Massive Galileons and Vainshtein screening: a numerical perspective

Daniela Saadeh

Institute of Cosmology and Gravitation, Portsmouth (UK)

arXiv:2008.01456
arXiv:2011.07037

with J. Braden, C. Burrage, B. Coltman, B. Elder, T.Padilla, T. Wilson

The problem

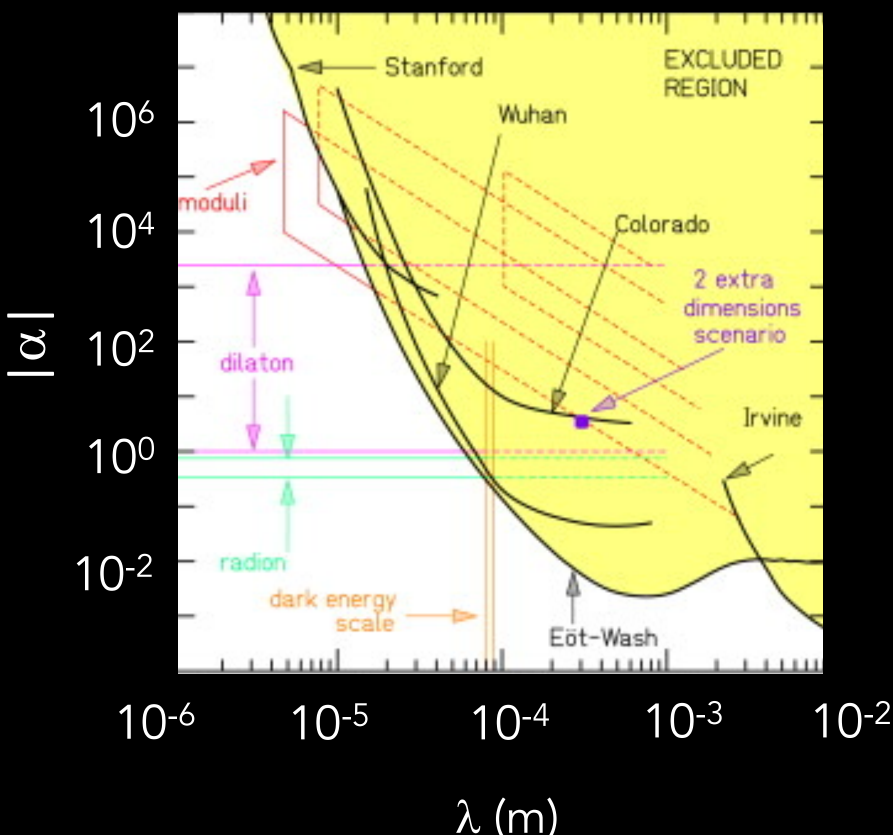
Vainstein screening and UV completion

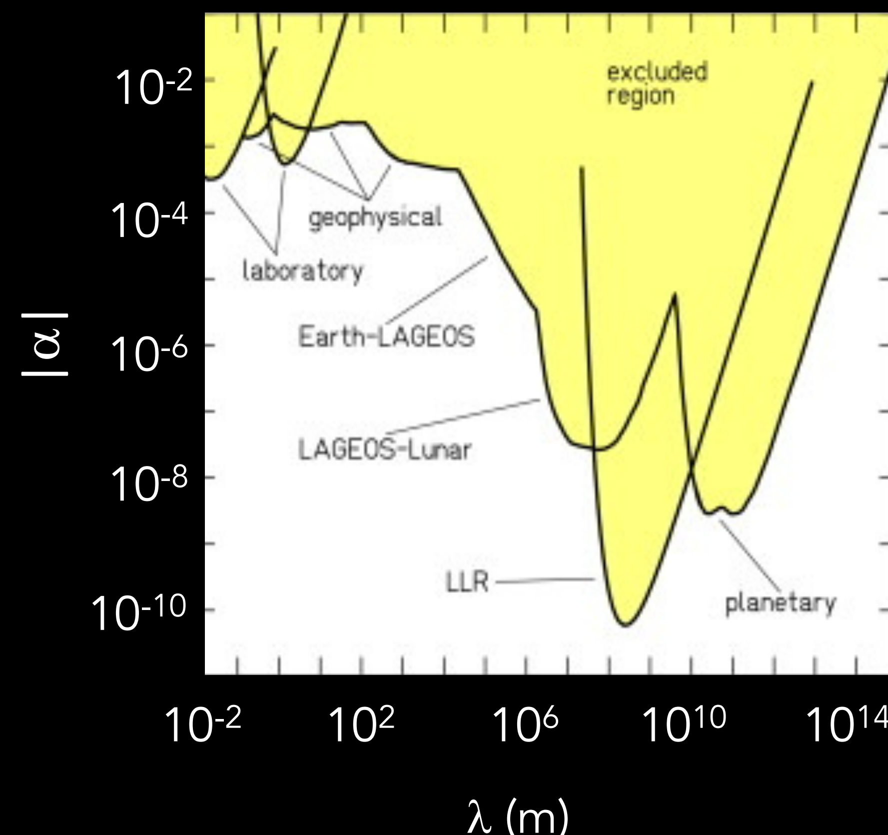
Scalar extensions and 5th forces

- Theories attempting to solve the cosmological constant problem come with extra scalar fields
- Scalar extensions come with extra mediated 5th forces
- Do we see extra forces experimentally?

Scalar extensions and 5th forces

$$V(r) = -\alpha G \frac{m_1 m_2}{r} e^{-\frac{r}{\lambda}}$$





E.G.Adelberger J.H.Gundlach B.R.Heckel S.Hoedl, S.Schlamminger, Progr. in Part. and Nucl. Phys., 62, 1, 2009, 102-134; C. Will, Living Reviews in Relativity 2006.

Scalar extensions and 5th forces

→ *Canonical* scalar fields *with small mass* disfavoured

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\phi}{M}T^{\mu}{}_{\mu}$$

E.G.Adelberger J.H.Gundlach B.R.Heckel S.Hoedl, S.Schlamminger, Progr. in Part. and Nucl. Phys., 62, 1, 2009, 102-134; C. Will, Living Reviews in Relativity 2006.

Scalar extensions and 5th forces

- If scalar field has non-trivial **self-interactions**, 5th force may be *screened* (= suppressed) where it is known to be small…
- … while still being able to mediate a **long-range force** where it can be relevant at current **cosmological scales**

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \ldots)\partial_\mu\partial_\nu\phi - V(\phi) + g(\phi)T^\mu{}_\mu$$

Scalar extensions and 5th forces

$$V(r) = -\alpha G \frac{M_s m}{r} e^{-\frac{r}{\lambda}}$$

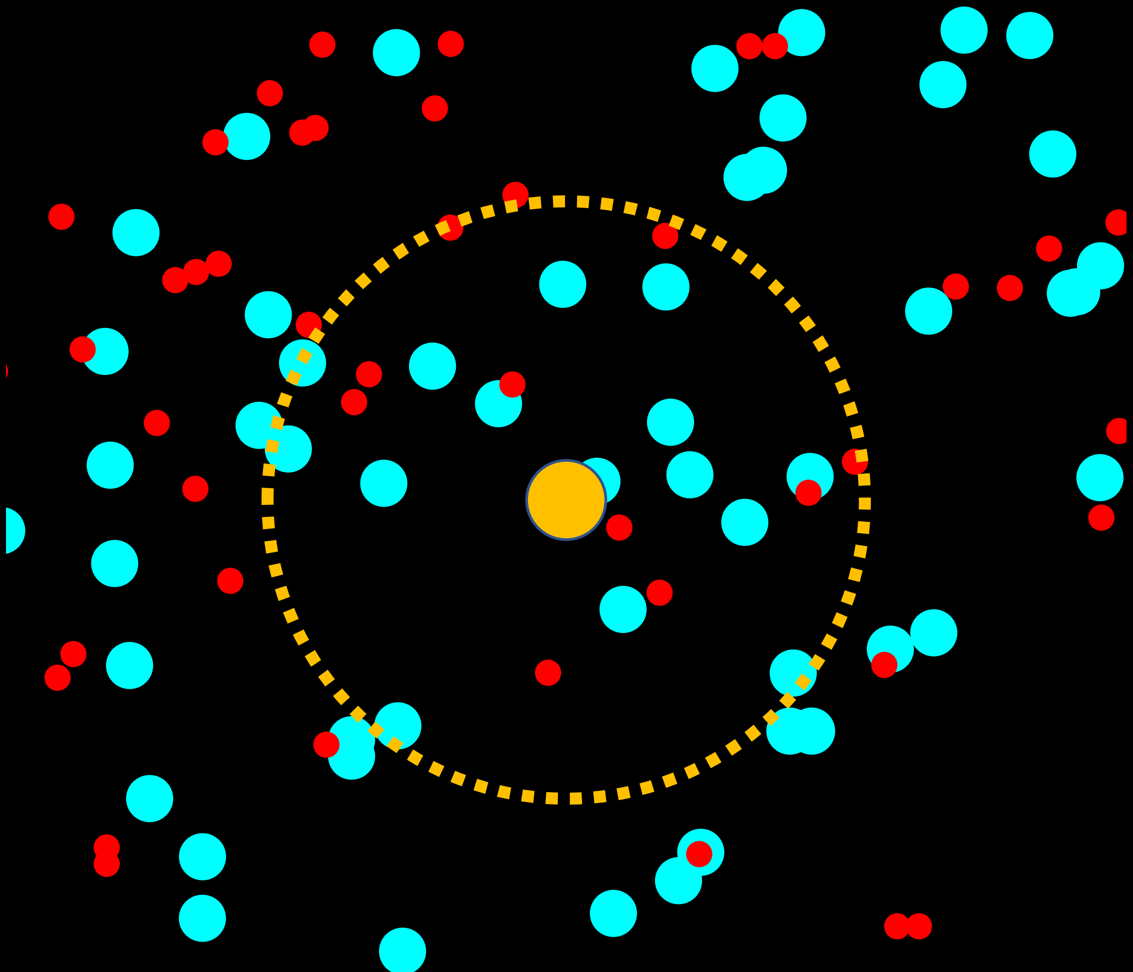
$$V(r) = -\alpha(\phi_{bg}) G \frac{f(M_s) m}{r} e^{-\frac{r}{\lambda(\phi_{bg})}}$$

Debye screening of electromagnetism

The influence of a charged particle in a plasma is only felt up to the Debye length ~1/m_{Debye}

$$m^2_{\mathrm{Debye}} = \frac{8\pi n e^2}{T}$$

n electron density
e electron charge
T plasma temperature



Credits: Clare Burrage

Chameleon screening

Inside galaxy: density high,
Chameleons short-ranged

Outside galaxy: density low,
Chameleons long-ranged

Credits: Ben Elder

Symmetron screening

$$V_{\text{eff}}(\phi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{\lambda}{4!}\phi^4$$

Inside galaxy: density high,
Symmetron decoupled

$$\phi \to 0$$

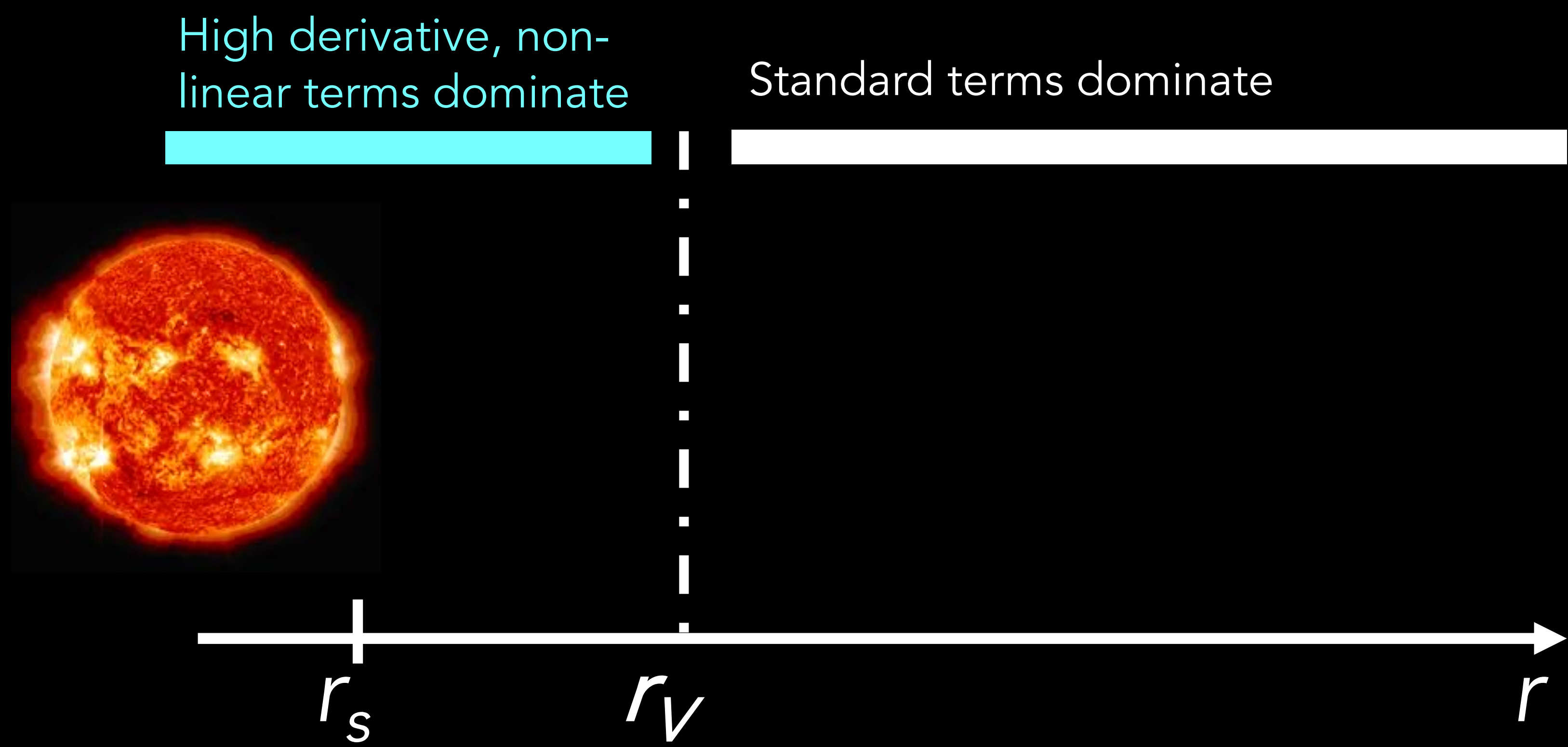
Outside galaxy: density low,
Symmetron coupled

$$\phi \to \mu/\sqrt{\lambda}$$

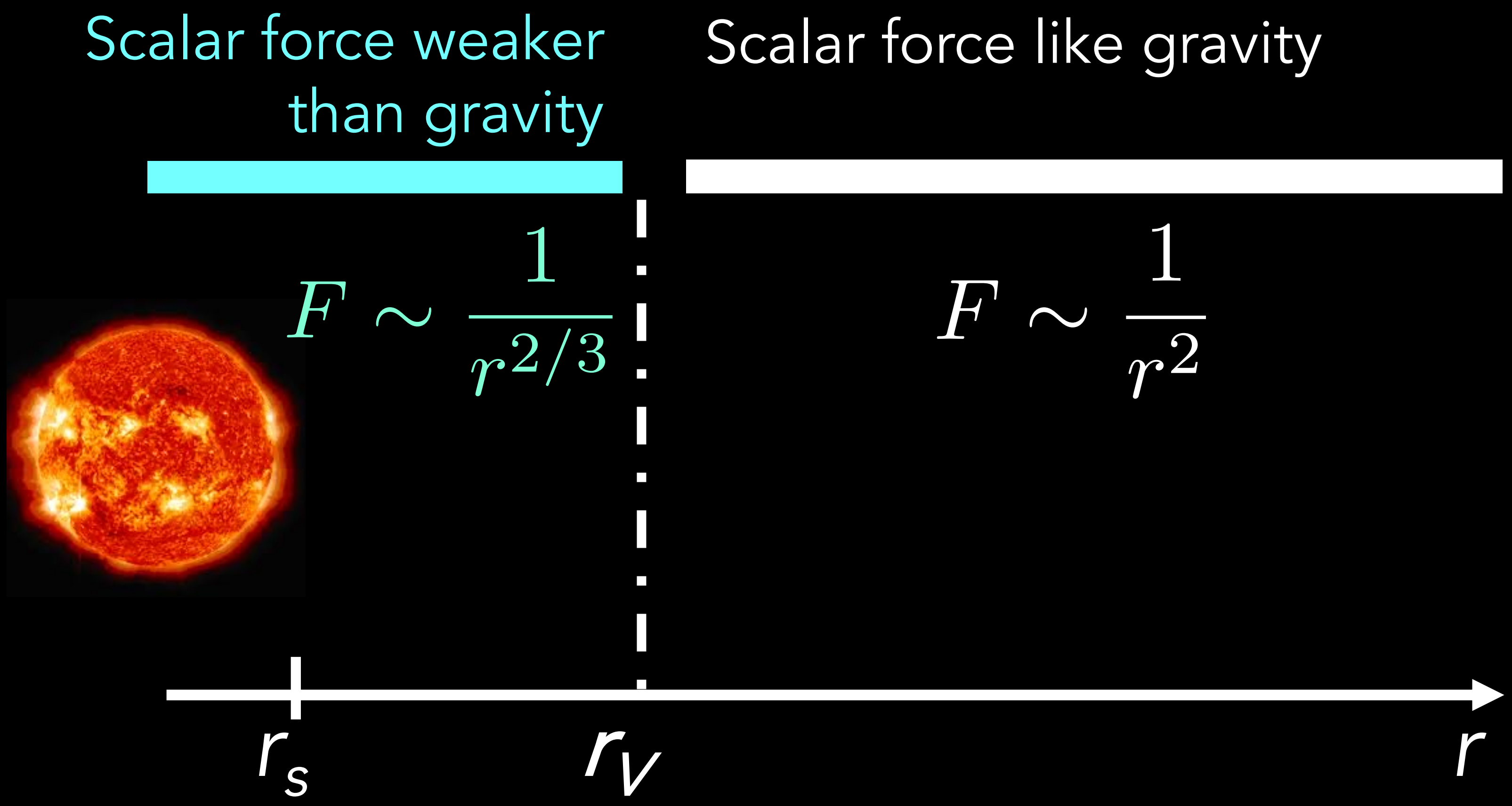
Local matter coupling: $\sim \phi/M^2$

Vainshtein screening

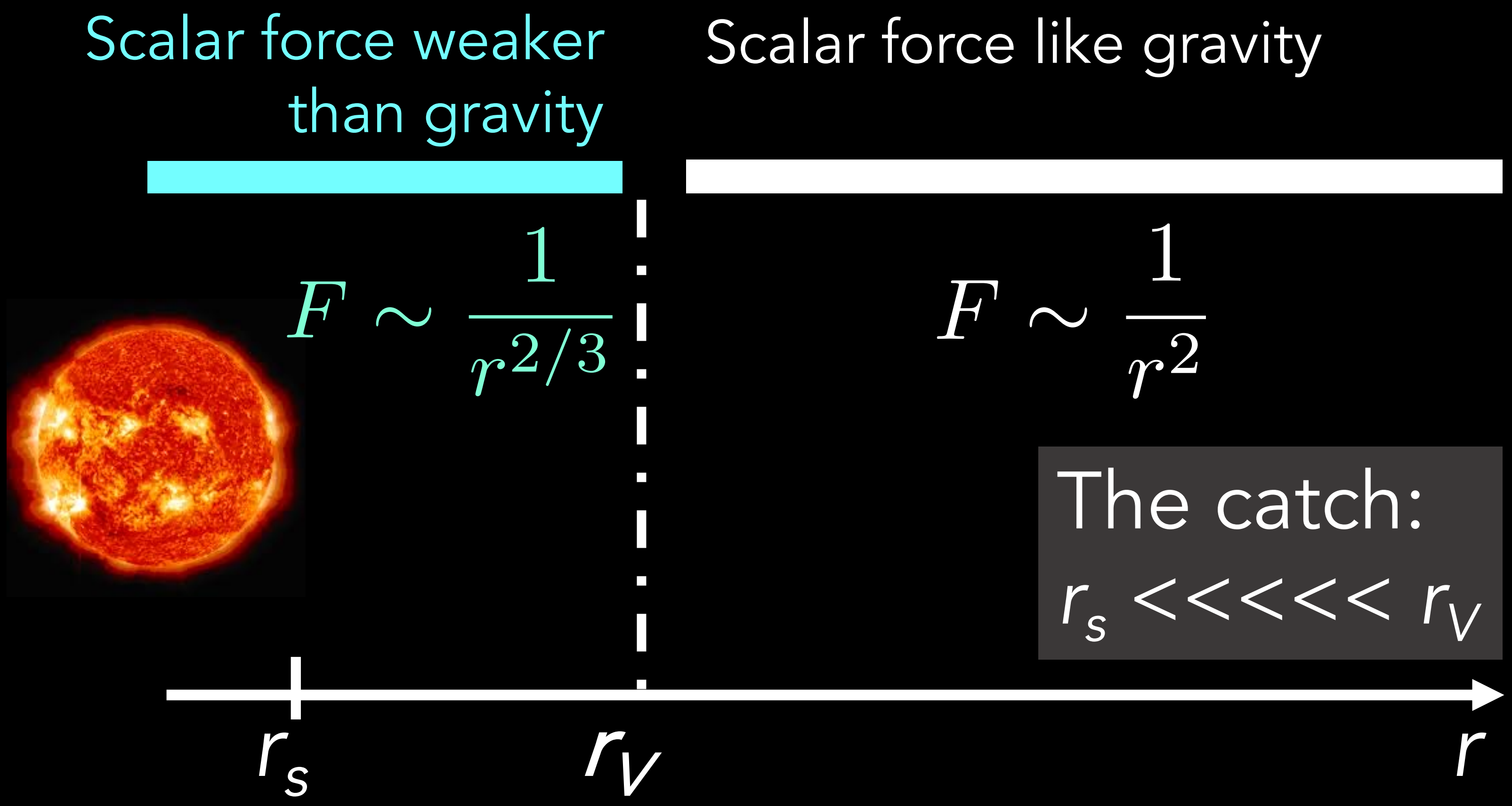
Standard gravity	$\Box\phi = \frac{\rho}{M}$
Cubic Galileons (with Vainshtein)	$\Box\phi + (\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2 = \frac{\rho}{M}$



Vainshtein screening



Vainshtein screening



Vainshtein screening

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \boxed{\frac{1}{2\Lambda^3}(\partial\phi)^2\Box\phi}$$

Typically, Vainshtein screening is taken to be an effective field theory…

…but if specific higher-order kinetic terms are large, then we are out of a perturbative regime…

A. Padilla and I. D. Saltas, JCAP 06 (2018) 039; M. A. Luty, M. Porrati, R. Rattazzi, JHEP 0309 (2003) 029; A. Nicolis and R. Rattazzi, JHEP 0406 (2004) 059

Vainshtein screening

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2\Lambda^3}(\partial\phi)^2\Box\phi$$

why are these higher-order kinetic term macroscopically important but others are not?

A. Padilla and I. D. Saltas, JCAP 06 (2018) 039; M. A. Luty, M. Porrati, R. Rattazzi, JHEP 0309 (2003) 029; A. Nicolis and R. Rattazzi, JHEP 0406 (2004) 059

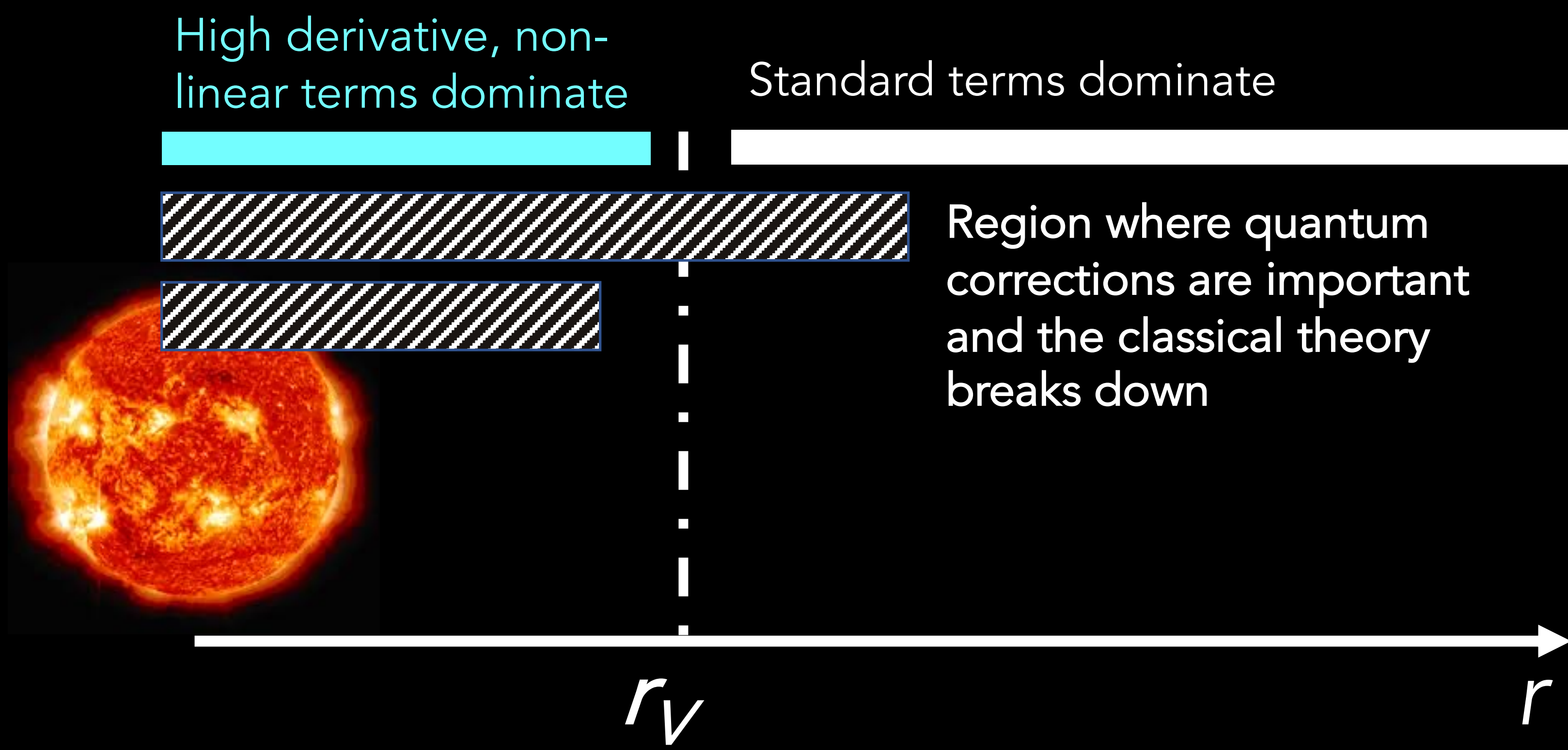
Vainshtein screening

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \boxed{\frac{1}{2\Lambda^3}(\partial\phi)^2\Box\phi}$$

You can only really tell if you know the full UV sector

A. Padilla and I. D. Saltas, JCAP 06 (2018) 039; M. A. Luty, M. Porrati, R. Rattazzi, JHEP 0309 (2003) 029; A. Nicolis and R. Rattazzi, JHEP 0406 (2004) 059

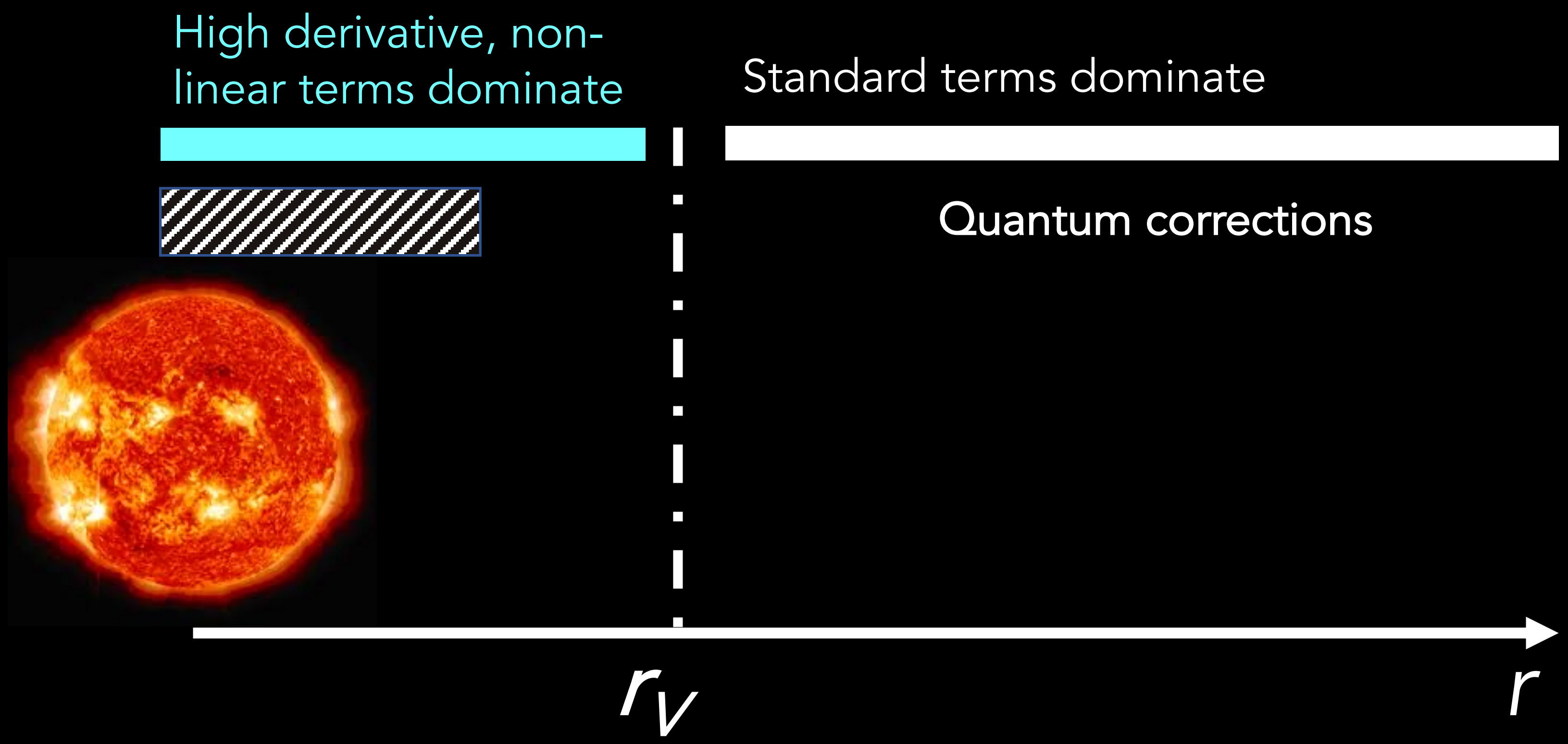
Vainshtein screening – more accurate picture



Hinterbichler, Rev. Mod. Phys. 84, 671-710 (2012)

Vainshtein screening – more accurate picture

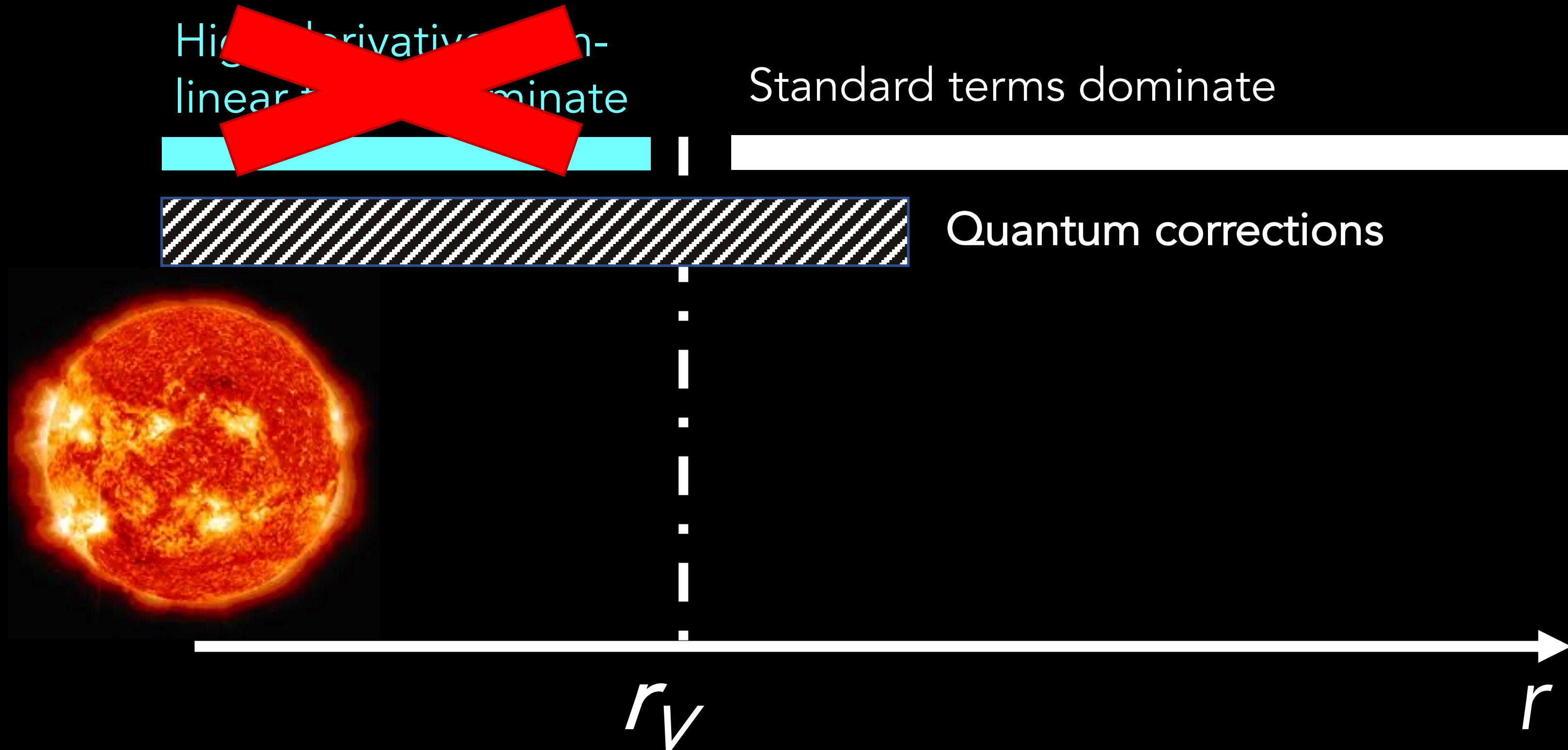
Λ_3 theories: classical nonlinearities kick in before quantum corrections;
however, standard UV completion not possible



Hinterbichler, Rev. Mod. Phys. 84, 671-710 (2012)

Vainshtein screening – more accurate picture

Λ_5 theories: standard UV completion possible;
however, quantum corrections kick in before classical nonlinearities



Hinterbichler, Rev. Mod. Phys. 84, 671-710 (2012)

The goal

- Compare a low-energy theory with Vainshtein screening against its UV completion
- Does the screening survive the extension?

A UV-complete theory of massive Galileons

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial H)^2 - \frac{1}{2}M^2H^2 - \alpha H\Box\phi - \frac{\lambda}{4!}H^4 - \frac{\rho}{M_{pl}}\phi$$

Theory of two coupled massive scalar fields, one self-coupled
Important higher-order kinetic terms

Does it screen?

C. de Rham, S. Melville, A. J. Tolley, and S.-Y. Zhou, JHEP 9 (2017) 72

A UV-complete theory of massive Galileons

$$\begin{cases} \Box\phi - m^2\phi - \alpha\Box H = \frac{\rho}{M_{pl}} \\ \Box H - M^2 H - \alpha\Box\phi - \frac{\lambda}{3!}H^3 = 0 \end{cases}$$

Theory of two coupled massive scalar fields, one self-coupled
Important higher-order kinetic terms

Does it screen?

C. de Rham, S. Melville, A. J. Tolley, and S.-Y. Zhou, JHEP 9 (2017) 72

Low-energy theory

From this high-energy theory, it is possible to obtain the IR theory

$$\Box\pi - m^2\pi - \frac{\lambda\alpha^4}{3!}\frac{\Box(\Box\pi^3)}{M^8} + \ldots = \frac{\rho}{M_{pl}}$$

which relies on the series of operators

$$O_n \propto (-1)^{n+1}\lambda^n\alpha^{2n+1}\frac{\Box(\Box\pi^{2n+1})}{M^{6n+2}}$$

to converge.

Does it look anything like the high-energy theory?

The goal

- Let's compute the scalar force for the high-energy and low-energy theories, to see which one, if any, displays Vainshtein screening

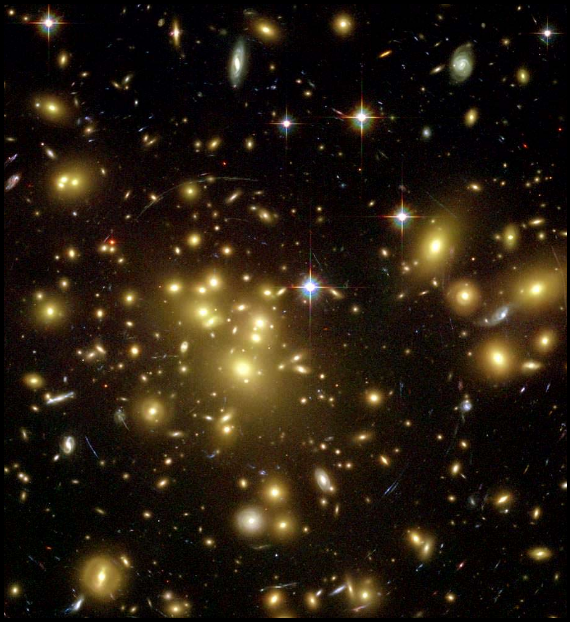
- Let's compute the O_n operators to see if the series converge

C. Burrage, B. Coltman, A. Padilla, DS, T. Wilson, JCAP 02 (2021) 050

The problem

- We need to solve complex equations characterised by large non-linearities, sharp transitions, complex boundary conditions

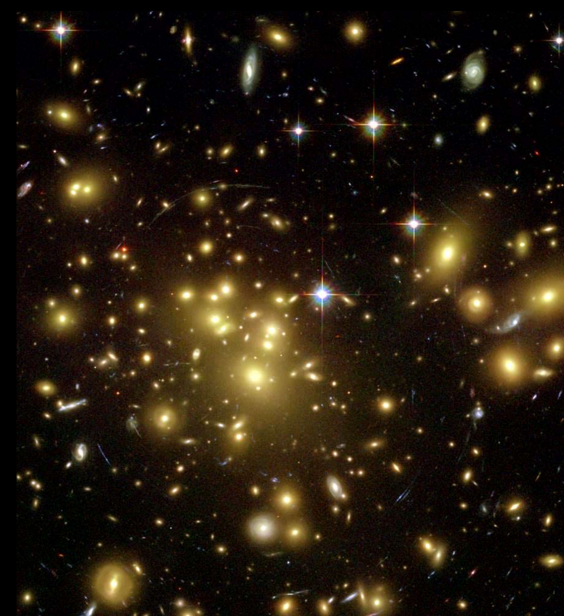
vs



The problem

- We need to solve complex equations characterised by large non-linearities, sharp transitions, complex boundary conditions

Finite element method



Method

φenics: screening with the finite element method

φenics

- Finite-element code for the solution of the full equation of motion of models of screening

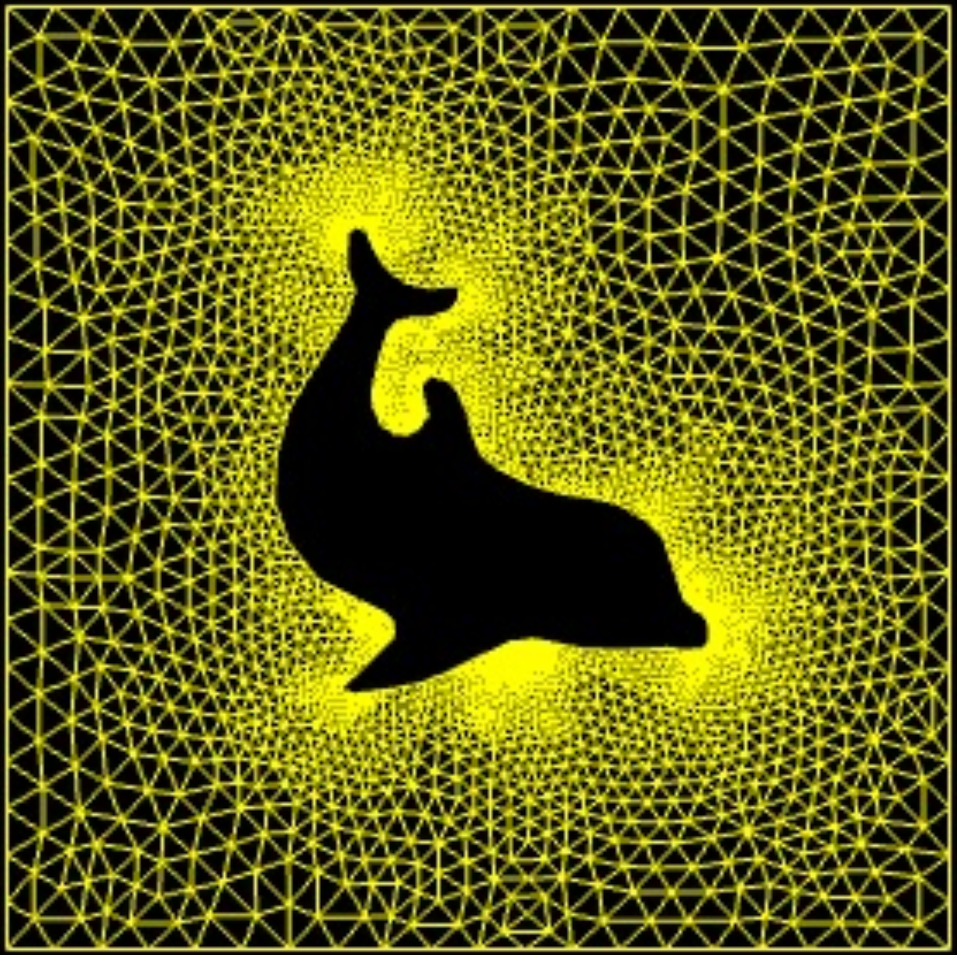
- Builds on FEniCS library

- Computes field profiles, fifth force, (horrible) high-order operators accurately

J. Braden, C. Burrage, B. Elder, DS, JCAP 03 (2021) 010

The finite element method

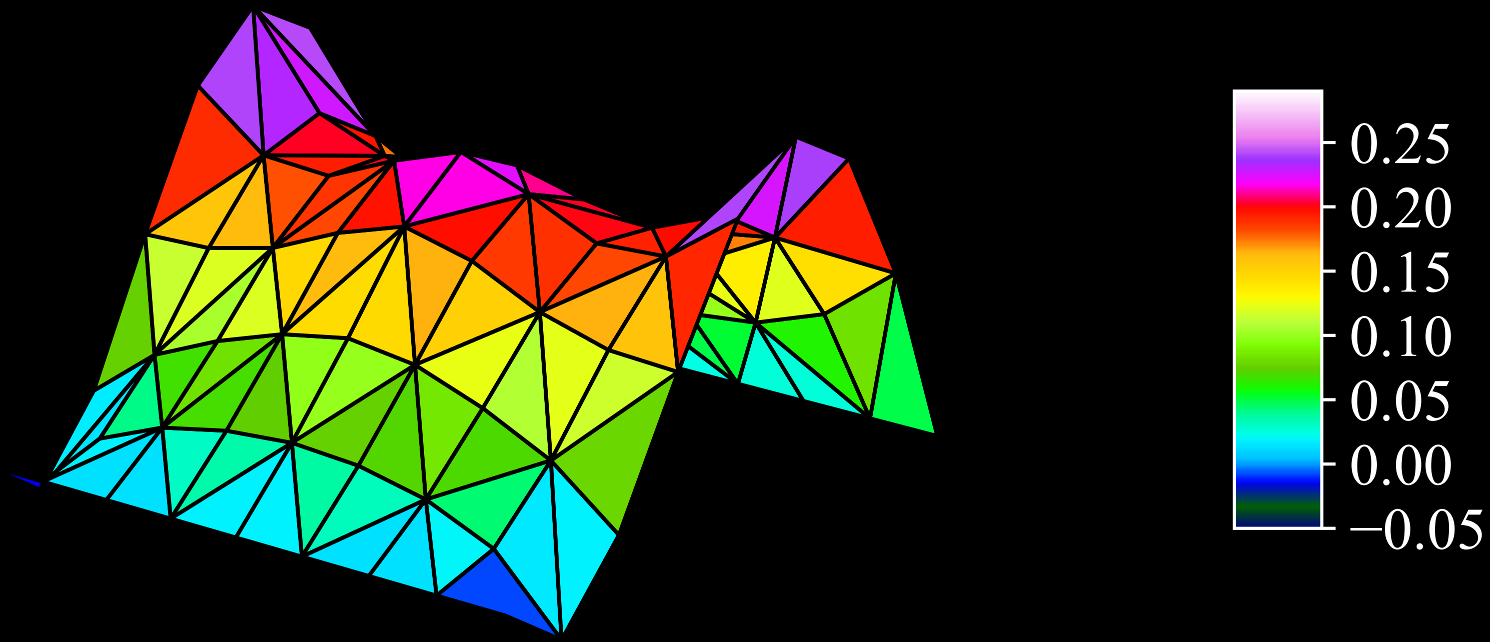
Step (1): discretise domain



Credits: FEniCS project

The finite element method

Functions can be approximated piecewise by polynomials:



$$f = \sum_{i}^{N} f(P_i) e_i$$

P_i vertices
e_i basis polynomials

The finite element method

Step (2): turn original *(strong)* equation into its **weak form**

- Example: Poisson equation

  Strong problem: find u such that

$-\nabla^2 u = f$	in Ω (spatial domain)
$u = u_D$	in $\partial\Omega_D$ (Dirichlet boundary)
$-\frac{\partial u}{\partial n} = g$	in $\partial\Omega_N$ (Neumann boundary),
	n outward normal direction to the boundary

Weak formulation: find u such that

$-< \nabla^2 u, v > = < f, v > \qquad \forall v \in \hat{V},$

$\hat{V}$ space of 'test' functions,

$<\cdot,\cdot>$ inner product

With the inner product $< f, g >= \int_\Omega f g \, dx$:

$$-\int_\Omega \nabla^2 u \, v = \int_\Omega f v \; \forall v \in \hat{V}$$

using integration by parts:

$$\int_\Omega \nabla u \nabla v - \int_{\partial\Omega} \frac{\partial u}{\partial n} v \, ds = \int_\Omega f v$$

The finite element method

Integrating by parts and applying Neumann boundary conditions…

$$\int_\Omega \nabla u \nabla v = -\int_{\partial\Omega_N} gv\, ds + \int_\Omega fv$$

$$a(u,v) := \int_\Omega \nabla u \nabla v$$ bilinear form

$$L(v) := -\int_{\partial\Omega_N} gv\, ds + \int_\Omega fv$$ linear form

The finite element method

Step (3): turn complicated equation into **algebraic system**

Because a and L are bilinear and linear, respectively, they only need to be defined on basis vectors:

find u such that:

$$a(u, v) = L(v) \quad \forall v \in \hat{V}$$

Expanding in the discretised basis:

$$a(u, e_i) = L(e_i) \quad \forall\, i = 1, \dots, N$$

$$\sum_{j=1}^{N} u_j\, a(e_j, \hat{e}_i) = L(\hat{e}_i) \quad \forall\, i = 1, \dots, N \quad \Rightarrow \quad \boxed{AU = b}$$

Coefficients of u in discretised basis

The finite element method

Step (4): if equation is nonlinear, solve iteratively using e.g. Newton's method

$$F(u) = 0$$

$$\Downarrow$$

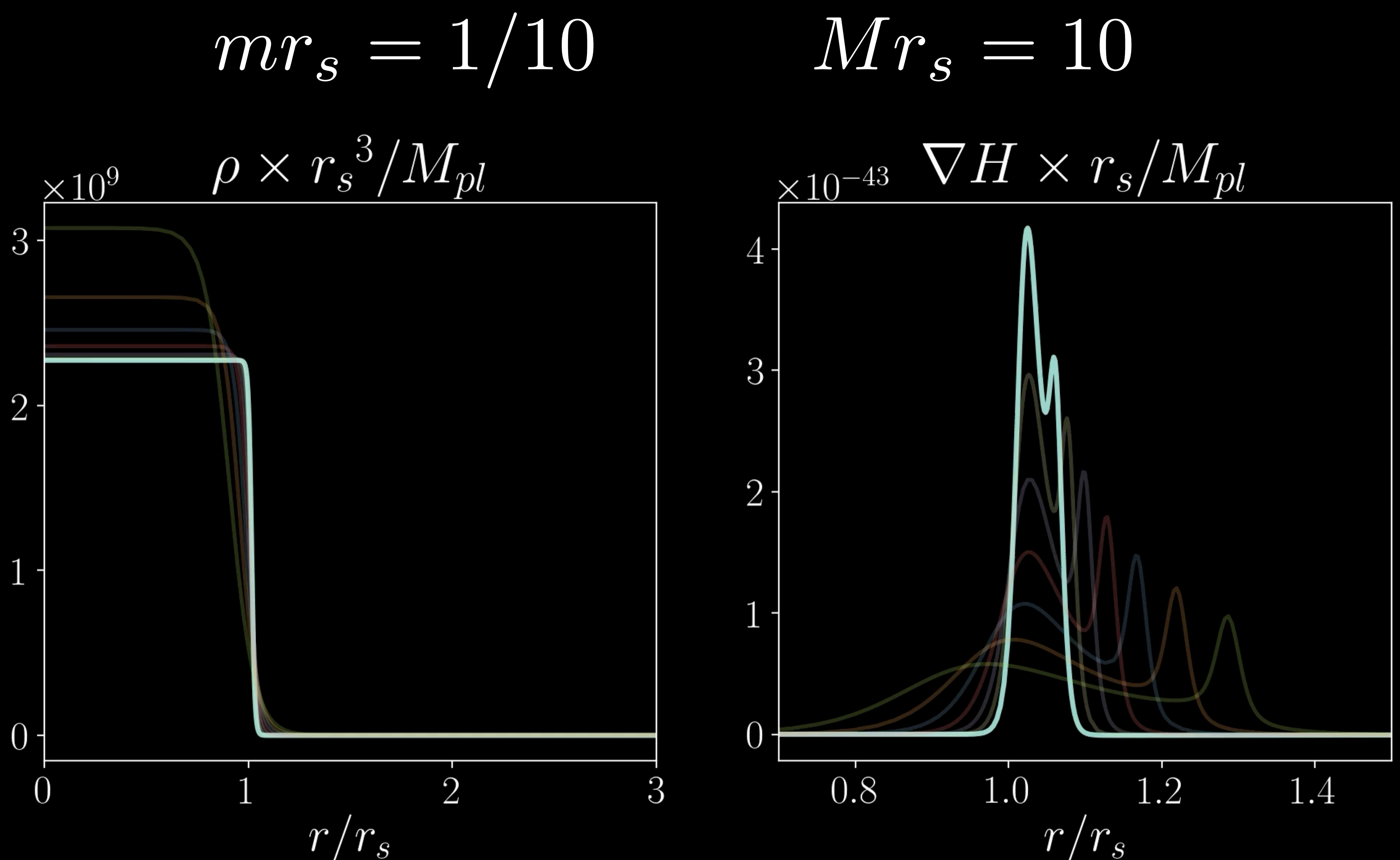
$$F(u_k) + J(u_k)(u - u_k) = 0$$

Starting from some initial guess u_k,
Then substituting $u \rightarrow u_k$ until convergence

Prelude to results

φenics: screening with the finite element method

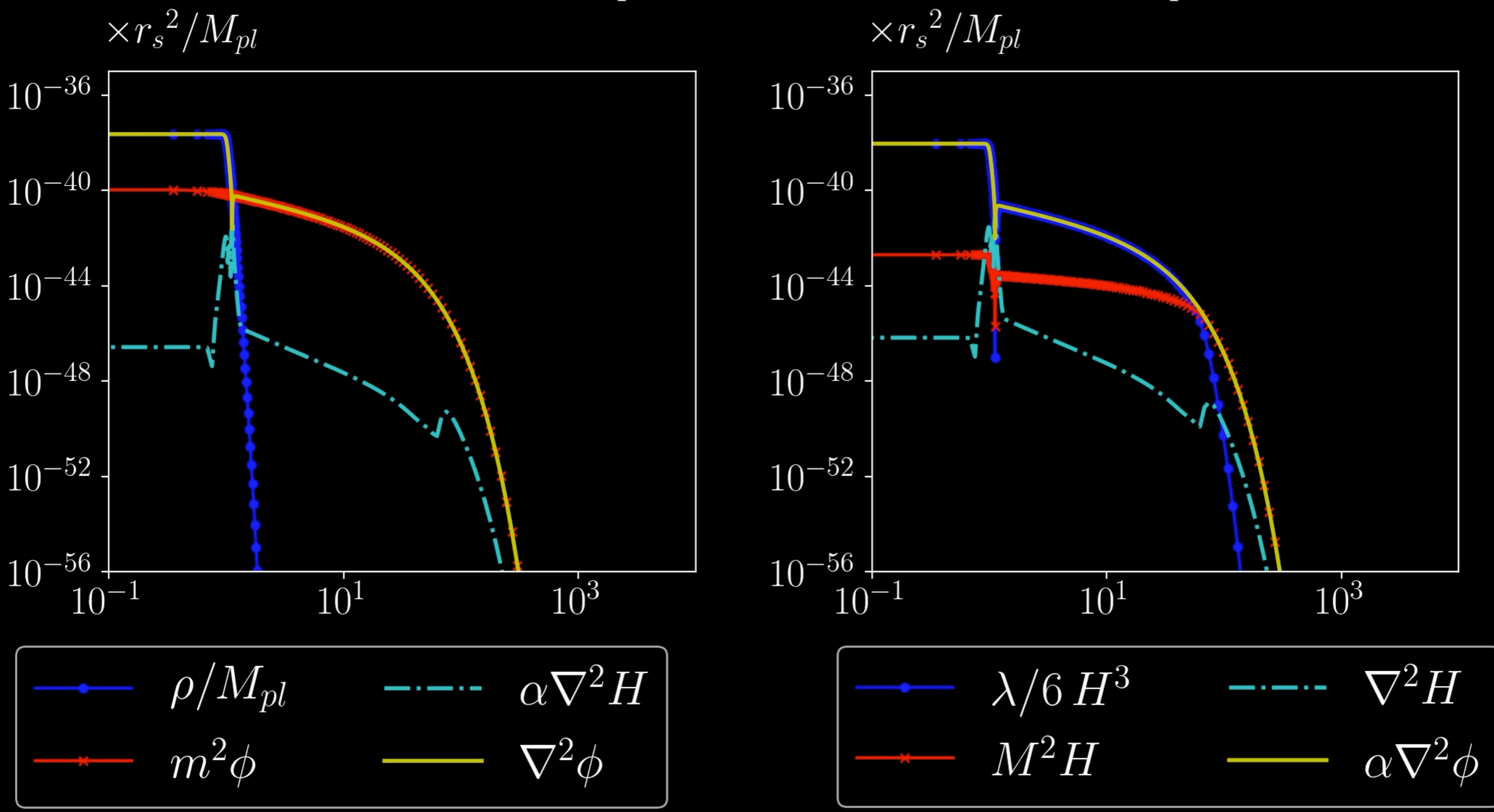
Features at the source-vacuum transition



J. Braden, C. Burrage, B. Elder, DS, JCAP 03 (2021) 010

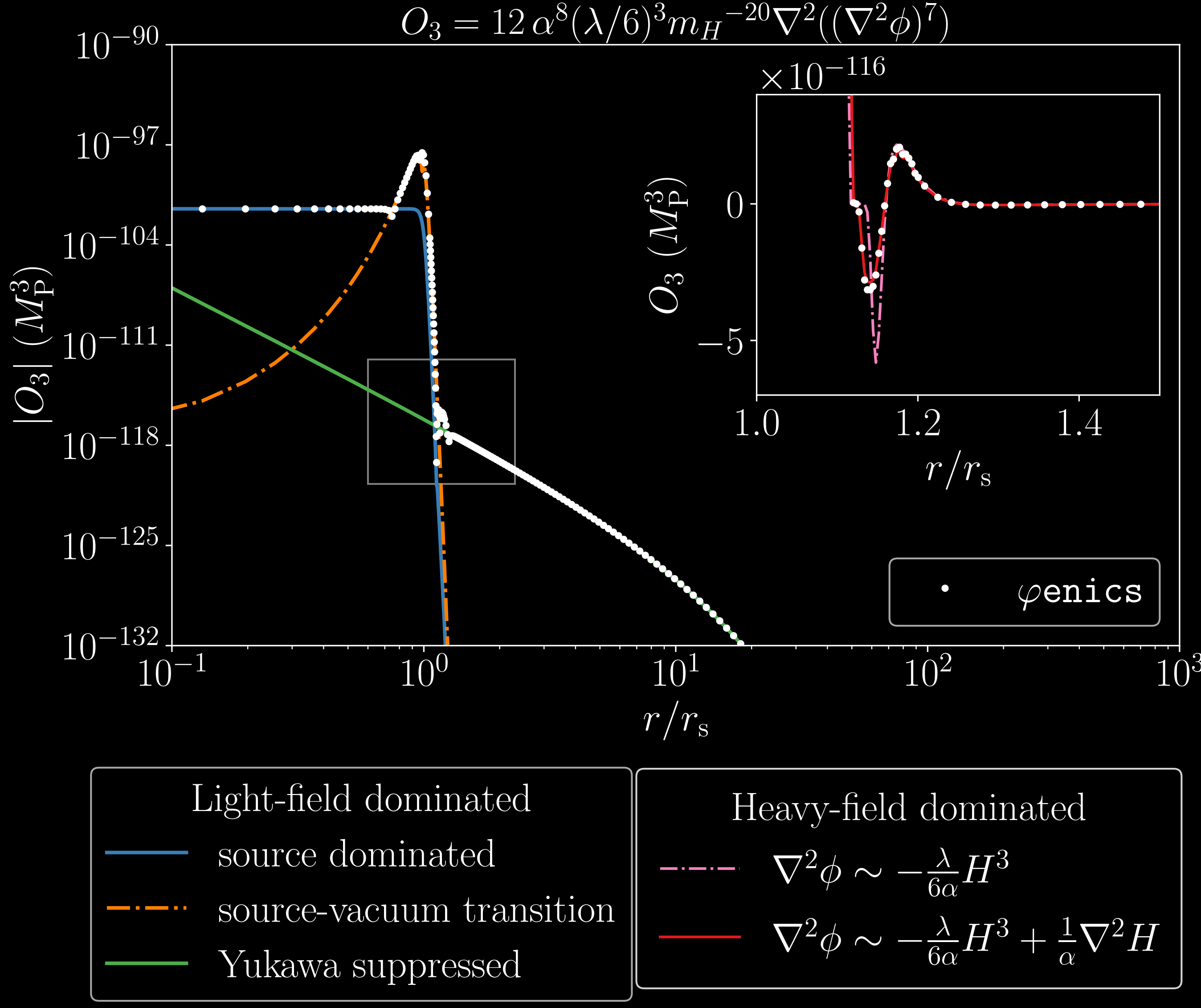
Terms in the equations across all regimes

$$m = 10^{-48} M_{pl} \qquad M = 3.2 \times 10^{-47} M_{pl}$$



J. Braden, C. Burrage, B. Elder, DS, JCAP 03 (2021) 010

High order operators

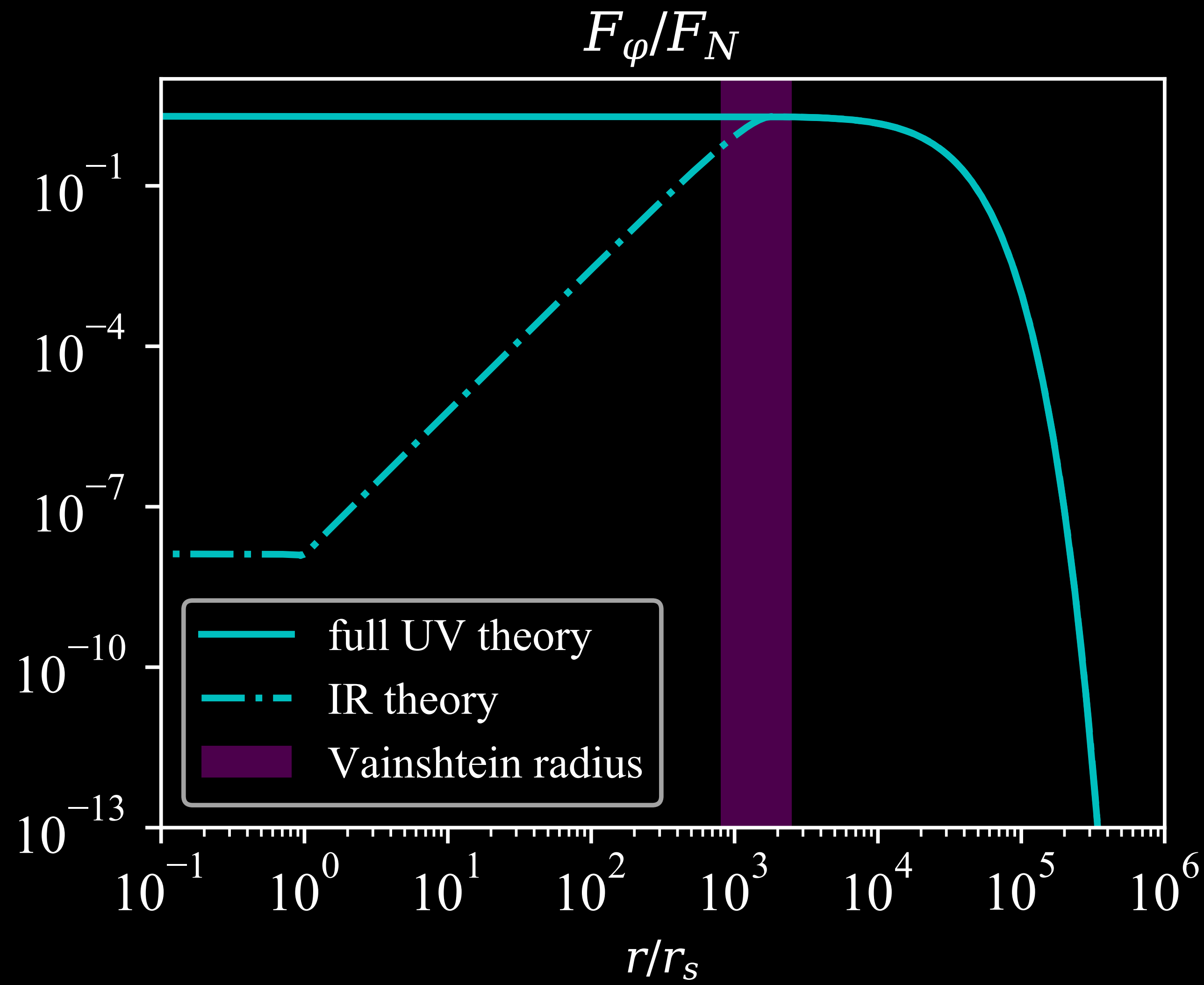


J. Braden, C. Burrage, B. Elder, DS, JCAP 03 (2021) 010

Results

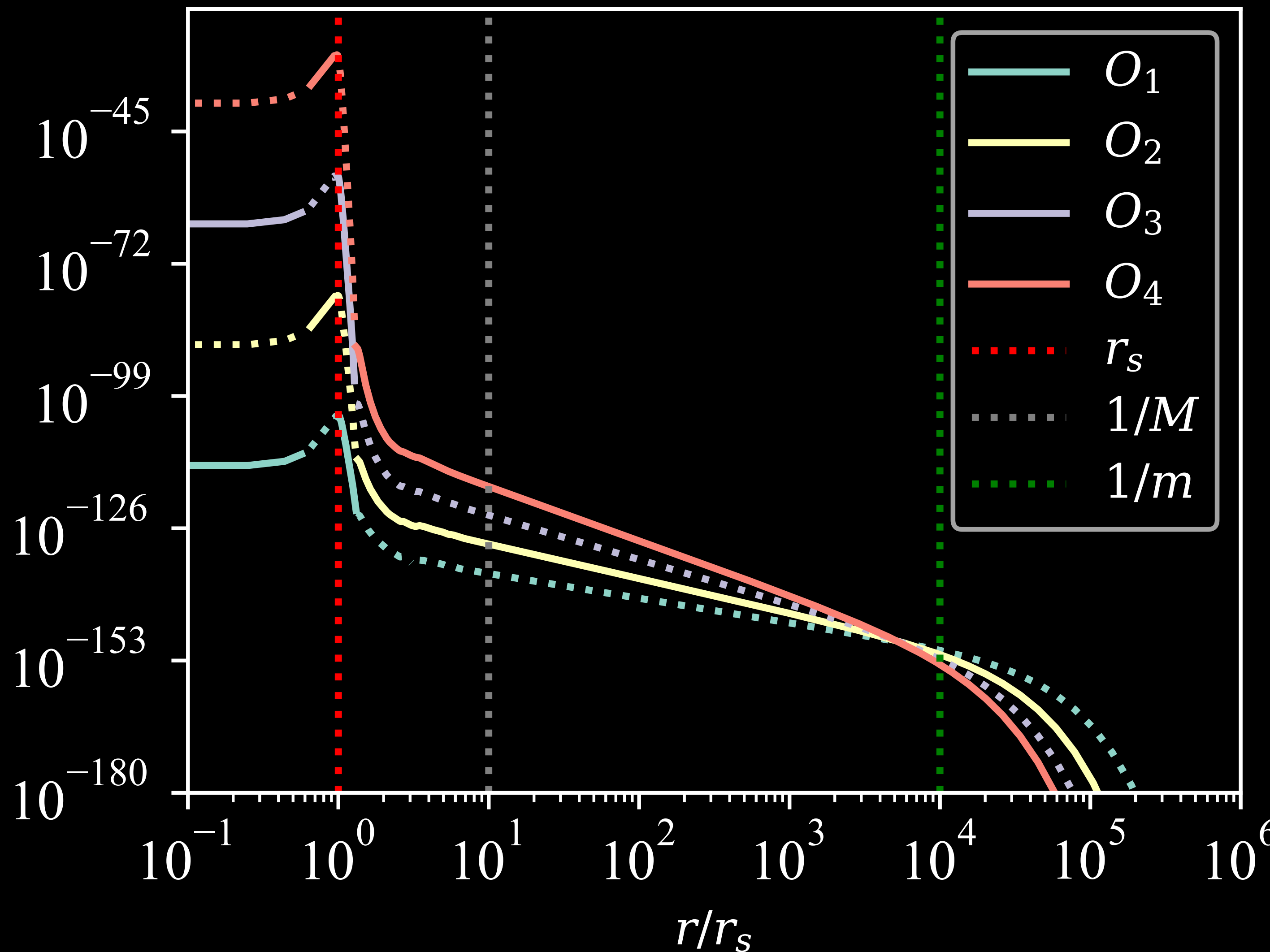
Vainstein screening and UV completion

Vainshtein screening – is it viable?



C. Burrage, B. Coltman, A. Padilla, DS, T. Wilson, JCAP 02 (2021) 050

Vainshtein screening – is it viable?



C. Burrage, B. Coltman, A. Padilla, DS, T. Wilson, JCAP 02 (2021) 050

Summary and conclusions

Summary and conclusions

- Vainshtein screening is an effective mechanism to hide the fifth force in high-density regions; it is characterised by complicated equations that are impossible to solve analytically and difficult to solve numerically;

- We developed φenics, finite-element code for the full numerical solution of equations of screening

- We compared a theory of massive Galileons against its UV completion: the IR theory screens, the UV theory doesn't!

- It is unclear Vainshtein screening can be invoked within the limits of validity of the theories that make use of it


Thank you for
your attention!