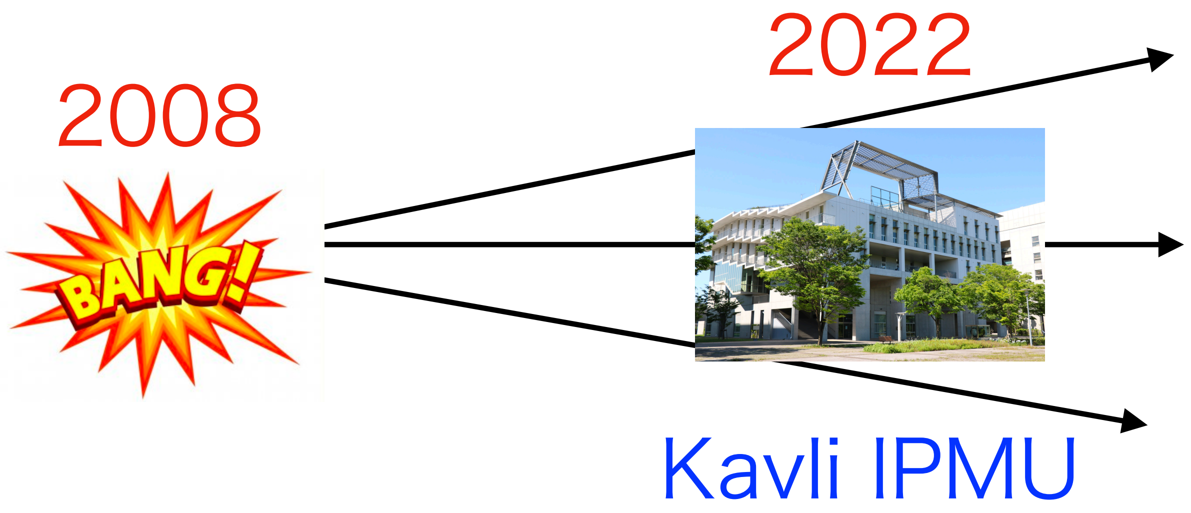
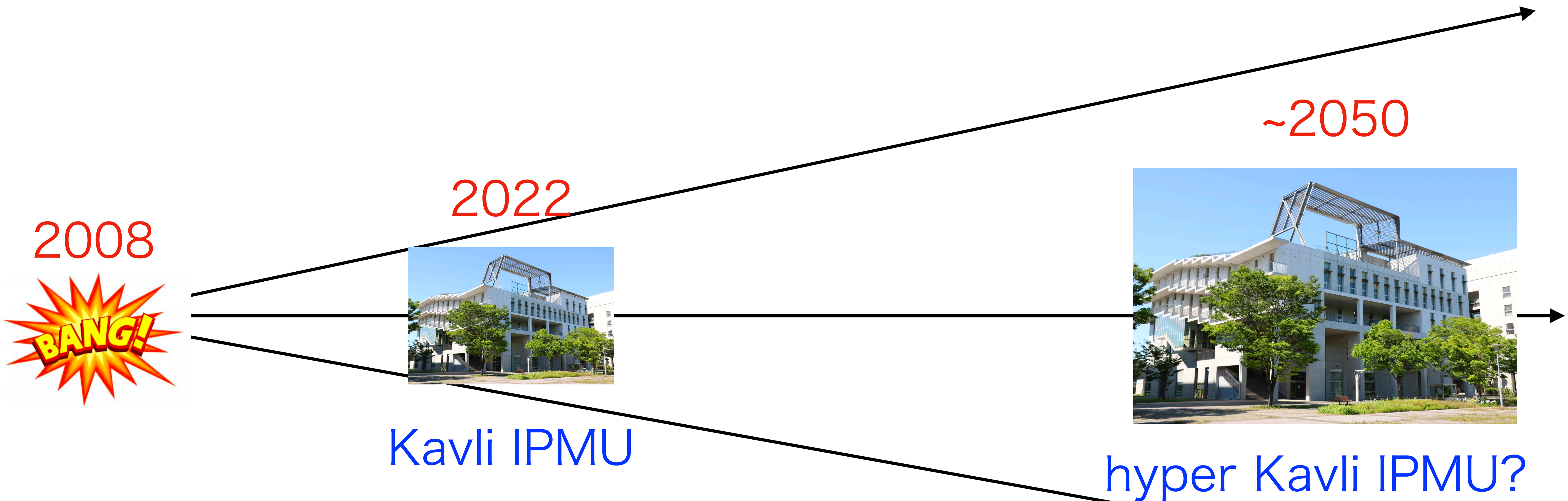
Exploring θ-vacua of Yang-Mills Theories

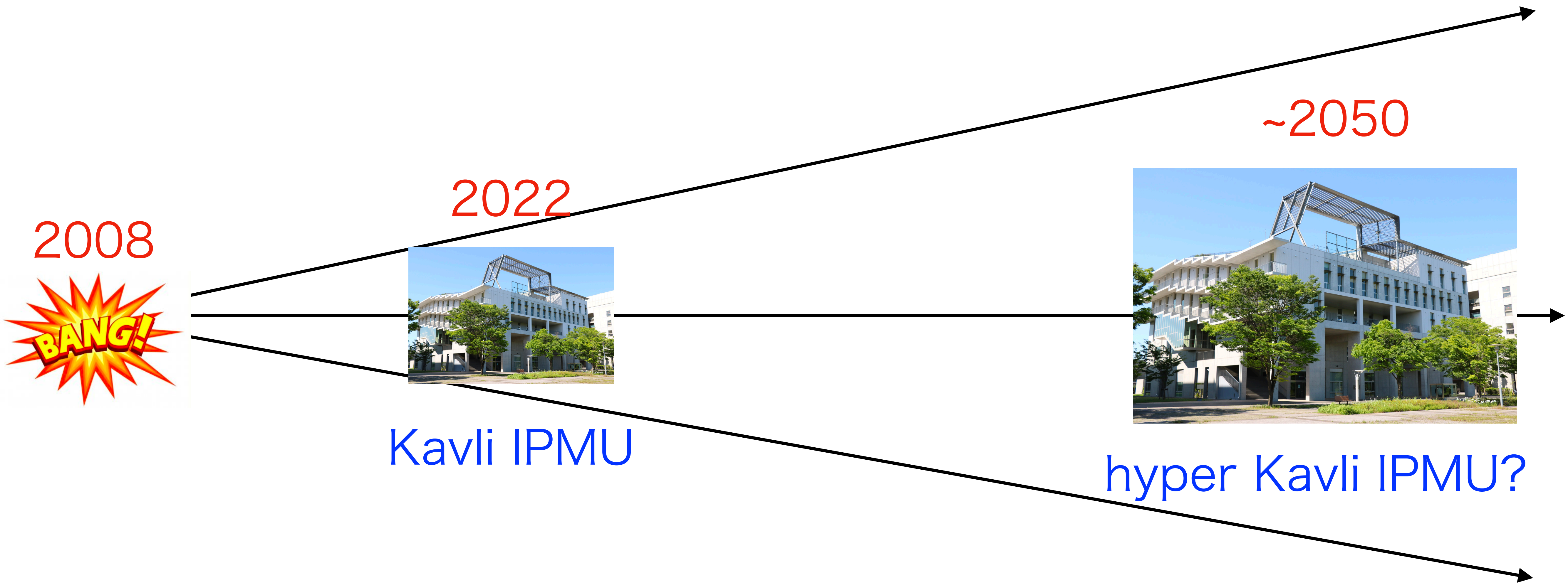
Masahito Yamazaki

Feb/9/2021 KAVLI IPMU colloquium

1. Introduction

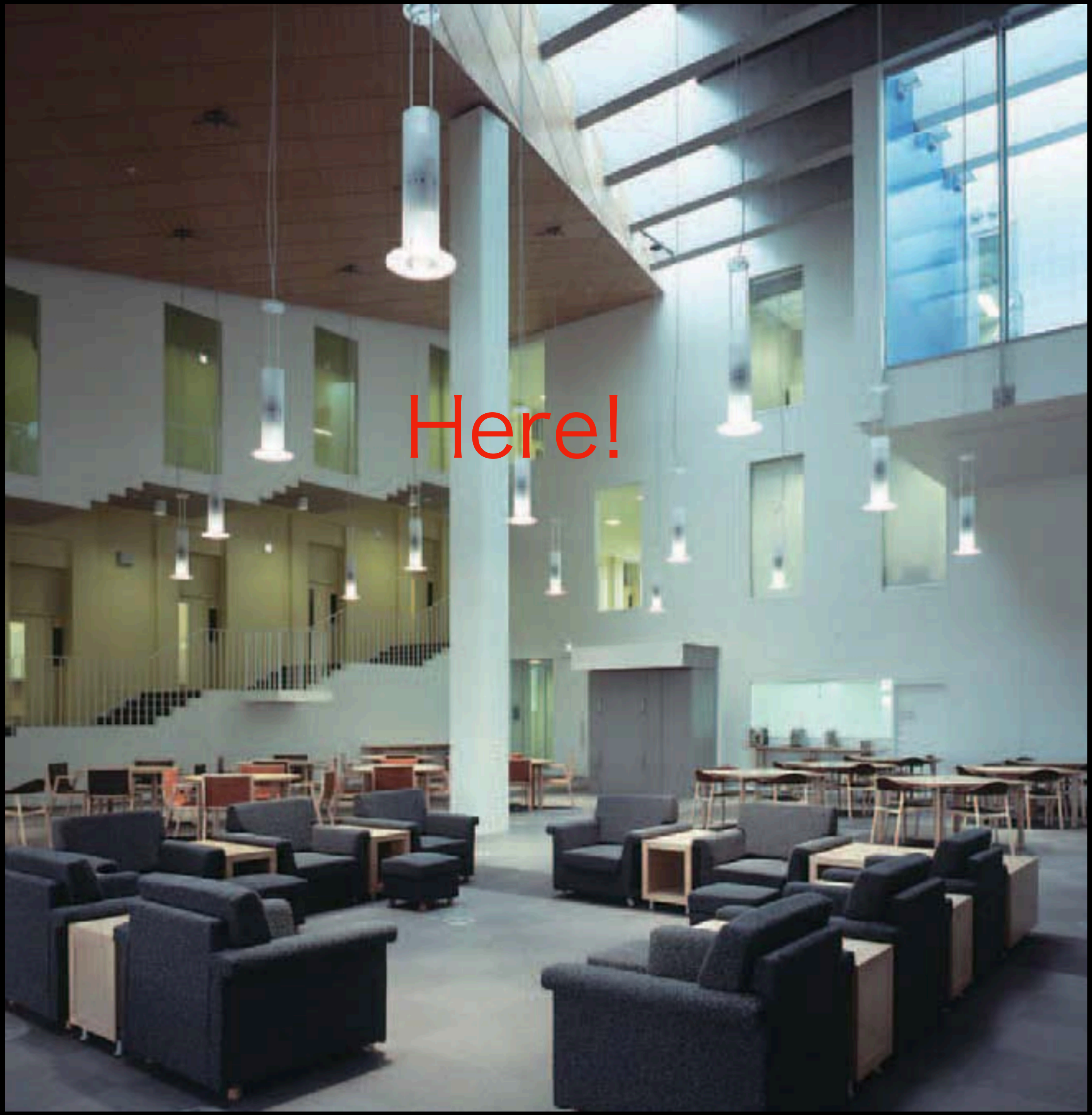

2008
BANG!
2022
Kavli IPMU


~2050
2022
2008
BANG!
Kavli IPMU
hyper Kavli IPMU?



「初心忘るべからず」

“Remember the spirits that you started with”



Galileo

“The Universe is written in the language of mathematics”

… except this is sometimes not obvious!

⋯ except this is sometimes not obvious!

Non-Abelian gauge theory 1954

(Yang & Mills, also Shaw, Utiyama)

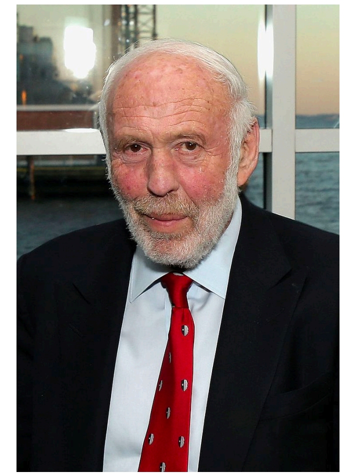
… except this is sometimes not obvious!

Non-Abelian gauge theory 1954

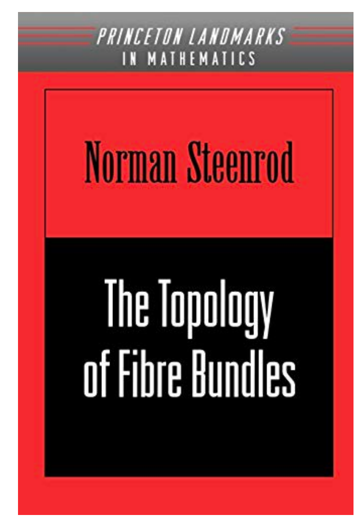
(Yang & Mills, also Shaw, Utiyama)

Conversation with Jim Simons

late 60's - early 70's



connections on fiber bundles



Steenrod's textbook 1951

··· except this is sometimes not obvious!

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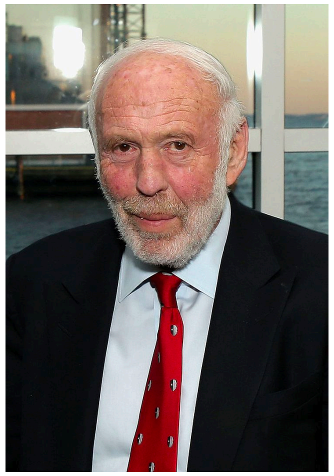

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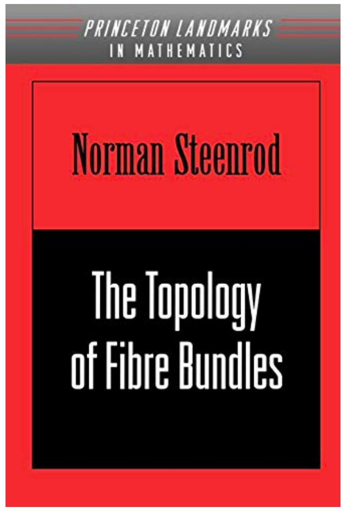


Steenrod's textbook 1951

. . . we are concerned with the necessary *concepts to describe the physics of gauge theories.* It is remarkable that these concepts have already been studied as mathematical constructs.

Tai Tsun Wu and Chen Ning Yang (1975)

Abelian gauge theory goes back to electromagnetism

··· but only as a "tool"

$$\mathbb{E} = -\nabla\phi - \frac{\partial \mathbb{A}}{\partial t}, \quad \mathbb{B} = \nabla \times \mathbb{A}$$

electric field ($\mathbb{E}$), scalar potential (ϕ), magnetic field ($\mathbb{B}$), vector potential ($\mathbb{A}$)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (F = dA)$$

gauge "symmetry" as a principle

Weyl '29

$$\phi(x) \to e^{i\alpha}\phi(x)$$ global symmetry

$$\phi(x) \to e^{i\alpha(x)}\phi(x)$$ gauge symmetry (redundancy)

Yang-Mills theory: non-Abelian gauge group

physicists "catching up" with mathematicians

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$\mathcal{L} = \frac{1}{4g^2} \int \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}$$

$$A_\mu = \sum_a A_\mu^a \underbrace{t^a}_{\text{matrix}}$$

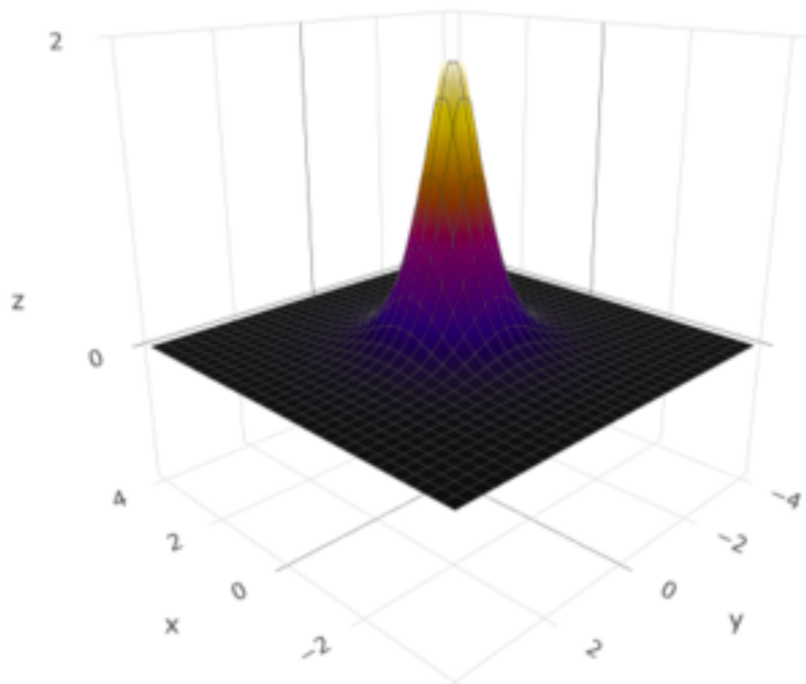
Initially a "wrong theory" but Higgs mechanism and confinement helps

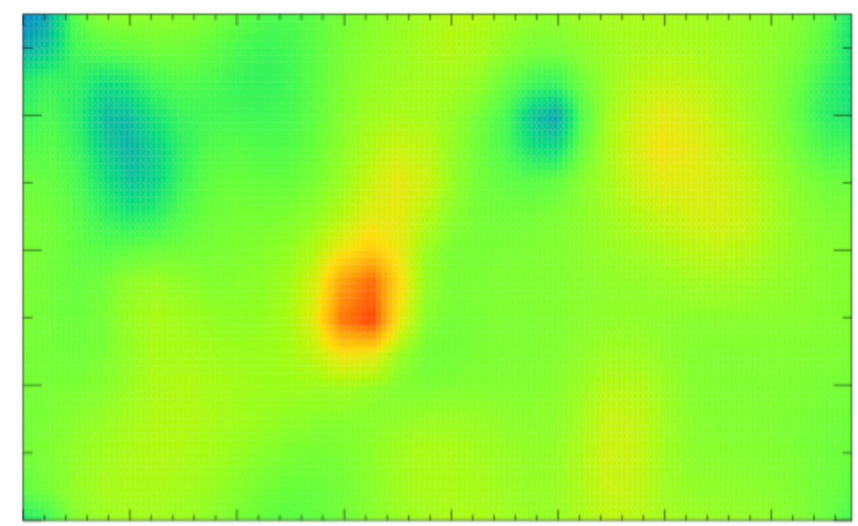
$$G_{SM} = \Big(U(1) \times \underbrace{SU(2)}_{\text{weak force}} \times \underbrace{SU(3)}_{\text{strong force}} \Big) / \mathbb{Z}_6$$

Topological Sectors: Instantons

Belavin-Polyakov-Schwarz-Tyupkin '75
also Atiyah-Hitchin-Singer '77 '78
Atiyah-Drinfeld-Hitchin-Manin '78

localized both in space and time





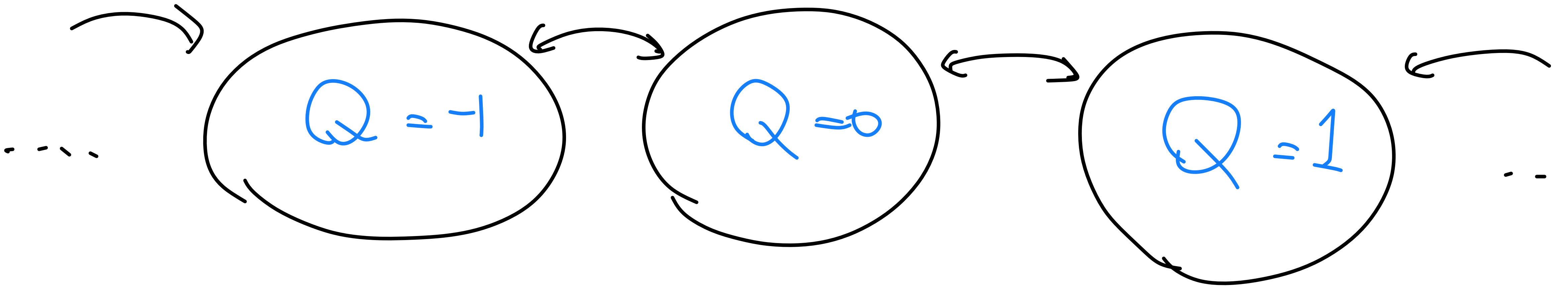
Topological charge: instanton number

$$Q = \frac{1}{32\pi^2}\int_{\mathbb{R}^4\cup\{\infty\}} \mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \int_{\mathbb{R}^4\cup\{\infty\}} ch_2 \in \mathbb{Z}$$

cf. Chern-Weil theory for characteristic classes

Different topological sectors weighted by the θ-angle

Callan-Dashen-Gross '76



$$\mathcal{L} \supset \frac{\theta}{32\pi^2}\int \mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \theta Q$$

$$\Downarrow$$

$$e^{i\mathcal{L}} \supset e^{i\theta Q}$$

- $\theta \sim \theta + 2\pi$
- ~~CP~~ unless $\theta = 0, \pi$!

2. Revisiting θ-vacua of Yang-Mills Theories

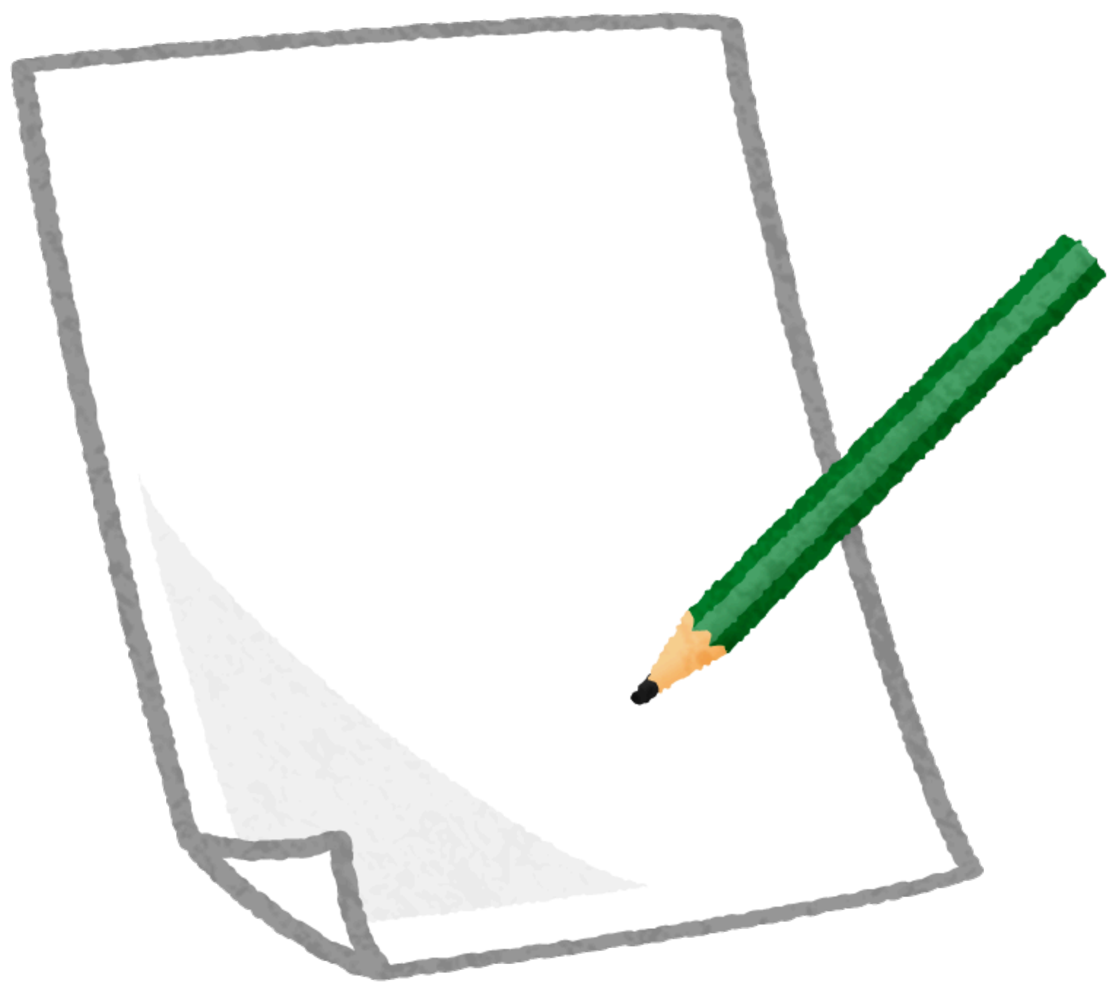
Unsolved Problems Discussed Today

Consider 4d SU(N) pure YM theory w/ θ-angle

Q: Free Energy $F(\theta) = -\frac{1}{V} \ln \frac{Z(\theta)}{Z(0)}$ as a function of θ?

Q: Fate of CP-sym. @ $\theta = \pi$?

Q: gapped (confinement) or gapless?



Instanton Analysis

t' Hooft '76

$$F(\theta) \sim \int^{\rho \to \infty : IR} \frac{d\rho}{\rho^5} e^{-\frac{8\pi^2}{g^2(\mu)}} (\mu\rho)^{\frac{11N}{3}} (1 - \cos\theta) + \cdots\cdots$$

(multi instanton)

※ 2π - periodic, CP preserved

※ Not correct in general ! (for $T \ll T_c$)

divergence as $\rho \to \infty$; IR problem

Large N

't Hooft '73,⋯, Witten '80

$$\mathcal{L} \sim \frac{1}{g^2} \mathrm{Tr}\, F \wedge * F + \theta\, \mathrm{Tr}\, F \wedge F$$

"$\hbar \sim \frac{1}{N} \ll 1$"

$$\sim \frac{1}{N^{-1}} \left(\underbrace{\frac{1}{g^2 N}}_{\text{fixed}} \mathrm{Tr}\, F \wedge * F + \underbrace{\frac{\theta}{N}}_{\text{fixed}} \mathrm{Tr}\, F \wedge F \right)$$

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$$\sim \frac{1}{N^{-1}} \Big(\underbrace{\frac{1}{g^2 N}}_{\text{fixed}} \mathrm{Tr}\, F \wedge * F + \underbrace{\frac{\theta}{N}}_{\text{fixed}} \mathrm{Tr}\, F \wedge F \Big)$$

$$E(\theta) = N^2 f\left(\frac{\theta}{N}\right) = \frac{1}{2} \chi \theta^2 \left(1 + b_2 \theta^2 + \cdots\right)$$

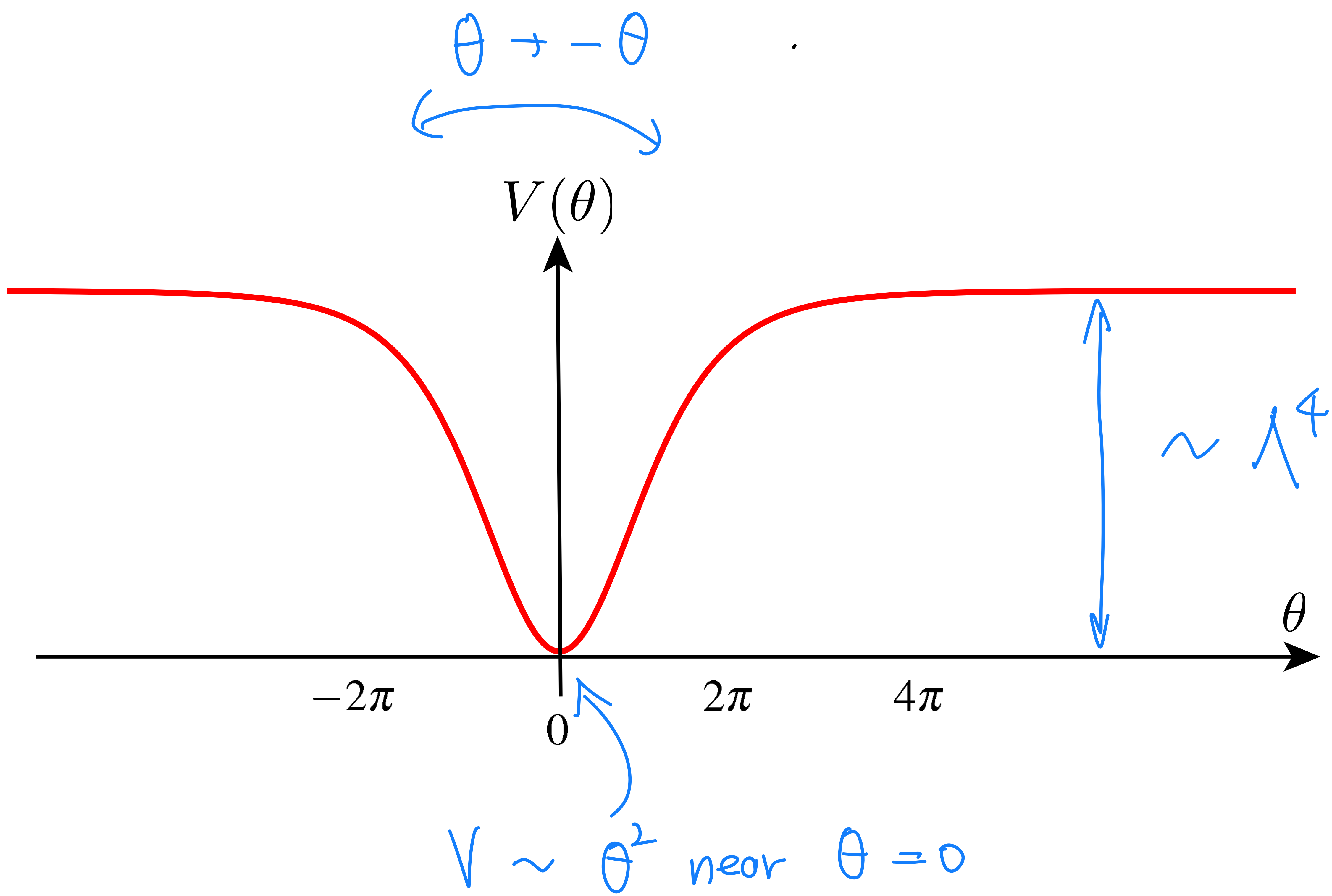
$$\chi = \chi^{(0)} + O\left(\frac{1}{N^2}\right) \to \chi^0$$

$$b_{2n} = \frac{b_{2n}^{(0)}}{N^{2n}} + O\left(\frac{1}{N^{2n+2}}\right) \to 0$$

※ NOT 2π-periodic

Expectation for large but finite N

Based on several papers by MY and collaborators



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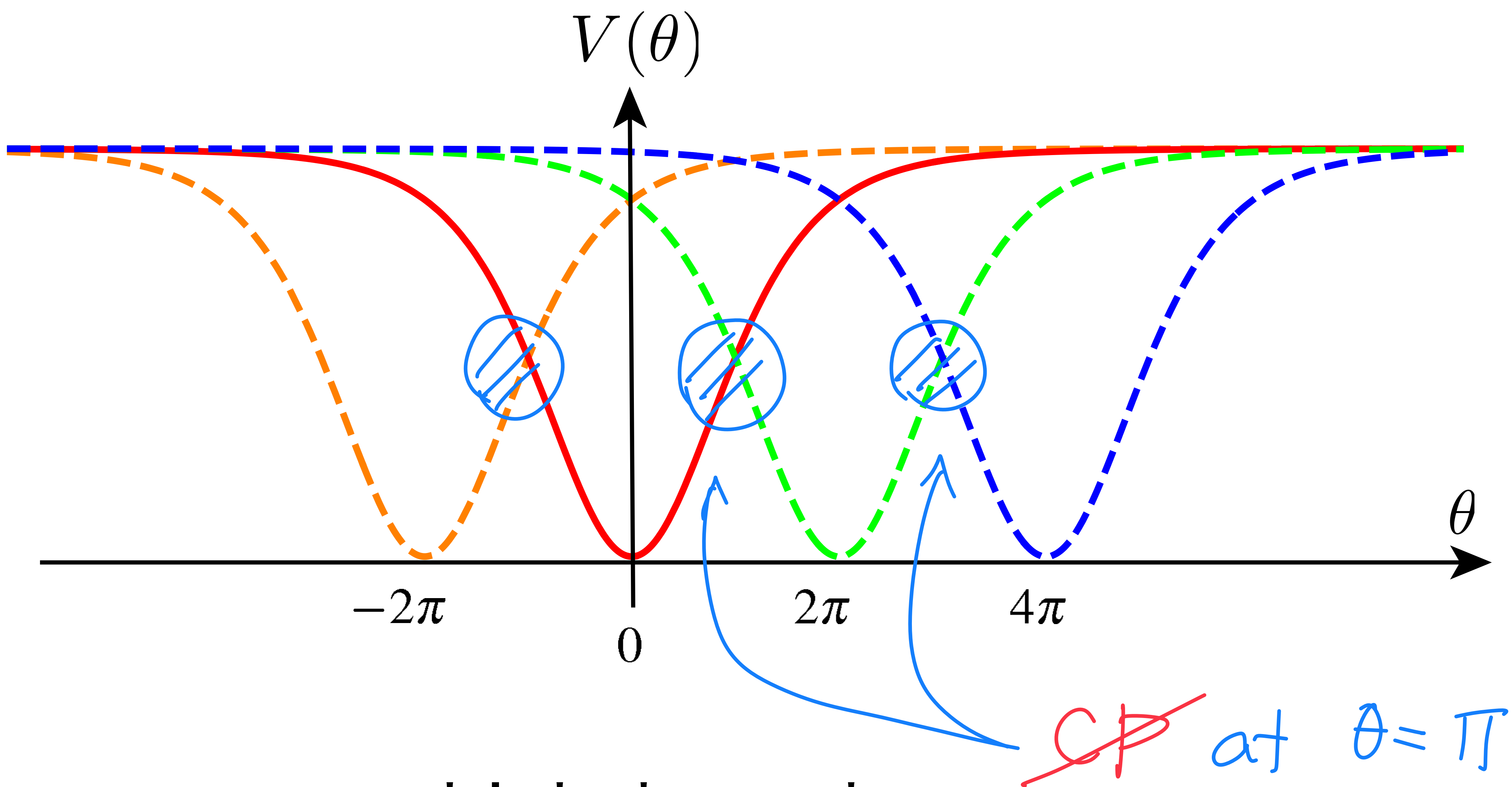


multiple branches

Small N ?

4d $SU(N)$ YM $(\theta=\pi)$

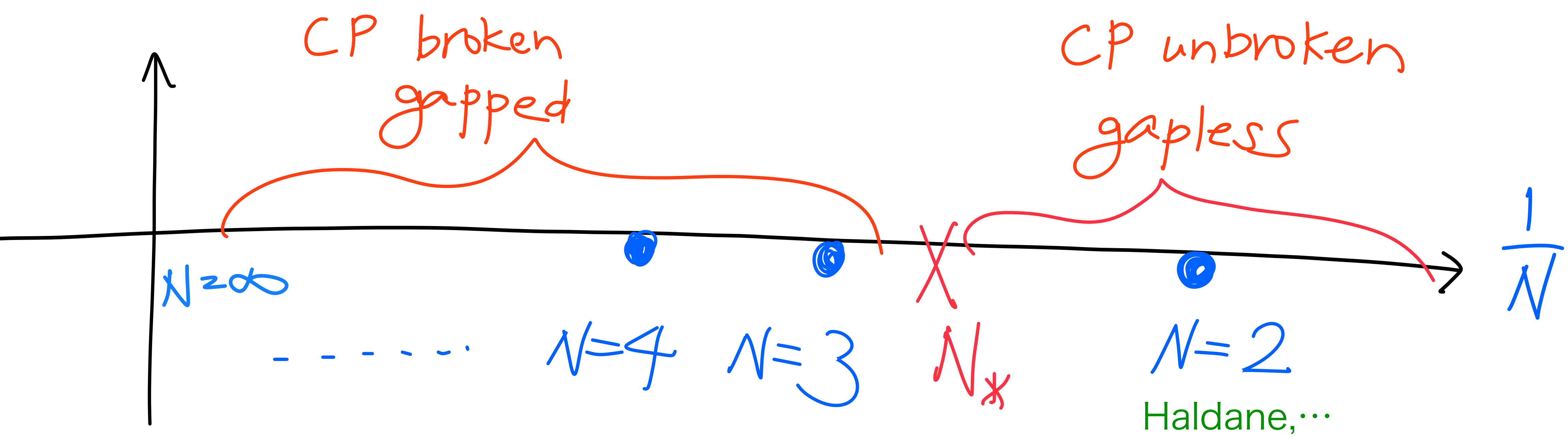
$\updownarrow$ similarity (e.g. mass gap)

2d CP^{N-1} model $(\theta=\pi)$

Small N ?

4d SU(N) YM $(\theta=\pi)$

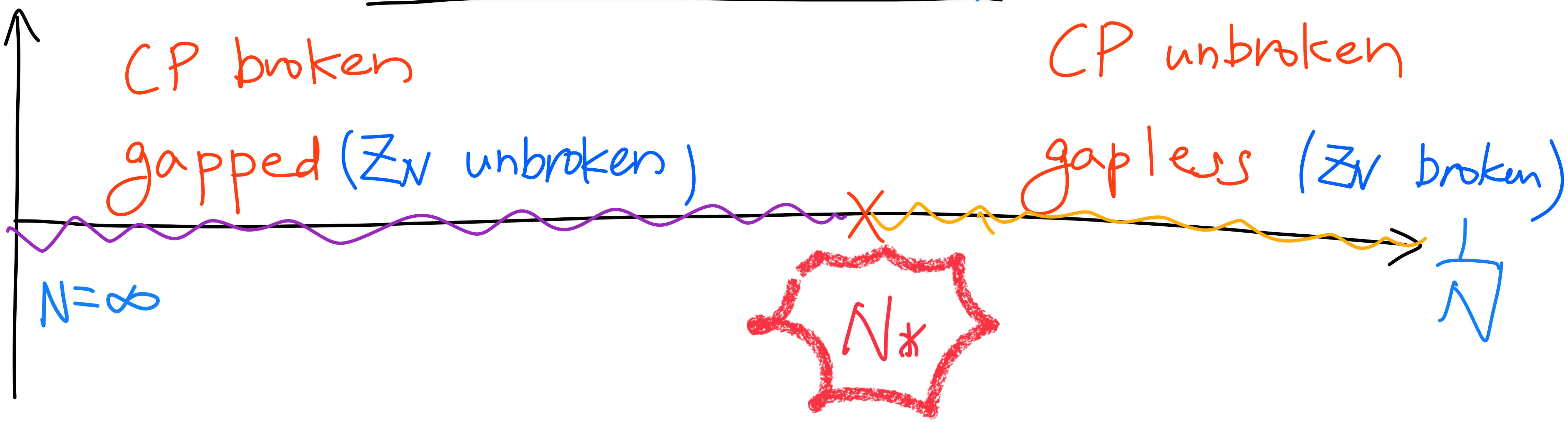
2d CP^{N-1} model $(\theta=\pi)$



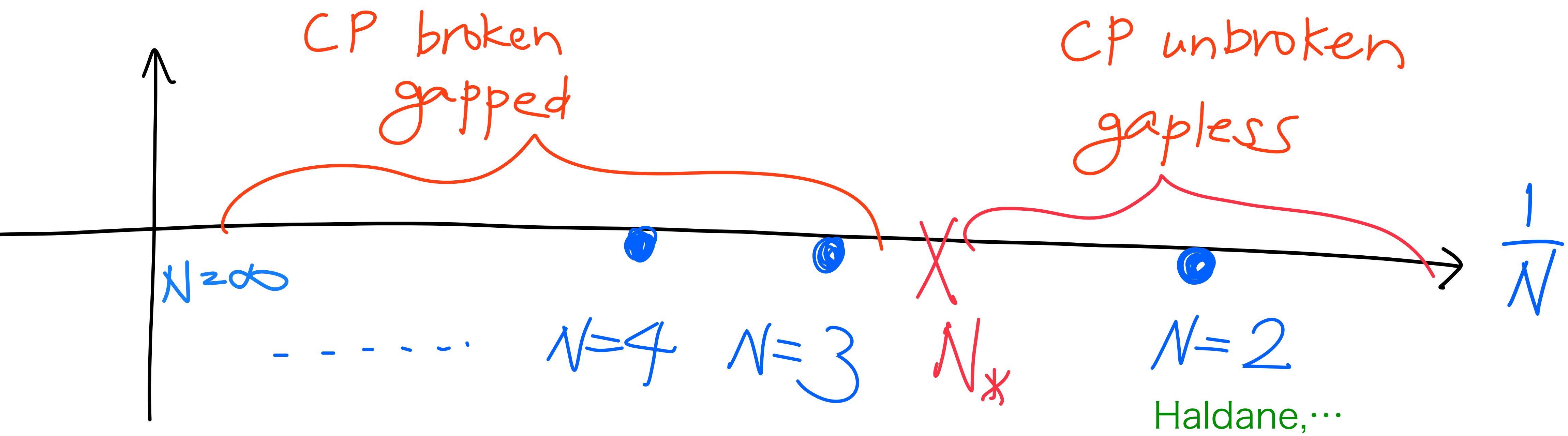
Small N ?

cf. Gaiotto-Kapustin-Komargodski-Seiberg '17

4d $SU(N)$ YM $(\theta=\pi)$



2d CP^{N-1} model $(\theta=\pi)$



4d vs. 2d

MY + Yonekura, MY '17

4d $SU(N)$ pure YM ($\mathbb{Z}_N$ center sym.)

$\downarrow$ T^2

cf. Atiyah '84

Looijenga '77, '80

$\mathcal{M}^{SU(N)}_{flat}(T^2) \simeq \mathbb{CP}^{N-1}$ + (singularities)

2d "$\mathbb{CP}^{N-1}$ - model" ($\mathbb{Z}_N$ flavor sym.)

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2d "$\mathbb{CP}^{N-1}$ - model" ($\mathbb{Z}_N$ flavor sym.)

$\downarrow$ S^1 (+ $\mathbb{Z}_N$ twist)

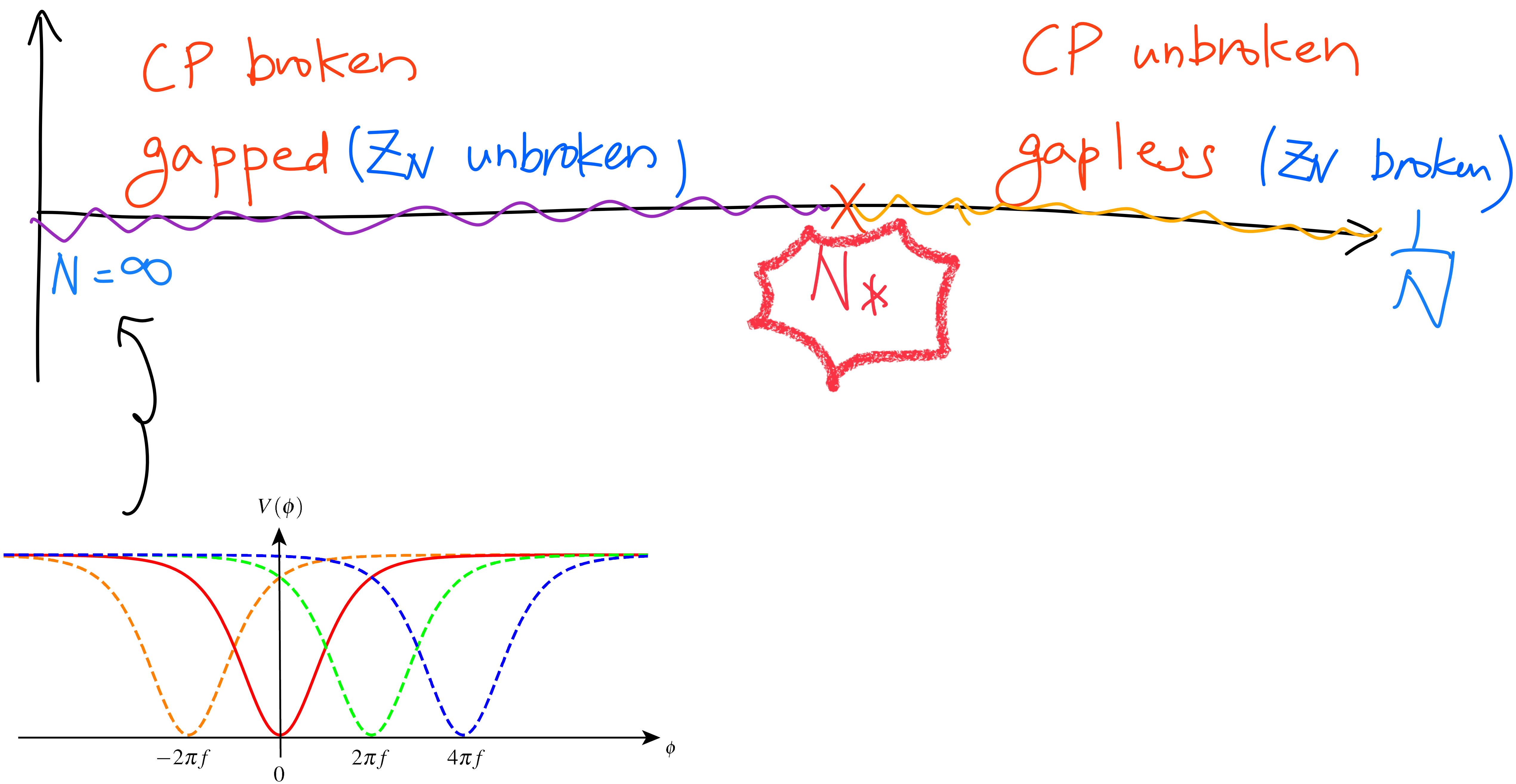
1d quantum mechanics

fractional instanton $\doteqdot$ renormalon ?

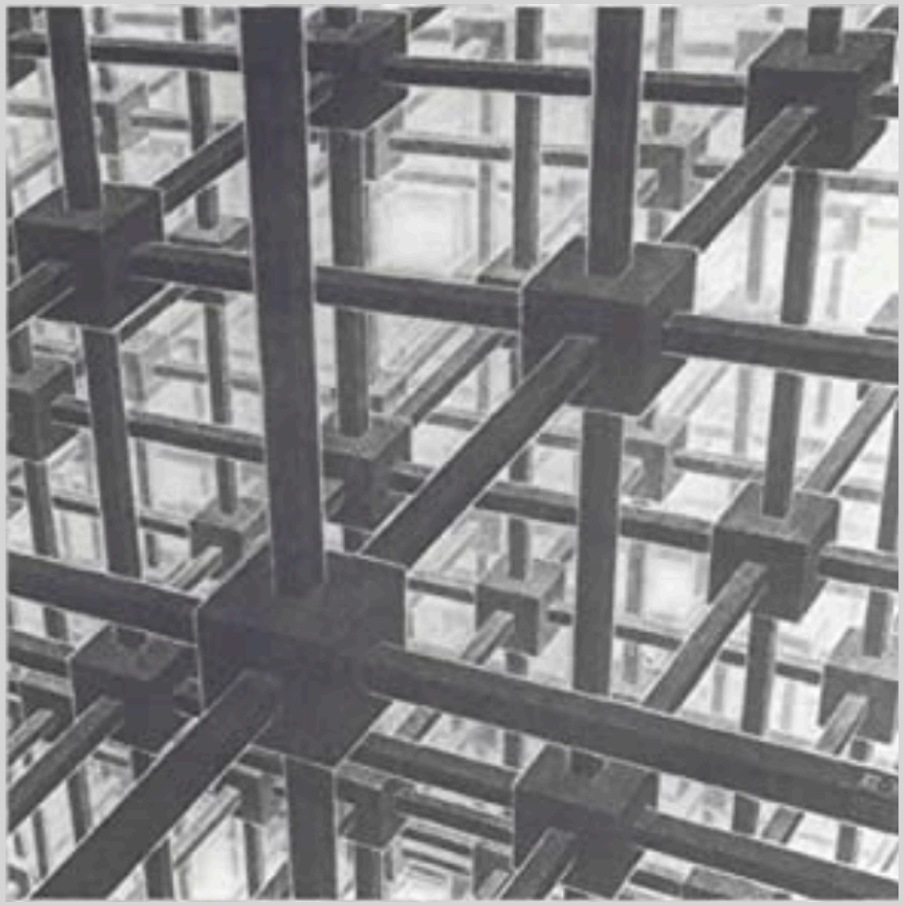
cf. Dunne-Unsal '12 for CP^N model

Summarizing···

4d SU(N) YM @ $\theta = \pi$:



Computer Simulations



… requires computational resources
(and several years of my research time!)

Oakforest-PACS in Kashiwa

Cygnus in Tsukuba

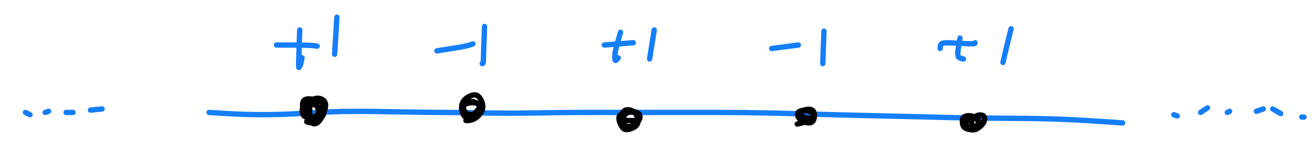
“Just do it” on the lattice? However…

⚠ Q is not quantized on the lattice

(short-distance fluctuations)

Luscher ‘82

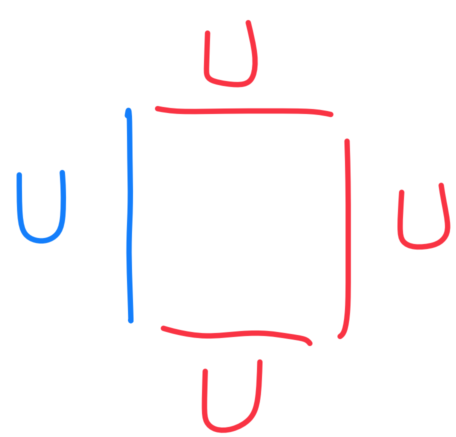
e.g.



Practical solution: smearing

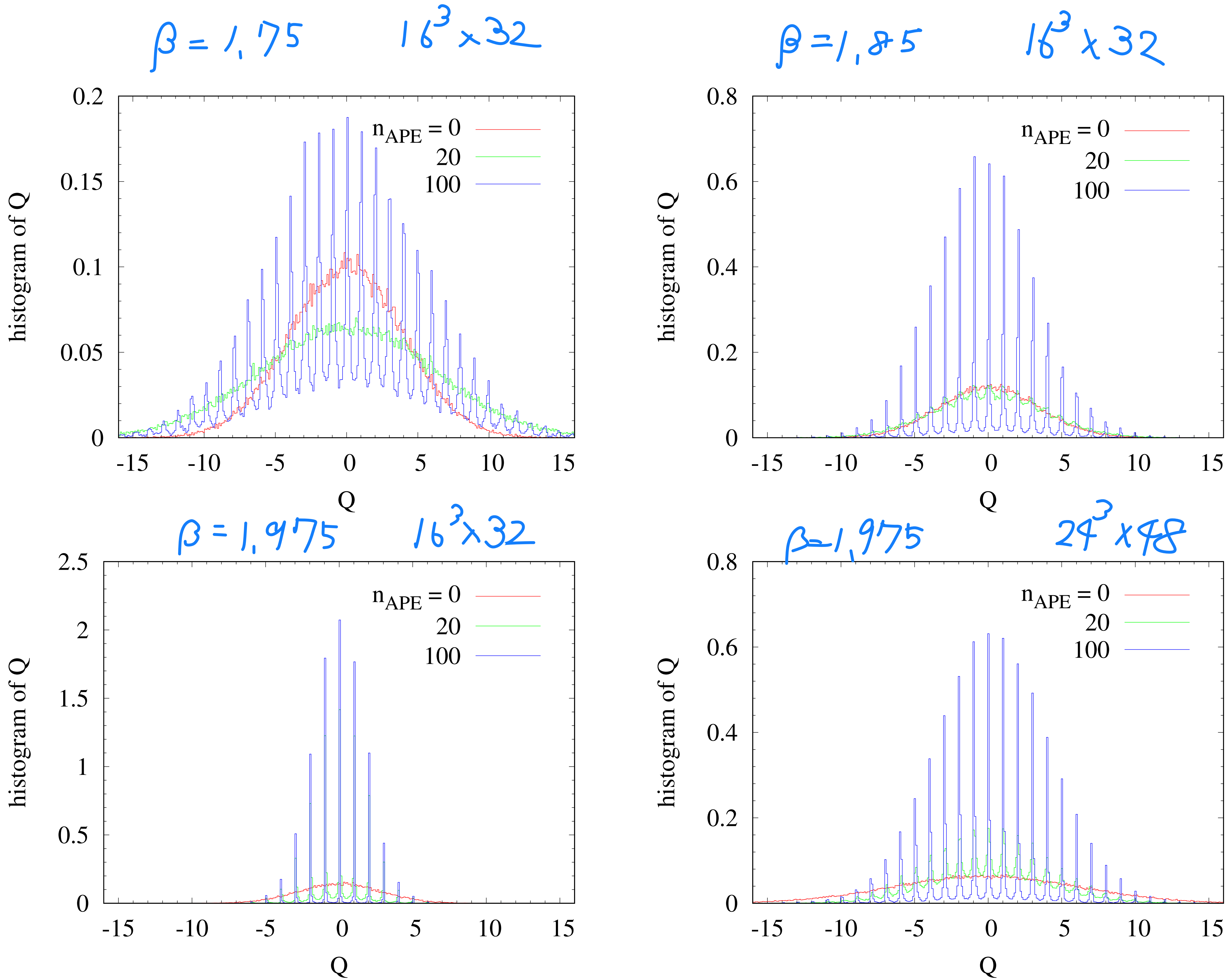
We use APE smearing & gradient flow

Albanese+ ‘87



Luscher ‘10

$$\frac{\partial A_\mu(s)}{\partial s} = \frac{\delta \mathcal{L}}{\delta A_\mu(s)}$$



β = 1.75 16^3 x 32
β = 1.85 16^3 x 32
β = 1.975 16^3 x 32
β = 1.975 24^3 x 48
histogram of Q
Q
n_APE = 0
20
100

“Just do it” on the lattice? However…

sign problem

$$e^{-S_g + i\theta Q}$$

widely fluctuating @ $\theta \sim \pi$

expansion around $\theta = 0$

Kitano-Yamada-MY ‘20

sub volume method

Kitano-Yamada-Matsudo-MY ’21

Expansion around $\theta = 0$

generate gauge conf. at $\theta = 0$ ← no sign problem

↓

measure top. charge Q

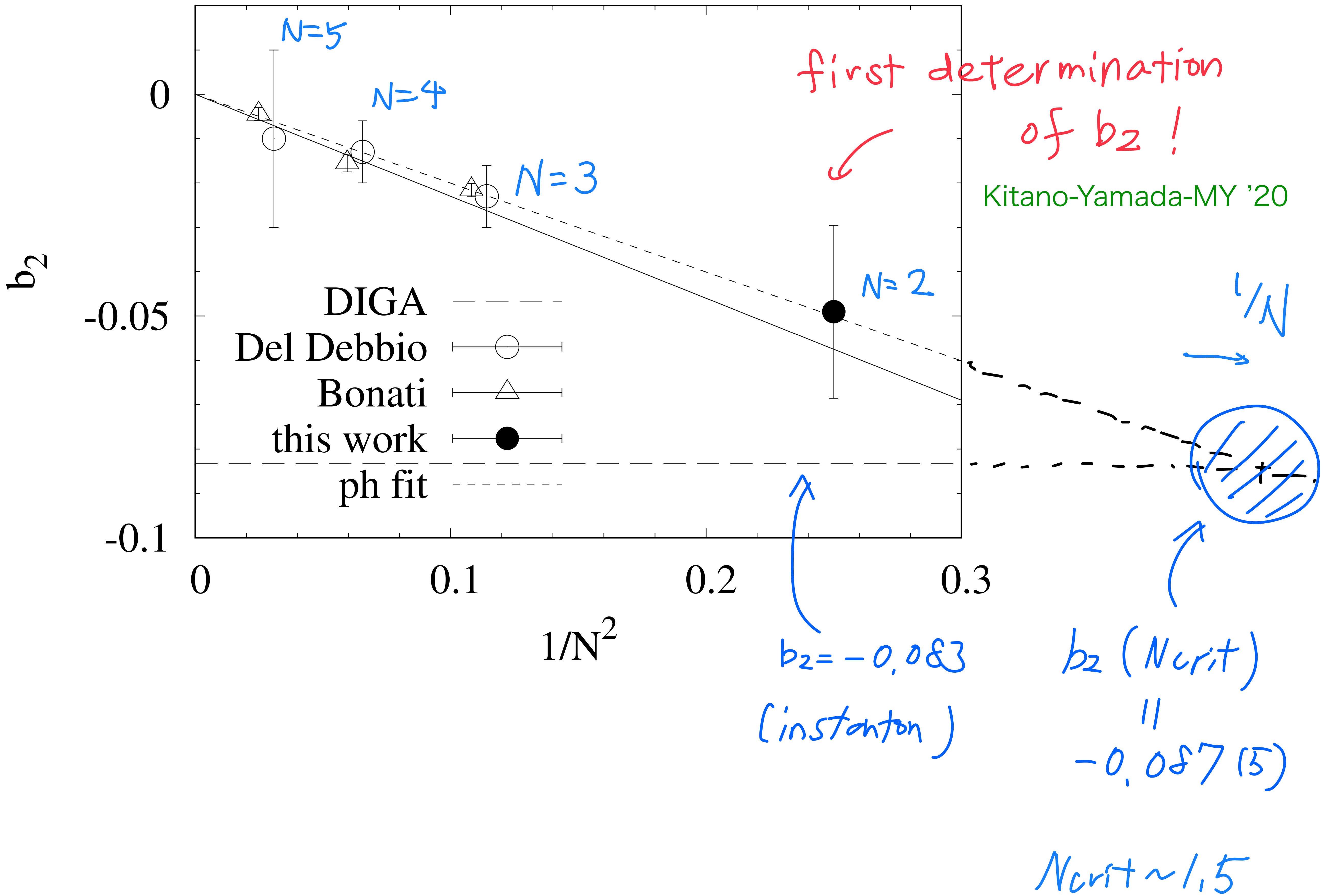
↓

$$\chi = \frac{\langle Q^2 \rangle_{\theta=0}}{V} ,$$

$$b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3\langle Q^2 \rangle_{\theta=0}^2}{12\langle Q^2 \rangle_{\theta=0}} ,$$

$$b_4 = \frac{\langle Q^6 \rangle_{\theta=0} - 15\langle Q^2 \rangle_{\theta=0}\langle Q^4 \rangle_{\theta=0} + 30\langle Q^2 \rangle_{\theta=0}^3}{360\langle Q^2 \rangle_{\theta=0}} ,$$

$$F(\theta) = \frac{1}{2}\chi\theta^2\left(1 + b_2\theta^2 + b_4\theta^4 + \cdots\right)$$

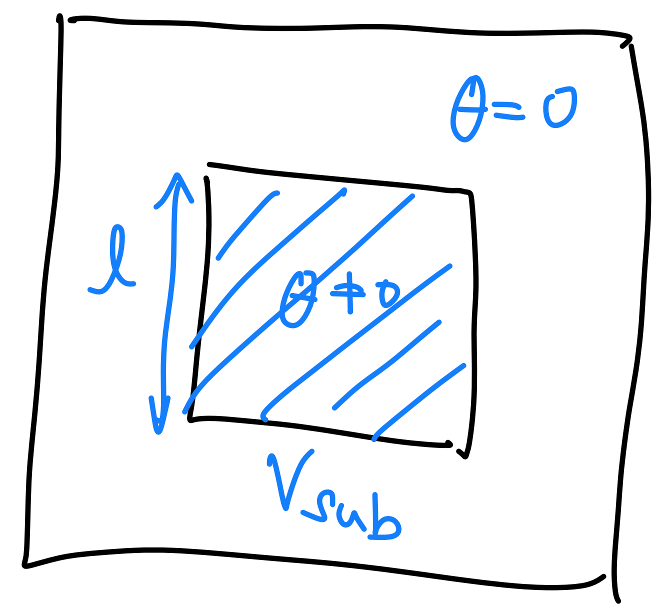

first determination of b2 !
Kitano-Yamada-MY '20
N=5
N=4
N=3
N=2
DIGA
Del Debbio
Bonati
this work
ph fit
b2
0
-0.05
-0.1
0
0.1
0.2
0.3
1/N2
1/N
b2 = -0.083 (instanton)
b2 (Ncrit) = -0.087(5)
Ncrit ~ 1.5

Subvolume Method

Kitano-Yamada-Matsudo-MY '21
cf. Keith-Hynes & Thacker '08
for 2d CP^1 model

$$e^{-\overbrace{V_{sub}}^{l^4} F_{sub}(\theta)} = \frac{1}{Z(0)} \int \mathcal{D}U \, e^{-S_g + i\theta Q_{sub}}$$

$$= \left\langle e^{i\theta Q_{sub}} \right\rangle = \left\langle \cos(\theta Q_{sub}) \right\rangle$$



Fit

$$F_{sub}(\theta) \sim F(\theta) + \frac{s(\theta)}{l} + O\left(\frac{1}{l^2}\right)$$

surface tension

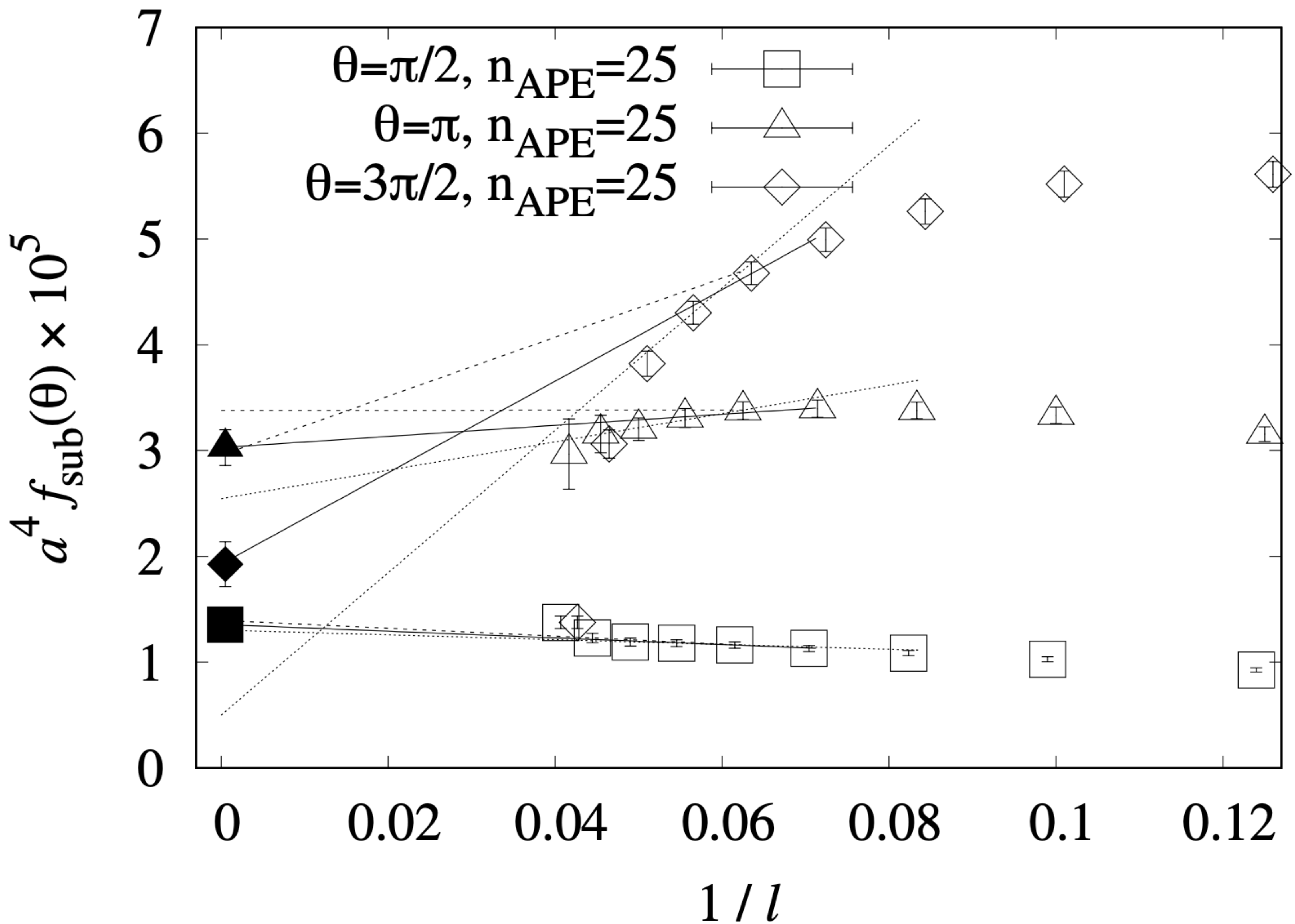
$Q_{sub} \notin \mathbb{Z}$

better w/ sign problem!

inside region

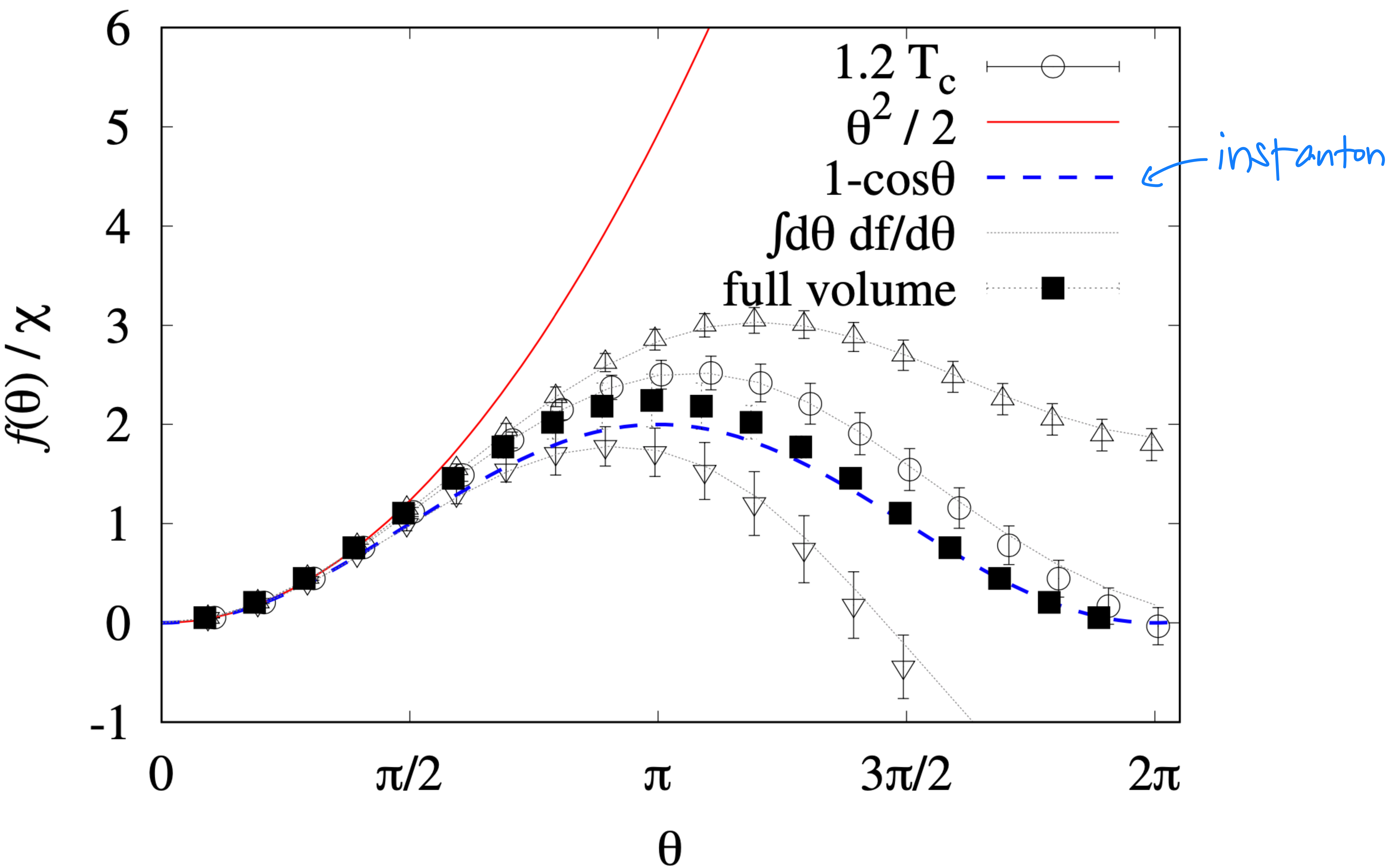
$$(aT_c)^{-4} \ll V_{sub} \ll V_{full}$$

Kitano-Yamada-Matsudo-MY '21



F(θ) @ T= 1.2 Tc > Tc

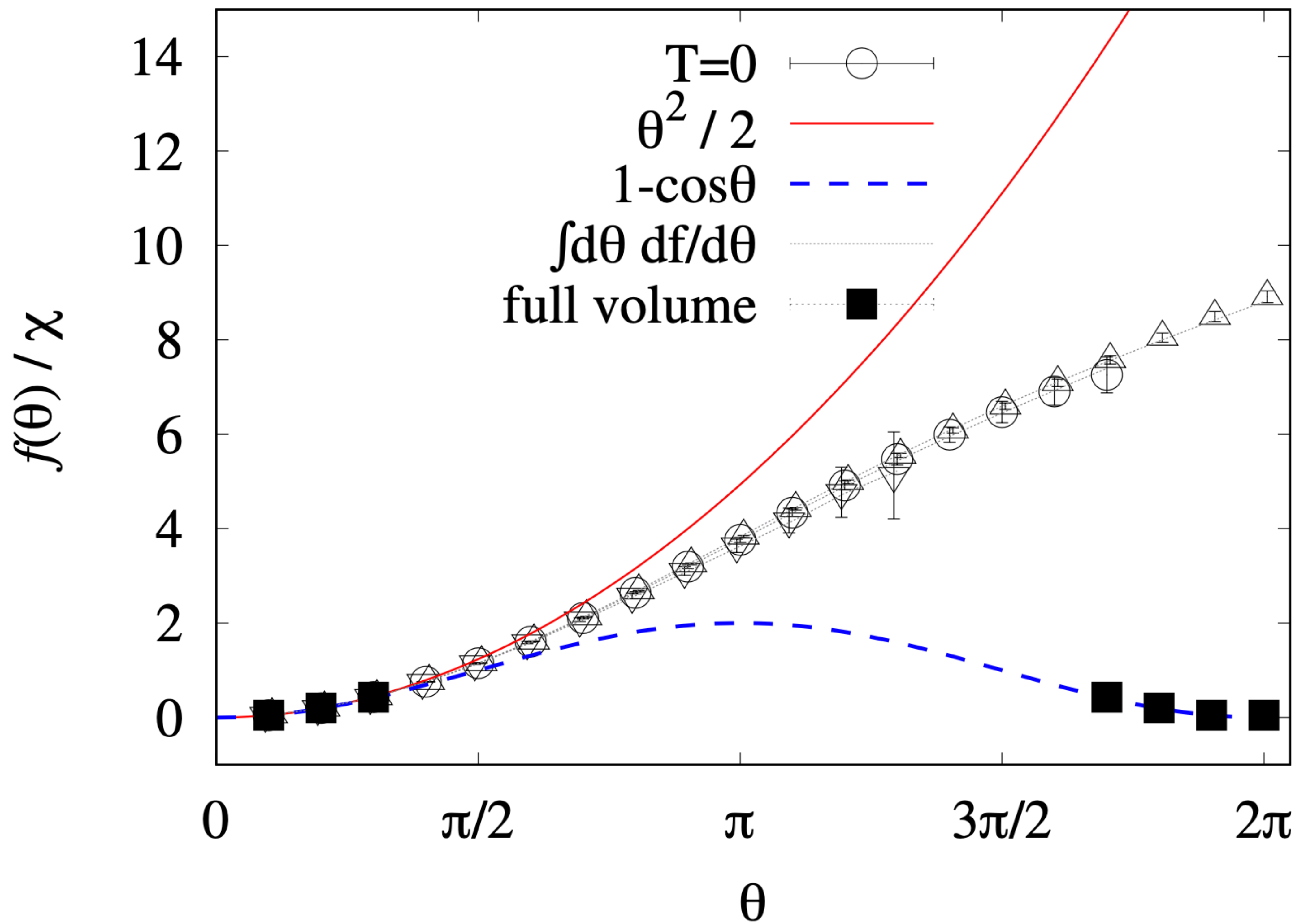
Kitano-Yamada-Matsudo-MY '21



Kitano-Yamada-Matsudo-MY '21

$F(\theta)$ @ $T=0$

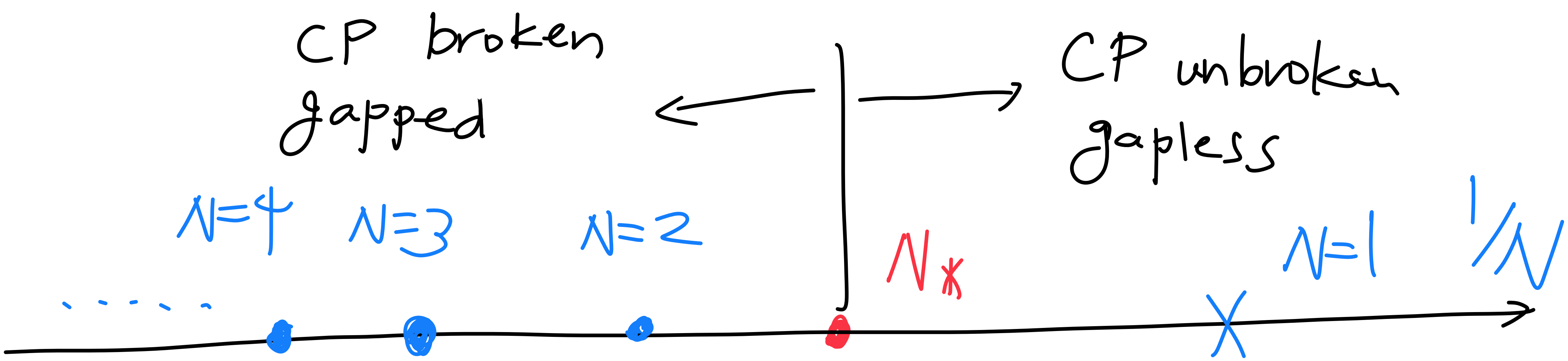
clearly NOT 2π-periodic !!



Summarizing…

4d SU(2) YM @ $\theta = \pi$: still "large N"

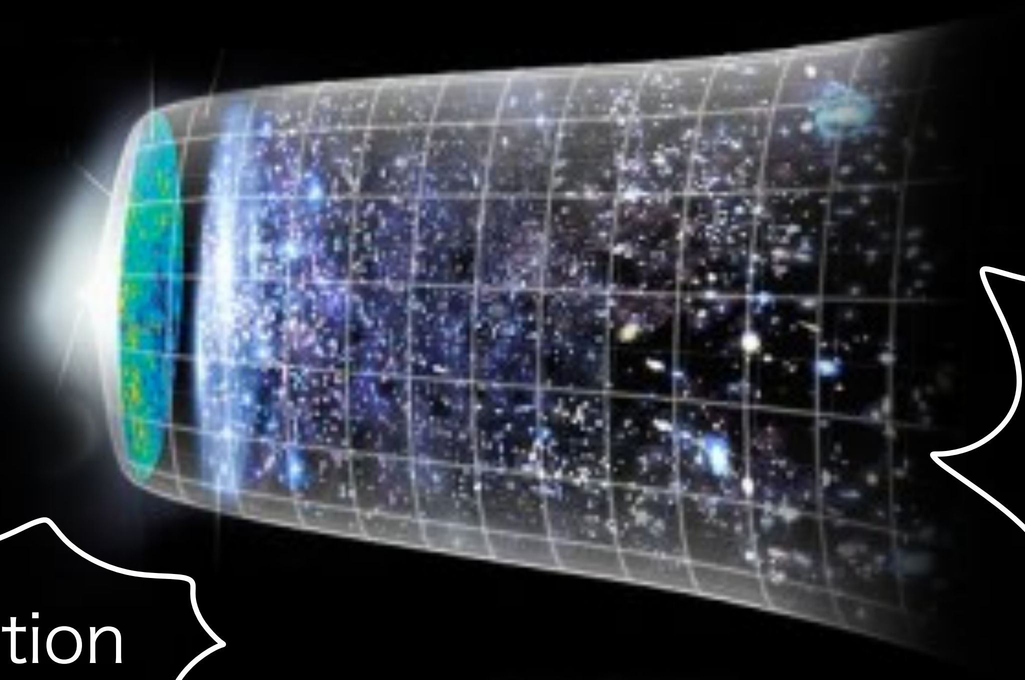
Spontaneous CP breaking, mass gap



Stay Tuned!

- Improve systematics
- Explore
  - (T, θ) - phase diagram [e.g. $T_c(\theta)$]
  - different matter contents
- (T, μ) - phase diagram? quark-gluon plasma

3. Axions “in the sky”?



dark energy

inflation

dark matter

“U” of IPMU…

Axions

Axion: promotion to a dynamical scalar

$$\mathcal{L} = \frac{1}{32\pi^2}\frac{a}{f}\int \mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu} + \int (\partial_\mu a)^2$$

Peccei-Quinn '77
Weinberg, Wilzcek '78

Original motivation: strong CP problem

$$|\theta_{QCD}| \lesssim 10^{-13}$$

axion potential important

Axion / axion-like particles under search

(inflation, strong CP, dark matter, dark energy ...)

Theoretical Elegance of Axions

axion
- shift sym. $a \to a + (\text{const.})$
- non-perturbative shift sym breaking $V \sim \mathcal{O}(\Lambda^4)$

☺ Simple

☺ protected against (quantum) corrections

☺ Within EFT

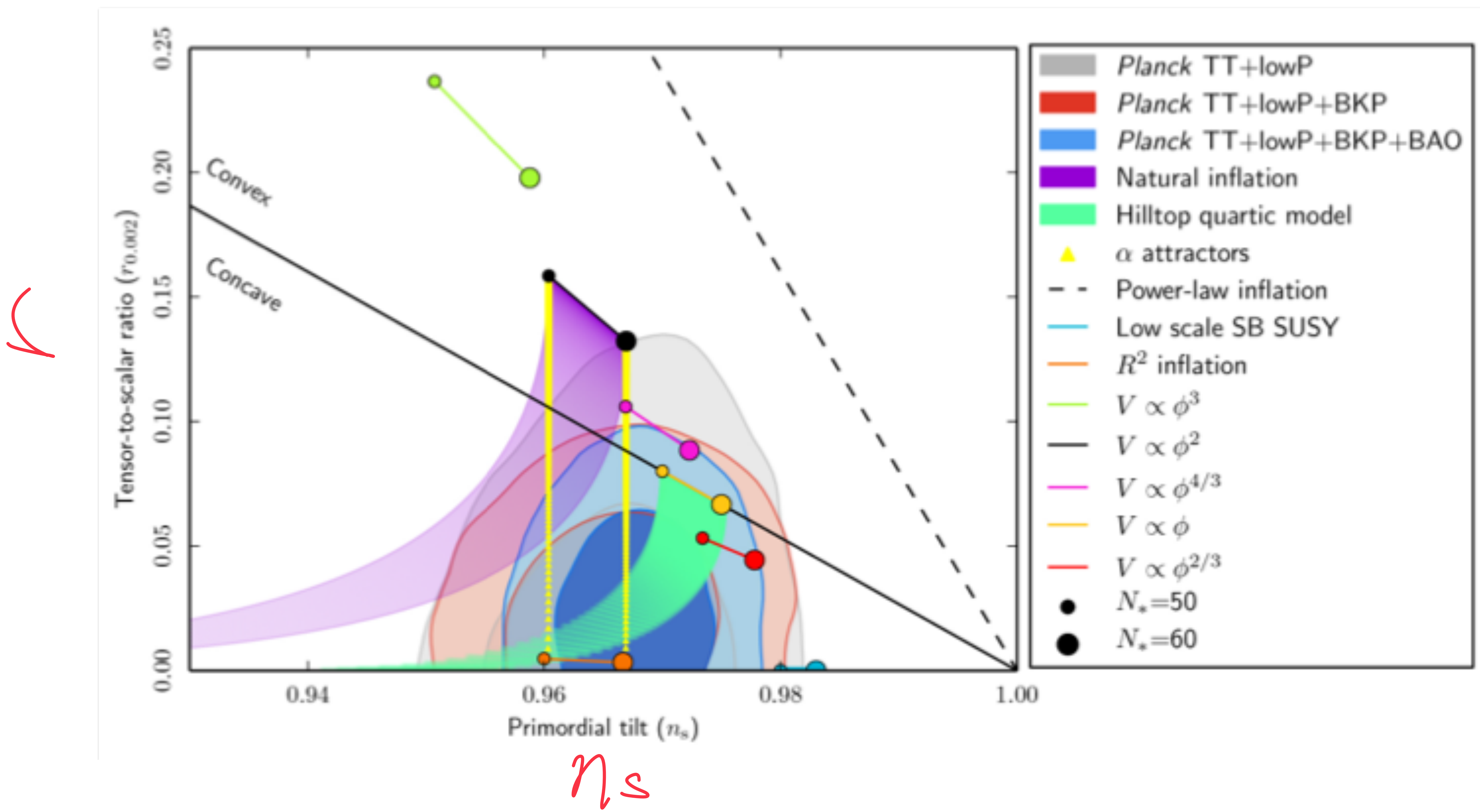
☺ "String Axiverse"

Inflation

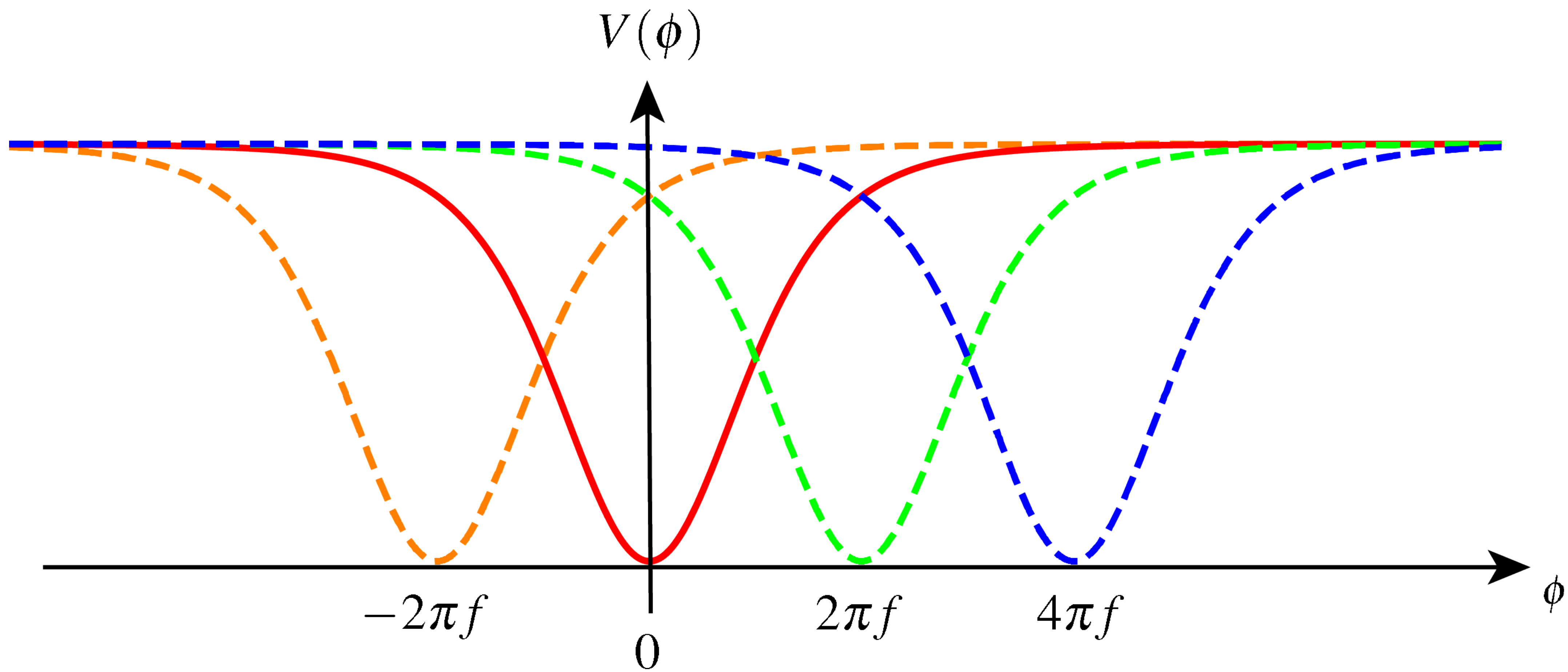
$$V(\phi) = \Lambda^4 \left(1 - \cos\frac{\phi}{f}\right)$$

Natural inflation
Freese-Frieman-Olinto '90

being excluded by CMB ☹



Expected potential for pure YM



parametrize

$$V(\phi) = M^4 \left[1 - \left(1 - \left(\frac{\phi}{f} \right)^2 \right)^{-p} \right]$$

Nomura-Watari-MY '17
"Pure Natural Inflation"

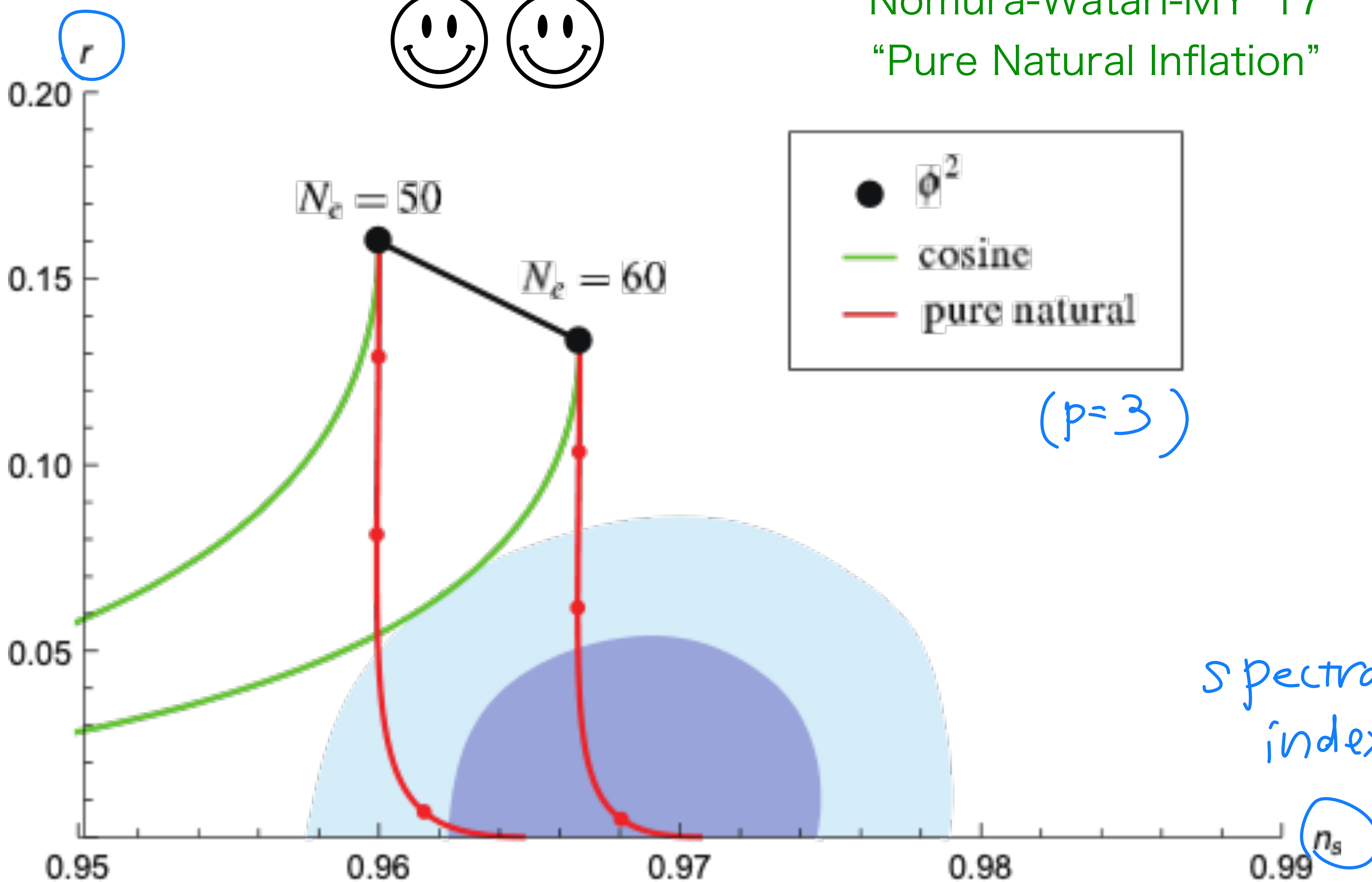
- has all the expected properties
- motivated by holographic Yang-Mills

Dubovsky-Lawrence-Roberts '11

tensor-to scalar ratio

Surprise!

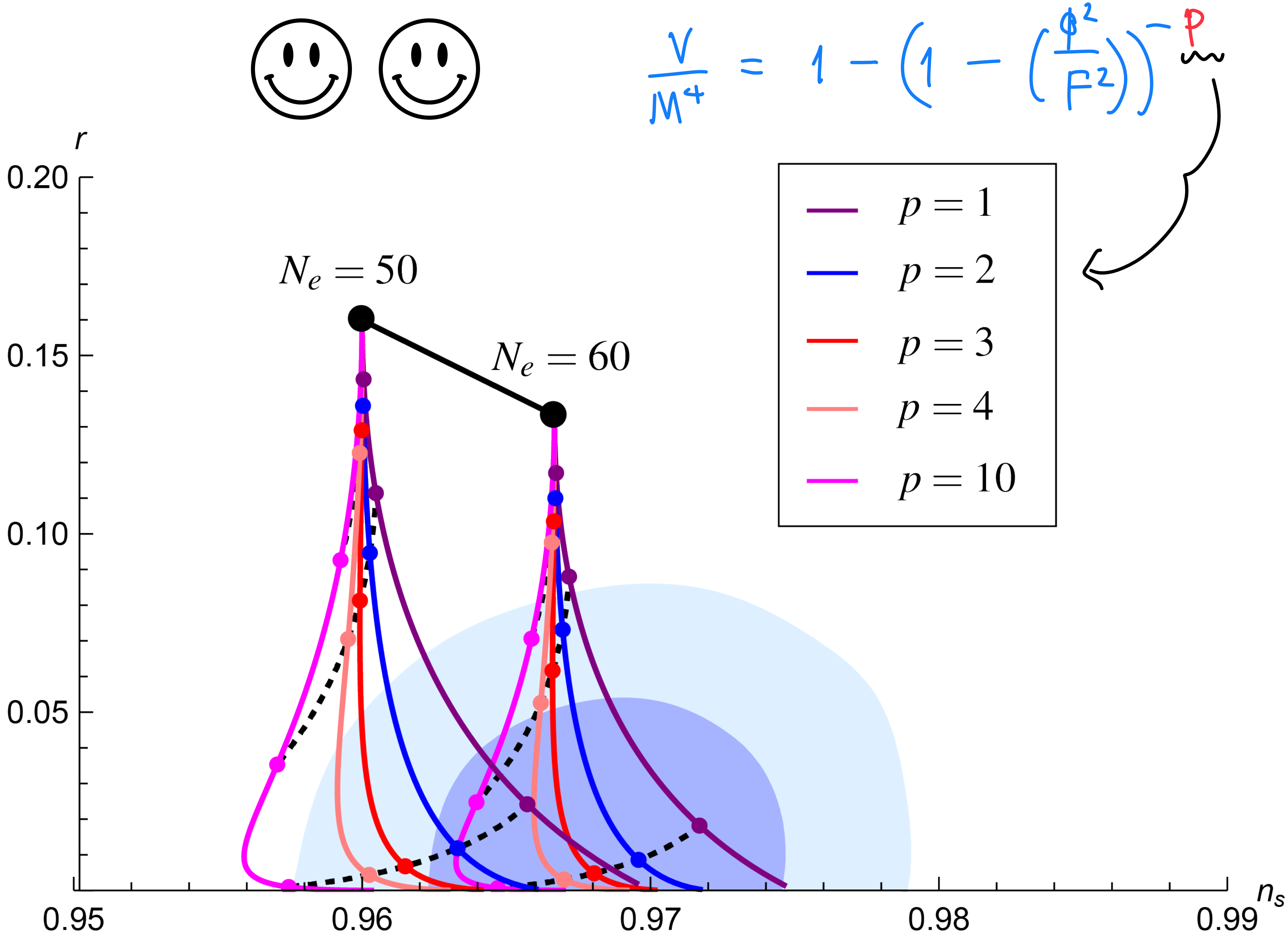
Nomura-Watari-MY '17
"Pure Natural Inflation"



spectral index

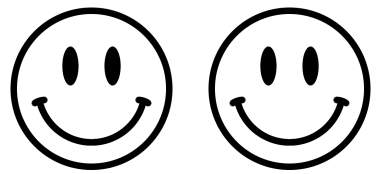
Nomura-Watari-MY '17

"Pure Natural Inflation"

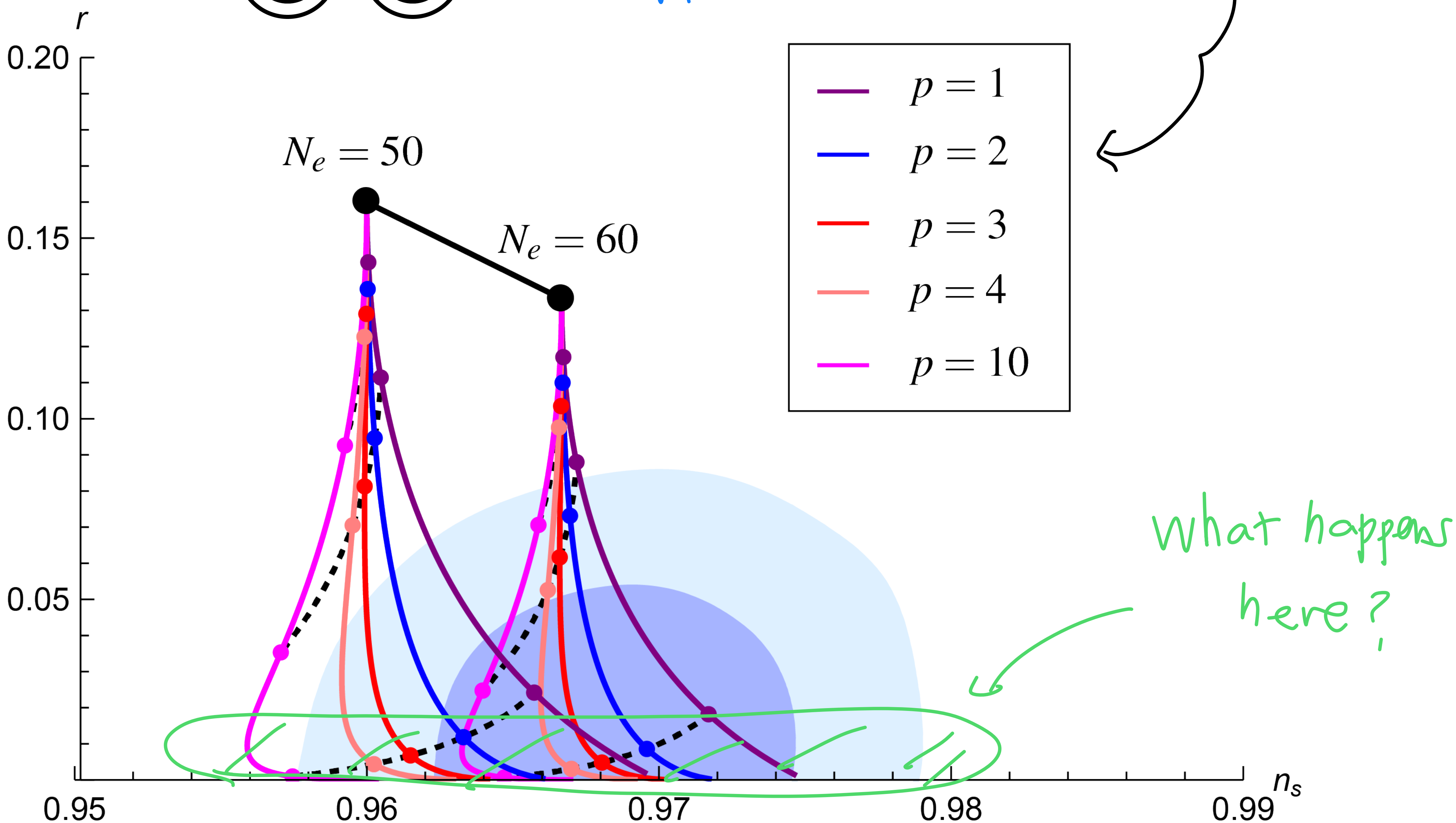


Nomura-Watari-MY '17

"Pure Natural Inflation"



$$\frac{V}{M^4} = 1 - \left(1 - \left(\frac{\phi^2}{F^2}\right)\right)^{-p}$$



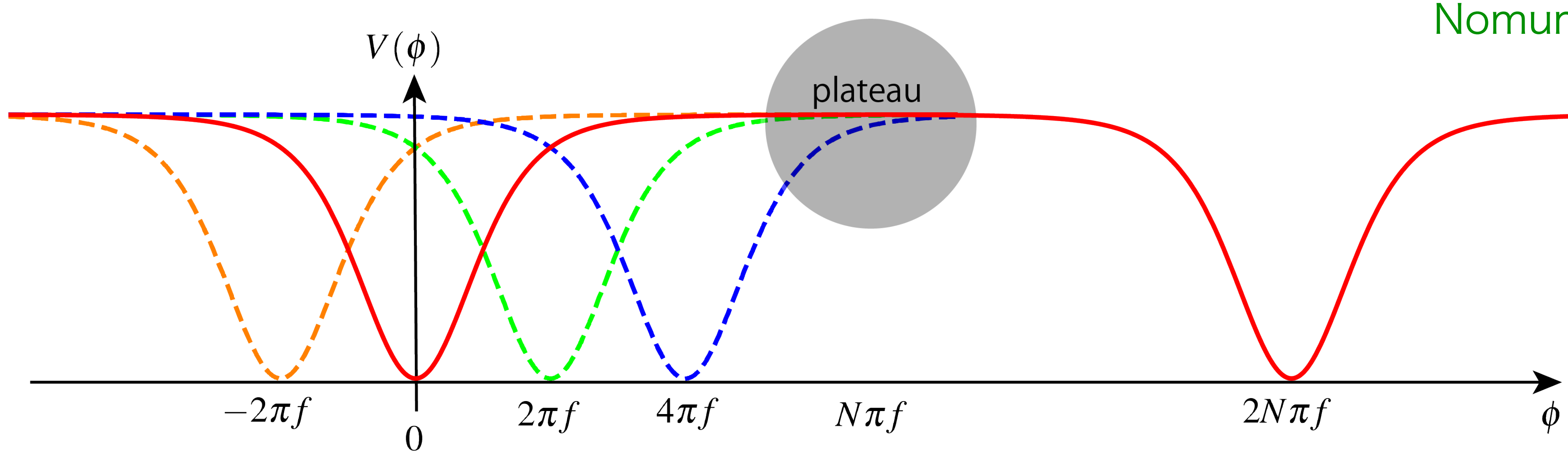
finite number (N) metastable branches

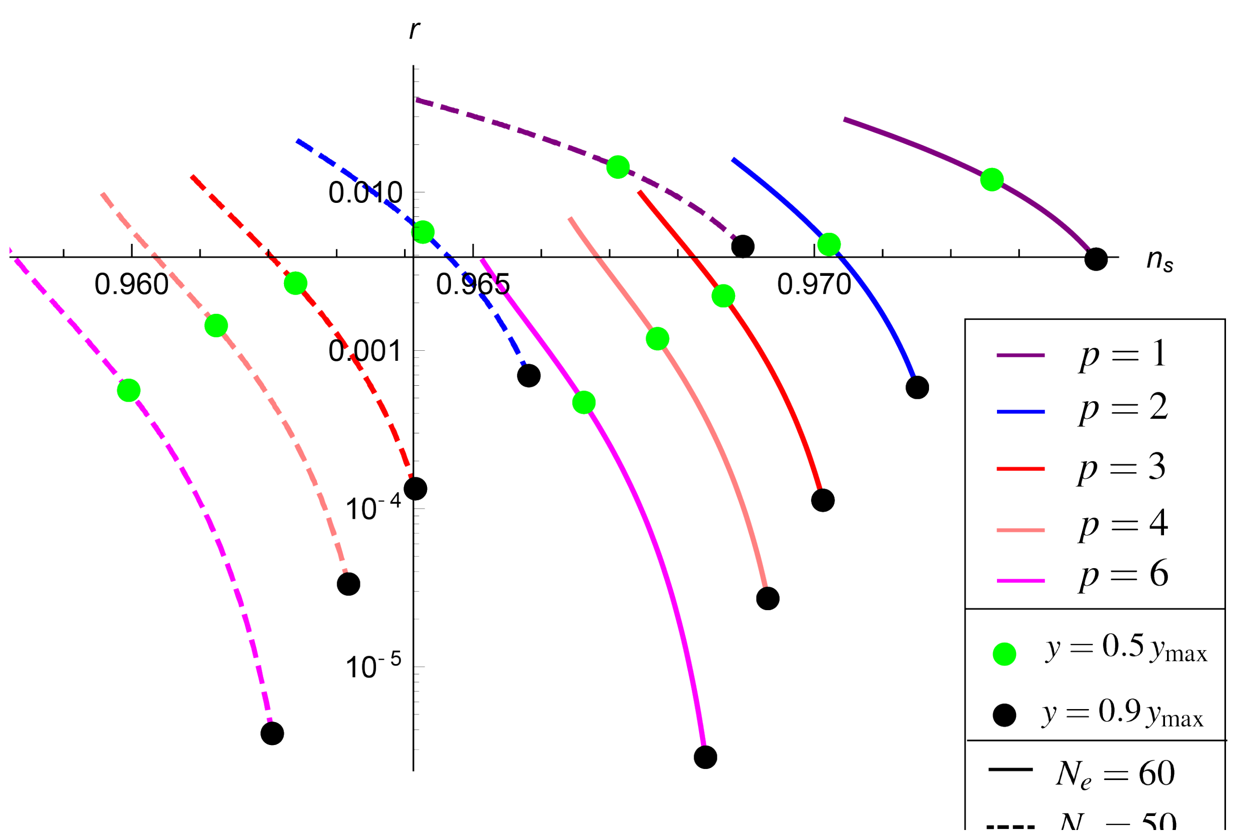
Yonekura-MY '17

↓

plateau is cutoff at $\phi \lesssim N\pi f$

Nomura-MY '17





Dark Energy

UV versus IR?

- Recall : for $SU(N)$ pure YM $\quad (\alpha \equiv g^2/4\pi)$

$$F(\theta) \sim \int \frac{d\rho}{\rho^5} e^{-\frac{2\pi}{\alpha(\mu)}} (\mu\rho)^{\frac{11N}{3}} (1-\cos\theta)$$

IR divergent for $N \gtrsim N_* \sim \frac{11}{15}$

- For EW $SU(2)$ in Standard Model

UV divergent

$$F(\theta) \sim M_{pl}^4 e^{-\frac{2\pi}{\alpha(M_{pl})}} (1-\cos\theta)$$

← UV sensitive

$$\alpha_2(M_Z) \simeq \frac{1}{29} \overset{\text{RG}}{\rightsquigarrow} \alpha_2(M_{pl}) \simeq \frac{1}{48}$$

$$\Lambda^4 \simeq M_{pl}^4 \, e^{-\frac{2\pi}{\alpha_2(M_{pl})}} \simeq \mathcal{O}(10^{-130}) M_{pl}^4 \;!!$$

$$\left[\Lambda_{c.c.}^4 \simeq \mathcal{O}(10^{-120}) M_{pl}^4 \right]$$

dark energy from EW axionic quintessence?

☺ ☺

Nomura-Watari-Yanagida '00
Mclerran-Pisarski-Skokov '12
Ibe-Yanagida-MY '18

UV dependence is a feature, not a bug

- $\Lambda_{c.c}$ expected to be UV-sensitive
- θ-angle in EW $SU(2)$ can be rotated away: $(B+L)$-global sym.
- We expect, however, $(B+L)$-sym. to be broken by higher-dim. operators

$$\mathcal{L}_{\cancel{B+L}} \supset \frac{1}{M_{pl}^2} qqql \rightsquigarrow V(\theta) \neq 0$$

In string theory ?

Weak gravity conjecture implies

Arkani-Hamed+Motl +Nicolis+Vafa '06

$$f \lesssim \frac{M_{Pl}}{S_{inst}} \sim \mathcal{O}(10^{-2}) M_{Pl} \ll M_{Pl}$$

$$S_{inst} = \frac{2\pi}{\alpha_2(M_{Pl})} \simeq 300$$

However, we need small quintessence mass

$$m^2 \simeq \frac{\Lambda^4}{f^2} \simeq \frac{H_0^2 M_P^2}{f^2} \lesssim H_0^2 \rightsquigarrow f \gtrsim M_{Pl}$$

Svrcek '06

Ibe-Yanagida-MY '18

→ Better in
- Hilltop
- SUSY

Ibe-Yanagida-MY '18

Summary

- Yang-Mills theory : fascinating example of physics/mathematics interaction
- Yang-Mills theory + θ-term :
  still basic questions yet to be answered
  ... but interesting progress recently
- Implications for axion search
  inflation, dark energy, ...

IPMU 2050 as a Utopia for Physics and Mathematics of the Universe