F–theory GUTs

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Outline

What is F–theory?

F–theory GUTs and local models

Why this is not the whole story

F–theory Facts I

- F–theory is type IIB string theory (with D–branes) with non–constant string coupling. [Vafa '96]
  - The complexified string coupling constant is given by the axio–dilaton:

$$\tau = C_0 + ie^{\phi} \qquad g_s = e^{\phi}$$

- In F–theory the back reaction of the $D7$ branes on the geometry is taken into account.
- F–theory is non–perturbative.
  - There is no world sheet formulation of F–theory.

F–theory Facts II

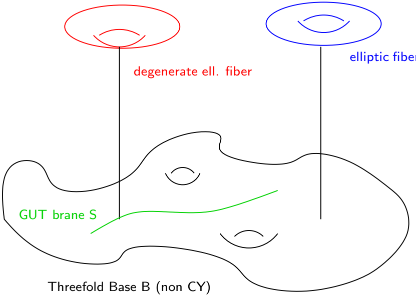
- For IIB to be invariant under S–duality τ has to transform under the S–duality group $SL(2,\mathbb{Z})$:

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

  - $a, b, c, d \in \mathbb{Z}$, $ad - bc = 1$
  - $SL(2,\mathbb{Z})$ is the modular group of a two–torus T^2.
  - τ can be viewed as the complex structure of a torus.
- We can geometrize the complexified string coupling.
  - IIB with varying coupling $\tau \Leftrightarrow$ 'F–theory' on an elliptically fibered Calabi–Yau fourfold

Geometric Picture

- F–theory on elliptically fibered CY_4:



- What happens at the loci where the torus fiber degenerates?

Elliptic Fibration

- An elliptic fibration is given by a Weierstrass equation:

$$y^2 = x^3 + f(y_i)xz^3 + g(y_i)z^6$$

  - $(x, y, z)\ldots$ coords. on T^2–fiber, $y_i \ldots$ coords. in base
- The torus degenerates at the zeros of the discriminant Δ:

$$\Delta = 4f^3 + 27g^2$$

- The singularity structures of the elliptic fiber at the degeneration loci have been classified.

[Kodaira '63][Tate '75][Bershadsky et al '96][Katz,Vafa '96]

Singular Fibers

- The degeneration loci S of the elliptic fiber in the base are the positions of (p, q) 7–branes
    - When (p, q) 7–branes collide there will be enhanced gauge symmetry on the worldvolume.
    - Gauge groups correspond to singularities of the elliptic fiber.
    - ADE Lie groups.
- Depending on the singularity type (p, q) 7–branes have an interpretation in IIB:
    - A_n: $D7$ branes with SU(n) gauge symmetry
    - D_n: $D7$ branes and $O7$ planes with SO(2n) gauge symmetry
    - $E_{6,7,8}$: no IIB interpretation
- F–theory makes it possible to realize non–abelian gauge groups.
    - This is essential for constructing GUTs in string theory.

F-theory GUTs

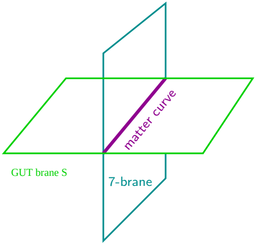
- One can realize GUT models along S where the elliptic fiber degenerates.

[Donagi, Wijnholt '08][Beasley, Heckman, Vafa '08]

- The GUT brane S wraps a divisor in the three–dimensional base B.
  - The GUTs group (typically SU(5) or SO(10)) is determined by how the elliptic fiber degenerates.
- Decoupling limit
  - Decoupling of gravity
  - This is possible if S is a del Pezzo surface.
  - Then the theory can be described by the SUSY gauge theory on the world volume of S. $\Rightarrow$ local model

Matter Curves

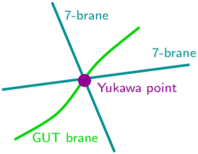
- Chiral matter arises at order 1 enhancements of the GUT singularity.
- Geometrically matter is localized on curves Σ in S where S intersects with a further $U(1)$ 7–brane.



- For an $SO(10)$ GUTs there are the following enhancements:
  - E_6: 16 matter curves
  - $SO(12)$: 10 matter curves

Yukawa couplings

- Yukawa couplings arise at order 2 enhancements of the GUT singularity.
- Geometrically this corresponds to triple intersections of the GUT brane with further 7–branes.



- For an $SO(10)$ GUTs there are the following enhancements:
  - E_7: 16 16 10 Yukawas
  - $SO(14)$: 10 10 1 Yukawas – not really necessary for a minimal $SO(10)$ GUT

Phenomenological Features

- For $SU(5)$ GUTs many details on phenomenology have been worked out.
  - Break to the Standard Model gauge group by $U(1)$ (hypercharge) flux:
    $\Rightarrow$ This solves the doublet-triplet splitting problem.
  - no μ–problem, no dimension 4 proton decay operators
  - SUSY breaking, neutrino masses, flavor hierarchy from instantons.
  - ...
- $SO(10)$ models cannot be broken directly to the Standard Model.
  - First break to $SU(5) \times U(1)$ (flipped $SU(5)$).
    $\Rightarrow$ no doublet–triplet splitting problem
  - No problem with proton decay up to dimension 6 operators.
  - ...

What's nice about local models?

- Local F–theory GUTs elegantly solve many problems GUT models usually have.
- In the local model there are not many parameters which can be adjusted.
  - Matching one quantity with experimental results fixes many others.
  - The numbers match very well with experimental data (e.g. neutrino masses).
- BUT...

What's missing?

- The GUT model has to be embedded into a consistent string compactification.
  - The local geometry has to 'fit' into a Calabi–Yau fourfold.
  - One should not get too many exotics or a hidden sector that is too large.
  - One has to worry about moduli stabilization.
  - One has to worry about global consistency constraints such as tadpole cancellation.
  - The decoupling limit must be realized explicitly.

Outline of part 2

- Construction of global models.
  - Toric geometry
  - Construction of the base manifold B
  - Identification of GUT divisors
  - Existence of the decoupling limit
  - Construction of the elliptically fibered fourfold
- $SO(10)$ GUTs
  - Realization in toric geometry
  - Spectral cover
- Some phenomenology results
  - Split spectral cover to generate chiral matter on the 10 curves
  - Flipped SU(5) models
  - Three generation models

Global $SO(10)$ F–theory GUTs

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joint work with C.-M. Chen, M. Kreuzer, C. Mayrhofer: `arXiv:1005.xxxx[hep-th]`

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Outline

Setup

- Construction of compact CY fourfolds which are suitable for F–theory model building.
  - Fourfolds are complete intersections in a six–dimensional toric ambient space: [Blumenhagen et al. '09][Grimm et al. '09]

$$P_B(y_i, w) = 0 \qquad P_W(x, y, z, y_i, w) = 0$$

$\Rightarrow$ P_B describes the geometry of the base manifold B:
$(y_i, w)\ldots$ base coordinates
$\Rightarrow$ P_W describes the elliptic fibration: $(x, y, z)\ldots$ fiber coordinates
$\Rightarrow$ $w = 0$ describes the GUT divisor S.

  - The Weierstrass model in Tate form:

$$P_W = x^3 - y^2 + xyza_1 + x^2z^2a_2 + yz^3a_3 + xz^4a_4 + z^6a_6$$

$\Rightarrow$ The $a_n(y_i, w)$ are sections of K_B^{-n}.

Goals – Geometry

- Systematically construct fourfold geometries.
- Use toric geometry as a tool.
- First construct a 3-dimensional non–CY base B:
  - Point/curve blowups in Fano threefolds
- Identify candidates for GUT divisors S in B:
  - del Pezzo
  - Decoupling limit
- Make an elliptic fibration over the base B that is a CY fourfold.
- This talk: focus on the geometric aspects

Goals – Model building

- Construct $SO(10)$ F–theory GUTs on our geometries.
  - Construct an $SO(10)$ Weierstrass model in toric geometry.
  - Describe matter curves and fluxes using the spectral cover construction
- Use a split spectral cover to get chiral matter on the $\mathbf{10}$ curves.
  - new degrees of freedom to adjust to get three generations
  - Generate Higgs from chiral matter on $\mathbf{10}$ curves
- Use abelian fluxes to break to flipped $SU(5)$.
- Discuss examples of three–generation models.

Toric Geometry I

- Toric geometry encodes the geometry of a (toric) manifold in the combinatorics of lattice polytopes.
- A toric variety X of dimension n is defined as:

$$X = (\mathbb{C}^r - Z)/((\mathbb{C}^*)^{r-n} \times G)$$

  - $(\mathbb{C}^*)^{r-n}$ acts by coordinate-wise multiplication.
  - Z is an exceptional set encoding which of the coordinates in $\mathbb{C}^r$ cannot vanish simultaneously.
  - G is the action of a discrete group (will not play a role here).
- E.g. $\mathbb{CP}^2$:
  - $(z_1, z_2, z_3) \sim (\lambda z_1, \lambda z_2, \lambda z_3)$, $\lambda \in \mathbb{C}^*$, $Z = \{z_1 = z_2 = z_3 = 0\}$

$$\mathbb{CP}^2 = (\mathbb{C}^3 - \{z_1 = z_2 = z_3 = 0\})/((z_1, z_2, z_3) \sim (\lambda z_1, \lambda z_2, \lambda z_3))$$

Toric Geometry II

- The information about the geometry is encoded in dual pairs of cones and (reflexive) lattice polytopes.
- N–lattice
  - Polytope $\Delta^\circ \subseteq N$, Points $y_i \in N$, Vertices $v_i \in N$, $\dim N = n$.
  - Vertices encode information about coordinates z_i and divisors $D_i = \{z_i = 0\}$.
  - Encode homogeneous weights $\vec{q} = (q_1, \ldots, q_r)$ in $r - n$ linear relations

$$\sum_{i=1}^{r} q_i v_i = 0$$

Toric geometry III

- M–lattice
  - Polytope $\Delta \subseteq M$, Points $y_i \in M$.
  - Points encode information about monomials and hypersurface equations.
  - Regular monomials are:

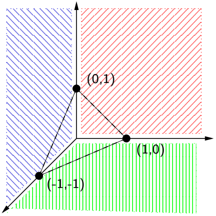
$$\chi^m = \prod_{j=1}^{r} z_j^{\langle m, v_j \rangle + 1} \quad m \in M, v_j \in N$$

  - CY Hypersurface in toric space: $f = \sum_m c_m \chi^m$
    Hypersurfaces are divisors in X.
- Δ and Δ° are dual (polar) polytopes:

$$x_i \in N, y_j \in M : \quad \langle x_i, y_j \rangle \geq -1$$

Example: $\mathbb{CP}^2$

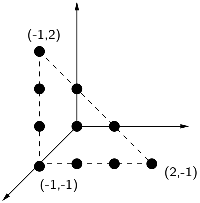
- N-lattice



- Weights: $\vec{q} = (1,1,1) \Leftrightarrow (z_1, z_2, z_3) \sim (\lambda z_1, \lambda z_2, \lambda z_3)$
- Relations: $\sum_i q_i v_i = 0$
- Divisors:

z_1	z_2	z_3
1	1	1
D	D	D

- M-lattice



- Monomials:

$$z_j^{\langle m, v_j \rangle + 1}:$$

$$z_1^3, z_2^3, z_3^3,$$
$$z_1^2 z_2, z_1^2 z_3, z_2^2 z_3,$$
$$z_1 z_2^2, z_1 z_3^2, z_2 z_3^2, z_1 z_2 z_3$$

Complete intersections

- Generalization of the construction:

$$\Delta = \Delta_1 + \ldots + \Delta_r \qquad \Delta^\circ = \langle \nabla_1, \ldots, \nabla_r \rangle_{\mathrm{conv}}$$

$$(\nabla_n, \Delta_m) \geq -\delta_{nm}$$

$$\nabla^\circ = \langle \Delta_1, \ldots, \Delta_r \rangle_{\mathrm{conv}} \qquad \nabla = \nabla_1 + \ldots + \nabla_r$$

  - Minkowski sum: $\Delta = \Delta_1 + \ldots + \Delta_r$
  - nef partition: $\langle \nabla_1, \ldots \rangle_{conv}$ ($\langle \ldots \rangle_{conv}$: convex hull)
  - Each Δ_k yields a hypersurface equation for the complete intersection.
- Analyze toric data with PALP. [Kreuzer,Skarke '02]
  - Input: weight systems or vertices in M/N–lattice
  - Output: Polytope data, Hodge numbers, nef–partitions,. . .

Blowups in Fano threefolds

- Construct non–CY threefold base B as point/curve blowups of a Fano threefold which is a hypersurface of some toric space.
  - Fanos themselves are not good base manifolds because there can be no decoupling limit. [Cordova '09]
- Starting point: Fano threefolds with one Kähler class

$$\mathbb{P}^4[d] = \{P_d(y_1, \ldots, y_5) = 0 | [y_i : \ldots : y_5] \in \mathbb{P}^4\} \qquad d = 2, 3, 4,$$

- Blowups:
  - More Kähler moduli $\Leftrightarrow$ "size" of blowup
  - New exceptional divisors

Weight matrices

- Blowups can be encoded in the weight matrices describing the toric space X:
  - $\mathbb{CP}^4$:

	y_1	y_2	y_3	y_4	y_5	$\sum$
w_1	1	1	1	1	1	5

  - Curve blowup:

	y_1	y_2	y_3	y_4	y_5	w	$\sum$
w_1	1	1	1	1	1	0	5
w_2	0	0	0	1	1	1	3
	J_1					J_2	

  - Point blowup:

	y_1	y_2	y_3	y_4	y_5	w	$\sum$
w_1	1	1	1	1	1	0	5
w_2	1	0	0	0	0	1	2
		J_1				J_2	

Hypersurfaces

- The weight systems only define the ambient space.
- The hypersurface B is specified by a divisor B.
- Example: Hypersurface of degree $(4,2)$ in curve blowup in $\mathbb{P}^4$ [Blumenhagen et al. '09]
    - Class of B: $[B] = 4J_1 + 2J_2$
    - How to see the curve blowup:
        1. Tune complex structure in $\mathbb{P}^4[4] = p_4(y_1, \ldots, y_5)$:

$$P = f_4 + y_4 f_3 + y_5 g_3 + y_4^2 f_2 + y_5^2 g_2 + y_4 y_5 h_2$$

$\Rightarrow$ singular curve at $(0,0,0,y_4,y_5) \sim \lambda(0,0,0,y_4,y_5)$.

        2. Blow up by introducing a new coordinate w:

$$\tilde{P} = w^2 f_4 + w y_4 f_3 + w y_5 g_3 + y_4^2 f_2 + y_5^2 g_2 + y_4 y_5 h_2$$

$\Rightarrow$ new weight vector $(y_1, y_2, y_3, y_4, y_5, w) \sim (y_1, y_2, y_3, \lambda y_4, \lambda y_5, \lambda w)$.
$\Rightarrow$ $w = 0$ defines a dP_7.

GUT divisors

- Having specified a base B we must look for a GUT divisor S inside B.
- The GUT divisor should be del Pezzo. [Donagi,Wijnholt '08][BHV '08]
  - Del Pezzos are two–dimensonal Fanos.
  - They are $\mathbb{P}^1 \times \mathbb{P}^1$ and $dP_n, n = 0, \ldots, 8$ which is $\mathbb{P}^2$ with up to eight points blown up.
- The GUT divisor should satisfy a decoupling limit.
  - physical: the volume of S should stay finite when the volume of B goes to infinity.
  - mathematical: the volume of S goes to 0 whereas the volume of B does not

Del Pezzo divisors

- Identify a del Pezzo by its topological data.
  - Chern class of S in B:

$$c(S) = \frac{\prod_i (1 + D_i)}{(1 + B)(1 + S)}$$

  - $$\int_S c_1(S)^2 = 9 - n \qquad \int_S c_2(S) = n + 3 \qquad \Rightarrow \qquad \chi_h = \int_S \mathrm{Td}(S) = 1,$$
  - Integrals of $c_1(S)$ over all torically induced curves on S have to be positive:

$$D_i \cdot S \cdot c_1(S) > 0 \qquad D_i \neq S \qquad \forall D_i \cdot S \neq \emptyset\,.$$

- Input data: Divisor classes, exceptional set (Stanley-Reisner ideal), intersection ring [Kreuzer,Walliser unpublished]

Decoupling Limit

- Calculate the volumes of B and S explicitly.
  - Find a basis K_i of the Kähler cone such that the Kähler form J can be decomposed as:

$$J = \sum_i r_i K_i \quad r_i > 0$$

  - Volumes of B and S:

$$\mathrm{Vol}(B) = J^3 \quad \mathrm{Vol}(S) = S \cdot J^2$$

  - Condition for the decoupling limit:
    physical: $\mathrm{Vol}(S)$ is independent of at least one of the r_i
    mathematical: tune parameters to get $\mathrm{Vol}(S) \to 0$ while still keeping non–zero terms in $\mathrm{Vol}(B)$
- Input data: previous data+Mori cone (dual of the Kähler cone)

CY fourfold

- Construct a Calabi–Yau fourfold by fibering a torus $\mathbb{CP}_{123}[6]$ over B.
  - Torus coordinates x, y transform as sections of K_B^{-2}, K_B^{-3}.
  - The weight system of the torus is:

y	x	z	$\sum$
3	2	1	6

- The data of the (6d) ambient space is encoded in the combined weight system of the base and the torus.
- Since the fourfold is a complete intersection we have to specify a nef–partition that is compatible with the elliptic fibration.

Example

- Base: blowup of one curve and one point in $\mathbb{P}^4[3]$:

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	$\sum$	deg
w_1	1	1	1	1	1	0	0	5	3
w_2	0	0	0	1	1	1	0	3	2
w_3	1	0	0	0	0	0	1	2	1

  - The two exceptional divisors are dP_3 and dP_4 and satisfy the physical decoupling limit.
- Fourfold:

	y	x	z	y_1	y_2	y_3	y_4	y_5	y_6	y_7	$\sum$
w_0	3	2	1	0	0	0	0	0	0	0	6
w_1	6	4	0	1	1	1	1	1	0	0	15
w_2	3	2	0	0	0	0	1	1	1	0	8
w_3	3	2	0	1	0	0	0	0	0	1	7

  - Use PALP to list all possible nef-partitions – one gives a Weierstrass model.

Weierstrass model

- Using the toric construction we can explicitly compute the Weierstrass equation for a given geometry:

$$P_W = x^3 - y^2 + xyza_1 + x^2z^2a_2 + yz^3a_3 + xz^4a_4 + z^6a_6$$

- We get explicit expressions for the a_i in terms of monomials in the base coordinates y_i.
- Using the toric data we can compute the Hodge numbers and the Euler number of the CY fourfold Y.
  - Works if there are no terminal singularities.
  - This data is needed for computing the $D3$ tadpole cancellation condition:

$$N_{D3} = \frac{\chi(Y)}{24} - \frac{1}{2}\int_Y G \wedge G$$

Results

- We have considered all weight systems which describe up to three point and curve blowups in $\mathbb{P}^4$.
  - We have looked at hypersurfaces of $\deg_i < \sum \text{weights}_i \Leftrightarrow$ blowups inside Fano threefolds.
  - 241 base geometries
- 208 of the base manifolds had at least one del Pezzo divisor with a decoupling limit
- For 86 models we could construct a CY fourfold Y which is described torically by reflexive polytopes.
  - Whenever this works the base is almost Fano.
    $\Rightarrow$ algebraic threefold which has a non–trivial anti-canonical bundle with at least one non–zero section

Status

- So far:
  - explicit toric construction of the base B and the fourfold Y
  - identification of possible GUT divisors S
- Next:
  - construct $SO(10)$ models
  - identify matter curves and Yukawa couplings
  - construct fluxes

$SO(10)$ Weierstrass model

- To get an $SO(10)$ GUT group the a_i in the Weierstrass equation have to factorize:

$$a_1 = b_5 w^1 \quad a_2 = b_4 w^1 \quad a_3 = b_3 w^2 \quad a_4 = b_2 w^3 \quad a_6 = b_0 w^5$$

- Matter curves:
  - $b_3 = 0$ $\quad$ **10** matter ($SO(12)$ enhancement)
  - $b_4 = 0$ $\quad$ **16** matter (E_6 enhancement)
- Yukawa couplings
  - $b_3 = 0 \cap b_4 = 0$ $\quad$ E_7 Yukawas: **16 16 10**
  - $b_2^2 - 4b_0 b_4 = 0 \cap b_3 = 0$ $\quad$ $SO(14)$ Yukawas: **10 10 1**

Toric realization

- The $SO(10)$ model can be realized in toric geometry.
- Remember: The points in the M–lattice correspond to monomials in the Weierstrass equation.
  - The a_n are coefficients of z^n.
- Remove all points on the M–lattice where the corresponding monomials do not satisfy the $SO(10)$ factorization condition.
- As a consequence one gets additional vertices in the dual N–lattice.
  - These correspond to new exceptional divisors which one obtains from resolving the $SO(10)$ singularity.
- The equations for the matter curves and Yukawas can be given explicitly.

Spectral cover I

- In the heterotic string the spectral cover is used to describe stable bundles on elliptically fibered threefolds.
- In F–theory the spectral cover describes bundles and fluxes in the vicinity of the GUT brane. [Donagi,Wijnholt '09]
  - Also for models without a heterotic dual the spectral cover seems to be valid beyond the local picture. [Blumenhagen et al. '09]
- For $SO(10)$ models we must look at an $SU(4)$ spectral cover.
- The spectral cover is a divisor on an auxiliary compact non–CY threefold $\bar{X}$ whose base is the GUT brane S:

$$\bar{X} = \mathbb{P}(\mathcal{O}_S \oplus K_S)$$

Spectral Cover II

- The base S is the vanishing locus of the section σ in $\bar{X}$.
  - One can show that: $\sigma \cdot \sigma = -\sigma \cdot c_1(S)$
- The spectral cover C_V is associated to the fundamental representation V of $G = SU(4)$ is:

$$C_V : \; b_0 s^4 + b_2 s^2 + b_3 s + b_4$$

where

$$b_i \sim \eta - i\, c_1(S) \sim (6-i) c_1(S) + c_1(\mathcal{N}_{S|B}) \quad i = 0, \ldots, 4$$

- Defining a projection $\pi_C : C_V \to S$ the class of C_V is:

$$[C_V] = 4\sigma + \pi_C \eta$$

Spectral Cover III

- Matter curves are intersections of the spectral cover with σ.
- Fluxes are encoded in a non–trivial spectral line bundle $\mathcal{N}$ on C_V that gives rise to a rank 4 bundle $V = \pi_C \mathcal{N}$ on S.
    - 16 matter curves:

$$\Sigma_{\mathbf{16}} = C_V \cap \sigma$$

    - Flux γ:

$$\gamma = \frac{1}{4}\pi_C^* c_1(V) + \gamma_u \quad \gamma_u = 4[\Sigma_V] - \pi_C^*(\eta - nc_1(S))$$

    - Chiral Matter:

$$n_{\mathbf{16}} = -\eta \cdot (\eta - 4c_1(S))$$

10 curves

- **10** curves are obtained from a $\wedge^2 V$ spectral cover:

$$\Sigma_{10} = C_{\wedge^2 V} \cap \sigma$$

- Problem: There is no chiral matter on the **10** curves. [Hayashi et al. 08]
  - For phenomenological reasons the electroweak Higgs should come from the **10** curves.
  - We need extra degrees of freedom to build three generation models.
- Proposed solution: split spectral cover:
  - Factorize: $C_V \to C^{(1)} + C^{(3)}$
    $\Rightarrow S(U(3) \times U(1))$ spectral cover

Split spectral cover

- From the split spectral cover we get two types of **16** matter curves:

$$\Sigma_a = C^{(1)} \cap \sigma \quad \Sigma_b = C^{(3)} \cap \sigma$$

- The spectral cover for the **10** curves gets mixed contributions from $C^{(1)}$ and $C^{(3)}$.
- By turning on different fluxes on Σ_a and Σ_b we can generate chiral matter on the **10** curves.
- Using the split spectral cover, we worked out several $SO(10)$ models with three generations.

Further results

- Flipped $SU(5)$:
  - The $SO(10)$ GUT group cannot be broken directly to the Standard Model gauge group by $U(1)$ flux. [BHV '08]
  - We can use $U(1)$ flux to break $SO(10)$ to $SU(5) \times U(1)$.
  - We show that we can get three generation models and realistic Yukawa couplings.
- Tadpole cancellation:
  - The D3 tadpole cancellation condition requires to know the Euler number of the CY fourfold.
  - We compare the Euler numbers computed from the fourfold geometry with a conjectured formula. [Blumenhagen et al. '09]
  - For some examples we find a discrepancy for the Euler numbers computed by the two methods.

Summary

- Making use of toric geometry have systematically constructed fourfold geometries which can support global F–theory GUTs.
- We have constructed $SO(10)$ GUTs on these geometries.
- Using a split spectral cover we have generated chiral matter on the $\mathbf{10}$ curves and were able to produce three generation models.

Open Problems

- Moduli stabilization
- Classification of fourfold geometries (computer search).
- Standard Model from $SO(10)$ GUTs.
- Explain the discrepancy in the Euler numbers.
- Is the split spectral cover globally defined for our models?

  [Hayashi et al. '10]