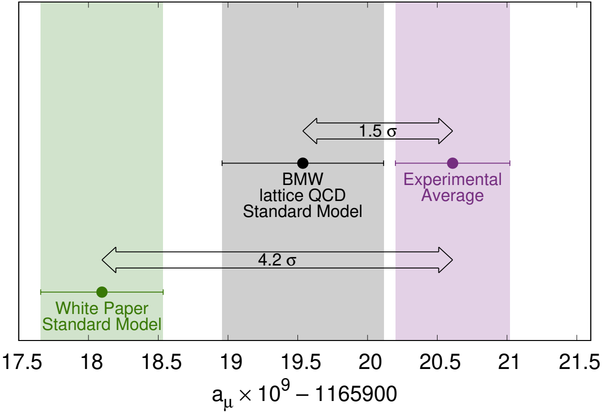
$(g-2)_\mu$ from lattice QCD and experiments: 4.2 sigma, indeed?

Zoltan Fodor

Budapest–Marseille–Wuppertal collaboration
(BMW)

KAVLI-IPMU, 19 May, 2021

Tensions in $(g-2)_\mu$

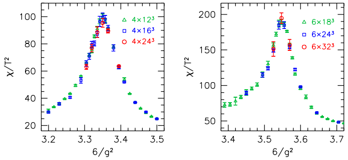


[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

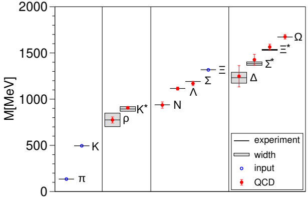
[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

Lattice QCD: examples

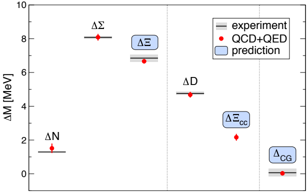
- Wuppertal–Budapest-collaboration,
  The order of the quantum chromodynamics transition predicted by the standard model of particle physics,
  Nature 443 (2006) 675-678



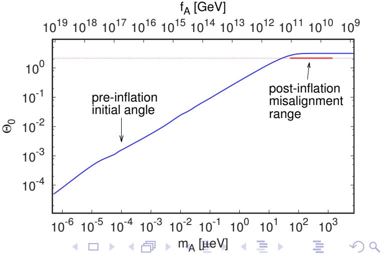
- Budapest–Marseille–Wuppertal-collaboration,
  Ab-initio Determination of Light Hadron Masses,
  Science 322 (2008) 1224-1227



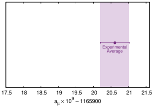
- Budapest–Marseille–Wuppertal-collaboration,
  Ab initio calculation of the neutron-proton mass difference, Science 347 (2015) 1452-1455

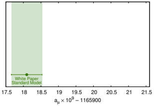


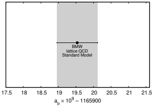
- Wuppertal–Budapest-collaboration,
  Lattice QCD for Cosmology, Nature 539 (2016) 7627, 69-71



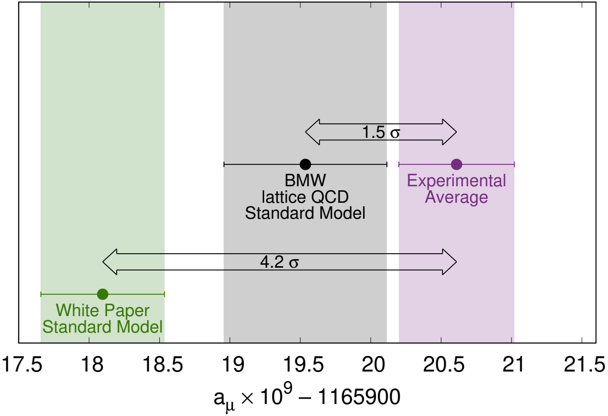
Outline

1. $(g-2)_\mu$
2. 

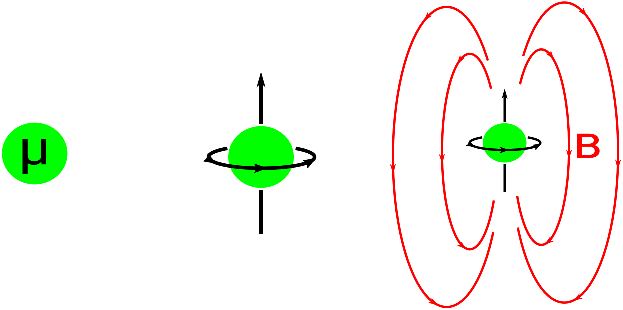
3. 

4. 

5. Summary



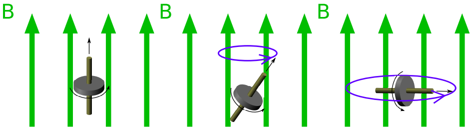
The muon



- Subatomic particle, first discovered in 1936
- Created cosmic rays entering the atmosphere
- Elementary particle, belongs to the family of charged leptons: e^-, μ^-, τ^-
  - same electric charge
  - same spin ($s = \frac{1}{2}$)
  - $\approx$ 207 times heavier than electron
  - finite lifetime: $\tau = 2.2\,\mu$s
- Acts like a tiny magnet. How strong? $\mu = ?$

Anomalous magnetic moment

- Charged top, with homogeneous charge and mass distribution, Q,M



- Circular current $\longrightarrow$ magnetic moment $\boldsymbol{\mu}$

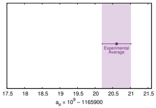
$$\boldsymbol{\mu} = \frac{Q}{2M}\mathbf{L}$$

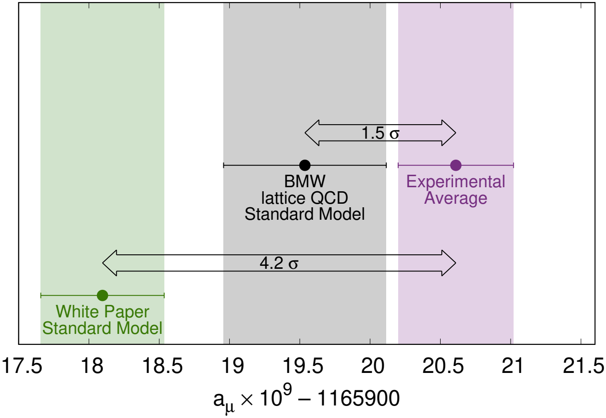
- $\boldsymbol{\mu}$ is proportional to angular momentum $\mathbf{L}$
- Muon has spin $\frac{1}{2}$ $\longrightarrow$ described by the Dirac equation

$$\boldsymbol{\mu}_\mu = 2\,\frac{e}{2m_\mu}\mathbf{S}_\mu = g\,\frac{e}{2m_\mu}\mathbf{S}_\mu$$

- Quantum mechanics predicts: $g = 2$
- In reality: $g_\mu \approx 2.00233\ldots$
- Deviation: Anomalous Magnetic Moment $a = \dfrac{g-2}{2}$

Outline

2. 
Experimental
Average
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$


1.5 σ
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Experimental result

- Newly announced result at Fermilab

$$a_\mu(\text{FNAL}) = 11\,659\,204.0(5.4)\cdot 10^{-10} \quad (0.46\,\text{ppm})$$

- Equivalent to: bathroom scale sensitive to weight of a single eyelash.

- Fully agrees with the BNL E821 measurement

$$\begin{aligned} a_\mu(\text{BNL}) &= 11\,659\,209.1(6.3)\cdot 10^{-10} \quad (0.54\,\text{ppm}) \\ a_\mu(\text{combined}) &= 11\,659\,206.1(4.1)\cdot 10^{-10} \quad (0.35\,\text{ppm}) \end{aligned}$$

- Target uncertainty: (1.6)

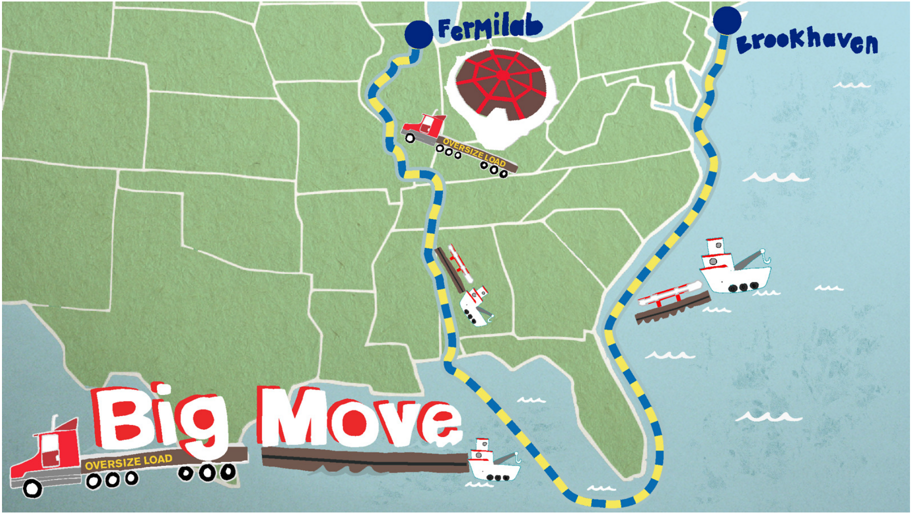
History of $(g-2)_\mu$ experiments

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL	1997	μ^+	11 659 251(150)	13
BNL	1998	μ^+	11 659 191(59)	5
BNL	1999	μ^+	11 659 202(15)	1.3
BNL	2000	μ^+	11 659 204(9)	0.73
BNL	2001	μ^-	11 659 214(9)	0.72
Average			11 659 208.0(6.3)	0.54

- Brookhaven had the $30 million magnet
- Fermilab had the most intense beam of muons in the world

$\longrightarrow$ Bring the 14 m diameter magnet ring to Fermilab

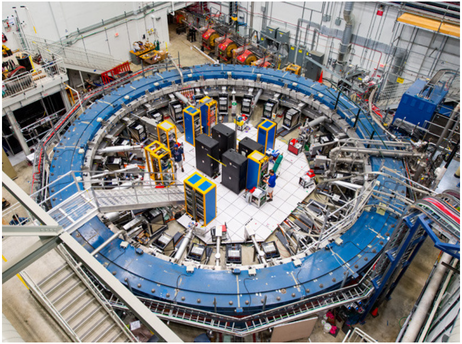
The route



Transportation

Experiment at FNAL

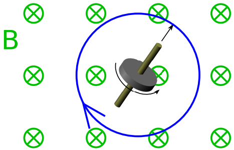
- Storage ring at Fermilab

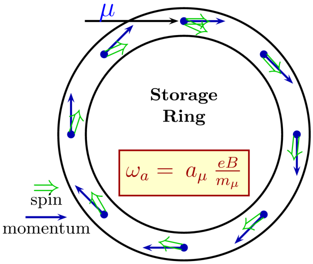


- Ring diameter: 14 m
- Homogeneous magnetic field: $B = 1.45\,\mathrm{T}$
- Momentum of the muon: $p_\mu = 3.094\,\mathrm{GeV}$

Measurement principle

- Cyclotron motion frequency: $\omega_c = \frac{eB}{m_\mu \gamma}$ with $\gamma = \frac{1}{\sqrt{1-v^2}}$
- Spin precession frequency: $\omega_s = \frac{eB}{m_\mu \gamma} + a_\mu \frac{eB}{m_\mu}$





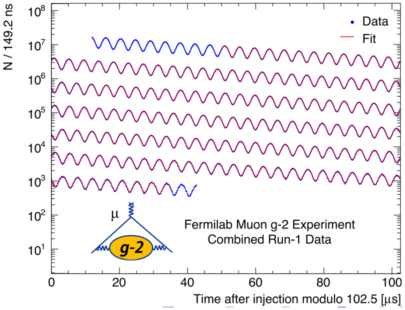
- Measure: $\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{m_\mu}$
- Gives $a_\mu = \frac{g_\mu - 2}{2}$ directly
- During each circle spin axis changes $12'$

Measurement principle

- Pions produced at fixed target $p + p \to p + n + \pi^+$
- Pion decays through weak interaction $\longrightarrow$ highly polarized muons

$$\pi^+ \to \mu^+ + \nu_\mu$$

- Muons decay after several circles: $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$
- Strong correlation between μ^+ spin and e^+ momentum
- Detect emitted e^+



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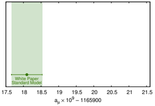
3. 
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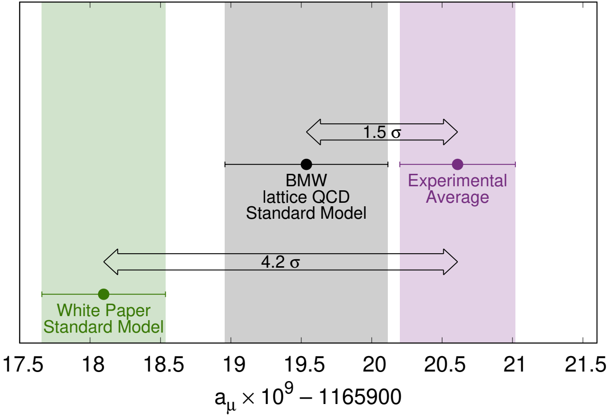
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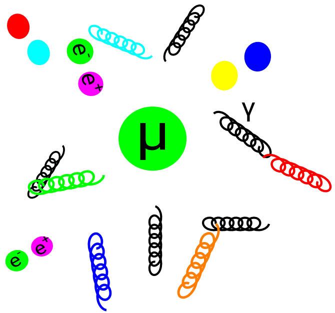
Theory: Standard Model



Sum over all known physics:

1. quantum electrodynamics (QED): photons, leptons
2. electroweak (EW): W, Z bosons, neutrinos, Higgs
3. strong (QCD): quarks and gluons

- [2006.04822] White Paper of Muon g-2 Theory Initiative

	$a_\mu \times 10^{-10}$
QED	11658471.9(0.0)
electroweak	15.4(0.1)
strong	693.7(4.3)
total	11659181.0(4.3)

Theory: QED

- $\alpha = \frac{e^2}{4\pi} \ll 1$ → rapidly converging series, was a key to success of QED

$$\left(\frac{g-2}{2}\right) = \left(\frac{\alpha}{\pi}\right) a^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 a^{(2)} + \left(\frac{\alpha}{\pi}\right)^3 a^{(3)} + \dots$$

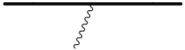
- all contributions with photons and leptons (e, μ, τ)

n-loop	$a_\mu^{\text{QED}} \times 10^{-10}$
1	11614097.330(0.008)
2	41321.762(0.010)
3	3014.190(0.000)
4	38.081(0.030)
5	0.448(0.140)
total	11658471.811(0.160)

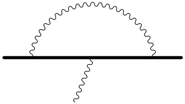
- inputs are m_μ/m_e, m_μ/m_τ and α

Theory: QED

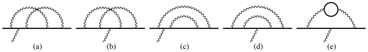
$\alpha^0 \to g = 2$



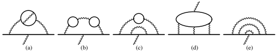
$\alpha^1 \to \alpha/(2\pi)$



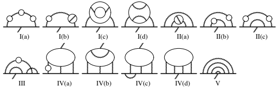
α^2



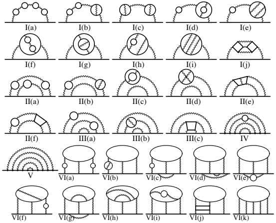
α^3



α^4



α^5

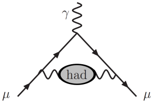


Five loops by [Kinoshita et al '15]

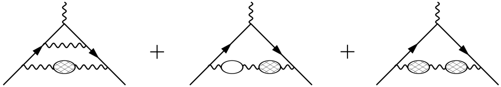
- 12672 diagrams
- automated diagram generation, numerical evaluation of integrals, only some diagrams known analytically
- not yet confirmed by other groups

Hadronic contributions

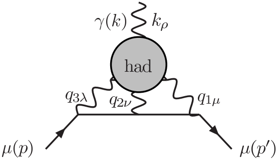
- LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)



- NLO hadron vacuum polarization (NLO-HVP, $(\frac{\alpha}{\pi})^3$)



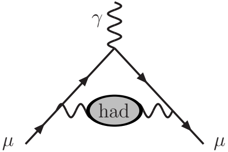
- Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)



- pheno $a_\mu^{\text{HLbL}} = 9.2(1.9)$
  [Colangelo, Hoferichter, Kubis, Stoffer et al '15–'20]
- lattice a_μ^{HLbL}=7.9(3.1)(1.8) or 10.7(1.5) [RBC/UKQCD '19 and Mainz '21]

$\mathcal{O}(\alpha^2)$ hadronic contribution

- Hadronic vacuum polarization (HVP) of photon



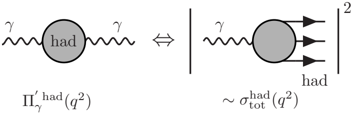
- In QED, $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137} \ll 1 \quad \longrightarrow$ rapidly converging series

$$\left(\frac{g-2}{2}\right) = \left(\frac{\alpha}{\pi}\right) a^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 a^{(2)} + \left(\frac{\alpha}{\pi}\right)^3 a^{(3)} + \dots$$

- In QCD, at low energies: $\alpha_s = \mathcal{O}(1)$
- Perturbative approach fails

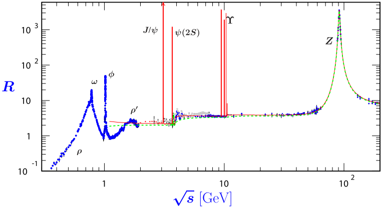
HVP from R-ratio

- Optical theorem



Use $e^+e^- \to$ had data of CMD, SND, BES, KLOE, BABAR, ...
systematics limited

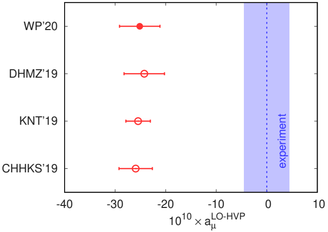
$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$



LO	[Jegerlehner '18]	688.1(4.1)	0.60%
LO	[Davier et al '19]	693.9(4.0)	0.58%
LO	[Keshavarzi et al '19]	692.78(2.42)	0.35%
LO	[Hoferichter et al '19]	692.3(3.3)	0.48%
NLO	[Kurz et al '14]	-9.87(0.09)	
NNLO	[Kurz et al '14]	1.24(0.01)	

Discrepancy

- $a_\mu^{\text{exp}} - a_\mu^{\text{theory}} = 25.1(6.0)$ around 4.2σ significance



error budget:

$(4.1)_{\text{exp}}(0.1)_{\text{QED}}(0.1)_{\text{weak}}(4.0)_{\text{HVP}}(1.8)_{\text{HLbL}}$

- HUGE: is about $2\times$ electroweak contribution

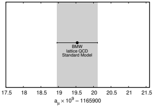
For new physics:

- FNAL(plan) + same theory errors 6σ
- FNAL(plan) + HLbL 10% + HVP 0.2% 11σ

For no new physics:

- 4% larger HVP, $a_\mu^{\text{LO-HVP}} = 720.0(6.8)$
- 360% larger HLbL, $a_\mu^{\text{HLbL}} = 37.9(7.1)$

Outline

4. 
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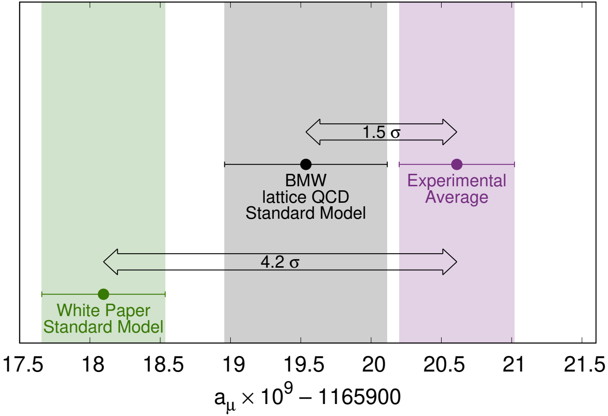
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Lattice QCD

- Integrate over all classical field configurations

$$\int [\mathrm{d}U]\,[\mathrm{d}\overline{\psi}]\,[\mathrm{d}\psi]\;\mathcal{O}\;e^{-S_g(U)-\overline{\psi}\,M(U)\,\psi}$$

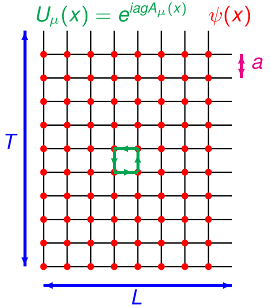
- E.g. $96^3 \times 144$ lattice $\longrightarrow$ $\approx 4 \cdot 10^9$ dimensional integral
- Stochastic integration

- 100000 years for a laptop $\longrightarrow$ 1 year for supercomputer

Lattice QCD

- Lattice gauge theory: systematically improvable, non-perturbative,

  1st principles method
- Discretize space-time with lattice spacing: a



- quarks on sites, gluons on links
- discretize action + operators

$$\int d^4x \longrightarrow a^4 \sum_x$$

$$\partial_\mu \longrightarrow \text{finite differences}$$

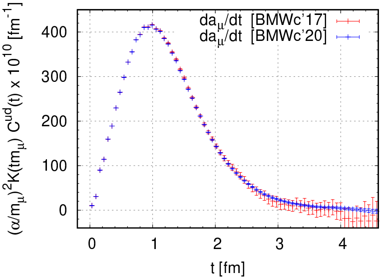
- To get physical results, need to perform:
  1. Infinite volume limit $(V \to \infty)$ $\longrightarrow$ numerically or analytically
  2. Continuum limit $(a \to 0)$ $\longrightarrow$ min. 3 different a

$a_\mu^{\text{LO-HVP}}$ from lattice QCD

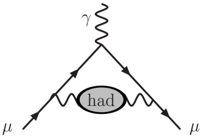
- Compute electromagnetic current-current correlator

$$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$$

$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt\, K(t)\, C(t)$$



$K(t)$ describes the leptonic part of diagram

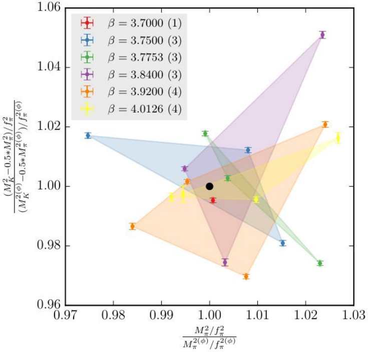


Simulation setup

- 6 lattice spacings: 0.13 fm – 0.064 fm $\longrightarrow$ controlled continuum limit
- Box size: $L \sim 6$ fm

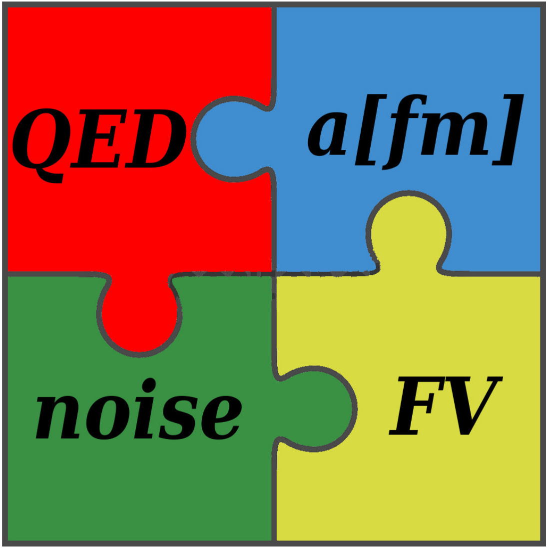
  $L \sim 11$ fm at one lattice spacing $\longrightarrow$ FV effects

  1 fm $= 10^{-15}$ m $\sim$ size of proton
- Quark masses bracketing their physical values



β	a[fm]	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980

New challenges



Scale determination

Lattice spacing a enters into a_μ determination:

- physical value of m_μ
- physical values of m_π, m_K

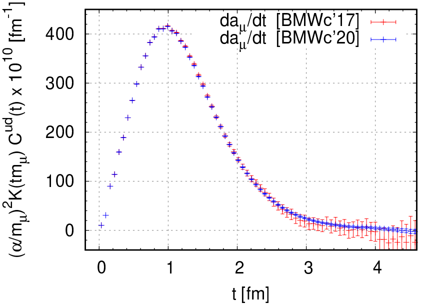
$\longrightarrow$ $\Delta_{\text{scale}} a_\mu \sim 2 \cdot \Delta(\text{scale})$

1. For final results: M_Ω scale setting $\longrightarrow$ $a = (aM_\Omega)^{\text{lat}} / M_\Omega^{\text{exp}}$
   - Experimentally well known: 1672.45(29) MeV [PDG 2018]
   - Moderate m_q dependence
   - Can be precisely determined on the lattice
2. For separation of isospin breaking effects: w_0 scale setting
   - Moderate m_q dependence
   - Can be precisely determined on the lattice
   - No experimental value
     $\longrightarrow$ Determine value of w_0 from $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70]\ \text{fm}$$

Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances



- Low Mode Averaging: use exact (all2all) quark propagator in IR and stochastic in UV
- decrease noise by replacing $C(t)$ by upper/lower bounds above t_c

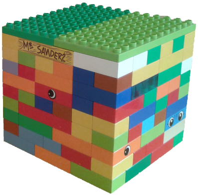
$$0 \leq C(t) \leq C(t_c)e^{-E_{2\pi}(t-t_c)}$$

$\longrightarrow$ few permil level accuracy on each ensemble

Finite-size effects

- Typical lattice runs use $L < 6\,\mathrm{fm}$, earlier model estimates gave $O(2)\%$ FV effect.

$L_{\mathrm{ref}} = 6.272\,\mathrm{fm}$



$L_{\mathrm{big}} = 10.752\,\mathrm{fm}$

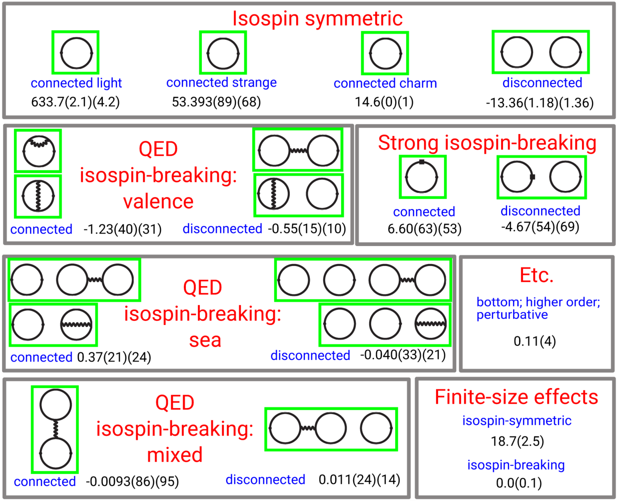
1. $a_\mu(\mathrm{big}) - a_\mu(\mathrm{ref})$
   - perform numerical simulations in $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$
   - perform analytical computations to check models

lattice	NLO XPT	NNLO XPT	MLLGS	HP	RHO
$18.1(2.0)_{\mathrm{stat}}(1.4)_{\mathrm{cont}}$	11.6	15.7	17.8	16.7	15.2

2. $a_\mu(\infty) - a_\mu(\mathrm{big})$
   - use models for remnant finite-size effect of “big” $\sim 0.1\%$

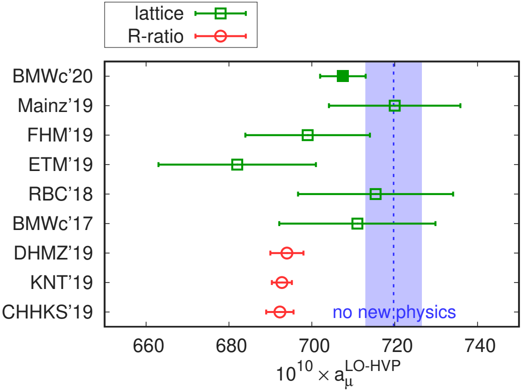
Isospin breaking effects

- Include leading order IB effects: $\mathcal{O}(e^2)$, $\mathcal{O}(\delta m)$



$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$

Final result

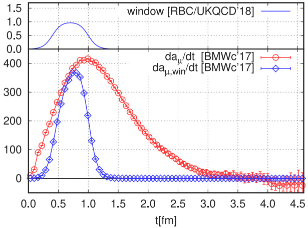


- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- consistent with new FNAL experiment
- 2.0σ larger than [DHMZ'19], 2.5σ than [KNT'19]

Window observable

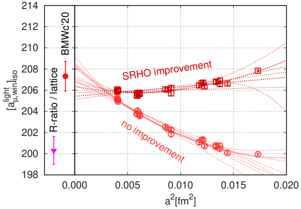
- Restrict correlator to window between $t_1 = 0.4\,\text{fm}$ and $t_2 = 1.0\,\text{fm}$

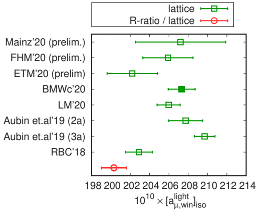
[RBC/UKQCD'18]



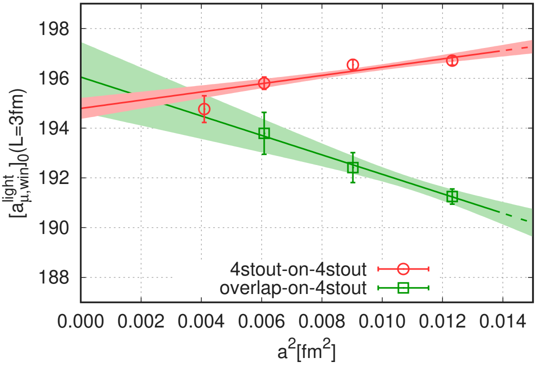
(144 × 96^3, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

- Less challenging than full a_μ
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)



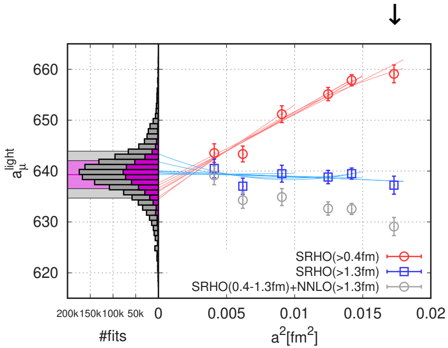


Crosscheck – overlap

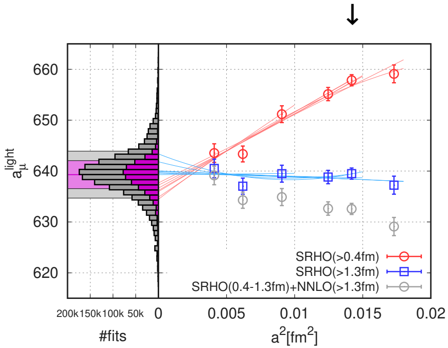


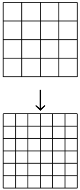
- compute $a_{\mu,\text{win}}$ with overlap valence
- local current instead of conserved $\longrightarrow$ had to compute Z_V
- cont. limit in $L = 3\,\text{fm}$ box consistent with staggered valence

Continuum limit

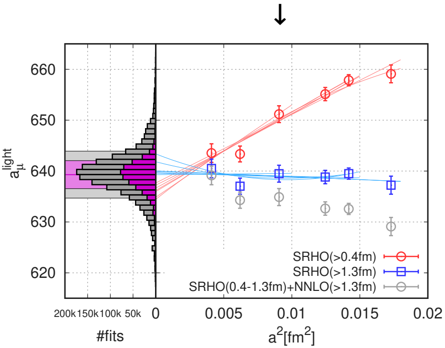


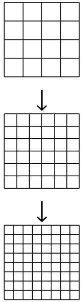
Continuum limit



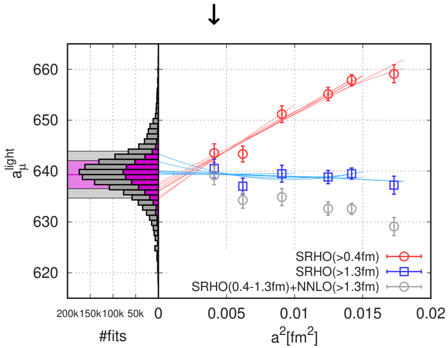


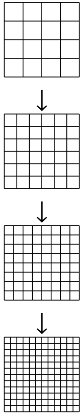
Continuum limit



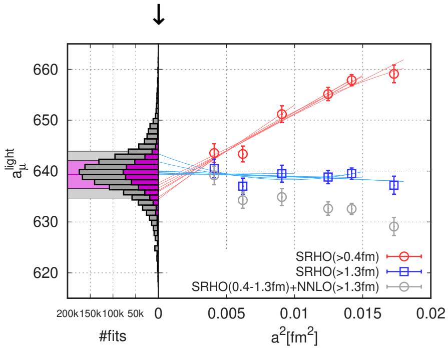


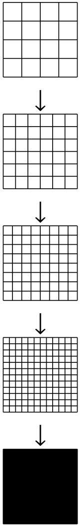
Continuum limit





Continuum limit

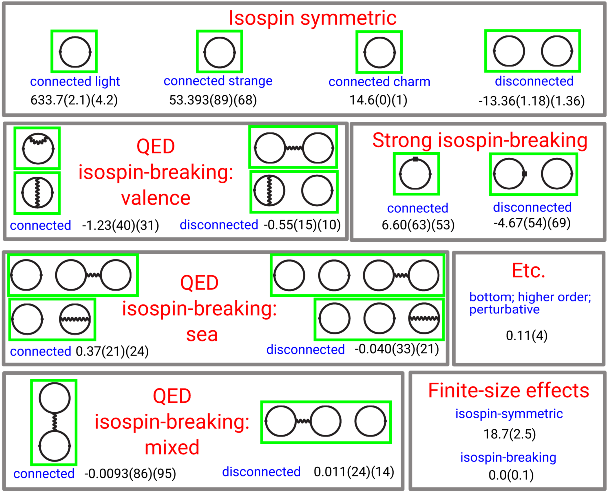




Outline

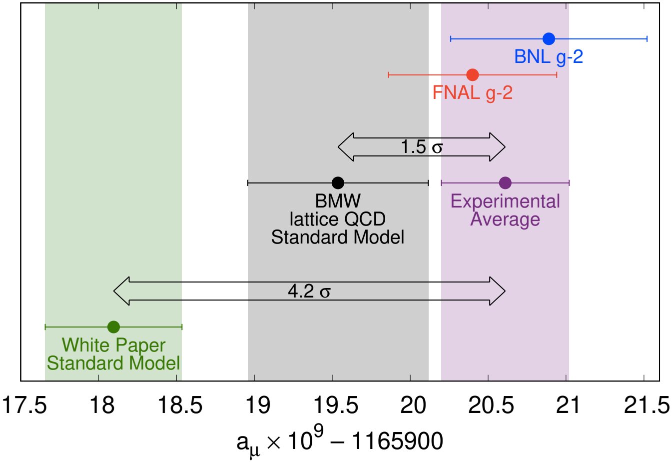
5. Summary

Final result



$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

Tensions


BNL g-2
FNAL g-2
1.5 σ
BMW
lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$