An Invitation to Exotic Theories

Ho Tat Lam

MIT

Based on

[2108.00020] w/ Pranay Gorantla, Nathan Seiberg and Shu-Heng Shao

[2110.09529] w/ Fiona Burnell, Trithep Devakul, Pranay Gorantla, and Shu-Heng Shao

IPMU Math-String Seminar, Nov 2nd 2021

Introduction

- **Common lore:**
  A lattice model with short-range interactions can be described by an effective continuum quantum field theory at low-energy/long-distance.

- However, some lattice models pose a challenge to this lore, e.g.
  - XY plaquette model [Paramekanti-Balents-Fisher 2002; ...]
  - Fracton models [Chamon 2004; Haah 2011; Vijay-Haah-Fu 2016; ...]

  These lattice models do not admit standard continuum field theory descriptions. They will be examined later in the talk.

- **Our goal:**
  - Understand why these lattice models do not admit standard continuum limit.
  - Expand the framework of quantum field theory to incorporate them.

This Talk

- In this talk, we will discuss 2+1d XY plaquette model as a concrete example of exotic theories and at the end briefly comment on models of fractons.

  For comparison, we will also review the well-known 1+1d XY model.

- We will try our best to find a continuum description for these exotic theories. But as we will see, occasionally we will need to restore the lattice. We will find that these continuum theories exhibit various peculiarities:
  - Exact or emergent exotic symmetries
    with 't Hooft anomalies [2110.09529 Gorantla-HTL-Seiberg-Shao]
  - UV/IR mixing – IR behavior is sensitive to UV details.
    [2108.00020 Burnell-Devakul-Gorantla-HTL-Shao]

Outline

- 1+1d XY model
- 2+1d XY plaquette model
- fracton theories (brief)

Outline

- 1+1d XY model
- 2+1d XY plaquette model
- fracton theories (brief)

1+1d XY model

[... ; Jose-Kadanoff-Kirkpatrick-Nelson 1977; ...]

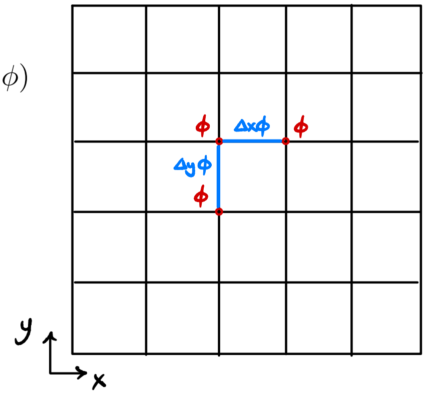
- We work in the Euclidean Lagrangian formalism.
- On a 2d Euclidean lattice, there is a phase $\exp(i\phi)$ at each site with the Euclidean action

$$S = -\beta \sum_{\text{link}} \cos(\Delta_\mu \phi) , \quad \mu = \tau, x .$$

- Exact $U(1)$ momentum symmetry

$$\phi \to \phi + \alpha .$$

- It is gapped at small β and gapless at large β.



1+1d compact boson

- In the continuum limit, the gapless phase is described by the 1+1d theory of compact boson, with the Euclidean Lagrangian

$$\mathcal{L} = \frac{\beta}{2}\left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2\right] , \quad \phi \sim \phi + 2\pi$$

- Global symmetries:
  - $U(1)$ momentum symmetry $\phi \to \phi + \alpha$ with current $J_\mu = i\beta\partial_\mu \phi$
  - $\widetilde{U}(1)$ winding symmetry with current $\tilde{J}_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu}\partial^\nu \phi$

    The symmetry does not act on the field ϕ but winding configurations of ϕ, such as $\phi = 2\pi x/\ell$, carry nontrivial charges.
  - Invariant under scale transformation $(\tau, x) \to (\lambda\tau, \lambda x) \;\Leftrightarrow\; [\beta] = 0$

    It is enhanced to conformal symmetry.
- T-duality: $\phi \to \widetilde{\phi}$, $\beta \leftrightarrow 1/(4\pi^2\beta)$, $U(1)$ momentum $\leftrightarrow$ $\widetilde{U}(1)$ winding

1+1d compact boson

- The $U(1) \times \widetilde{U}(1)$ symmetry has a mixed 't Hooft anomaly. It can be seen by coupling the symmetry to background gauge fields A_μ, $\tilde{A}_\mu$:

$$\mathcal{L} \to \frac{R^2}{2\pi}(\partial_\mu \phi - A_\mu)^2 + \frac{i}{2\pi}\epsilon_{\mu\nu}\tilde{A}_\mu(\partial_\nu \phi - A_\nu)\ .$$

  The partition function is NOT invariant under background gauge transformation

$$\phi \to \phi + \alpha\ , \quad A_\mu \to A_\mu + \partial_\mu \alpha\ , \quad \tilde{A}_\mu \to \tilde{A}_\mu + \partial_\mu \tilde{\alpha}\ .$$

$$\mathcal{L} \to \mathcal{L} - \frac{i}{2\pi}\tilde{\alpha}(\partial_\tau A_x - \partial_x A_\tau)\ .$$

- Anomaly inflow: the anomaly can be canceled by coupling the system to a 2+1d symmetry-protected topological phase (SPT) described by the classical theory

$$\mathcal{L}_{\mathsf{SPT}} = \frac{i}{2\pi}\tilde{A}dA \ \to\ \mathcal{L}_{\mathsf{SPT}} + \frac{i}{2\pi}d(\tilde{\alpha}dA)\ .$$

1+1d compact boson

$$\mathcal{L} = \frac{\beta}{2}\left[(\partial_\tau\phi)^2 + (\partial_x\phi)^2\right] \ , \quad \phi \sim \phi + 2\pi$$

- On a circle of size ℓ

$$\phi(x,\tau) = \phi_0(\tau) + 2\pi W\frac{x}{\ell} + \sum_{k\neq 0} a_k(\tau)\exp\left(\frac{2\pi i k x}{\ell}\right)$$

  - Plane waves $\quad E \sim 1/\ell$
  - States charged under the momentum symmetry $\quad E \sim 1/(\beta\ell)$
  - States charged under the winding symmetry $\quad E \sim \beta/\ell$

- The spectrum is gapless – the energies of all these states vanish as $\ell \to \infty$. They vanish equally fast as $1/\ell$.

- Two-point functions of local operators, such as $\partial_\tau\phi$, $\exp(i\phi)$ etc, decay as a power law on infinite space. (Consequence of the conformal symmetry)

Outline

- 1+1d XY model
- 2+1d XY plaquette model
- fracton theories (brief)

2+1d XY plaquette model

[Paramekanti-Balents-Fisher 2002]

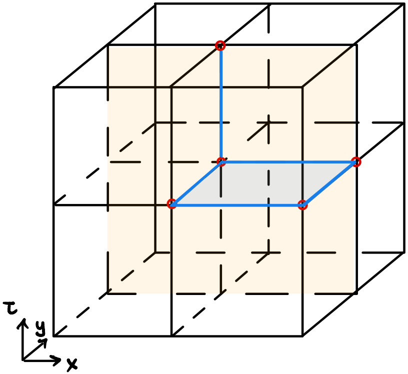
- We work in the Euclidean Lagrangian formalism.
- On a 3d Euclidean lattice, there is a phase $\exp(i\phi)$ at each site with the Euclidean action

$$S = -\beta_0 \sum_{\tau\text{-link}} \cos(\Delta_\tau \phi) - \beta \sum_{xy\text{-plaq}} \cos(\Delta_x \Delta_y \phi) \ .$$

- Exact $U(1)$ momentum subsystem symmetry

$$\phi \to \phi + \alpha_x(\hat{x}) + \alpha_y(\hat{y}) \ .$$

- The model is gapped at small β_0, β and gapless at large β_0, β.



2+1d ϕ-theory

[Paramekanti-Balents-Fisher 2002; Seiberg-Shao 2020]

- In the continuum limit, the gapless phase is described by the continuum ϕ-theory

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_\tau \phi)^2 + \frac{1}{2\mu}(\partial_x \partial_y \phi)^2 \,, \qquad \phi \sim \phi + 2\pi$$

- Exact $U(1)$ momentum subsystem symmetry $\phi \to \phi + \alpha_x(x) + \alpha_y(y)$

$$\partial_\tau J_\tau = \partial_x \partial_y J_{xy} \,, \quad (J_\tau, J_{xy}) = \left(i\mu_0 \partial_\tau \phi, \frac{i}{\mu} \partial_x \partial_y \phi \right)$$

  Discontinuous field configurations, such as $\phi(\tau, x, y) = \phi_x(\tau, x) + \phi_y(\tau, y)$, are not suppressed. As we will see, they lead to UV/IR mixing.

Subsystem global symmetry

[Seiberg-Shao 2020]

$$\partial_\tau J_\tau = \partial_x \partial_y J_{xy}$$

- The conserved charges are line operators

$$Q^x(x) = \oint dy\, J_\tau\ , \quad Q^y(y) = \oint dx\, J_\tau$$

$$\partial_\tau Q^x(x) = \oint dy\, \partial_\tau J_\tau = \oint dy\, \partial_y(\partial_x J_{xy}) = 0$$

At every point in x (and similarly in y) there is an independent charge

$$Q^x(x_1) \neq Q^x(x_2) \ \text{ if } \ x_1 \neq x_2$$

This leads to infinitely many charges in the continuum.

$2{+}1\mathrm{d}\ \phi$-theory

[Seiberg-Shao 2020]

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_\tau \phi)^2 + \frac{1}{2\mu}(\partial_x \partial_y \phi)^2 \ , \qquad \phi \sim \phi + 2\pi$$

- Emergent $\widetilde{U}(1)$ winding subsystem symmetry

$$\partial_\tau \widetilde{J}_\tau = \partial_x \partial_y \widetilde{J}_{xy} \ , \quad (\tilde{J}_\tau, \widetilde{J}_{xy}) = \left(\frac{1}{2\pi} \partial_x \partial_y \phi, \frac{1}{2\pi} \partial_\tau \phi \right)$$

  The winding subsystem symmetry does not act on the field ϕ but winding configurations of ϕ carry nontrivial charges.

- A scale symmetry: invariant under $(\tau, x, y) \ \rightarrow \ (\lambda_x \lambda_y \tau, \lambda_x x, \lambda_y y)$ [Gromov 2018; Karch-Raz 2020]

- Self-duality: $(\mu_0, \mu) \rightarrow (\mu/4\pi^2, 4\pi^2 \mu_0)$, $U(1)$ momentum $\leftrightarrow$ $\widetilde{U}(1)$ winding

$2{+}1$d ϕ-theory

[Gorantla-HTL-Seiberg-Shao 2021; Burnell-Devakul-Gorantla-HTL-Shao 2021]

- The $U(1) \times \widetilde{U}(1)$ subsystem symmetry has a mixed 't Hooft anomaly.
  It can be seen by coupling the subsystem symmetry to background tensor gauge fields (2+1d version of [Xu-Wu 2008]):

$$(A_\tau, A_{xy}) \to (A_\tau + \partial_\tau \alpha, A_{xy} + \partial_x \partial_y \alpha) \;, \qquad \phi \to \phi + \alpha$$
$$(\tilde{A}_\tau, \tilde{A}_{xy}) \to (\tilde{A}_\tau + \partial_\tau \tilde{\alpha}, \tilde{A}_{xy} + \partial_x \partial_y \tilde{\alpha}) \;.$$

  The tensor gauge symmetry is due to the conservation equation $\partial_\tau J_\tau = \partial_x \partial_y J_{xy}$.

- The partition function is NOT gauge invariant

$$\begin{aligned}\mathcal{L} &= \frac{\mu_0}{2}(\partial_\tau \phi - A_\tau)^2 + \frac{1}{2\mu}(\partial_x \partial_y \phi - A_{xy})^2 + \frac{i}{2\pi}\tilde{A}_\tau \partial_x \partial_y \phi + \frac{i}{2\pi}\tilde{A}_{xy}\partial_\tau \phi \\ &\to \mathcal{L} + \frac{i}{2\pi}\alpha(\partial_x \partial_y \tilde{A}_\tau - \partial_\tau \tilde{A}_{xy}) \;.\end{aligned}$$

Anomaly inflow

[Burnell-Devakul-Gorantla-HTL-Shao 2021]

- The bulk geometry is $\Sigma_3 \times \mathbb{R}_{z\leq 0}$, and the boundary theory is placed at $z=0$.
- The background gauge fields are extended to bulk as

$$(A_\tau, A_{xy}, A_z) \to (A_\tau + \partial_\tau \alpha, A_{xy} + \partial_x\partial_y \alpha, A_z + \partial_z \alpha) \ ,$$
$$(\tilde{A}_\tau, \tilde{A}_{xy}, \tilde{A}_z) \to (\tilde{A}_\tau + \partial_\tau \tilde{\alpha}, \tilde{A}_{xy} + \partial_x\partial_y \tilde{\alpha}, \tilde{A}_z + \partial_z \tilde{\alpha}) \ .$$

- Anomaly inflow: the anomaly can be canceled by coupling the system to a 3+1d $U(1) \times \widetilde{U}(1)$ subsystem SPT described by the classical theory

$$\begin{aligned}\mathcal{L} &= \frac{i}{2\pi}\left[A_{xy}(\partial_\tau \tilde{A}_z - \partial_z \tilde{A}_\tau) + A_z(\partial_\tau \tilde{A}_{xy} - \partial_x\partial_y \tilde{A}_\tau) - A_\tau(\partial_z \tilde{A}_{xy} - \partial_x\partial_y \tilde{A}_z)\right] \\ &\to \mathcal{L} - \frac{i}{2\pi}\partial_z\left[\alpha(\partial_x\partial_y \tilde{A}_\tau - \partial_\tau \tilde{A}_{xy})\right] + \cdots \ .\end{aligned}$$

1+1d compact boson vs 2+1d ϕ-theory

Properties	1+1d compact boson	2+1d ϕ-theory
Momentum Sym. Winding Sym.	$U(1)$ symmetry $\widetilde{U}(1)$ symmetry	$U(1)$ subsystem symmetry $\widetilde{U}(1)$ subsystem symmetry
Scale Sym.	$(\lambda\tau, \lambda x)$	$(\lambda_x\lambda_y\tau, \lambda_x x, \lambda_y y)$
Mixed Anomaly	$U(1) \times \widetilde{U}(1)$ SPT	$U(1) \times \widetilde{U}(1)$ subsystem SPT
Self-duality	$\beta \to 1/(4\pi^2\beta)$	$(\mu_0, \mu) \to (\mu/4\pi^2, 4\pi^2\mu_0)$

Spectrum

[Seiberg-Shao 2020]

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_\tau \phi)^2 + \frac{1}{2\mu}(\partial_x \partial_y \phi)^2 \,, \qquad \phi \sim \phi + 2\pi$$

We now discuss the spectrum on a torus of size $\ell_x = \ell_y = \ell$.

- Plane waves with quantized momenta $p_x = 2\pi k_x/\ell$, $p_y = 2\pi k_y/\ell_y$:

$$\phi(x,y,\tau) = \sum_{k_x, k_y \neq 0} a_{k_x,k_y}(\tau) \exp\left[2\pi i \left(\frac{k_x x}{\ell} + \frac{k_y y}{\ell}\right)\right] \,.$$

They have a non-standard dispersion relation

$$\omega^2 = \frac{p_x^2 p_y^2}{\mu\mu_0} = \frac{(2\pi)^4 k_x^2 k_y^2}{\mu\mu_0 \ell^4} \,.$$

- $\omega \sim 1/\ell^2$ (not $\omega \sim 1/\ell$ as in 1+1d compact boson)
- UV/IR mixing (more below): large momentum does not imply large energy, e.g. small ω at large p_x with a sufficiently small p_y.

Spectrum

[Seiberg-Shao 2020]

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_\tau \phi)^2 + \frac{1}{2\mu}(\partial_x \partial_y \phi)^2 \;, \qquad \phi \sim \phi + 2\pi$$

- Momentum states: states charged under the momentum subsystem symmetry.

  They are obtained by quantizing the plane waves with $k_x = 0$ or $k_y = 0$.

$$\phi(x, y, \tau) = \phi_x(x, \tau) + \phi_y(y, \tau)$$

  The momentum subsystem symmetry shifts

$$\begin{aligned} \phi_x(x, \tau) &\to \phi_x(x, \tau) + f_x(x) \;, \\ \phi_y(y, \tau) &\to \phi_y(y, \tau) + f_y(y) \;. \end{aligned}$$

  The symmetry appears to be spontaneously broken classically.
  But we will soon see that the symmetry is restored quantum mechanically.

Spectrum

[Seiberg-Shao 2020]

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_\tau \phi)^2 + \frac{1}{2\mu}(\partial_x \partial_y \phi)^2 \,, \qquad \phi \sim \phi + 2\pi$$

- We now quantize the momentum states. For simplicity, ignore the common zero mode of $\phi_x(x,\tau), \phi_y(y,\tau)$.
- These modes are independent rotors at different x, y with point-wise periodicity

$$S = \frac{\mu_0 \ell}{2} \int d\tau \left[\int dx \left(\partial_\tau \phi_x(x,\tau)\right)^2 + \int dy \left(\partial_\tau \phi_y(y,\tau)\right)^2 \right]$$

Restore the lattice $\int dx \to a \sum_{\hat{x}}$. The Hamiltonian is

$$H = \frac{1}{2\mu_0 \ell a} \left[\sum_{\hat{x}} \pi_x(x)^2 + \sum_{\hat{y}} \pi_y(y)^2 \right] \,, \quad \pi_x(x), \pi_y(y) \in \mathbb{Z}$$

Spectrum

[Seiberg-Shao 2020]

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_\tau \phi)^2 + \frac{1}{2\mu}(\partial_x \partial_y \phi)^2 \,, \qquad \phi \sim \phi + 2\pi$$

- Winding states: states charged under the winding subsystem symmetry.

  They are obtained from winding configurations

$$\phi = 2\pi \left[\frac{x}{\ell}\Theta(y - y_0) + \frac{y}{\ell}\Theta(x - x_0) - \frac{xy}{\ell^2} \right]$$

$$\widetilde{J}_\tau = \frac{1}{2\pi}\partial_x \partial_y \phi = 2\pi \left[\frac{1}{\ell}\delta(y - y_0) + \frac{1}{\ell}\delta(x - x_0) - \frac{1}{\ell^2} \right]$$

  These states have infinite energies in the continuum. They can be regularized by restoring the lattice $\delta(0) \to 1/a$

$$E = \frac{4\pi^2 \delta(0)}{\mu \ell} \qquad \to \qquad E = \frac{4\pi^2}{\mu \ell a}$$

Spectrum

[Seiberg-Shao 2020]

- Spectrum on a torus of size $\ell_x = \ell_y = \ell$ (lattice with $L = \ell/a$ sites)

  - Plane waves, created by $\partial_x\partial_y\phi$, $\partial_\tau\phi$ $\qquad E \sim \frac{1}{\sqrt{\mu\mu_0}\ell^2} \sim \frac{1}{L^2a^2}$

  - Momentum states, created by $\exp(i\phi)$ $\qquad E \sim \frac{1}{\mu_0 a\ell} \sim \frac{1}{La^2}$

  - Winding states, created by $\exp(i\widetilde{\phi})$ $\qquad E \sim \frac{4\pi^2}{\mu a\ell} \sim \frac{1}{La^2}$

- States charged under the subsystem symmetries have infinite energies in the continuum limit $a \to 0$. They are excluded from the continuum Hilbert space.

  This implies that the charged operators are infinitely irrelevant. The continuum theory is robust under small deformations violating the subsystem symmetries!

UV/IR mixing

[Gorantla-HTL-Seiberg-Shao 2020]

- We can regularize the infinite energies of the momentum and the winding states by restoring the lattice.
  - Plane waves $$E \sim \frac{1}{\ell^2} = \frac{1}{L^2 a^2}$$
  - Momentum and winding states $$E \sim \frac{1}{a\ell} = \frac{1}{La^2}$$
- There are two interesting limits:
  - Continuum limit $a \to 0$ with $\ell = aL$ fixed: we zoom into the plane waves and push the charged states to infinity. This recovers the continuum theory.
  - Thermodynamic limit $L \to \infty$ with a fixed: all states become light.
- The continuum limit and the thermodynamic limit do not commute – UV/IR mixing.

Correlation functions

- We now study Euclidean correlation functions of local operators on the lattice. The list of local operators include $\partial_\tau\phi$, $\partial_x\partial_y\phi$, $\exp(i\phi)$, $\exp(i\widetilde{\phi})$, ...

  For simplicity, we will set $\mu_0 = \mu = 1$ and drop constants of order one.

- We will consider two limits:
  - Continuum limit: $a \to 0$ with $\ell = aL$ fixed. The operators are separated further and further apart as $a \to 0$ such that continuum coordinates are fixed.
  - Thermodynamic limit: $L \to \infty$ with a fixed. The operators are separated finite lattice spacing apart. We can later take them to be far separated.

$\langle \partial_\tau \phi \partial_\tau \phi \rangle$

[Gorantla-HTL-Seiberg-Shao 2020]

- The two-point function of $\partial_\tau \phi$ at generic positions is independent of the order of the continuum limit and the thermodynamic limit. After taking the two limits,

$$\left\langle \partial_\tau \phi(0,0,0) \partial_\tau \phi(x,y,\tau) \right\rangle \sim \begin{cases} -\dfrac{1}{x^2 y^2} & |\tau| \ll |xy| \\ -\dfrac{1}{\tau^2} \log \left| \dfrac{\tau}{xy} \right| & |\tau| \gg |xy| \end{cases}$$

  Check: covariant under $(\tau, x, y) \ \to \ (\lambda_x \lambda_y \tau, \lambda_x x, \lambda_y y)$.

- The two-point function diverges at non-coincidental points, $xy \to 0$ at finite τ. Such singularities are not present in ordinary Euclidean correlators.

- The non-coincidental singularity is a consequence of the non-standard dispersion relation $\omega^2 \sim p_x^2 p_y^2$. For example, there is a singularity at $x \to 0$ because states with a large p_x can have a small ω as long as p_y is sufficiently small.

$\langle \partial_\tau \phi \partial_\tau \phi \rangle$

[Gorantla-HTL-Seiberg-Shao 2020]

- To better understand the singularity, let us start with the two-point function with $x = 0$ on lattice. There is no divergence. The divergence shows up only after we take both the continuum and the thermodynamic limit.

- Thermodynamic limit first: a UV divergence regularized by a, $|p_x| < 1/a$

$$\left\langle \partial_\tau \phi(0,0,0) \partial_\tau \phi(0,y,\tau) \right\rangle \sim -\frac{1}{\tau^2} \log \left| \frac{\tau}{ay} \right|$$

  Continuum limit first: an IR divergence regularized by ℓ, $|p_y| > 1/\ell$

$$\left\langle \partial_\tau \phi(0,0,0) \partial_\tau \phi(0,y,\tau) \right\rangle \sim -\frac{1}{\tau^2} \log \left| \frac{\ell}{y} \right|$$

- The non-coincidental divergence can be regularized both by a UV regulator and a IR regulator. This reflects the UV/IR mixing in the spectrum of plane waves.

$$\langle \exp(i\phi) \exp(-i\phi) \rangle$$

[Gorantla-HTL-Seiberg-Shao 2020]

- Because of the subsystem symmetry, the two-point function of $\exp(i\phi)$ vanishes unless the two operators are at the same position in space.
  (Discontinuities in the observables)

  The behavior of the two-point function depends on the order of the continuum limit and the thermodynamic limit – UV/IR mixing.

- If we take the continuum limit first,

$$\left\langle \exp(i\phi(0,0,0)) \exp(-i\phi(0,0,\tau)) \right\rangle \sim \exp\left(-\frac{|\tau|}{a\ell}\right)$$

  The exponent represents the infinite energy of the lowest momentum states in the continuum limit. This demonstrates that the operator $\exp(i\phi)$ acts trivially in the continuum theory – they are redundant operators.

$\langle \exp(i\phi) \exp(-i\phi) \rangle$

[Gorantla-HTL-Seiberg-Shao 2020]

- If we take the thermodynamic limit first [Paranekanti-Balents-Fisher 2002],

$$\Big\langle \exp(i\phi(0,0,0)) \exp(-i\phi(0,0,\tau)) \Big\rangle \sim \exp\left[- \left(\log \left| \frac{\tau}{a^2} \right| \right)^2 \right]$$

  It decays faster than any power law, but is not exponentially suppressed as in the continuum limit.

  This reflects the fact that energies of the momentum states go to zero in the thermodynamic limit as $L \to \infty$, but is slower than the plane waves.

- Identical analysis for the winding operators $\exp(i\widetilde{\phi})$.

UV/IR mixing

[Gorantla-HTL-Seiberg-Shao 2020]

spectrum	Two-point functions
plane waves	$\partial_\tau \phi, \partial_x \partial_y \phi$
- non-standard dispersion $\omega^2 \sim p_x^2 p_y^2$ - small ω at large p_x or p_y	- non-coincidental singularity at $xy = 0$ - can be viewed as a UV/IR divergence
momentum and winding states	$\exp(i\phi), \exp(i\widetilde{\phi})$
- $E \sim 1/(a\ell)$ - the thermodynamic limit and the continuum limit do not commute	- continuum limit: $\exp(-\tau/a)$ - thermodynamic limit: $\exp[-(\log\|\tau\|)^2]$ - different behaviors in the two limits

Outline

- 1+1d XY model
- 2+1d XY plaquette model
- fracton theories (brief)

X-cube model

[Vijay-Haah-Fu 2016]

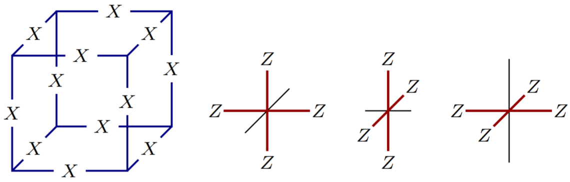


Figure from [Shirley-Slagle-Wang-Chen 2017]

- The theory is gapped and robust – new gapped phases beyond TQFT description
- More peculiar UV/IR mixing: ground state degeneracy depends sensitively on the number of lattice sites, G.S.D $= N^{2L_x+2L_y+2L_z-3}$ (infinite in the continuum limit).
- Gapped excitations have restricted mobility: fractons (immobile excitations)

QFT for Fractons

- We can construct fracton theories by considering gauge theories of subsystem symmetries. These are dynamical theories of tensor gauge fields.

- As an example, the tensor gauge field for the 2+1d $U(1)$ subsystem symmetry is

$$(A_\tau, A_{xy}) \sim (A_\tau + \partial_\tau \alpha, A_{xy} + \partial_x \partial_y \alpha) \ .$$

  The gauge invariant defect $\exp(i \int d\tau A_\tau)$ respresents the worldline of a particle. It is an immobile fracton because the curve cannot be deformed due to the lack of $A_x \sim A_x + \partial_x \alpha$ or $A_y \sim A_y + \partial_y \alpha$.

  There are other gauge invariant operators supported on strips wrapping around the x- or the y-cycle: $\exp(i \int_{x_1}^{x_2} dx \oint dy A_{xy})$ and $\exp(i \int_{y_1}^{y_2} dy \oint dx A_{xy})$.

QFT for Fractons

[Seiberg-Shao 2020]

- The $\mathbb{Z}_N$ version of this tensor gauge theory describes a gapped fracton theory
$$\mathcal{L} = \frac{N}{2\pi}\phi^{xy}(\partial_\tau A_{xy} - \partial_x\partial_y A_\tau)$$
The theory has a $\mathbb{Z}_N$ subsystem symmetry
$$\phi^{xy} \to \phi^{xy} + \frac{2\pi i}{N}m_x(x) + \frac{2\pi i}{N}m_y(y)\ , \quad m_x(x), m_y(y) \in \mathbb{Z}\ .$$
It is generated by $\exp(i\int_{x_1}^{x_2} dx \oint dy A_{xy})$ and $\exp(i\int_{y_1}^{y_2} dy \oint dx A_{xy})$.
The subsystem symmetry is spontaneously broken. After restoring the lattice, it leads to a finite number of degenerate ground states G.S.D $= N^{L_x+L_y-1}$.

- The X-cube model is described by a 3+1d $\mathbb{Z}_N$ tensor gauge theory [Slagle-Kim 2017, Seiberg-Shao 2020]. It reproduces the G.S.D after restoring the lattice.

Summary

- The low-energy limit of a lattice theory with short-range interactions is expected to be a continuum quantum field theory.
- Exotic lattice models are challenging counter examples because of
  - Subsystem global symmetry (It can have 't Hooft anomaly)
  - UV/IR mixing
  - Excitations with restricted mobility
  - Sub-extensive ground state degeneracy
  - Discontinuous and even singular observables in the continuum limit
- Some exotic continuum theories can capture these facts, but occasionally we need to restore the lattice.
- We have also studied many other exotic theories with different $U(1)$ or $\mathbb{Z}_N$ subsystem symmetries in 3+1d.

Thank you!