Amplification of Primordial Perturbations from the Rise or Fall of the Inflaton

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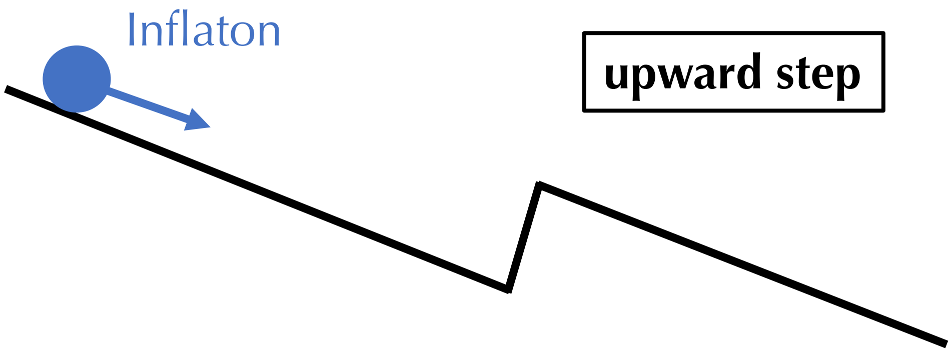
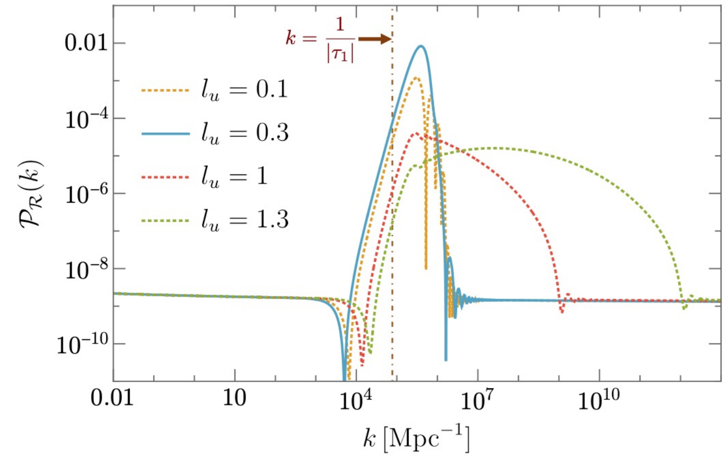
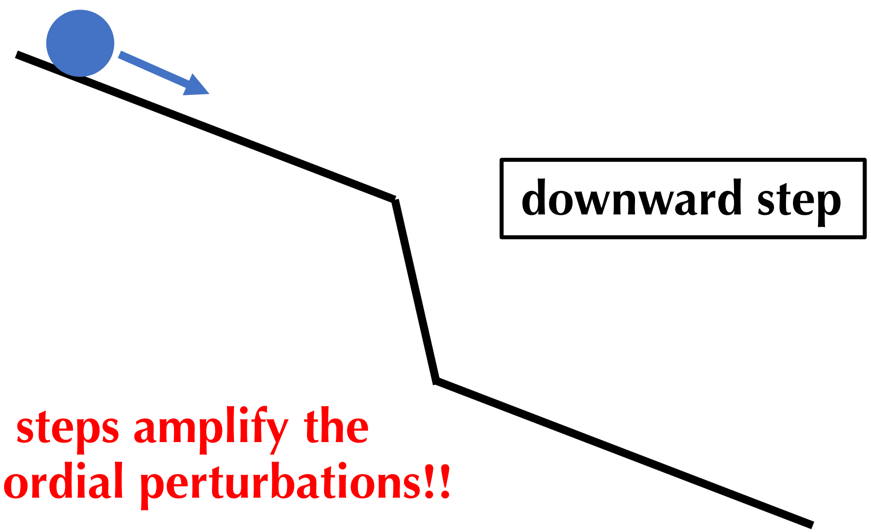
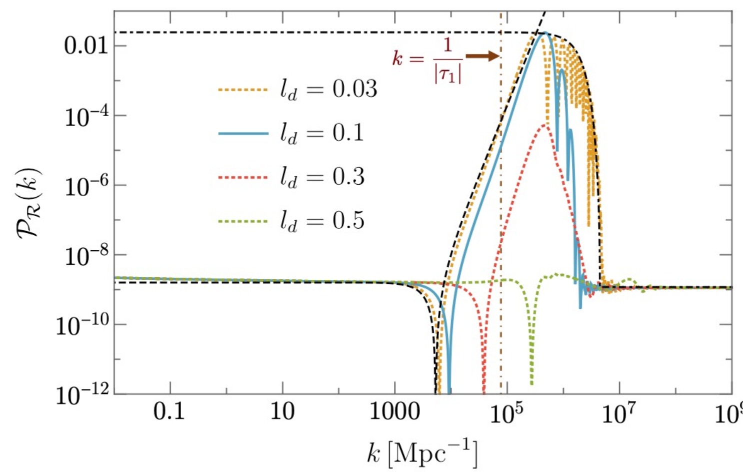
Overview

We discuss amplification of the primordial perturbation with an upward/downward step feature in the inflaton potential.









Both steps amplify the primordial perturbations!!

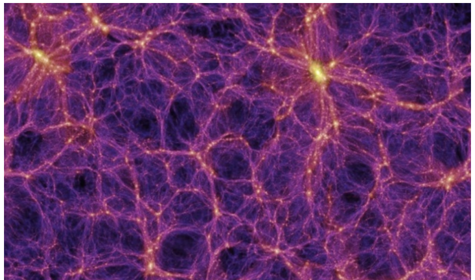
Outline

- Introduction
- Amplification of the perturbations
- Strong coupling of perturbations
- Inflaton trapping
- Summary

Cosmological perturbations

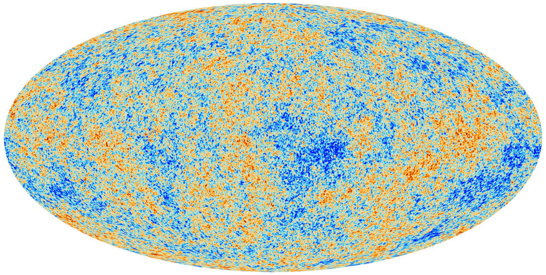
Cosmological perturbations originate from vacuum fluctuations of fields during inflation era.

Examples



(Credit: Millennium Simulation Project)

Large Scale Structure



(Credit: ESA and the Planck Collaboration)

CMB anisotropies

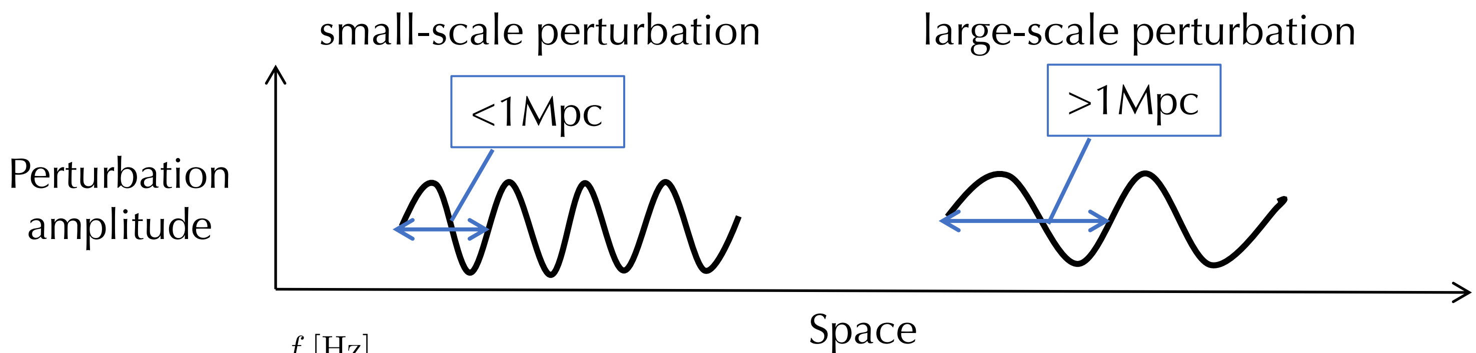
From the observations, we already know the amplitude of the perturbations on large scale ($k < 1\mathrm{Mpc}^{-1}$).

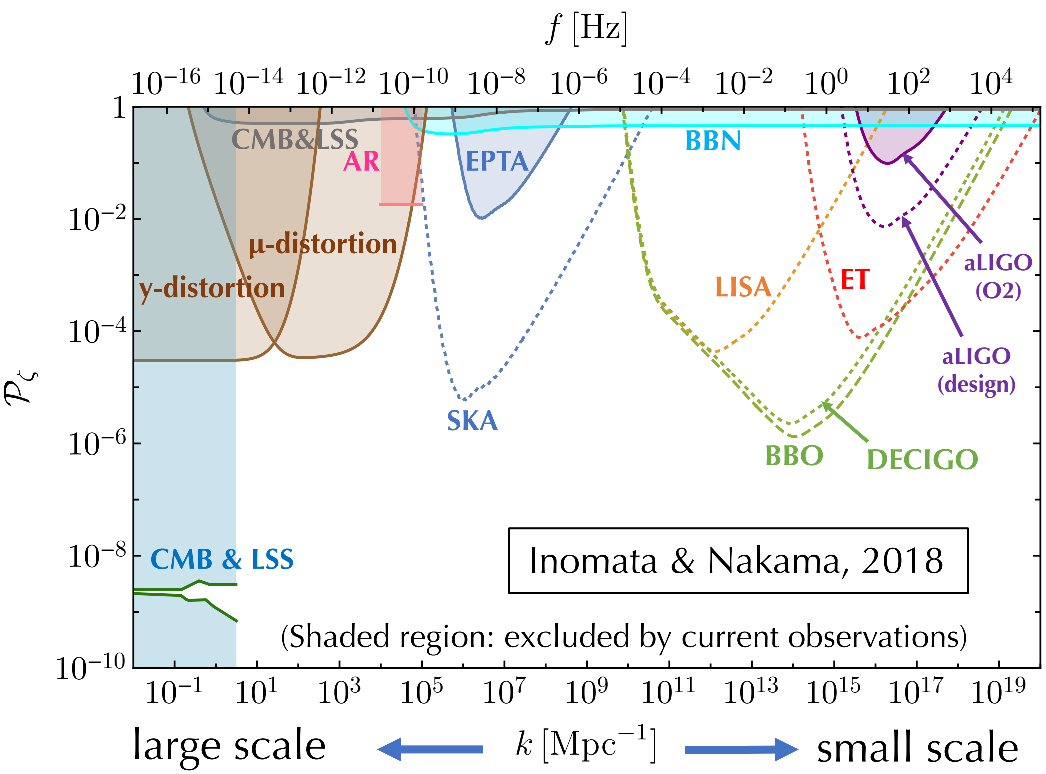
$$\mathcal{P}_{\mathcal{R}} = 2.1 \times 10^{-9} \quad \text{(Planck 2018)}$$

$$(\delta\rho/\rho \sim 10^{-5})$$

Note that the power spectrum depends on the shape of inflaton potential.

Small scale perturbations





It is difficult to measure small-scale perturbations due to Silk damping or non-linear growth of large scale structure(LSS).

On the other hand, the small-scale perturbations have attracted attention.

They can be sources of

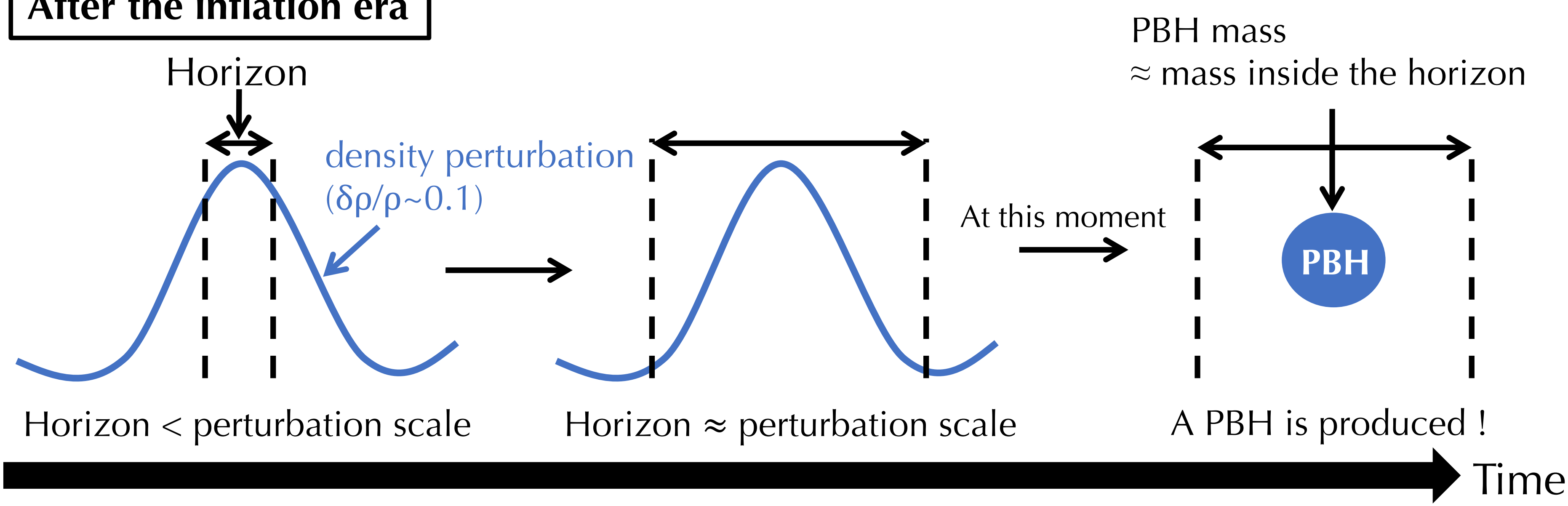
- primordial black holes
- ultra-compact mini-halos
- induced GWs
- CMB distortion

Primordial Black Hole

Primordial Black Hole (PBH) (Hawking 1971)

BH produced by a large density perturbation in the early Universe.

After the inflation era



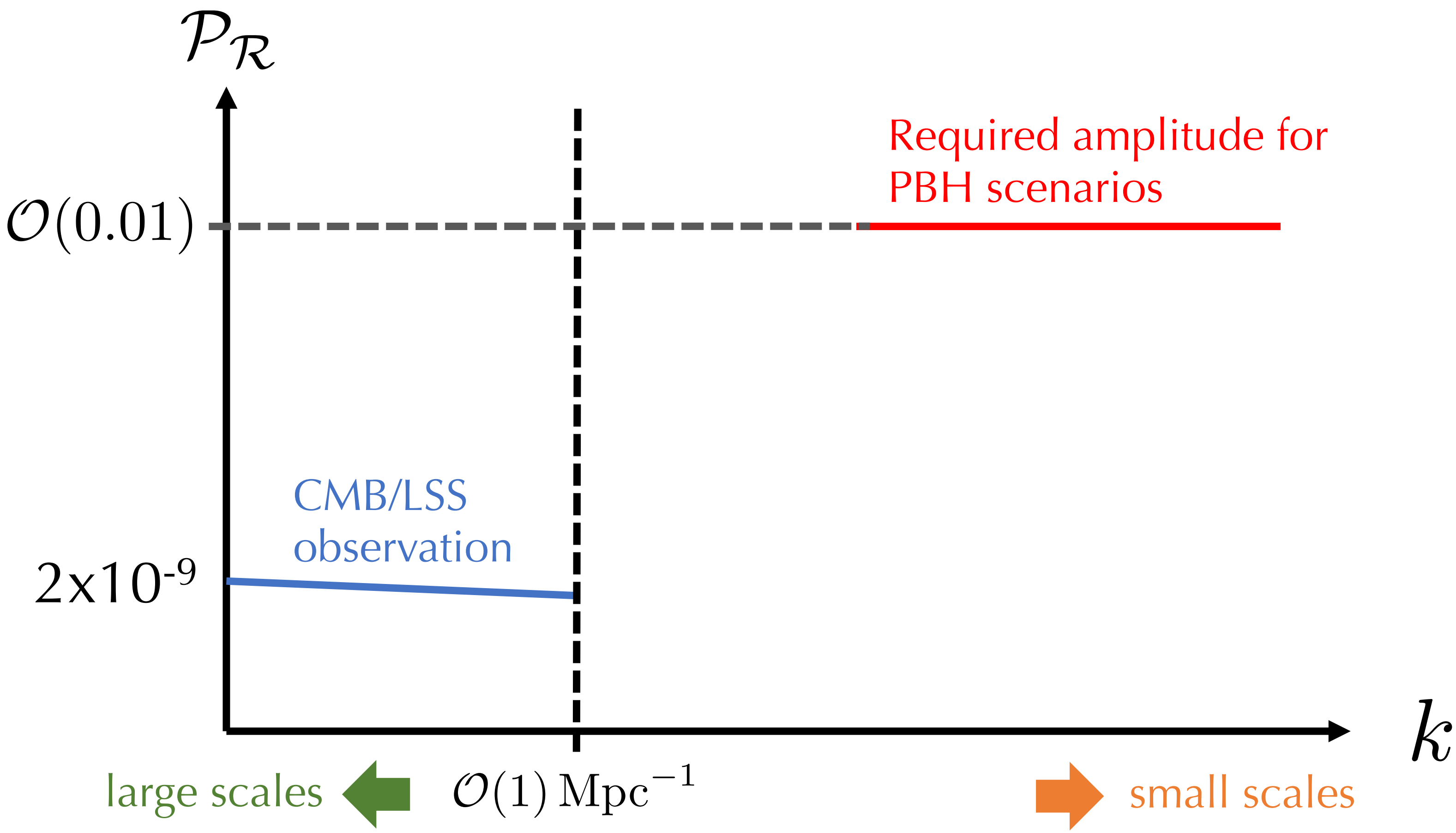
PBH mass is determined by the perturbation scales.

PBHs can have various masses:

M_{PBH}~10^{20}g for DM

M_{PBH} ~30 $M_{\odot}$ for BHs detected by LIGO-Virgo collaboration

Large perturbations for PBH scenarios



For the PBH scenarios, the enhancement of the power spectrum on small scales should be O(10^7).

Enhancement in previous works

Review (Throughout this work, we focus on single field models.)

Under the slow-roll approximation, power spectrum is given by

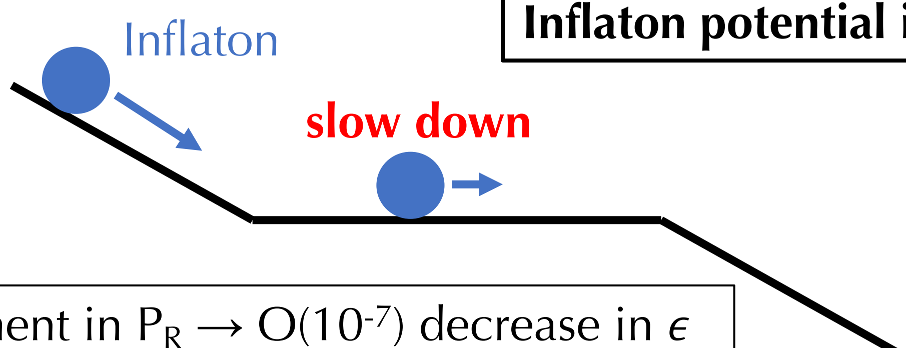
$$\mathcal{P}_\mathcal{R} = \frac{H^2}{8\pi^2 M_{\rm Pl}^2 \epsilon}, \quad \epsilon = \frac{\dot\phi^2}{2H^2 M_{\rm Pl}^2}$$

slow-roll condition
$\ddot\phi \ll V'$

Slow-down of inflaton leads to the enhancement of the power spectrum.

The large enhancement can be realized by a flat region of inflaton potential (even if the slow-roll approximation is violated). (Ivanov *et al*. 1994, Gracia-Bellido and Ruiz Morales 2017)

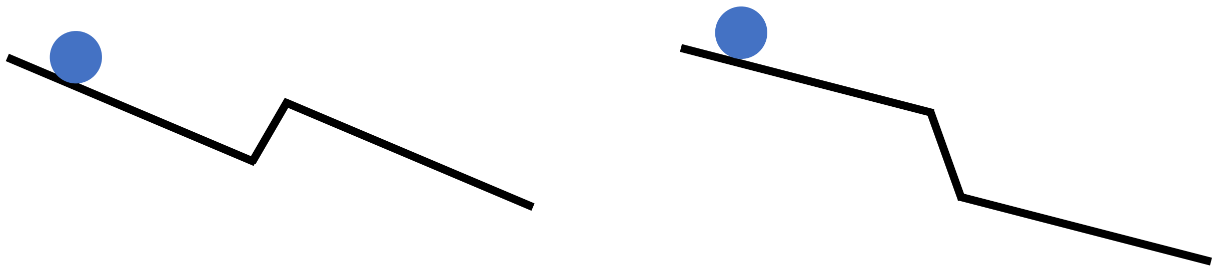
Inflaton potential in previous works



O(10^7) enhancement in P_R → O(10^{-7}) decrease in ϵ

What we discuss

In this work, putting the origin aside, we focus on the power spectrum enhancement associated with a step in the inflaton potential.



This step-like feature has been studied in the context of CMB spectrum modulation. (Chen et al. 2006, Adshead et al. 2011, Cai et al. 2015)

In this work, we discuss a step-like feature that realizes the large ($O(10^7)$) enhancement required for PBH scenarios.

Distinct feature from previous PBH studies:
The $O(10^7)$ enhancement can be realized without the $O(10^{-7})$ decrease in ϵ compared to CMB value even in single field inflation models.

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Equation for curvature perturbations

Mukhanov-Sasaki equation:

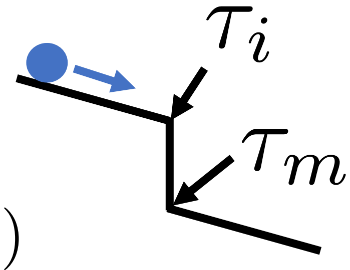
$$\mathcal{R}_k'' + (2+\eta)\, aH\mathcal{R}_k' + k^2\mathcal{R}_k = 0$$

$$\eta \equiv \frac{\mathrm{d}\ln\epsilon}{\mathrm{d}N} = \frac{\epsilon'}{aH\epsilon}$$

During the rapid rolling up/down of the step (large η),

$$\mathcal{R}_k'' + \frac{\epsilon'}{\epsilon}\mathcal{R}_k' \approx 0$$

$$\Rightarrow (\mathcal{R}_k'\epsilon)' \simeq 0$$



Then, we obtain

$$\mathcal{R}(\tau_m) = \mathcal{R}(\tau_i),\ \mathcal{R}'(\tau_m) = (\epsilon_i/\epsilon_m)\mathcal{R}'(\tau_i)$$

R′ gets
- enhanced (upward step)
- suppressed (downward step)

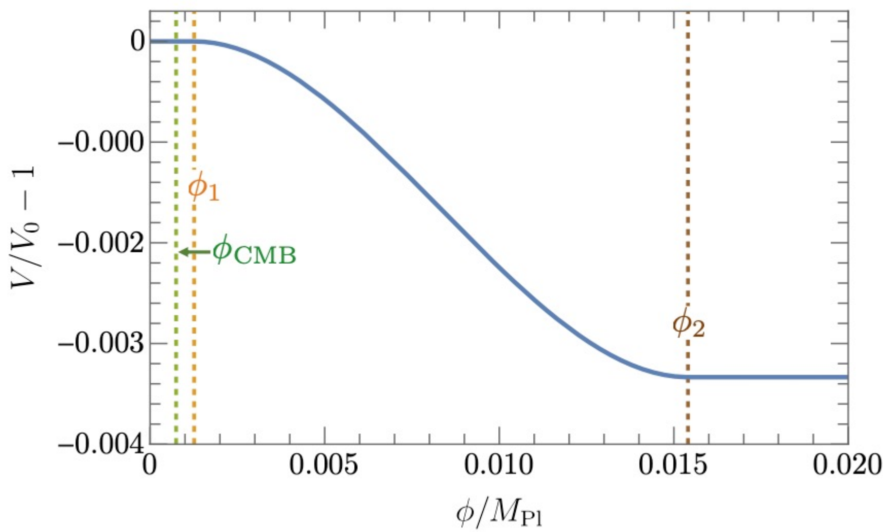
In the following, we consider the case where the potential derivative (V′) is almost the same before and after the step for simplicity.

Fiducial inflaton potential

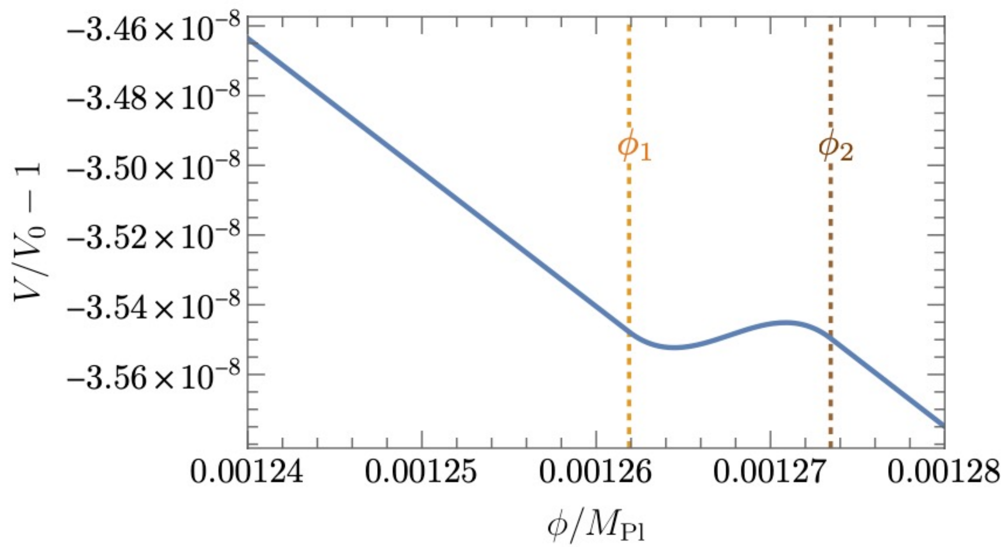
$$V(\phi) = V_b(\phi) F(\phi; \phi_1, \phi_2, h)$$

$$F(\phi; \phi_1, \phi_2, h) = 1 + h \left[S\left(\frac{\phi - \phi_1}{\phi_2 - \phi_1} \right) \Theta(\phi - \phi_1)\Theta(\phi_2 - \phi) + \Theta(\phi - \phi_2) \right]$$

$$S(x) = x^2(3 - 2x)$$

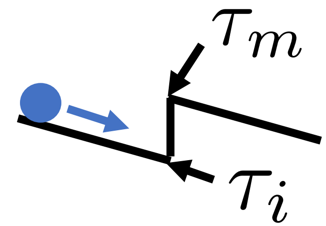


downward step



upward step

Enhancement in upward step



The epsilon changes as

instantaneously **acceleration**

$$\epsilon_i \rightarrow \epsilon_m (\ll \epsilon_i) \rightarrow \epsilon_f (\sim \epsilon_i)$$

$(\epsilon = \dot{\phi}^2/(2H^2 M_{\rm Pl}^2))$

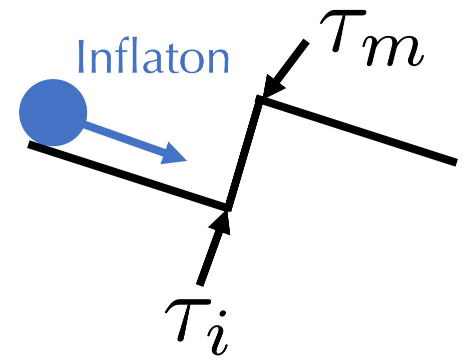
The R′ is enhanced.

$\mathcal{R}(\tau_m) = \mathcal{R}(\tau_i), \ \mathcal{R}'(\tau_m) = (\epsilon_i/\epsilon_m)\mathcal{R}'(\tau_i)$

The curvature perturbation grows following the enhancement of R′.
On the other hand, the acceleration phase decreases R, but this effect shuts off after the horizon exit.

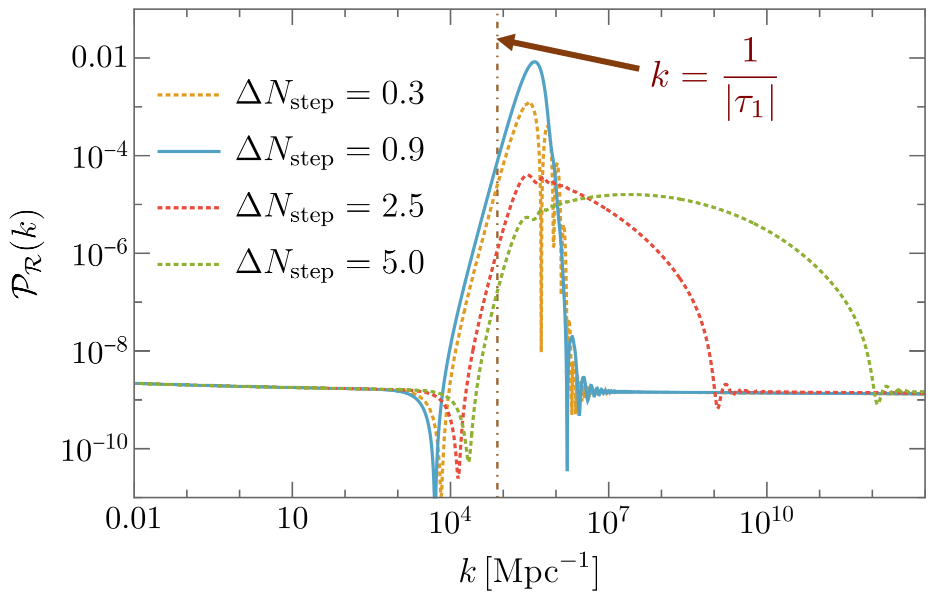
The final power spectrum is enhanced by $O((\epsilon_i/\epsilon_m)^2)$ on the peak scale up to the effect of the acceleration phase.

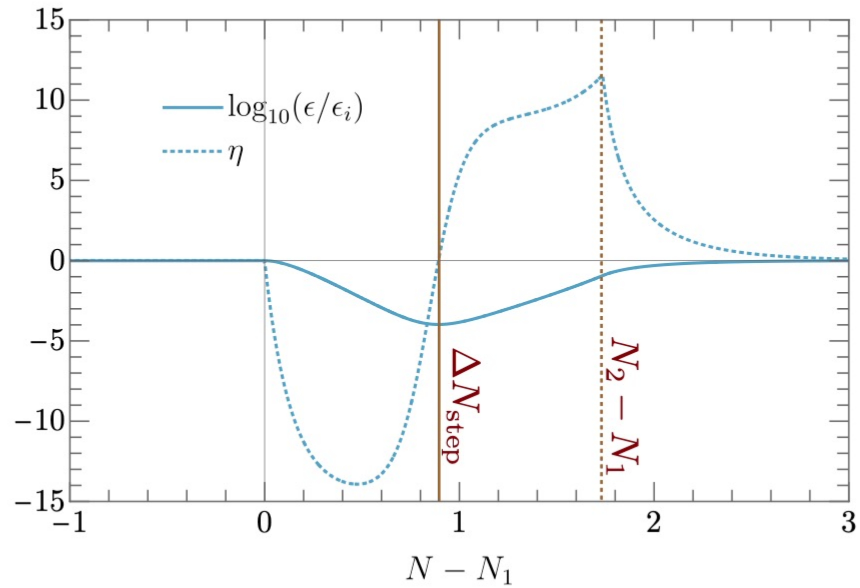
Numerical results for upward step



$\Delta N_{\rm step}$ is the e-folds for the change from $\epsilon_{\rm i}$ to $\epsilon_{\rm m}$.

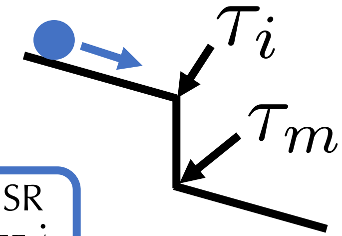
$$\epsilon_i/\epsilon_m = 10^4$$





If the step is sharp, the enhancement can be much larger than $O(\epsilon_{\rm i}/\epsilon_{\rm m})$.

Enhancement in downward step



After the downward step, too much kinetic energy of the inflaton realizes the ultra-slow-roll (USR) phase.

The epsilon changes as

during USR
$V' \ll 3H\dot{\phi}$

$$\epsilon_i \xrightarrow{\text{instantaneously}} \epsilon_m (\gg \epsilon_i) \xrightarrow{\text{USR phase}} \epsilon_f (\sim \epsilon_i)$$

$(\epsilon = \dot{\phi}^2/(2H^2 M_{\mathrm{Pl}}^2))$

Before and after the downward step, the curvature perturbation holds

$$\mathcal{R}(\tau_m) = \mathcal{R}(\tau_i),\ \mathcal{R}'(\tau_m) = (\epsilon_i/\epsilon_m)\mathcal{R}'(\tau_i)$$

$$\Rightarrow \mathcal{P}_{\mathcal{R}} = \text{const.}$$

The curvature perturbation grows during the USR period even on superhorizon scales.

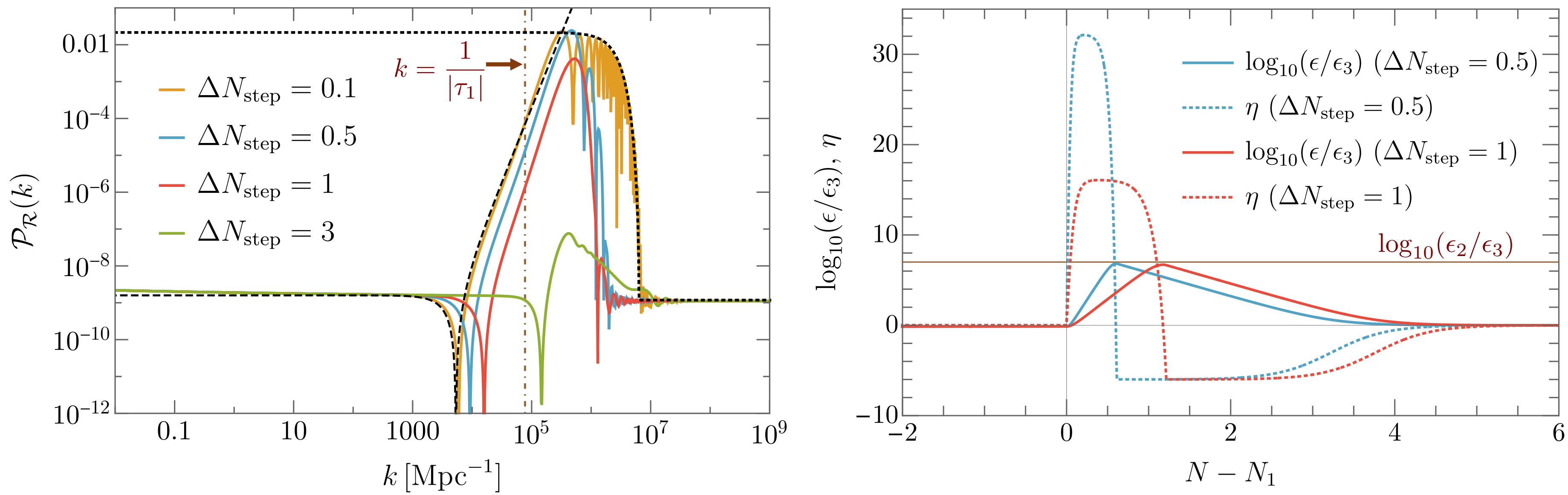
$$\Rightarrow \mathcal{P}_{\mathcal{R}} \propto \epsilon^{-1}$$

The final power spectrum is enhanced by $(\epsilon_m / \epsilon_f)$.

Numerical results for downward step

$$\epsilon_m/\epsilon_f = 10^7$$

$$\eta \equiv \frac{\mathrm{d}\ln\epsilon}{\mathrm{d}N}$$



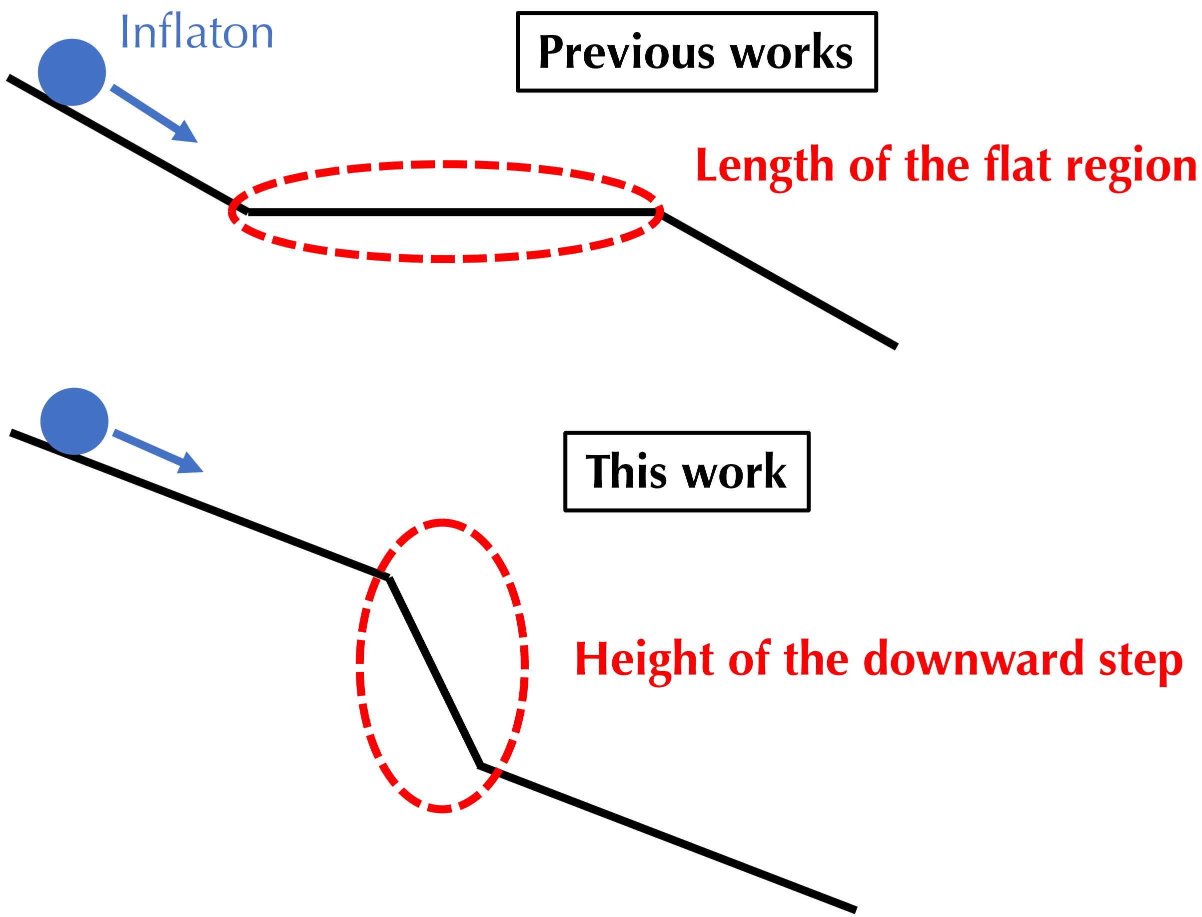
The epsilon after the transition never becomes smaller than the value before the transition. Still, power spectrum is enhanced by ($\epsilon_m/\ \epsilon_f$).

determined by the step height.

The damping of the small-scale side of the peak is determined by the cut-off of the particle production.

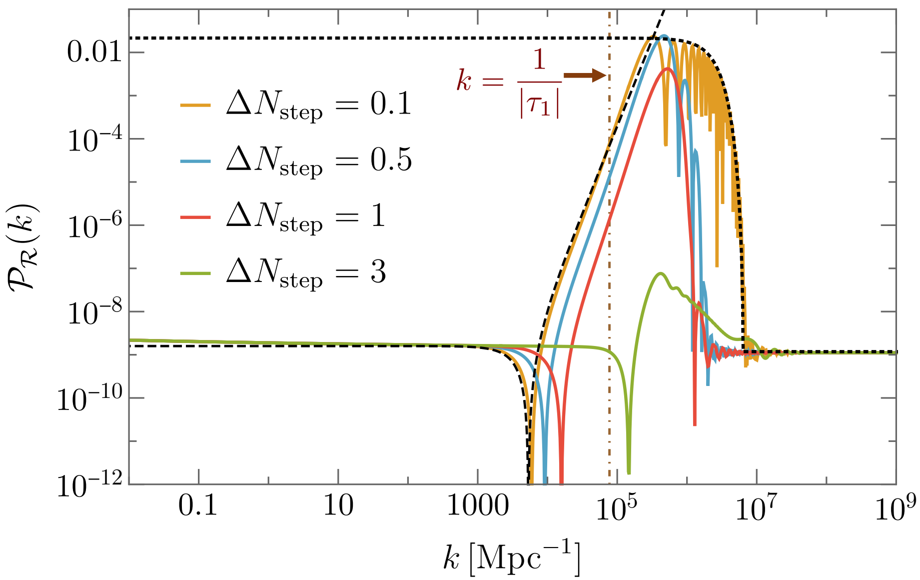
Difference from previous works 1

The order of the perturbation enhancement is determined by

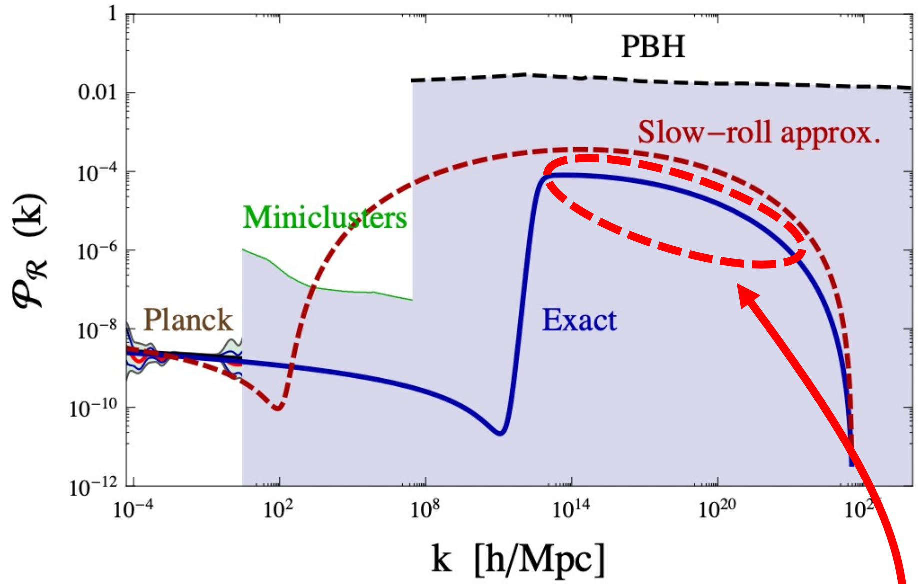


Difference from previous works 2

our work

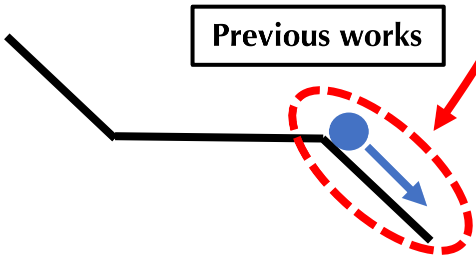


previous work (Garcia-Bellido, Ruiz Morales, Phys.Dark Univ. 18 (2017) 47-54)



Our enhancement mechanism can realize a sharp damping on the small-scale side of the peak.

Unlike in our model, the damping in previous works is mainly determined by the potential after the flat region.



Physical interpretation

This enhancement can be associated with the particle production due to the non-adiabatic evolution of the inflaton at the step.

particle production = excitation of negative mode ($e^{ik\tau}$) from positive mode ($e^{-ik\tau}$)

Without the particle production, the enhancement cannot be much larger than $O(\epsilon_i / \epsilon_m)$ in the upward step and no enhancement occurs in the downward step.

The ``non-adiabatic'' means

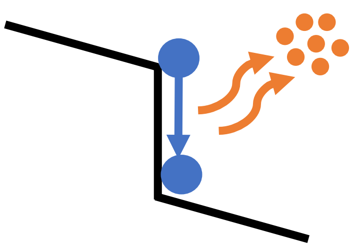
(time scale of the step transition) $\ll$ **(oscillation time scale of perturbation: 1/k)**

curvature perturbation → $\mathcal{R} \simeq \frac{\delta\phi}{2\sqrt{\epsilon}}$ ← inflaton fluctuation

$$(\epsilon = \dot{\phi}^2/(2H^2 M_{\rm Pl}^2))$$

This phenomenon can be interpreted as the inflaton particle production.

On very small scales, the particle production does not occur, which leads to the sharp cutoff of the enhancement.



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Strong coupling of perturbations

Sharp features of the potential can lead to the strong coupling of the perturbations.

E.o.m for the inflaton perturbations:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n=3}\frac{1}{(n-1)!}V^{(n)}(\delta\phi)^{n-1}$$

The right-hand side comes from the higher order contribution, which cannot be neglected in the case of the potential with a very sharp step feature.

In perturbations are strongly coupled, the perturbation theory is no longer reliable. In this case, lattice calculation is required.

Our concern:

We might not be able to conclude that O(10^7) enhancement can be realized by a step-like feature with the linear theory due to the strong coupling.

What we will see:

Our fiducial model can avoid the strong coupling issue with appropriate modification.

In other words, there is at least one potential that can realize the O(10^7) enhancement without strong coupling.

Check of strong coupling 1

The higher-order Lagrangian of the curvature perturbation is given by

$$\mathcal{L}_n(\pi) \simeq -\frac{M_{\rm Pl}^2}{(n-2)!} H^{(n-1)} \pi^{n-2} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) \qquad (H^{(n)} \equiv \partial_t^n H)$$

$\mathcal{R} \simeq -H\pi$ (The approximation is valid in the case of $\epsilon << 1$)

We impose the following condition for the weak coupling:

$$\left| \frac{\mathcal{L}_n}{\mathcal{L}_2} \right| < 1 \quad (\text{for all } n (> 2))$$

$$\Rightarrow \quad \left| \frac{H^{(n-1)}}{(n-2)! H^{n-2} \dot{H}} \mathcal{R}^{n-2}(\tau) \right| < 1 \quad (\text{for all } n(> 2)).$$

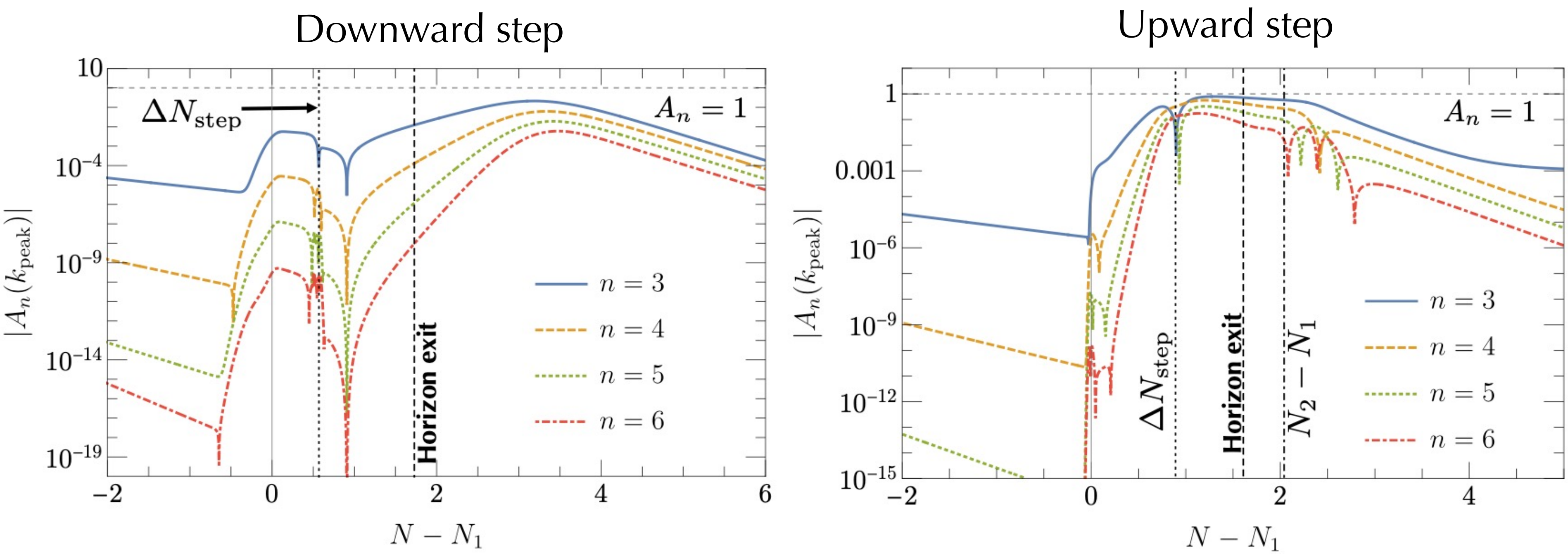
Here, we take the following approximation:

$$\mathcal{R}^{n-2} \to \left\langle \mathcal{R}^{2(n-2)} \right\rangle^{1/2} = \sqrt{(2n-5)!!} \left\langle \mathcal{R}^2 \right\rangle^{(n-2)/2} \simeq \sqrt{(2n-5)!!} \mathcal{P}_{\mathcal{R}}^{(n-2)/2}(k_{\rm peak})$$

Finally, we obtain the following condition:

$$A_n(k_{\rm peak}, \tau) < 1 \qquad A_n(k,\tau) \equiv \frac{H^{(n-1)}}{(n-2)! H^{n-2} \dot{H}} \sqrt{(2n-5)!!}\, \mathcal{P}_{\mathcal{R}}^{(n-2)/2}(k,\tau)$$

Check of strong coupling 2



After the smoothing, both cases avoid the strong coupling, though the upward step case is marginal.

Note for experts

For the case of n>4, A_n includes third or higher derivative of potential. To realize $A_n<1$ for n>4, the smoothing of the potential around ϕ_1 and ϕ_2 must be performed.
(The above figures are the results after the smoothing. We have checked the smoothing does not affect the power spectrum.)

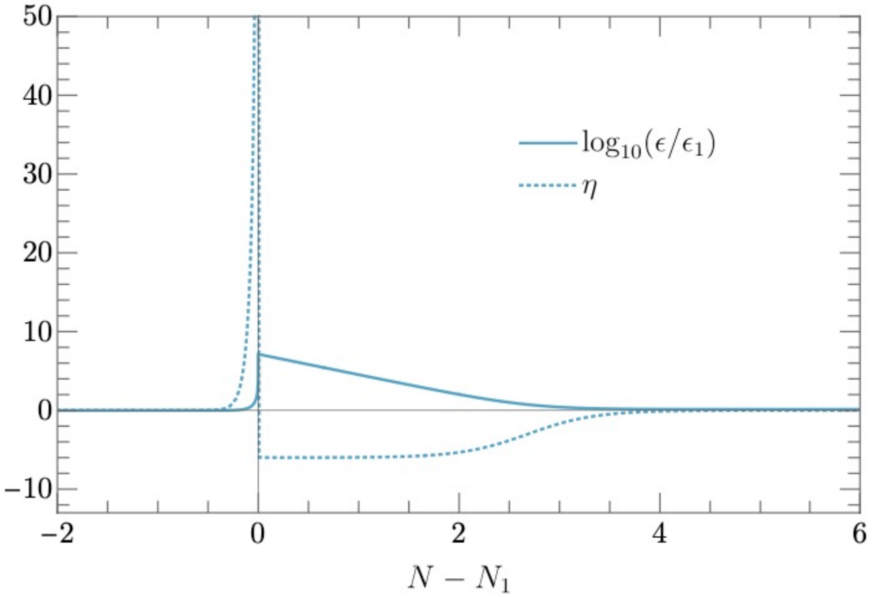
Tanh step

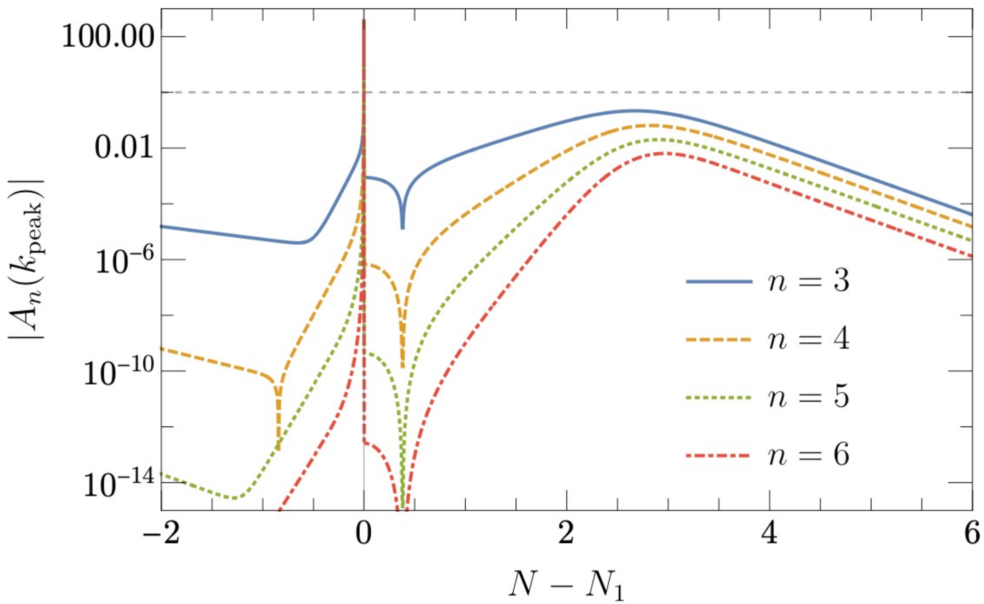
For comparison, we consider the often-used step form: (e.g. Adshead et al. 2011)

$$V(\phi) = V_b(\phi) F(\phi; \phi_1, \phi_2, h)$$

$$F(\phi; \phi_1, \phi_2, h) = 1 + \frac{h}{2}\left[1 + \tanh\left(\frac{\phi - \phi_1}{\phi_2 - \phi_1}\right)\right]$$

The epsilon in the downward step changes with the timescale much smaller than the Hubble time at that time due to the acceleration of the rolling down inflaton.





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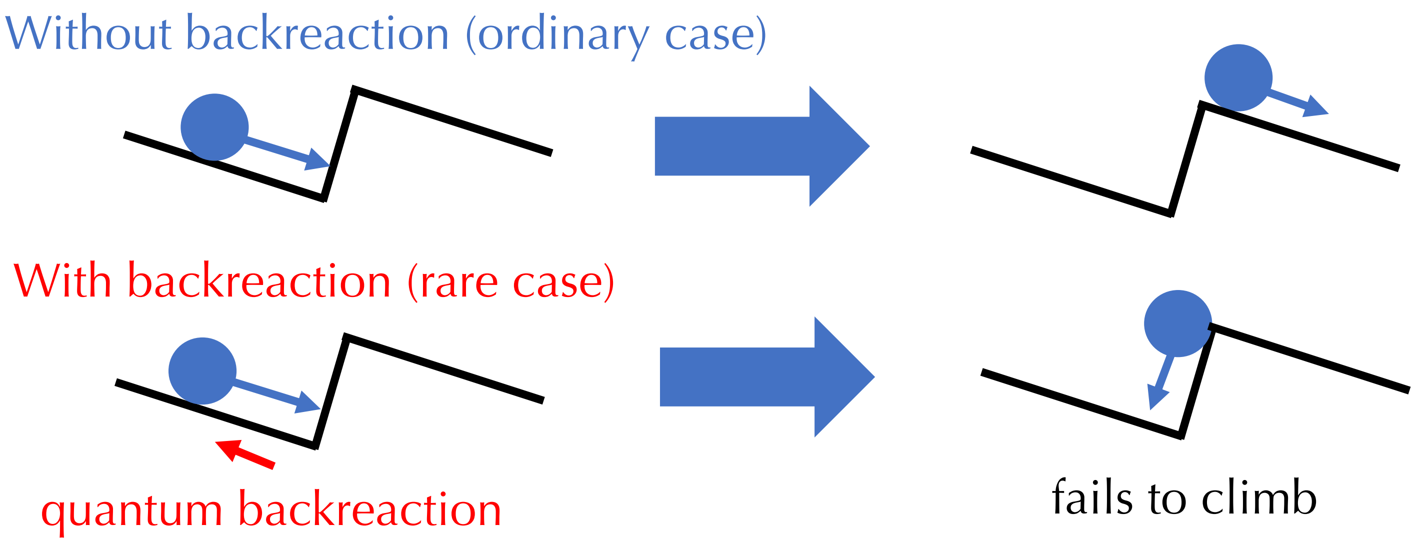
Trapping of inflaton

In the upward step case, the inflaton can be trapped because the quantum backreaction to the classical dynamics decrease the kinetic energy at the beginning of the step.

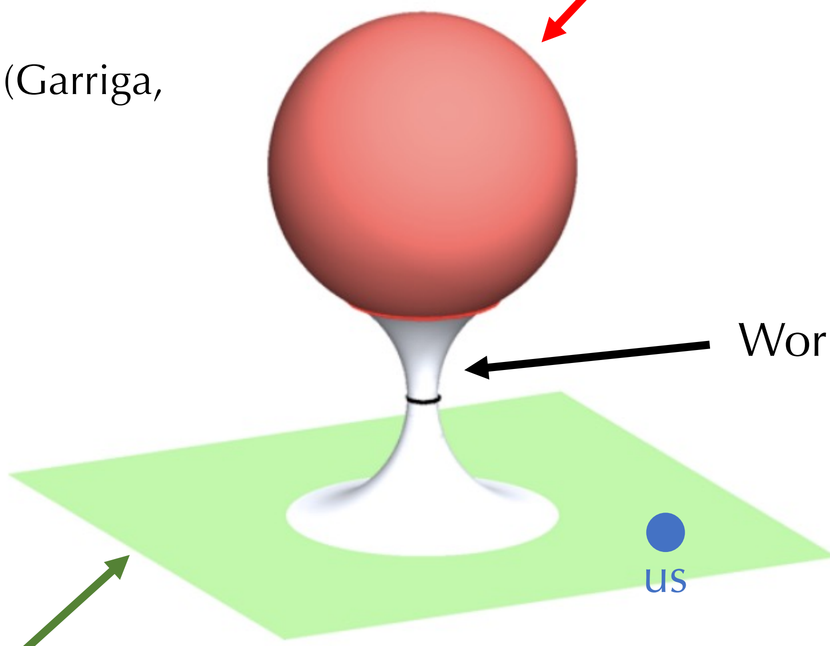


While the inflation does not end in the Inflaton-trapped region, it does in the other ordinary and the RD or MD era begins there.

PBH production

Exponentially expanding region (trapped-inflaton region)

From JCAP 02 (2016) 064 (Garriga, Vilenkin, Zhang)



Worm hole

RD or MD era region (The other ordinary region)

The trapped-inflaton region can be finally seen as a PBH from an outside observer in the other ordinary areas. (Atal et al. 2019)

Inflaton trapping rate = PBH production rate

Quantum backreaction

The variance of the kinetic energy fluctuation caused by quantum backreaction is

$$\left\langle (\Delta K)^2 \right\rangle \equiv \left\langle \left(\frac{1}{2} \left(\dot{\bar{\phi}} + \Delta\dot{\phi} \right)^2 - \frac{1}{2} \dot{\bar{\phi}}^2 \right)^2 \right\rangle$$
$$\simeq \dot{\bar{\phi}}^2 \left\langle (\Delta\dot{\phi})^2 \right\rangle$$
$$= \frac{3H^4}{4\pi^2} \bar{K}$$

From Biagetti et al. (2018)

$$\left\langle (\Delta\dot{\phi})^2 \right\rangle = \frac{3H^4}{8\pi^2}$$

$$\bar{K} \equiv \frac{1}{2} \dot{\bar{\phi}}^2 (= \epsilon_i H^2 M_{\rm Pl}^2)$$

If we neglect the backreaction and tune the potential so that the epsilon changes as $\epsilon_i \rightarrow \epsilon_m$, the inflaton has a redundant kinetic energy for the climbing the step by $O(\epsilon_m H^2 M_{Pl}^2)$.

So, if the backreaction is

$$-\Delta K > \epsilon_m H^2 M_{\rm Pl}^2$$

the inflaton fails to climb the step and is trapped at the local minimum.

PBH production rate 1

The inflaton trapping condition:

$$-\Delta K > \epsilon_m H^2 M_{\rm Pl}^2$$

For convenience, we define

$$E_K \equiv \Delta K / \bar{K}$$

The variance is given by

$$\sigma_{E_K}^2 \equiv \langle E_K^2 \rangle = \frac{3H^2}{4\pi^2 \epsilon_i M_{\rm Pl}^2}$$
$$= 6\mathcal{P}_{\mathcal{R}}(k \ll -1/\tau_1) \simeq 6\mathcal{P}_{\mathcal{R}}(k_{\rm CMB})$$

The PBH production rate is given by

$$\beta_{\rm trap} = \int_{-\infty}^{-\epsilon_m/\epsilon_i} {\rm d}E_K \frac{1}{\sqrt{2\pi}\sigma_{E_K}} \exp\left(-\frac{E_K^2}{2\sigma_{E_K}^2}\right)$$
$$\simeq \frac{1}{\sqrt{2\pi}} \frac{\sigma_{E_K}}{\epsilon_m/\epsilon_i} \exp\left(-\frac{(\epsilon_m/\epsilon_i)^2}{2\sigma_{E_K}^2}\right)$$

PBH production rate 2

The large perturbations themselves can also produce PBHs when they enter the horizon. The production rate is given by

$$\beta_p(M_{\text{peak}}) \simeq \frac{(2.0\mathcal{P}_{\mathcal{R}}(k_{\text{peak}}))^{1/2}}{\sqrt{2\pi}} \exp\left(-\frac{1}{3.9\mathcal{P}_{\mathcal{R}}(k_{\text{peak}})}\right)$$

On the other hand, the PBH production rate from the inflaton trapping is

$$\beta_{\text{trap}} \simeq \frac{1}{\sqrt{2\pi}} \frac{\sigma_{E_K}}{\epsilon_m/\epsilon_i} \exp\left(-\frac{(\epsilon_m/\epsilon_i)^2}{2\sigma_{E_K}^2}\right)$$

$$\sigma_{E_K}^2 \equiv \langle E_K^2 \rangle = \frac{3H^2}{4\pi^2 \epsilon_i M_{\text{Pl}}^2} = 6\mathcal{P}_{\mathcal{R}}(k \ll -1/\tau_1)$$

Since $\mathcal{P}_{\mathcal{R}}(k_{\text{peak}}) < (\epsilon_i/\epsilon_m)^2 \mathcal{P}_{\mathcal{R}}(k \ll -1/\tau_1)$,

we have $\beta_{\text{trap}} > \beta_p$

Note that this conclusion can be changed if we take into account the non-Gaussianity of the perturbations (see e.g. arXiv: 2112.13836 (Cai et al.)).

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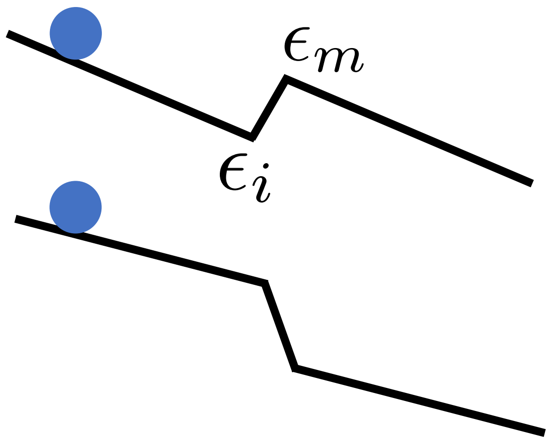
Summary

- We have discussed the perturbation amplification with a step-like feature.



In the upward step case, the perturbation enhancement can be $O((\epsilon_i/\epsilon_m)^2)$, which is much larger than the slow-roll case, $O(\epsilon_i/\epsilon_m)$.

In the downward step case, the perturbation enhancement can be $O(\epsilon_m/\epsilon_f)$ even if ϵ never becomes smaller than the CMB value.

These large enhancements are associated with the particle production caused by the non-adiabatic evolution of the inflaton.

- We have checked that our fiducial potential can avoid the strong coupling issue, though the often-used tanh downward step cannot.

- We have calculated the PBH production rate from the inflaton trapping due to the quantum backreaction in the upward step case.