On compatibility between conformal symmetries and continuous higher form symmetries

Yunqin Zheng

Kavli IPMU, ISSP, U.Tokyo

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Global symmetries and 't Hooft anomalies
impose strong constraints on dynamics of quantum many body systems.

A well known example:
A $1+1$d spin chain with
$SO(3)$ and translation symmetry
and spin $\frac{1}{2}$ per unit cell
$\Rightarrow$ can not be symmetric trivially gapped.

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no symmetric trivially gapped ground state.

Question: Are there more implications on dynamics?

Dynamics are often characterized by RG fixed points.

Gapped: (IR fixed points)

- Trivially gapped = invertible TQFT;
- TQFT with anyons (possible only when $d \geq 2 + 1$.)
- Multiple vacua (discrete SSB)

Gapless: (UV and IR fixed points)

- Conformal field theory;
- Scale invariant but non-conformally invariant.

Constraint on gapped phases:

Not all anomalies can be saturated by
unitary TQFTs which preserves all the global symmetries:
Symmetry enforced gaplessness. [Wang,Senthil, 1401.1142]
[Cordova,Ohmori, 1910.04962, 1912.13069]

- Two free Dirac fermions in $(2+1)$d
  can not be symmetrically gapped
  preserving $SU(2) \times \mathcal{T}$.

- $SU(N)$ QCD with 1 adjoint fermion in $(3+1)$d
  can not be symmetrically gapped
  preserving $\mathbb{Z}_{2N}$ chiral symmetry.

This talk: constraint on gapless theories:

Question:

Unitary Conformal field theory $\stackrel{?}{\longleftrightarrow}$ Continuous higher form symmetry

Answer:

not always compatible.

Unitary Conformal field theory:

- Conformal symmetry generator: $P_\mu, M_{\mu\nu}, D, K_\mu$
- Local operator $\mathcal{O}$ and states $|\mathcal{O}\rangle$ labeled by
  scaling dimension $\Delta_{\mathcal{O}}$,
  Lorentz quantum number (spin) $h_{\mathcal{O}}$
- States organized into conformal tower:
  primary state: $|\mathcal{O}\rangle$, $K_\mu |\mathcal{O}\rangle = 0$
  descendant state: $P_\mu |\mathcal{O}\rangle, P_\mu P_\nu |\mathcal{O}\rangle, \ldots$
- Unitarity: all states have non-negative norm $||PP|\mathcal{O}\rangle||^2 \geq 0$
  Unitary bound: $\Delta_{\mathcal{O}} \geq f(h_{\mathcal{O}})$
  Completely determined. [Minwalla, hep-th/9712074]

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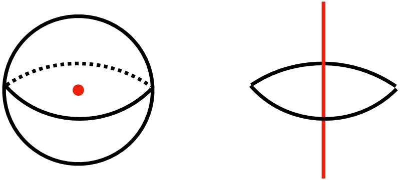
d	Lorentz Algebra	Representation	Unitarity Bound $\Delta_{\mathcal{O}} \geq$	
3	$\mathfrak{so}(3)$	$[h]_{\mathcal{O}}$	0	$(h=0)$
			1	$(h=\frac{1}{2})$
			$h+1$	$(h \geq 1)$
4	$\mathfrak{so}(4)$	$[h_1,h_2]_{\mathcal{O}}$	0	$(h_1=h_2=0)$
			h_1+1	$(h_1>0, h_2=0)$
			h_2+1	$(h_1=0, h_2>0)$
			h_1+h_2+2	$(h_1>0, h_2>0)$
5	$\mathfrak{so}(5)$	$[h_1,h_2]_{\mathcal{O}}$	0	$(h_1=h_2=0)$
			2	$(h_1=h_2=\frac{1}{2})$
			h_1+2	$(h_1=h_2\neq 0,\frac{1}{2})$
			h_1+3	$(h_1>h_2)$
6	$\mathfrak{so}(6)$	$[h_1,h_2,h_3]_{\mathcal{O}}$	0	$(h_1=h_2=h_3=0)$
			h_1+2	$(h_1=h_2=\|h_3\|\neq 0)$
			h_1+3	$(h_1=h_2>\|h_3\|)$
			h_1+4	$(h_1>h_2)$

Continuous higher form symmetry:

[Gaiotto,Kapustin,Seiberg,Willett, 1412.5148]

- Natural generalization of continuous 0-form symmetry, e.g. $U(1)$ charge conservation symmetry.
- 0-form symmetry:
  Charged operator: point like, 0d.
  1-form conserved current J^{μ}, $\partial_{\mu} J^{\mu} = 0$
- p-form symmetry:
  Charged operator: p dimensional brane
  $(p+1)$-form conserved current $J^{\mu_1 \cdots \mu_{p+1}}$, $\partial_{\mu_1} J^{\mu_1 \cdots \mu_{p+1}} = 0$



- Scaling dimension: $\Delta_J = d - p - 1$
- Lorentz quantum number is also determined.

	$d=3$		$d=4$		$d=5$		$d=6$	
	Δ_J	$[h]_J$	Δ_J	$[h_1,h_2]_J$	Δ_J	$[h_1,h_2]_J$	Δ_J	$[h_1,h_2,h_3]_J$
$p=0,\ J^{\mu}$	2	$[1]$	3	$[\frac{1}{2},\frac{1}{2}]$	4	$[1,0]$	5	$[1,0,0]$
$p=1,\ J^{\mu\nu}$	1	$[1]$	2	$[1,0]\oplus[0,1]$	3	$[1,1]$	4	$[1,1,0]$
$p=2,\ J^{\mu\nu\rho}$			1	$[\frac{1}{2},\frac{1}{2}]$	2	$[1,1]$	3	$[1,1,1]\oplus[1,1,-1]$
$p=3,\ J^{\mu\nu\rho\sigma}$					1	$[1,0]$	2	$[1,1,0]$
$p=4,\ J^{\mu\nu\rho\sigma\eta}$							1	$[1,0,0]$

Combining

- From unitary CFT:
  Unitary bound: $\Delta_J \geq f(h_J)$
- From higher form symmetry:
  Scaling dimension: $\Delta_J = d - p - 1$

They are not always compatible!

	$d=3$	$d=4$	$d=5$	$d=6$
$p=0$	✓	✓	✓	✓
$p=1$	✗	✓ : if chiral ✗ : otherwise	✓	✓
$p=2$		✗	✗	✓ : if chiral ✗ : otherwise
$p=3$			✗	✗
$p=4$				✗

Theorem:

A unitary CFT cannot have the “forbidden” *p*-form symmetry (✗) whose conserved current is the conformal primary operator.

	$d=3$	$d=4$	$d=5$	$d=6$
$p=0$	✓	✓	✓	✓
$p=1$	✗	✓ : if chiral ✗ : otherwise	✓	✓
$p=2$		✗	✗	✓ : if chiral ✗ : otherwise
$p=3$			✗	✗
$p=4$				✗

N.B. When considering supersymmetry, in 6d, 1-form symmetry is also forbidden. [Cordova,Dumitrescu,Intriligator, 1612.00809]

If a theory has a forbidden p-form symmetry (✗), its RG fixed points should have either of the following scenarios:

1. a unitary CFT, but the p-form symmetry G is decoupled.
2. scale invariant but not conformal, and the p-form symmetry G may or may not decouple.
3. non-unitary.
4. gapped TQFT (including a trivial theory).

Four examples:

- Free compact scalar in d dimensions.
- Free Maxwell theory in d dimensions.
- Free four derivative Maxwell theory in 6d.
- Interacting modified QED_6.

Free compact scalar

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2, \qquad \phi \sim \phi + 2\pi R$$

Symmetry:

- $U(1)^{(0)}$ electric shift symmetry
- $U(1)^{(d-2)}$ topological symmetry. (Forbidden (✗) when $d \geq 3$.)

Dynamics:

- UV fixed point: $R \to 0$.
  $U(1)^{(0)}$ is trivial, $U(1)^{(d-2)}$ survives.
  Can not be a unitary CFT! Scale invariant but non-CFT!
- IR fixed point: $R \to \infty$.
  $U(1)^{(0)}$ survives, $U(1)^{(d-2)}$ is trivial.
  It is a CFT of non-compact scalar.

[El-Showk,Nakayama,Rychkov, 1101.5385]
[Nakayama, 1302.0884]

Free Maxwell theory:

$$\mathcal{L} = -\frac{1}{8\pi^2} F \wedge *F, \qquad A \sim A + \frac{2\pi\eta}{R}$$

Symmetry:

- $U(1)^{(1)}$ electric shift symmetry. (Forbidden (✗) when $d = 3$)
- $U(1)^{(d-3)}$ topological symmetry. (Forbidden (✗) when $d \geq 5$.)

Dynamics:

- $d = 3$: UV: Scale inv, non-CFT; IR: non-compact scalar CFT.
- $d = 4$: CFT.
- $d \geq 5$:
  - UV fixed point: $R \to \infty$.
    $U(1)^{(1)}$ is trivial, $U(1)^{(d-3)}$ survives.
    Can not be a unitary CFT! Scale invariant but non-CFT!
  - IR fixed point: $R \to 0$.
    Can not rule out unitary CFT.

Four derivative Maxwell theory in 6d

$$\mathcal{L} = \frac{1}{4e^2} G_{\mu\nu} \nabla^2 G^{\mu\nu}$$

Symmetry:

- $U(1)^{(1)}$ electric shift symmetry
- $U(1)^{(3)}$ topological symmetry. (Forbidden (✗)!)

Dynamics:

- It was shown to be CFT. [Tseytlin,1310.1795] [Giombi,Klebanov,Tarnopolsky,1508.06354]
- Forbidden symmetry $U(1)^{(3)}$ enforces non-unitary. Supported by $\langle T_{\mu\nu} T_{\mu\nu} \rangle < 0$. [Giombi,Tarnopolsky, Klebanov,1602.01076]

A variant of QED_6

$$\mathcal{L} = \frac{1}{4e^2} G_{\mu\nu} \nabla^2 G^{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i (\partial\!\!\!/ - i A\!\!\!/) \psi_i .$$

[Giombi,Klebanov,Tarnopolsky,1508.06354]

Symmetry:

- $U(1)^{(3)}$ topological symmetry. (Forbidden (✗)!)

Dynamics:

- Beta function: $\beta_e = -\frac{\epsilon}{2} e - \frac{N_f}{120\pi^3} e^3 + \mathcal{O}(e^5)$
- UV: fermion decouples from gauge field.
  non-unitary CFT ⊗ free fermion
- IR: RG can be unitary when N_f is sufficiently large.

Summary:

- A unitary CFT cannot have the "forbidden" p-form symmetry (✗) whose conserved current is the conformal primary operator.
- Nontrivial dynamical consequences.
- Streamlines previous discussions on scale invariant but non-conformal invariant theories from the higher form symmetry point of view.

Further problems:

Q1: Are there more incompatibilities between:

1. the full set of conformal algebra (without unitarity)
2. 't Hooft anomaly of continuous higher form symmetry / 2-group symmetry?

A1: We haven't find inconsistency.

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Q1: Are there more incompatibilities between:

1. the full set of conformal algebra (without unitarity)
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A1: We haven't find inconsistency.

Q2: Can we find tension between

1. conformal symmetries (with or without unitarity)
2. discrete higher form symmetries / anomalies?

A2: It is an open question.