vec{n}_1\right) T_{00}\left(0, \vec{n}_2\right) \right| \Omega \rangle$$

Regge trajectory : Single particle states

- Consider a CFT on cylinder with a U(1).
- Consider a state with large charge Q.

- There is a separation of energy scale in Q being very large, the UV scale is $\rho^{-1/(d-1)}$, while the IR scale is R.

- There is a EFT description of the physics in between these scale with 1/Q being cut off.

- Symmetry breaking pattern is $SO(d+1,1) \times U(1) \mapsto SO(d) \times \tilde{U}(1)$.

Superfluid

S. Hellerman, D. Orlando, S. Reffert and M. Watanabe ;
A. Monin, D. Pirtskhalava, R. Rattazzi and F. K. Seibold

$$\langle \Omega \left| T_{00} \left(\tau, \vec{n}_1 \right) T_{00} \left(0, \vec{n}_2 \right) \right| \Omega \rangle$$

Regge trajectory : Single particle states

- Consider a CFT on cylinder with a U(1).
- Consider a state with large charge Q.

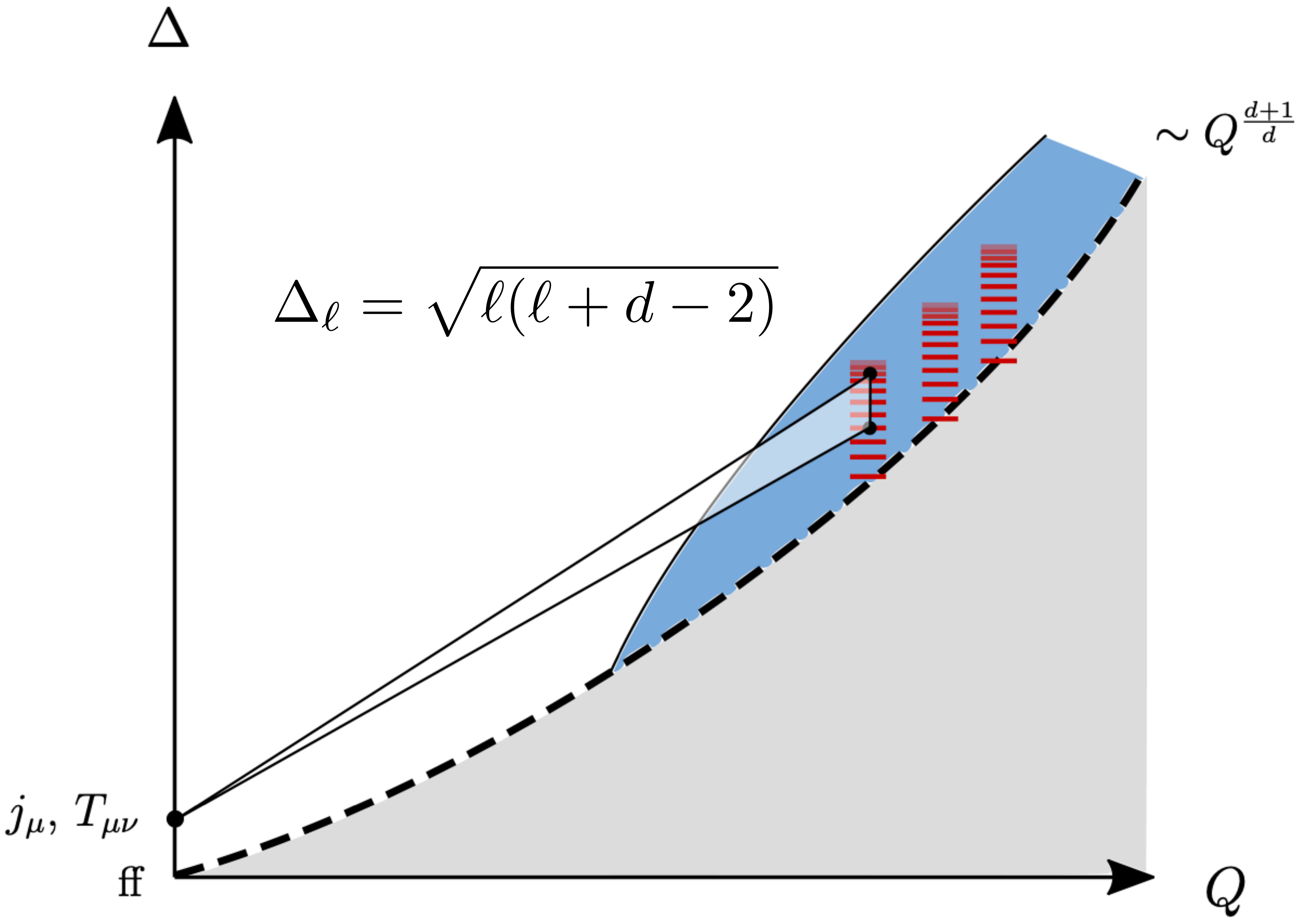
- There is a EFT description of the physics in between these scale with 1/Q being cut off.
- The action of the effective field theory can be constructed in the CCWZ way in terms of a field χ and its fluctuation π around the symmetry breaking saddle $\chi = i\mu t + \pi$

$$S_\pi = \frac{d(d-1)}{2} c_1 \mu^{d-2} \int d^d x \sqrt{g} \left(\dot{\pi}^2 + \frac{1}{d-1} \partial_i \pi \partial^i \pi \right) + \cdots$$

Superfluid

$$\langle \Omega \, | T_{00}(\tau, \vec{n}_1) \, T_{00}(0, \vec{n}_2) | \, \Omega \rangle$$

Regge trajectory : Single particle states

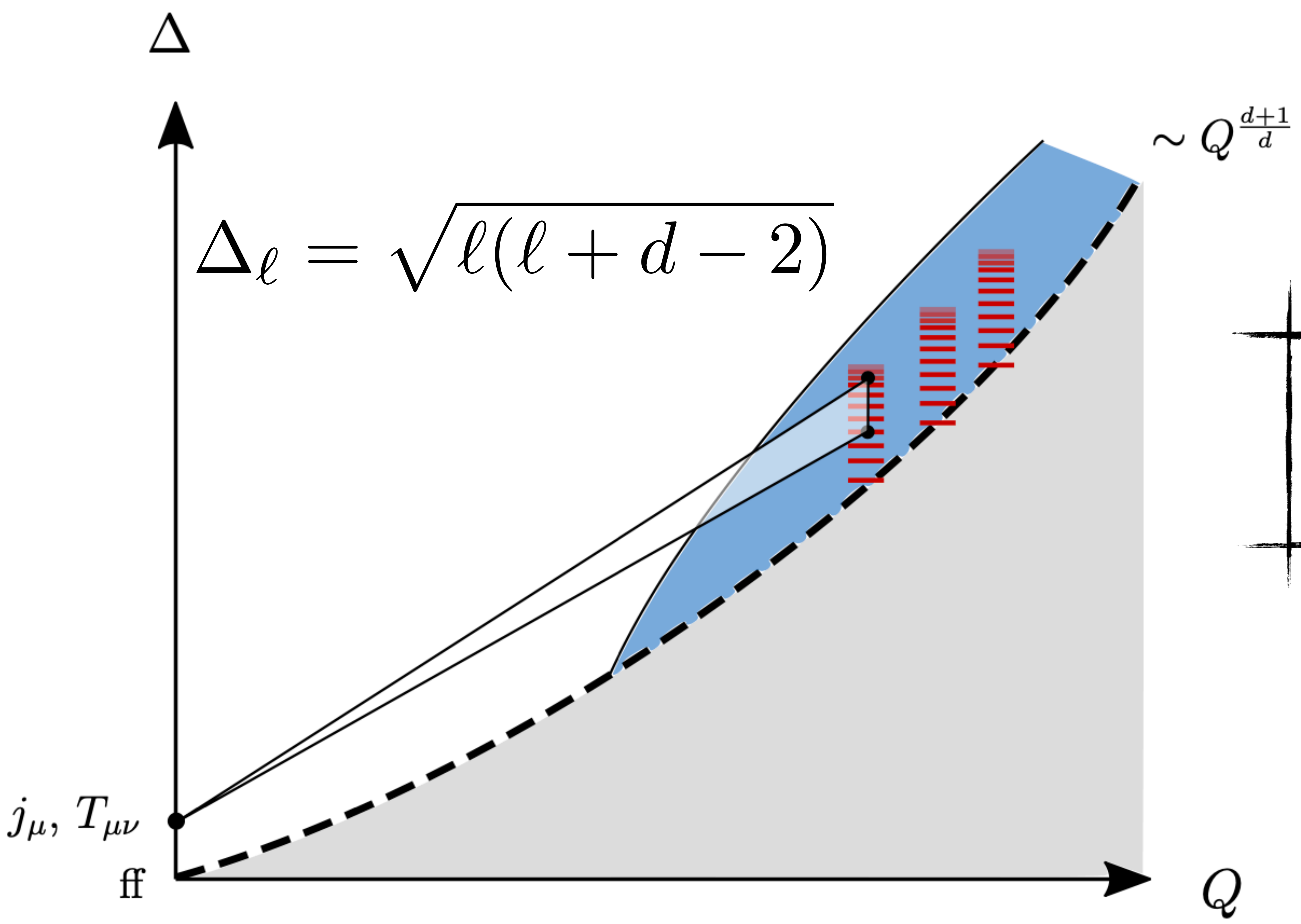


Picture taken from arXiv: 2102.05046
(G Cuomo, L Delacratez, U Mehta)

Superfluid

$$\langle \Omega \left| T_{00}\left(\tau, \vec{n}_1\right) T_{00}\left(0, \vec{n}_2\right) \right| \Omega \rangle$$

Regge trajectory : Single particle states



Picture taken from arXiv: 2102.05046
(G Cuomo, L Delacratez, U Mehta)

$$C^2_{\Omega T_{00}\Delta} = \frac{d\Delta_\Omega}{s^2_{d-1}} \left(\frac{2\ell + d - 2}{2(d-2)}\right) \sqrt{\ell(\ell + d - 2)/(d-1)}$$

Superfluid

$$\langle \Omega \,| T_{00}\,(\tau, \vec{n}_1)\, T_{00}\,(0, \vec{n}_2) |\, \Omega \rangle$$

Regge trajectory : Single particle states

$$C^2_{\Omega T_{00}\Delta} = \frac{d\Delta_\Omega}{s^2_{d-1}} \left(\frac{2\ell + d - 2}{2(d-2)} \right) \sqrt{\ell(\ell + d - 2)/(d-1)}$$

$$\rho_{T^{00}T^{00}}(\omega, p) = \frac{d\epsilon}{2\sqrt{d-1}} p \left[\delta\left(\omega - \frac{p}{\sqrt{d-1}} \right) - \delta\left(\omega + \frac{p}{\sqrt{d-1}} \right) \right]$$

Take the soft limit .

Superfluid

$$\langle \Omega \left| T_{00}(\tau, \vec{n}_1) T_{00}(0, \vec{n}_2) \right| \Omega \rangle$$

Regge trajectory : Single particle states

$$C^2_{\Omega T_{00} \Delta} = \frac{d\Delta_\Omega}{s^2_{d-1}} \left(\frac{2\ell + d - 2}{2(d-2)} \right) \sqrt{\ell(\ell + d - 2)/(d-1)}$$

$$\rho_{T^{00}T^{00}}(\omega, p) = \frac{d\epsilon}{2\sqrt{d-1}} p \left[\delta\left(\omega - \frac{p}{\sqrt{d-1}}\right) - \delta\left(\omega + \frac{p}{\sqrt{d-1}}\right) \right]$$

$$\rho_{T^{00}T^{00}}(\omega, p) \underset{p \to 0}{\simeq} -\frac{d\epsilon}{d-1} p^2 \delta'(\omega) = -(\epsilon + P)\delta'(\omega)$$

Superfluid: Parity violating set up

arXiv: 2102.05046. (G Cuomo, L Delacratez, U Mehta)

$$\langle \Omega \left| T_{00} \left(\tau, \vec{n}_1 \right) T_{00} \left(0, \vec{n}_2 \right) \right| \Omega \rangle$$

Regge trajectory : Single particle states

$$\Delta_{\text{min}}(Q, J) = \Delta_{\text{min}}(Q) + \frac{\sqrt{2\pi\chi_0}}{4} \frac{J(J+1)}{Q^{3/2}} + \cdots$$

- There are softer modes but they can not contribute to the sum rule since the phonon modes already saturate the sum rule. This is corroborated by the OPE coefficients found in arXiv: 2102.05046 and the fact the vortex solutions can not survive the infinite volume limit.

2D CFT

$$\langle \Omega \,| T_{00} \left(\tau, \vec{n}_1 \right) T_{00} \left(0, \vec{n}_2 \right) |\, \Omega \rangle$$

Regge trajectory : Single particle states

- U(1) can not be spontaneously broken.
- Nonetheless, one can consider heavy states. The boost can be spontaneously broken and the sum rules still holds.

$$\langle \Omega | T(u) T(0) | \Omega \rangle = \frac{h_\Omega^2}{R^4} + \frac{h_\Omega}{R^4(\cosh(u/R) - 1)} + \frac{c}{8R^4(\cosh(u/R) - 1)^2}$$

- Everything is controlled Virasoro Identity module, hence we know the full answer and verify the macroscopic limit exists.

$$\rho_{T_{tt}T_{tt}} \underset{k \to 0}{\simeq} -2\epsilon k^2 \delta'(\omega) - \frac{k^4}{6}\left[c\delta'(\omega) + 2\epsilon \delta'''(\omega)\right] + \cdots$$

2D CFT: Can we have EFT?

$$\langle \Omega \left| T_{00} \left(\tau, \vec{n}_1 \right) T_{00} \left(0, \vec{n}_2 \right) \right| \Omega \rangle$$

Regge trajectory : Single particle states

$$\langle \Omega | T(u) T(0) | \Omega \rangle = \pi^2 \epsilon^2 + \frac{2\pi\epsilon}{u^2} + \frac{c}{2u^4}$$

- A candidate EFT could be the superfluid EFT specialized to d=2.

$$\frac{\kappa}{\pi} \int d^2x \ \partial\varphi \bar{\partial}\varphi + \frac{c-1}{12\pi} \int d^2x \ \frac{\partial^2\varphi \bar{\partial}^2\varphi}{\partial\varphi\bar{\partial}\varphi}$$

- The story resembles EFT of long string by Polchinski-Strominger .

2D CFT: Can we have EFT?

- A candidate EFT could be the superfluid EFT specialized to d=2.

$$\frac{\kappa}{\pi}\int d^2x\ \partial\varphi\bar{\partial}\varphi + \frac{c-1}{12\pi}\int d^2x\ \frac{\partial^2\varphi\bar{\partial}^2\varphi}{\partial\varphi\bar{\partial}\varphi}$$

- Can we push this to higher order ?

- The crucial observation is that all the EFT terms that could be written down vanish on-shell. Thus one can write down a field redefinition which makes the EFT Lagrangian

$$S = \frac{\kappa}{\pi}\int d^2x \partial\tilde{\varphi}\bar{\partial}\tilde{\varphi}$$

- Of course, stress energy tensor is not the usual free field one, rather it gets correction under field redefinition.

2D CFT: Can we have EFT?

$$S = \frac{\kappa}{\pi}\int d^2x \partial\tilde{\varphi}\bar{\partial}\tilde{\varphi}$$

- Of course, stress energy tensor is not the usual free field one, rather it gets correction under field redefinition so that TT ope is the one for the CFT with central charge c.

$$\begin{aligned} T(u) =& \frac{\kappa\mu^2}{4} + i\kappa\mu\partial\tilde{\pi} - \kappa(\partial\tilde{\pi})^2 + \frac{1}{\mu}\left[\frac{i(c-1)}{12}\partial^3\tilde{\pi} + \gamma_1\partial\tilde{\pi}\partial^2\tilde{\pi}\right] \\ &+ \frac{1}{\mu^2}\left[\frac{(c-1)}{6}\left(\left(\partial^2\tilde{\pi}\right)^2 + \partial\tilde{\pi}\partial^3\tilde{\pi}\right) + \gamma_2\partial^4\tilde{\pi} + \gamma_3(\partial\tilde{\pi})^2\partial^2\tilde{\pi}\right] + O\left(1/\mu^3\right) \end{aligned}$$

2D CFT: Can we have EFT?

$$S = \frac{\kappa}{\pi} \int d^2x \partial\tilde{\varphi}\bar{\partial}\tilde{\varphi}$$

- Of course, stress energy tensor is not the usual free field one, rather it gets correction under field redefinition so that TT ope is the one for the CFT with central charge c.

$$T(u) = \frac{\kappa\mu^2}{4} + i\kappa\mu\partial\tilde{\pi} - \kappa(\partial\tilde{\pi})^2 + \frac{1}{\mu}\left[\frac{i(c-1)}{12}\partial^3\tilde{\pi} + \gamma_1\partial\tilde{\pi}\partial^2\tilde{\pi}\right] + \frac{1}{\mu^2}\left[\frac{(c-1)}{6}\left(\left(\partial^2\tilde{\pi}\right)^2 + \partial\tilde{\pi}\partial^3\tilde{\pi}\right) + \gamma_2\partial^4\tilde{\pi} + \gamma_3(\partial\tilde{\pi})^2\partial^2\tilde{\pi}\right] + O\left(1/\mu^3\right)$$

- Prediction of the EFT

$$\boxed{\Delta_Q = \frac{Q^2}{2\kappa} + \frac{c-1}{12}}$$

2D CFT: Can we have EFT?

$$T(u) = \frac{\kappa\mu^2}{4} + i\kappa\mu\partial\tilde{\pi} - \kappa(\partial\tilde{\pi})^2 + \frac{1}{\mu}\left[\frac{i(c-1)}{12}\partial^3\tilde{\pi} + \gamma_1\partial\tilde{\pi}\partial^2\tilde{\pi}\right]$$
$$+ \frac{1}{\mu^2}\left[\frac{(c-1)}{6}\left((\partial^2\tilde{\pi})^2 + \partial\tilde{\pi}\partial^3\tilde{\pi}\right) + \gamma_2\partial^4\tilde{\pi} + \gamma_3(\partial\tilde{\pi})^2\partial^2\tilde{\pi}\right] + O\left(1/\mu^3\right)$$

- We immediately face a problem if we want to promote U(1) to the Kac-Moody one.
  If we try to impose JJ and TJ ope, we are forced to have c=1.

- Either we give up on Kac-Moody symmetry, such CFTs must have continuous spectra or we are forced to make c=1, in which case, the EFT story is trivial.
- A more interesting application could perhaps be large charge sector of boundary 3D CFT, where one needs to take into account the boundary trace anomaly.

THANK YOU