

Asymmetric Dark Matter from Scattering

Satyanarayan Mukhopadhyay
IACS, Kolkata

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Plan:

1. Asymmetric dark matter from semi-annihilation
2. The interplay of self-scattering and annihilation in DM cosmology

Based on:

1. arXiv: 2004.07705
2. arXiv: 2103.14009

Both of them with [Avirup Ghosh](#) (PhD HRI -> Postdoc IACS)
and [Deep Ghosh](#) (PhD IACS, ongoing)

Particle Dark Matter Density from Thermal Mechanisms:

The present DM density may depend upon initial conditions in the early Universe (such as the inflaton decay branching ratio)

DM density may also be determined independent of any initial conditions if the DM sector thermalised.

DM sector can thermalise within itself with a temperature different than the SM bath temperature

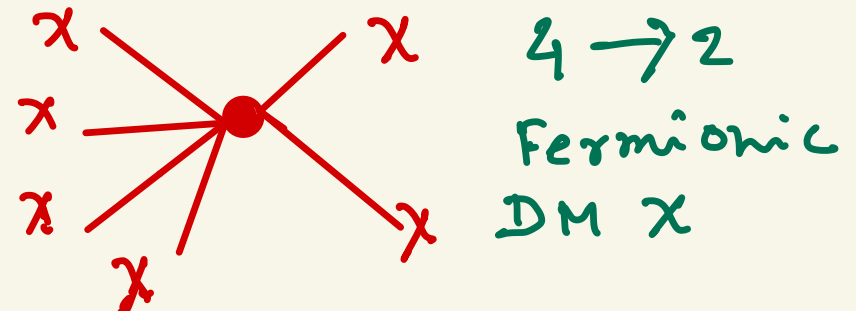
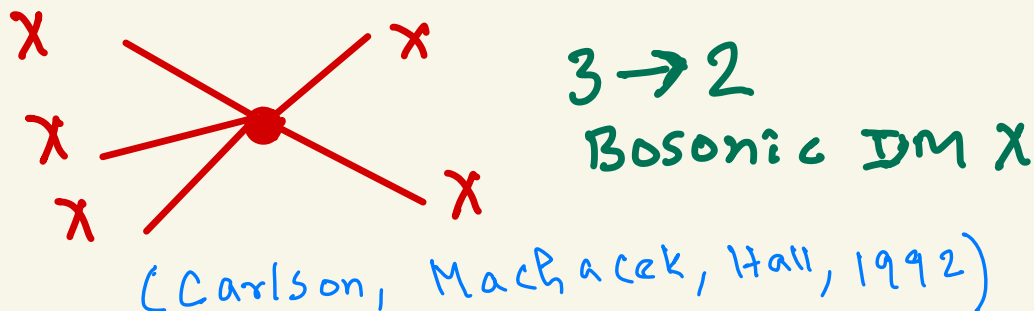
We shall assume that the DM and SM sectors reach a common temperature through elastic/inelastic scatterings

In the absence of any number changing reactions, thermal number densities may be too high even after dilution due to expansion of the Universe

Therefore, some number-changing reactions become necessary to achieve a relic density of DM consistent with observations

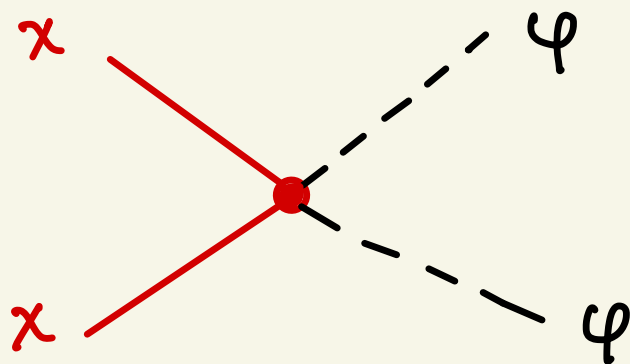
Such reactions can either be within the DM sector itself, or it can involve the SM sector

Assuming only one new particle in the DM sector (along with its anti-particle if not self-conjugate) the number changing reactions can be of $n \rightarrow m$ type, with $n > m$.



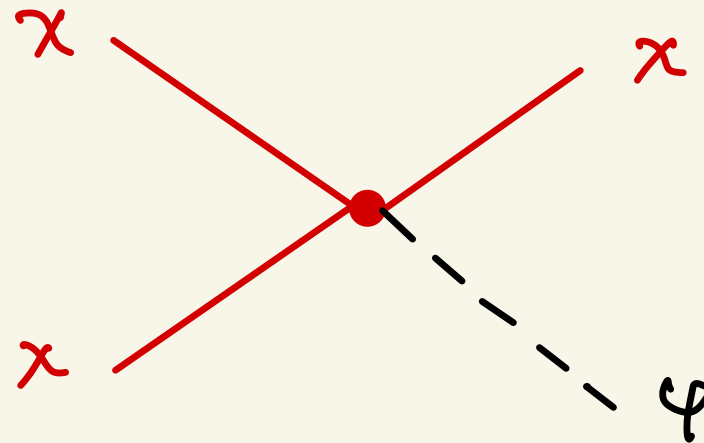
Number-changing reactions involving the SM fields can also be characterized in terms of such topologies

With one DM particle (+anti-particle) such topologies of $2 \rightarrow 2$ type can be



$$\Delta N_\chi = -2$$

Annihilation

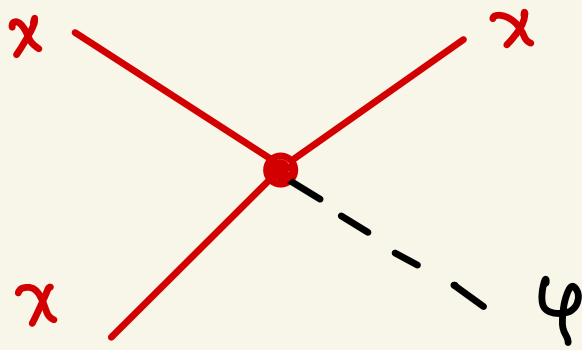


$$\Delta N_\chi = -1$$

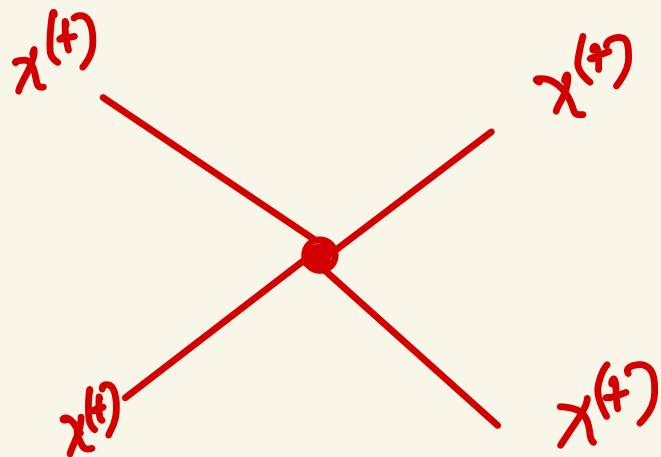
Semi-annihilation (D'Erramo, Taler, 2010)

φ : Either SM particle, or mixes with/decays to SM

This talk: the role of new / less explored topologies in DM cosmology



semi-annihilation : previous studies looked at WIMP-like cosmology, which is similar to $2 \rightarrow 2$ standard WIMP. It offers much richer phenomena in its full generality.



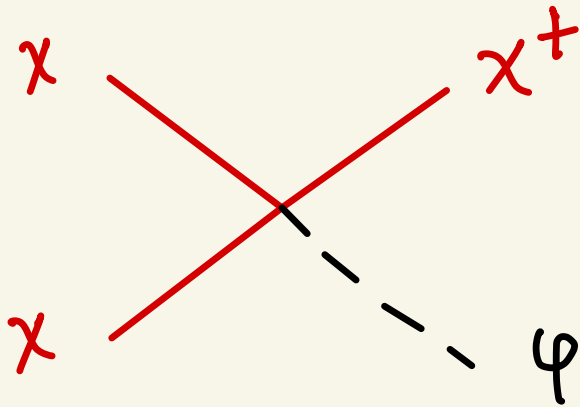
DM self-scattering/conversion : Its role in determining DM density and asymmetry is novel and can occur naturally in a simple scenario such as complex scalar DM.

Semi-annihilation: minimal scenario with a complex scalar DM

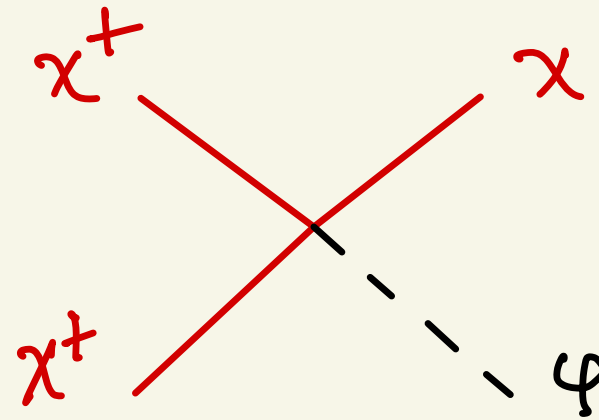
To ensure DM stability, an effective low energy \mathbb{Z}_3 symmetry is needed with charge assignment:

$\chi : \omega$, $\chi^\dagger : \omega^2$ such that $\omega^3 = 1$.

↑
Complex
scalar



$$\Delta N = -3$$



$$\Delta N = +3$$

DM number:

$$N(\chi) = +1$$

$$N(\chi^\dagger) = -1$$

$$\Delta(n_\chi + n_{\chi^\dagger}) = -1$$

Semi-annihilation can change both the DM number and net DM-antiDM density in one reaction.

In general the two CP-conjugate reactions can have different rates

$$\sigma(\chi\chi \rightarrow \chi^*\psi) \neq \sigma(\chi^*\chi^* \rightarrow \chi\psi) \text{ in general.}$$

Hence semi-annihilation can lead to particle-antiparticle asymmetry in DM

Q: 1. How much asymmetry can we generate?

Q: 2. How do the DM mass predictions change in the presence of an asymmetry?

Q: 3. What kind of semi-annihilation rates are necessary?

All three questions have relevance in the observational tests of such a scenario.

First let us understand these in a model-independent manner and then we shall show an explicit model realisation.

Ingredients for model-independent analysis: Boltzmann Equations

$$\begin{aligned} \frac{dn_\chi}{dt} + 3Hn_\chi = & - \int \prod_{i=1}^4 \frac{d^3 p_i}{(2\pi)^3 2E_{p_i}} g_\chi^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \left[2f_\chi(p_1)f_\chi(p_2) \overline{|M|^2}_{\chi\chi \rightarrow \chi^\dagger\phi} \right. \\ & - 2f_{\chi^\dagger}(p_3)f_\phi(p_4) \overline{|M|^2}_{\chi^\dagger\phi \rightarrow \chi\chi} - f_{\chi^\dagger}(p_1)f_{\chi^\dagger}(p_2) \overline{|M|^2}_{\chi^\dagger\chi^\dagger \rightarrow \chi\phi} \\ & \left. + f_\chi(p_3)f_\phi(p_4) \overline{|M|^2}_{\chi\phi \rightarrow \chi^\dagger\chi^\dagger} \right], \end{aligned} \quad (2.2)$$

In terms of $Y_i = \frac{n_i}{s}$ and $x = \frac{m_\chi}{T}$, with
 s : entropy density and H : Hubble parameter:

$$\begin{aligned} \frac{dY_\chi}{dx} &= -\frac{s}{Hx} \left[A_S \left(Y_\chi^2 + \frac{Y_0 Y_\chi}{2} \right) - B_S \left(\frac{Y_{\chi^\dagger}^2}{2} + Y_0 Y_{\chi^\dagger} \right) \right] \\ \frac{dY_{\chi^\dagger}}{dx} &= -\frac{s}{Hx} \left[B_S \left(Y_{\chi^\dagger}^2 + \frac{Y_0 Y_{\chi^\dagger}}{2} \right) - A_S \left(\frac{Y_\chi^2}{2} + Y_0 Y_\chi \right) \right]. \end{aligned}$$

$$A_S = \langle \sigma v \rangle_S + \langle \epsilon \sigma v \rangle_S$$

$$B_S = \langle \sigma v \rangle_S - \langle \epsilon \sigma v \rangle_S$$

$$\epsilon = \frac{|M|_{\chi\chi \rightarrow \chi^\dagger\phi}^2 - |M|_{\chi^\dagger\chi^\dagger \rightarrow \chi\phi}^2}{|M|_{\chi\chi \rightarrow \chi^\dagger\phi}^2 + |M|_{\chi^\dagger\chi^\dagger \rightarrow \chi\phi}^2},$$

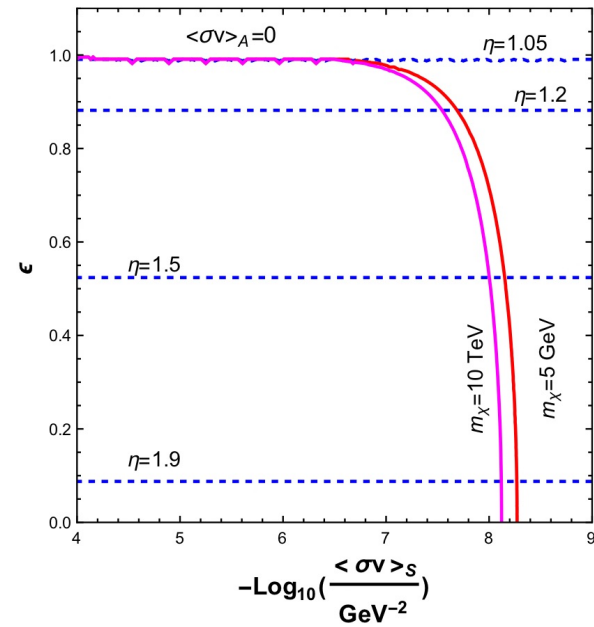
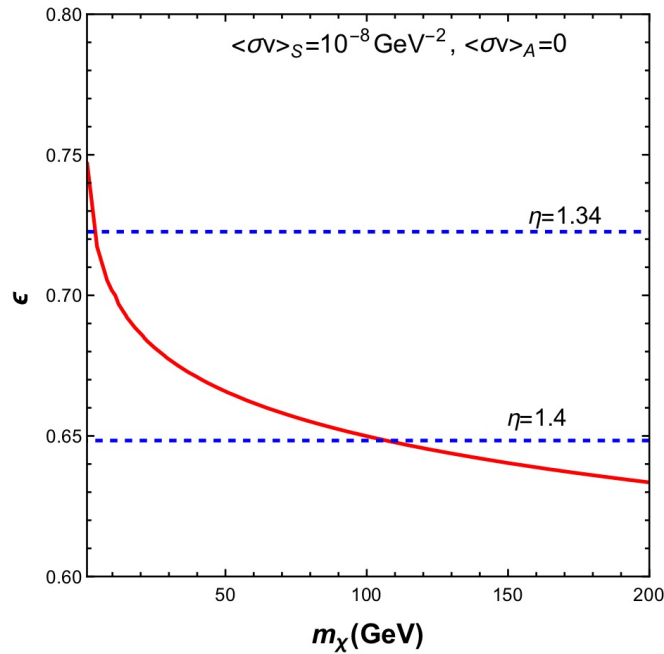
$$\langle \epsilon \sigma v \rangle_s = \frac{\int \prod_{i=1}^4 \frac{d^3 p_i}{(2\pi)^3 2E_{p_i}} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \epsilon(p_i) \overline{|M_0|^2} f_0(p_1) f_0(p_2)}{\int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} f_0(p_1) f_0(p_2)}$$

$$|M_0|^2 = |M|_{\chi\chi \rightarrow \chi^\dagger\phi}^2 + |M|_{\chi^\dagger\chi^\dagger \rightarrow \chi\phi}^2.$$

$$\epsilon \equiv \langle \epsilon \sigma v \rangle_s / \langle \sigma v \rangle_s$$

$$\epsilon \rightarrow 1 \Rightarrow \eta \rightarrow 1$$

Larger ϵ
 \Rightarrow larger
 pseudo
 chemical
 potential
 \Rightarrow larger
 number
 density
 \Rightarrow lower mass



Larger
 ϵ , larger
 $\langle \sigma v \rangle_s$
 required.

$$\Omega h^2 = 0.12 \text{ contours}$$

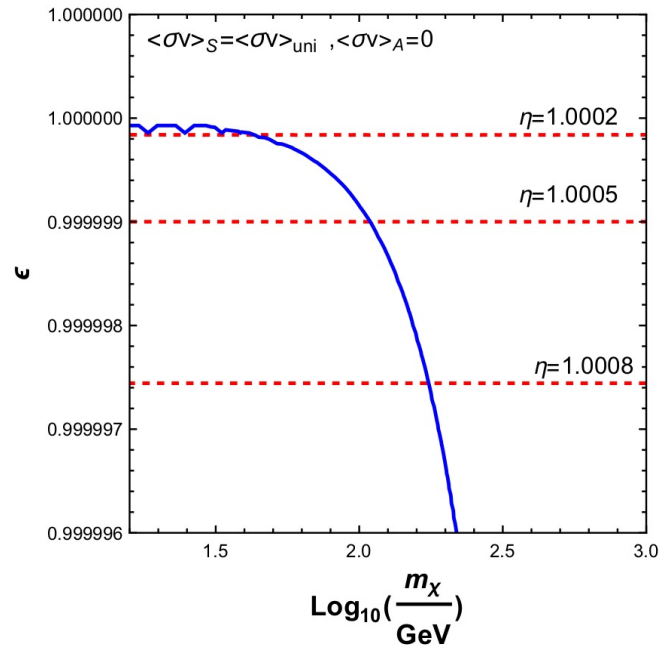
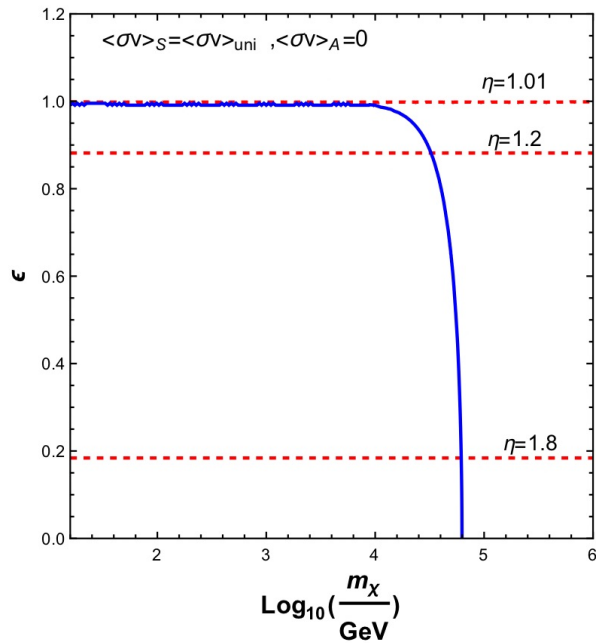
Present day relative abundance

$$\eta = \frac{Y_X(\infty) + Y_{X^+}(\infty)}{Y_{X^+}(\infty)} \quad \text{for } \epsilon > 0, X^+ \text{ survives}$$

Completely asymmetric limit : $\eta \rightarrow 1$

Completely symmetric limit : $\eta \rightarrow 2$

What is the maximum DM mass allowed?



Use the unitarity upper limit on $\langle\sigma v\rangle_S$

$$= \frac{4\pi}{m_\chi^2} \left(\frac{x_F}{\pi} \right)^{\frac{1}{2}}$$

$$x_F = \frac{m_\chi}{T_F}$$

$\epsilon \rightarrow 0$: purely symmetric semi-annihilating DM
 $m_\chi^{\text{max.}} \simeq 80 \text{ TeV}$

$\epsilon \rightarrow 1$: $m_\chi^{\text{max}} \simeq 15 \text{ GeV}$

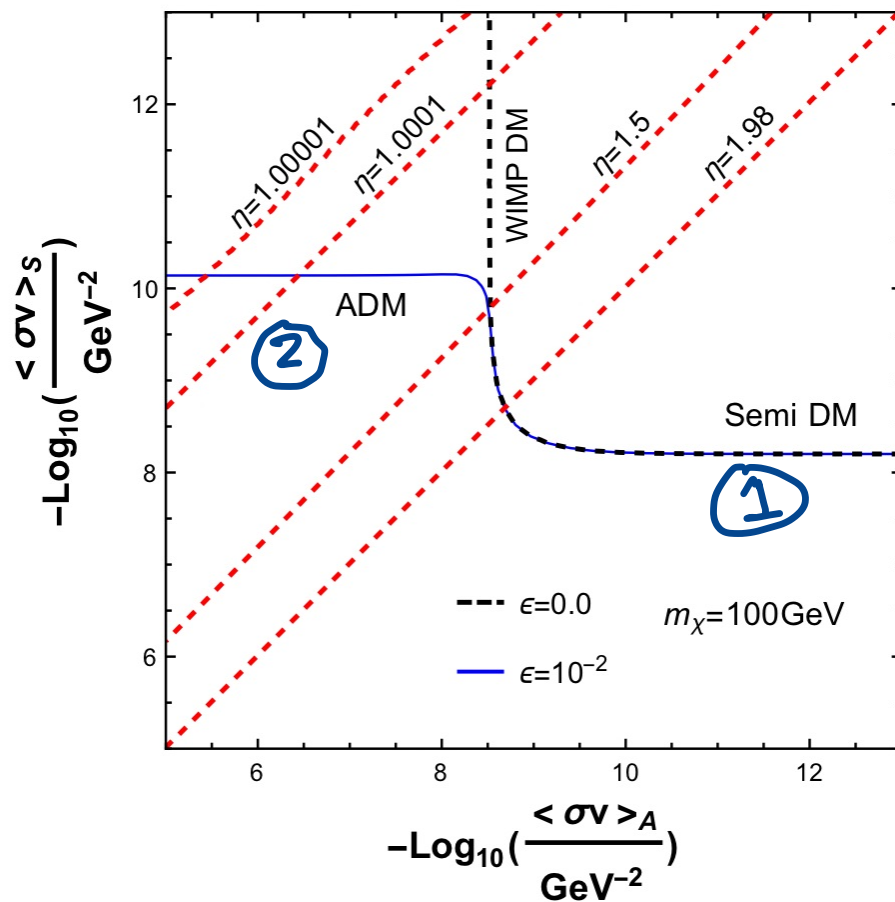
Much stronger than ADM scenarios with subsequent pair annihilation.

What if additional pair-annihilations are present?

The Boltzmann equations are modified: **interplay of the processes**

$$\frac{dY_\chi}{dx} = -\frac{s}{Hx} \left[A_S \left(Y_\chi^2 + \frac{Y_0 Y_\chi}{2} \right) - B_S \left(\frac{Y_{\chi^\dagger}^2}{2} + Y_0 Y_{\chi^\dagger} \right) + \langle \sigma v \rangle_A (Y_\chi Y_{\chi^\dagger} - Y_0^2) \right]$$

$$\frac{dY_{\chi^\dagger}}{dx} = -\frac{s}{Hx} \left[B_S \left(Y_{\chi^\dagger}^2 + \frac{Y_0 Y_{\chi^\dagger}}{2} \right) - A_S \left(\frac{Y_\chi^2}{2} + Y_0 Y_\chi \right) + \langle \sigma v \rangle_A (Y_\chi Y_{\chi^\dagger} - Y_0^2) \right]$$



Phases of DM:

$\epsilon = 0 \Rightarrow$ no asymmetry
 typical WIMP-like rates
 ($\sim 10^{-8} \text{ GeV}^{-2}$) for both
 semi and pair annihilation

$\epsilon \neq 0$: ① Symmetric phase:
 $\langle \sigma v \rangle_A$ small, $\eta \rightarrow 2$

② Asymmetric phase:
 $\langle \sigma v \rangle_A$ large, $\eta \rightarrow 1$

This can be realised in a simple scenario with the following interaction Lagrangian

$$\mathcal{L} \supset \frac{1}{3!} (\underline{\mu\chi^3} + \text{h.c.}) + \frac{1}{3!} (\underline{\lambda\chi^3\phi} + \text{h.c.}) + \frac{\lambda_1}{4} (\chi^\dagger\chi)^2 + \frac{\lambda_2}{2} \phi^2 \chi^\dagger\chi + \mu_1 \phi \chi^\dagger\chi + \frac{\mu_2}{3!} \phi^3 + \frac{\lambda_3}{4!} \phi^4. \quad (4.1)$$

μ, λ can be complex; redefine χ to make μ real

χ : complex scalar with charge ω under a Z_3 symmetry ($\omega^3 = 1$)

ϕ : real SM-singlet scalar, Z_3 even.

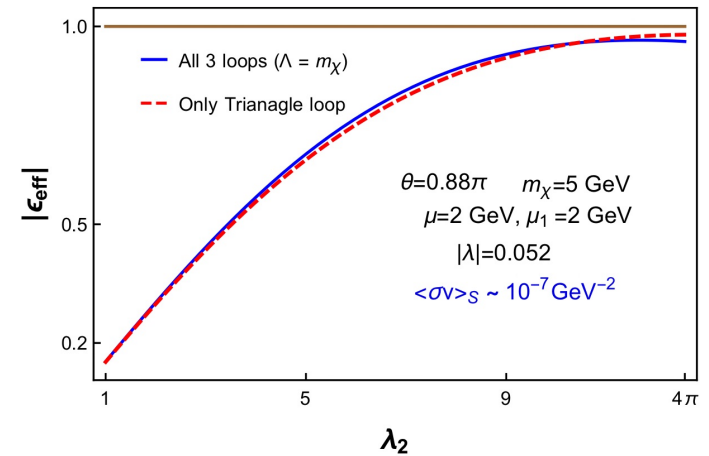
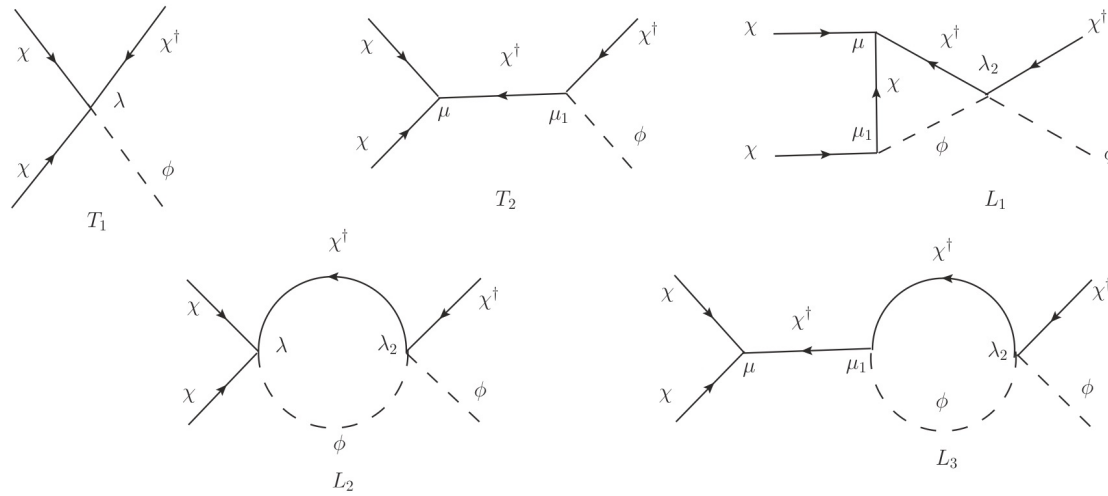
There can be additional interactions with the SM Higgs doublet

$$\lambda_{H\chi} (\chi^\dagger\chi |H|^2) + \lambda_{H\phi} (\phi^2 |H|^2) + \mu_{H\phi} (\phi |H|^2)$$

$\lambda_{H\chi} \rightarrow 0$ (to avoid direct detection; cosmology similar to $\phi^2 \chi^\dagger\chi$ term)

can thermalize ϕ with the SM sector; $\mu_{H\phi}$ small to avoid direct detection

The processes of our interest are



For $\frac{\mu}{m_\chi}, \frac{\mu_1}{m_\chi} \ll 1$, $2 \rightarrow 2$ processes dominate

$\epsilon_{\text{eff}} \equiv \frac{\langle \epsilon \sigma v \rangle}{\langle \sigma v \rangle}$
 $\epsilon_{\text{eff}} \rightarrow 1$ within perturbative couplings

The interference of the tree and loop amplitudes can lead to CP violation characterized by

$$|M|_{\chi\chi \rightarrow \chi^\dagger\phi}^2 - |M|_{\chi^\dagger\chi^\dagger \rightarrow \chi\phi}^2 = \frac{4|\lambda|\mu\mu_1\lambda_2 \sin\theta}{16\pi\sqrt{s(s-4m_\chi^2)}} \log \left[\frac{m_\chi^2 + m_\phi^2 - s + \beta_1}{m_\chi^2 + m_\phi^2 - s - \beta_1} \right]$$

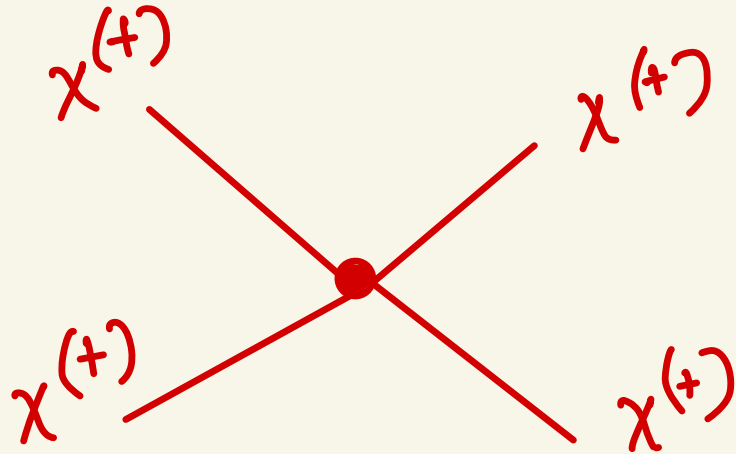
$$\beta_1 = \sqrt{\frac{(s-4m_\chi^2)(m_\chi^4 + (m_\phi^2 - s)^2 - 2m_\chi^2(s + m_\phi^2))}{s}}$$

$$\theta = \arg(\lambda)$$

From T_i and L_i interference

PART-2 : Interplay of self-scattering and annihilation in cosmology

Usually, the following self-scattering topologies are not relevant in determining the DM density and composition, simply because they cannot change the net DM number



$$\Delta n(\chi + \chi^\dagger) = 0$$

DM - anti DM self-scattering
or conversion.

However, in the presence of asymmetries, they can play a role in determining both the DM density and composition.

We shall show this by taking a simple example: the most general model for a complex scalar, where the Lagrangian breaks the $U(1)$ symmetry of DM number, but preserves the DM stabilising Z_2 symmetry

With a complex scalar odd under a \mathbb{Z}_2 symmetry:

$$\mathcal{L} \supset (\partial_\mu \chi)^\dagger \partial^\mu \chi - m_\chi^2 \chi^\dagger \chi - \frac{1}{2} (\tilde{\mu}^2 \chi^2 + \text{h.c.}) - \lambda_{\chi H} |\chi|^2 |H|^2,$$

constrained by direct detection
take $\lambda_{\chi H} \simeq 0$

breaks $U(1)_\chi$ (charge $\chi: +1, \chi^\dagger: -1$)

Take $|\tilde{\mu}|^2 \ll \frac{m_\chi^2}{M_{Pl}}$: mass and charge eigenstates essentially same during the cosmological evolution timescale

Particle-antiparticle oscillations may occur in late epochs if $\tilde{\mu}^2$ is sufficiently large. (Buckley, Profumo, 2012)

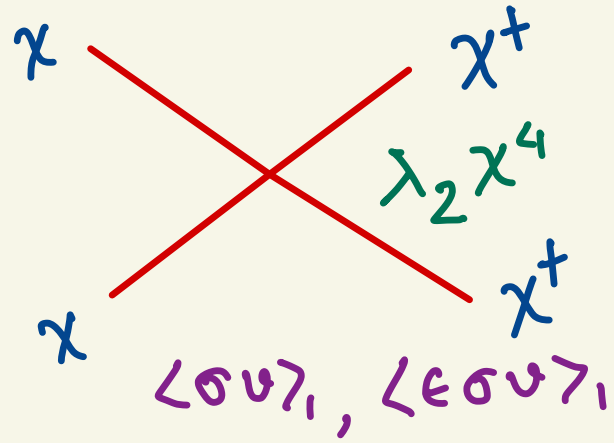
Need extra dof for thermalisation: minimal addition of a real SM singlet scalar ϕ :

constrained by direct detection: set $\mu \simeq 0, \tilde{\mu} \simeq 0$

$$-\mathcal{L}_{\text{int}} \supset \mu \chi^\dagger \chi \phi + \left(\frac{\mu_1}{2} \chi^2 \phi + \text{h.c.} \right) + \frac{\lambda_1}{4} (\chi^\dagger \chi)^2 + \left(\frac{\lambda_2}{4!} \chi^4 + \text{h.c.} \right) + \left(\frac{\lambda_3}{4} \chi^2 \phi^2 + \text{h.c.} \right) \\ + \left(\frac{\lambda_4}{3!} \chi^3 \chi^\dagger + \text{h.c.} \right) + \frac{\lambda_5}{2} \phi^2 \chi^\dagger \chi + \frac{\mu_\phi}{3!} \phi^3 + \frac{\lambda_\phi}{4!} \phi^4 + \frac{\lambda_{\phi H}}{2} \phi^2 |H|^2 + \mu_{\phi H} \phi |H|^2$$

Thermalises ϕ with SM sector
constrained by Higgs data (2.2)

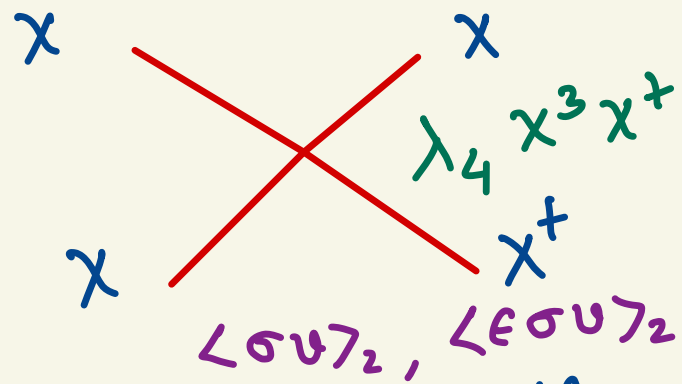
Four important types of scattering processes:



$$\Delta N_\chi = -4 \quad (\text{self-scattering with } \chi \rightarrow \chi^+ \text{ conversion})$$

$$\Delta(n_\chi + n_{\chi^+}) = 0$$

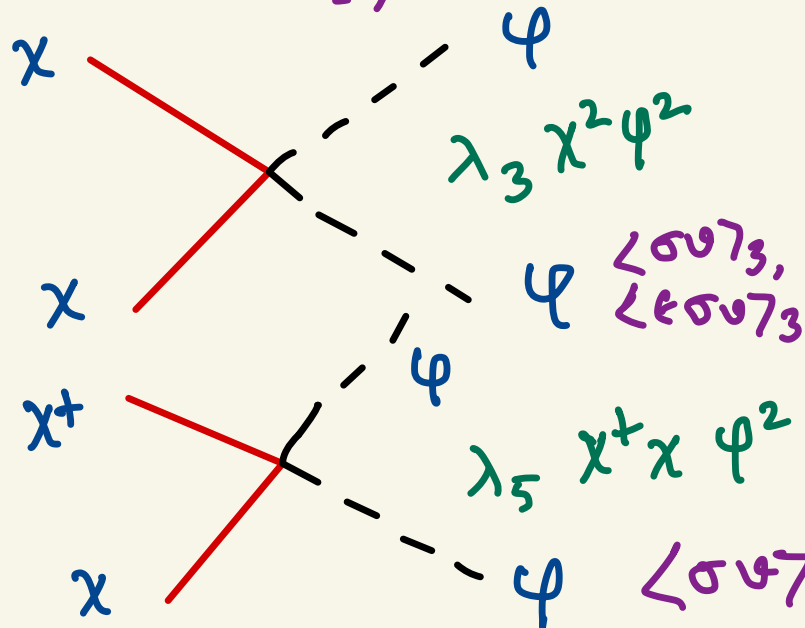
$\lambda_2 : \text{complex} \rightarrow \text{may violate CP}$



$$\Delta N_\chi = -2 \quad (\text{self-scattering with } \chi \rightarrow \chi^+ \text{ conversion})$$

$$\Delta(n_\chi + n_{\chi^+}) = 0$$

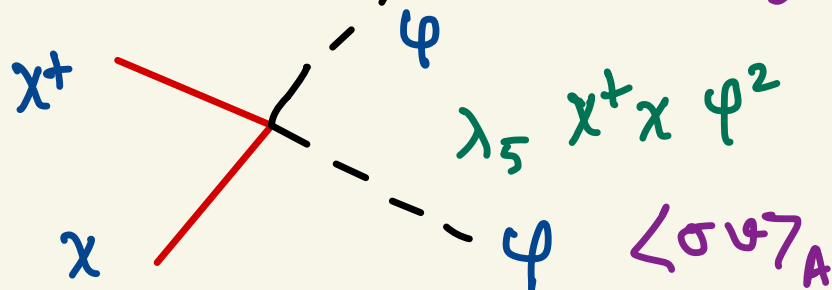
$\lambda_4 : \text{complex} \rightarrow \text{may violate CP}$



$$\Delta N_\chi = -2 \quad (\text{DM number violating annihilation})$$

$$\Delta(n_\chi + n_{\chi^+}) = -2$$

$\lambda_3 : \text{complex} \rightarrow \text{may violate CP}$



$$\Delta N_\chi = 0$$

$$\Delta(n_\chi + n_{\chi^+}) = -2$$

CP-conserving

Boltzmann Equations:

$$\begin{aligned}\frac{dY_\chi}{dx} &= -\frac{s}{2Hx} \left[\left(\langle \sigma v \rangle_1 + \frac{\langle \sigma v \rangle_2}{2} + \langle \sigma v \rangle_3 \right) (Y_\chi^2 - Y_{\chi^\dagger}^2) + \langle \sigma v \rangle_3 (Y_{\chi^\dagger}^2 - Y_0^2) \right. \\ &\quad \left. + \underline{\langle \epsilon \sigma v \rangle_1} (Y_{\chi^\dagger}^2 - Y_0^2) + \underline{\langle \epsilon \sigma v \rangle_2} \left(Y_{\chi^\dagger} Y_\chi + \frac{Y_{\chi^\dagger}^2}{2} - \frac{Y_\chi^2}{2} - Y_0^2 \right) + 2 \langle \sigma v \rangle_A (Y_{\chi^\dagger} Y_\chi - Y_0^2) \right] \\ \frac{dY_{\chi^\dagger}}{dx} &= -\frac{s}{2Hx} \left[\left(\langle \sigma v \rangle_1 + \frac{\langle \sigma v \rangle_2}{2} + \langle \sigma v \rangle_3 \right) (Y_{\chi^\dagger}^2 - Y_\chi^2) + \langle \sigma v \rangle_3 (Y_\chi^2 - Y_0^2) \right. \\ &\quad \left. - \underline{\langle \epsilon \sigma v \rangle_1} (Y_\chi^2 - Y_0^2) - \underline{\langle \epsilon \sigma v \rangle_2} \left(Y_{\chi^\dagger} Y_\chi + \frac{Y_\chi^2}{2} - \frac{Y_{\chi^\dagger}^2}{2} - Y_0^2 \right) + 2 \langle \sigma v \rangle_A (Y_{\chi^\dagger} Y_\chi - Y_0^2) \right].\end{aligned}$$

Asymmetric Thermal Averages:

$$\langle \epsilon \sigma v \rangle_f = \frac{\int \prod_{i=1}^4 \frac{d^3 p_i}{(2\pi)^3 2E_{p_i}} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \epsilon_f(p_i) |M_0|_f^2 f_0(p_1) f_0(p_2)}{\int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} f_0(p_1) f_0(p_2)}$$

$$|M_0|_f^2 = |M|_{\chi\chi \rightarrow f}^2 + |M|_{\chi^\dagger\chi^\dagger \rightarrow f^\dagger}^2$$

$$\epsilon_f(p_i) = \frac{|M|_{\chi\chi \rightarrow f}^2 - |M|_{\chi^\dagger\chi^\dagger \rightarrow f^\dagger}^2}{|M|_{\chi\chi \rightarrow f}^2 + |M|_{\chi^\dagger\chi^\dagger \rightarrow f^\dagger}^2}.$$

CP violation in final state f

Parametrization: CP-violating

source terms - $\langle \epsilon \sigma v \rangle_1$ and $\langle \epsilon \sigma v \rangle_2$.

$\langle \epsilon \sigma v \rangle_3$ eliminated (not independent) \rightarrow Unitarity sum rules

$$\langle \epsilon \sigma v \rangle_1 + \langle \epsilon \sigma v \rangle_2 + \langle \epsilon \sigma v \rangle_3 = 0,$$

Key relation to understand the interplay of processes

Unitarity Sum Rules: amplitudes for CP violating processes are related by S-matrix unitarity

→ Sum over final states f from $\chi\chi$ initial state

$$\sum_f \int dPS_f |M|_{\chi\chi \rightarrow f}^2 = \sum_f \int dPS_f |M|_{f \rightarrow \chi\chi}^2 = \sum_f \int dPS_f |M|_{\chi^\dagger \chi^\dagger \rightarrow f^\dagger}^2,$$

Sum over momenta and discrete labels in f .

S-matrix unitarity

CPT

This implies

with $f = \{\chi^\dagger \chi^\dagger, \chi \chi^\dagger, \psi \psi\}$

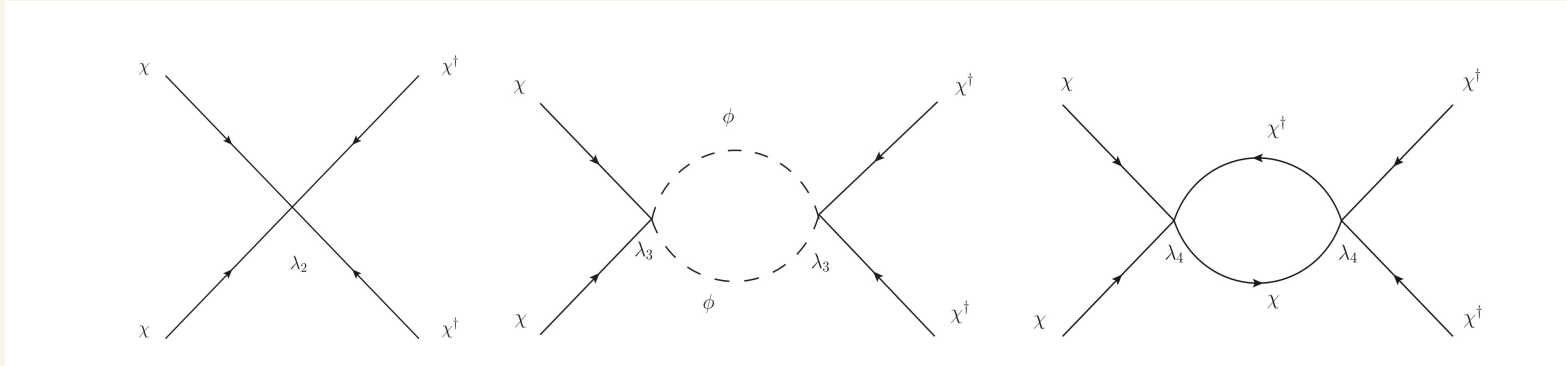
$$\sum_f \int dPS_f \epsilon_f |M_0|_f^2 = 0, \Rightarrow \langle \epsilon \sigma v \rangle_1 + \langle \epsilon \sigma v \rangle_2 + \langle \epsilon \sigma v \rangle_3 = 0,$$

Hence the CP-violation in all three channels are related.

We cannot have a CP-violating annihilation without the CP-violating self-scatterings of comparable magnitude.

Rates of processes:

① self-scattering: $\chi + \chi \rightarrow \chi^\dagger + \chi^\dagger$



$$\epsilon'_{\text{eff}} \equiv \frac{\langle \epsilon \sigma v \rangle_1}{\langle \sigma v \rangle_1}$$

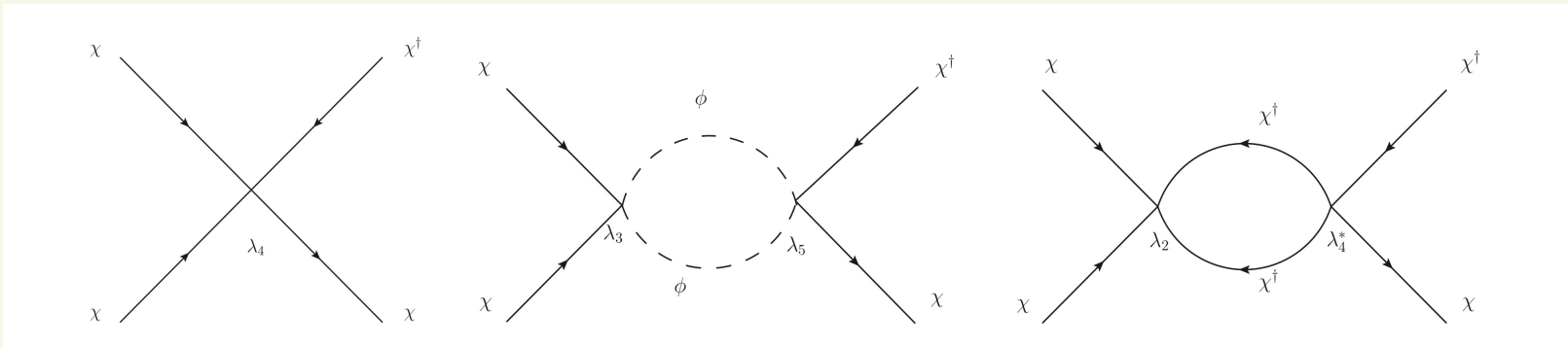
$$\simeq \frac{1}{16\pi} \text{ for } x \gg 1$$

with $\lambda_i \simeq \mathcal{O}(1)$
 $\arg(\lambda_2^* \lambda_3^2) = \pi/2$

$$\langle \epsilon \sigma v \rangle_1 = \frac{1}{(16\pi m_\chi)^2} \left[\frac{1}{\sqrt{\pi x}} \text{Im}(\lambda_2^* \lambda_3^2) + \frac{3}{2x} \text{Im}(\lambda_2^* \lambda_4^2) \right].$$

$$\langle \sigma v \rangle_1 = \frac{|\lambda_2|^2}{16\pi m_\chi^2} \frac{1}{\sqrt{\pi x}}, \quad \leftarrow \text{velocity dependent}$$

② Self-scattering: $\chi + \chi \rightarrow \chi^\dagger + \chi$



$$\epsilon_{\text{eff}}^2 \simeq \frac{\lambda_5}{16\pi}$$

for $x \gg 1$
 all other $\lambda_i \sim \mathcal{O}(1)$
 For $\lambda_5 \sim \mathcal{O}(4\pi)$

$$\langle \epsilon \sigma v \rangle_2 = \frac{1}{(16\pi m_\chi)^2} \left[\frac{3}{2x} \text{Im}(\lambda_2 \lambda_4^{*2}) + \frac{2}{\sqrt{\pi x}} \text{Im}(\lambda_4^* \lambda_3) \lambda_5 \right].$$

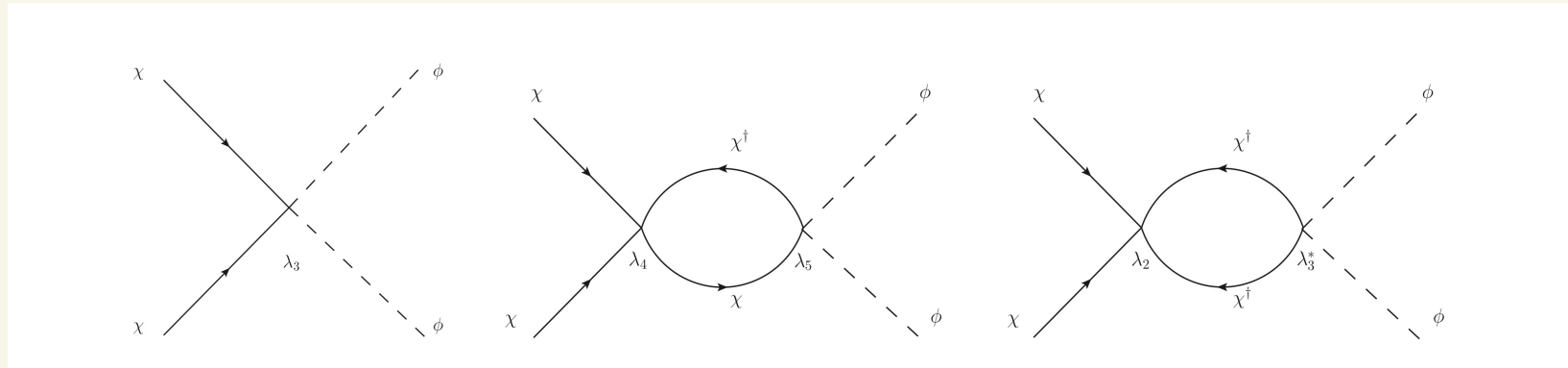
$$\langle \sigma v \rangle_2 = \frac{|\lambda_4|^2}{8\pi m_\chi^2} \frac{1}{\sqrt{\pi x}}$$

$$\epsilon_{\text{eff}}^2 \simeq 0.25$$

$\leftarrow v$ -dependent

Rates of processes:

③ CP -violating annihilations : $\chi + \chi \rightarrow \varphi + \varphi$



$$\epsilon_{\text{eff}}^3 \simeq -\frac{1}{8\pi} \times \frac{1+2\lambda_5}{\sqrt{\pi x}} \quad \text{for } x \gg 1$$

Other $\lambda_i \sim \mathcal{O}(1)$

$$\langle \epsilon \sigma v \rangle_3 = \frac{1}{(16\pi m_\chi)^2} \frac{1}{\sqrt{\pi x}} \left[\text{Im}(\lambda_2 \lambda_3^{*2}) + 2 \text{Im}(\lambda_3^* \lambda_4) \lambda_5 \right].$$

$$\langle \sigma v \rangle_3 = \frac{|\lambda_3|^2}{32\pi m_\chi^2} \left(1 + \frac{3}{4x} \right).$$

The unitarity sum rule can be explicitly checked at this order in perturbation theory:

$$\langle \epsilon \sigma v \rangle_1 + \langle \epsilon \sigma v \rangle_2 + \langle \epsilon \sigma v \rangle_3 = 0$$

The solutions to the rate equations: Define $Y_S = Y_x + Y_{x^+}$
 $Y_{\Delta x} = Y_x - Y_{x^+}$

$$\langle \sigma v \rangle_{\text{ann}} = 2\langle \sigma v \rangle_1 + \langle \sigma v \rangle_2 + \langle \sigma v \rangle_3$$

$$\begin{aligned} \frac{dY_S}{dx} &= -\frac{s}{2Hx} \left[\langle \sigma v \rangle_A (Y_S^2 - Y_{\Delta x}^2 - 4Y_0^2) + \langle \sigma v \rangle_3 \left(\frac{Y_S^2 + Y_{\Delta x}^2 - 4Y_0^2}{2} \right) - \langle \epsilon \sigma v \rangle_S Y_S Y_{\Delta x} \right] \\ \frac{dY_{\Delta x}}{dx} &= -\frac{s}{2Hx} \left[\langle \epsilon \sigma v \rangle_S \left(\frac{Y_S^2 - 4Y_0^2}{2} \right) + \langle \epsilon \sigma v \rangle_D \frac{Y_{\Delta x}^2}{2} + \langle \sigma v \rangle_{\text{all}} Y_S Y_{\Delta x} \right]. \end{aligned} \quad (3.7)$$

$$\begin{aligned} \langle \epsilon \sigma v \rangle_S &= \langle \epsilon \sigma v \rangle_1 + \langle \epsilon \sigma v \rangle_2 \leftarrow \text{Source terms for the asymmetry } Y_{\Delta x} \\ \langle \epsilon \sigma v \rangle_D &= \langle \epsilon \sigma v \rangle_1 - \langle \epsilon \sigma v \rangle_2 \end{aligned}$$

Approximate analytical solutions:

$$Y_S(x) = 2Y_0 \left[1 + \frac{Hx}{sY_0} \frac{\langle \sigma v \rangle_{\text{all}}}{2 \langle \sigma v \rangle_{\text{ann}} \langle \sigma v \rangle_{\text{all}} + \langle \epsilon \sigma v \rangle_S^2} \right] \quad (\text{for } 1 \leq x \leq x_F),$$

$$|Y_{\Delta x}(x)| = \frac{2Hx}{s} \frac{\langle \epsilon \sigma v \rangle_S}{2 \langle \sigma v \rangle_{\text{ann}} \langle \sigma v \rangle_{\text{all}} + \langle \epsilon \sigma v \rangle_S^2}$$

Both yields
 $\propto \frac{1}{\langle \sigma v \rangle_{\text{ann}}}$

$$\langle \sigma v \rangle_{\text{ann}} = \langle \sigma v \rangle_A + \frac{\langle \sigma v \rangle_3}{2}$$

$$Y_{\Delta x} \propto \langle \epsilon \sigma v \rangle_1 + \langle \epsilon \sigma v \rangle_2$$

The solutions to the rate equations:

For $x > x_F$, self-scatterings that generate the asymmetry are mostly decoupled due to velocity suppression

For $\langle \sigma v \rangle_{\text{all}} < \langle \sigma v \rangle_A$, with $\langle \sigma v \rangle_A$ dominant, $Y_{\Delta\chi}$ essentially remains frozen at $Y_{\Delta\chi}(x_F)$.
dominantly s-wave $\langle \sigma v \rangle_A$:

Then, with a dual role of CP-conserving processes.

$$Y_S(x > x_F) = |Y_{\Delta\chi}^F| \frac{1 + r_F \exp \left[\lambda |Y_{\Delta\chi}^F| (x^{-1} - x_F^{-1}) \right]}{1 - r_F \exp \left[\lambda |Y_{\Delta\chi}^F| (x^{-1} - x_F^{-1}) \right]},$$

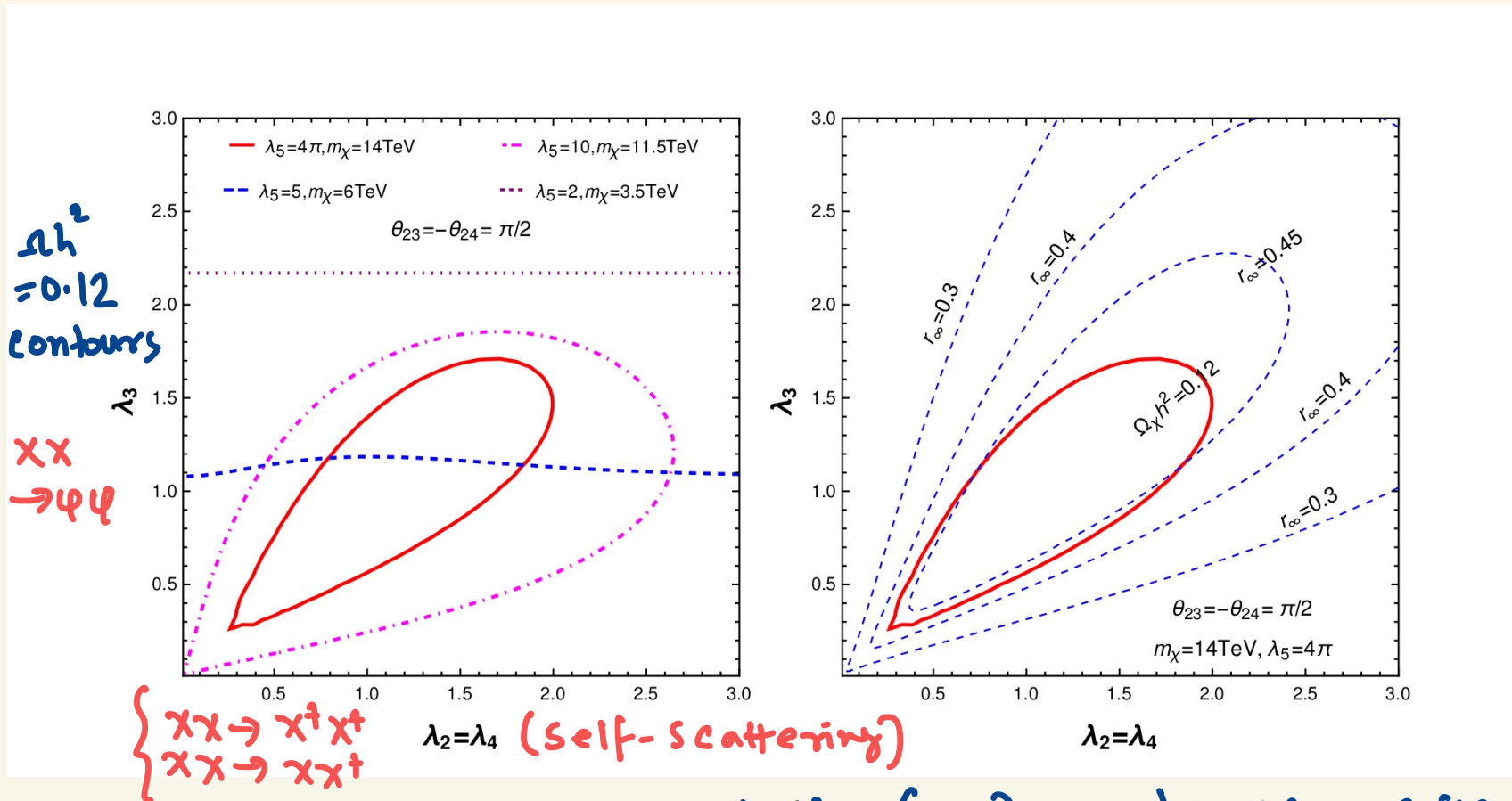
$$r_F = \frac{Y_S^F - |Y_{\Delta\chi}^F|}{Y_S^F + |Y_{\Delta\chi}^F|}.$$

↑
yields at $x = x_F$

$$\lambda = 1.32 m_\chi M_{Pl} g_*^{1/2} \langle \sigma v \rangle_A,$$

Thus $Y_S \propto |Y_{\Delta\chi}^F|$
for $x > x_F$, and s is $\Omega_\chi h^2$.

The interplay of self-scattering and annihilation:



$$\gamma_\infty = \frac{|Y_x - Y_{x^*}|}{Y_x + Y_{x^*}}$$

Y_{x,x^*} : asymptotic yields

Symmetric DM

$$\gamma_\infty = 0$$

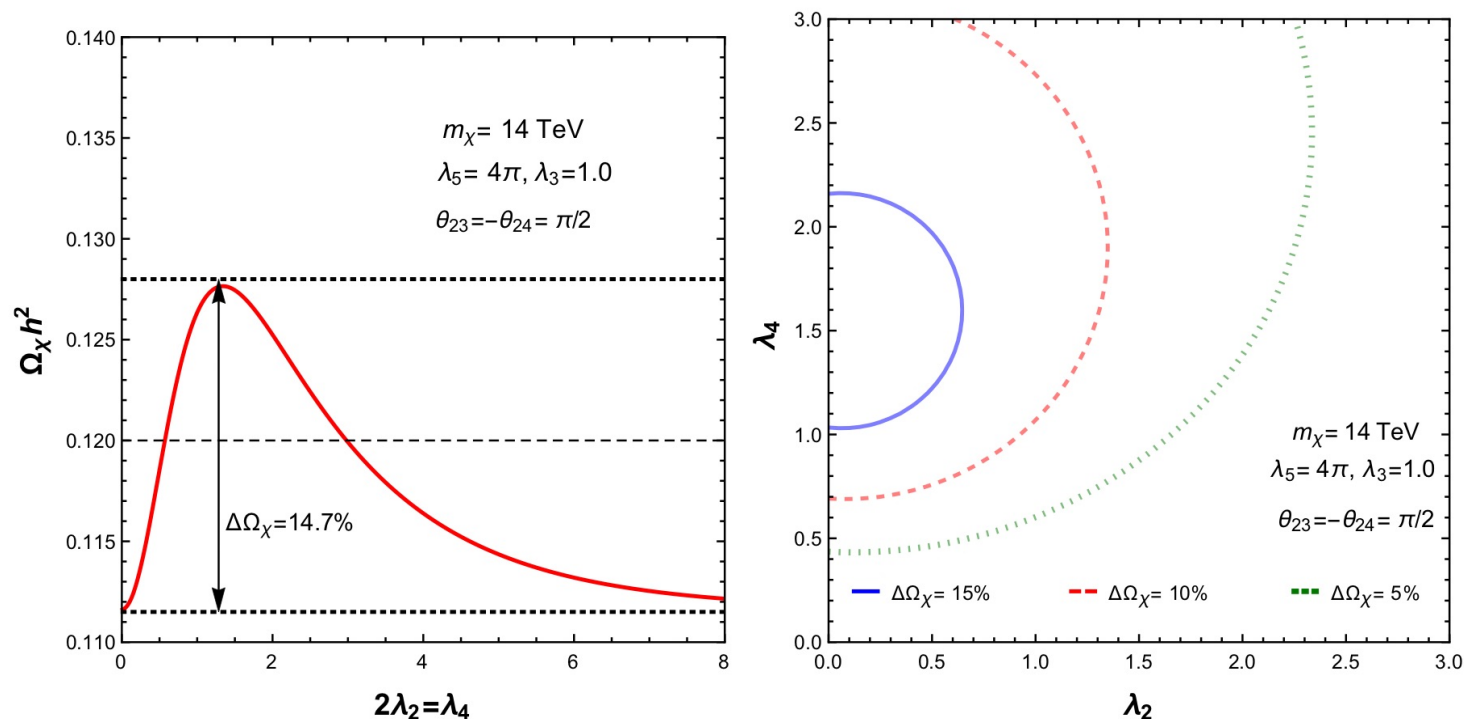
completely Asymmetric DM

$$\gamma_\infty = 1$$

- Larger pair-annihilation (λ_5) \Rightarrow larger asymmetry \Rightarrow Larger correlations

- $\gamma_\infty \simeq 0.5 \Rightarrow \frac{Y_x}{Y_{x^*}} \simeq 3.$

Self-scattering and relic abundance:



$$\Delta \Omega_\chi(\lambda_2, \lambda_4) = \frac{\Omega_\chi - \Omega_\chi(0,0)}{\Omega_\chi(0,0)}$$

For largish asymmetry
 $\Omega_\chi \propto Y_{\Delta\chi}$
 \downarrow
 determined by
 self-interactions

- The relic density can change in the presence of non-zero self-interaction
- Such a change is found for a wide range of values of the self-interaction couplings

Conclusions:

There are interesting, less explored dark matter scattering topologies which lead to novel phenomena in DM cosmology

Semi-annihilation can change DM number and reduce the net density at the same time: can lead to almost completely asymmetric DM without the need for subsequent pair annihilations

There can be a close interplay of DM self-scatterings with annihilations in deciding its present density and composition

The role of DM self-interactions in determining its density through an asymmetry is a novel effect, which should be explored in other scenarios