Testing the mean field description of scalar field dark matter

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In collaboration with:
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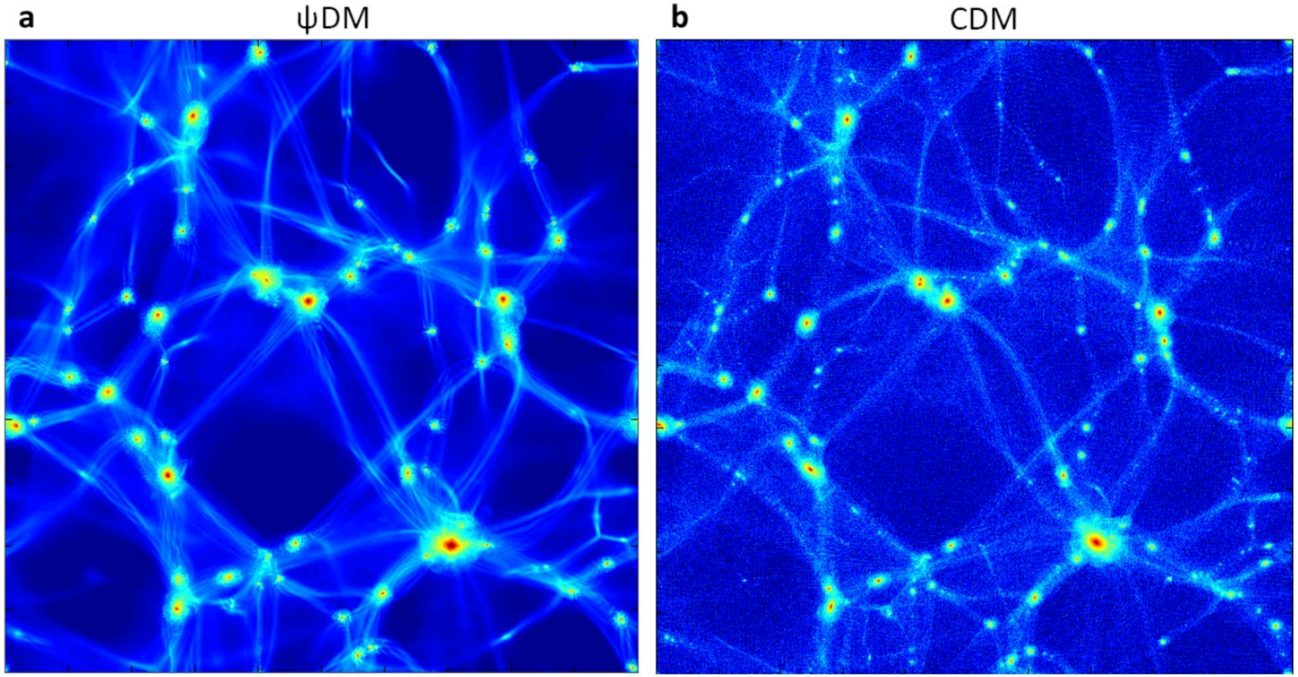
Outline

- What model are we discussing and why?
- How is this model traditionally approached?
- Why may this approach be problematic?
- How are we testing the problem?
- Results, conclusions, and limitations

The model

- Scalar field dark matter



Schive et al (Nature 2014)

The model

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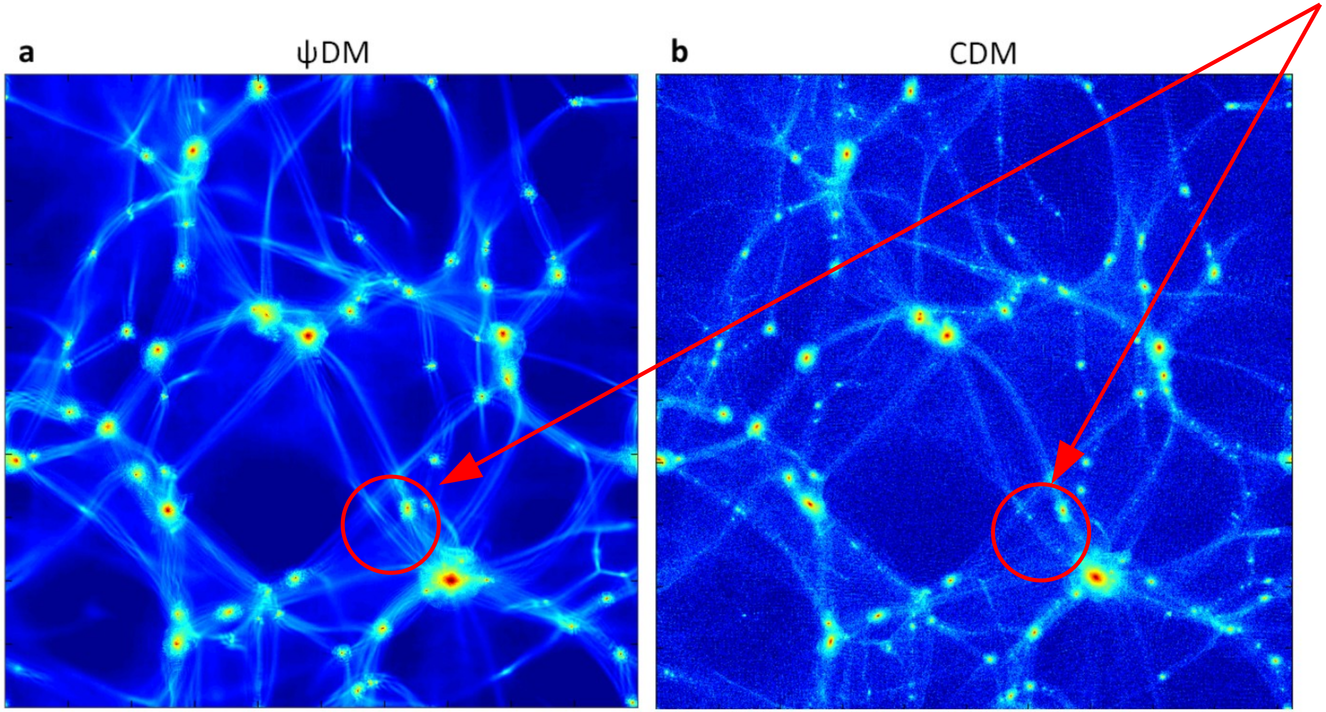

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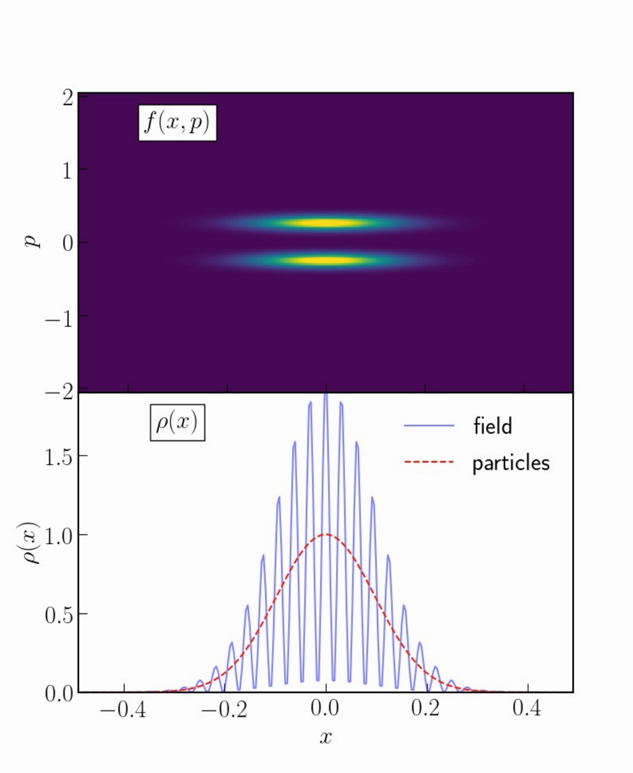
The model

- Scalar field dark matter
  - Produces a cutoff length scale for structure formation



The model

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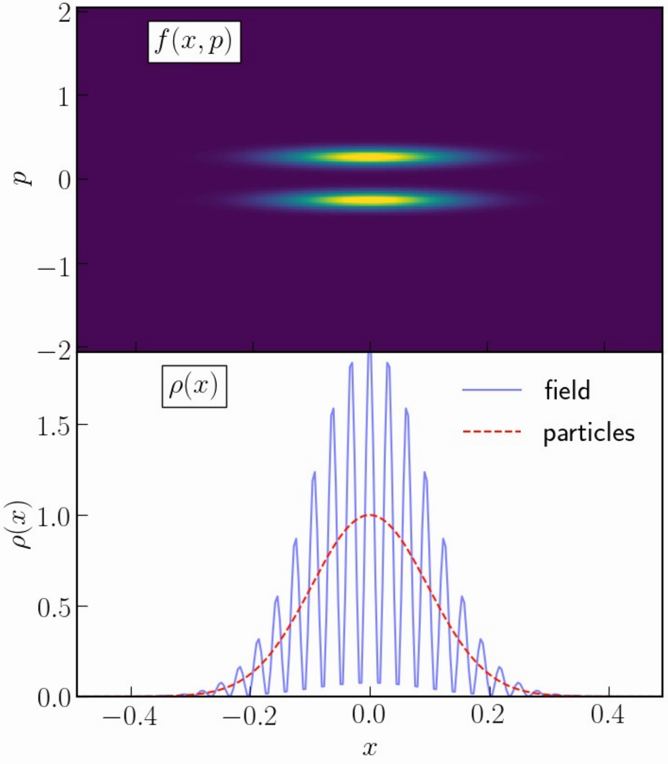
Changing mass of DM particle



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  - Large occupation numbers

$$n_{tot} \sim 10^{100}$$

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- “Ultralight” implies:
  - Large occupation numbers
  - Non thermal production mechanism (e.g. misalignment)

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- "Ultralight" implies
- Initial conditions described by a coherent state with large parameter

$|\vec{z}\rangle$ Coherent state

$|\vec{z}|^2 \gg 1$

How is this model traditionally approached?

- “Ultralight” implies
- Initial conditions described by a coherent state with large parameter
- Expectations values are large compared to fluctuations

$$E[\hat{N}_i] \sim n_{tot}$$

$$\mathrm{Var}[\hat{N}_i] \sim n_{tot}$$

$$E[\hat{N}_i] \gg \sqrt{\mathrm{Var}[\hat{N}_i]}$$

How is this model traditionally approached?

- “Ultralight” implies
- Initial conditions described by a coherent state with large parameter
- Expectations values are large compared to fluctuations
- (nonrelativistic) Classical field is used

$$\hat{\psi} \rightarrow \psi$$

How is this model traditionally approached?

- "Ultralight" implies
- Initial conditions described by a coherent state with large parameter
- Expectations values are large compared to fluctuations
- (nonrelativistic) Classical field is used
- Computational complexity of mean field theory is dramatically smaller

C-nums needed

$$\partial_t \hat{\psi} = -i\left[\frac{-\nabla^2}{2m} + \hat{V}\right]\hat{\psi} \quad \sim (10^{100})^{100}$$

$$\partial_t \psi = -i\left[\frac{-\nabla^2}{2m} + V\right]\psi \quad \sim 100$$

How is this model traditionally approached?

- Mean field theory is the following approximation

$$\partial_t \left(\text{occupations}\right) \approx f \left[\text{means}\right]$$

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  - Exact when the quantum state is a coherent state

$$\partial_t \left(\text{occupations}\right) = f\left[\text{means}\right]$$

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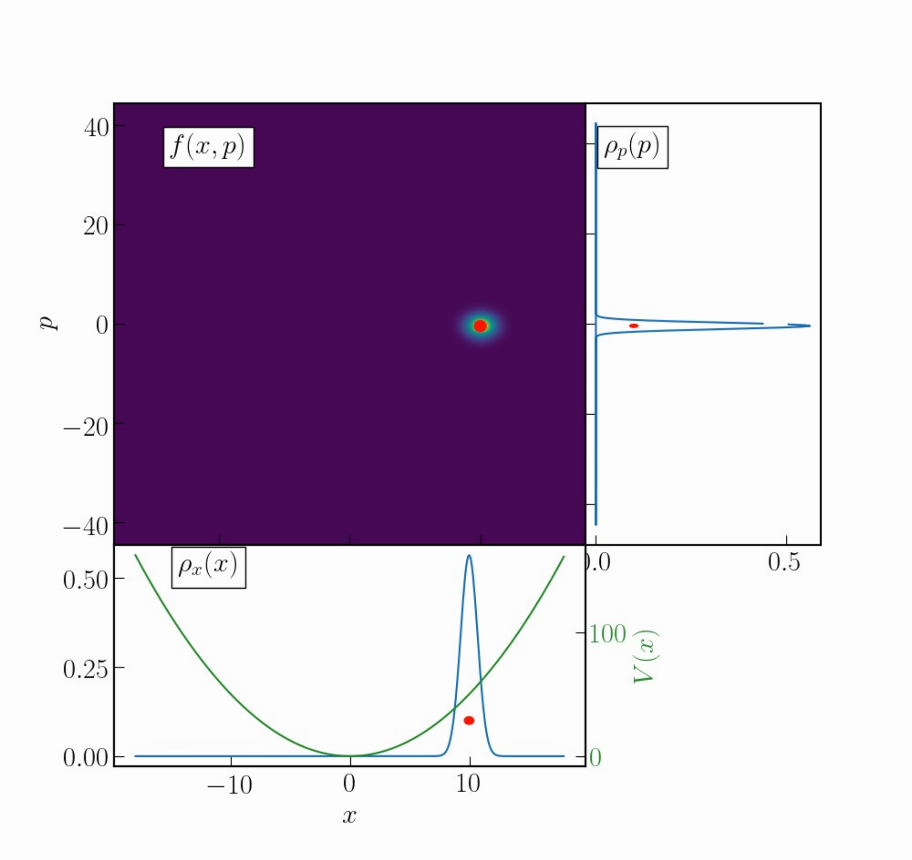
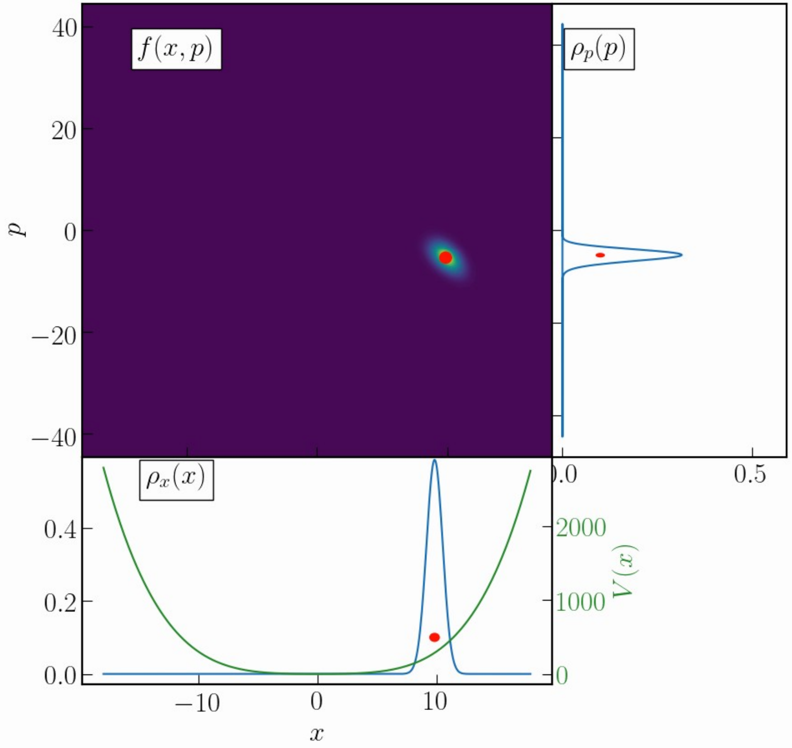
Why might this approach be problematic?

- For linear Hamiltonians the mean field theory description survives indefinitely



Why might this approach be problematic?

- For linear Hamiltonians the mean field theory description survives indefinitely
- Nonlinearities will introduce quantum corrections on some time scale (quantum breaktime)



Why might this approach be problematic?

- Mean field theory is the following approximation
  - Exact when the quantum state is a coherent state ← Will no longer be true on some timescale

$$\partial_t \left(\text{occupations}\right) = f\left[\text{means}\right]$$

Why might this approach be problematic?

- In reality the spread of the wavefunction may eventually introduce significant quantum corrections

$$\partial_t \left(\text{occupations}\right) = f^1 \left[\text{means}\right] + f^2 \left[\text{covariances}\right] + \dots$$

Why might this approach be problematic?

- In reality the spread of the wavefunction may eventually introduce significant quantum corrections
- How long does the classical description of scalar field dark matter survive?

$$\partial_t \left(\text{occupations}\right) = f^1 \left[\text{means}\right] + f^2 \left[\text{covariances}\right] + \dots$$

How can this question be approached?

Other approaches

How can this question be approached?

- Classical description remains accurate due to large occupation numbers per deBroglie wavelength

$$\frac{\delta\hat{\psi}}{\psi} \sim \frac{1}{\sqrt{\mathcal{N}}}$$

Guth, A. H., Hertzberg, M. P., & Prescod-Weinstein, C. (n.d.). Do Dark Matter Axions Form a Condensate with Long-Range Correlation?

How can this question be approached?

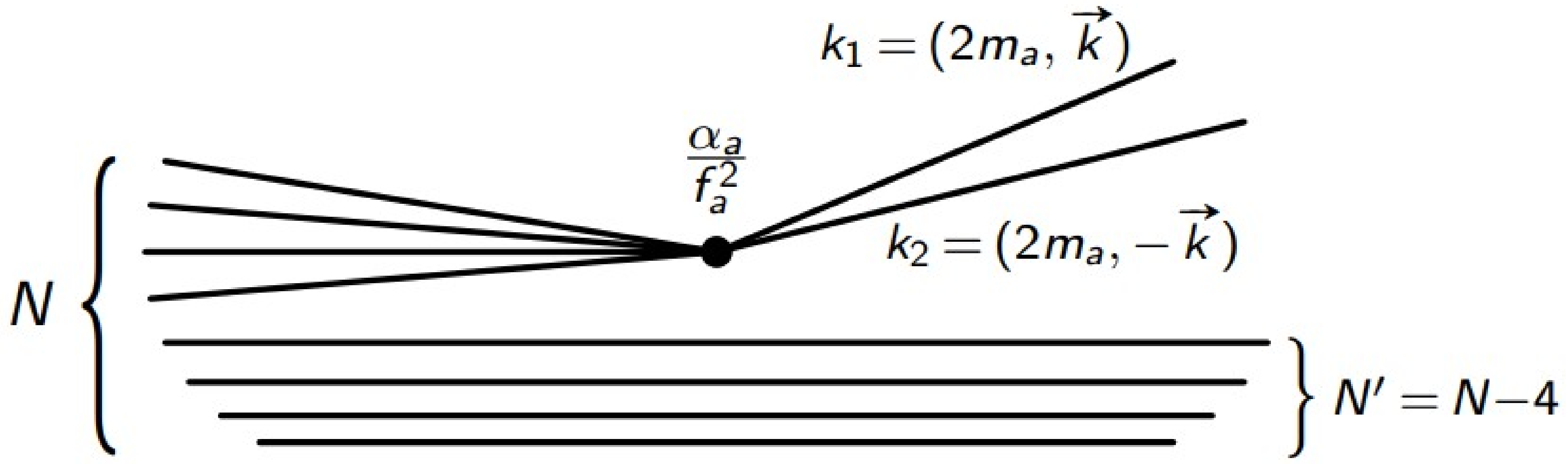
- Classical description is extended due to “log(n) enhancement”

$\sim \tau \ln \bar{N}$, as one expects $\sim \ln \bar{N}$ collisions for the small initial quantum uncertainty $\sim 1/\sqrt{\bar{N}}$ to grow to be $\mathcal{O}(1)$

Hertzberg, M. P. (2016). Quantum and classical behavior in interacting bosonic systems.

How can this question be approached?

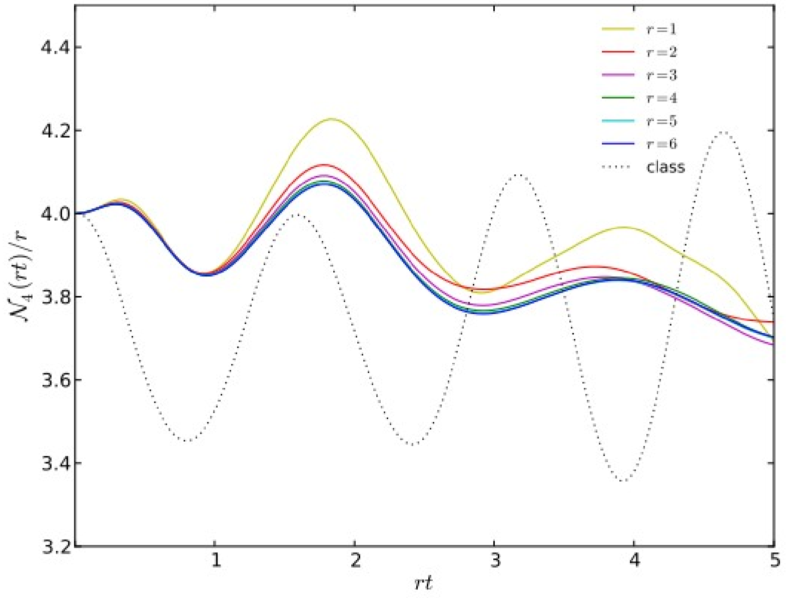
- Non-classical diagrams are inefficient



Dvali, G., & Zell, S. (2018). Classicality and quantum break-time for cosmic axions.

How can this question be approached?

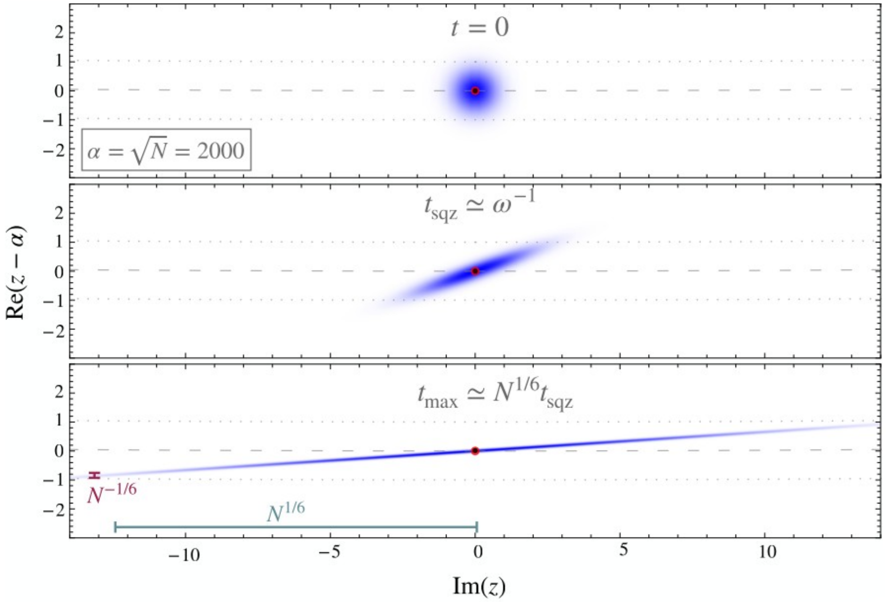
- Classical description fails on dynamical timescale (for number eigenstates)



Sikivie, P., & Todarello, E. M. (2017). Duration of classicality in highly degenerate interacting Bosonic systems.

How can this question be approached?

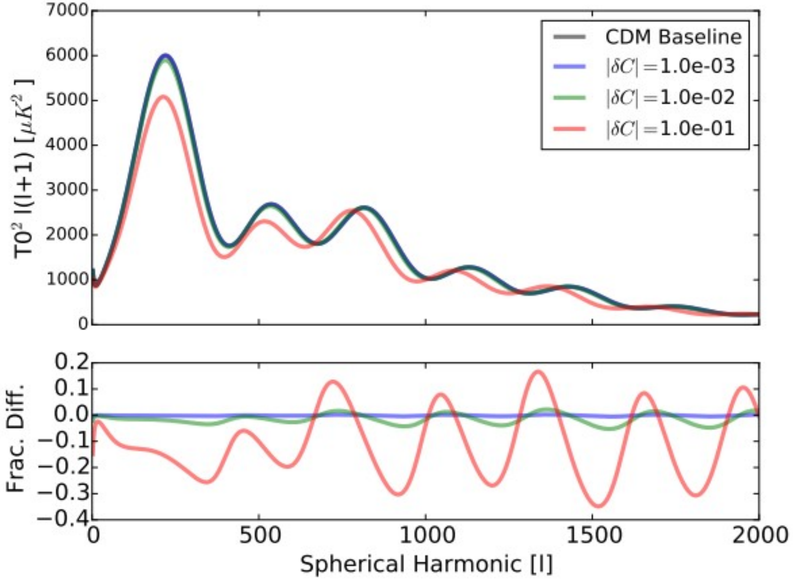
- Classical state efficiently undergoes quantum squeezing



Kopp, M., Fragkos, V., & Pikovski, I. (2021). Nonclassicality of axion-like dark matter through gravitational self-interactions.

How can this question be approached?

- Classical state admits quantum corrections during nonlinear growth due to inter-particle correlations



Lentz, E. W., Quinn, T. R., & Rosenberg, L. J. (2019). Axion structure formation – I: The co-motion picture.

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Our approach

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- Study the behavior of quantum corrections as total particle number is increased

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Penrose-Onsager Criterion

$$\langle \hat{\psi}^\dagger(x)\,\hat{\psi}(y)\rangle$$

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Penrose-Onsager Criterion

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Penrose-Onsager Criterion

$$\langle \hat{\psi}^\dagger(x)\, \hat{\psi}(y) \rangle = n_{tot}\, \xi_p^*(x)\, \xi_p(y)$$

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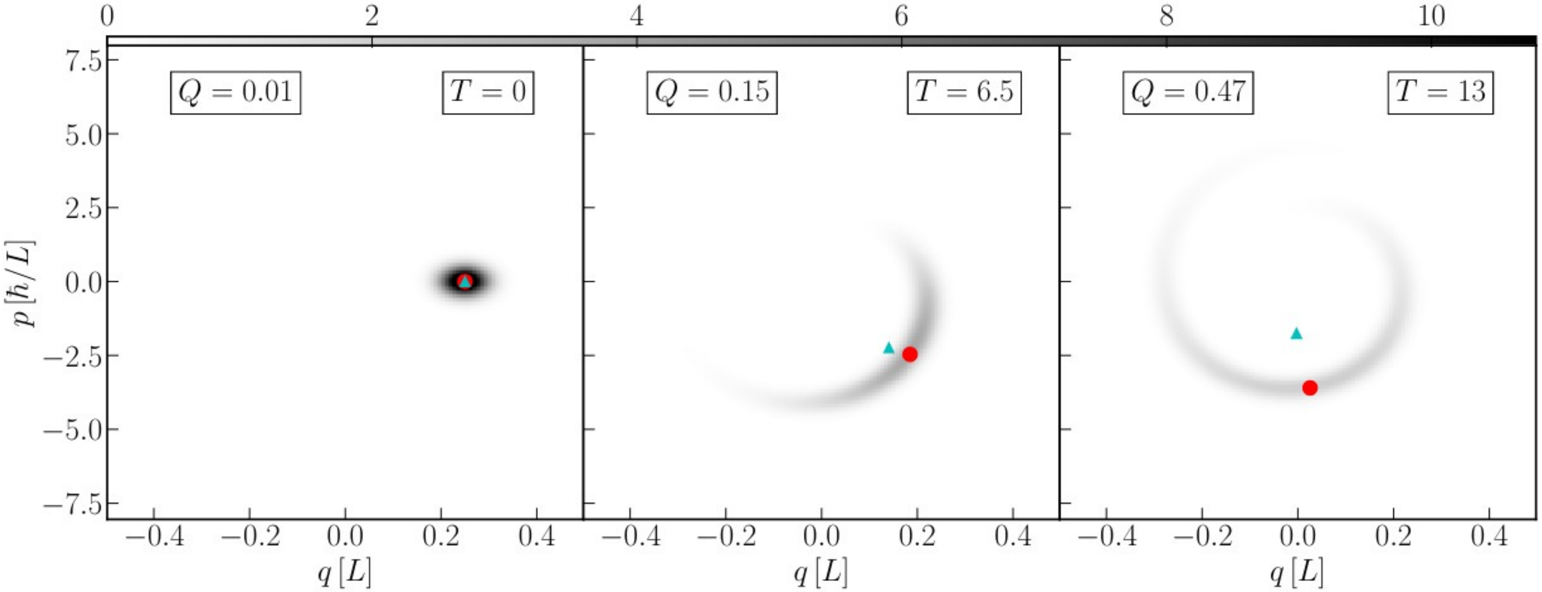
Spread of wavefunction

$$Q = \frac{\sum_i \delta\hat{a}_i^\dagger \delta\hat{a}_i}{n_{tot}}$$



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Small Systems: Large Systems:

$$M = 5,\, n_{tot} < 100 \longrightarrow M = 256,\, n_{tot} < 10^{10}$$

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- For small systems direct integration is possible

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Total Hilbert space: $\mathcal{H}^T$

Small Systems:

$$M = 5,\, n_{tot} < 100$$

- The total relevant Hilbert space is quite large

$$N_s \sim \mathcal{O}(10^8)$$

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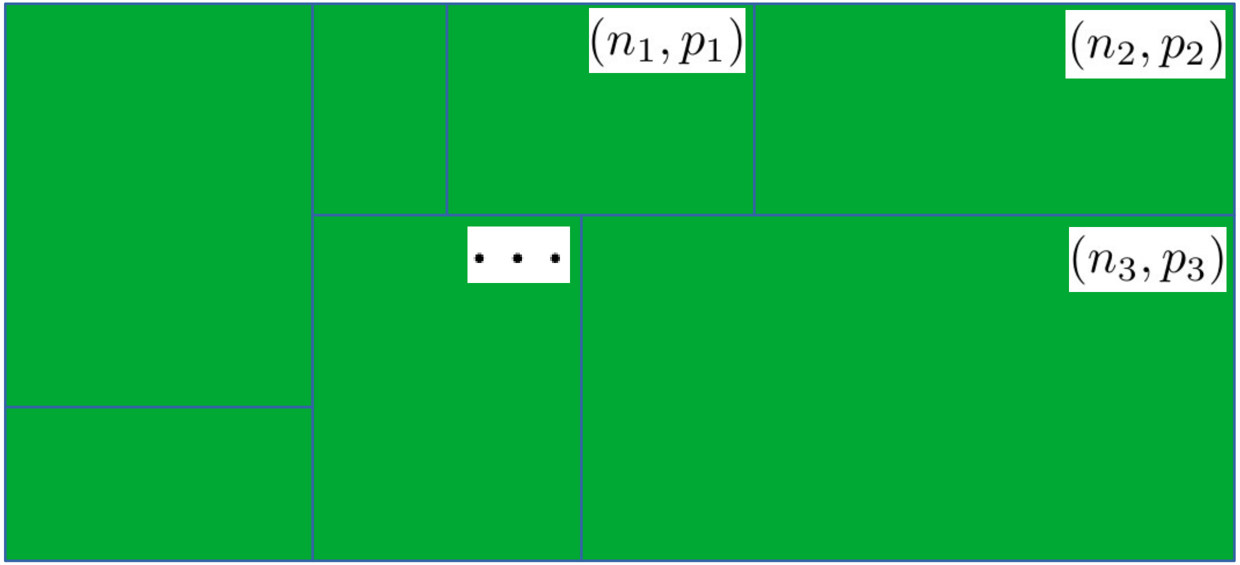

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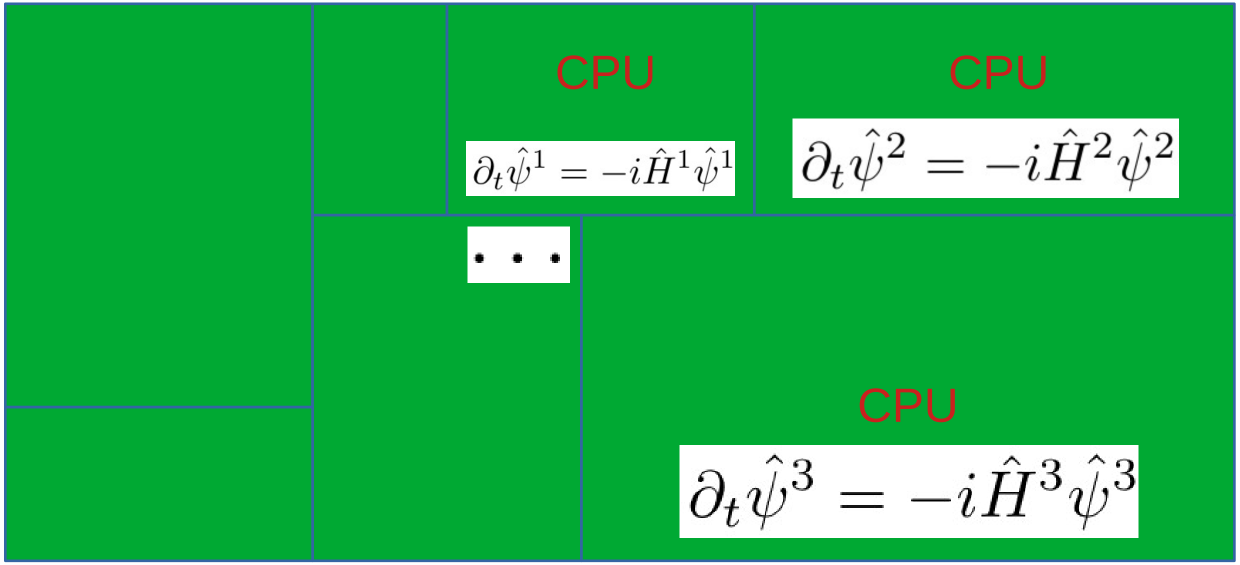

Small Systems:

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- The total relevant Hilbert space is quite large
- We can partition it into many (often thousands) subspaces using the conserved quantities of the Hamiltonian
- The evolution of the state component in each subspace is independent of the other spaces and can be done in entirely in parallel

Results: small systems

- Ran increasing total occupation keeping the classical solution fixed and compared quantum and classical evolution

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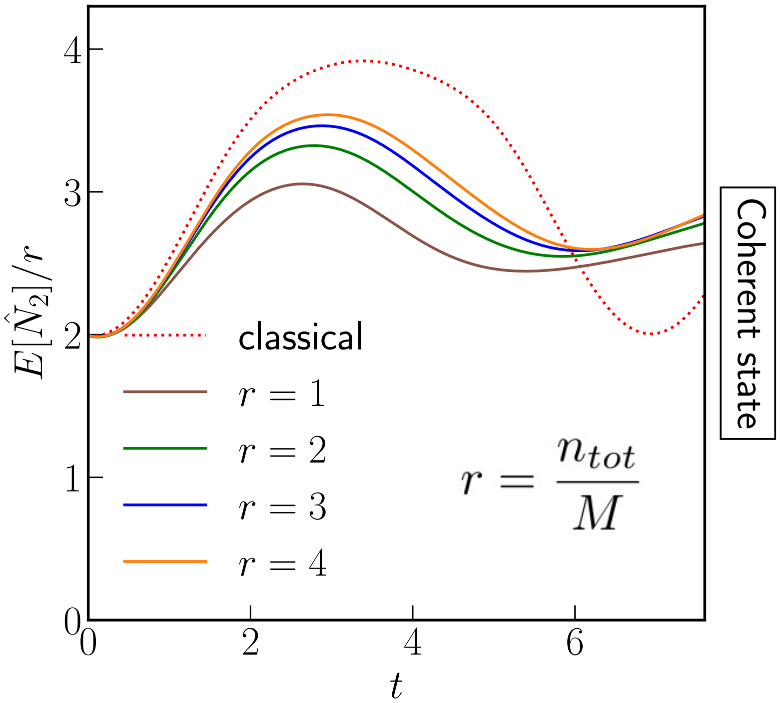

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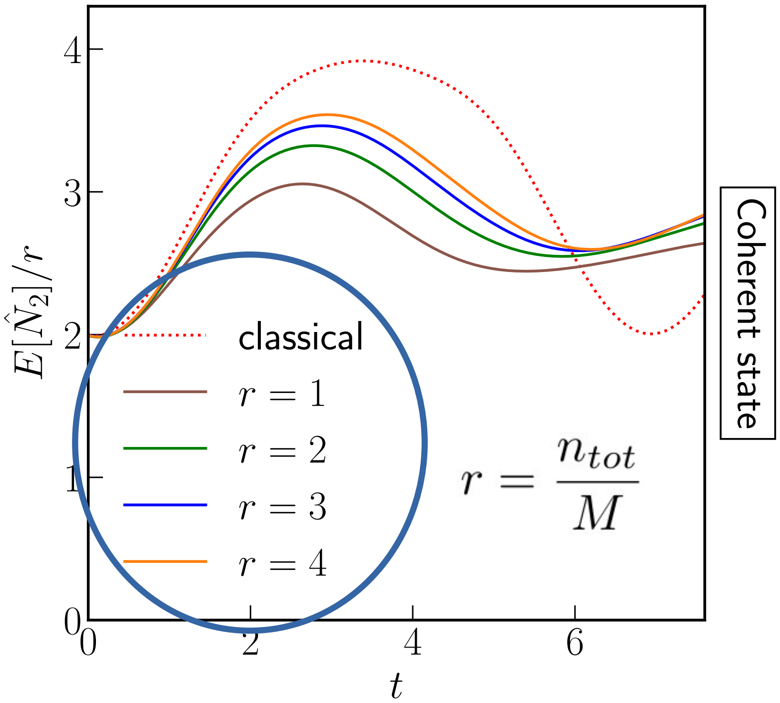

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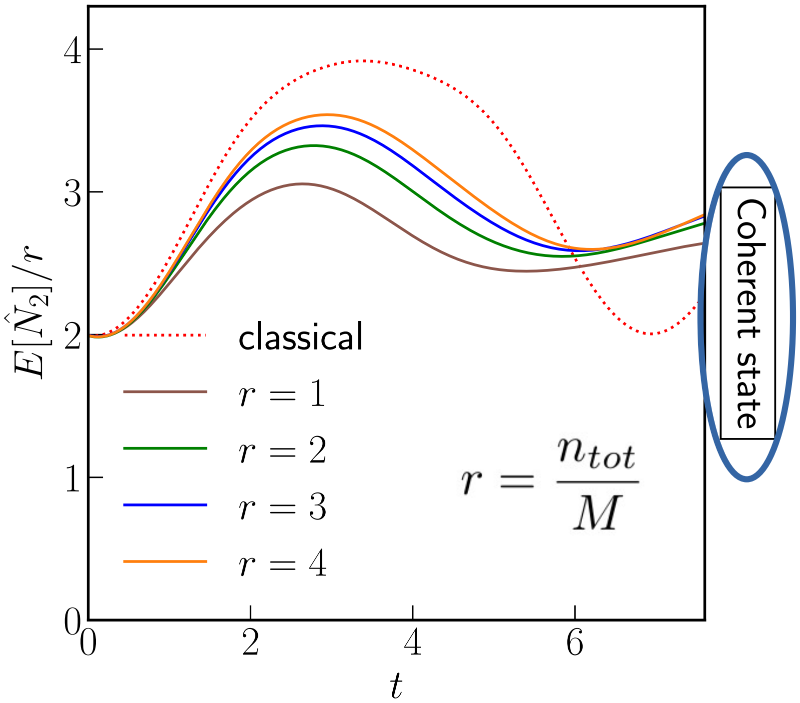

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- In general we see that the quantum solution converges as occupations are increased

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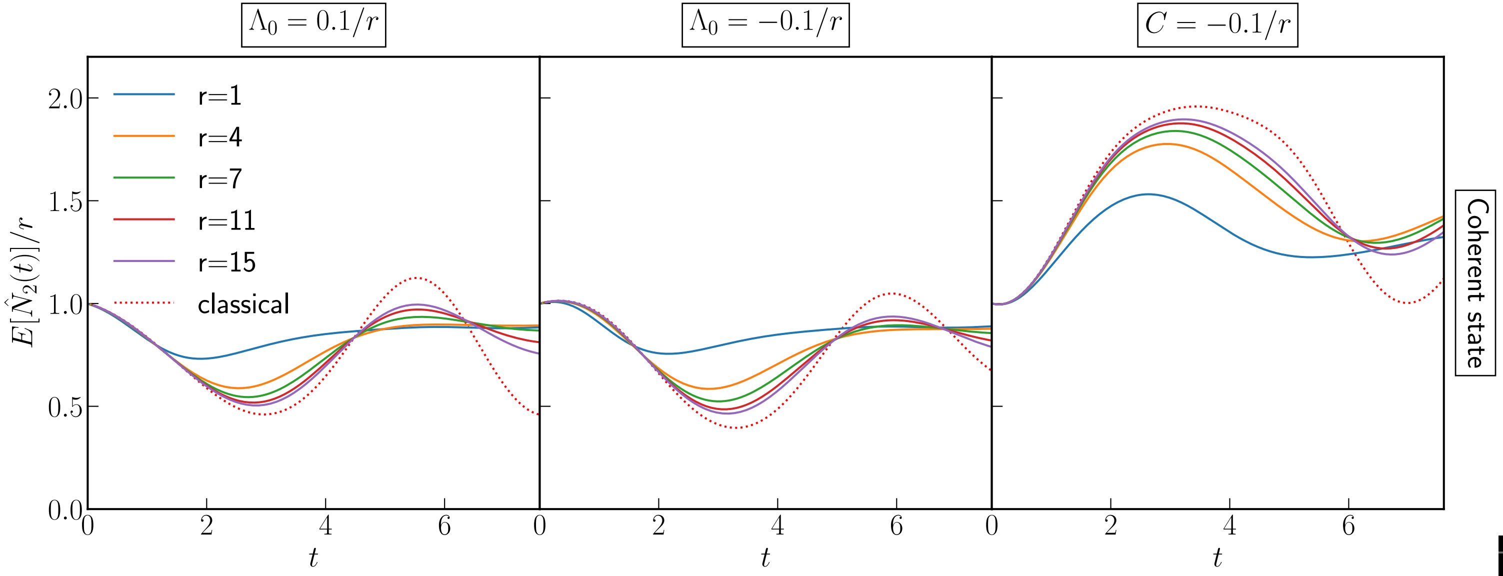

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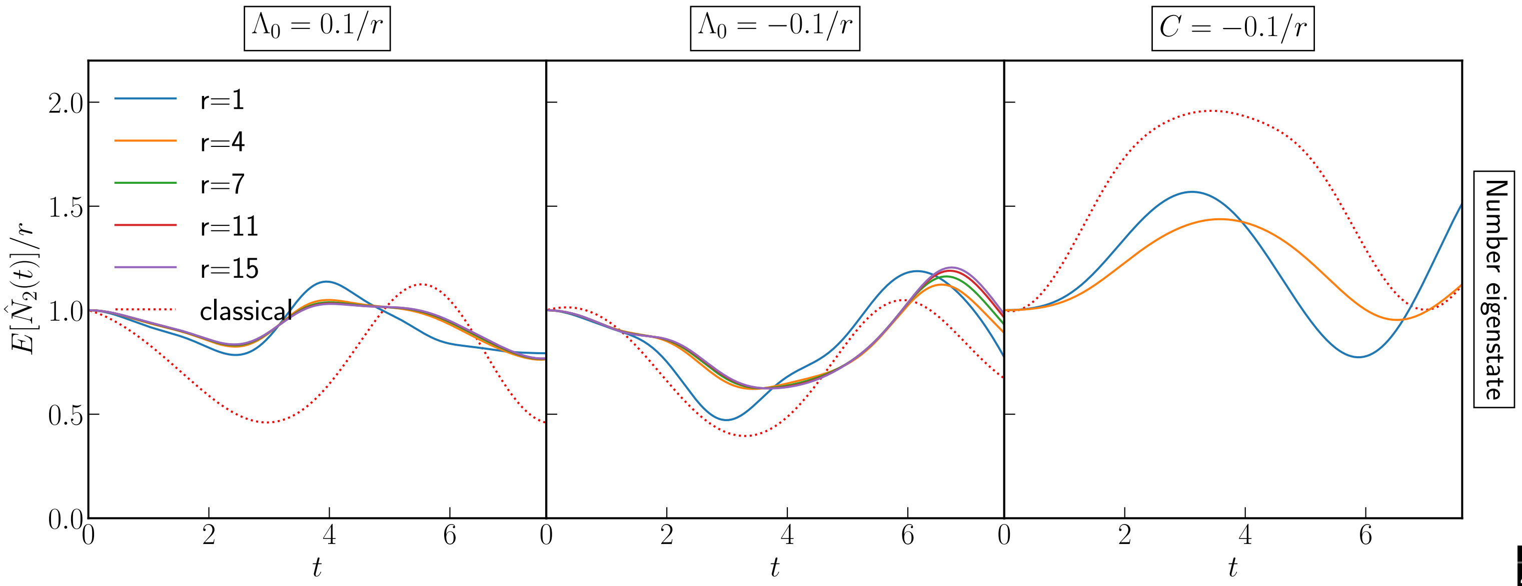

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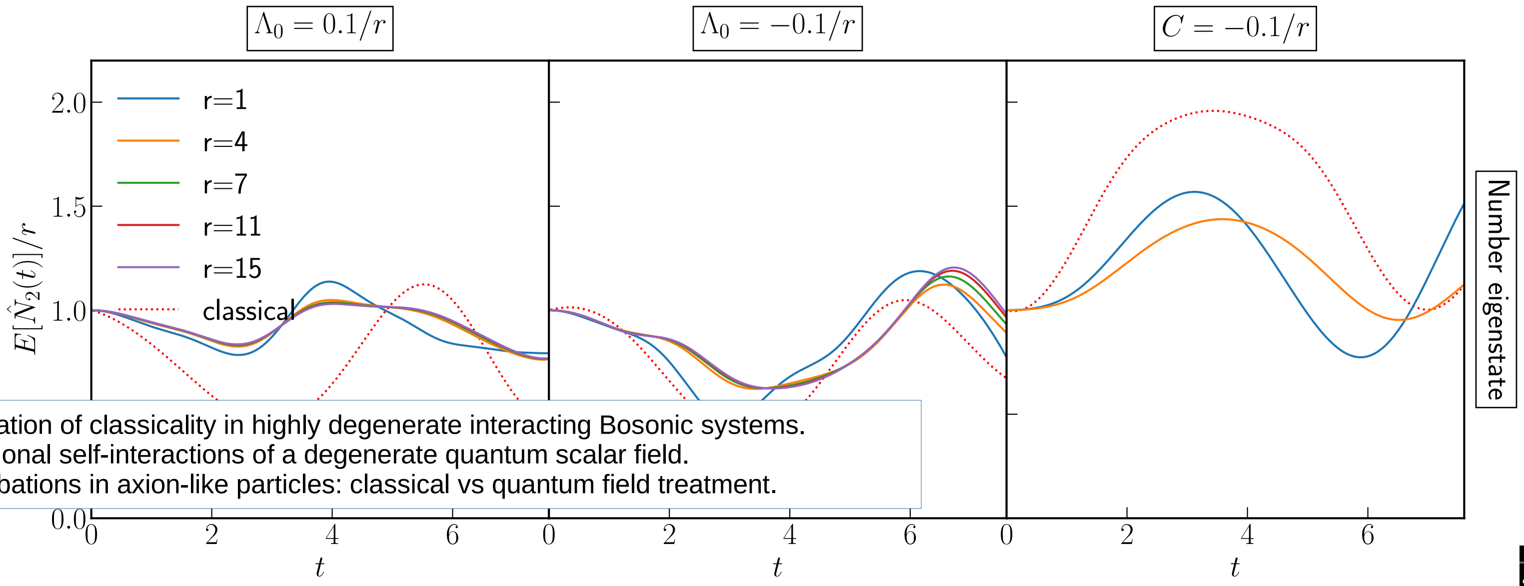

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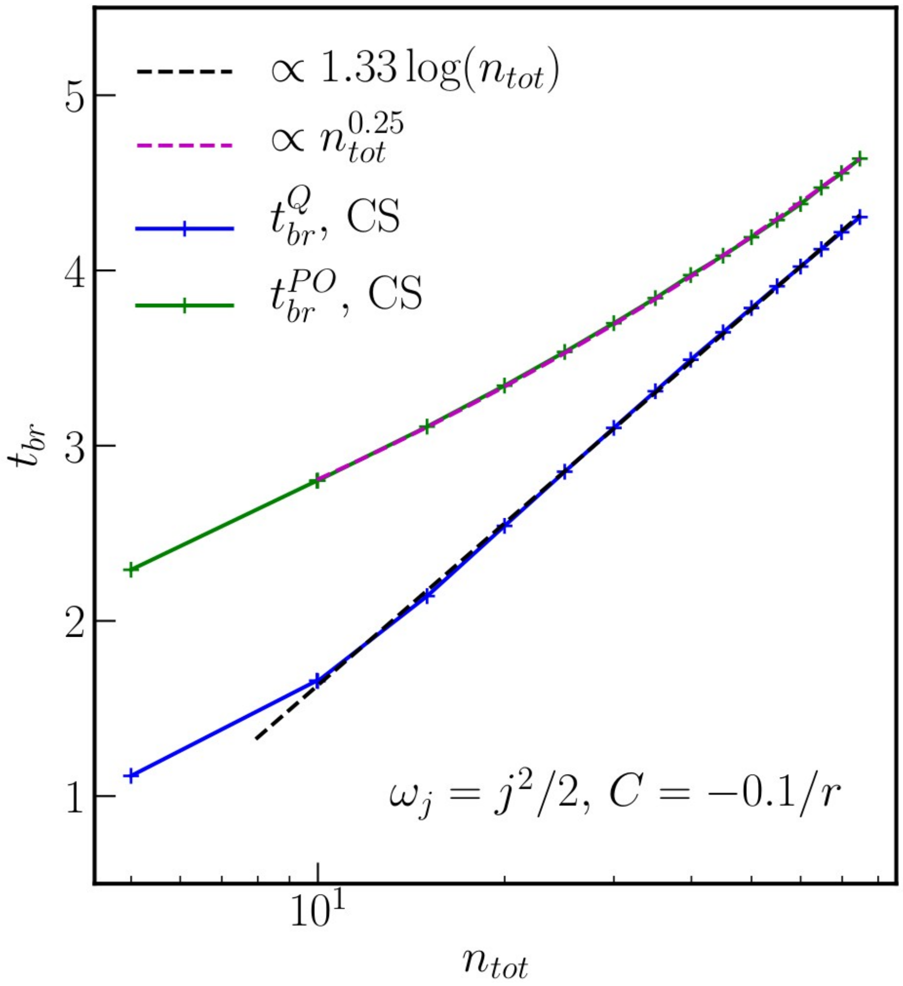
Chakrabarty, S. S. (2021). Density perturbations in axion-like particles: classical vs quantum field treatment.

Results: small systems

- We can study the rate of this convergence using our classicallity criteria



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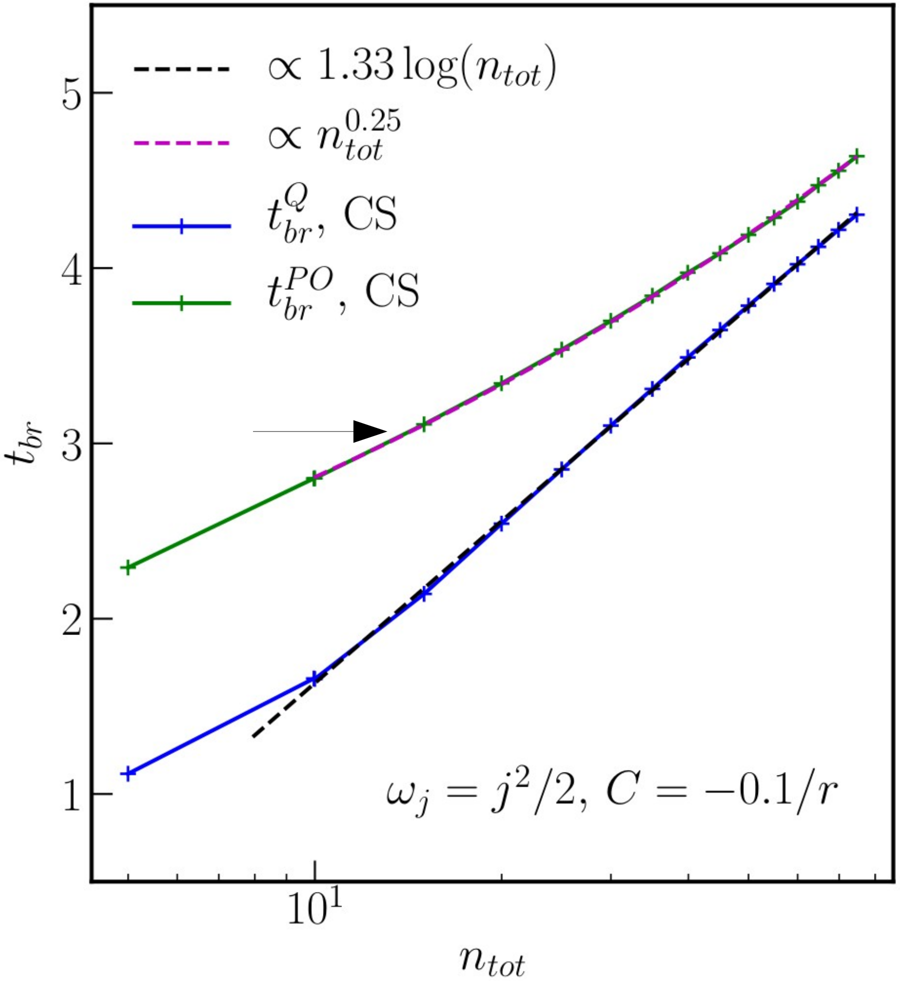

Results: small systems

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- log(n) enhancement in time it takes wavefunction to spread

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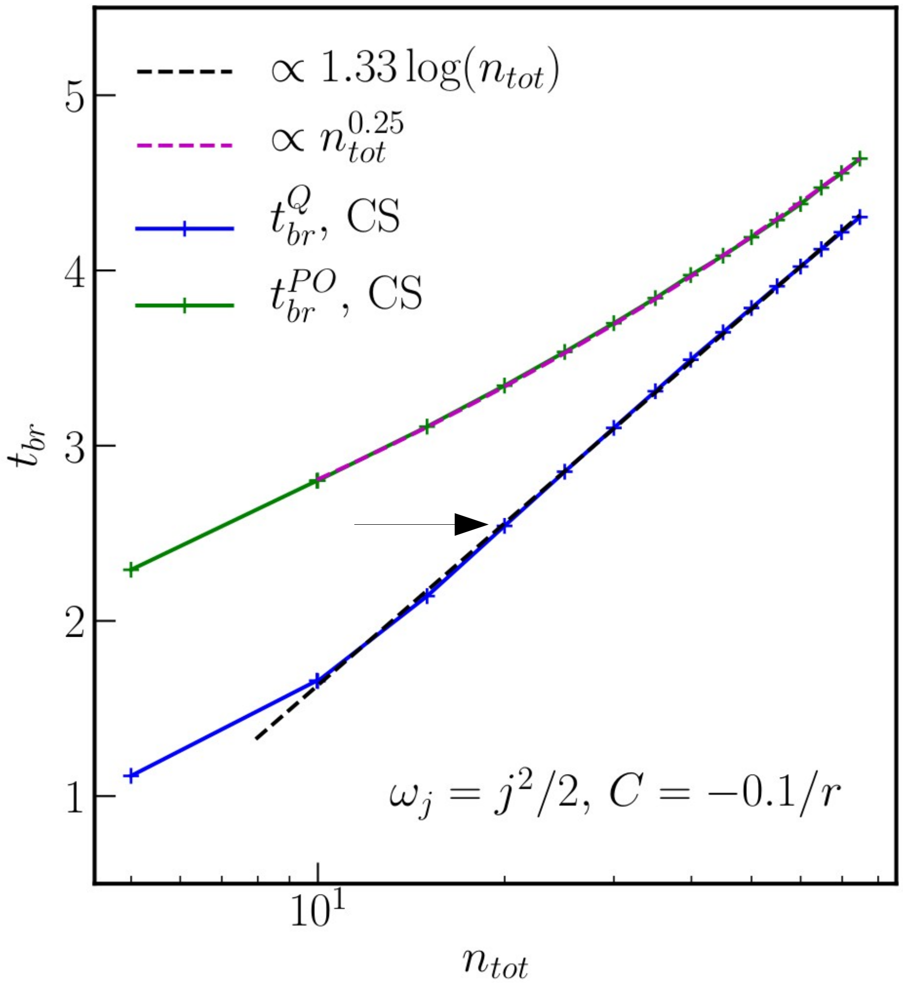

Results

- Study the behavior of quantum corrections as total particle number is increased

Small Systems: $M = 5,\ n_{tot} < 100$ → Large Systems: $M = 256,\ n_{tot} < 10^{10}$

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Large Systems:

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Our method: Large systems

- For large systems the full quantum state is intractably large

Large Systems:

$$M = 256,\ n_{tot} < 10^{10}$$

$$N_s \sim (10^{100})^{100}$$

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- Instead we simulate the MFT + leader order corrections

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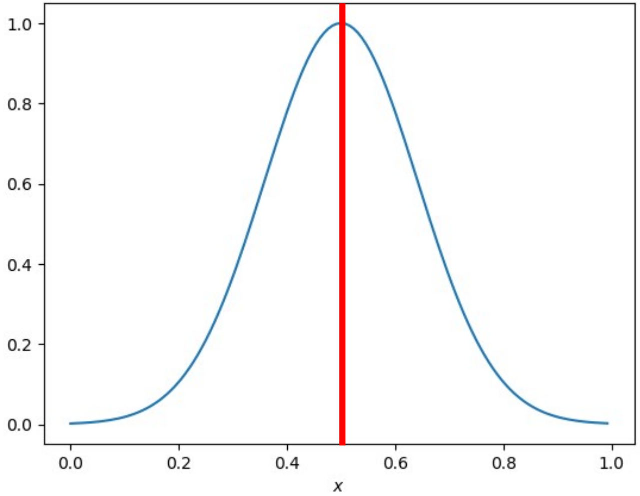
MFT:

$$\partial_t \left(\text{occupations}\right) \approx f\left[\text{means}\right]$$



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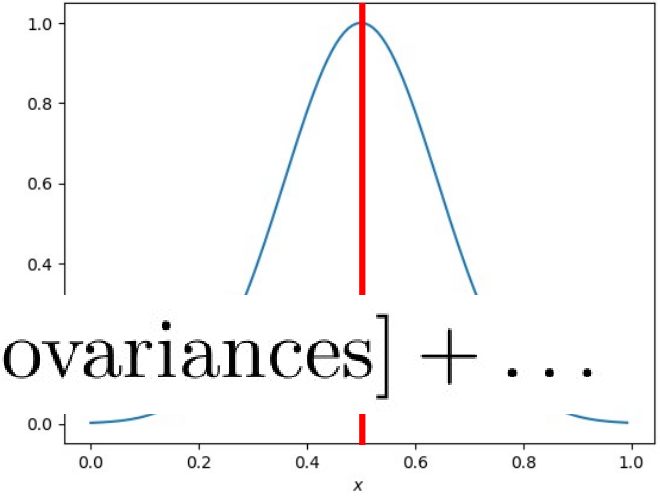

MFT:

$$\partial_t\,(\text{occupations}) = f^1\,[\text{means}] + f^2\,[\text{covariances}] + \dots$$

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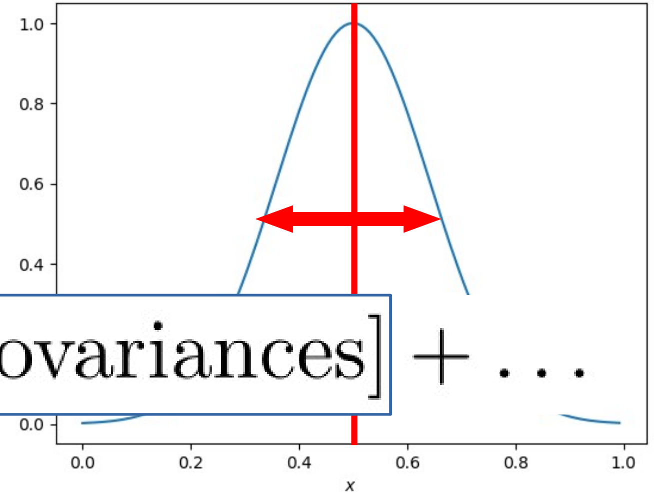


FME:

$$\partial_t\left(\text{occupations}\right) = f^1\left[\text{means}\right] + f^2\left[\text{covariances}\right] + \ldots$$

Our method: Large systems

- Extension should be more accurate than mean field theory until the quantum breaktime assuming:

Large Systems:

$$M = 256,\ n_{tot} < 10^{10}$$

$$\partial_t \psi \approx -i \left[\frac{-\nabla^2}{2m} + V \right] \psi$$

↓

$$\partial_t \psi \approx -i \left[\frac{p^2}{2m} + V \right] \psi + \text{corrections}$$

Our method: Large systems

Large Systems:

$$M = 256,\ n_{tot} < 10^{10}$$

- Extension should be more accurate than mean field theory until the quantum breaktime assuming:
  - The system is initially well described by mean field theory
  - Central moment growth is hierarchical

$$f^1|_{t=0} \gg f^2|_{t=0}$$

Mean | Root covariance | Root coskewness

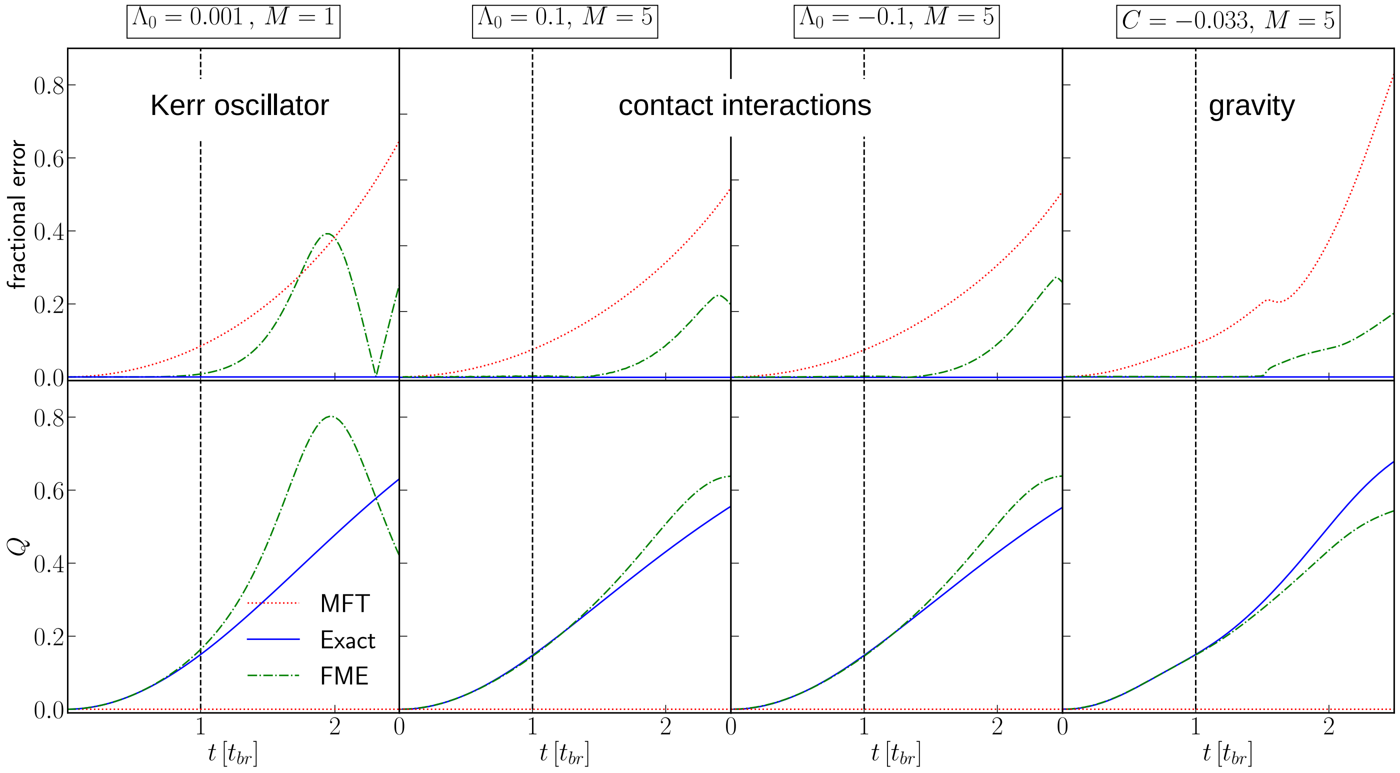
$$\langle\hat{\psi}\rangle \gg \sqrt{\langle\delta\hat{\psi}^\dagger\delta\hat{\psi}\rangle} \gg \sqrt[3]{\langle\delta\hat{\psi}^\dagger\delta\hat{\psi}\,\delta\hat{\psi}\rangle} \gg \ \ldots$$

Method: Large systems

- FME is generally able to accurate predict quantum corrections until our breaktime

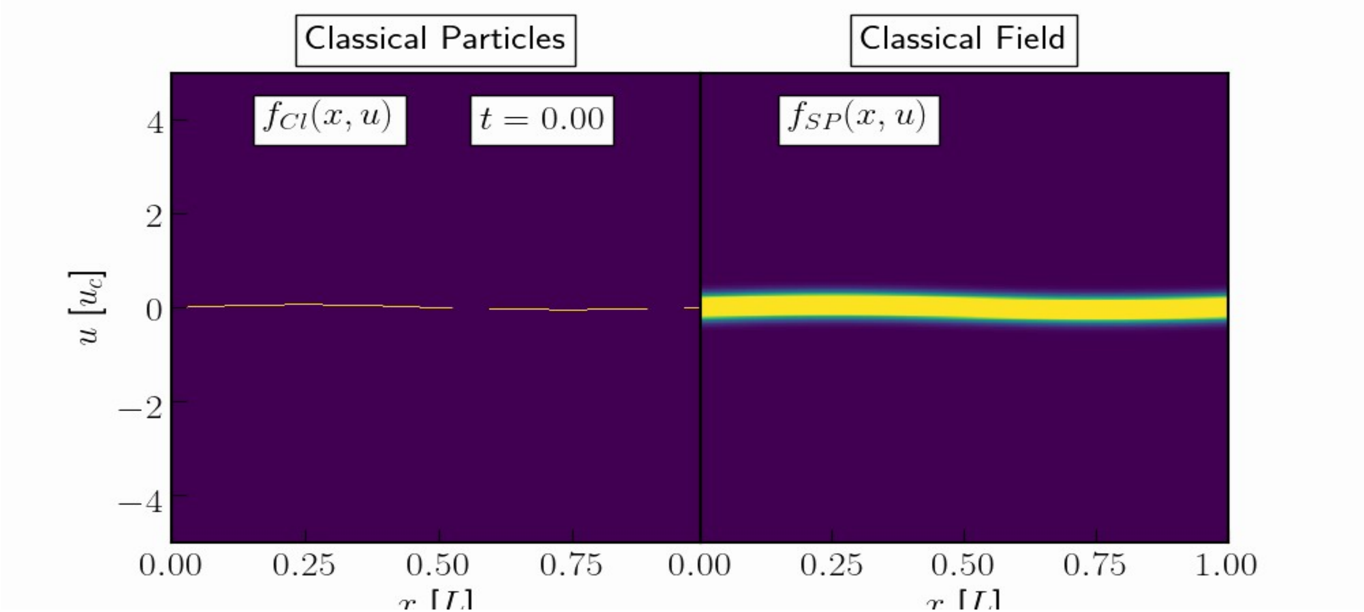
$$\hat{H} = \sum_{j}^{M} \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_{ijkl}^{M} \frac{\Lambda_{kl}^{ij}}{2} \hat{a}_k^\dagger \hat{a}_l^\dagger \hat{a}_i \hat{a}_j$$

$$\Lambda_{pl}^{ij} = \left(\frac{C}{2(p_p - p_i)^2} + \frac{C}{2(p_p - p_j)^2} + \Lambda_0 \right) \delta_{pl}^{ij}$$



Results: Large systems

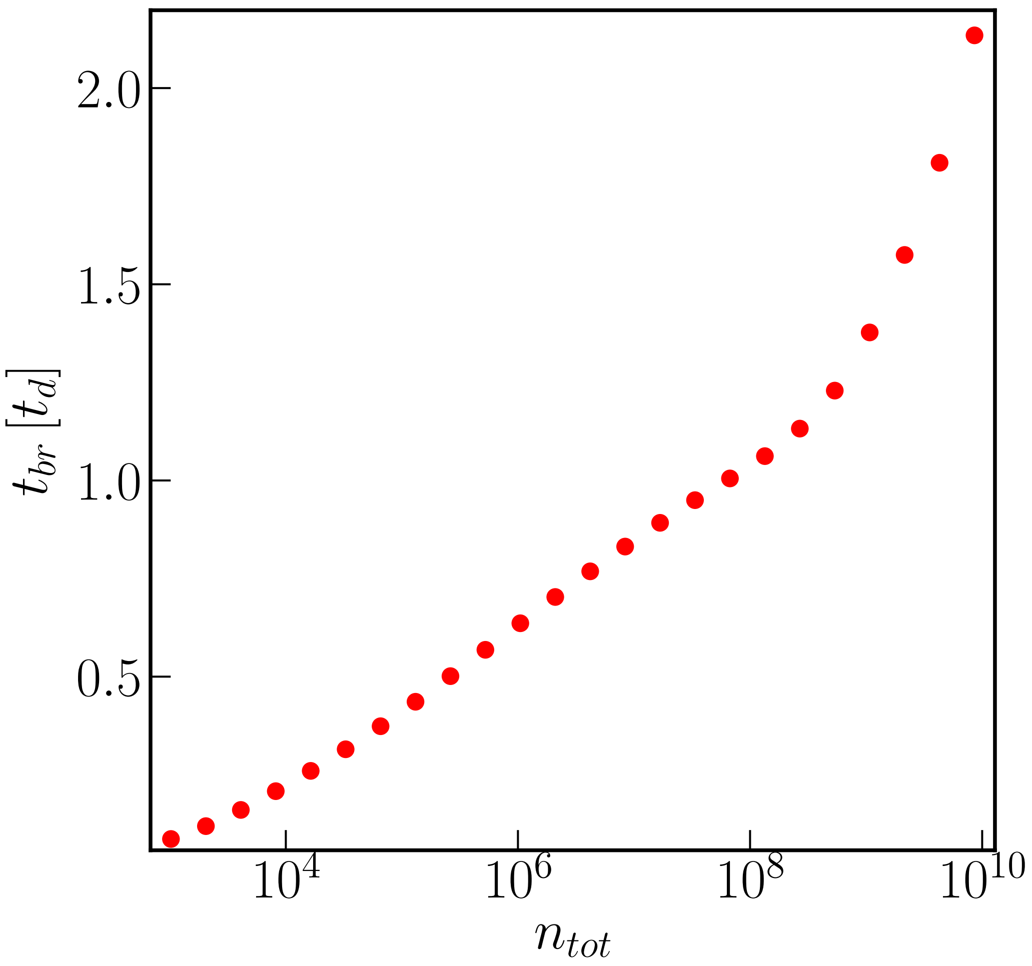
- Calculate the breaktime for a common cosmo test problem



Gravitational collapse of overdensity

Results: Large system

- Calculate the breaktime for a more reasonable cosmological system



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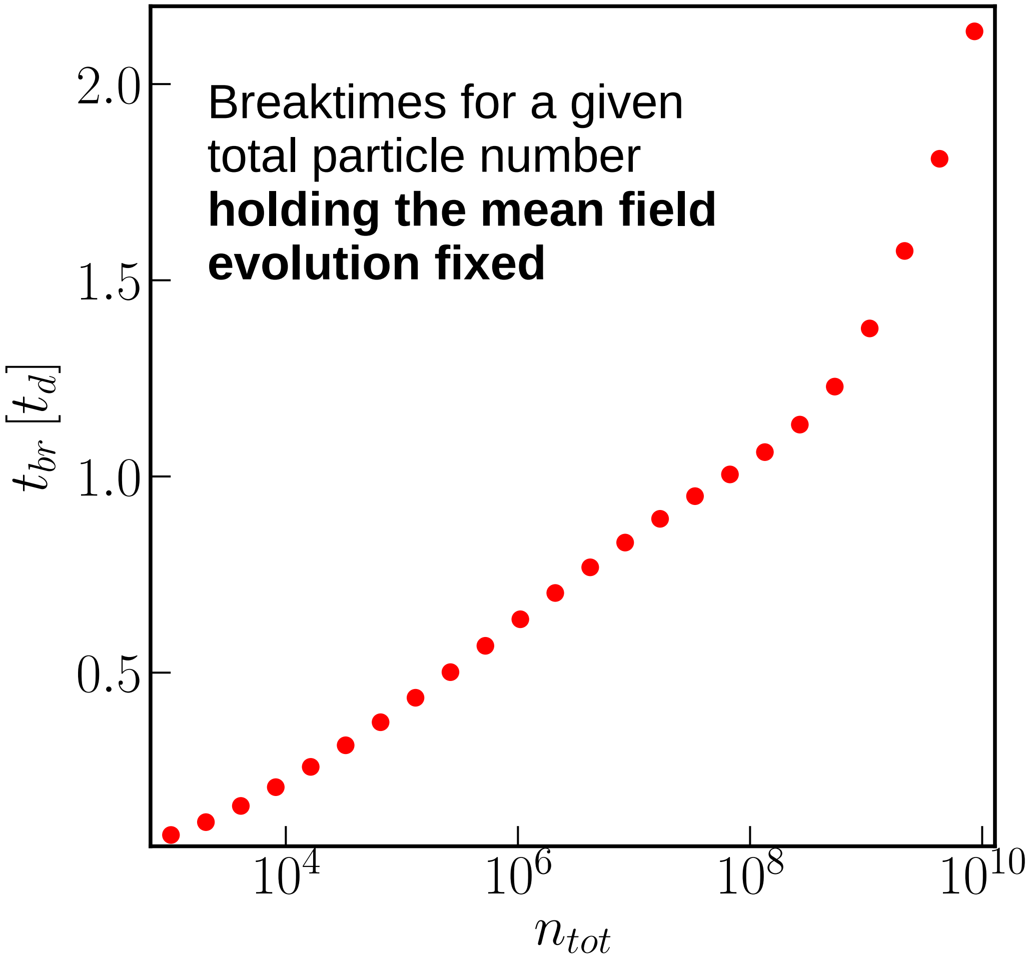
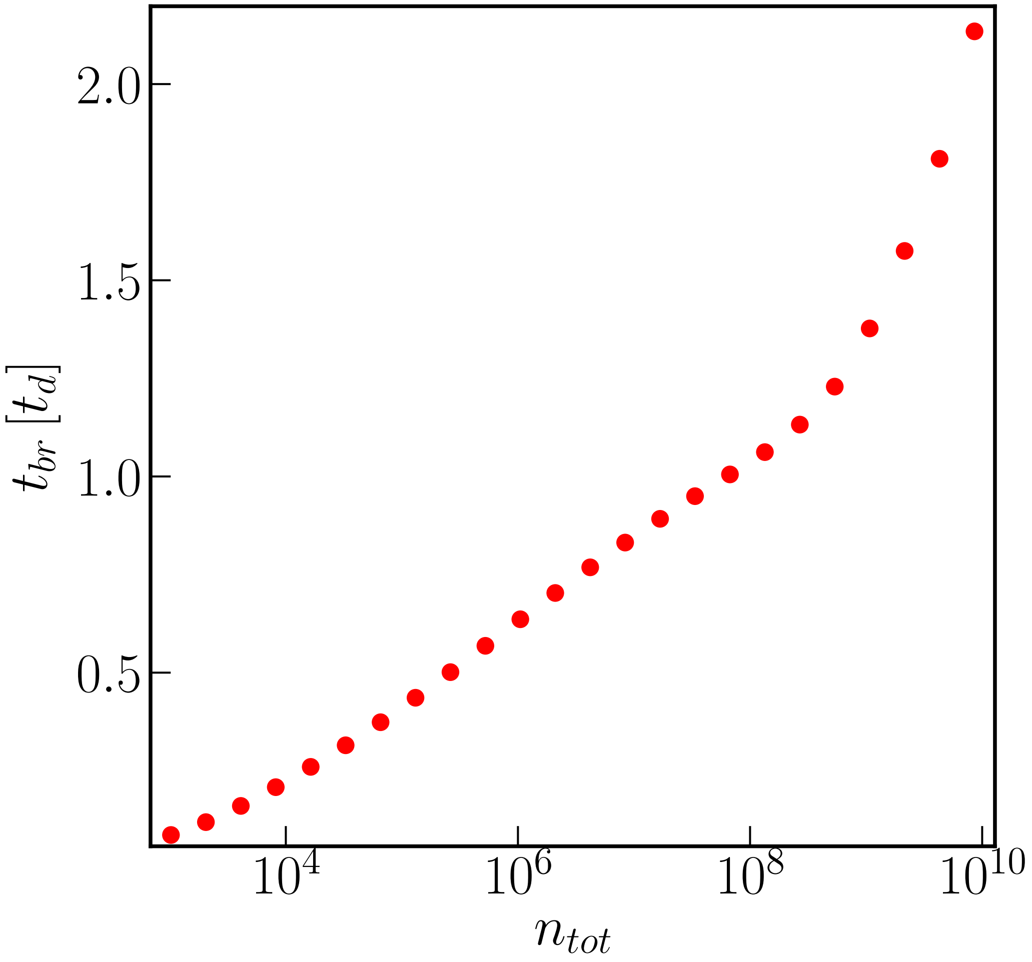


Results: Large system

- Calculate the breaktime for a more reasonable cosmological system
- Quantum corrections enter more quickly during nonlinear growth



Results: Large systen

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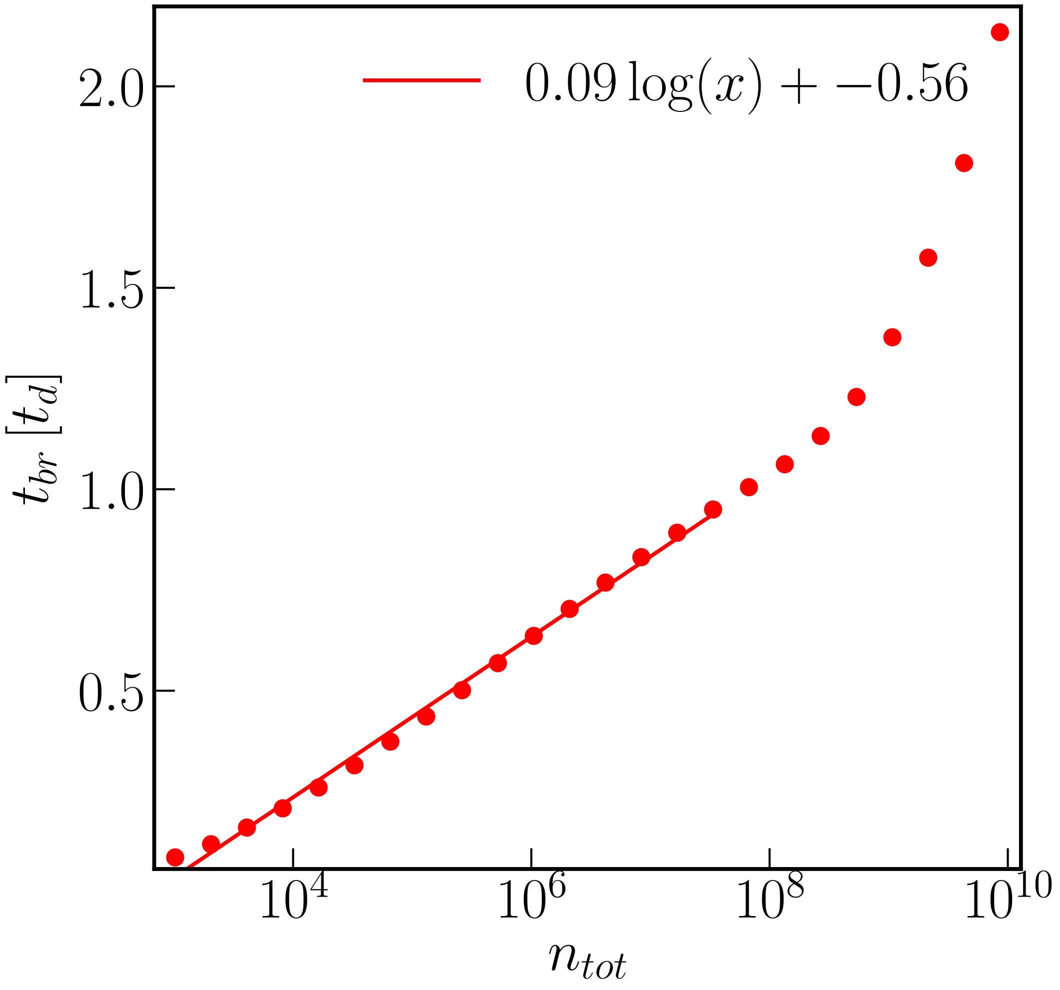
- “Logarithmic enhancement” with particle number



Results: Large systems

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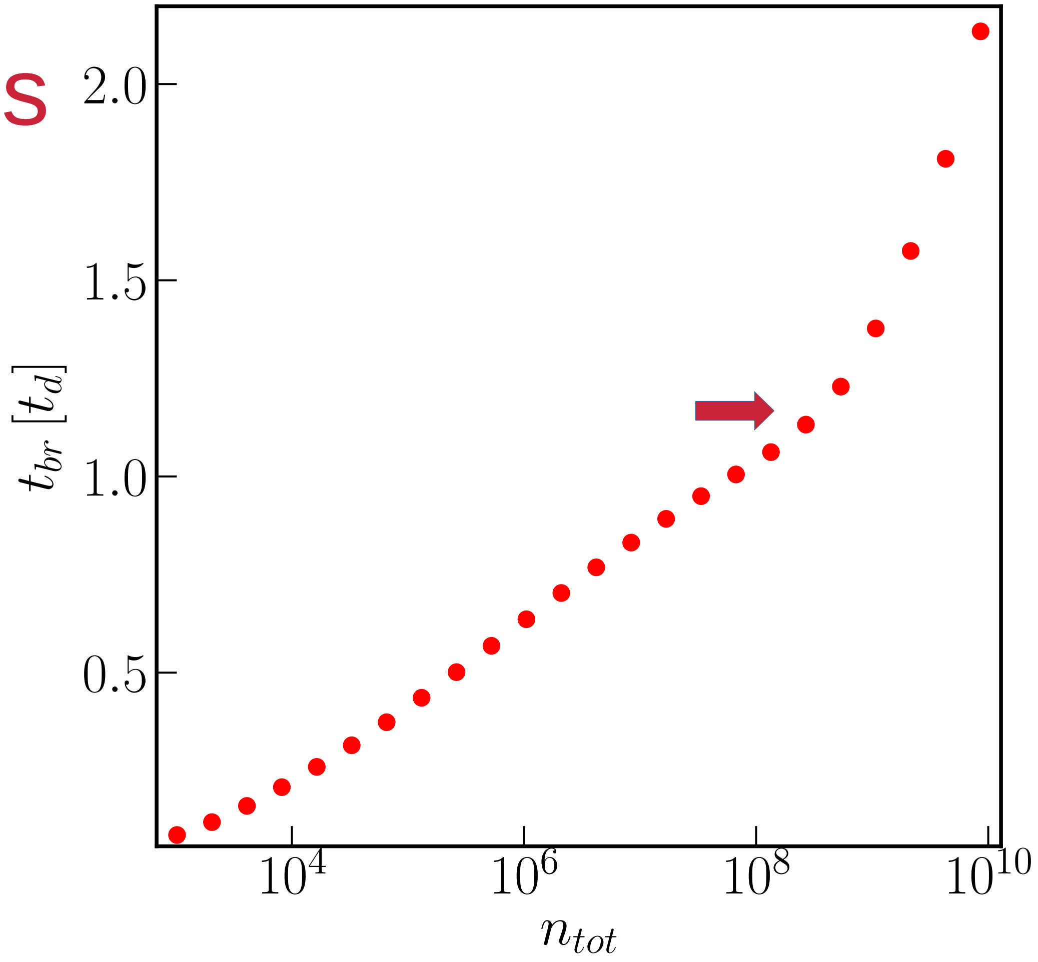

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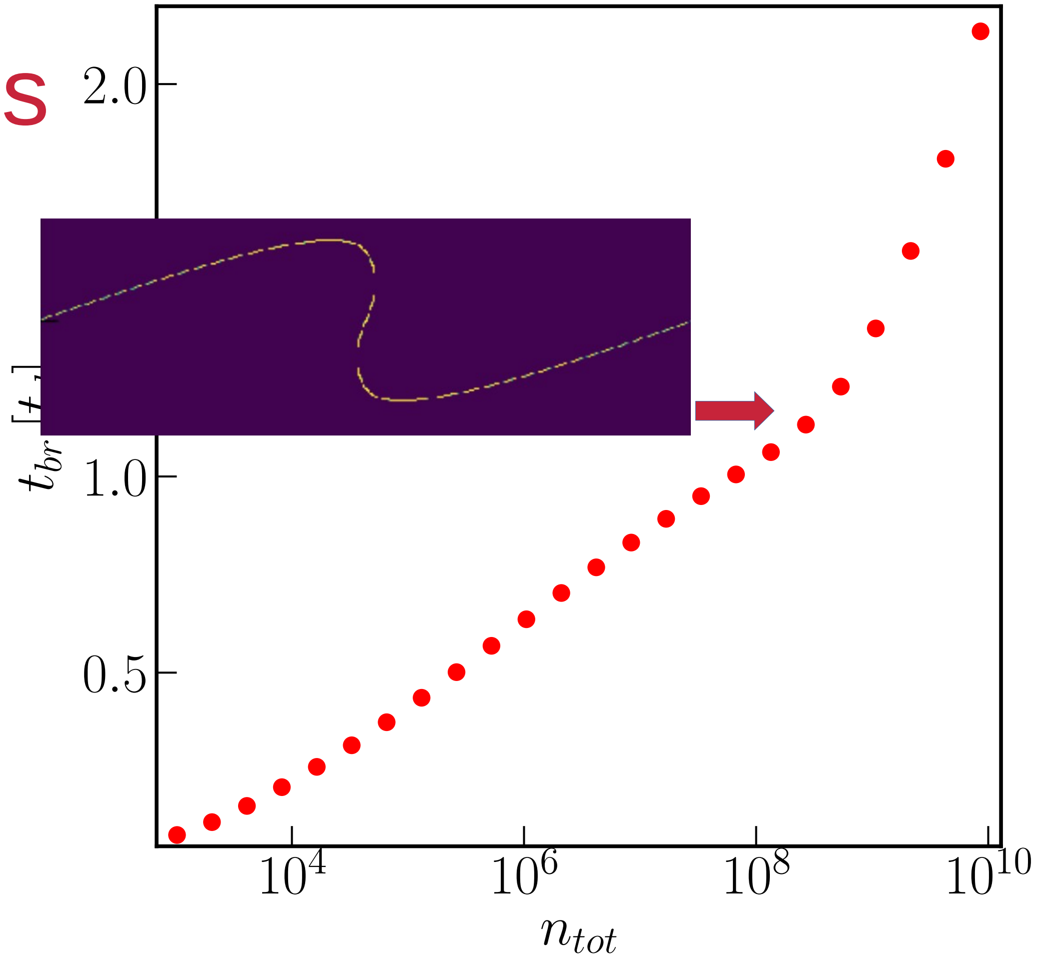

Results: Large system

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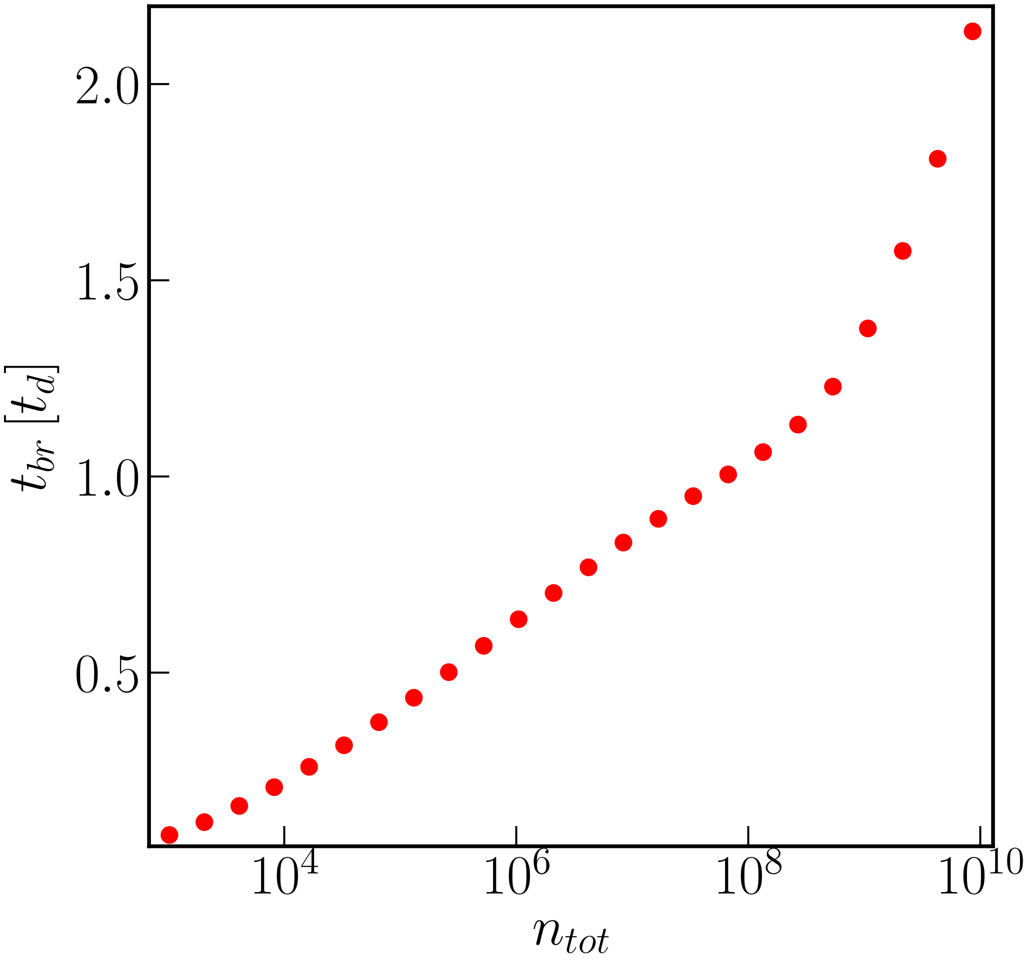
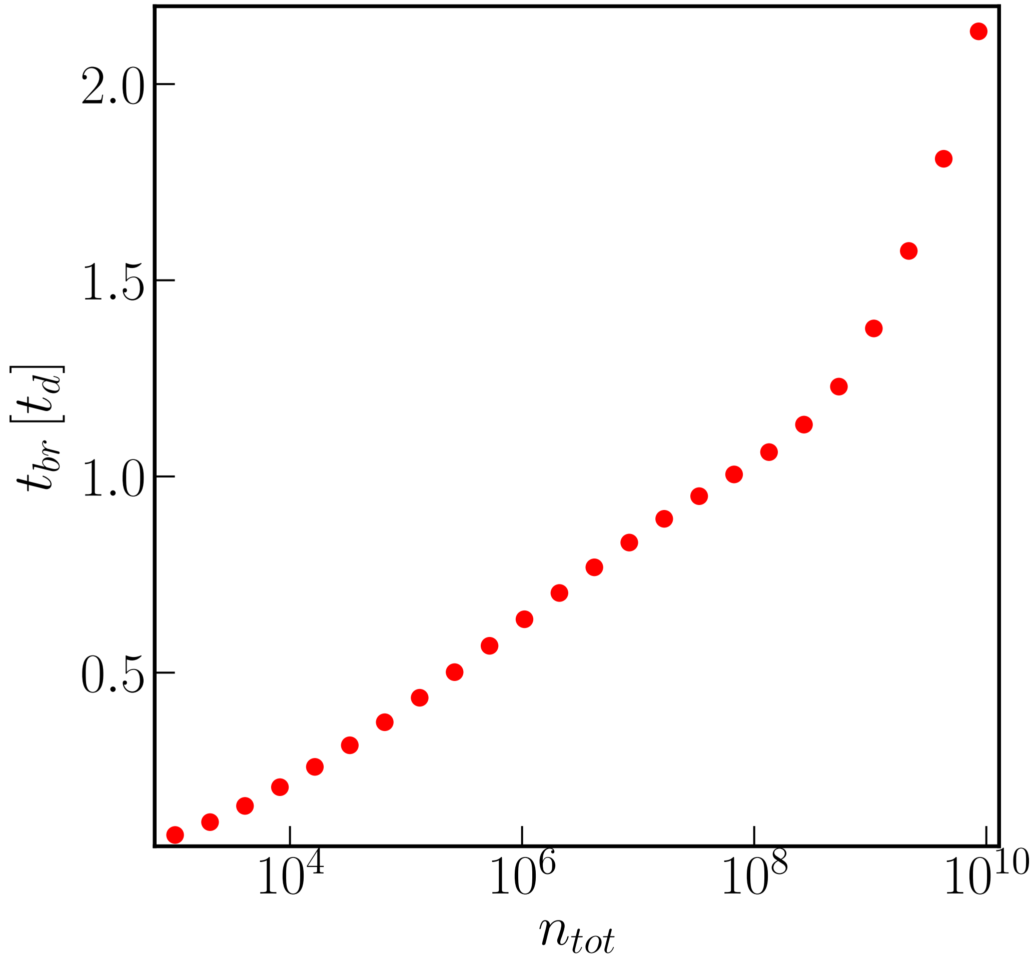
- During nonlinear growth we expect quantum corrections to start becoming non-subleading at ~300 Myr



Conclusions

- Quantum corrections introduced in some systems on a timescale less than the age of the universe



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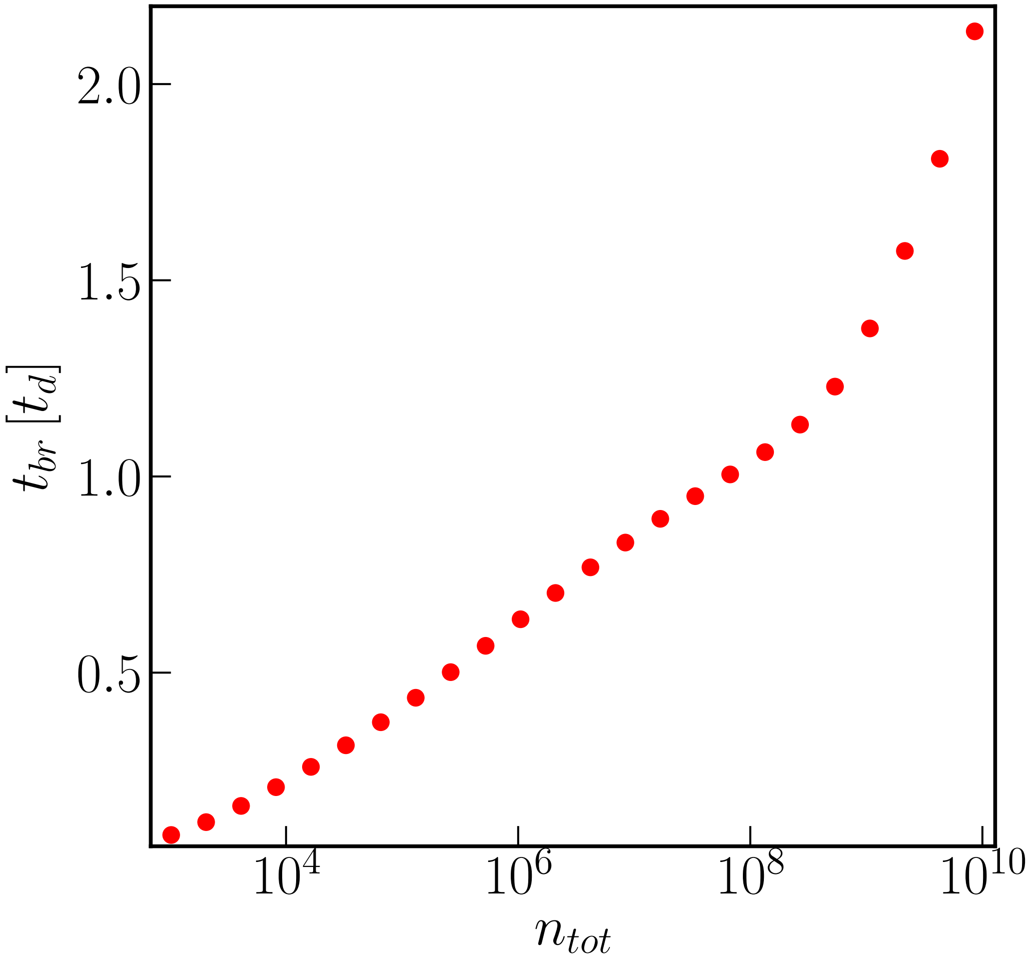

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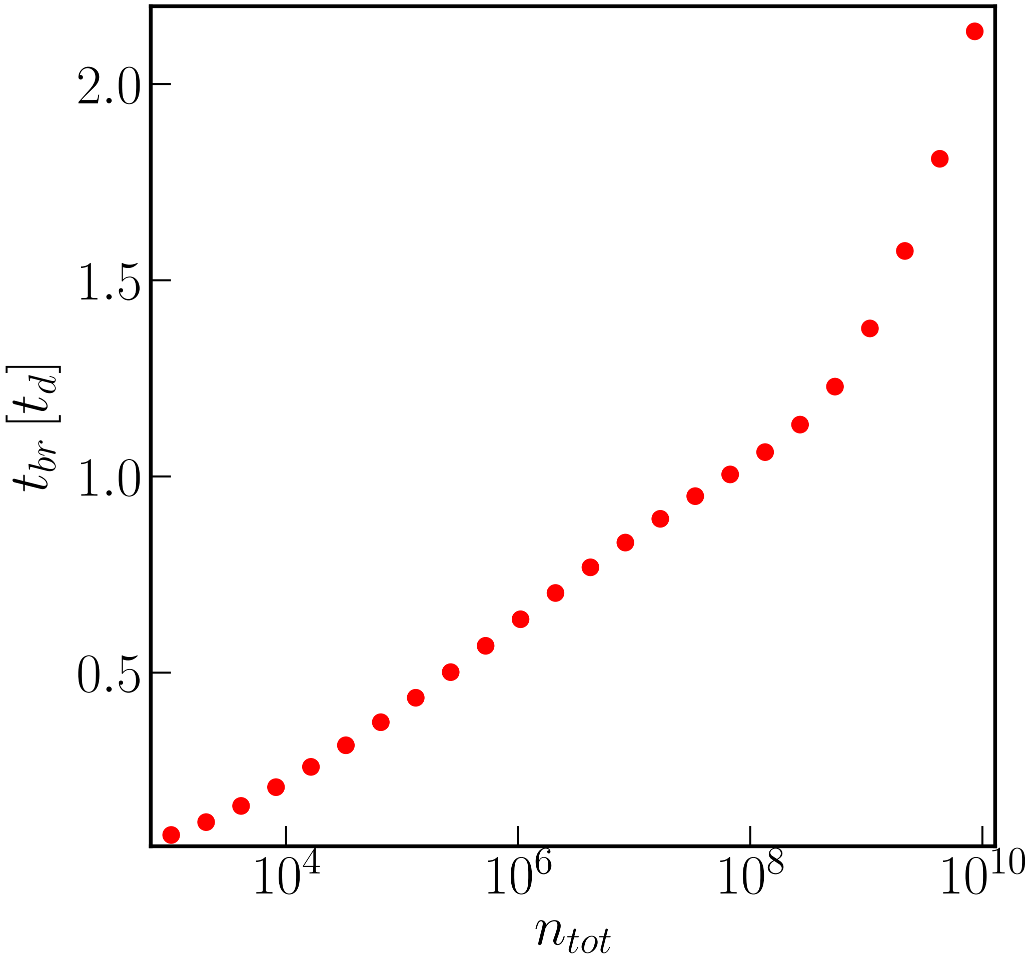
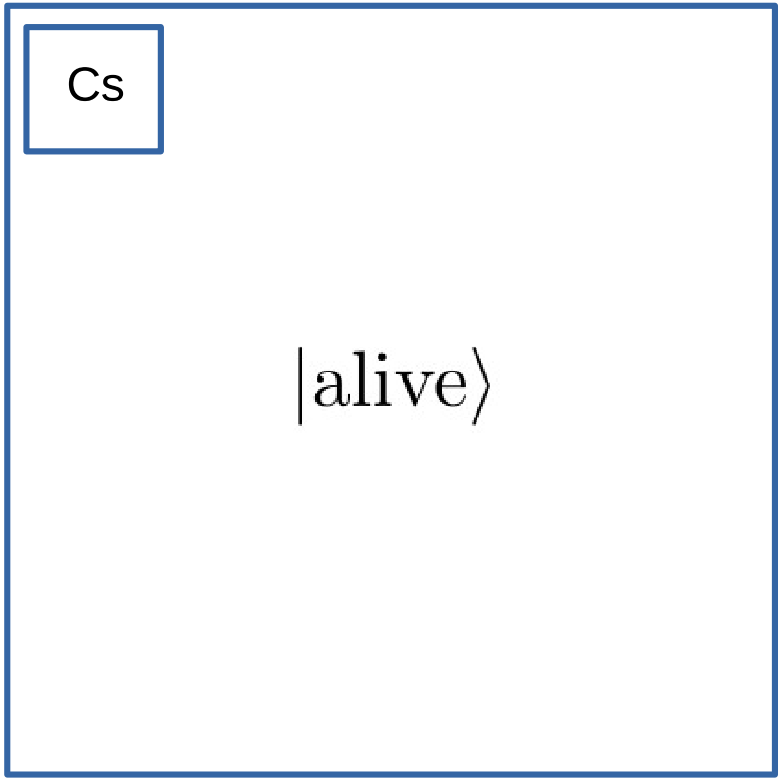
- Quantum corrections introduced in some systems on a timescale less than the age of the universe
- Does this imply that classical field simulations are wrong? Maybe



Limitations

Limitations

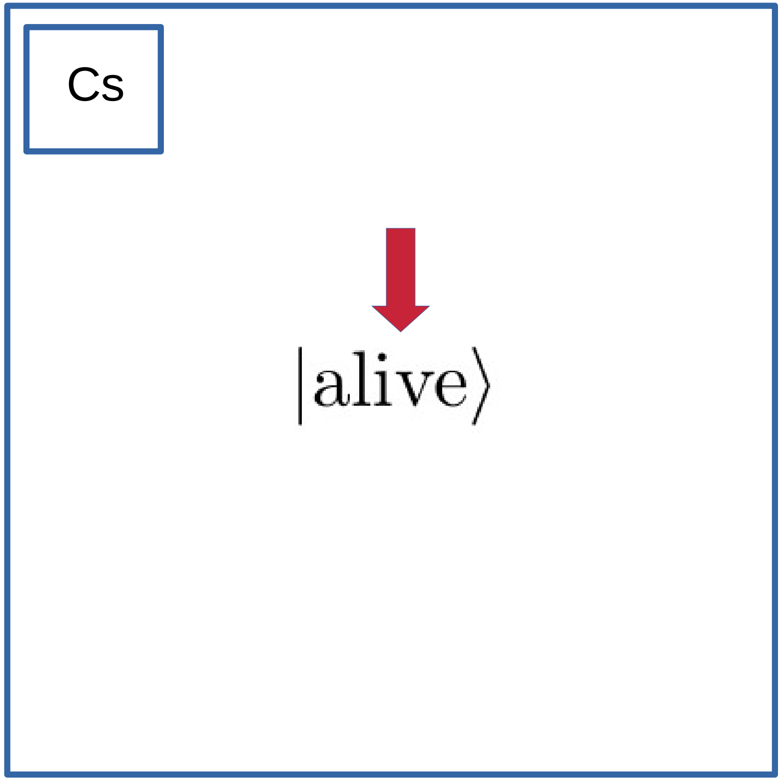
- Let’s look at an analogous system



Limitations

$|\psi(t=0)\rangle$

- The system starts in a state well described by classical mechanics



Limitations

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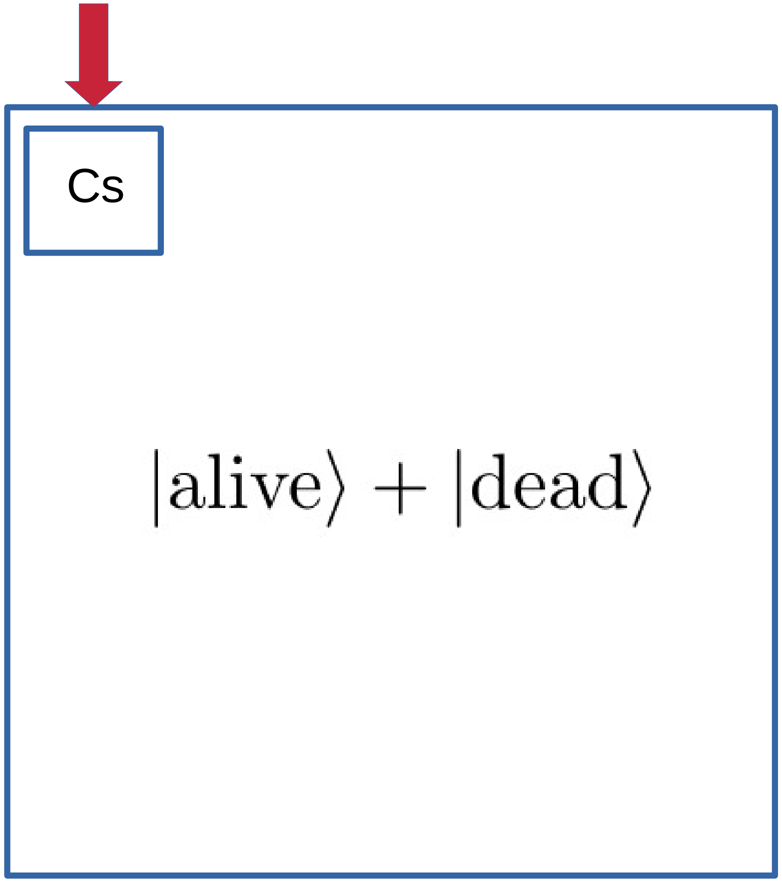
- The system starts in a state well described by classical mechanics

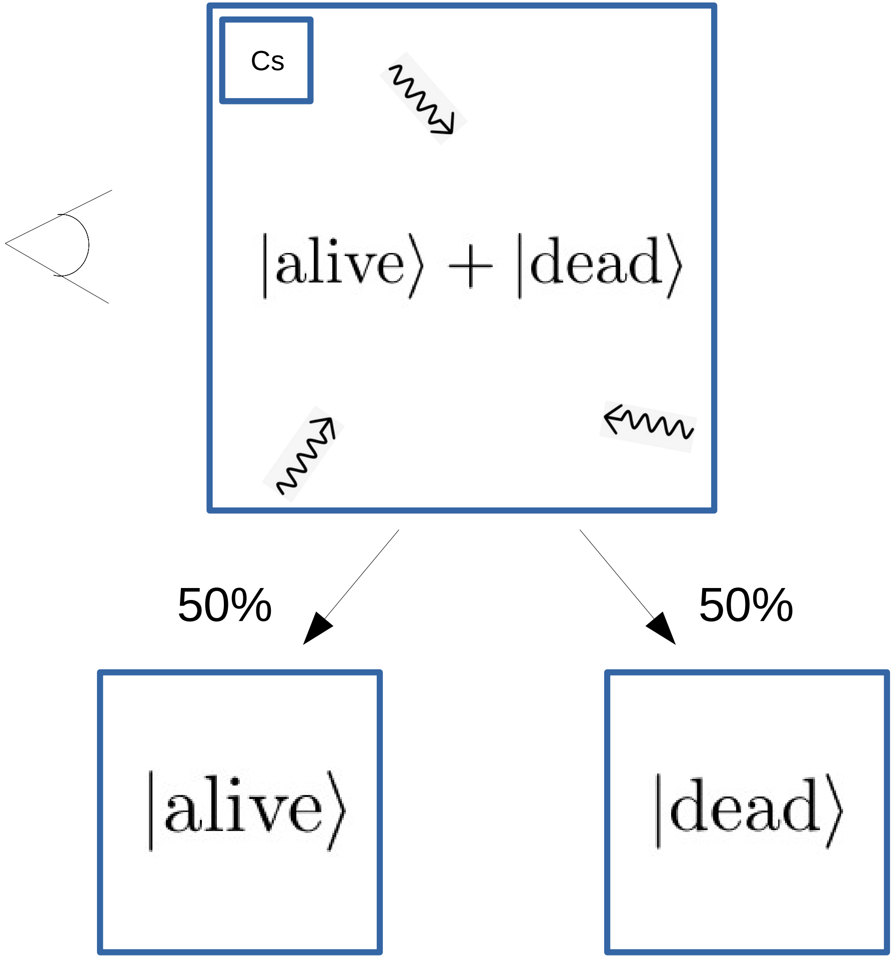
τ_{NL}

- On some timescale nonlinear interactions will create a system poorly described by classical mechanics



Limitations

- $|\psi(t=0)\rangle$ The system starts in a state well described by classical mechanics
- τ_{NL} On some timescale nonlinear interactions will create a system poorly described by classical mechanics
- τ_{env} On some timescale environmental interactions (“observers”) will send this system to its pointer states



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50% $|\text{alive}\rangle$

50% $|\text{dead}\rangle$

Pointer states

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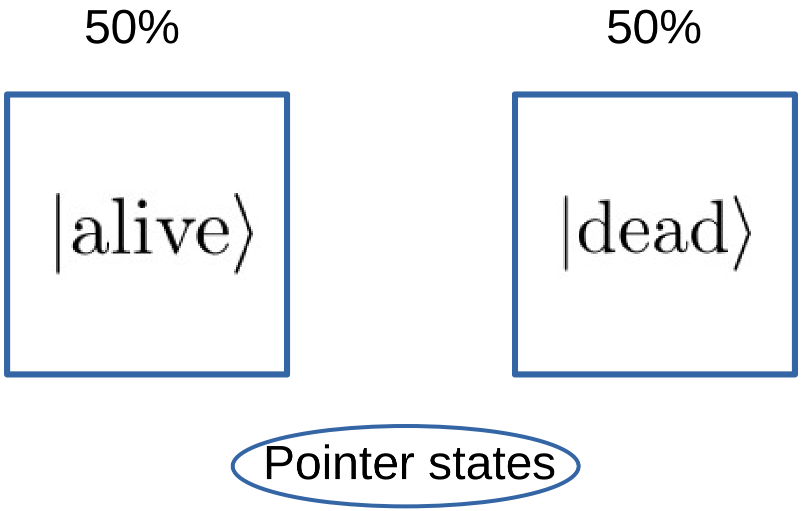

Future work

- Estimate the decoherence time and pointer states numerically

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- Estimate the decoherence time and pointer states numerically
- More realistic systems (3D) / higher order approximation

Questions?