Cosmological Constant Problem on the Horizon

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arXiv: 2201.02016

Outline

arXiv: 2201.02016, H. F.

A brief review of CC problems

Highlight the roles of dS horizon of zero point energy

The roles of light and heavy fields in zero point energy

Indications of cracks on cosmological constant:

Inhomogeneities of the distributions of vacuum zero point energy.

Cosmological Implications:

Dark Energy

Dark Matter

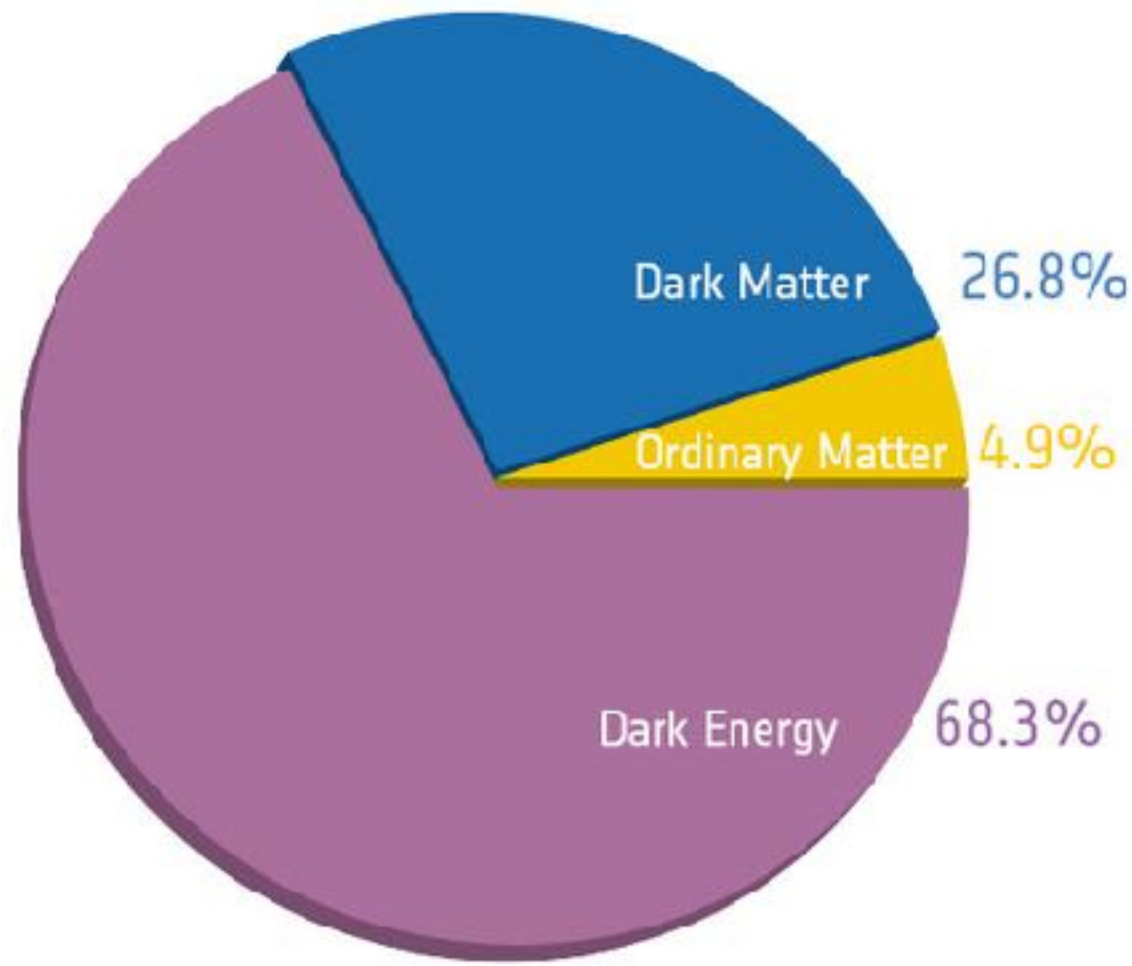
H_0 tension

Speculations?!

The Standard Model of Cosmology

ΛCDM

- Ordinary Atoms:(Baryons) 5%
- Dark Matter: 26%
- Dark Energy: 69%
- Spatial Curvature ~ 0



Planck Collaboration

Photo: Lawrence Berkeley National Lab

Saul Perlmutter

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

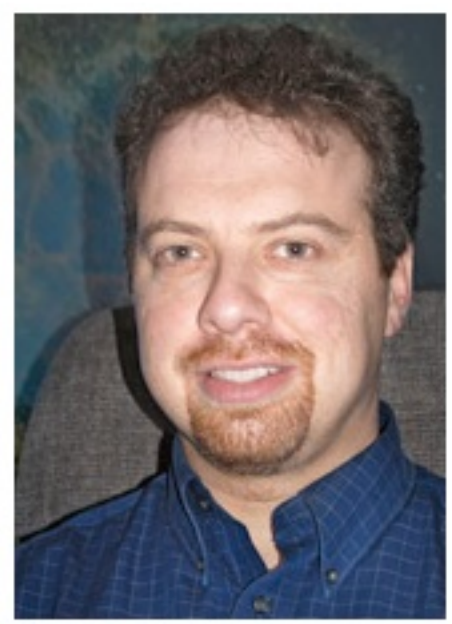


Photo: Scanpix/AFP

Adam G. Riess

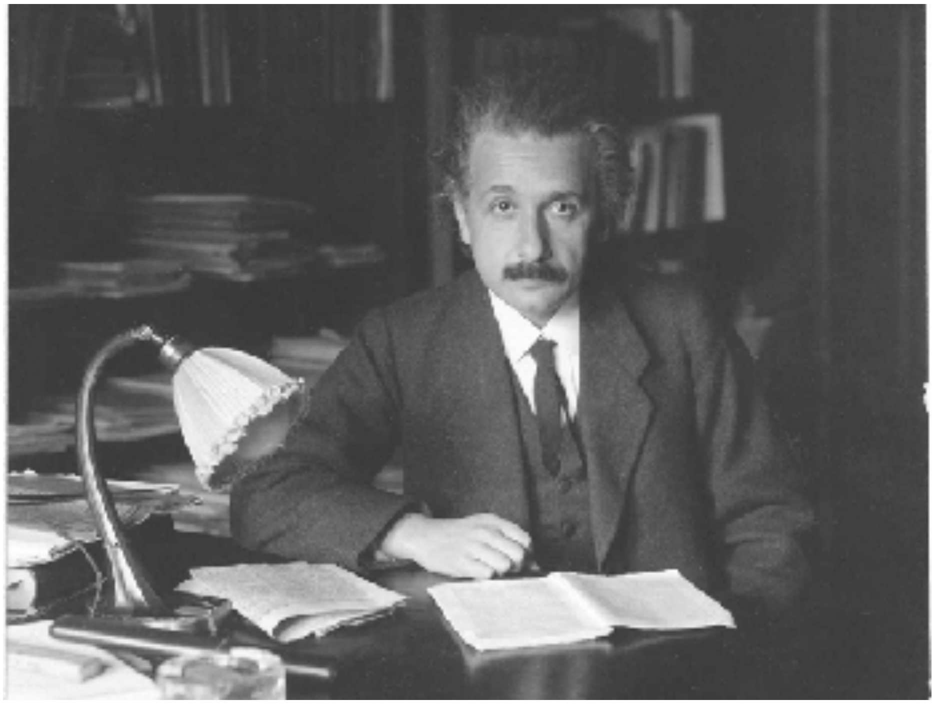
Nobel 2011 (dark energy)

Cosmological Constant Problems

A constant term, Λ, is consistent with the Einstein Field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein added this term to obtain a static cosmological solution.



After the observation of the expansion of the Universe by Hubble,Einstein discarded this CC term, calling it:

"the biggest blunder of his life."

ΛCDM is working well and it seems that something like a cosmological constant term is the source of Dark Energy.

On the other hand, in QFT vacuum carries energy.

In the semi-classical approximations where the matter fields are quantized while gravity is a classical field, we can have the quantum contribution: $\langle 0 | T_{\mu\nu} | 0 \rangle$

$$G_{\mu\nu} + \Lambda_b \, g_{\mu\nu} = 8\pi G \langle 0 | T_{\mu\nu} | 0 \rangle$$

The effective "physical" cosmological constant term is:

$$\Lambda_{\text{eff}} = \Lambda_b + 8\pi G \rho_v \quad , \quad \rho_v \equiv \langle 0 | T_{00} | 0 \rangle$$

The Cosmological constant problems

Weinberg, Rev. Mod. Phys. , 1989

The Old Cosmological Constant Problem:

Why is not ρ_V very large?

$$\langle \rho \rangle = \frac{1}{2} \int_0^M d^3\mathbf{k} \sqrt{k^2 + m^2} \sim M^4$$

examples:

$$M \sim m_e \rightarrow \langle \rho \rangle \sim 10^{24}\, eV^4$$

$$M \sim TeV \rightarrow \langle \rho \rangle \sim 10^{48}\, eV^4$$

$$M \sim M_P \rightarrow \langle \rho \rangle \sim 10^{108}\, eV^4$$

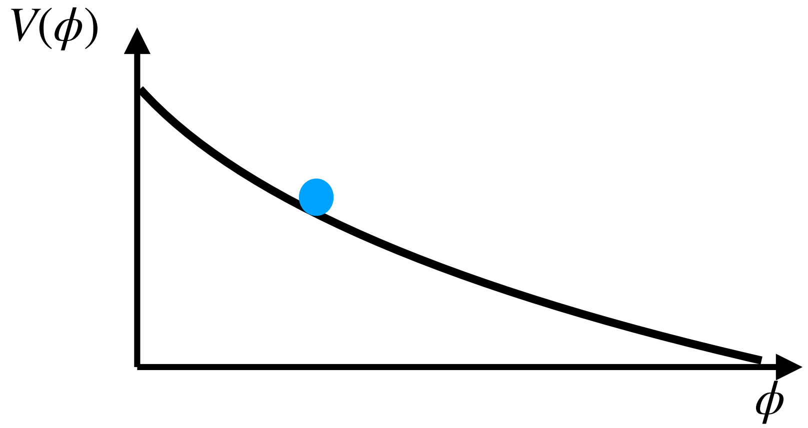
The new Cosmological Constant Problem:

Why is dark energy comparable to the current matter energy density?

$$\rho_V \sim \rho_C \sim 10^{-29} gr/cm^3 \sim 10^{-12} eV^4$$

Some Proposals for CC Problems

1- The Quintessence:



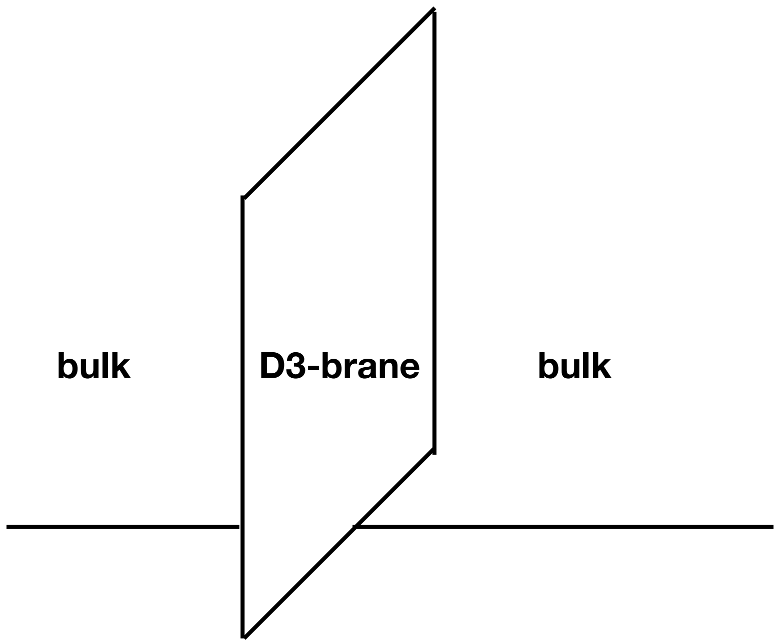
There is an extremely light field which rolls slowly towards its minima at $\phi \to \infty$ with $V \to 0$.

2- Supersymmetry :

$$\sum_{B,F} \left(\rho_F + \rho_B\right) = 0.$$

SUSY is not observed in Nature for $E \lesssim TeV$.

3- Self-tuning in Extra Dimensions:



As in RS scenario, one can have a D3 brane where the SM fields are confined. Gravity can leak to extra dimension.

4- Anthropic Considerations:

The CC should have a value in the "Landscape" to allow formation of gravitationally bound object like galaxies in order for the intelligent life to form.

Weinberg has employed this type of reasoning and concluded the existence of $\rho_V \lesssim 10^2 \rho_0$.

Troubles with hard cutoff

J. Martin, arXiv: 1205.3365

The conventional approach leading to $\langle \rho_V \rangle \sim M^4$ suffers from serious flaws.

1- The hard momentum cutoff violates underlying Lorentz invariance.
It only respects the spatial $O(3)$ invariance.

2- It predicts a wrong equation of state for the pressure:

$$\langle p \rangle = \frac{1}{3} \langle \rho \rangle$$

One has to use a regularization scheme which respects the underlying Lorentz invariance.

For example, using the dimensional regularization approach, for a real scalar field of mass m we obtain

$$\langle \rho \rangle = -\langle p \rangle = \frac{m^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right)$$

Akhmedov, hep-th/0204048

in which μ is the scale of renormalization.

Koksma, Prokopec, arXiv:1105.6296

The contributions from all fields (bosons and fermions) are given by

$$\langle\rho\rangle = \sum_i n_i \frac{m_i^4}{64\pi^2} \ln\left(\frac{m_i^2}{\mu^2}\right)$$

J. Martin, arXiv: 1205.3365

Koksma, Prokopec, arXiv:1105.6296

n_i: the degree of freedom (polarizations) of each field.

real scalar : $n = 1$

massive vector : $n = 3$

Dirac fermion : $n = -4$.

Some notable conclusions:

1- The massless fields (gravitons, photons, gluons) do not contribute in ρ_v.

2- The contributions of a given field can be either positive (dS) or negative (AdS).

3- For EW scale $m \sim 10^2 GeV \rightarrow |\langle\rho_V\rangle| \sim (10^{11})^4 eV^4$.

This leads to a hierarchy 10^{52} instead of the infamous 10^{120}.

Questions of dS horizon

arXiv: 2201.02016, H. F.

It is assumed that a homogenous zero point energy covers the entire spacetime.

In the presence of gravity, a positive vacuum energy possesses a dS horizon.

Denote the vacuum energy by the field with mass m by $\rho_v(m) \sim m^4$.

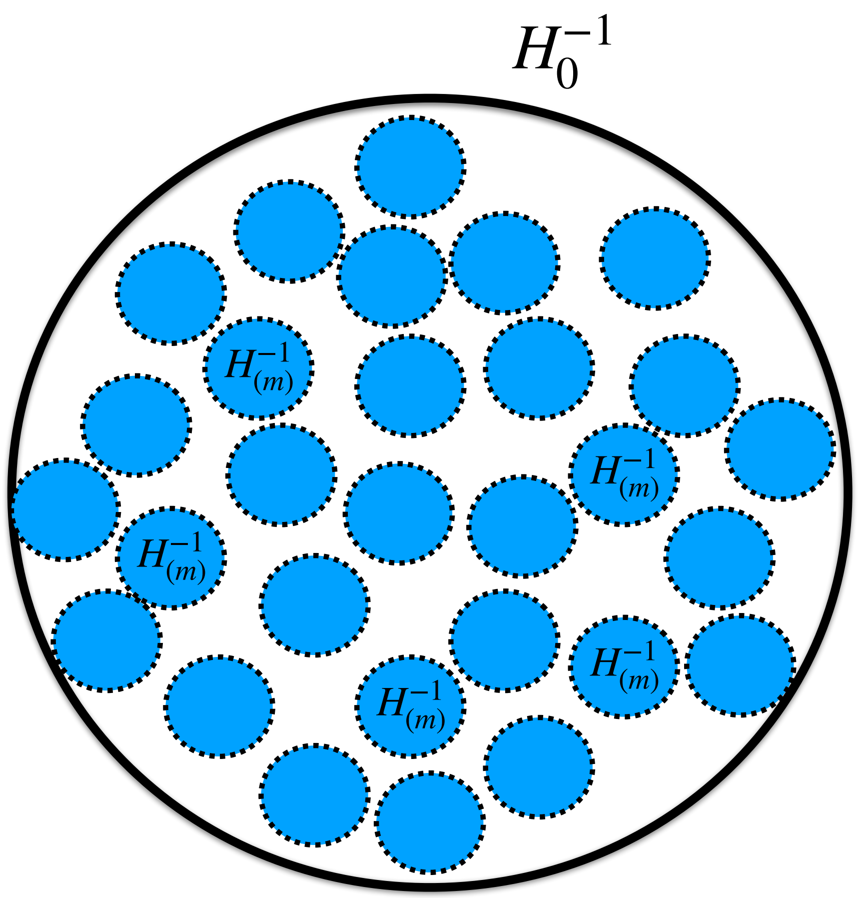
Define the associated Hubble expansion rate by $H_{(m)}$:

$$H_{(m)} \simeq \frac{m^2}{M_P}$$

Example: $m = m_{(e)}$ then $H_{(m)}^{-1} \sim 10^9 m$.

Compared to current FLRW horizon $H_0 \sim 10^{26} m$:

$$\left(\frac{H_0^{-1}}{H_{(m_e)}^{-1}}\right)^3 \sim 10^{51}$$



For a field of mass m, we require $N_{\rm patches} \gg 1$ dS patches of size $H_{(m)}^{-1}$ to cover the observable universe

$$N_{\rm patches} \sim \left(\frac{H_{(0)}^{-1}}{H_m^{-1}}\right)^3 \sim \left(\frac{m}{10^{-2}{\rm eV}}\right)^6$$

The cracks on cosmological constant

Correlation length

Later we show that the correlation length ξ is $\xi \sim m^{-1}$:

$$\frac{\xi}{H_{(m)}^{-1}} \sim \frac{m}{M_P} \ll 1$$

Vast hierarchy of scales for **heavy fields**

$$\xi \ll H_{(m)}^{-1} \ll H_0^{-1}$$

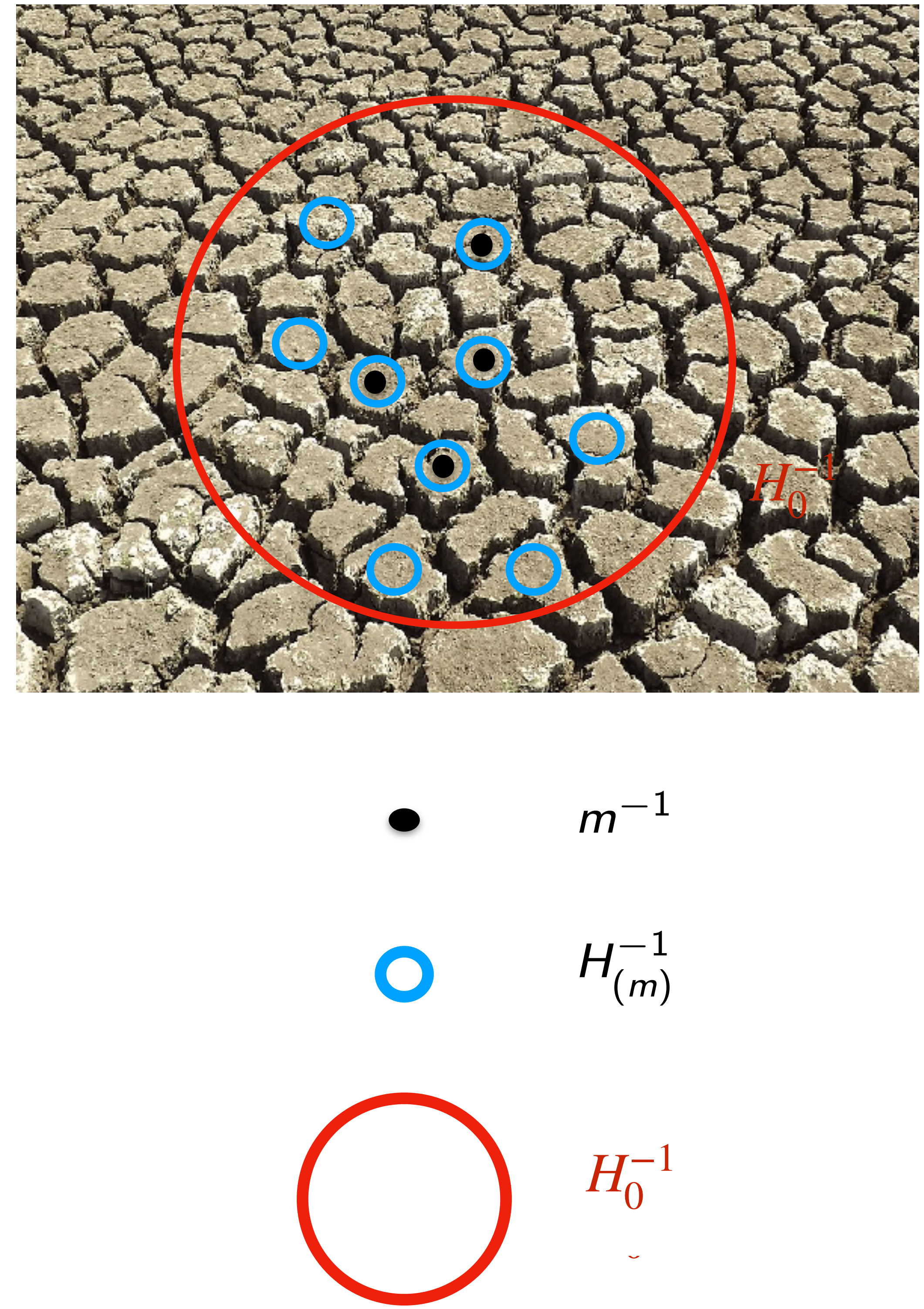
The energy density for super Compton length scales, i.e. the long modes with $k \leq m$:

$$\delta\rho_C(m) = \frac{1}{2}\int_0^m \frac{d^3\mathbf{k}}{(2\pi)^3}\sqrt{k^2+m^2} \sim \frac{6m^4}{64\pi^2}$$

Comparing to the would-be background ρ_v

$$\frac{\delta\rho_C(m)}{\rho_v(m)} \sim 1$$

Another hint for the inhomogeneities of background covered by the zero point energy.



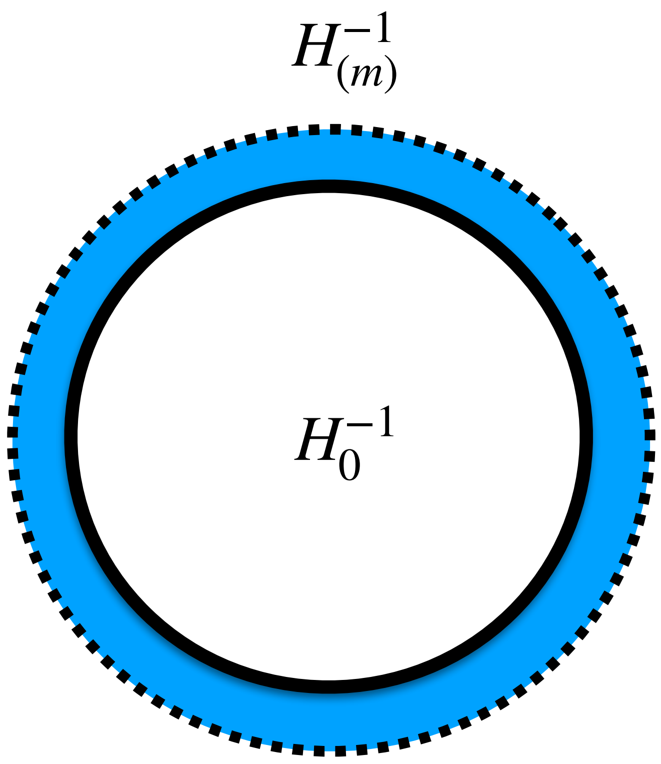
Light fields

arXiv: 2201.02016, H. F.

Suppose we have a very light fields such that $H_{(m)} \sim H_0$.

$$H_{(m)} \sim H_0 \quad \rightarrow \quad \frac{m^2}{M_P} \sim H_0 \quad \rightarrow \quad m \sim 10^{-2} eV$$

The entire universe is within one dS horizon!



This also seen from our expression for N_{patches}

$$N_{\text{patches}} \sim \left(\frac{H_{(m)}}{H_0}\right)^3 \sim \left(\frac{m}{10^{-2}\text{eV}}\right)^6$$

In the spectrum of SM, neutrino can do the job!

The universe is within one patch created by neutrino.

Neutrino is the source of observed dark energy today!!

Density Contrast

To justify our conclusion of inhomogeneities in vacuum zero point energy here we calculate the variance and density contrast $\frac{\delta\rho_v}{\rho_v}$.

We need to calculate $\delta\rho^2 = \langle\rho^2\rangle - \langle\rho\rangle^2$.

$$\phi(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\mathbf{k}}{\sqrt{2\omega(k)}} \left[e^{ik\cdot x} a_{\mathbf{k}} + e^{-ik\cdot x} a^{\dagger}_{\mathbf{k}} \right]$$

The energy density is given by $\rho \equiv \rho_1 + \rho_2 + \rho_3$:

$$\rho_1 \equiv \frac{\dot{\phi}^2}{2}, \qquad \rho_2 \equiv \frac{1}{2}\delta^{ij}\partial_i\phi\partial_j\phi, \qquad \rho_3 \equiv \frac{m^2}{2}\phi^2$$

Using dimensional regularization, we obtain

$$\langle\rho_1\rangle = \frac{1}{2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{\omega(q)}{2} = \frac{m^4}{128\pi^2}\ln\left(\frac{m^2}{\mu^2}\right),$$

$$\langle\rho_2\rangle = \frac{1}{2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{\mathbf{q}^2}{2\omega(q)} = \frac{-3m^4}{128\pi^2}\ln\left(\frac{m^2}{\mu^2}\right) \qquad \langle\rho_1\rangle = -\frac{1}{3}\langle\rho_2\rangle = \frac{1}{4}\langle\rho_3\rangle .$$

$$\langle\rho_3\rangle = \frac{1}{2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{m^2}{2\omega(q)} = \frac{4m^4}{128\pi^2}\ln\left(\frac{m^2}{\mu^2}\right).$$

The variance in density distribution is given by $\delta\rho_v^2 \equiv \langle \rho_v^2 \rangle - \langle \rho_v \rangle^2$:

$$\begin{aligned}\delta\rho^2 &= \langle \rho_1^2 \rangle + \langle \rho_2^2 \rangle + \langle \rho_3^2 \rangle + 2\langle \rho_1 \rho_2 \rangle + 2\langle \rho_1 \rho_3 \rangle + 2\langle \rho_2 \rho_3 \\ &- \left(\langle \rho_1 \rangle^2 + \langle \rho_2 \rangle^2 + \langle \rho_3 \rangle^2 \right) - 2\langle \rho_1 \rangle \langle \rho_2 \rangle - 2\langle \rho_1 \rangle \langle \rho_3 \rangle - 2\langle \rho_2 \rangle \langle \rho_3 \rangle\end{aligned}$$

We have

$$\langle \rho_i^2 \rangle = 3\langle \rho_i \rangle^2 \,, \qquad i = 1, 3.$$

and

$$\begin{aligned}\langle \rho_2^2 \rangle &= \langle \rho_2 \rangle^2 + (2)\frac{1}{4}\int \frac{d^3\mathbf{q}_1}{(2\pi)^3 2\omega(q_1)} \frac{d^3\mathbf{q}_2}{(2\pi)^3 2\omega(q_2)} (\mathbf{q}_1 \cdot \mathbf{q}_2)^2 \\ &= \langle \rho_2 \rangle^2 + (2 \times \frac{1}{3}) \langle \rho_2 \rangle^2 \,,\end{aligned}$$

Also

$$\langle \rho_1 \rho_2 \rangle = \frac{1}{4}\int \frac{d^3\mathbf{q}_1}{(2\pi)^3} \frac{d^3\mathbf{q}_2}{(2\pi)^3} \frac{\omega(q_1)}{2} \frac{\mathbf{q}_2^2}{2\omega(q_2)} = \langle \rho_1 \rangle \langle \rho_2 \rangle \,.$$

Similarly:

$$\langle \rho_2 \rho_3 \rangle = \langle \rho_2 \rangle \langle \rho_3 \rangle \,, \qquad \langle \rho_1 \rho_3 \rangle = \langle \rho_1 \rangle \langle \rho_3 \rangle$$

Combining all, we obtain

$$\delta\rho^2 = 10\,\langle\rho\rangle^2$$

and

$$\frac{\delta\rho}{\langle\rho\rangle} = \pm\sqrt{10}$$

The background constructed purely from the zero point energy is highly inhomogeneous.

Parts of spacetime becomes AdS.

These patches can collapse to black holes.

One can repeat these analysis for fermions.
For a Dirac fermion field we obtain

$$\left|\frac{\delta\rho}{\langle\rho\rangle}\right| = \frac{\pm\sqrt{10}}{4}$$

The spacetime filled purely by vacuum zero point energy is highly inhomogeneous.

One can also calculate the variance in $T^{\mu}_{\mu} \equiv T$.

This is a well-motivated question as T^{μ}_{μ} measures the Ricci scalar of the spacetime: $M_P^2 R = -T^{\mu}_{\mu}$.

From the perfect fluid form, we have $T^{\mu}_{\mu} = \rho + 3p$ in which

$$\rho = \rho_1 + \rho_2 + \rho_3 \qquad , \qquad p = \rho_1 - \frac{\rho_2}{3} - \rho_3 \, .$$

Performing the analysis as before, we obtain

$$\delta T^2 = \langle (T^{\mu}_{\mu})^2 \rangle - \langle T^{\mu}_{\mu} \rangle^2 = 40 \langle \rho \rangle^2 \, .$$

Correspondingly:

$$\frac{\delta T}{T} = \pm\sqrt{10} \, .$$

Intuitively speaking, at each point in space

$$\rho(x) = \langle \rho \rangle + \delta\rho(x) = \langle \rho \rangle \left(1 \pm \sqrt{10} \right)$$

$$T(x) = \langle T \rangle + \delta T(x) = \langle T \rangle \left(1 \pm \sqrt{10} \right)$$

In the previous analysis we have neglected the conventional matter and radiation energy density of the FLRW universe, ρ_F

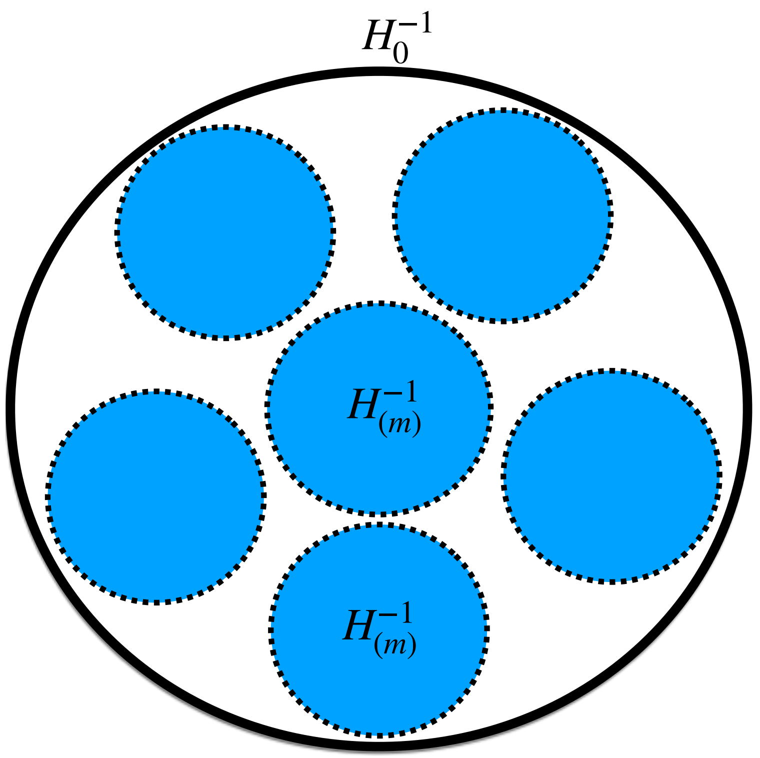
Now, let us look at the total energy density: $\rho_T = \rho_F + \rho_v$

Also note that $\langle \rho_F \rangle = \rho_F$ and $\delta\rho_T = \delta\rho_v$. Therefore,

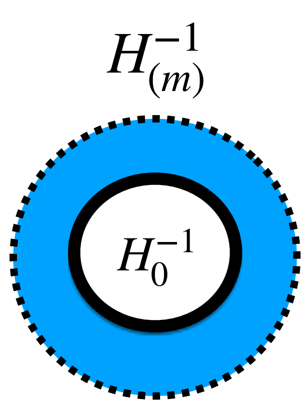
$$\frac{\delta\rho_T}{\langle\rho_T\rangle} = \frac{\delta\rho_v}{\rho_F + \langle\rho_v\rangle} = \pm\sqrt{10}\frac{\langle\rho_v\rangle}{\rho_F + \langle\rho_v\rangle}\,.$$

Demanding that $|\frac{\delta\rho_T}{\langle\rho_T\rangle}| < 1$, we obtain

$$\langle\rho_v\rangle < \left(\sqrt{10} - 1\right)^{-1}\rho_F \sim \frac{\rho_F}{2}\,.$$



To have a consistent background one requires $\langle\rho_v\rangle \lesssim \rho_F$.



$H_{(m)} \ll H_0$

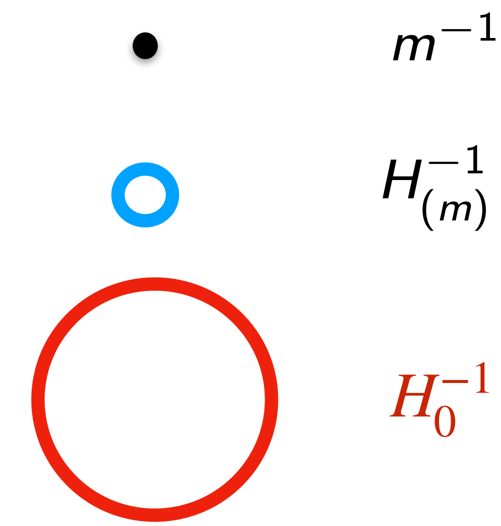
Consistent

$H_{(m)} \gg H_0$

Inconsistent

Correlation Length

Define the connected correlation function $\left\langle \rho(\mathbf{x})\rho(\mathbf{0})\right\rangle_c \equiv \left\langle \rho(\mathbf{x})\rho(\mathbf{0})\right\rangle - \langle \rho^2 \rangle$



The correlation length ξ is defined such that

$$\left\langle \rho(\mathbf{x})\rho(\mathbf{0})\right\rangle_c \to e^{-\frac{r}{\xi}}\,, \qquad (r \to \infty)\,.$$

For the scalar field, we typically have

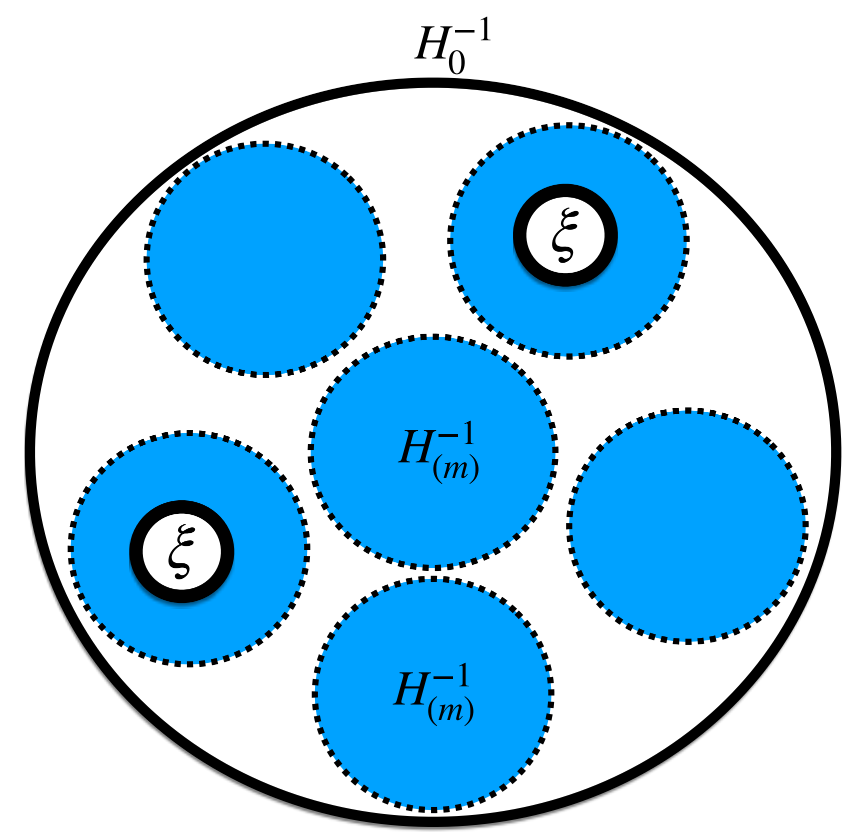
$$\left\langle \rho(\mathbf{x})\rho(\mathbf{0})\right\rangle_c = \frac{m^4}{2}\left(\int \frac{d^3\mathbf{q}}{(2\pi)^3 2\omega(q)} e^{-i\mathbf{q}\cdot\mathbf{x}}\right)^2 .$$

The above integral is well-known in QFT:

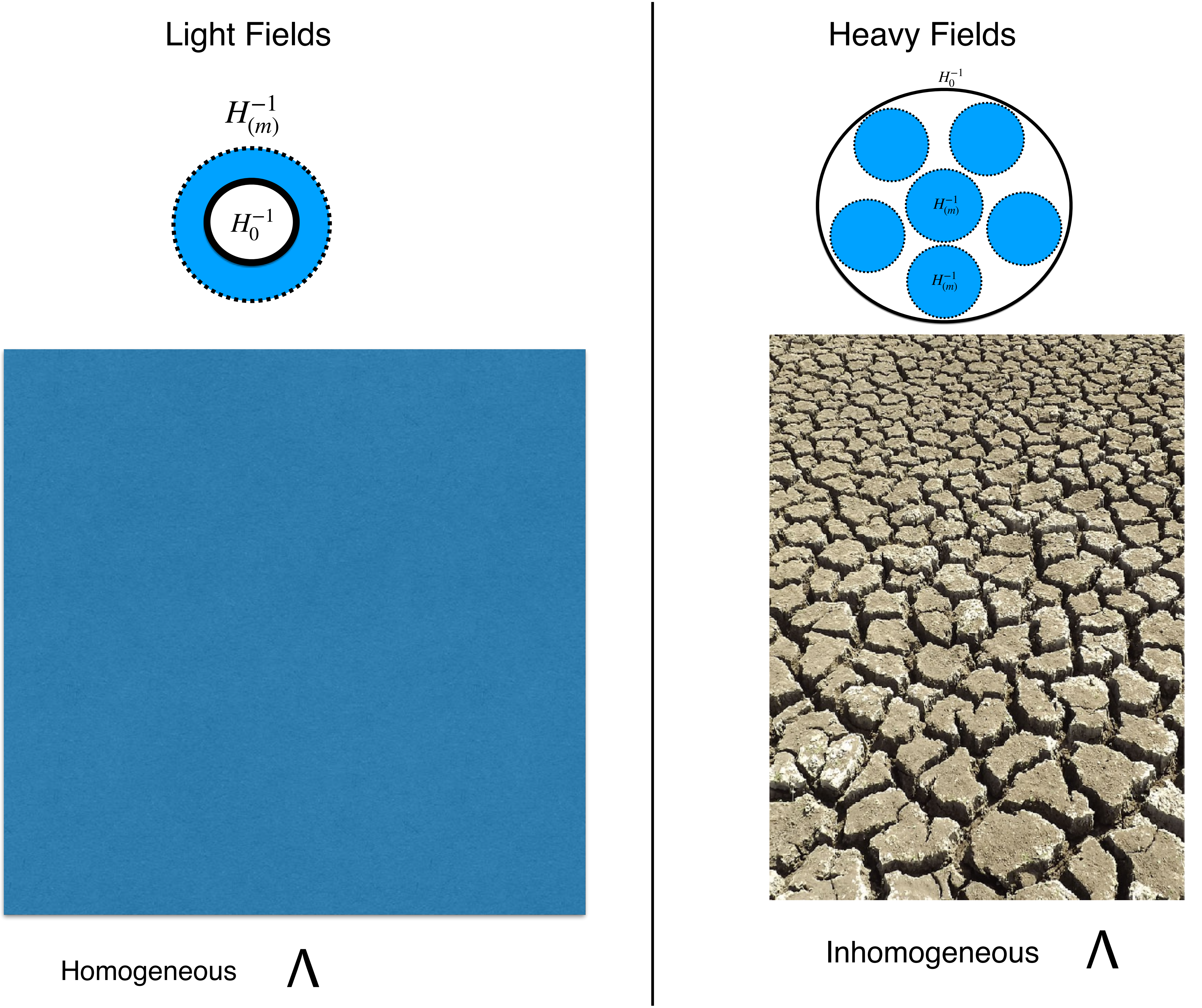
$$\left\langle \rho_3(\mathbf{x})\rho_3(\mathbf{0})\right\rangle_c = \frac{m^8}{32\pi^4}\left(\frac{K_1(mr)}{mr}\right)^2 ,$$

Using the asymptotic behaviour $K_1(x) \sim e^{-x}$, we obtain

$$\xi \simeq m^{-1} \qquad \to \qquad \xi \ll H_{(m)}^{-1}$$



One can not cover the entire FLRW horizon by a vast number of uncorrelated dS patches and yet expect them to behave as a uniform cosmological constant.


Light Fields
$H_{(m)}^{-1}$
H_0^{-1}
Homogeneous
Λ
Heavy Fields
H_0^{-1}
$H_{(m)}^{-1}$
$H_{(m)}^{-1}$
Inhomogeneous
Λ

Zero Point Energy in dS Background

The analysis of vacuum zero point energy yielding to $\langle \rho_v \rangle \sim m^4$ have been performed in a flat background.

Question: is the result reliable in a curved background?

Answer: The combination of the Lorentz invariance and the equivalence principle guarantees that the above result should be valid in a curved spacetime.

We examine this conclusion for a dS spacetime!

Dimensional regularization for a scalar field in a dS background:

$$S = \int d^d x \sqrt{-g} \Big[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 \Big].$$

defining the canonically normalized field σ via $\sigma \equiv a^{\frac{d-2}{2}} \phi$ the mode function is given by

$$\sigma_k'' + \Big[k^2 + \frac{m^2}{H^2 \tau^2} - \frac{(d-2)^2}{2\tau^2} \Big] \sigma_k = 0 \, .$$

Imposing the Minkowski (Bunch-Davies) vacuum deep inside the horizon:

$$\sigma_k = \frac{1}{\sqrt{2k}} e^{-ik\tau} ,$$

the solution of the mode function is given in terms of the Hankel function of the first type as follows

$$\phi_k(t) = (-H\tau)^{\frac{d-1}{2}} \left(\frac{\pi}{4H}\right)^{\frac{1}{2}} e^{-\frac{\pi}{2}\nu} H^{(1)}_{i\nu}(-k\tau) ,$$

where

$$\nu \equiv \sqrt{\frac{m^2}{H^2} - 1} \simeq \frac{m}{H} .$$

The energy density is given by

$$\langle\rho\rangle = \frac{\pi\mu^{4-d} e^{-\pi\nu} H^d}{8(2\pi)^{d-1}} \left(\int d^{d-2}\Omega\right) \int_0^\infty dx\, x^d \left[\left|\frac{d}{dx} H^{(1)}_{i\nu}(x)\right|^2 + \left(1 + \frac{\nu^2}{x^2}\right)\left|H^{(1)}_{i\nu}(x)\right|^2 \right]$$

The integral over the azimuthal directions is:

$$\int d^{d-2}\Omega = \frac{2\pi^{(d-1)/2}}{\Gamma(\frac{d}{2} - \frac{1}{2})} ,$$

The final step is to perform the following integral

$$I \equiv \int_0^\infty x^d \Big[\Big| \frac{d}{dx} H_{i\nu}^{(1)}(x) \Big|^2 + (1 + \frac{\nu^2}{x^2}) \Big| H_{i\nu}^{(1)}(x) \Big|^2 \Big]$$

Fortunately the above integral can be taken exactly:

$$I = \frac{(1-d+2i\nu)}{4\pi^{5/2}\sinh^2(\nu\pi)} \Gamma(-i\nu - \frac{1}{2} + \frac{d}{2}) \Gamma(i\nu + \frac{1}{2} + \frac{d}{2}) \Gamma(\frac{d}{2} - \frac{1}{2}) \Gamma(-\frac{d}{2}) \times \mathcal{C}$$

where $\mathcal{C}$ is given by

$$\begin{aligned} \mathcal{C} \equiv\ & 2\cosh(\nu\pi)\cos(\frac{\pi(2i\nu - d)}{2})\cos(\frac{\pi(2i\nu + d)}{2}) \\ - & \cos(\frac{\pi d}{2}) \Big[\cos(\frac{\pi(2i\nu + d)}{2}) + \cos(\frac{\pi(2i\nu - d)}{2}) \Big] \end{aligned}$$

Expanding $d = 4 - \epsilon$ in $\langle\rho\rangle$, we obtaining

$$\langle\rho\rangle = \frac{H^4}{2048\pi^2}(4\nu^2+1)(4\nu^2+9)\Big[-\frac{4}{\epsilon} + 2\ln\left(\frac{H^2}{4\pi\mu^2}\right) + \Delta + \mathcal{O}(\epsilon) \Big],$$

where $\Delta(\nu) \to 4\ln(\nu)$ for large ν limit.

Expanding $\langle \rho \rangle$ to leading order in ϵ and taking $\nu \simeq \frac{m}{H} \gg 1$, we obtain

$$\langle \rho \rangle = -\frac{\nu^4 H^4}{64\pi^2} \left[\frac{2}{\epsilon} - \ln\left(\frac{H^2 \nu^2}{4\pi \mu^2} \right) + ... \right] ,$$

Performing the regularization we obtain the original result:

$$\langle \rho \rangle = \frac{m^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2} \right)$$

Like in flat background, the correlation length is $\xi \sim m^{-1}$.

The energy of long modes beyond the correlation length:

$$\delta\rho_C \simeq \frac{H^4}{16\pi} \frac{\nu^4}{\pi} \simeq \frac{m^4}{16\pi^2} .$$

$$\rightarrow \quad \frac{\delta\rho_C}{\langle \rho_v \rangle} \sim 1$$

The background is highly inhomogeneous!

Cosmological Implications

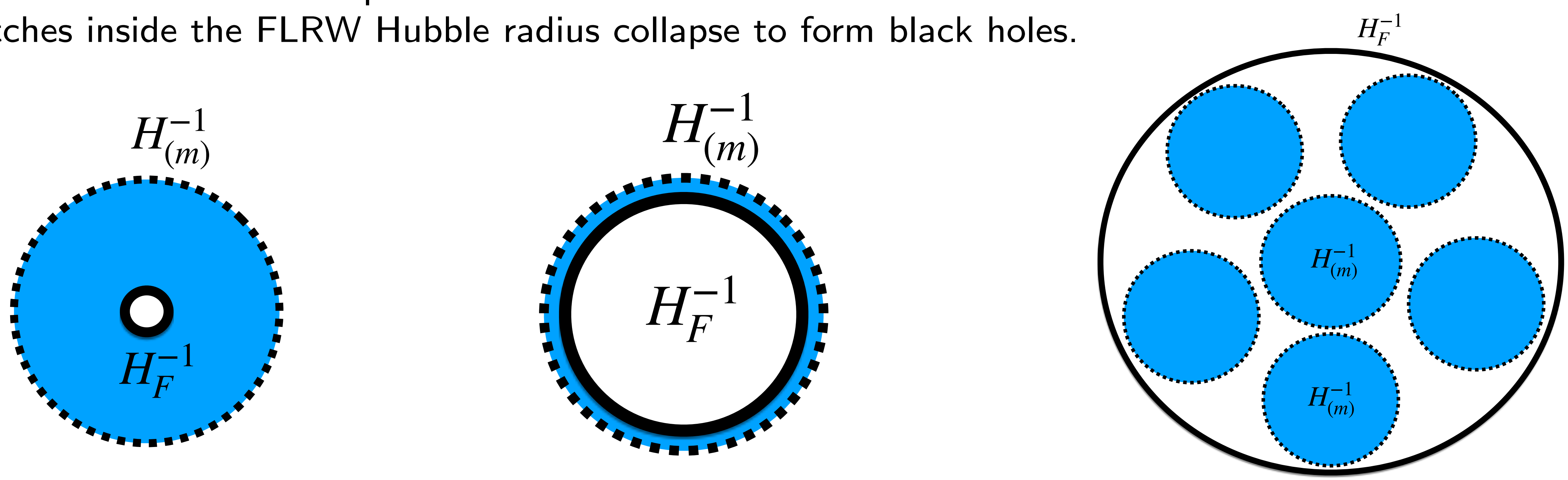
The heavy fields with $\langle \rho_v \rangle \gg \rho_F$ can not contribute to the observed dark energy today. Question: What roles in cosmology their vacuum energy density play?

Stage 1: Early expansion history when $\rho_F \gg \rho_v$ so $H_F^{-1} \ll H_{(m)}^{-1}$, then ρ_v is irrelevant.

Stage 2: ρ_F decreases until $\rho_F \sim \rho_v$ and $H_F^{-1} \sim H_{(m)}^{-1}$ so ρ_v becomes relevant.

Stage 3: ρ_F falls off rapidly and $\rho_v \gg \rho_F$. The regions filled with the zero point energy develop strong inhomogeneities while falling into the FLRW Hubble horizon. It takes time of about $1/H_{(m)}$ for each dS horizon to enter the FLRW horizon.

As more and more of dS patches enter the FLRW horizon the mass inside the dS patches inside the FLRW Hubble radius collapse to form black holes.



Stage 1 Stage 2 Stage 3

Once the masses inside these patches collapse into black holes, they behave as **dark matter** or the **seeds of dark matter.**

We do not have a deep understanding of the mechanism of collapse for small dS patches with $H_{(m)}^{-1} \ll H_F^{-1}$.

Phenomenological Fluid Description

We assume to have two fluid components: $\rho_F, \tilde{\rho}_m$ and $\rho_T = \rho_F + \tilde{\rho}_m$ with:

$$\dot{\rho}_F + 3H(1+w_F)\rho_F = 0, \qquad \dot{\tilde{\rho}}_m + 3H(1+w_m)\tilde{\rho}_m = 0$$

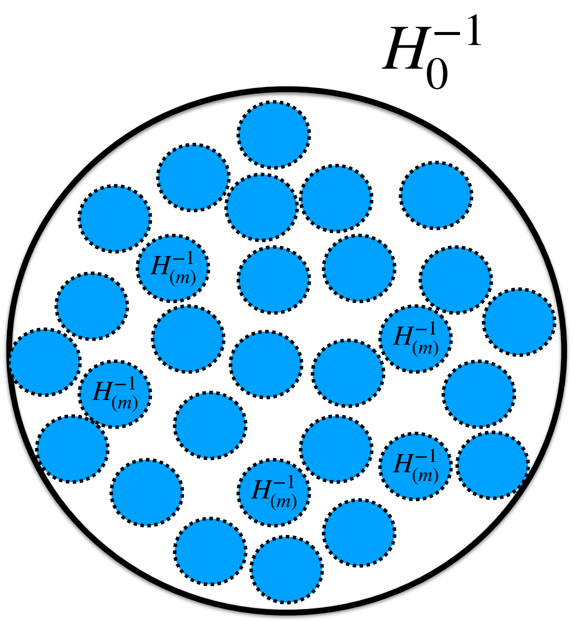
The expansion of FLRW background is given by

$$\rho_T(t) = \left[\rho_m \left(\frac{a(t)}{a(t_m)}\right)^{-3(1+w_m)} + \rho_F(t_m)\left(\frac{a(t)}{a(t_m)}\right)^{-3(1+w_F)}\right]$$

in which t_m is the time when $\rho_F(t_m) \sim \rho_m$. Alternatively:

$$\rho_T(t) = \rho_F(t_m)\left(\frac{a(t)}{a(t_m)}\right)^{-3(1+w_F)}\left[1 + \kappa' \left(\frac{a(t)}{a(t_m)}\right)^{3(w_F - w_m)}\right]$$

where $\tilde{\rho}_m(t_m) \equiv \kappa' \rho_F(t_m)$



arXiv: 2201.02016, H. F.

Selection Rules:

At each stage in cosmic epoch, only field with $H_{(m)} \sim H_F$ can be the source of D. E.

Fields which are much lighter, $H_{(m)} \ll H_F$, are irrelevant in cosmic expansion.

Heavy fields with $H_{(m)} \gg H_F$ can contribute as the seeds of dark matter.

Solution to CC problems:

Old c.c problem: Why is not ρ_v large?

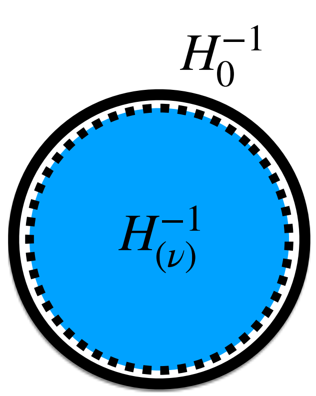
New c.c problem: Why it becomes comparable to the current matter energy density?

Answer: There is a field in the SM field content, the lightest neutrino with $\rho_{(\nu)} \sim m_\nu^4$ which happens to have a mass at the same order as $\rho_{F0}^{1/4} \simeq \rho_c^{1/4} \sim (10^{-3}\mathrm{eV})^4$.

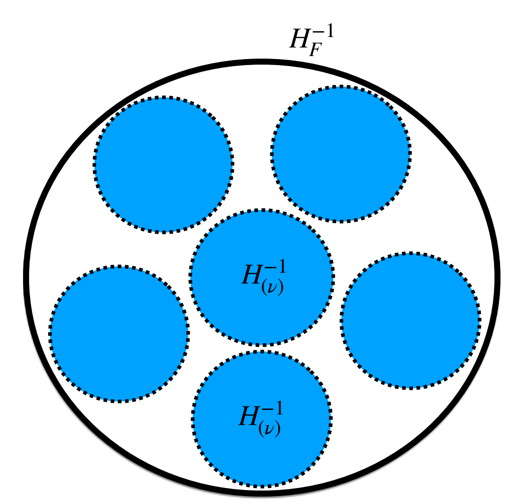
The entire FLRW universe is currently within a single patch of the lightest neutrino with the horizon radius $H_{(\nu)}^{-1}$.

Dark energy survives in future for roughly $H_{(\nu)}^{-1} \sim H_0^{-1} \sim 10^{10}$ years.

Current time



10 billion years later !



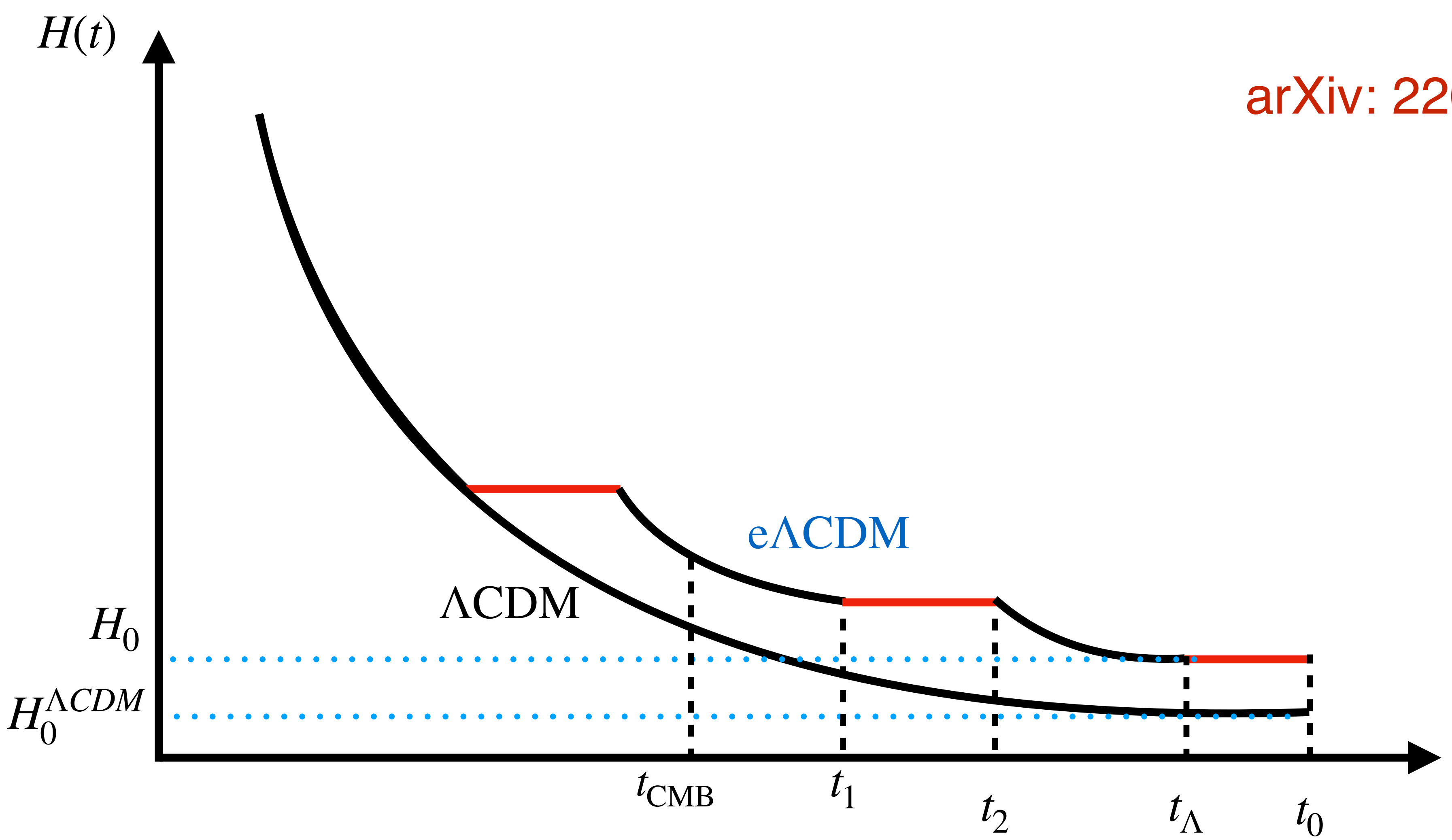
H_0 tension

The proposal predicts multiple transient period of dark energy, early and late time !

This happens when $m \sim T$ for a field in SM spectrum for 1-2 e-folds expansion.

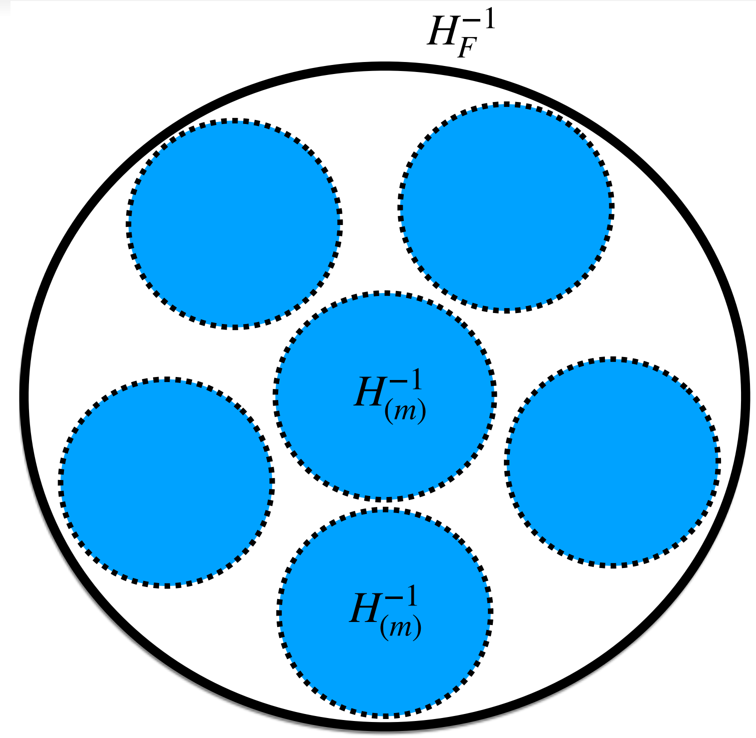
This proposal can resolve the H_0 tension problem.

As both Λ and CDM emerges dynamically, we may call this proposal emergent ΛCDM or **eΛCDM** for short!



arXiv: 2201.02016, H. F.

The Origin of Supermassive Black Holes?



The collapse of zero point energy may be the origin of the supermassive black holes (SMBHs)!

After the time when $T \sim m$ when $\frac{\delta\rho}{\rho} \sim 1$, the mass inside the (A)dS horizon may collapse to form BHs:

$$M_B(m) = \frac{4\pi}{3}\rho_v(m)H_{(m)}^{-3} \sim 10^2\frac{M_P^3}{m^2}$$

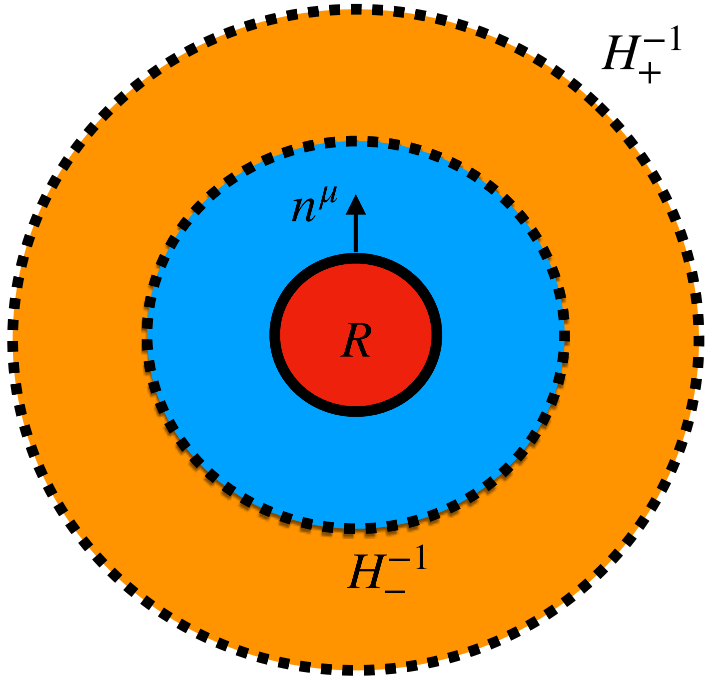
Surprisingly, the above formula is the same as the Chandrasekhar mass formula!!

For various fields in SM we have:

$$\begin{aligned}
m_t \simeq 170\text{GeV} &\rightarrow & M_B(m_t) &\simeq 4\times 10^{-5}M_\odot \\
m_W \simeq 80\text{GeV} &\rightarrow & M_B(W) &\simeq 2\times 10^{-4}M_\odot \\
m_\tau \simeq 1.7\text{GeV} &\rightarrow & M_B(\tau) &\simeq 0.4\times M_\odot \\
m_\mu \simeq 105\text{MeV} &\rightarrow & M_B(\mu) &\simeq 112\times M_\odot \\
m_e \simeq 1\text{MeV} &\rightarrow & M_B(e) &\simeq 5\times 10^6 M_\odot
\end{aligned}$$

Expansion of Bubble of Zero Energy

We can present the toy model of expansion of bubbles of zero point energy to capture our results.



We can imagine that bubbles of zero point energy are pupped up from the vacuum and expand in the ambient cosmological background.

This is somewhat similar to the question of expansion of bubbles of true vacuum in the false vacuum in the mechanism of false vacuum decay.

The interior and the exterior of the bubble are given by two FLRW metrics:

$$ds_\pm^2 = dt_\pm + a_\pm(t_\pm)^2(dr_\pm^2 + r_\pm^2 d\Omega^2)\ ,$$

Separated at the boundary by an expansing sphere of radius $R(\tau)$

$$ds^2 = -d\tau^2 + R(\tau)^2 d\Omega^2\ ,$$

We imagine that the surface of wall has the energy momentum tensor $S^{ab} = \mathrm{diag}(-\sigma, \sigma, \sigma)$.

The unit normal n^μ to the wall is

$$n_\mu^\pm = (-a_\pm r'_\pm\ , a_\pm t'_\pm, 0, 0)\ ,$$

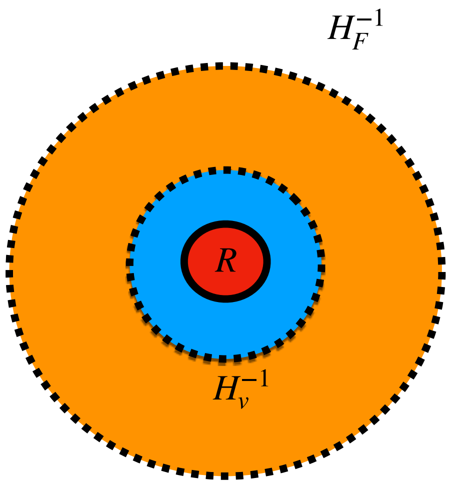
Imposing the Israel matching conditions we obtain $(\frac{dR}{d\tau})^2 = A^2 R^2 - 1$

in which

$$A^2 = \frac{\sigma^2}{16 M_P^4} + \frac{H_-^2 + H_+^2}{2} + \frac{M_P^4}{\sigma^2}(H_+^2 - H_-^2)^2 .$$

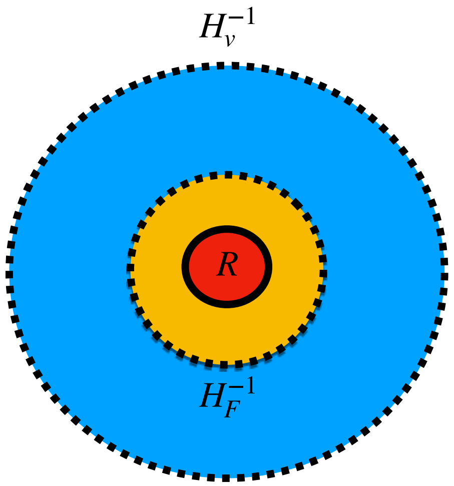
case 1: $H_{(m)}^{-1} < H_F^{-1}$

This is the case when the vacuum bubbles are (deep) inside the FLRW horizon. Typically this is the case where the field is massive, i.e. $m > T$.



case 2: $H_{(m)}^{-1} \gg H_F^{-1}$

This is the case where the FLRW horizon is deep inside the vacuum bubble and the universe expends. This corresponds to light fields, $m \ll T$.



Consider the case $H_{(m)}^{-1} \gg H_F^{-1}$:

$$\frac{dR}{dt_-} \simeq 1 + R\,H_-$$

Using the energy conservation equation, one obtains

$$H_- = H_{(m)} \coth\left(\frac{H_{(m)} t_-}{\lambda}\right)$$

We see that as $t_- \to \infty,\ H_- \to H_{(m)}$.

Correspondingly, for large t_-

$$R \simeq \frac{e^{H_{(m)} t_-}}{H_{(m)}}\,, \qquad (H_{(m)} t_- > 1)$$

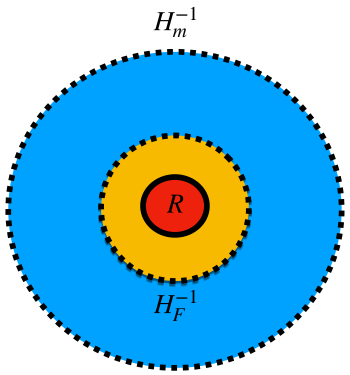
and

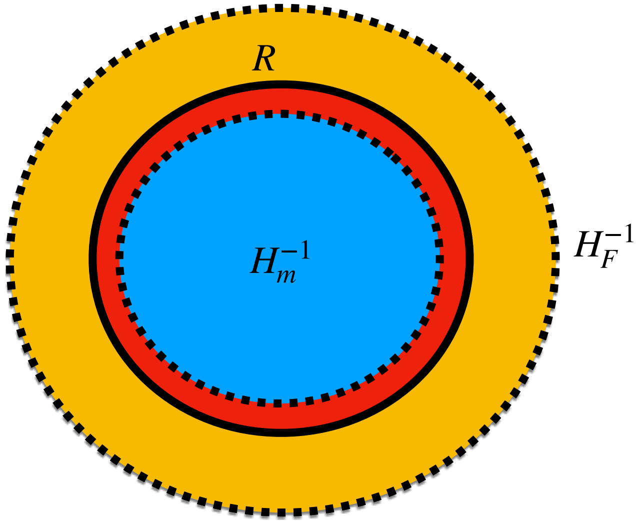
$$t_- \simeq \frac{1}{H_{(m)}} \ln\left(RH_{(m)}\right)$$

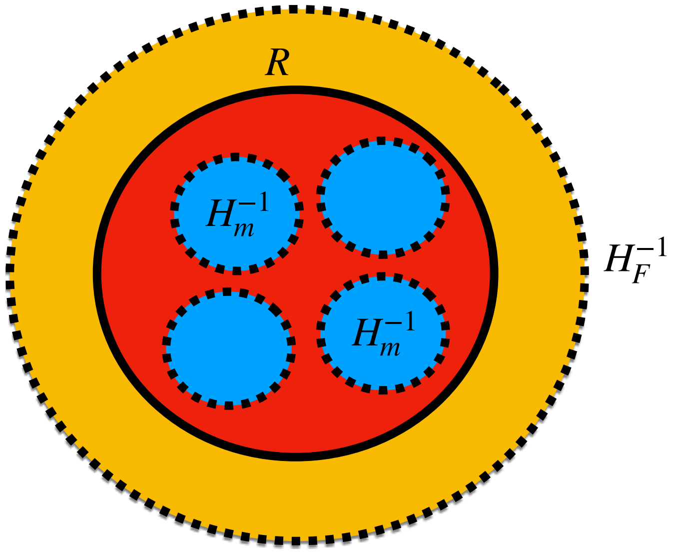
Interestingly, one obtains

$$R \simeq \frac{1}{H_{(m)}} \left(\frac{H_F}{2H_{(m)}}\right)^{-\frac{2}{3(1+w_F)}} \propto a_F$$

In other words, the radius of the bubble expands as the FLRW scale factor in the absence of DE.







Open Questions and Discussions

1- How the mechanism of collapse for the (A)dS patches of heavy fields inside the FLRW horizon works?

This is crucial for the understanding of **dark matter.**

2- The value of renormalization scale μ?

To solve the cosmological constant problems, we assumed that $\mu > m_\nu > \mathrm{eV}$. Another option may be simply $\mu \sim M_{EW} \sim 10^2\mathrm{GeV}$.

If $\mu \ll m_\nu$, then one needs a field of mass $m \sim 10^{-3}\mathrm{eV}$ in beyond SM sector to address the cosmological constant problems. **Axion?**

Works of Unruh and collaborators:

arXiv:1703.00543, arXiv:1805.12293, arXiv:1904.08599.

They have questioned the assumption of the homogeneity of the spacetime in the presence of the vacuum zero point energy. It was concluded that a uniform cosmological constant can not cover the large scale spacetime and the local spacetime is very inhomogeneous as in Wheeler's spacetime foam.

Conclusions

We have revisited the quantum cosmological constant problems.

As already known in the literature, the contribution of each field to CC is like m^4.

We highlighted the important roles played by the dS horizon associated with the vacuum zero point energy. It was argued that only fields which have dS horizon comparable to the FLRW horizon at that epoch can contribute to dark energy: **the dark energy selection rules.**

The proposal solves the old and new cosmological constant problems by noting that there exists a field in the SM spectrum, the (lightest) neutrino field, which happens to have a mass comparable to the photon temperature.

The spacetime created purely from the zero point energy is highly inhomogeneous. To have a stable cosmological background we require a classical source of energy not much smaller than dark energy.

Both dark energy and dark matter emerges dynamically in this setup so we have the **eΛCDM** setup instead of ΛCDM.

The **eΛCDM proposal** predicts multiple transient periods of dark energy which yield to a higher value of H_0. This can resolve the H_0 tension problem.

Thank You

سپاسگزارم