On 6D RG Flows

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Today:

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Next Week:

Speculations on Physical Discretization and Arithmetic Geometry

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Based On

hep-th/2103.13395 w/ Kundu and Zhang

+ in progress

As Well As:

Work with (many papers + in progress):

Apruzzi, Baume, Bhardwaj, Del Zotto, Fazzi,

Kundu, Hassler, Lawrie, Morrison, Rochais,

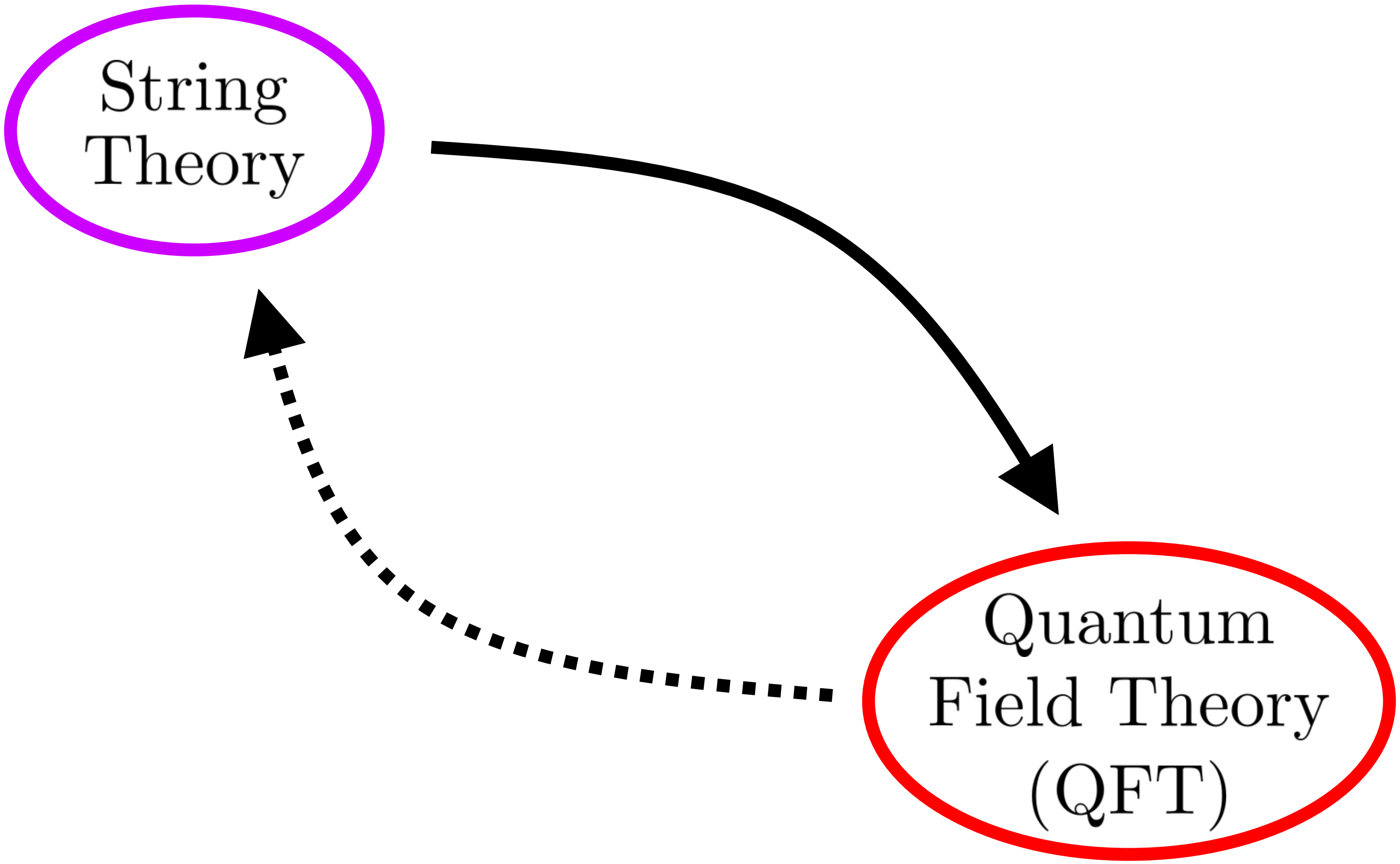
Rudelius, Tomasiello, Tizzano Vafa, Zhang

Plan of the Talk

- Motivation / Background
- Geometry of 6D SCFTs
- EFT for 6D Flows
- Conclusions / Future

Motivation / Background

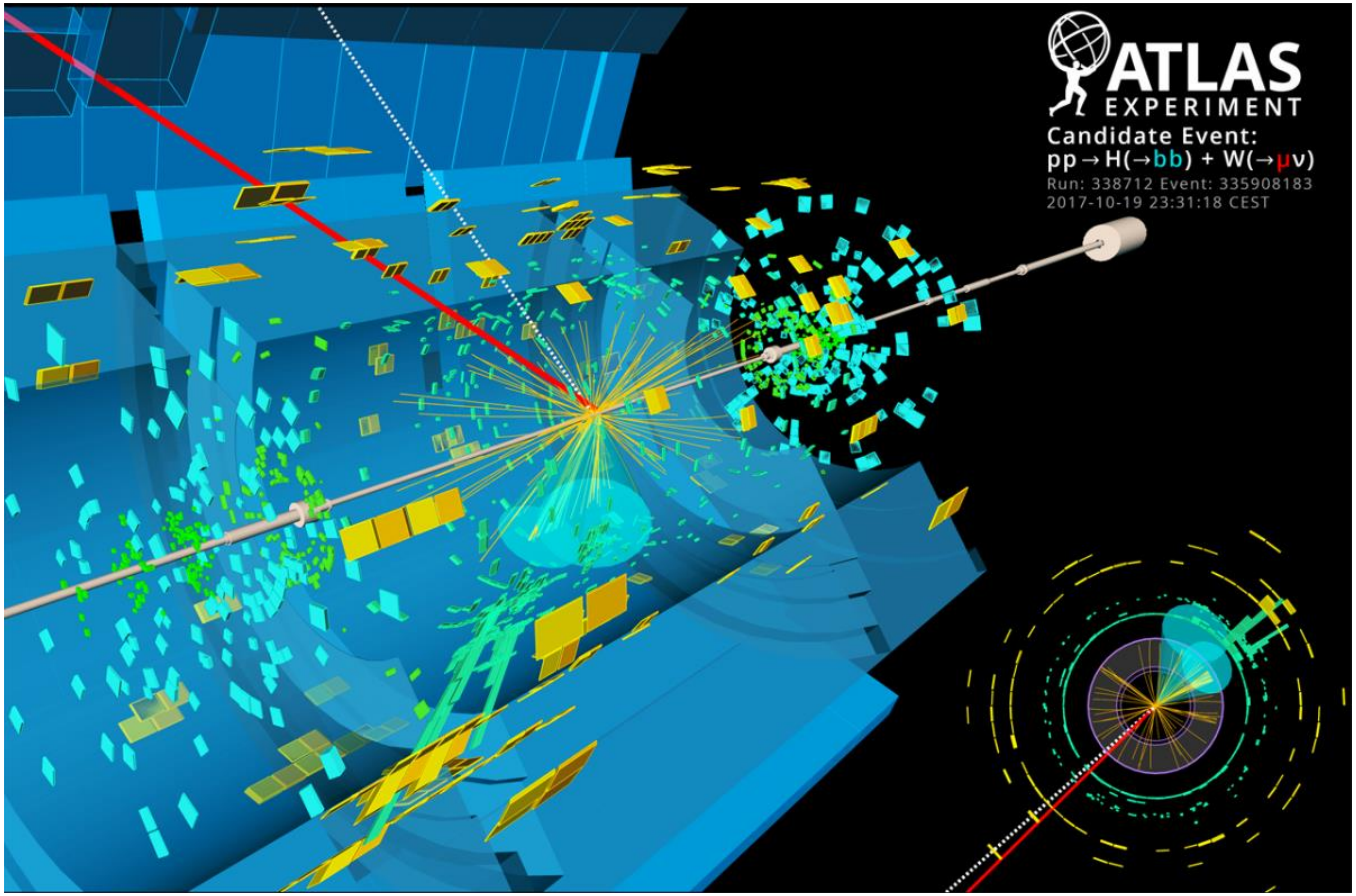
This Talk is About



Uses for QFT

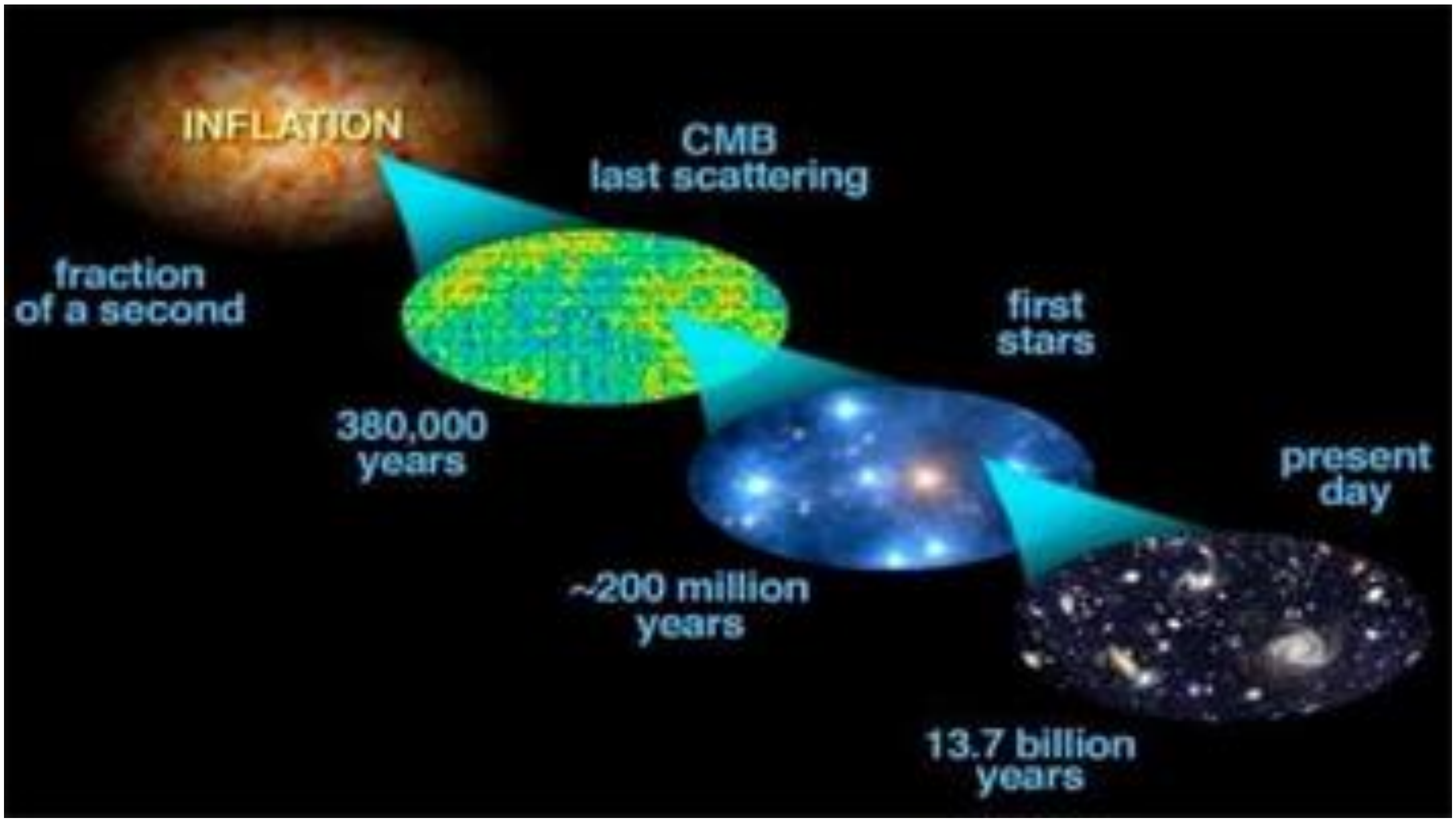
Huge Variety of Applications

Example: Particle Physics



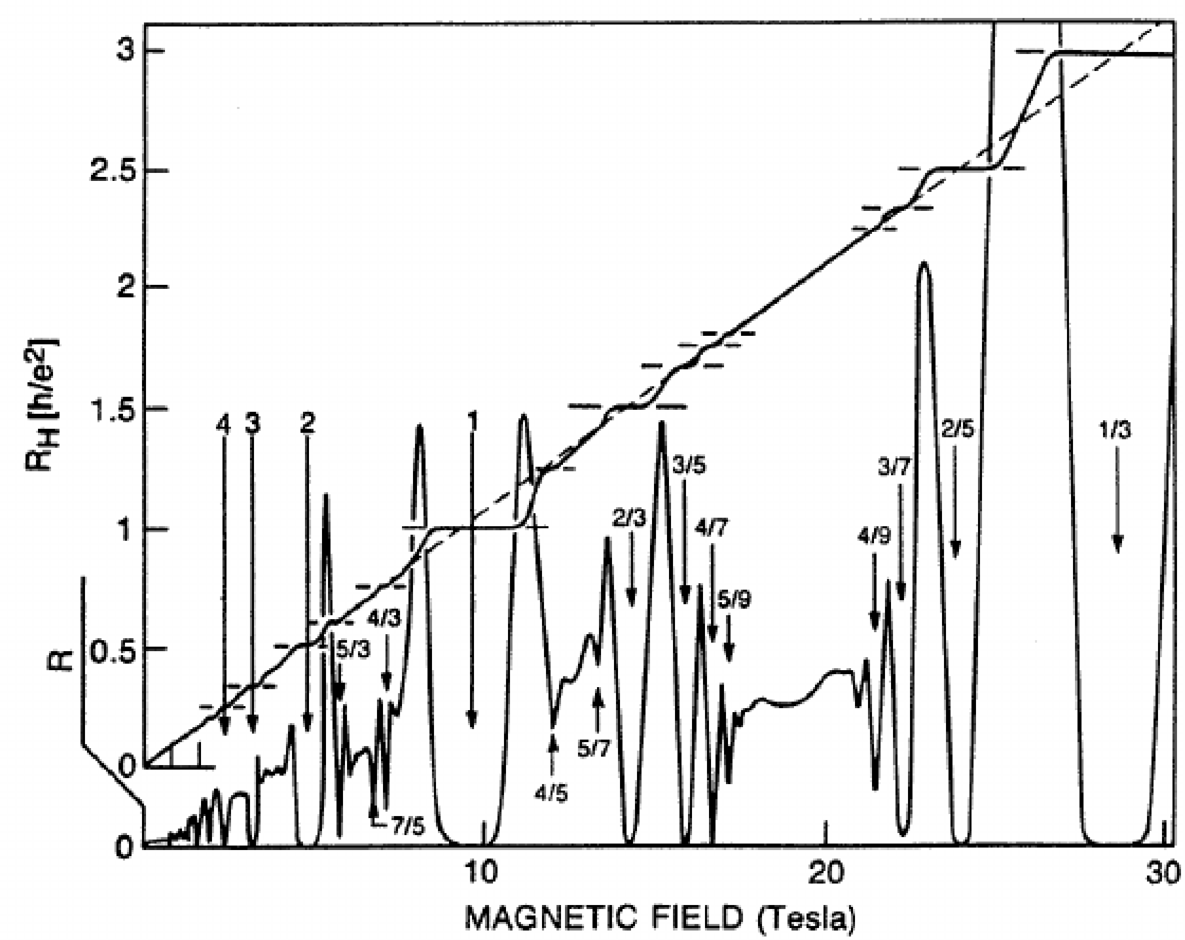
Source: ATLAS homepage

Example: Cosmology



Source: Hawking Centre for Theoretical Cosmology

Example: Condensed Matter



Source: Störmer *Physica* 1992

What Do I Mean By QFT?

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It's Quantum: States $|\Psi\rangle$ and Operators $\widehat{\mathcal{O}}$

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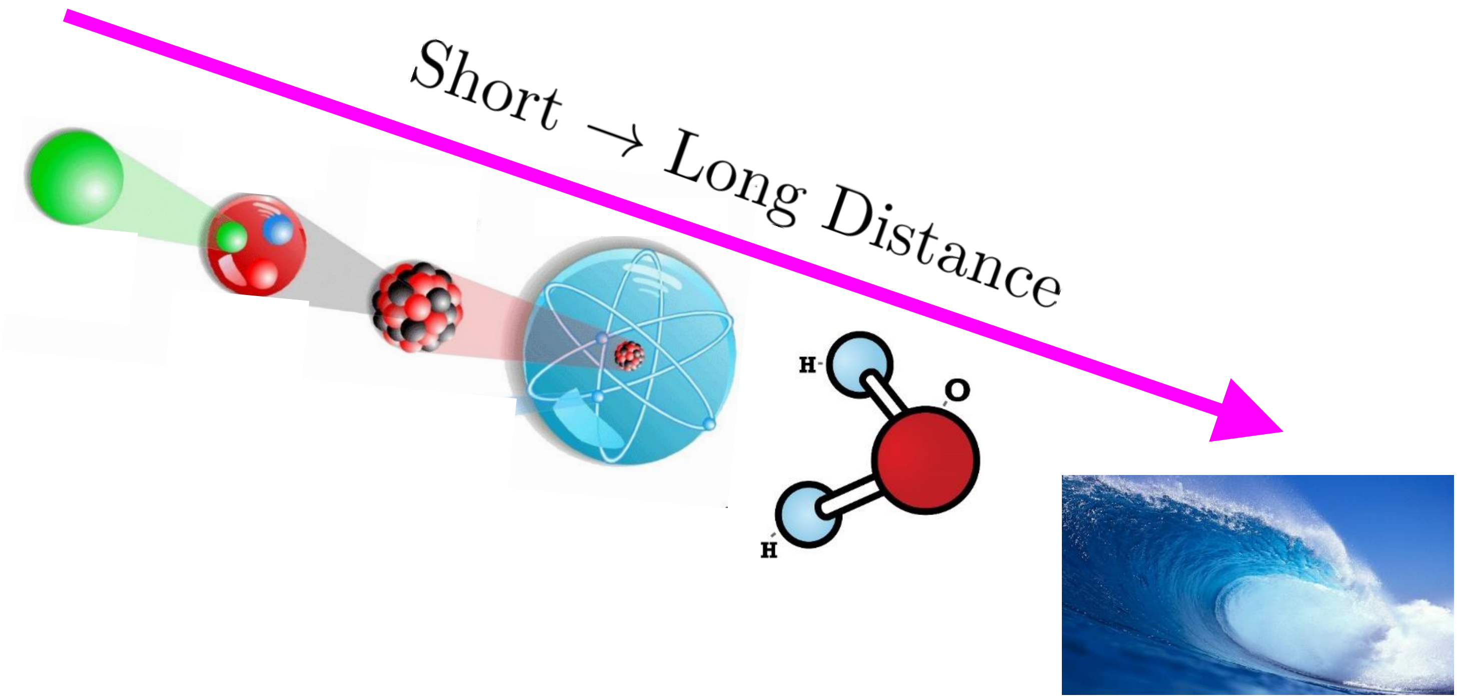
It's Local: "fields" $\widehat{\mathcal{O}}(\overrightarrow{x}, t)$
(e.g. quanta of $\overrightarrow{E}$ and $\overrightarrow{B}$ fields)

It Has Correlators: $\langle\Psi|\widehat{\mathcal{O}}_1 ... \widehat{\mathcal{O}}_m|\Psi'\rangle$

Length Scales

The states and operators of interest depend on scale

e.g. water:

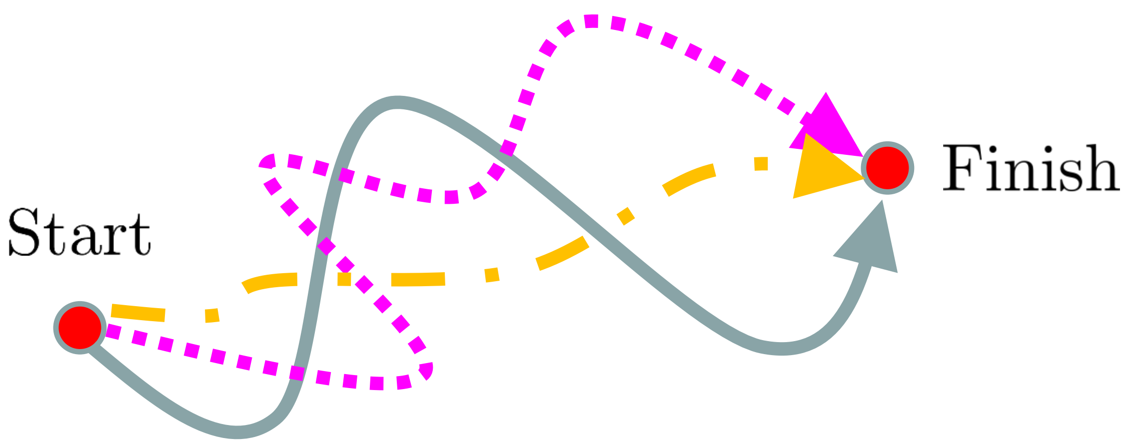


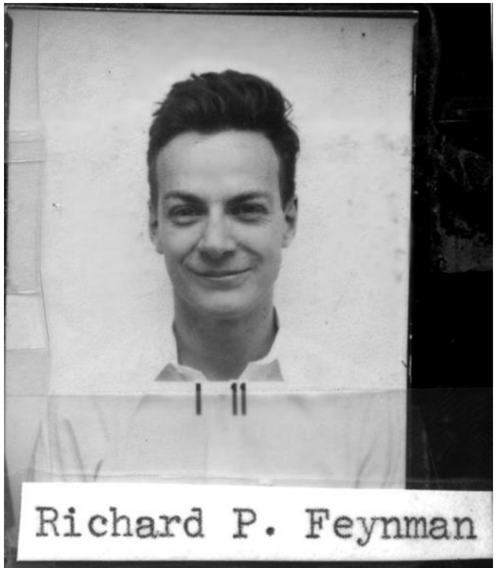
A Tool For Calculating

$$\langle \text{Finish} | \text{Start} \rangle = \sum_{\text{paths}} \exp(iS)$$

"Path Integral"

$\int dt$ Lagrangian





The Lagrangian

Useful for Classical *and* Quantum Physics!

The Lagrangian

"Action" $= S = \int dt\, L$

$L = \text{Kinetic Energy} - \text{Potential Energy}$

1D Example: $S_{\text{Harmonic Oscillator}} = \int dt\ \left(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2\right)$

Textbook Definition of QFT:
"Lots of Interacting Harmonic Oscillators"

QFT Questions

- Do all QFTs have a Lagrangian?

- Short $\rightarrow$ Long Distance?

- What can be calculated?

Surprises From Strings

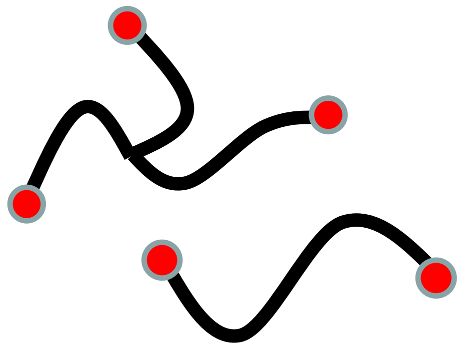
Surprise #1:

Only *tiny* fraction of QFTs have a Lagrangian!

Surprise #2:

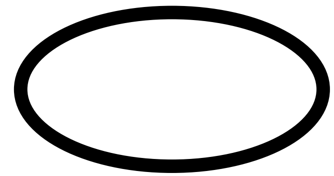
Can still calculate *even without* a Lagrangian!

What Are Strings?



Open Strings

Electrons, Quarks,...

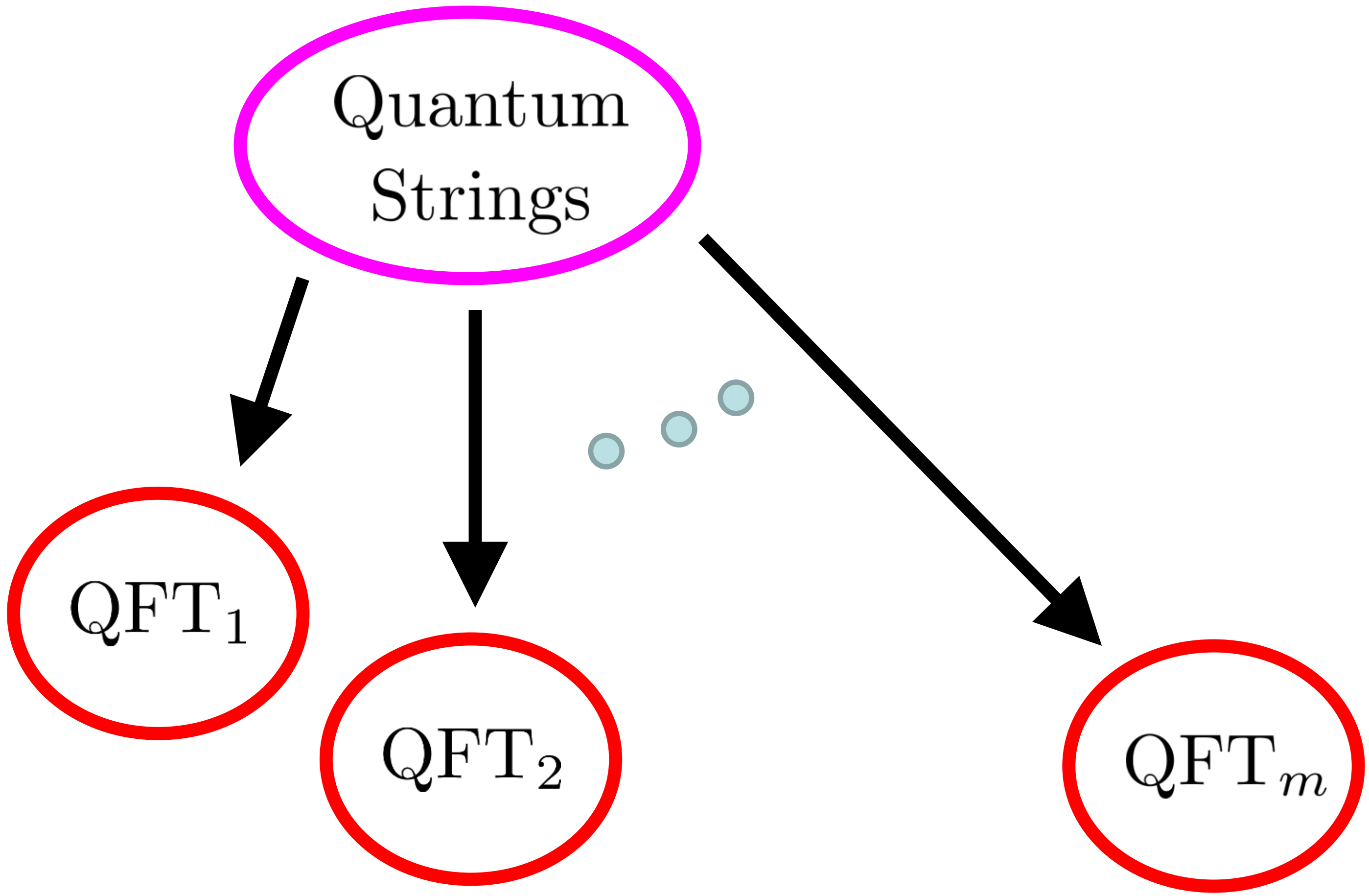


Closed Strings

Gravitons
(Quanta of Gravity)

Vibrational Modes $\Rightarrow$ Particles!

Long Distance Limits



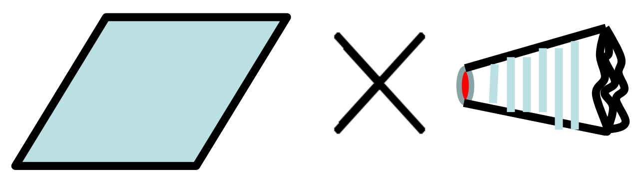
Geometry $\to$ Physics

Strings Predict: 10D $(9+1)$ spacetime

Geometry → Physics

Strings Predict: 10D $(9+1)$ spacetime

Example: 4D QFTs

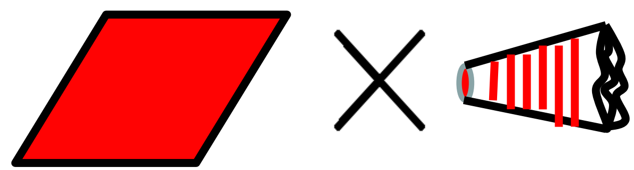


4D spacetime 6 extra

Geometry $\rightarrow$ Physics

Strings Predict: 10D $(9+1)$ spacetime

Example: 3D QFTs

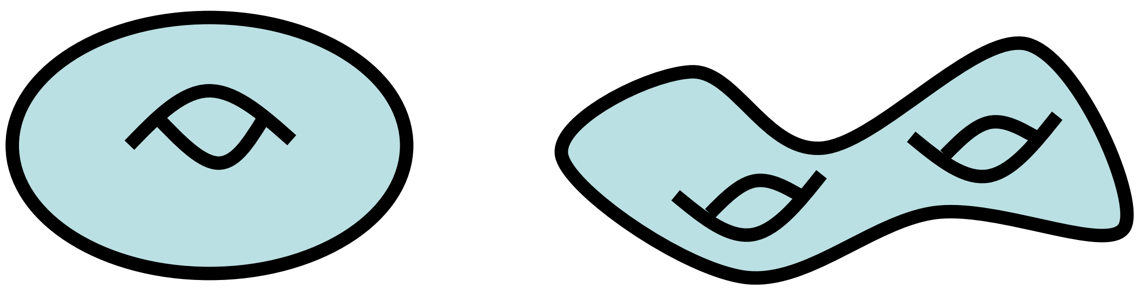


3D spacetime 7 extra

In Practice...

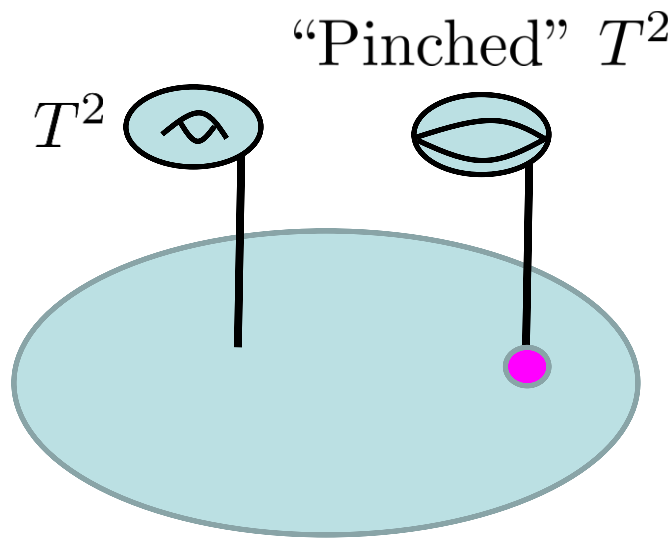
Characterize via Topology (Existence)

Examples:



In Practice...

Characterize via Topology (Existence)



More Elaborate Example:

“F-theory on elliptic Calabi-Yau”

Vafa ’96

Major advances in past 10 years

JJH et al. + many many more

Lagrangian Example(s)

- We know how to build Lagrangians for elementary particle physics

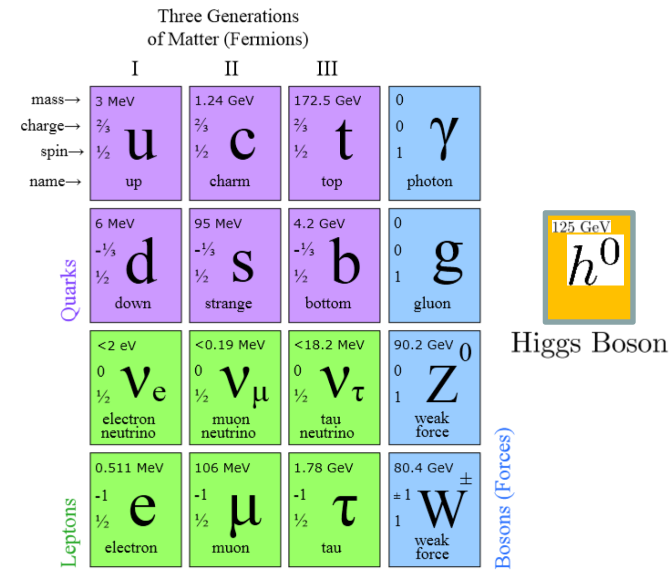
  Many groups around the world

  JJH, Vafa et al. "F-theory GUTs"

  Watari et al.

  UPenn String Pheno Groups

- + Many Many More



¿Beyond Textbook QFT?

Simplifying Assumptions

Common physics strategy: Start simple, then perturb

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- Keep Gravity “switched off”
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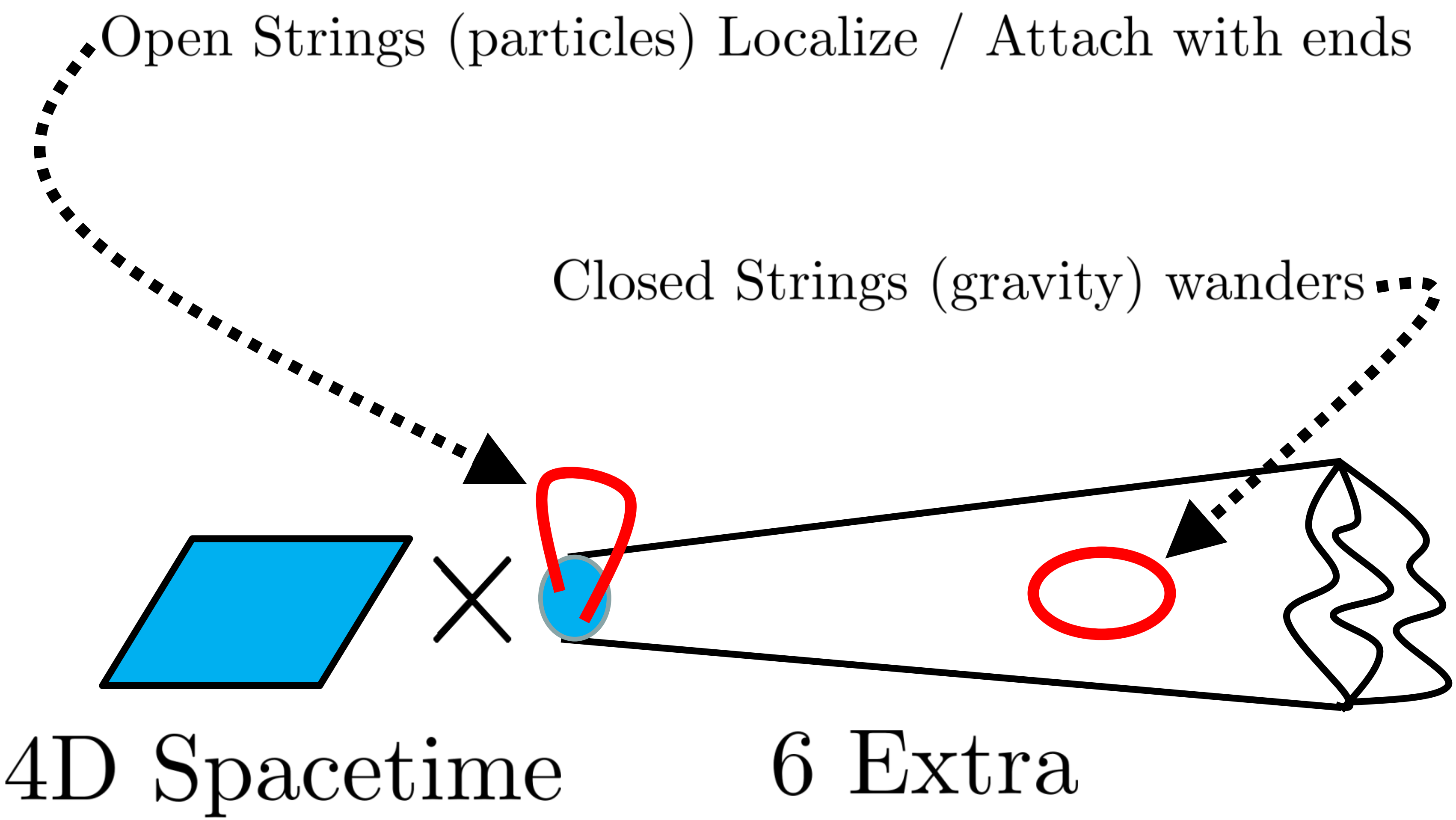
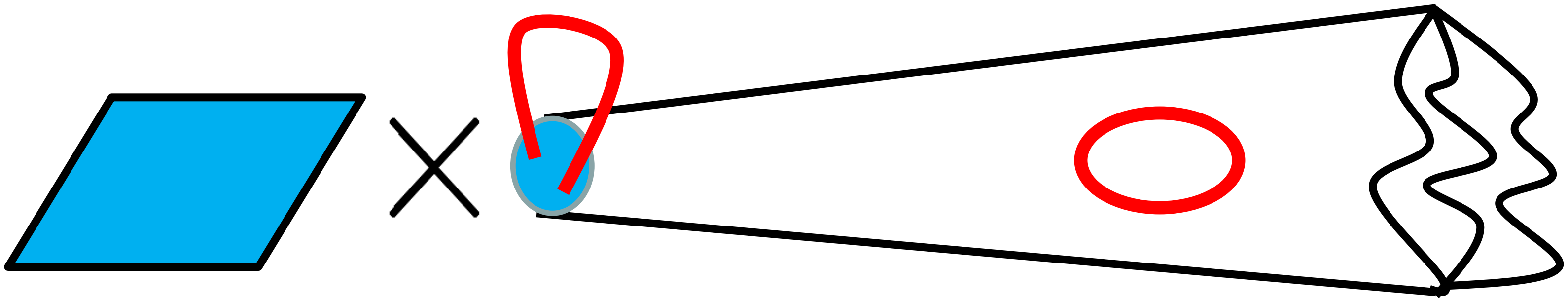
Turning Off Gravity



Turning Off Gravity

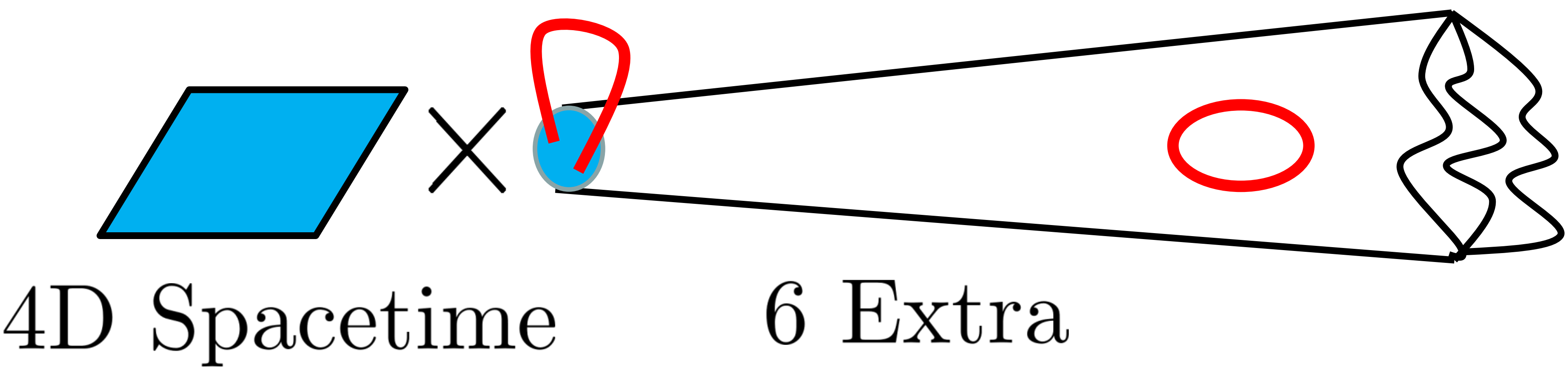
$$G_{\text{Newton}} \sim \frac{1}{\text{Vol(Extra Dimensions)}}$$



4D Spacetime 6 Extra

Turning Off G_{Newton}

$$G_{\text{Newton}} \sim \frac{1}{\text{Vol(Extra Dimensions)}} \to 0$$



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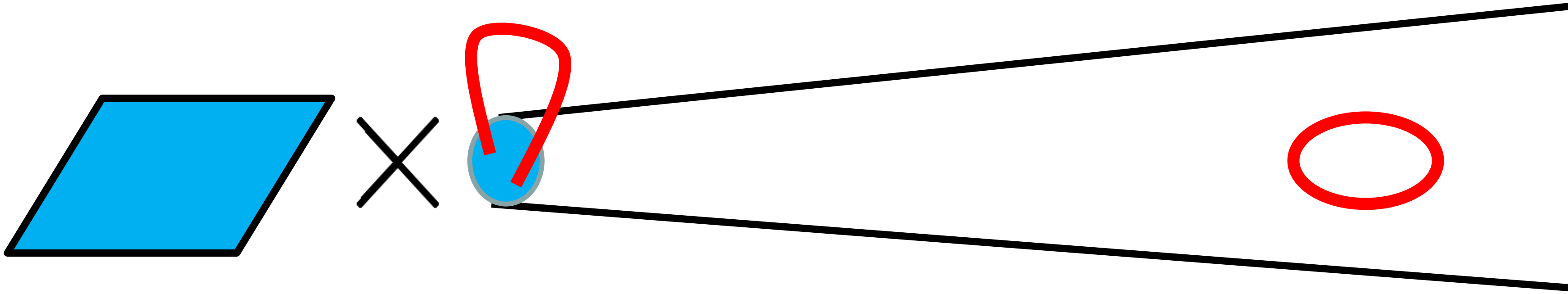

4D Spacetime

6 Extra

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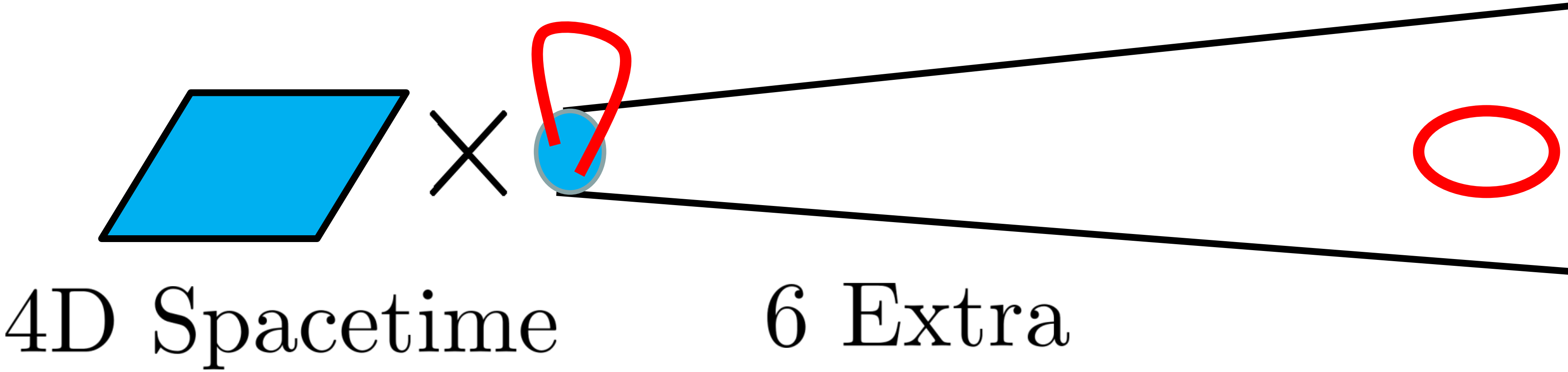

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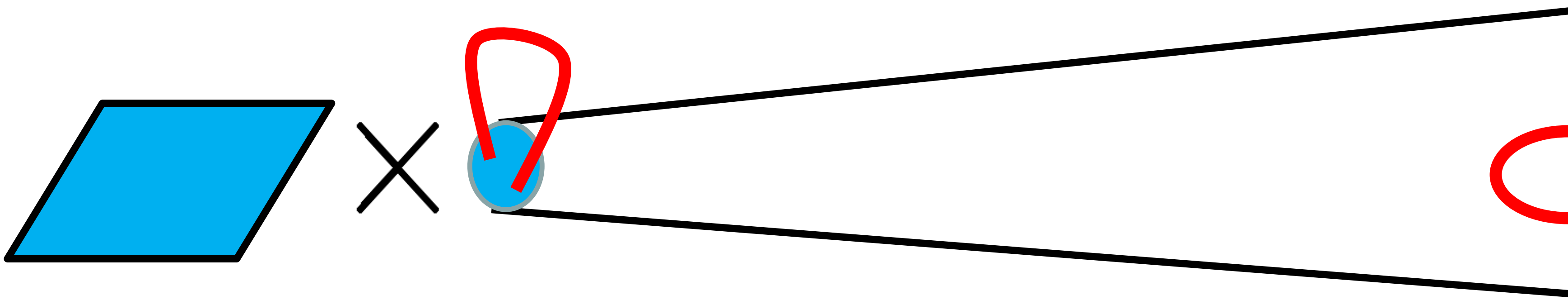

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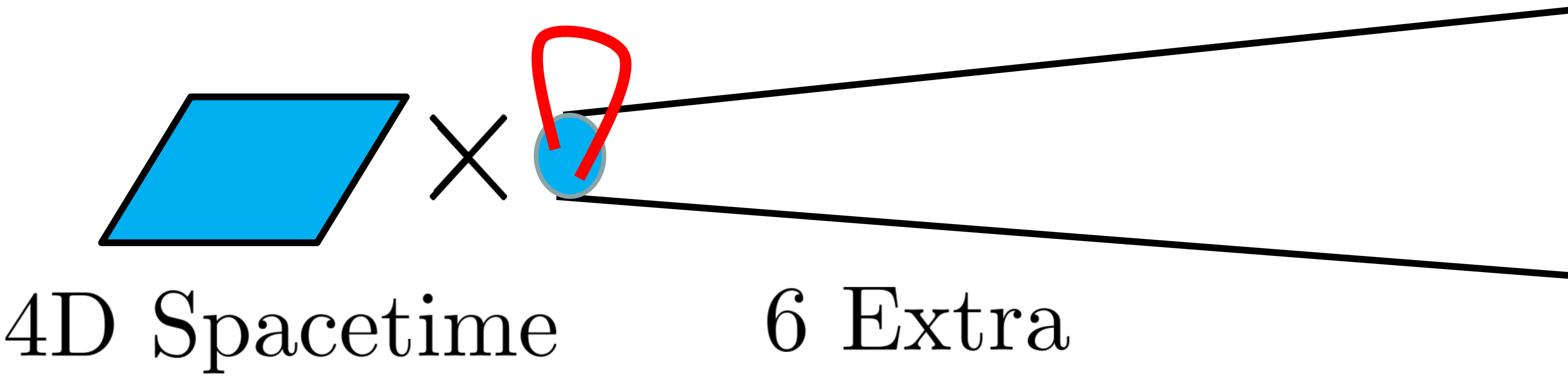

Simplifying Assumptions

Common physics strategy: Start simple, then perturb

- Keep Gravity "switched off"
- "Conformal Symmetry," i.e. no length scales
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Conformal Field Theories

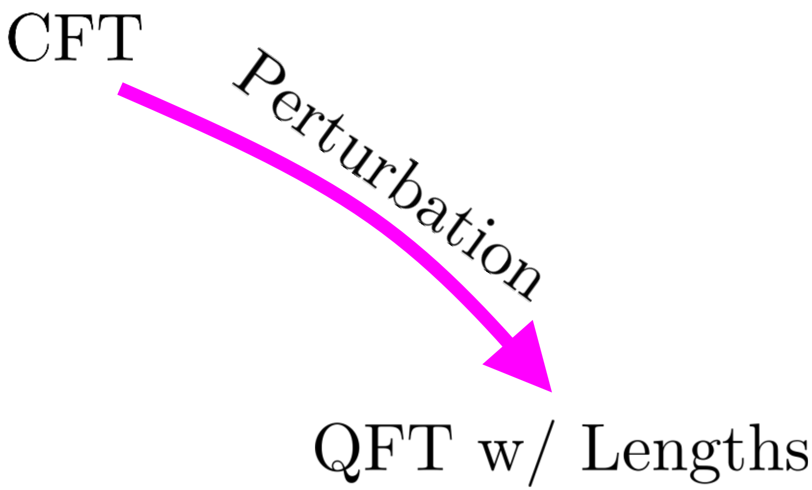
CFTs: No length scales, useful in modeling

- Phase Transitions / Critical Phenomena

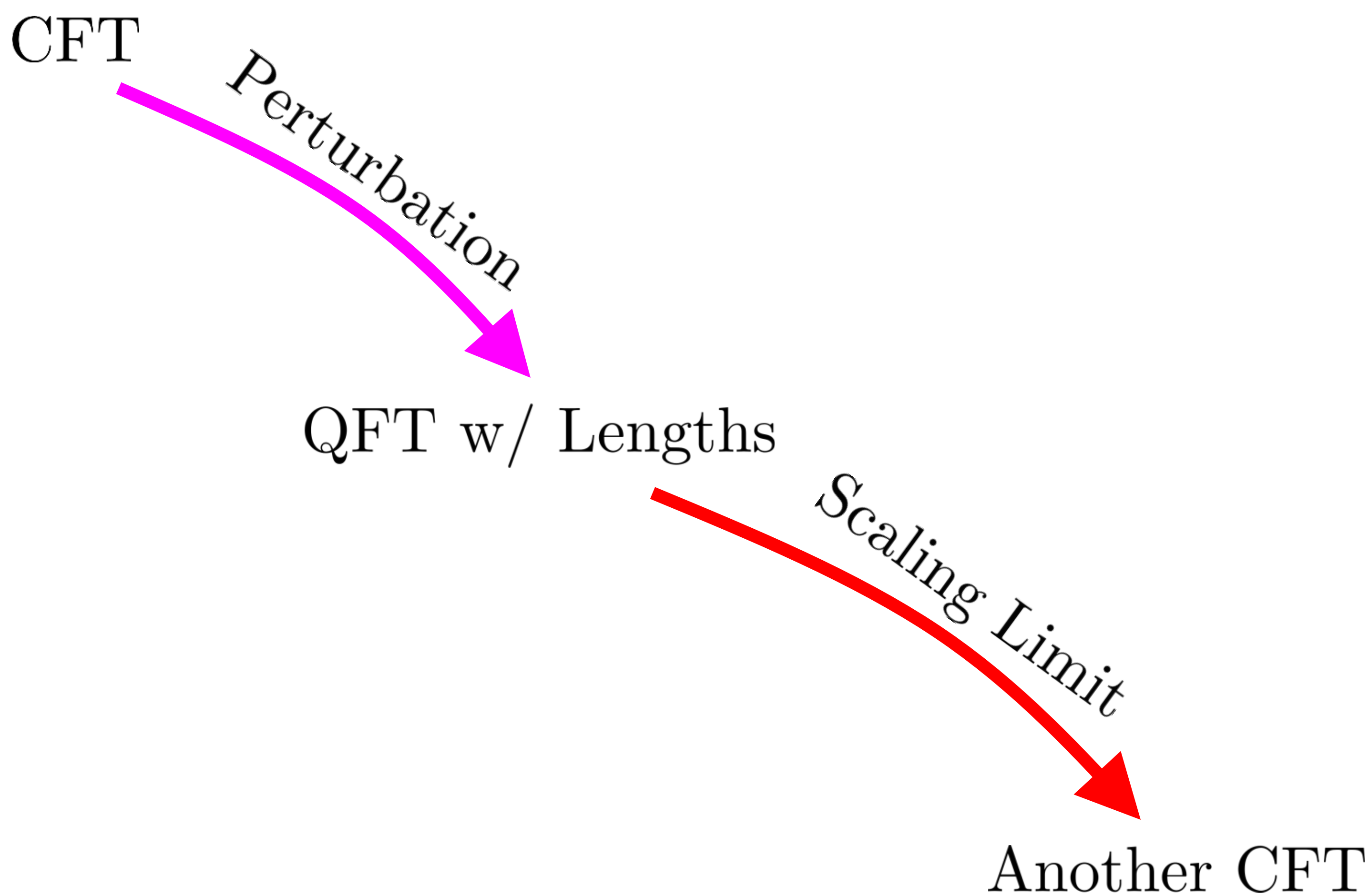
- New Elementary Particle Physics Scenarios

- Quantum Gravity (AdS / CFT)

General QFTs from CFTs



General QFTs from CFTs



Simplifying Assumptions

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Supersymmetry

Fundamental symmetry relating bosons $\leftrightarrow$ fermions

- Drives LHC @ CERN Searches

- Helps in studying quantum behavior

- Future: “Break” this symmetry

Superconformal Field Theories

- No Length Scales $\Rightarrow$ Conformal Field Theories (CFTs)

- Supersymmetry (SUSY)

$$\text{CFT} + \text{SUSY} = \text{SCFT}$$

- Nahm ('70's): $D \leq 6$

Simplifying Assumptions

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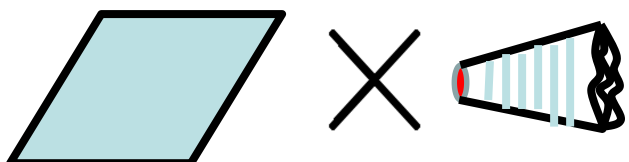
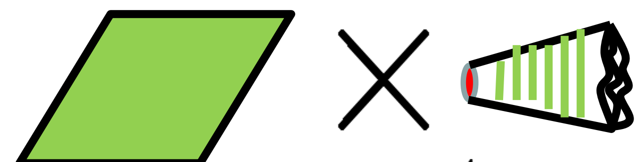
¿Why is 6D "Simple"?

More Extra Dimensions $\Rightarrow$ More Complicated!

Less Extra Dimensions $\Rightarrow$ Less Complicated!



More Complicated



Less Complicated

6D SCFTs:

Do they even exist?

A “No-Go” Answer

Try: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V_{\mathrm{small}}(\phi)$

Unstable: $V(\phi) = \phi^3$

Irrelevant: $V(\phi) = \phi^4 \;([\phi] = 2...)$

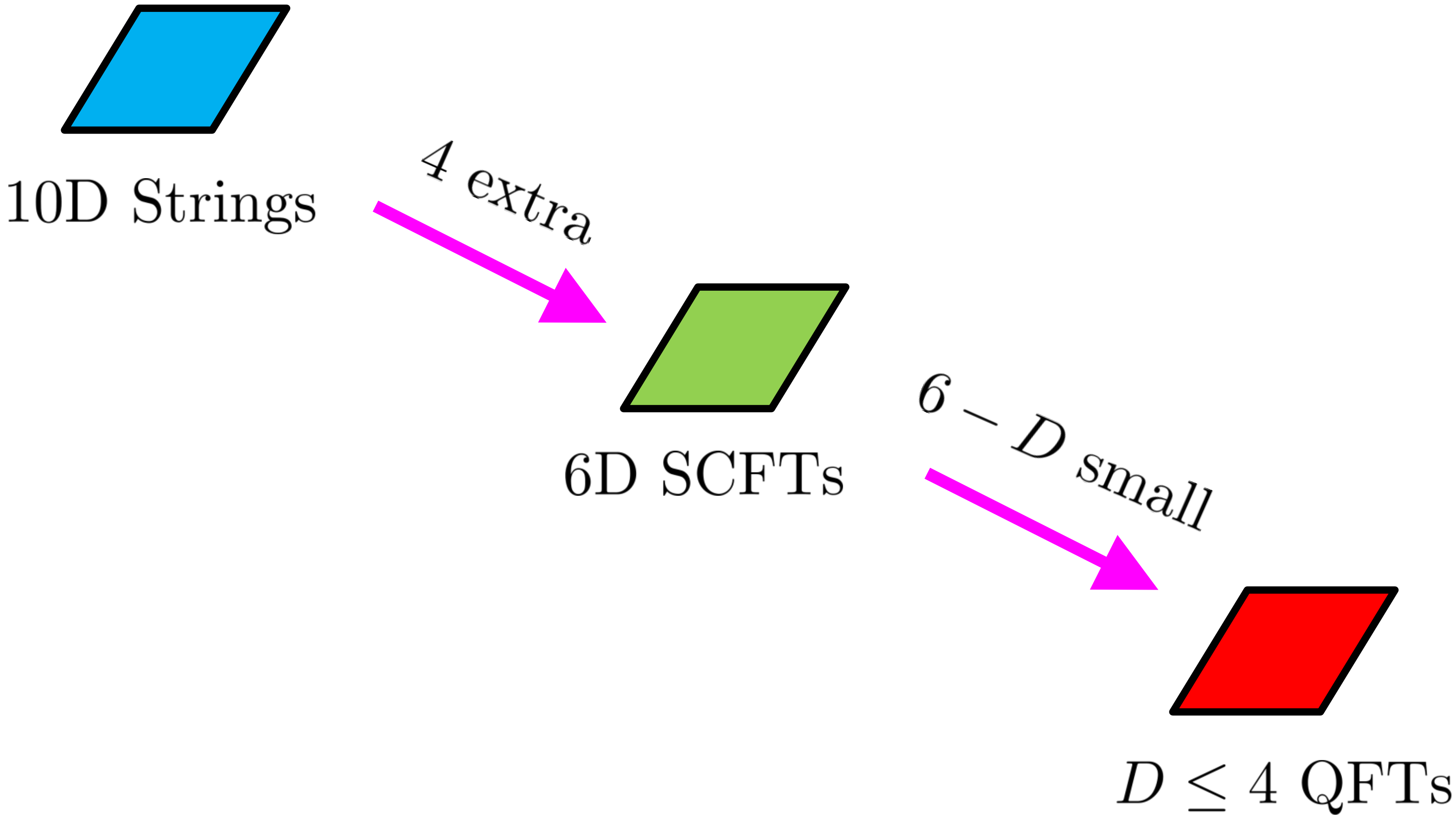
String Theory Says:

Yes, 6D CFTs Exist

Need: *Strong Coupling*

Need: *Singular* Geometry

Also Important in $D \leq 4$



Plan of the Talk

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- EFT for 6D Flows
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Geometry of 6D SCFTs

6D Theories and F-theory

Vafa '96, Vafa Morrison, I/II '96

All known 6D theories have F-theory avatar

(Note: "Frozen" singularities amount to a $\mathbb{Z}_2$ ambiguity)

IIB: $\mathbb{R}^{5,1} \times B_2$ with pos. dep. coupling $\tau(z_B)$

F-theory on $\mathbb{R}^{5,1} \times CY_3$

$$\begin{array}{ccc} T^2 & \rightarrow & CY_3 \\ & & \downarrow \\ & & B_2 \end{array}$$

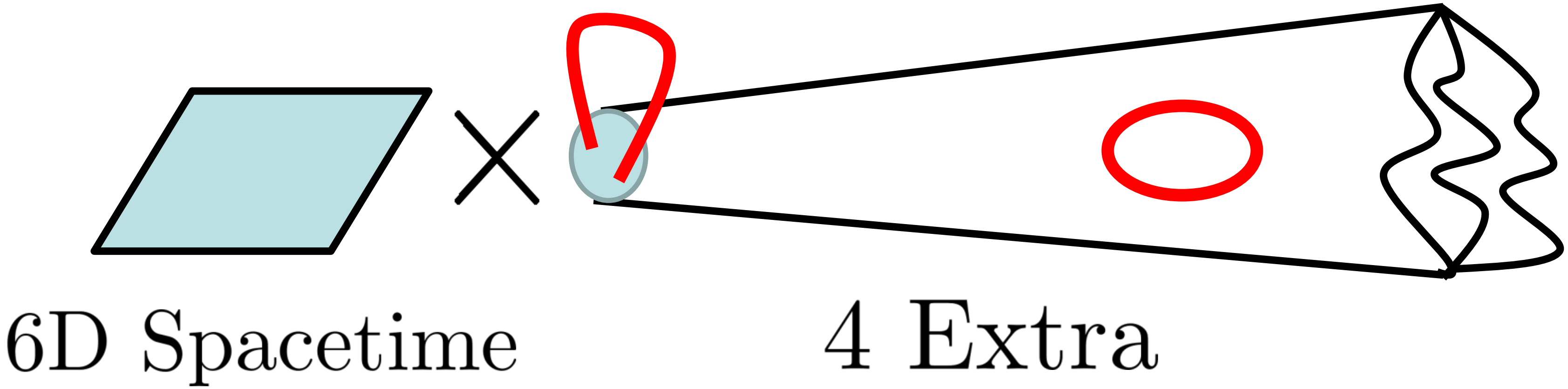
To Make a 6D SCFT...

- Start with 10D string geometry

- Write as (6D spacetime) $\times$ (4 extra)

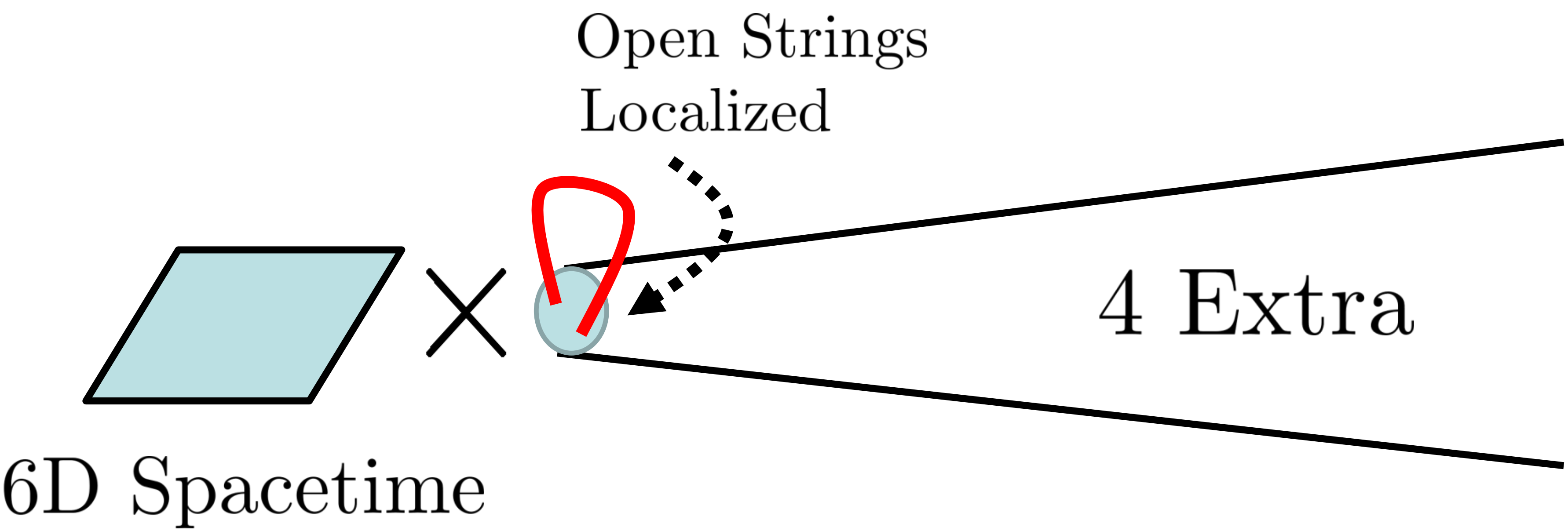
- CFT Limit: Take all lengths to 0 or ∞

Step 1: Turn off Gravity



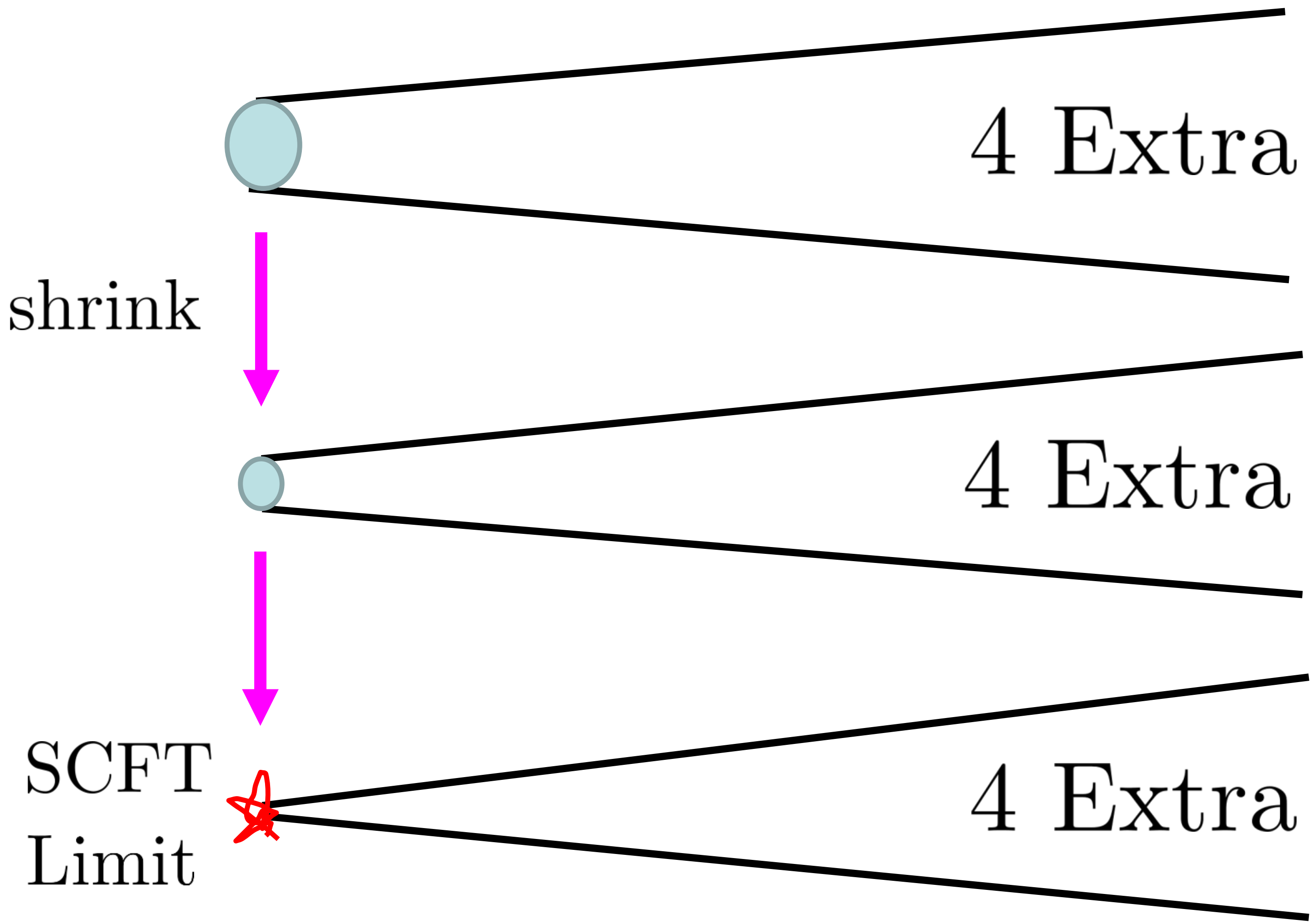
$$G_{\text{Newton}} \sim \frac{1}{\text{Vol(Extra Dimensions)}} \to 0$$

Step 1: Turn off Gravity



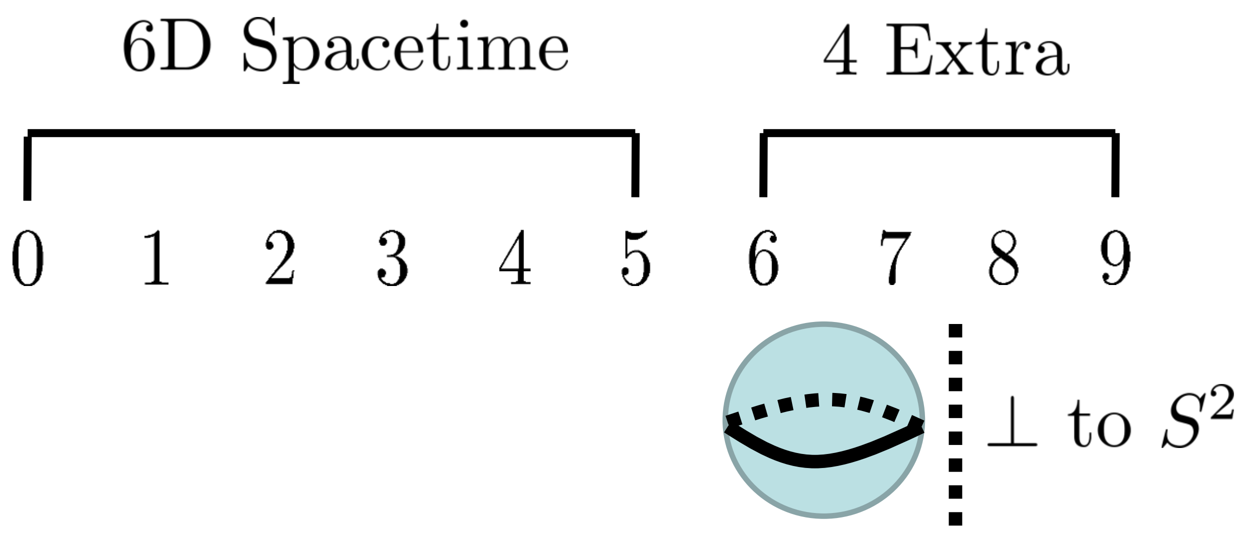
$$G_{\text{Newton}} \sim \frac{1}{\text{Vol(Extra Dimensions)}} \to 0$$

Step 2: Collapse Local Region



¿What's Collapsing?

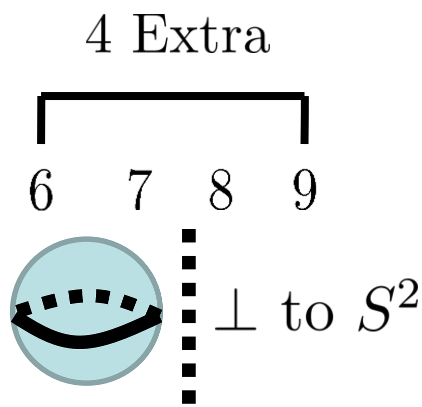
We are collapsing Two-Spheres (S^2) in 4_{extra}
e.g.: Surface of a balloon

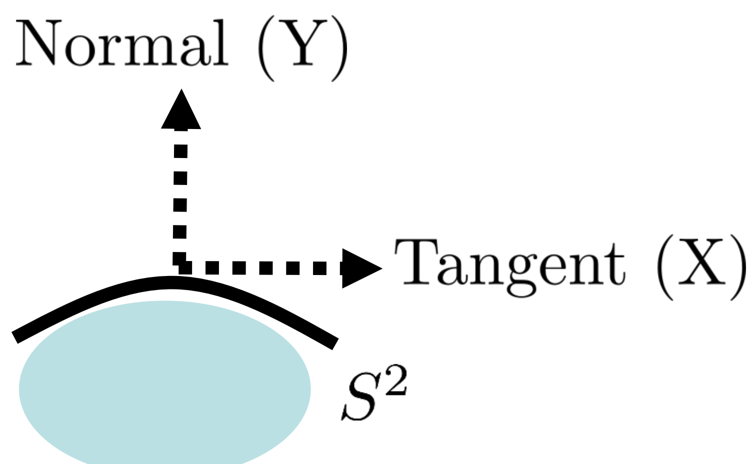


Mathematically,

Write $X = x_6 + ix_7$ as "one complex dimension"

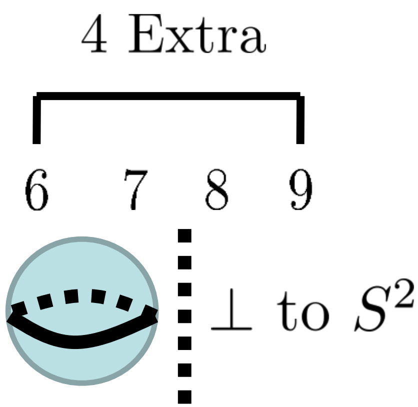
Write $Y = y_8 + iy_9$ as "one complex dimension"

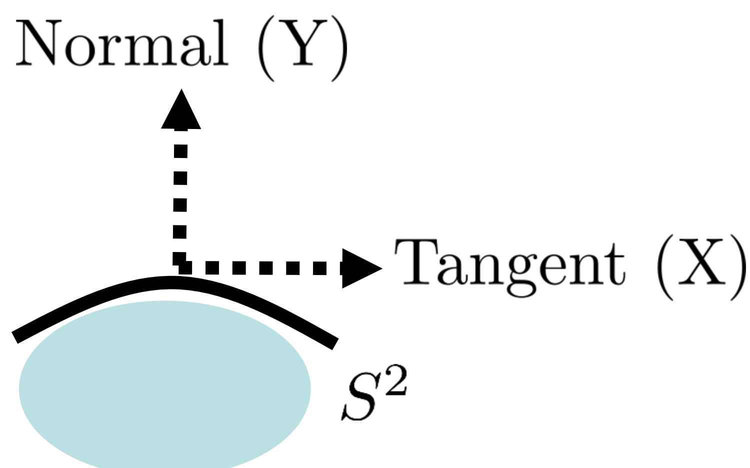




The “Simplest” 6D SCFTs

A Single Collapsing S^2

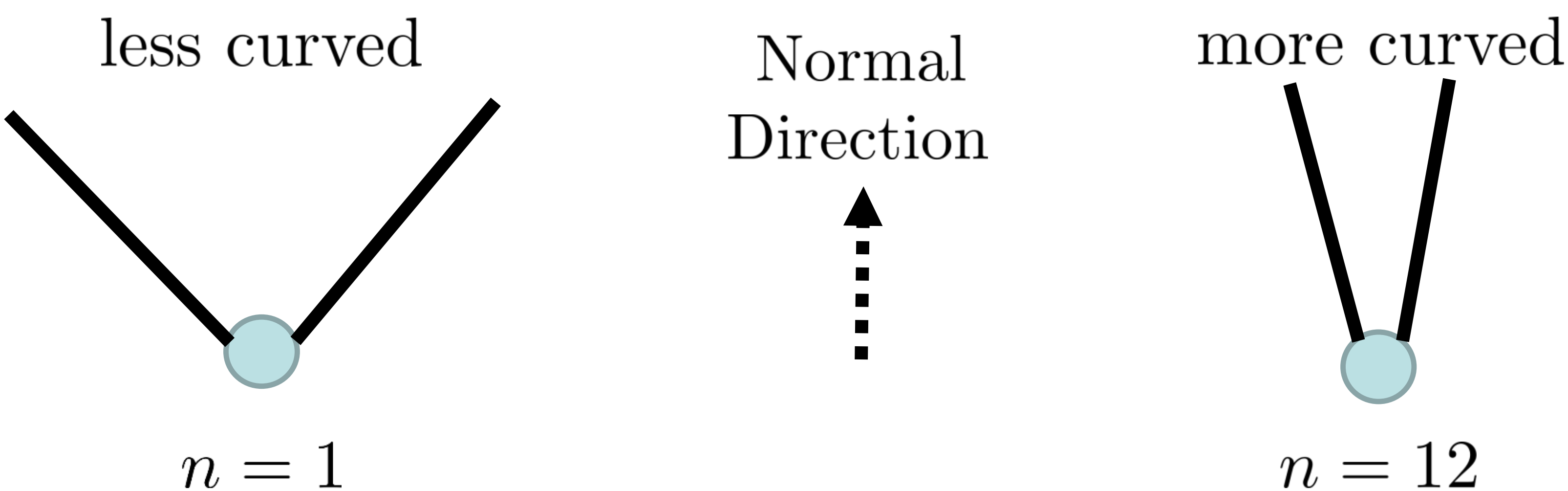




The "Simplest" 6D SCFTs

A Single Collapsing S^2

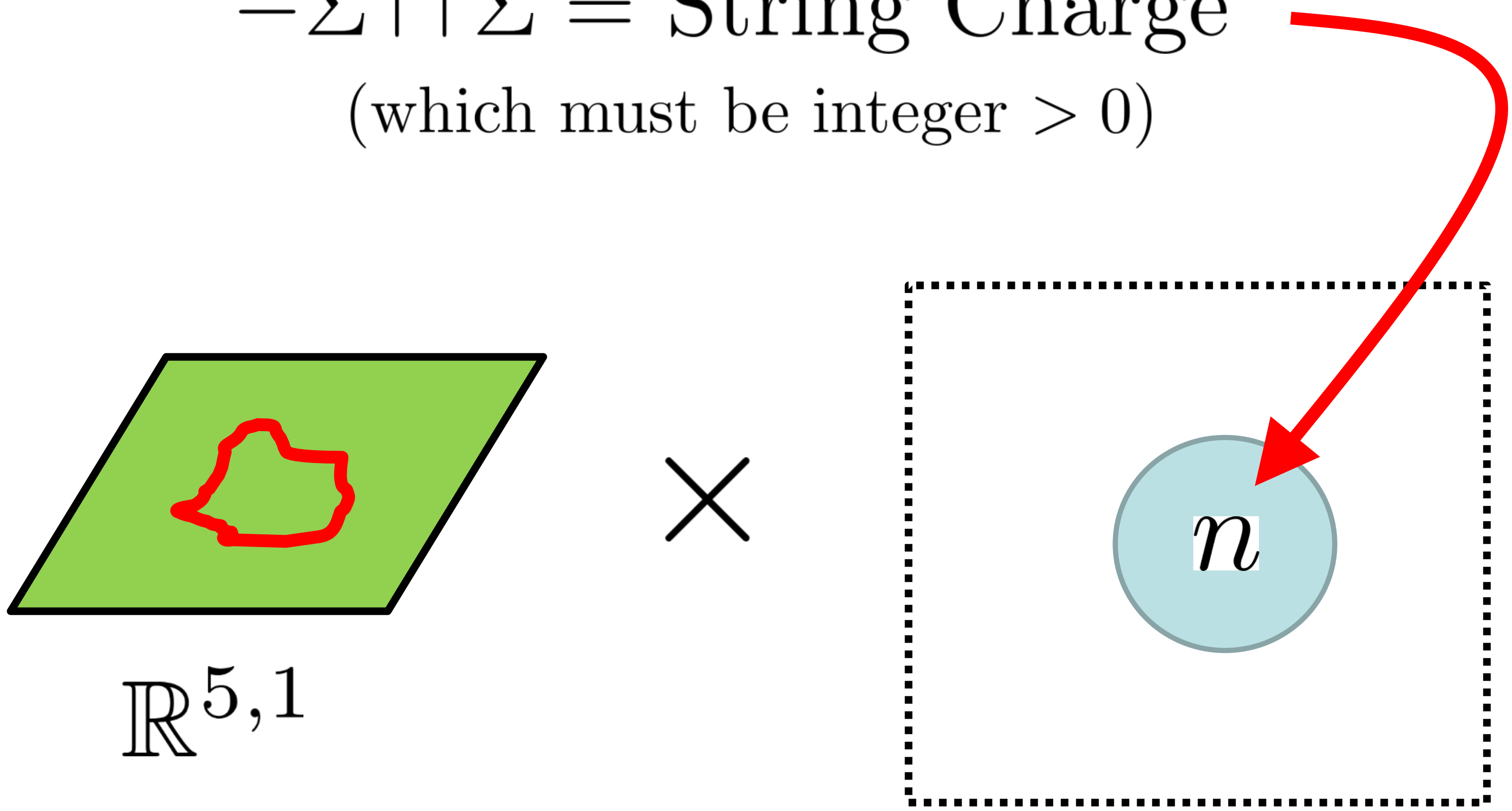
Normal Direction Curvature: Integer $1 \leq n \leq 12$



Eff. Strings from D3 / S^2

$$-\Sigma \cap \Sigma = \text{String Charge}$$

(which must be integer > 0)



Tensionless Strings in F-theory

- Realized by D3-brane on collapsing S^2

  Tension $= \mathrm{Vol}(S^2) \to 0$

$\mathbb{R}^{5,1}$ (0–5) S^2 (6–7)

	0	1	2	3	4	5	6	7	8	9
D3	×	×	•	•	•	•	×	×	•	•

Hallmark of $D > 4$ SCFTs:

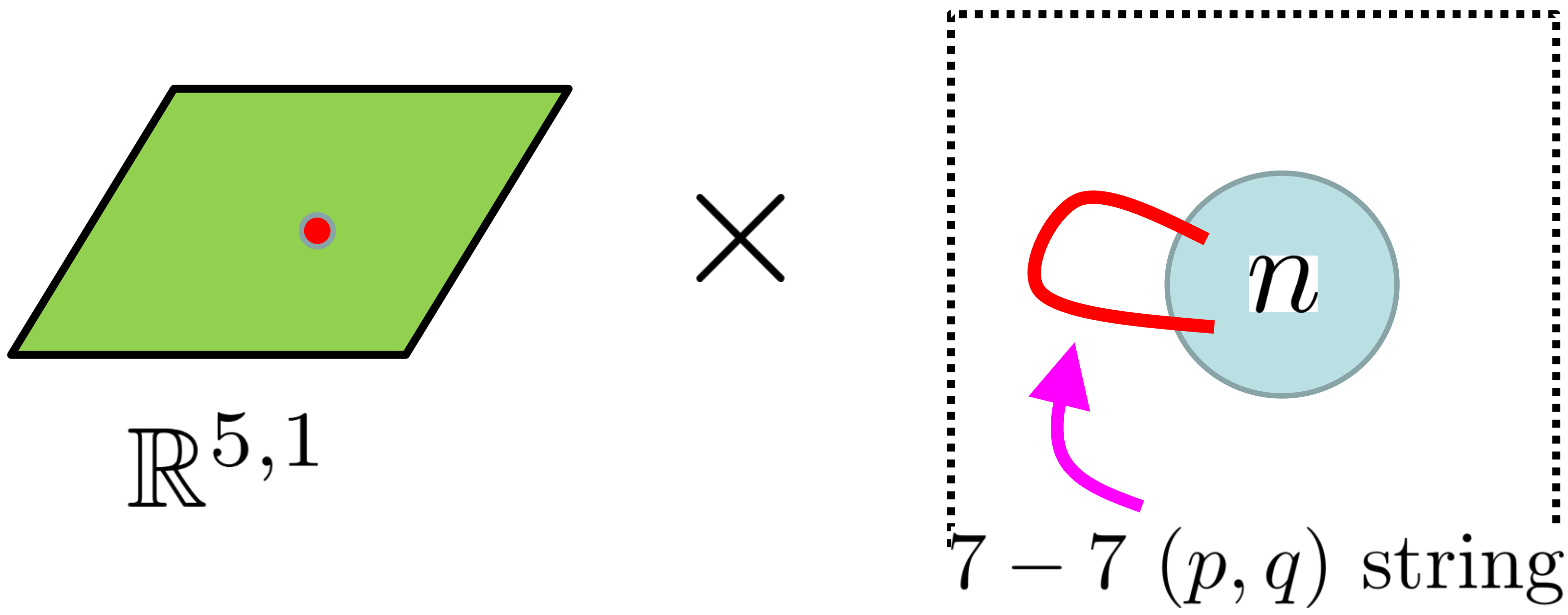
All known (stringy) constructions have:

Effective Strings with Tension $\to 0$ at the CFT

Particles from D7's on a S^2

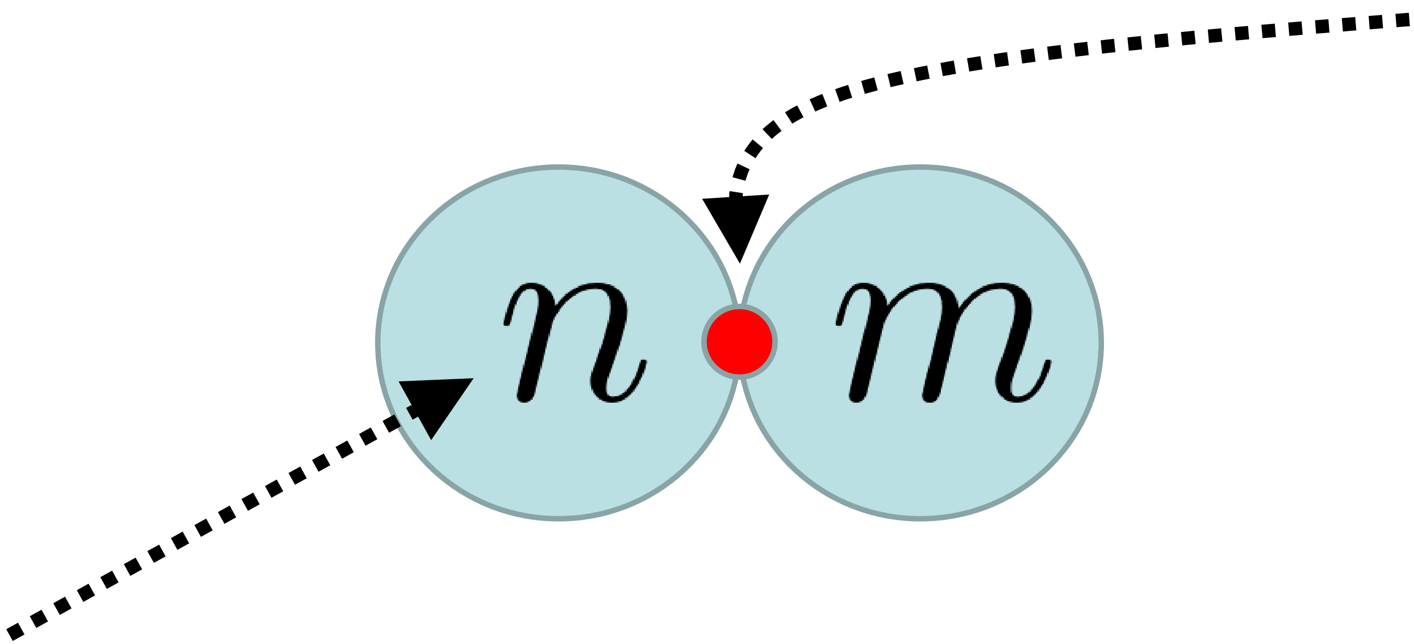
$3 \leq n \leq 12 \Rightarrow$ always have gauge fields

(elliptic fiber is singular: Morrison Taylor '12)



Next Simplest: Two S^2's

Two Collapsing S^2's touching at a common point

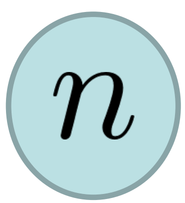


n and m: Data About Normal Direction

Building Blocks

Looks Like Chemistry

“Atoms”

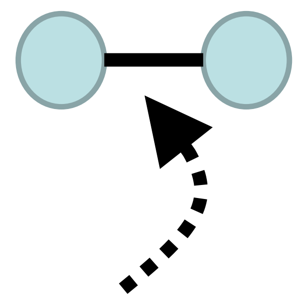


Restricted values of n

$n = 4$, $n = 6$, $n = 7$,

$n = 8$, $n = 12$

“Radicals”



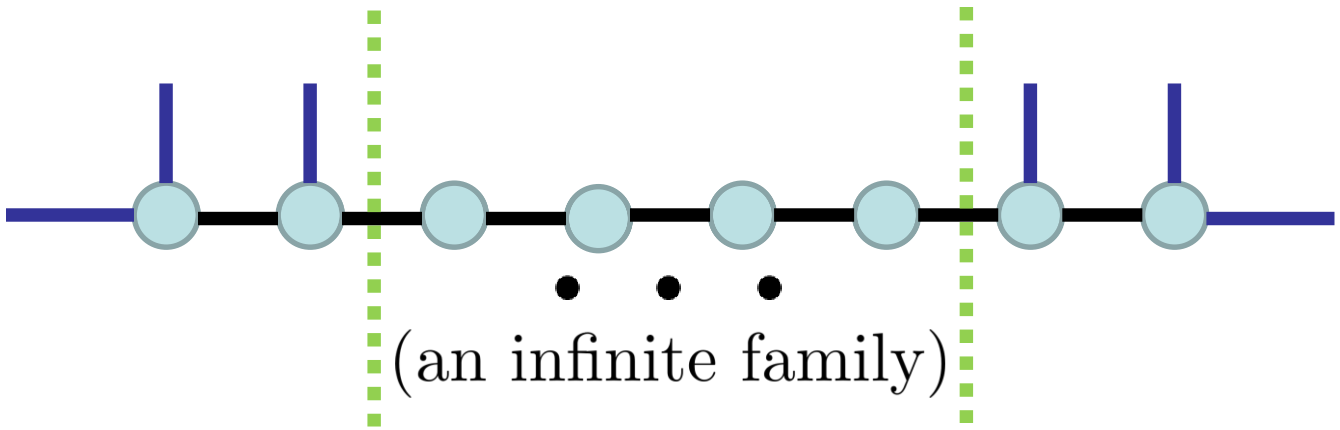
Repeating Pattern of S^2’s

e.g. 1,2,2,3,1,5,1,3,2,2,1

(normal direction data)

Classification of Geometries

JJH Morrision Vafa '13; JJH Morrison Rudelius Vafa '15



6D SCFTs = Simple "Molecules"

More Technically,

Classification of canonical singularities for non-compact $T^2 \to CY_3 \to B_2$

i) Classify Bases B_2

ii) Classify Fibrations over a base B_2

Calculability

Things We Know:

- Anomalies (Topology)
- Moduli Spaces (Holomorphy)
- Scaling Dimensions (T + H + Large $\mathcal{R}$)

Things We Know:

- Anomalies (Topology)

(Tachikawa et al. '14; JJH et al. '15; Cordova et al. '15; '20)

$$\mathcal{I}_8 = \alpha c_2(\mathcal{R})^2 + \beta c_2(\mathcal{R}) p_1(\mathcal{T}) + \gamma p_1(\mathcal{T})^2 + \delta p_2(\mathcal{T})$$

$$+ \text{ Flavor Invariants}$$

$$a_{6D} \sim \frac{8}{3}(\alpha - \beta + \gamma) + \delta$$

(Tachikawa et al. '14; JJH et al. '15; Cordova et al. '15; '20)
(6D a-theorem by brute force: JJH Rudelius '15)

Things We Know:

- Moduli Spaces (Holomorphy)

Via $T^2 \rightarrow CY_3 \rightarrow B_2 : y^2 = x^3 + fx + g$

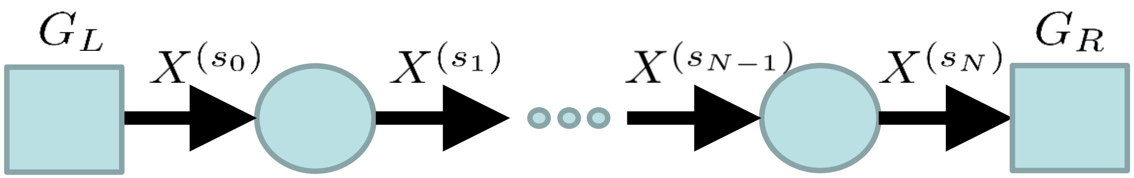
Tensor Branch: Blowups in Base B_2

Higgs Branch: Cplx Str. Def of CY_3

Things We Know:

Baume JJH Lawrie '20; to appear

- Scaling Dimensions (T + H + Large $\mathcal{R}$)



Controlled by $\mathfrak{osp}(8^*|1)$ open spin chain

(Large $\mathcal{R}$ + Bethe Ansatz)

$$\Delta(\mathcal{O}_{\text{eigen}}) = \Delta_{\text{BPS}} + \frac{\gamma}{N^2} + O(N^{-4})$$

(see also Berenstein Maldacena Nastase '02; Hellerman Orlando Reffert '15)

Geometry of Flows

JJH Rudelius Tomasiello ’16, JJH Rudelius Tomasiello ’18,
Apruzzi Fassler JJH Rochais ’18, JJH Hassler Rochais Rudelius Zhang ’19

Geometry of Flows

Complex Structure Deformation / Higgs Branch

Brane Recombination

Expand a curve in base to large size

Go to large tension / weak gauge coupling

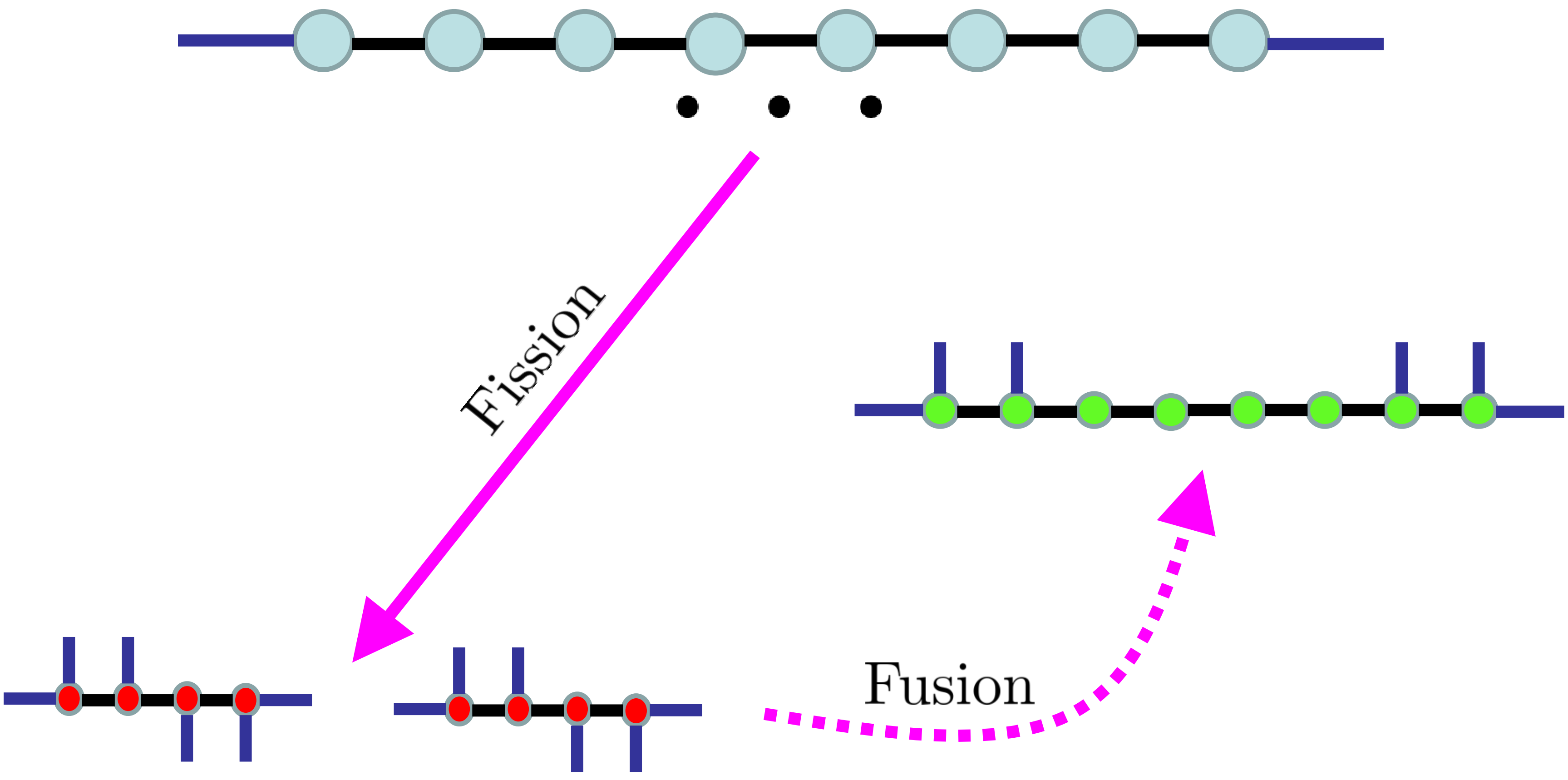
Claim

Every 6D SCFT via:

1) Fission of Progenitor Theories

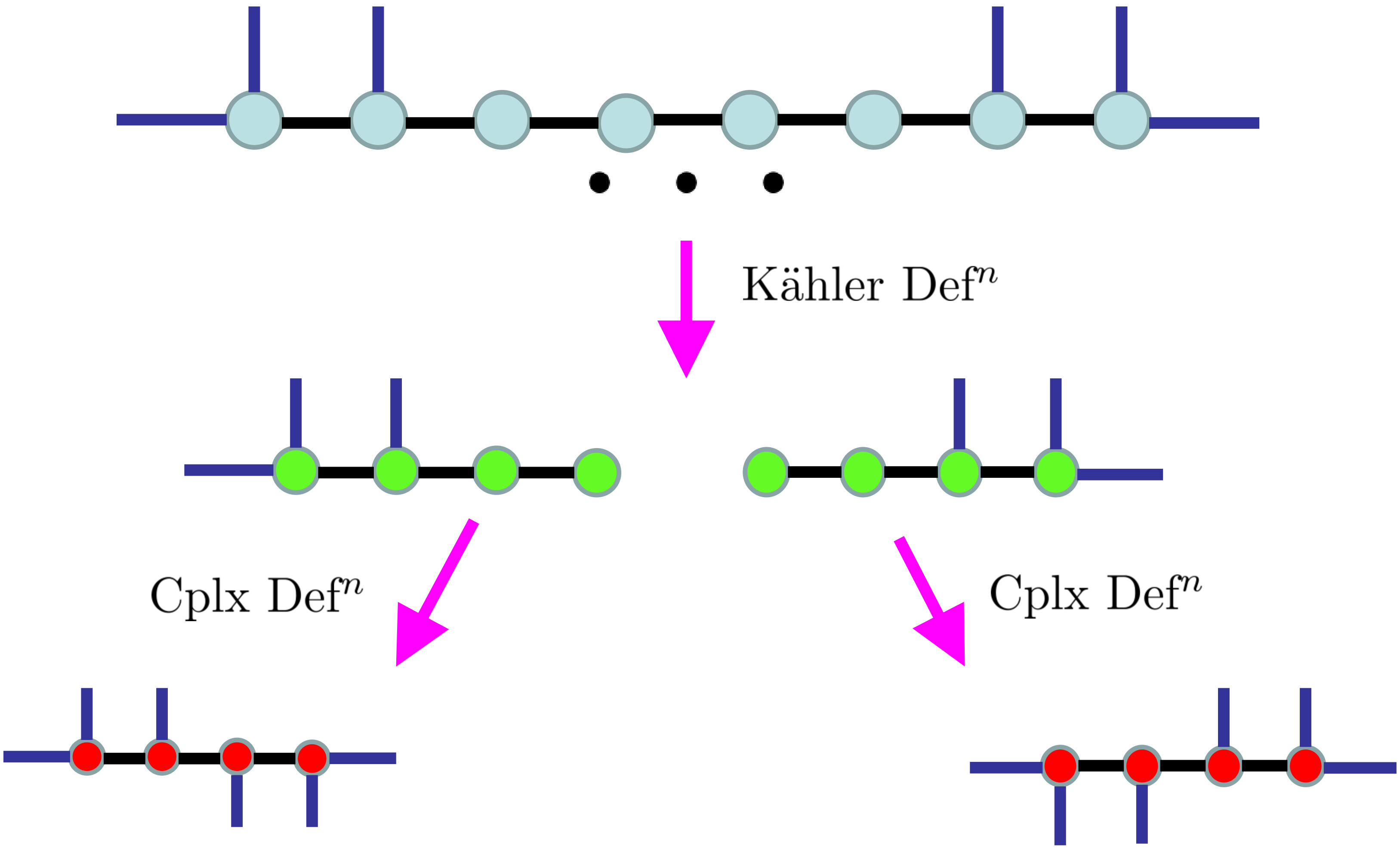
2) At Most One Step of Fusion

General Picture

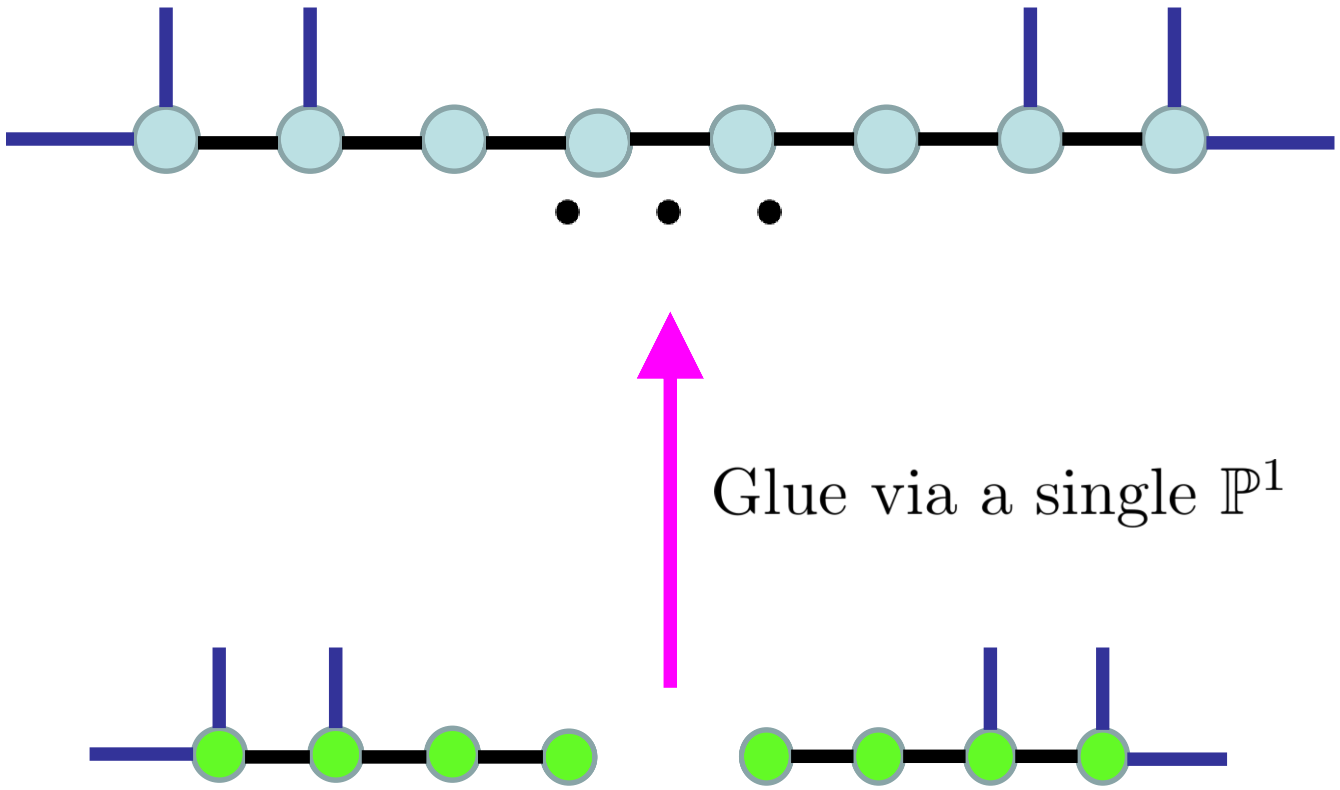


JJH Rudelius Tomasiello '18

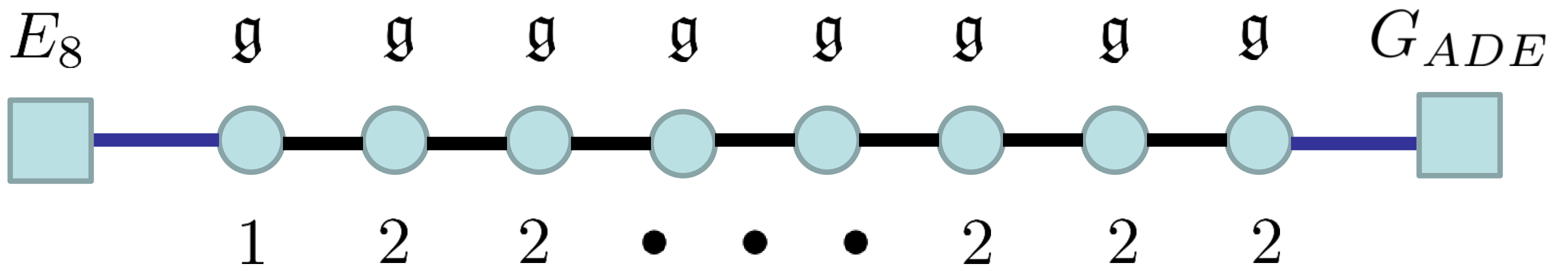
“Fission”



“Fusion”

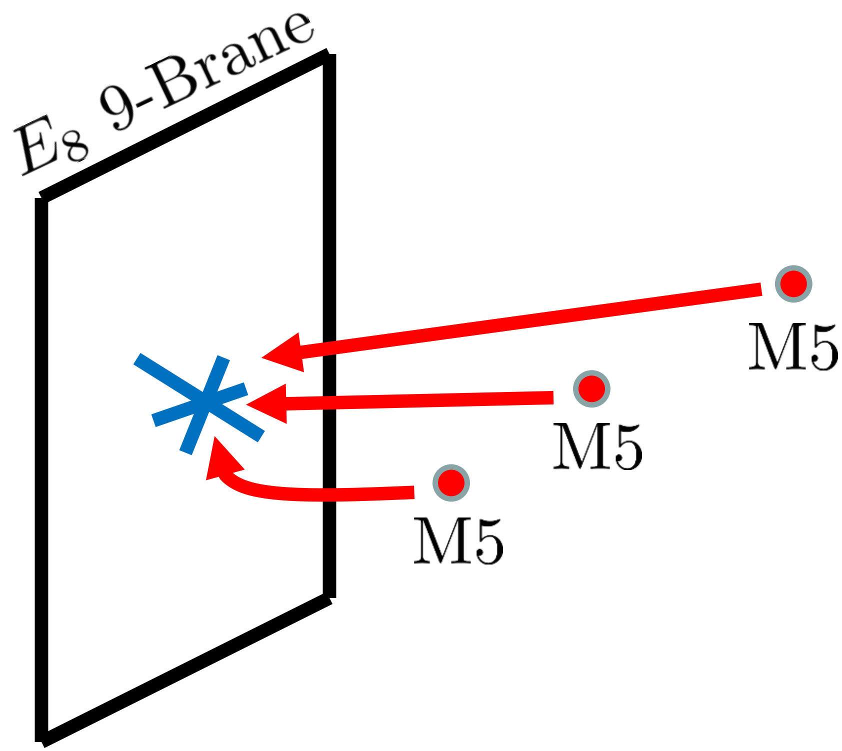


Progenitors



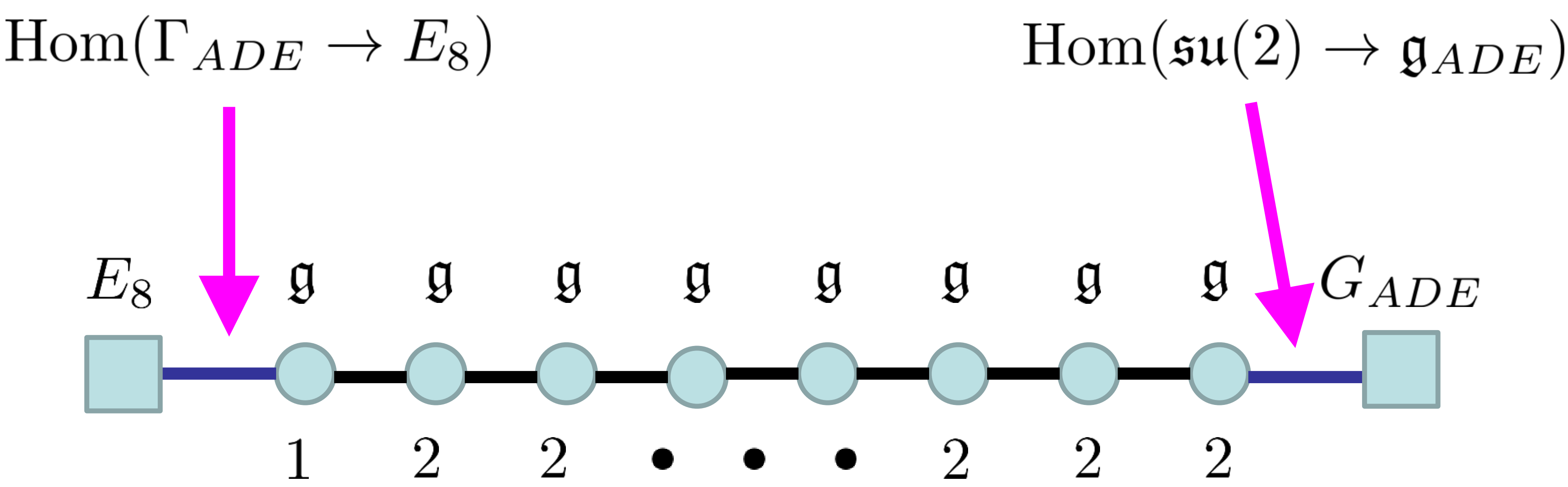
Heterotic / M-th Realization

M5's (small instantons) at $\mathbb{C}^2/\Gamma_{ADE}$ on E_8 wall



Smoothings

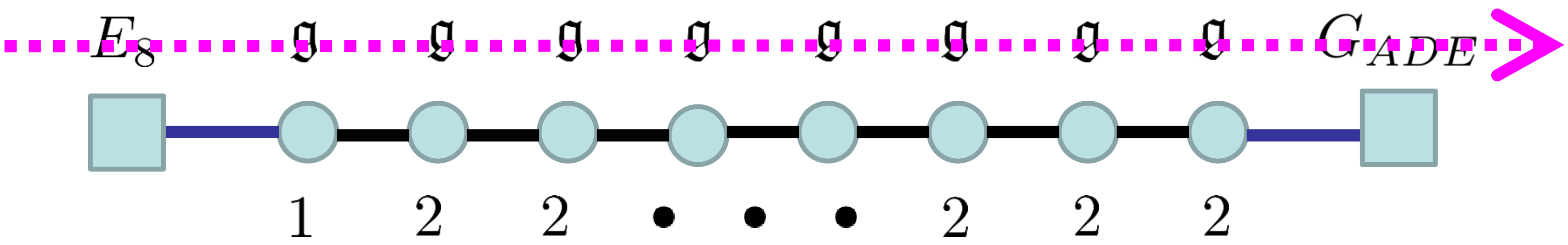
If it is *localized*, captured by:



Smoothings

If it is *not localized*, captured by

captured by common $U(1) \subset G_{\text{flav}}$



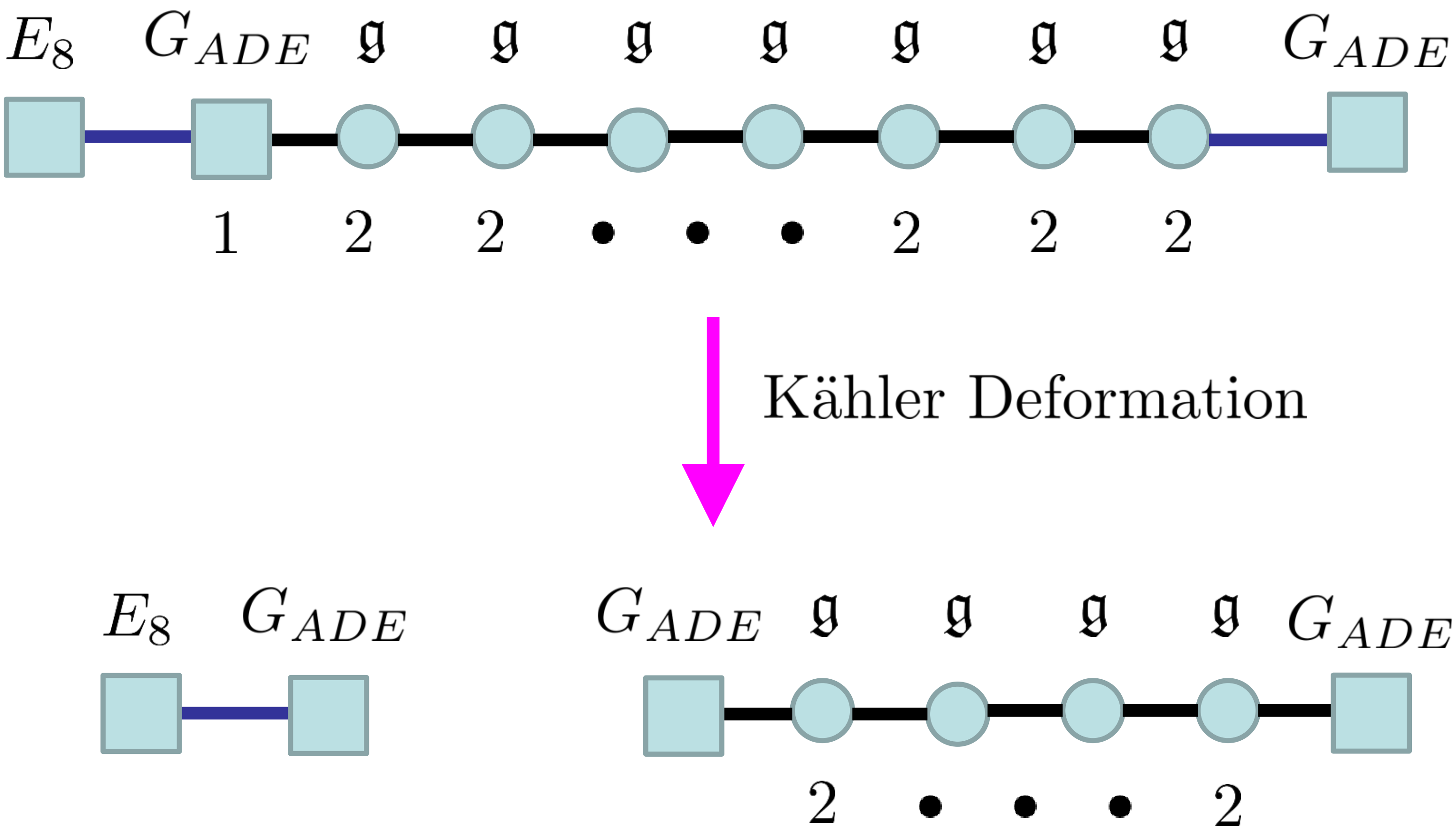
Partial Ordering

Known Ordering: $\mathrm{Hom}(\mathfrak{su}(2) \to \mathfrak{g}_L)$

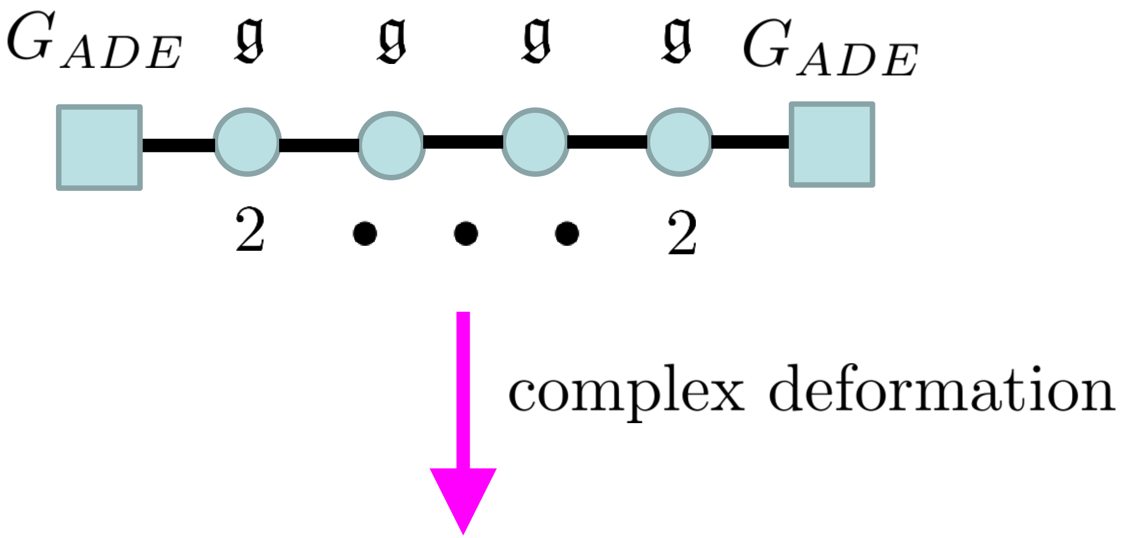
$$\mathrm{Orbit}(\mu) \subset \overline{\mathrm{Orbit}(\nu)}$$

Unknown Ordering (?): $\mathrm{Hom}(\Gamma_{ADE} \to E_8)$

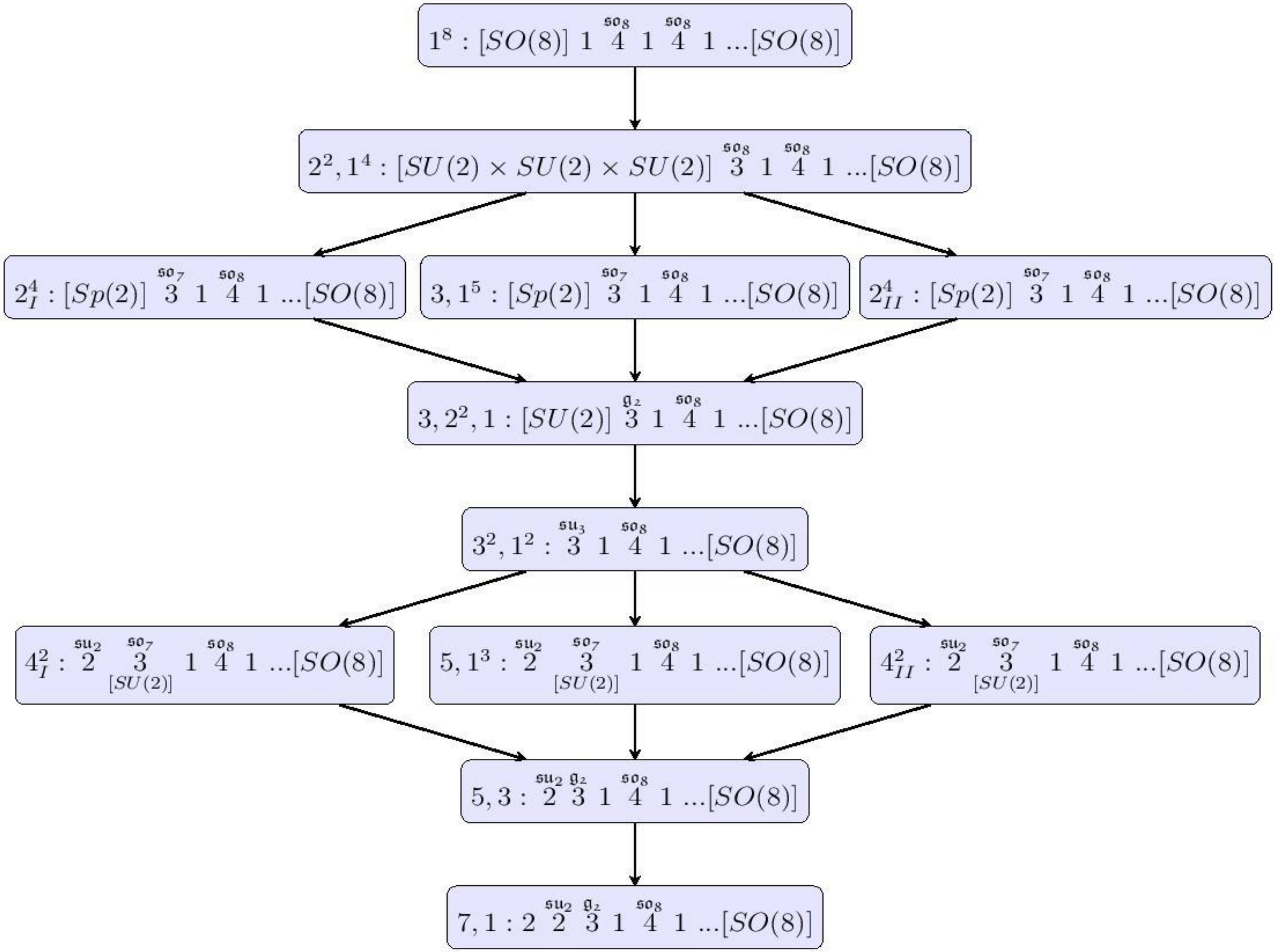
Examples: Decompactify −1



Further Smoothings?



Classify by μ_L and μ_R nilpotent
(if localized)


$1^8 : [SO(8)]\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$2^2, 1^4 : [SU(2) \times SU(2) \times SU(2)]\ \overset{\mathfrak{so}_8}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$2^4_I : [Sp(2)]\ \overset{\mathfrak{so}_7}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$3, 1^5 : [Sp(2)]\ \overset{\mathfrak{so}_7}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$2^4_{II} : [Sp(2)]\ \overset{\mathfrak{so}_7}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$3, 2^2, 1 : [SU(2)]\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$3^2, 1^2 :\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$4^2_I :\ \overset{\mathfrak{su}_2}{2}\ \underset{[SU(2)]}{\overset{\mathfrak{so}_7}{3}}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$5, 1^3 :\ \overset{\mathfrak{su}_2}{2}\ \underset{[SU(2)]}{\overset{\mathfrak{so}_7}{3}}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$4^2_{II} :\ \overset{\mathfrak{su}_2}{2}\ \underset{[SU(2)]}{\overset{\mathfrak{so}_7}{3}}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$5, 3 :\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$7, 1 : 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$

Another Example...

JJH Hassler Rochais Rudelius Zhang '19


[1^8] : 1 [SO(8)] so(8) 4 1 so(8) 4 1 so(8) 4 1 [SO(8)] : [1^8]
[2^2, 1^4] : so(8) 3 [SU(2)⊗3] 1 so(8) 4 1 so(8) 4 1 [SO(8)] : [1^8]
[2^2, 1^4] : so(8) 3 [SU(2)⊗3] 1 so(8) 4 1 so(8) 3 [SU(2)⊗3] : [2^2, 1^4]
[3, 1^5]; [2^4_{I,II}] : so(7) 3 [Sp(2)] 1 so(8) 4 1 so(8) 4 1 [SO(8)] : [1^8]
[3, 1^5]; [2^4_{I,II}] : so(7) 3 [Sp(2)] 1 so(8) 4 1 so(8) 3 [SU(2)⊗3] : [2^2, 1^4]
[3, 2^2, 1] : g_2 3 [SU(2)] 1 so(8) 4 1 so(8) 4 1 [SO(8)] : [1^8]
[3, 1^5]; [2^4_{I,II}] : so(7) 3 [Sp(2)] 1 so(8) 4 1 so(7) 3 [Sp(2)] : [3, 1^5]; [2^4_{I,II}]
[3, 2^2, 1] : g_2 3 [SU(2)] 1 so(8) 4 1 so(8) 3 [SU(2)⊗3] : [2^2, 1^4]
[3^2, 1^2] : su(3) 3 1 so(8) 4 1 so(8) 4 1 [SO(8)] : [1^8]
[3, 2^2, 1] : g_2 3 [SU(2)] 1 so(8) 4 1 so(7) 3 [Sp(2)] : [3, 1^5]; [2^4_{I,II}]
[3^2, 1^2] : su(3) 3 1 so(8) 4 1 so(8) 3 [SU(2)⊗3] : [2^2, 1^4]
[5, 1^3]; [4^2_{I,II}] : su(2) 2 so(7) 3 [SU(2)] 1 so(8) 4 1 [SO(8)] : [1^8]
[3, 2^2, 1] : g_2 3 [SU(2)] 1 so(8) 4 1 g_2 3 [SU(2)] : [3, 2^2, 1]
[3^2, 1^2] : su(3) 3 1 so(8) 4 1 so(7) 3 [Sp(2)] : [3, 1^5]; [2^4_{I,II}]
[5, 1^3]; [4^2_{I,II}] : su(2) 2 so(7) 3 [SU(2)] 1 so(8) 3 [SU(2)⊗3] : [2^2, 1^4]
[5, 3] : su(2) 2 g_2 3 1 so(8) 4 1 [SO(8)] : [1^8]
[3^2, 1^2] : su(3) 3 1 so(8) 4 1 g_2 3 [SU(2)] : [3, 2^2, 1]
[5, 1^3]; [4^2_{I,II}] : su(2) 2 so(7) 3 [SU(2)] 1 so(7) 3 [Sp(2)] : [3, 1^5]; [2^4_{I,II}]
[5, 3] : su(2) 2 g_2 3 1 so(8) 3 [SU(2)⊗3] : [2^2, 1^4]
[7, 1] : 2 su(2) 2 g_2 3 1 [F_4] : [1^8]
[3^2, 1^2] : su(3) 3 1 so(8) 4 1 su(3) 3 : [3^2, 1^2]
[5, 1^3]; [4^2_{I,II}] : su(2) 2 so(7) 3 [SU(2)] 1 g_2 3 [SU(2)] : [3, 2^2, 1]
[5, 3] : su(2) 2 g_2 3 1 so(7) 3 [Sp(2)] : [3, 1^5]; [2^4_{I,II}]
[7, 1] : 2 su(2) 2 g_2 2 [Sp(3)] : [2^2, 1^4]
[5, 1^3]; [4^2_{I,II}] : su(2) 2 so(7) 3 [SU(2)] 1 [SU(3)] su(3) 3 : [3^2, 1^2]
[5, 3] : su(2) 2 g_2 3 1 [SU(2)] g_2 3 [SU(2)] : [3, 2^2, 1]
[7, 1] : 2 su(2) 2 [N_f=1/2] su(3) 2 [SU(4)] : [3, 1^5]; [2^4_{I,II}]
[5, 1^3]/[4^2_{I,II}]/[4^2_{I,II}] : su(2) 2 [N_f=1/2] g_2 2 [Sp(2)] su(2) 2 [N_f=1/2] : [4^2_{I,II}]/[5, 1^3]/[4^2_{I,II}]
[5, 1^3] : su(2) 2 su(4) 2 [SU(4)] su(2) 2 : [5, 1^3]
[5, 3] : su(2) 2 g_2 3 1 [SU(3)] su(3) 3 : [3^2, 1^2]
[7, 1] : 2 su(2) 2 [N_f=3/2] su(2) 2 [SU(2)⊗2] : [3, 2^2, 1]
[5, 3] : su(2) 2 [N_f=1] su(3) 2 [SU(2)] su(2) 2 [N_f=1] : [5, 1^3]; [4^2_{I,II}]
[7, 1] : 2 su(2) 2 [SU(3)] 2 : [3^2, 1^2]
[5, 3] : su(2) 2 [SU(2)] su(2) 2 su(2) 2 [SU(2)⊗2] : [5, 3]

Plan of the Talk

- Motivation / Background
- Geometry of 6D SCFTs
- EFT for 6D Flows
- Conclusions / Future

EFT for 6D Flows

JJH Kundu Zhang ’21

Symmetry Breaking

Broken Symmetries $\Rightarrow$ Nambu-Goldstone Modes

- Conformal Symmetry $\Rightarrow$ Dilaton
- R-symmetry $\Rightarrow$ Axions
- Global Symmetries $\Rightarrow$ More Axions

Symmetry Breaking

Broken Symmetries ⇒ Nambu-Goldstone Modes

- Conformal Symmetry ⇒ Dilaton
  Elvang Freedman Hung Kiermaier Myers Theisen '11
- R-symmetry ⇒ Axions

- Global Symmetries ⇒ More Axions

UV $\rightarrow$ IR

(See Komargodski Schwimmer 4D a-theorem '11)

UV CFT

IR CFT + $S_{\text{eff}}[\tau, \beta_a]$

Nambu-Goldstone Modes

Couple to bkgnd $g_{\mu\nu}$ and $A_\mu^{\mathrm{R-symm}}$

$$g_{\mu\nu}(x) \to e^{2s(x)} g_{\mu\nu}(x)$$

$$\tau(x) \to \tau(x) + s(x)$$

$$A_\mu \to \Omega A_\mu \Omega^{-1} + i\Omega \nabla_\mu \Omega^{-1},\ \Omega = \exp(i\pi^a(x)\sigma^a)$$

$$\beta^a \to \beta^a + \pi^a$$

$$S_{\text{eff}}[\tau, \beta] =$$

$$\int d^6x \, (-2f^4 e^{-4\tau} \left((\partial\tau)^2 + \tfrac{1}{2}\gamma_0^2 \mathrm{Tr}(\partial e^{i\beta} \partial e^{-i\beta})\right)$$

$$+ \int d^6x \, 3(a_{UV} - a_{IR})\tau \Box^3 \tau$$

$$+ \int d^6x \, e^{-6\tau} \left(\mathcal{L}_{\text{conf}}[\widehat{g}_{\mu\nu}] + \mathcal{L}_{SU(2)}[\widehat{A}_\mu, \widehat{g}_{\mu\nu}]\right)_{\widehat{g}_{\mu\nu} = \eta_{\mu\nu}, \widehat{A}_\mu = 0}$$

$$+ \text{ higher order terms}$$

Introduce $\widehat{g}_{\mu\nu} \equiv e^{-2\tau} g_{\mu\nu}$ and $\widehat{A}_\mu = A_\mu - i e^{i\beta} \partial_\mu e^{-i\beta}$

Physical Modes

$$\exp(-2\tau)\exp(-2i\gamma_0\beta_a\sigma_a) = \mathbf{1}\left(1 - \tfrac{\phi}{f^2}\right) - \tfrac{i}{f^2}\sigma_a\xi_a + \ldots$$

$$S_{\text{eff}}[\phi,\xi] = \int d^6x\ -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\xi)^2$$

$$+\mathcal{L}_{\text{dilaton}} + \mathcal{L}_{\text{axion}} + \mathcal{L}_{\text{mix}}$$

$$\mathcal{L}_{\text{dilaton}} \supset \frac{\widehat{b}}{16f^4}\phi^2\Box^2\phi^2 + \frac{3}{16f^8}\left(\Delta a - \frac{2}{3}\widehat{b}^2\right)\phi^2\Box^3\phi^2 + \ldots$$

Non-Negativity

$$\mathcal{L}_{\text{dilaton}} \supset \frac{\widehat{b}}{16f^4}\phi^2\Box^2\phi^2 + \frac{3}{16f^8}\left(\Delta a - \frac{2}{3}\widehat{b}^2\right)\phi^2\Box^3\phi^2 + \ldots$$

Dispersion rel^n $\Rightarrow \widehat{b} \geq 0$

Adams Arkani-Hamed Dubovsky Nicolis Rattazzi '06

$\underline{\underline{\underline{IF}}}$ $\text{Res}_{s=\infty}\left(\frac{\mathcal{M}(\phi\phi\to\phi\phi)}{s^3}\right) = 0$, $\underline{\underline{\underline{THEN}}}$ $\widehat{b} > 0$

¿Generating $\widehat{b}$?

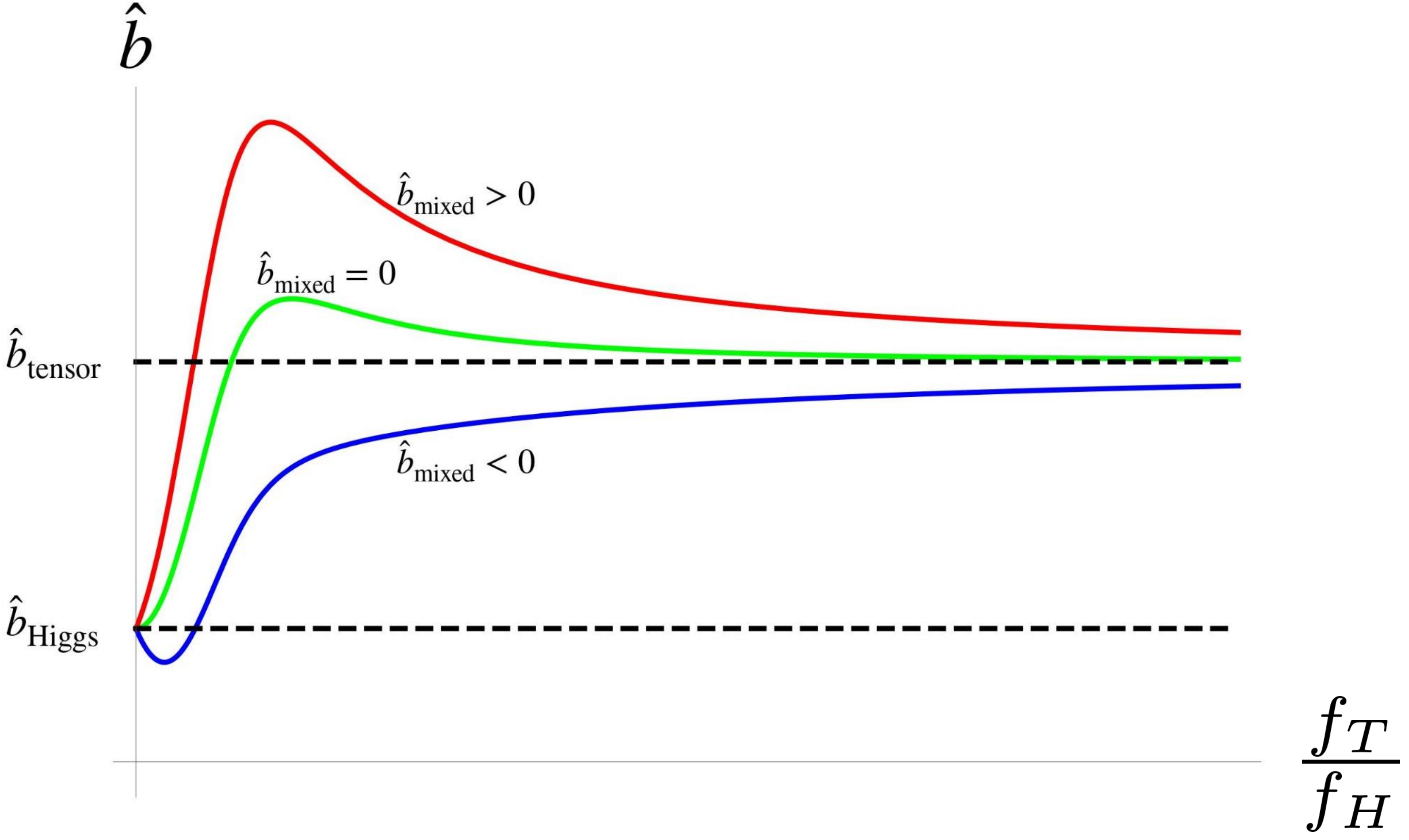
Define: $\widehat{b}_{\text{Tensor}}$ = Depends only on f_T

Define: $\widehat{b}_{\text{Higgs}}$ = Depends only on f_H

Define: $\widehat{b}_{\text{mixed}}$ = Everything else

$$\widehat{b} = \frac{1}{f^2}\left(f_T^2\widehat{b}_{\text{Tensor}} + 2f_H f_T\widehat{b}_{\text{mixed}} + f_H^2\widehat{b}_{\text{Higgs}}\right)$$

Mixed Branch Flow Possibilities



Tensor Branch Flows

$$\mathcal{L}_{\text{dilaton}} \supset \frac{\widehat{b}}{16f^4}\phi^2\Box^2\phi^2 + \frac{3}{16f^8}\left(\Delta a - \tfrac{2}{3}\widehat{b}^2\right)\phi^2\Box^3\phi^2 + \ldots$$

SUSY $\Rightarrow \Delta a = \frac{2}{3}\widehat{b}^2 \geq 0$

6-pt: (similar $\Delta a \propto \widehat{b}^2$) Cordova Dumitrescu Intriligator '15;

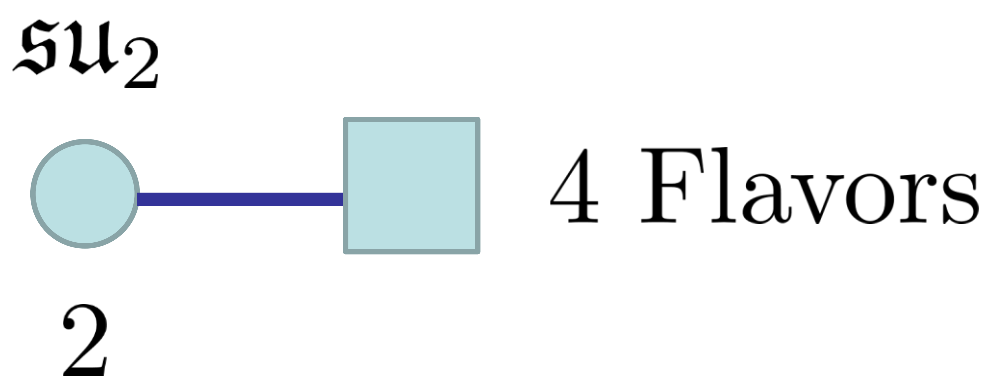
4-pt: JJH Kundu Zhang '21

Note: $\text{Res}_{s=\infty}\left(\frac{\mathcal{M}(\phi\phi\to\phi\phi)}{s^3}\right) = 0$, so $\Delta a > 0$

(effective strings integrated out)

¿Higgs Branch Flows?

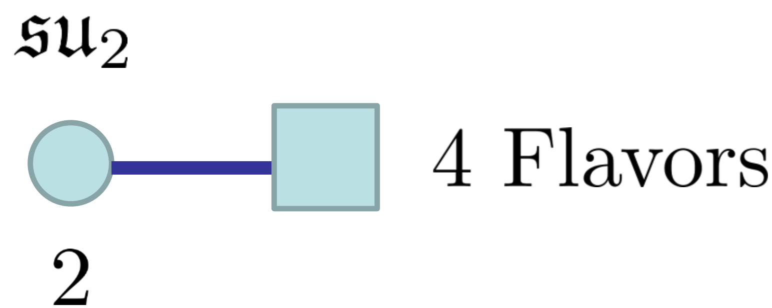
Consider An Example:



4D Reduction is also an SCFT

Higgsing on Tensor Branch

Consider 6D Example:

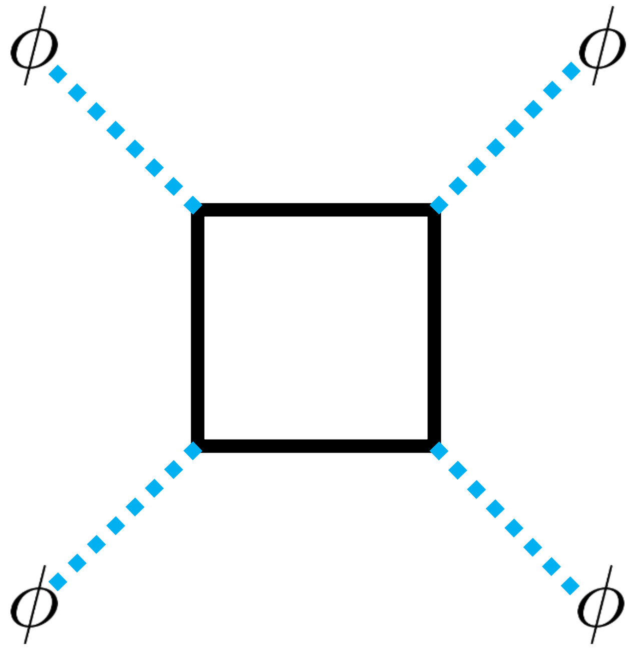


4D Quiver is also an SCFT

Note, in this case:

Higgs Branch(SCFT) = Higgs Branch(Quiver)

Generating $\widehat{b}_{\text{Higgs}}$



4D: $\sim \frac{g^4 s^2}{M_V^4} \sim \frac{s^2}{f_H^4}$

(Independent of g^2)

6D: $\sim \frac{g^4 s^2}{M_V^2} \sim \frac{s^2}{f_T^2 f_H^4}$

(Dependent on $g^2 = f_T^{-2}$)

¿¿¿Generating $\widehat{b}_{\text{Higgs}}$???

General EFT Arguments suggest $\widehat{b}_{\text{Higgs}} = 0$

F-th Geo extends to non-gauge theory cases also

Caveats:

We're working in limit $s \ll f_H^2 \ll f_T^2$

But on pure Higgs Branch, $f_T = 0$...

Caveats to Caveats:

F-th geo fine w/ general f_H, f_T

¿¿¿Conjecture: $\widehat{b}_{\text{Higgs}} = 0$???

Causality would require: $\text{Res}_{s=\infty}\left(\frac{\mathcal{M}(\phi\phi\to\phi\phi)}{s^3}\right) \neq 0$

Suggests Regge Trajectory $\mathcal{M}(\phi\phi \to \phi\phi) \sim s^2$

Can happen IF we have string tension $\to 0$...

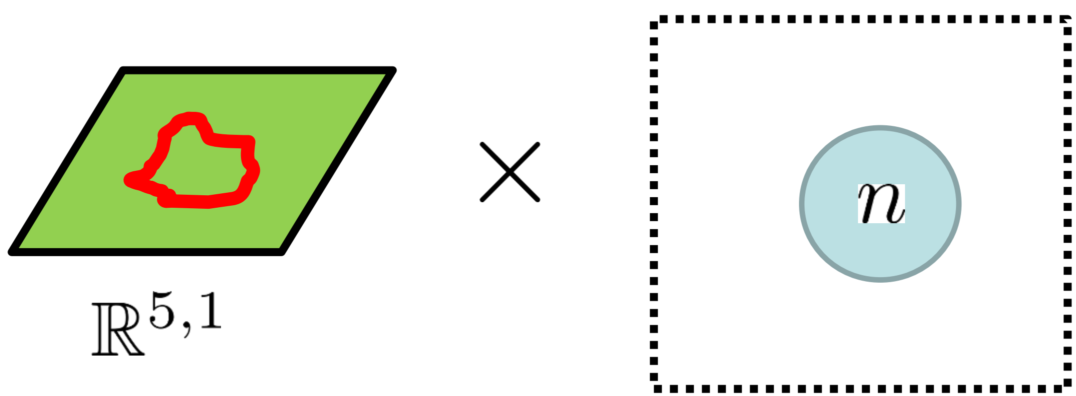
But, this is THE hallmark of CFTs in $D > 4$!?

Hallmark of $D > 4$ SCFTs:

All known (stringy) constructions have:

Effective Strings with Tension $\to 0$ at the CFT

$\exists$ a "spin 2 excitation" (Regge intercept?)



Conclusions / Future

Summary:

- 6D SCFTs Exist (and likely classified)
- 6D RG Flows and Fission / Fusion
- EFT for CFT and R-symm Breaking

Future:

- $\widehat{b}_{\mathrm{Higgs}} = 0$ and SUSY / Eff. Strings???
- Further Constraints on $\mathcal{L}_{\mathrm{dilaton}}$???