

Study of $B^+ \rightarrow \pi^+\pi^0\pi^0$ at Belle

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Kavli IPMU Postdoc Colloquium

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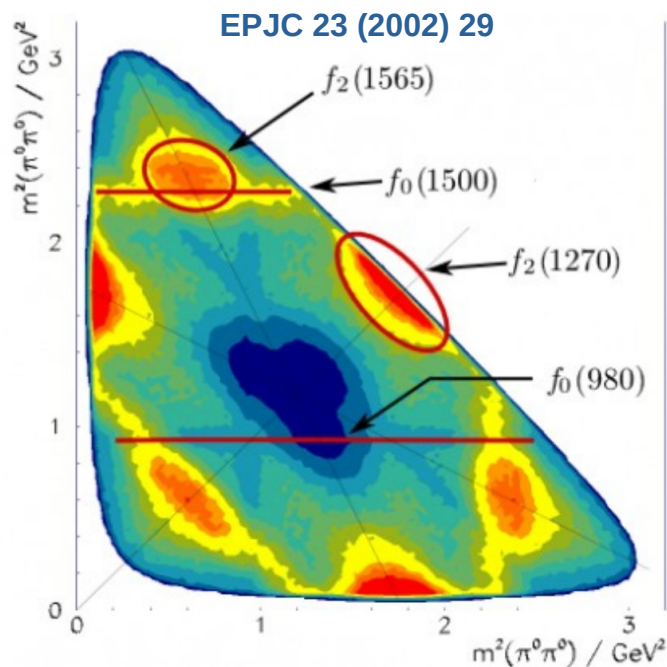
- Introduction.
- Study with Monte Carlo (MC):
 - Event selection and signal B reconstruction.
 - Background: continuum and $B\bar{B}$.
- Signal extraction and sPlot.
 - Fitter on sWeighted 2D $M_{\pi\pi}$ histogram: sub-decay and localized A_{CP} .
- Physics results.
- Summary

- Charmless three-body B decays are useful to study the properties of the weak interaction in the quark sector.
 - Complicated structure of decay amplitudes, interference in between, and CP asymmetry localized in multi-body phase space.
 - Dalitz plot: search for intermediate resonances and localized A_{CP} in the three-body phase space.
 - Result is helpful to constrain magnitudes and phases of the CKM matrix elements. For instance, $B \rightarrow \rho\pi$ for $\phi_2(\alpha)$ and also $B^+ \rightarrow \chi_{c0}\pi^+$ for $\phi_3(\gamma)$.
- Similar measurement on $B^+ \rightarrow \pi^+\pi^-\pi^+$ by BABAR and LHCb.
 - $BF = (15.2 \pm 0.6 \pm 1.2 \pm 0.4) \times 10^{-6}$: Reference for our blinded analysis.
 - Full amplitude analysis.
- Upper limit of $B^+ \rightarrow \pi^+\pi^0\pi^0$ was reported: 8.9×10^{-4} at 90 C.L. by CLEO.
 - [PRD 79, 072006 \(2009\)](#) [PRD 101, 012006 \(2020\)](#)
[PLB 241 278-282 \(1990\)](#)
- $B^+ \rightarrow \rho(770)^+\pi^0$: $(10.9 \pm 1.4) \times 10^{-6}$, by Belle and BABAR.
 - [PRL 94 031801 \(2005\)](#) [PRD 75 091103 \(2007\)](#)
 - Majority of the $B^+ \rightarrow \pi^+\pi^0\pi^0$ decays.

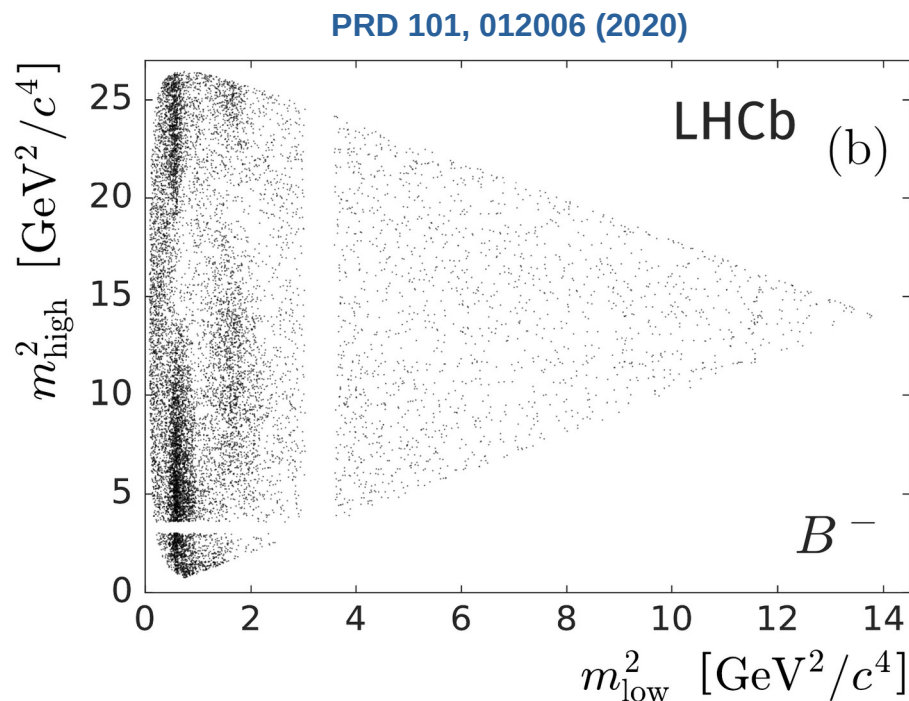
Dalitz plot

- A method to unfold the three-body decay phase space visually.
 - Structure within this decay: amplitudes, spin, and interference can be clearly seen.
- $B \rightarrow 1, 2, 3$.
 - One of the representation: m_{12}^2 vs m_{13}^2 .

Dalitz plot of $p\bar{p} \rightarrow \pi^0\pi^0\pi^0$ by Crystal Ball



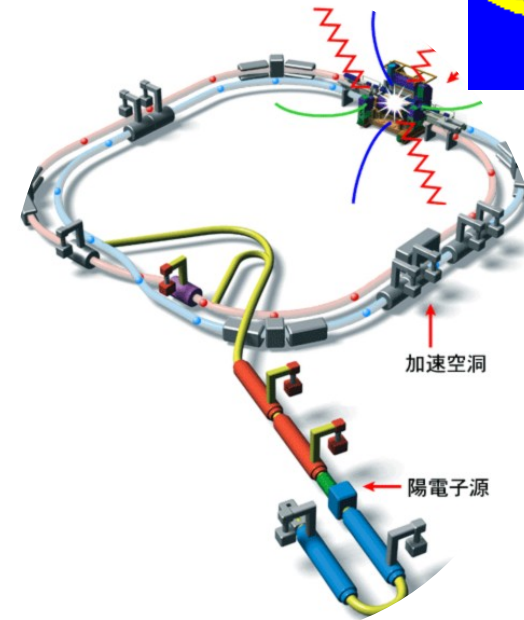
Dalitz plot of $B^- \rightarrow \pi^-\pi^+\pi^-$ by LHCb



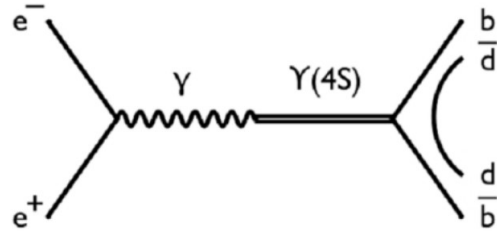
KEKB collider



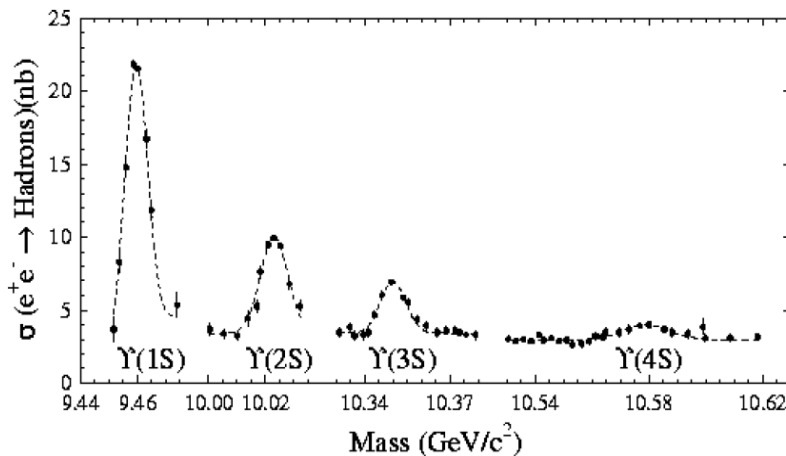
- An asymmetric energy e^+e^- collider at KEK.
 - LER(e^+) 3.5 GeV.
 - HER(e^-) 8 GeV.
 - Crossing angle: ± 11 mrad.



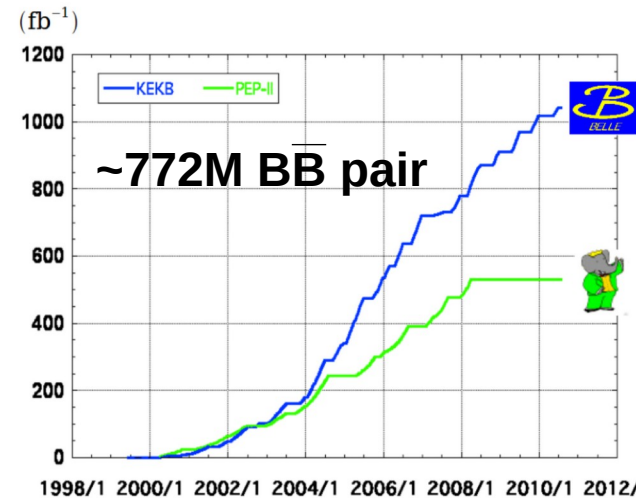
- Target: $e^+e^- \rightarrow Y(4S) \rightarrow B\bar{B}$ for B physics.



- Main background for B decay study:
 $e^+e^- \rightarrow q\bar{q}$, $q=u,d,s,c$ with 3 times larger cross section.



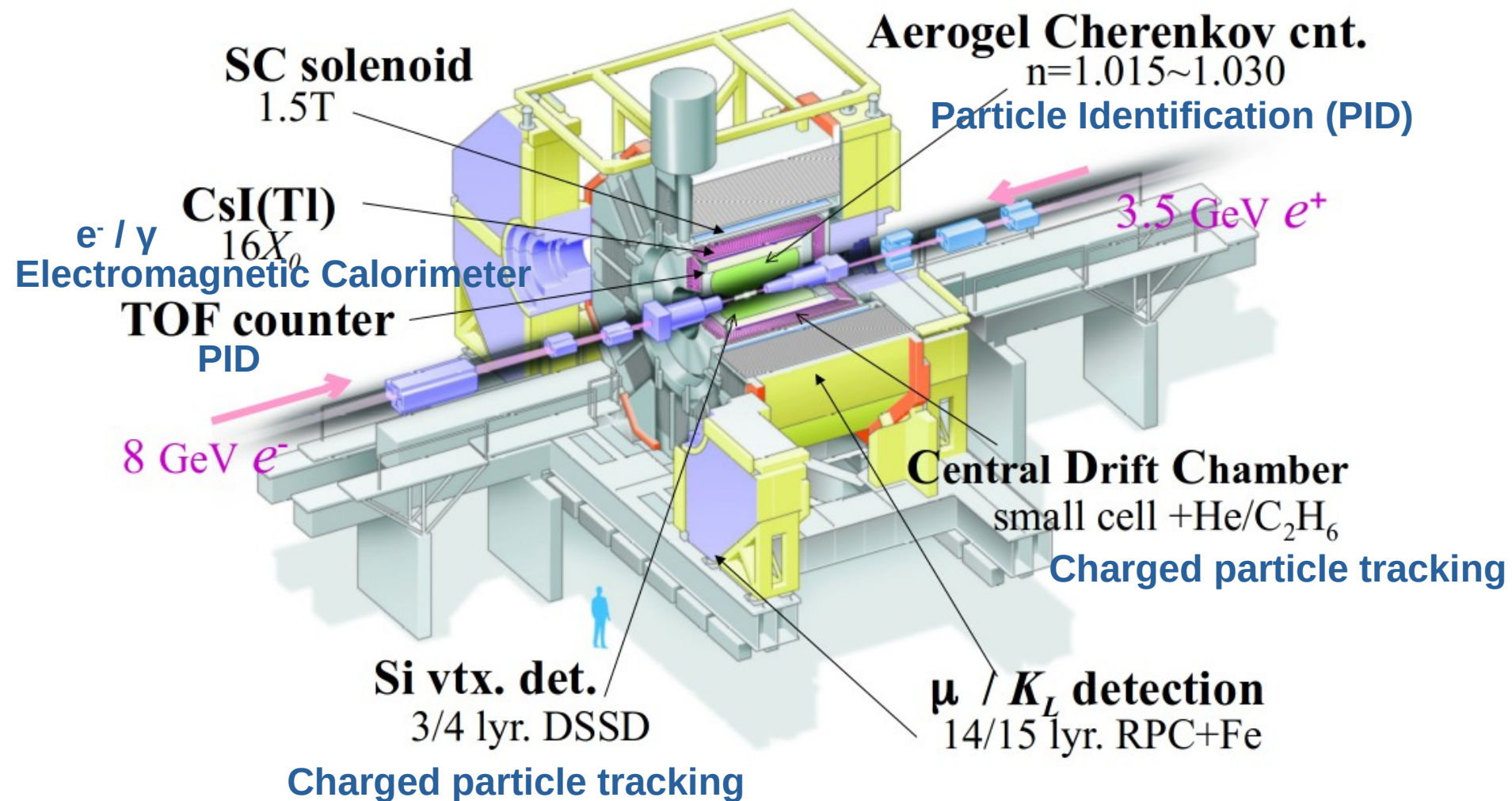
<http://www.lns.cornell.edu/public/lab-info/upsilon.html>



> 1 ab⁻¹
On resonance:
 $Y(5S)$: 121 fb⁻¹
 $Y(4S)$: 711 fb⁻¹
 $Y(3S)$: 3 fb⁻¹
 $Y(2S)$: 25 fb⁻¹
 $Y(1S)$: 6 fb⁻¹
Off reson./scan
 ~ 100 fb⁻¹

~ 550 fb⁻¹
On resonance:
 $Y(4S)$: 433 fb⁻¹
 $Y(3S)$: 30 fb⁻¹
 $Y(2S)$: 14 fb⁻¹
Off resonance:
 ~ 54 fb⁻¹

Belle detector



Event sample with Monte Carlo

- Blinded analysis: Prestudy with Monte Carlo (MC) samples for signal and backgrounds.
 - Signal: assumed BF (1.5×10^{-5}), reconstruction efficiency by MC → Expected number of events in data.
- Event samples categories expected to be seen in data:

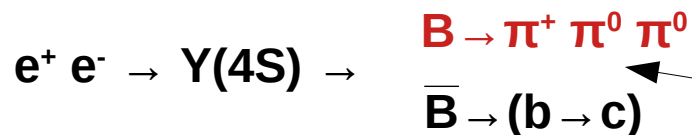
Signal	$e^+ e^- \rightarrow Y(4S) \rightarrow$ $\mathbf{B} \rightarrow \pi^+ \pi^0 \pi^0$ $\bar{\mathbf{B}} \rightarrow \text{randomly}$
Continuum background ($q\bar{q}$)	$e^+ e^- \rightarrow q\bar{q}, \quad q = u, d, s, c$
$B\bar{B}$ background	$e^+ e^- \rightarrow Y(4S) \rightarrow$ $\mathbf{B} \rightarrow \text{randomly}$ $\bar{\mathbf{B}} \rightarrow \text{randomly}$

Event sample with Monte Carlo (cont'd)

- In the signal reconstruction, there are truly reconstructed candidates ("true") and wrongly reconstructed candidates due to the decay of the other B:
 - Self-crossfeed (SCF).
- B decay backgrounds:
 - "Generic": $b \rightarrow c$. Larger BF. Well-measured.
 - "Rare": $b \rightarrow u, d, s$. Smaller BF (usually $< 10^{-5}$).

Within signal MC,
there would be wrong
reconstruction of
 $\pi^+ \pi^0 \pi^0$ candidates:
**Self-crossfeed
(SCF)**

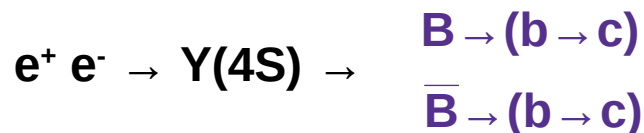
Signal



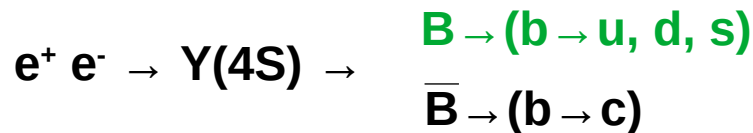
**Continuum background
($q\bar{q}$)**



Generic $B\bar{B}$ background



Rare $B\bar{B}$ background



Analysis overview

Blinded with Monte Carlo (MC)

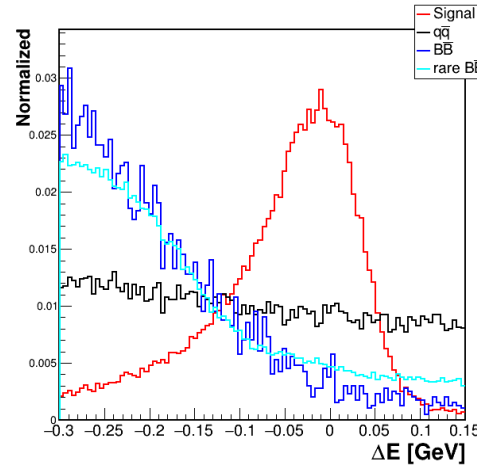
Assume $BF = 15 \times 10^{-6}$

- **Signal $B^+ \rightarrow \pi^+ \pi^0 \pi^0$ reconstruction:**
 - π^+ : Tracking & PID.
 - $\pi^0 \rightarrow \gamma\gamma$: Clusters in ECL (γ).
 - Details of signal study in Backup.
- **Background study:**
 - Continuum $e^+e^- \rightarrow q\bar{q}$.
Suppression with multivariate tools.
 - $B\bar{B}$.

Major variables to identify signal B (in CM frame)

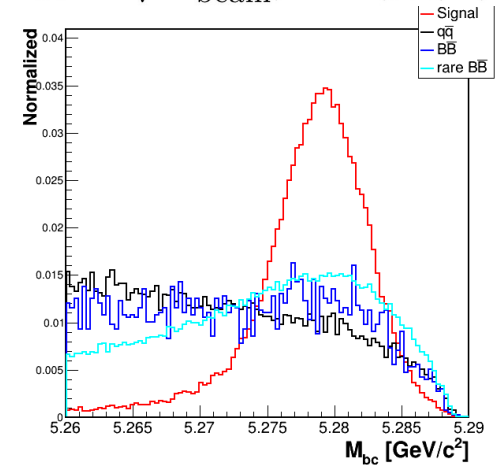
Energy difference

$$\Delta E \equiv E_B - E_{\text{beam}}$$



Beam-energy constrained mass

$$M_{bc} \equiv \sqrt{E_{\text{beam}}^2/c^4 - |\vec{p}_B/c|^2}$$

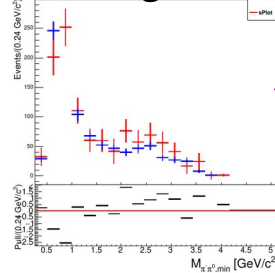


- **sPlot technique** M. Pivk and F.R. Le Diberder, Nucl. Instrum. Methods Phys. Res., Sect. A 555, 356 (2005)

- Use the sWeight from Laura++ to isolate signal in other dimensions.
- Study Dalitz plot and $M_{\pi\pi}$.

- **2D fit on sWeights $M_{\pi\pi}$**

Isolate signal on Y

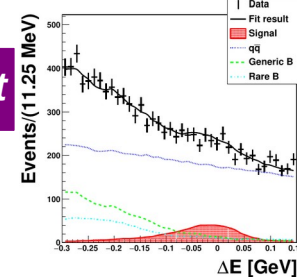


sPlot

- **Signal extraction by data fit**

- 3D extended unbinned likelihood fit by Laura++.

Fit on X

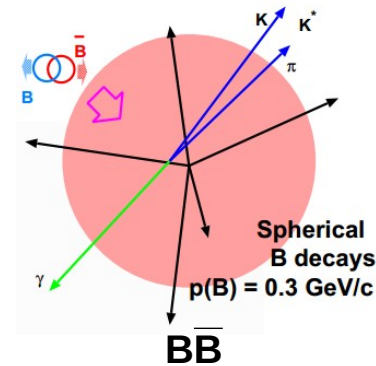
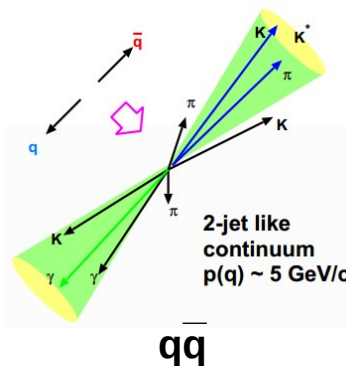


Sub-decays' BF & Localized A_{CP}

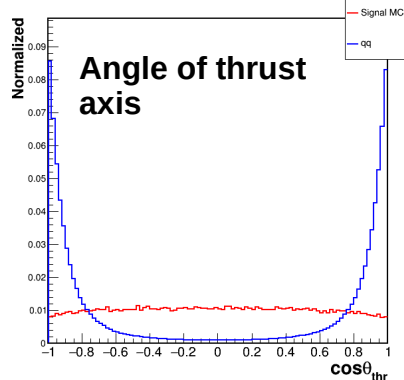
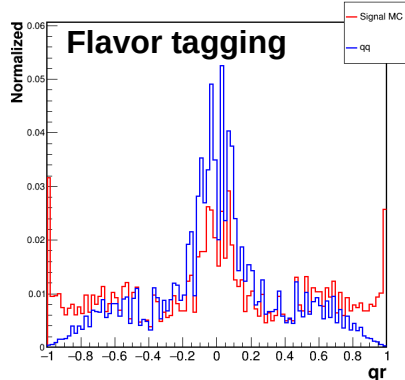
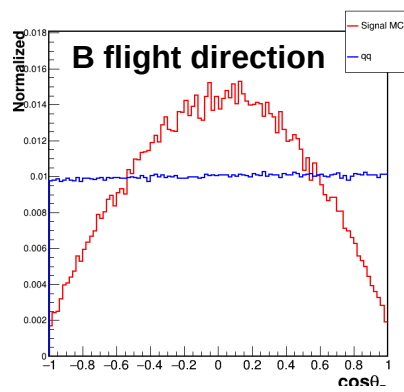
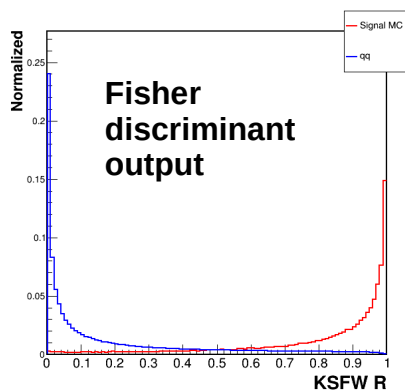
Total BF & Total A_{CP}

Continuum Background

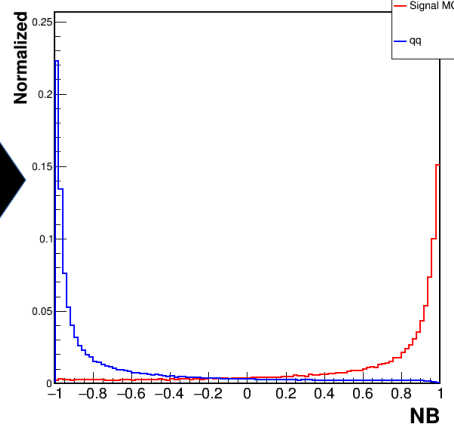
- Continuum production: $e^+e^- \rightarrow q\bar{q}$, $q=u,d,s,c$. Dominant background. Decay shape difference between $B\bar{B}$ and $q\bar{q}$.



- Fisher discriminant with 17 event topology variables.
- Then combine with 3 more variables with Neuro-Network.



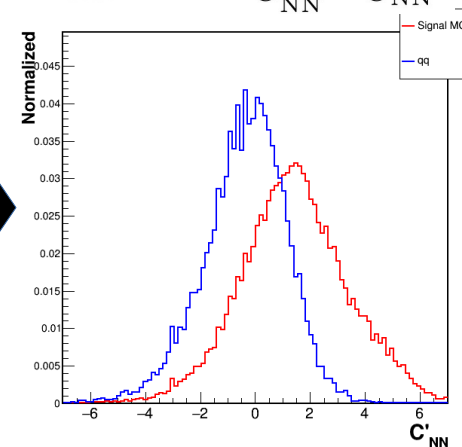
Output discriminant of Neuro-Network



Selection: $NN > 0.75$
Remove 98% of bkg while keeping 60% of signal.

Transformed output

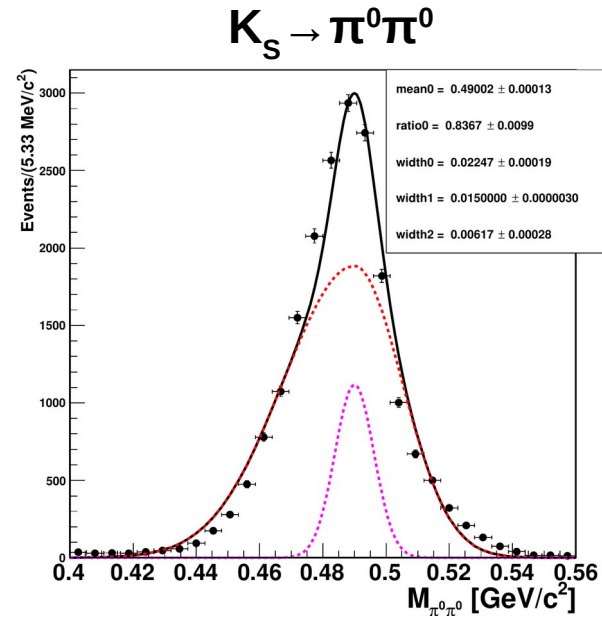
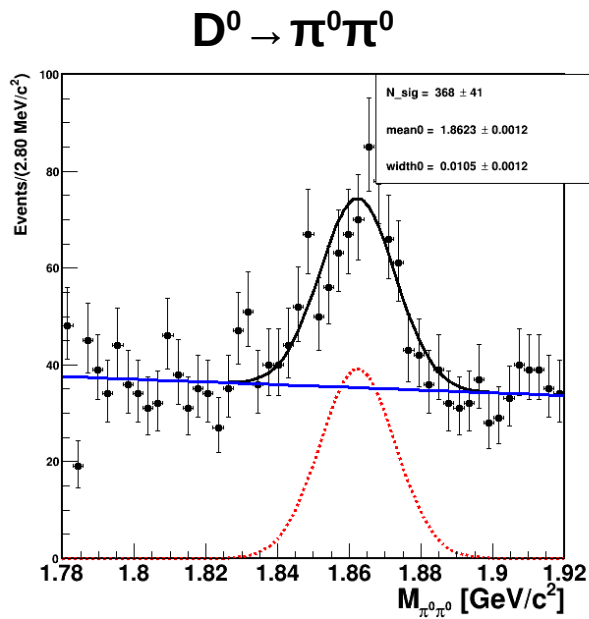
$$C'_{NN} \equiv \log\left(\frac{C_{NN} - C_{NN}^{\min}}{C_{NN}^{\max} - C_{NN}}\right)$$



Gaussian-like shape. Easier to model/fit.

$B\bar{B}$ Background

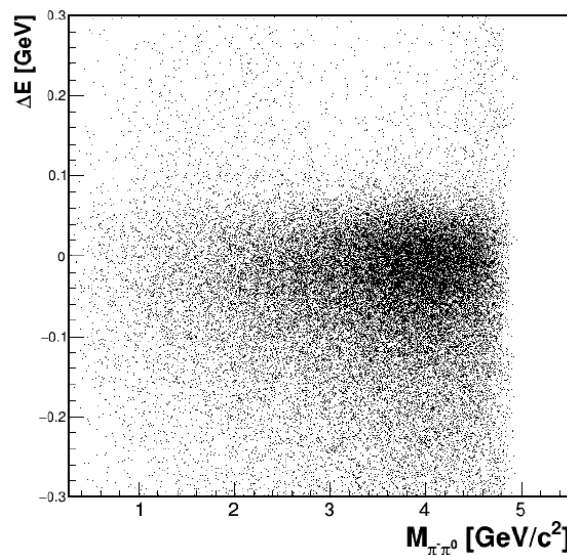
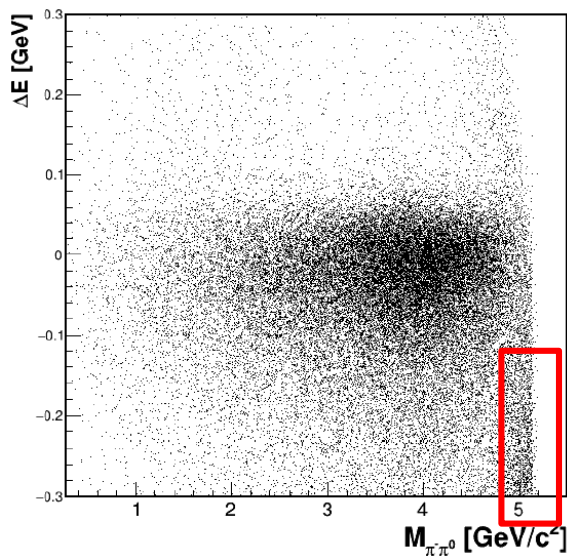
- Backgrounds from $B\bar{B}$ events.
- Dedicated MC samples with two categories:
 - Generic $B\bar{B}$: $b \rightarrow c$.
 - Rare $B\bar{B}$: $b \rightarrow u, d, s$.
- Look for B decay modes causing peaks in ΔE and M_{bc} :
reject the mass window (3σ of the resolution) around the nominal masses.



Soft- π^0 Background

- A structure in the ΔE v.s. $M_{\pi\pi}$ scattering plot due to soft- π^0 .
 - Found in the self-crossfeed (SCF) and other background samples.
 - ΔE v.s. $M_{\pi\pi}$ correlation: Distorts the sWeighted $M_{\pi\pi}$ histogram.
- π^0 momentum > 0.5 GeV/c is required.
 - $\sim 50\%$ of the SCF is reduced.
 - Non-uniform efficiency over Dalitz plot.

Signal MC

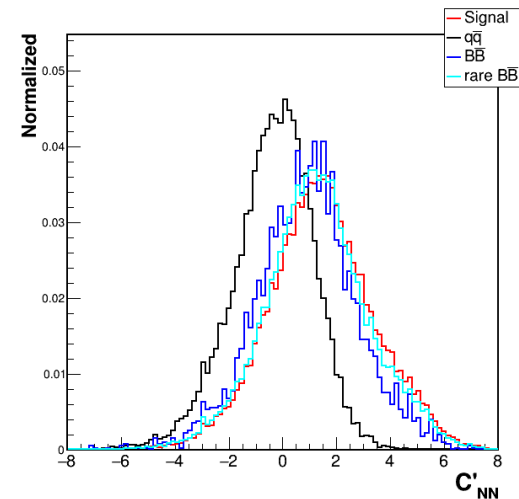
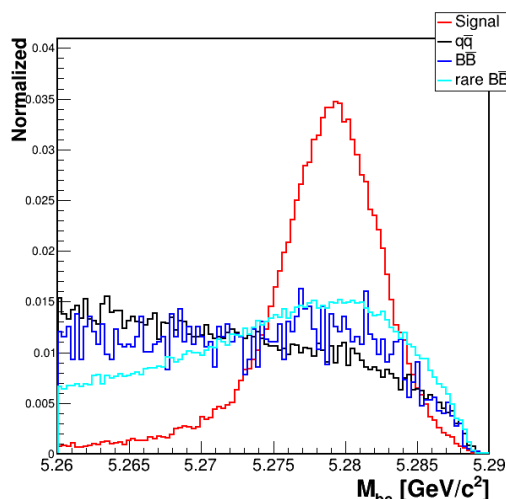
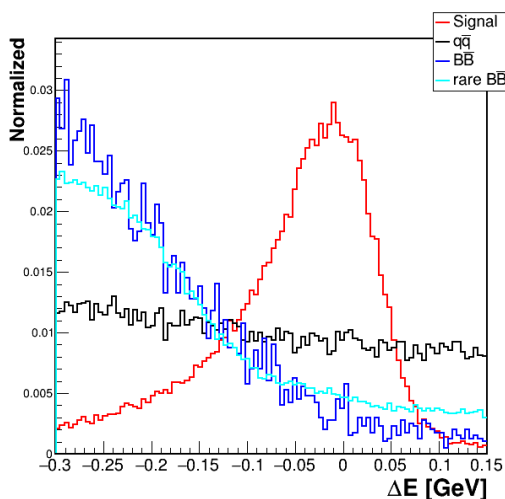


Signal extraction: Extended unbinned likelihood fit

- MC study: Understand the behavior of signal and backgrounds, and their expected numbers in data.
- Signal extraction: Extended unbinned likelihood fit.
 - Number of **signal yield** → **Inclusive BF**.
 - A_{CP} : signal yields of B^+ and B^- .
- **3D Fitter on ΔE , M_{bc} , C'_{NN}** :
 - The most distinctive variables.
 - Define each PDF based on MC distribution.

$$\mathcal{B} = \frac{N_{\text{sig}}}{\underbrace{\epsilon \times \eta}_{\text{Signal reconstruction eff. \& calibration.}} \times N_{B\bar{B}}}$$

Fitted signal yield
Total number of B events in Belle.

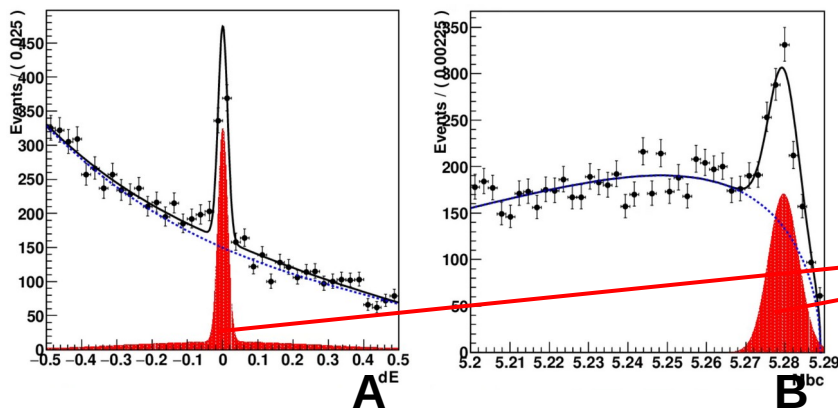


Signal extraction: Signal isolation by sPlot

- By the 3D unbinned fitter, we can obtain **the number of signal yield**.
 - To study the Dalitz plot, we need to further **isolate signal** within data: **sPlot** technique. M. Pivk and F.R. Le Diberder, Nucl. Instrum. Methods Phys. Res., Sect. A 555, 356 (2005)
 - Each event is assigned with a weight (**sWeight**) based on the likelihood.
 - Sum of sWeight = Fitted signal yield.
 - Sum of sWeight within other variables' binning: **Signal-isolated histogram** (sWeighted histogram).
- We use the sPlot to get the signal-isolated histogram of $M_{\pi\pi}$, Dalitz plot ($M^2_{\pi\pi}$ vs $M^2_{\pi\pi}$), helicity angles, etc.

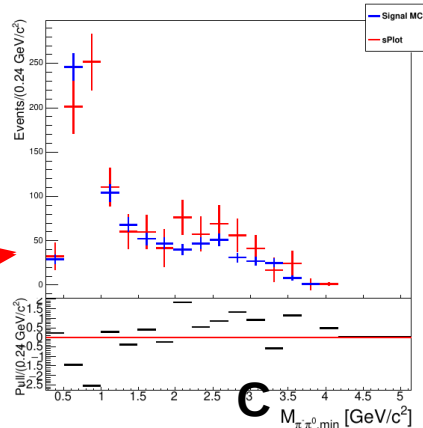
Illustration:

Unbinned fit on A & B



sPlot:
Event weight.

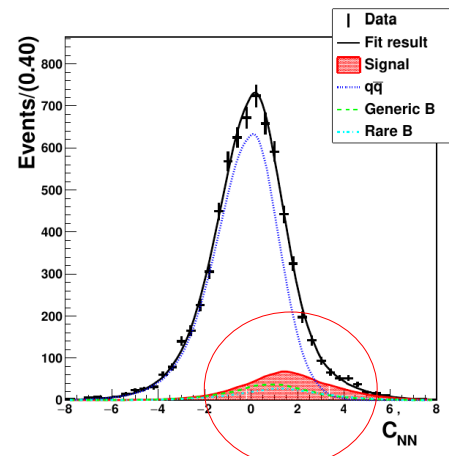
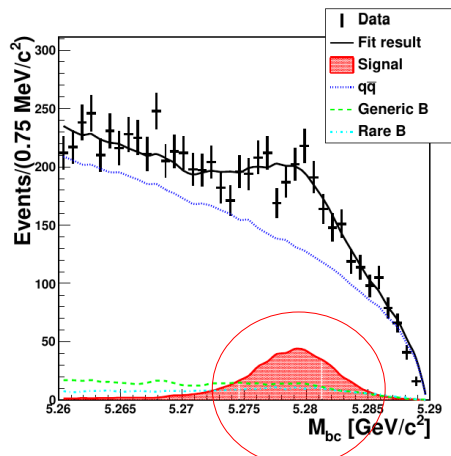
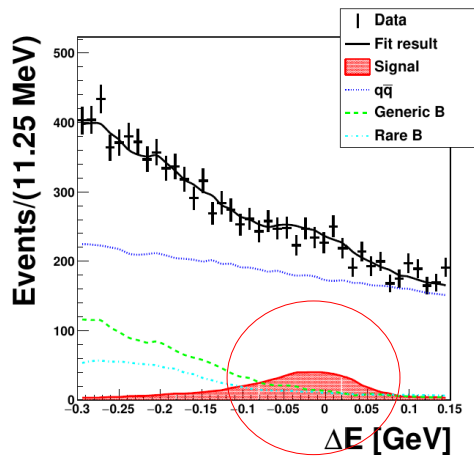
Isolate signal at C



Signal extraction: Signal isolation by sPlot (cont'd)

- An example with MC cocktail sample.

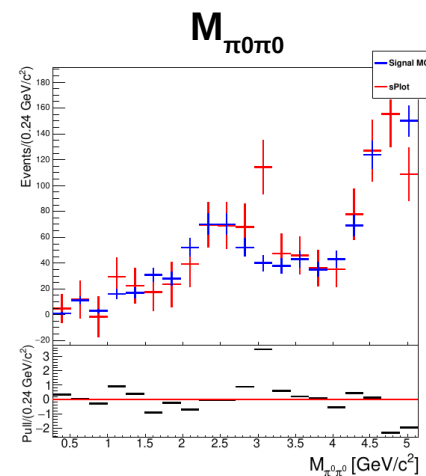
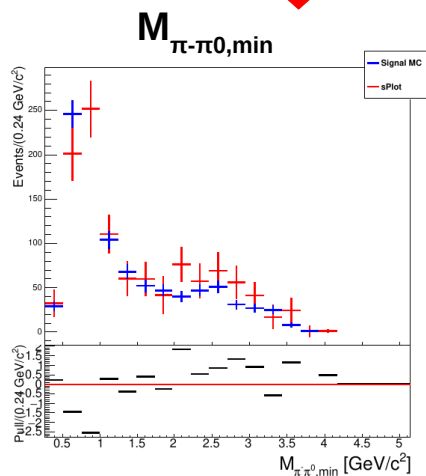
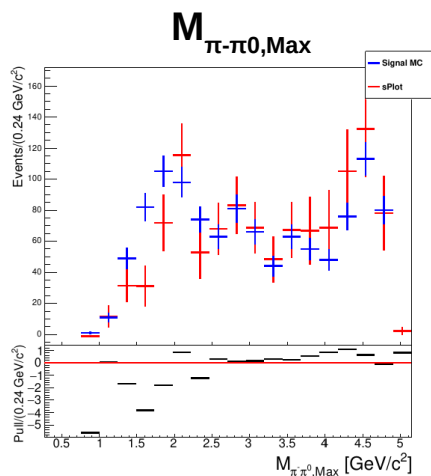
Example:
27% PHSP (400 events)+
73% $\rho(770)^+\pi^0$ (700 events)



Fitted
signal yield:
~1100.



Sum of sWeight after binning on $M_{\pi\pi}$.

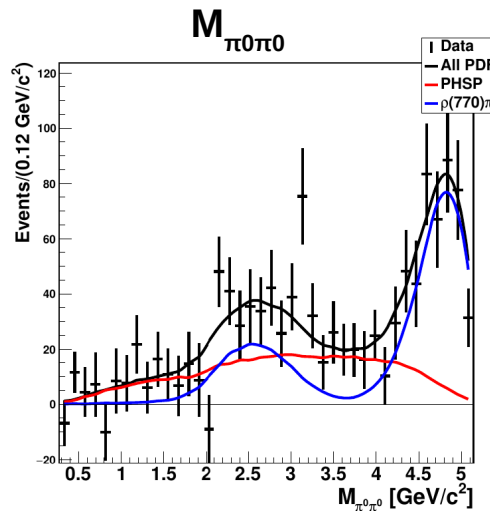
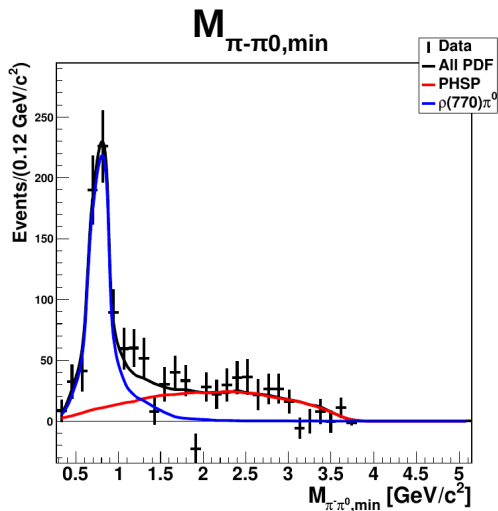


Entries of
histogram:
~1100.

Signal extraction: 2D binned fit on signal-isolated $M_{\pi\pi}$

- After getting the signal-isolated histogram of $M_{\pi\pi}$ (Dalitz plot):
 - We perform a **2D binned fit** on it.
 - To get the signal decay structure: Fraction of each modes.
 - PDF: using MC samples as template.
 - Fitting on charge-separated histograms: Obtain localized A_{CP} .

Example:
 27% PHSP (400 events)+
 73% $\rho(770)^+\pi^0$ (700 events)



Decay modes considered
 in the 2D fit:

PHSP (nonresonant)

$\rho(770)^-\pi^0$

$\rho(1450)^-\pi^0$

$f_0(980)^0\pi^-$

$f_2(1270)^0\pi^-$

$f_0(600)^0\pi^-$

$f_0(1370)^0\pi^-$

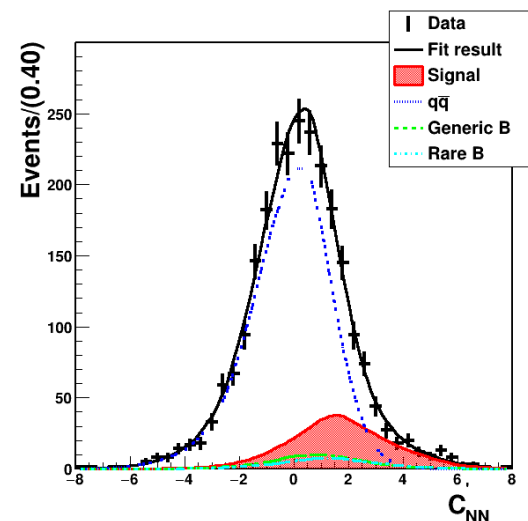
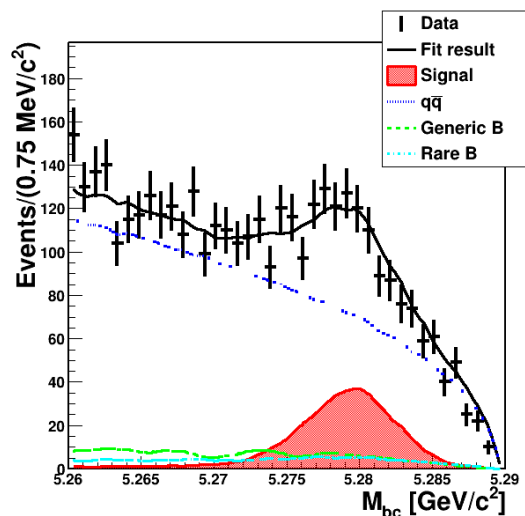
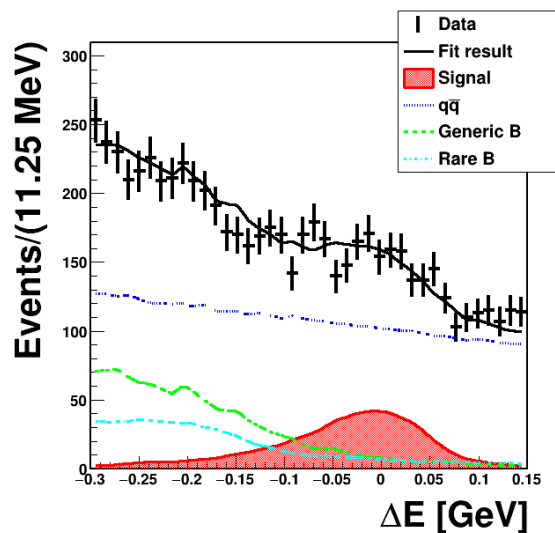
$\chi_{c0}\pi^-$

$\chi_{c2}\pi^-$

3D fit result on data

Preliminary

- $N_{\text{sig}} = 1062.8 +86.8 -85.4.$
 $A_{\text{CP}} = 0.092 +0.068 -0.068.$

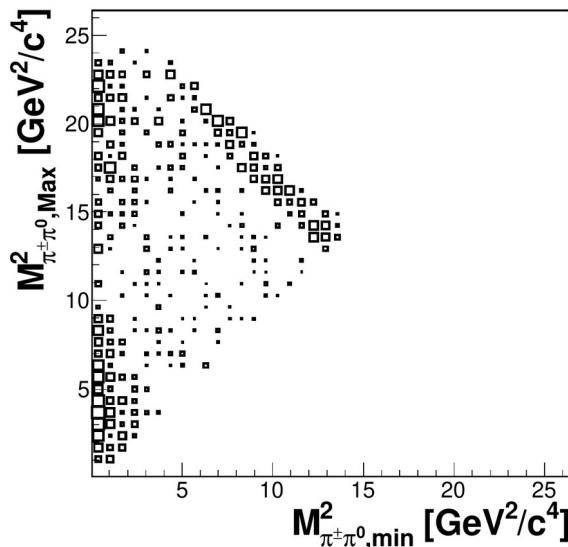


2D fit on sWeighted $M_{\pi\pi}$ histogram

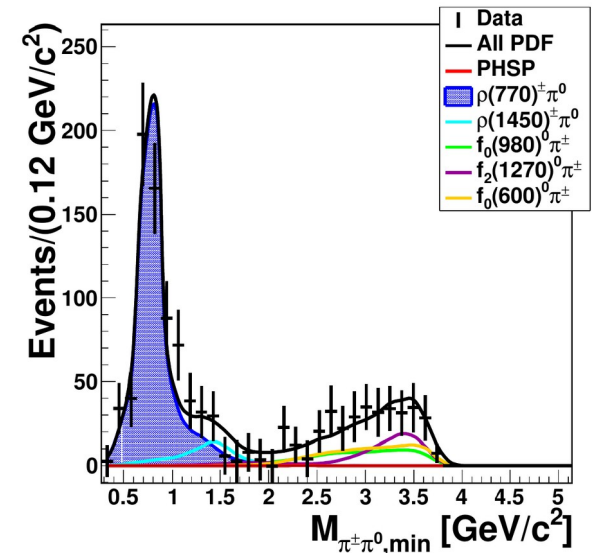
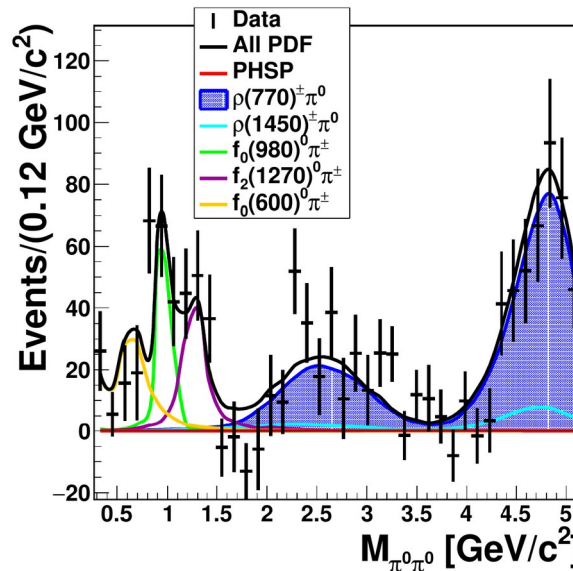
Preliminary

- Measured signal model: incoherent sum of 6 modes' PDF.
 - Interference is not considered in this study. (Different from the full amplitude analysis.)
 - $B^+ \rightarrow \rho(770)^+\pi^0$ with expected size and distribution.
 - Small contribution from non-resonant.
 - New structure found at low $M_{\pi^0\pi^0}$ region. Contribution from multiple resonances.
 - $f_0(1370)^0\pi^+$, $\chi_{c0}\pi^+$, $\chi_{c2}\pi^+$ not seen.
- 9.2 σ confirmed by 3D fit within selected region.

sWeighted Dalitz plot



2D fit result

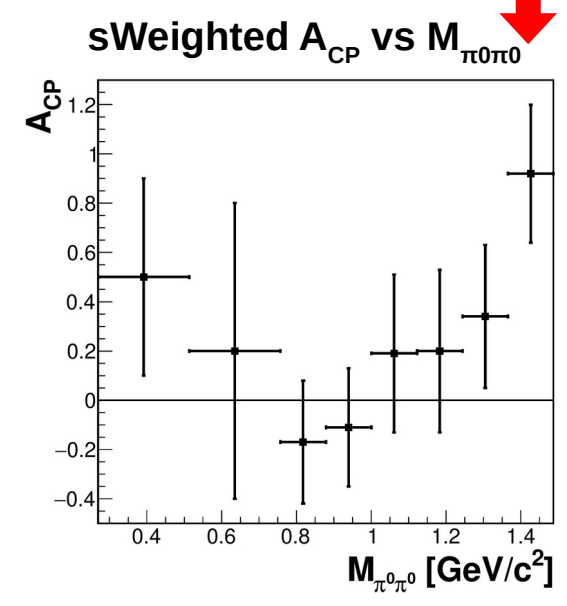
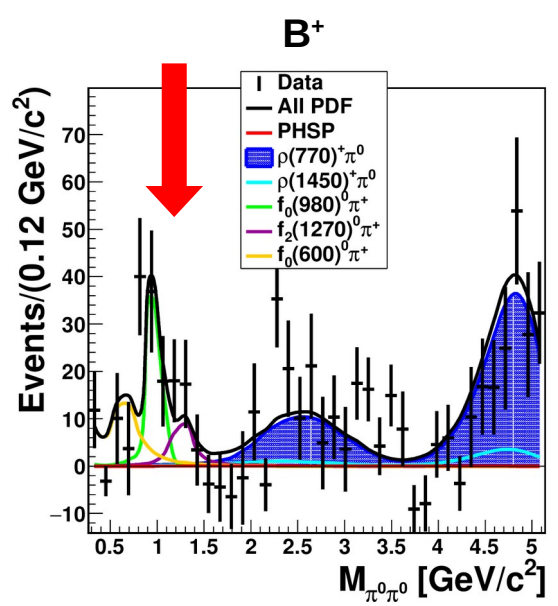
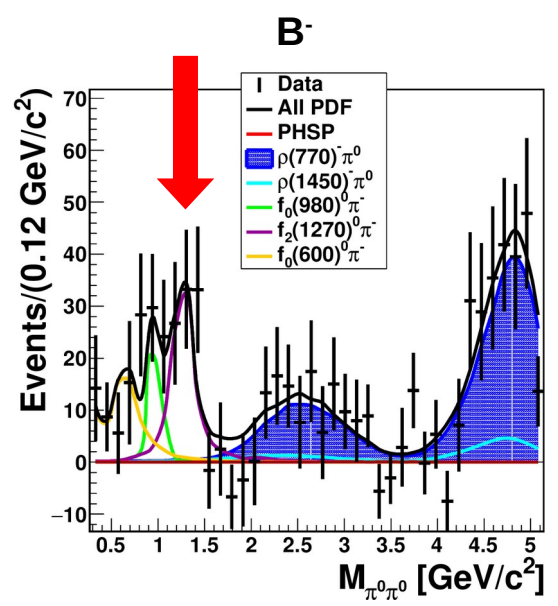


- 2D fit on the charge-separated $M_{\pi\pi}$ histograms.
- An asymmetry is found at $M_{\pi^0\pi^0} \sim 1.4 \text{ GeV}/c^2$.
 - Corresponding to $f_2(1270)^0\pi^+$.
 - Also seen in $B^+ \rightarrow \pi^+\pi^-\pi^+$ by Babar and LHCb.

3.2 σ confirmed by 3D fit within selected region.

PRD 79, 072006 (2009)

PRD 101, 012006 (2020)



Summary of measurement results

Preliminary

- Total BF: Efficiency is determined by the signal model from the last iteration
- $\text{BF}(B^+ \rightarrow \rho(770)^+\pi^0)$: We consider the interference effect with $B^+ \rightarrow \rho(1450)^+\pi^0$.
- BF of the $\pi^0\pi^0$ structure can't be reported separated due to lack of information: highly overlapping PDFs, large variations of masses and widths, interference.

Decay mode	Mass	Width	ϵ (%)	Fitted yield	\mathcal{B} (10^{-6})	\mathcal{A}_{CP} (%)
$\pi^+\pi^0\pi^0$ (total)			8.1	$1062.8^{+86.2}_{-85.4}$	$19.0 \pm 1.5 \pm 1.4$	$9.2 \pm 6.8 \pm 0.5$
Non-resonant			12.5	2.5 ± 13.5	$0.03 \pm 0.16^{+0.12}_{-0.15}$ (< 0.6)	-
$\rho(770)^+\pi^0, \rho(770)^+ \rightarrow \pi^+\pi^0$	775.5	150.3	8.5	636.5 ± 65.0	$11.2 \pm 1.1 \pm 0.9 \pm 1.4$	$8.0 \pm 15.0^{+2.2}_{-7.5}$
$\rho(1450)^+\pi^0, \rho(1450)^+ \rightarrow \pi^+\pi^0$	1465	400	9.9	79.7 ± 51.1	$1.2 \pm 0.6 \pm 0.2$ (< 2.5)	-
$f_0(980)^0\pi^+, f_0(980)^0 \rightarrow \pi^0\pi^0$	980	50	10.2	102.1 ± 30.0	-	$-27.0 \pm 30.0^{+44.8}_{-56.3}$
$f_2(1270)^0\pi^+, f_2(1270)^0 \rightarrow \pi^0\pi^0$	1275.4	185.1	6.6	119.3 ± 32.0	-	$57.0 \pm 23.0^{+11.4}_{-25.9}$
$f_0(600)^0\pi^+, f_0(600)^0 \rightarrow \pi^0\pi^0$	600	400	8.3	123.4 ± 37.4	-	$10 \pm 34^{+12.9}_{-22.6}$
$X\pi^+, X \rightarrow \pi^0\pi^0$	-	-	8.0	344.8 ± 47.5	$6.4 \pm 0.9 \pm 0.6$	-
$f_0(1370)^0\pi^+, f_0(1370)^0 \rightarrow \pi^0\pi^0$	1400	300	10.4	< 75.4	< 1.1	-
$\chi_{c0}\pi^+, \chi_{c0} \rightarrow \pi^0\pi^0$	3415.2	10.2	13.3	< 38.7	< 0.5	-
$\chi_{c2}\pi^+, \chi_{c2} \rightarrow \pi^0\pi^0$	3556.3	2.0	13.6	< 63.4	< 0.7	-

1st uncertainty: statistical.

2nd uncertainty: systematic.

3rd uncertainty (if any): interference effect.

Summary

- We perform the first measurement on the branching fraction and A_{CP} of the $B^+ \rightarrow \pi^+\pi^0\pi^0$ decay modes using the full data sample of 711 fb^{-1} collected by the Belle detector at the KEKB at $Y(4S)$ resonance.
 - Inclusive BF = $(1.90 \pm 1.5 \pm 1.4) \times 10^{-6}$.
- By using the sPlot technique, we study the structure in Dalitz plot, and measure the fraction and localized A_{CP} of sub-decay modes.
 - $BF(B^+ \rightarrow \rho(770)^+\pi^0) = (11.2 \pm 1.2 \pm 0.9 \pm 1.4) \times 10^{-6}$.
 - New structure at low $M_{\pi^0\pi^0}$ region: $BF(B^+ \rightarrow X\pi^+) = (6.4 \pm 0.9 \pm 0.6) \times 10^{-6}$.
 - An obvious asymmetry at $M_{\pi^0\pi^0} \sim 1.4 \text{ GeV}/c^2$.
- Journal article is under preparation and will be published soon.
- In near future, larger data set from Belle2 with full amplitude analysis is helpful to further understand this decay.

Backup

Event selection and signal B reconstruction

- **MC sample for signal study:**
 - 3-body PHSP, $\rho(770)^+\pi^0$, $\rho(1450)^+\pi^0$, $f_0(980)^0\pi^+$, $f_2(1270)^0\pi^+$, $f_0(600)^0\pi^+$, etc.

- **π^+ track:**
 - Tracking: SVD, CDC.
 - Impact parameter: $|dr| < 0.3$ cm, $|dz| < 5$ cm.
 - $L(\pi)/(L(\pi) + L(K)) > 0.6$.

- **Mass-constrain on B candidate:**
 - Correct pion momenta for $M_{\pi\pi}$.

- **B reconstruction:**

Two kinematic variables in CM frame.
For fitting purpose.

- **Energy difference:**

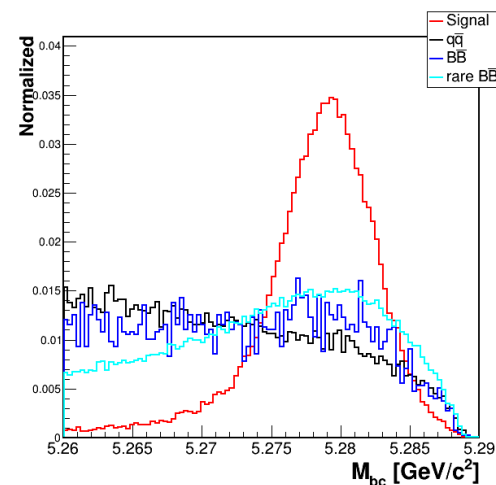
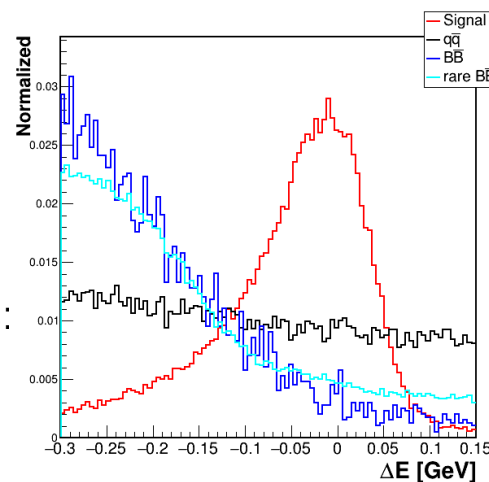
$$\Delta E \equiv E_B - E_{\text{beam}}$$

- **Beam-energy constrained mass:**

$$M_{bc} \equiv \sqrt{E_{\text{beam}}^2/c^4 - |\vec{p}_B/c|^2}$$

- **$\pi^0 \rightarrow \gamma\gamma$:**
 - Cluster on ECL crystal.
 - γ energy > 50 (100) MeV in barrel (endcap).
 - Mass-constrained fit on IP.
 - $115 < M_{\gamma\gamma} < 152$ MeV/ c^2 (3σ)
 - Momentum > 0.5 GeV/ c :
Soft- π^0 background.

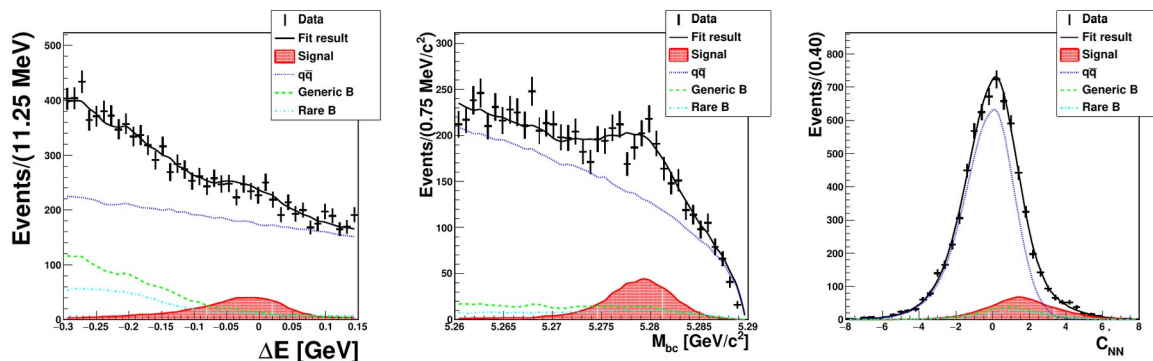
- **Multiple candidates in an event:**
 - Different π^0 : smaller sum of χ^2 .
 - Else: smaller $|dr|$.



Signal extraction: Example with a MC cocktail sample

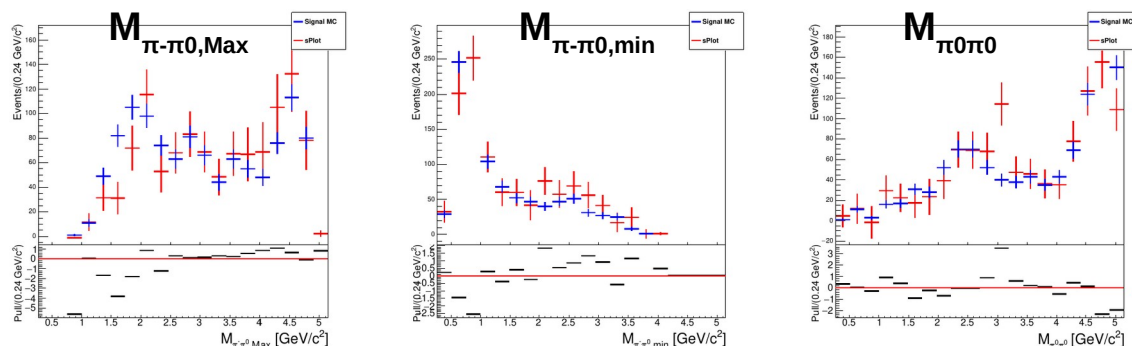
1. 3D ($\Delta E, M_{bc}, C'_{NN}$) extended unbinned maximum likelihood fit:

Based on Laura++ fitter.
Get total signal yield (BF)
and total A_{CP} .



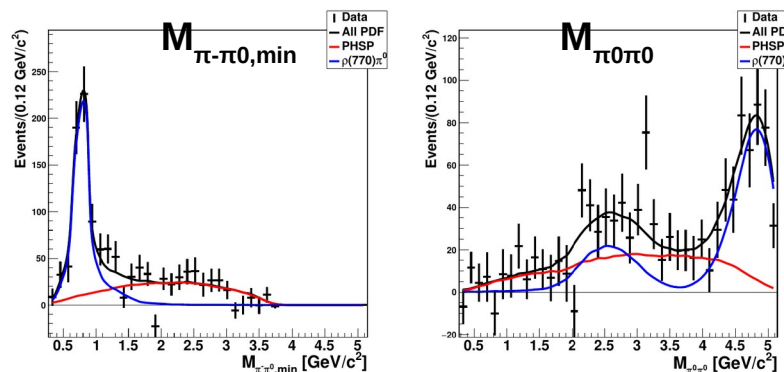
2. Background-subtraction with sPlot:

Get sWeighted Dalitz plot,
 $M_{\pi\pi}$ histograms.



3. 2D extended weighted binned likelihood fit on sWeighted $M_{\pi\pi}$ histograms:

Get fraction of each sub-decay mode
and localized A_{CP} .

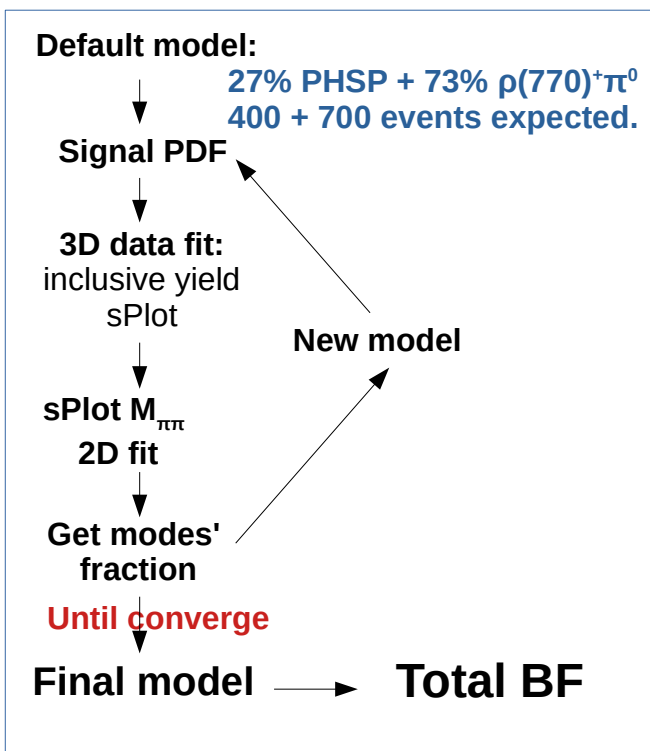


Example:
27% PHSP (400 events)+
73% $\rho(770)^+\pi^0$ (700 events)

Iteration on data measurement

- Signal model composition matters for efficiency, SCF rate, PDF shape. It affects the measurement results.
- Iterate the procedure until converging.
 - The final model is close to the real one.

Work flow:



Signal modes to be considered:

Model	eff.(all) (%)	eff.(true) (%)	$\mathcal{F}_{SCF} \equiv \frac{N_{SCF}}{N_{True}+N_{SCF}} \left(\frac{N_{SCF}}{N_{True}} \right)$ (%)
PHSP	12.5	11.6	7.4 (8.0)
$\rho(770)^-\pi^0$	8.4	6.2	25.9 (34.9)
$\rho(1450)^-\pi^0$	9.7	7.3	24.6 (32.6)
$f_0(980)^0\pi^-$	4.5	4.1	10.2 (11.3)
$f_2(1270)^0\pi^-$	6.4	5.8	10.0 (11.1)
$f_0(600)^0\pi^-$	8.3	7.4	11.5 (13.0)
$f_0(1370)^0\pi^-$	10.6	9.5	10.1 (11.3)
$\chi_{c0}\pi^-$	13.3	12.8	3.4 (3.5)
$\chi_{c2}\pi^-$	13.7	12.3	3.2 (3.3)
Mixed (default)	9.5	7.7	19.2 (23.8)

Systematic uncertainty

*: Calculated as 10^{-6} .

BF, common for all modes

Sources common to all decay modes	%
$N_{B\bar{B}}$	1.4
Tracking	0.35
π^+ identification	0.9
π^0 identification	4.8
Continuum suppression	1.4
Limited MC statistics	0.02
Signal PDF	4.8
Background PDF	2.7
Iteration procedure	0.7

BF, depends on modes

Decay mode	Fit bias	$sPlot$ bias	Resonance Shape	Dalitz model	Total
$\pi^+\pi^0\pi^0$ (total)	0.2	-	-	1.6	7.8
Non-resonant	+0.0 *	2.6	+0.12*	-	8.1 +0.12*
$\rho(770)^+\pi^0$	-0.13	2.6	-0.08	-	-0.15
$\rho(1450)^+\pi^0$	1.6	2.6	+0.7	-	8.3 ± 12.6
$X\pi^+$	-	2.6	-0.9	-	+14.7
$f_0(1370)^0\pi^+$	+0.0 *	2.6	+10.9	-	-15.4
$\chi_{c0}\pi^+$	-0.14	2.6	-11.9	-	8.6
$\chi_{c2}\pi^+$	+0.0 *	2.6	+0.28*	-	8.1, +0.28*
	-0.08	2.6	-0.42	-	-0.44
	+0.0 *	2.6	+0.003*	-	+0.003*
	-0.08	2.6	-0.04	-	-0.09
	+0.0 *	2.6	+0.01*	-	+0.01*
	-0.08	2.6	-0.03	-	-0.09

Total A_{CP}

Source for total A_{CP}	%
Detector bias	0.3
Signal PDF	0.2
Background PDF	0.4
Total	0.5

Sub-decay A_{CP}

Source for sub-decays	mode	%
Resonance shape	$\rho(770)^+\pi^0$	+2.2 -7.5
Resonance shape	$f_0(980)^0\pi^+$	+44.8 -56.3
Resonance shape	$f_2(1270)^0\pi^+$	+11.4 -25.9
Resonance shape	$f_0(600)^0\pi^+$	+12.9 -22.6

- Resonance shape of each sub-decay: Large variation of mass and width from $\rho(1450)^+$, $f_0(980)^0$, $f_0(600)^0$.
- Fit bias: Using ensemble test with Toy MC samples. Separated for 3D fit (total) and 2D fit (sub-decay).
- Interference between $\rho(770)^+\pi$ and $\rho(1450)^+\pi$: First time considered in BF of $\rho(770)^+\pi$ mode.