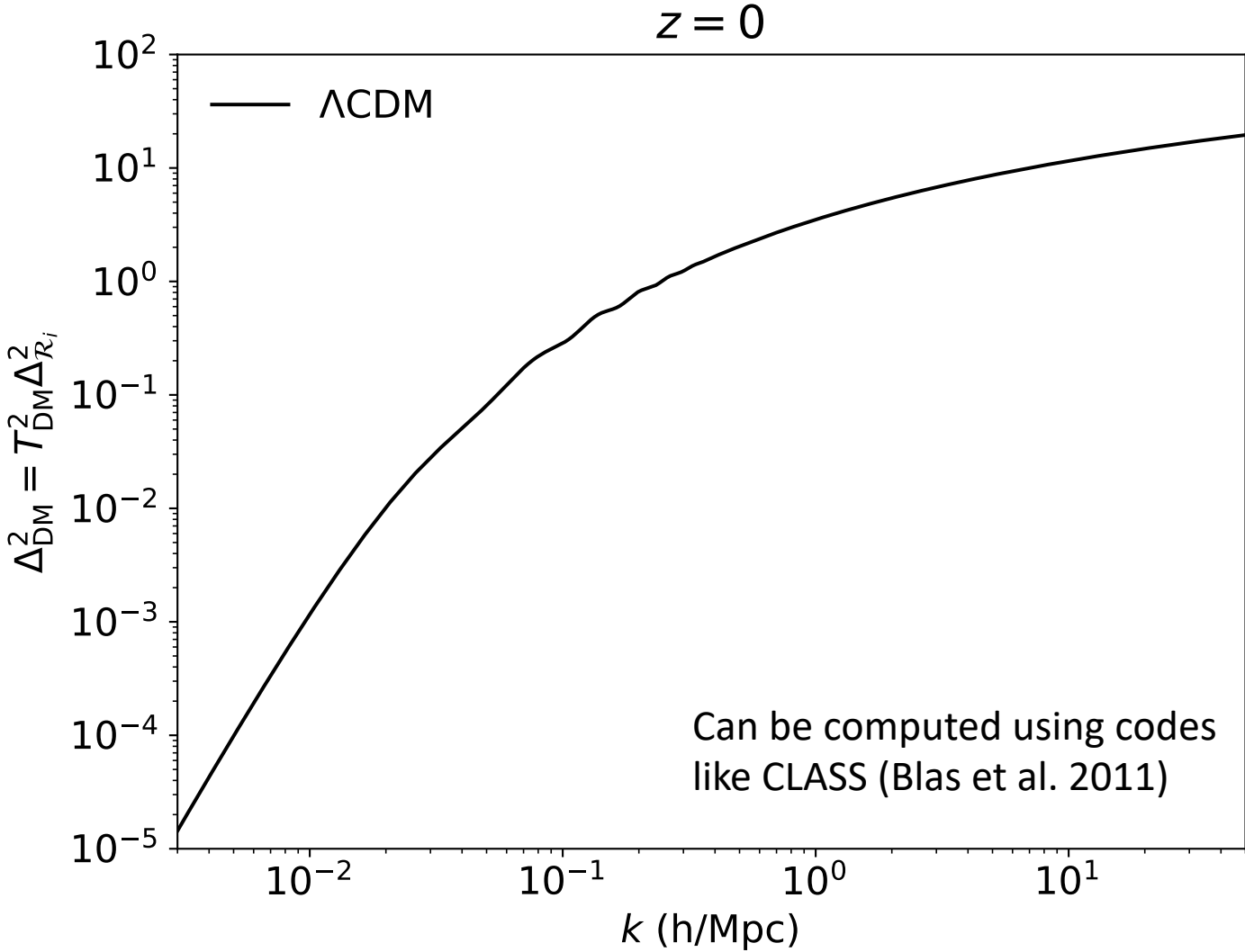


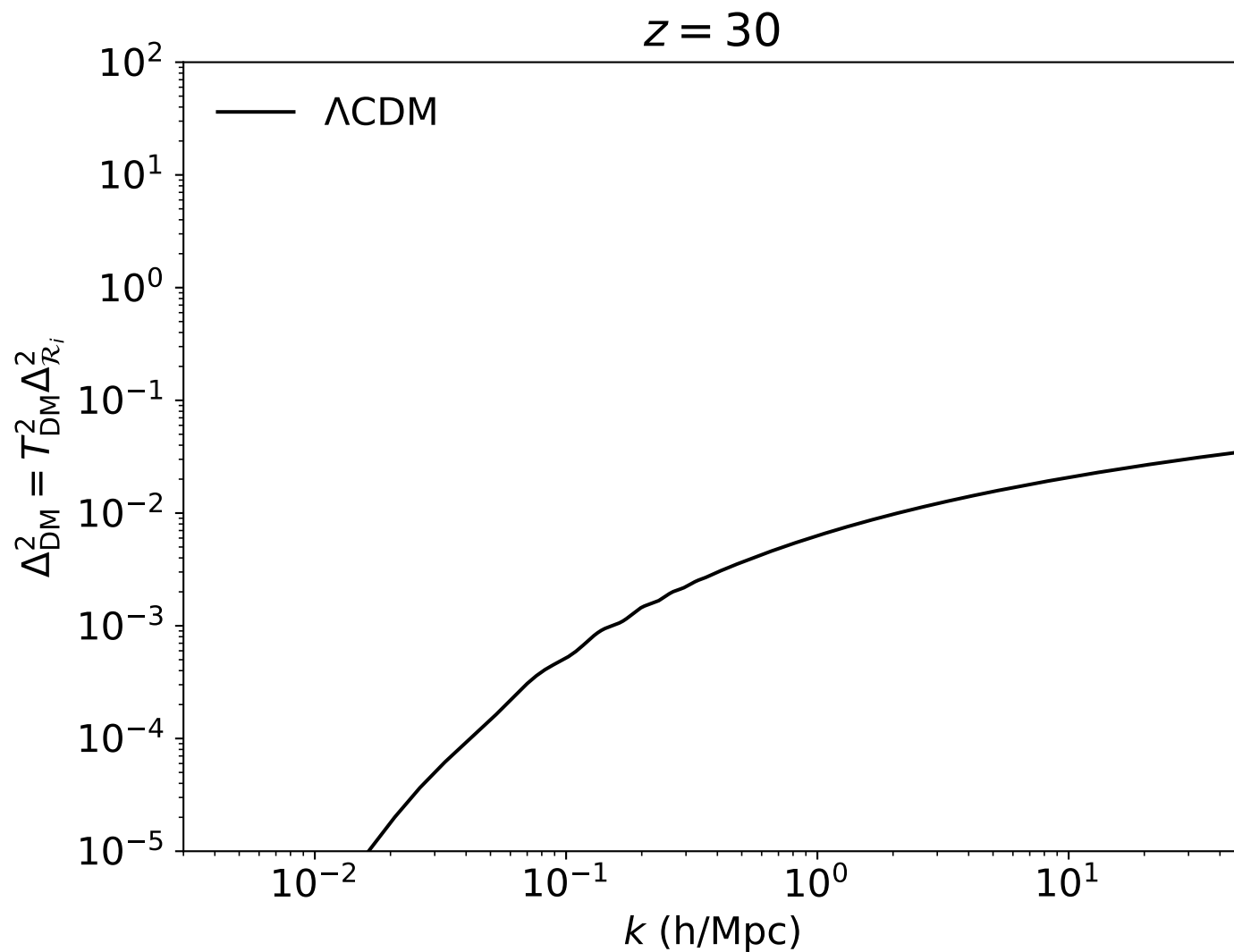
Halos throughout the Cosmic Dark Ages

Derek Inman Kavli IPMU Postdoc Colloquium October 6, 2022

Standard Λ CDM



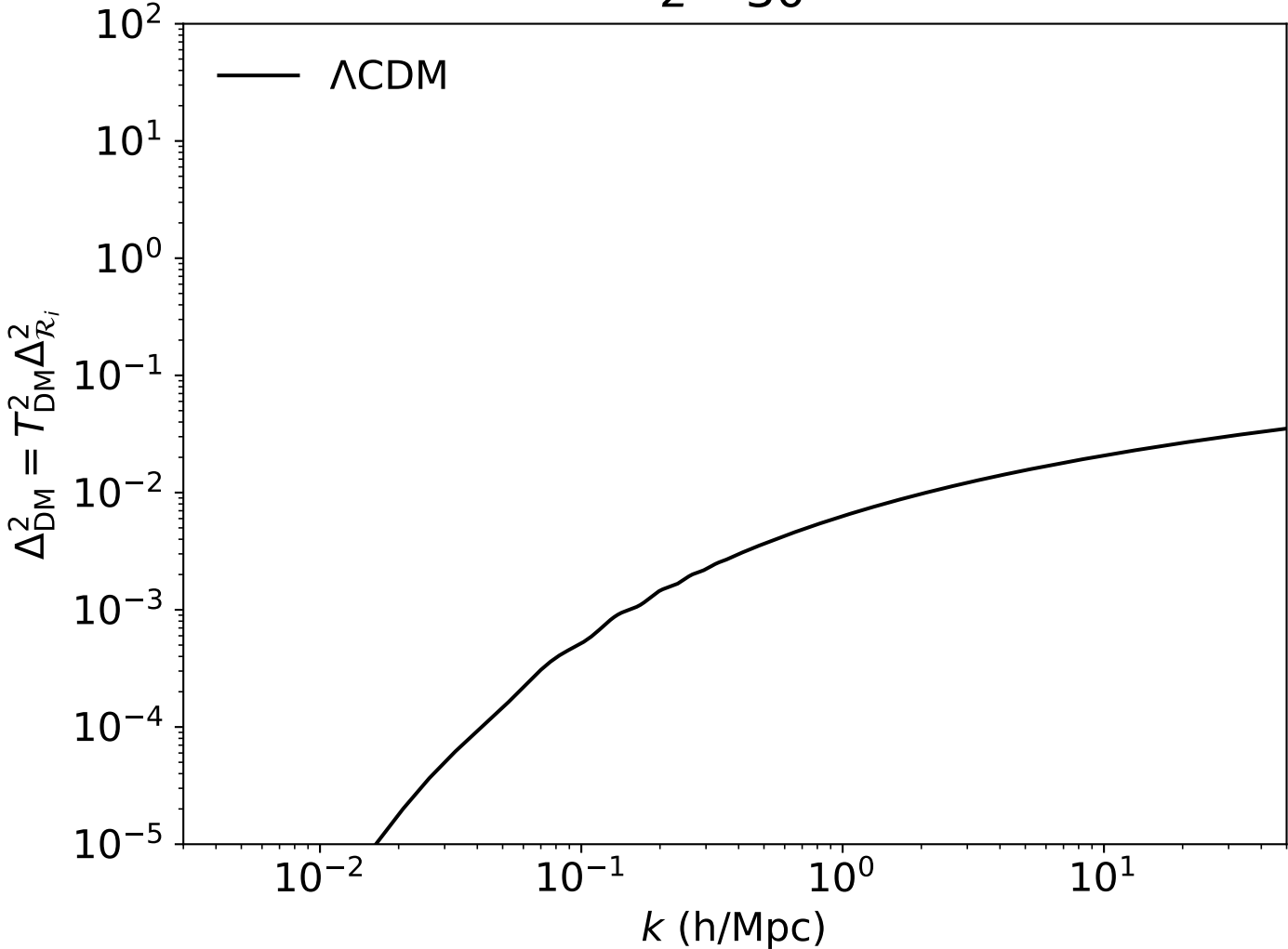
Standard Λ CDM – first halos



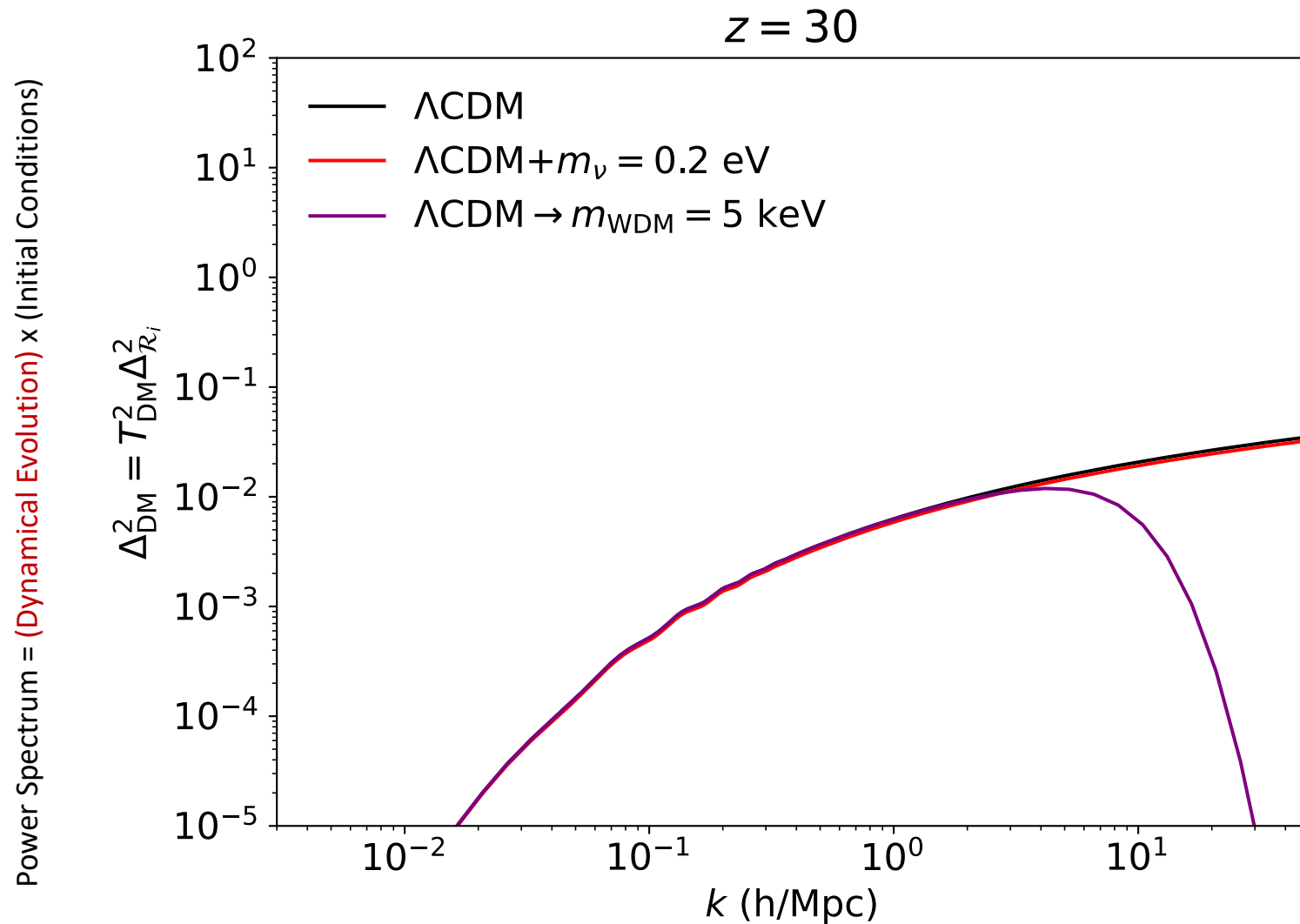
Standard Λ CDM – first halos

$z = 30$

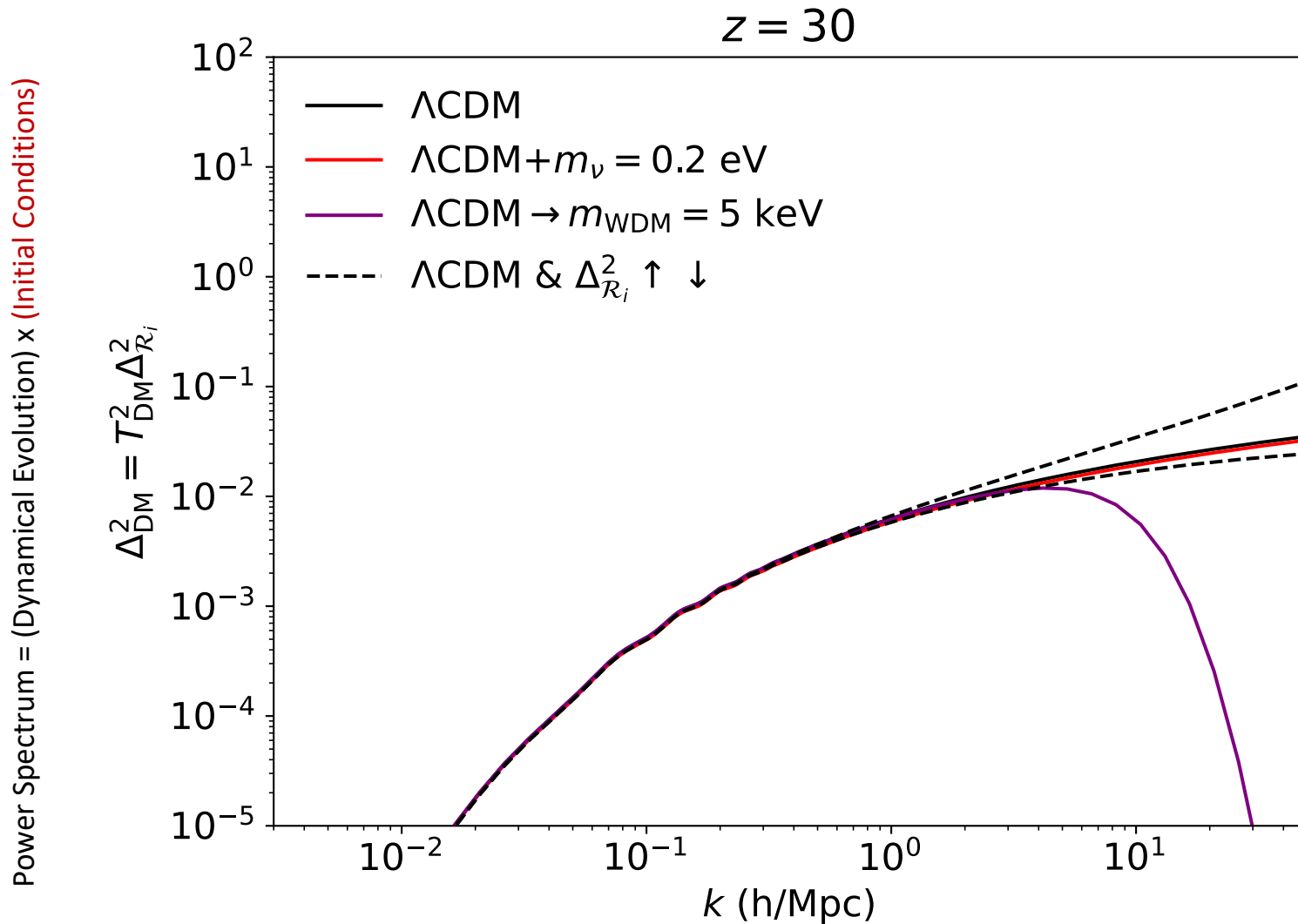
Power Spectrum = (Dynamical Evolution) x (Initial Conditions)



Beyond Λ CDM

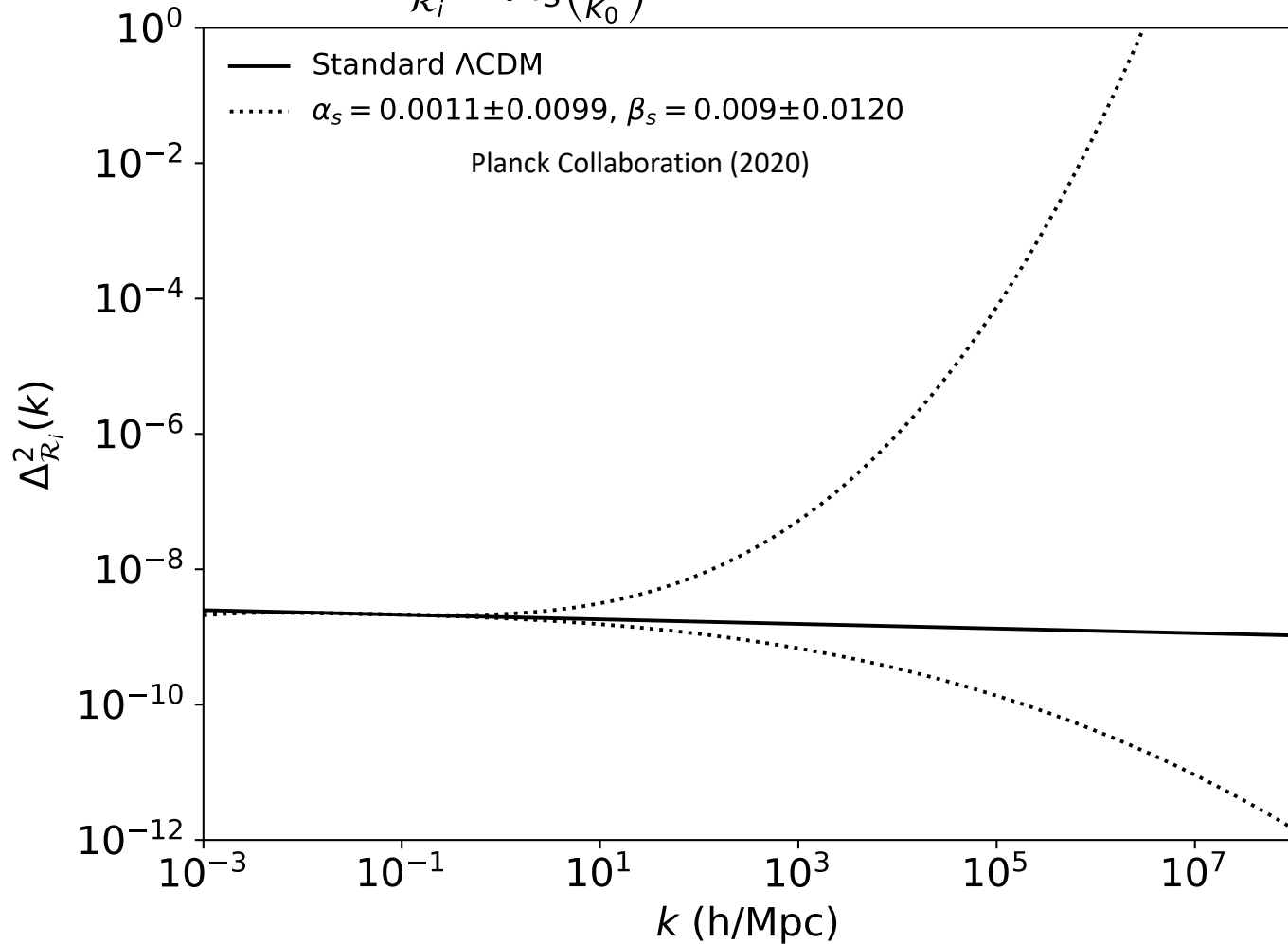


Beyond Λ CDM

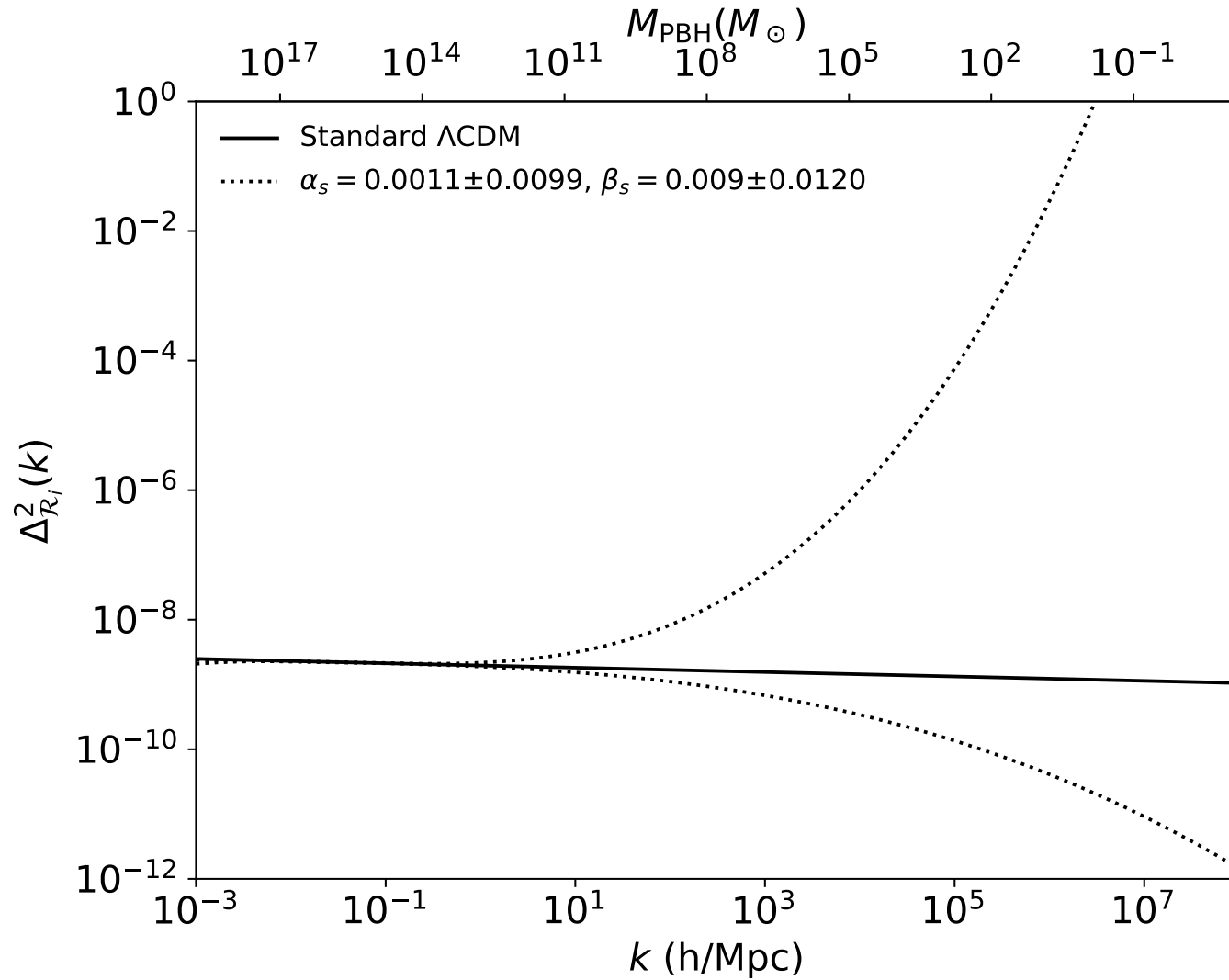


Primordial Power Spectrum

$$\Delta_{\mathcal{R}_i}^2 = \mathcal{A}_s \left(\frac{k}{k_0} \right)^{n_s - 1 + \frac{\alpha_s}{2} \log\left(\frac{k}{k_0}\right) + \frac{\beta_s}{6} \log^2\left(\frac{k}{k_0}\right)}$$

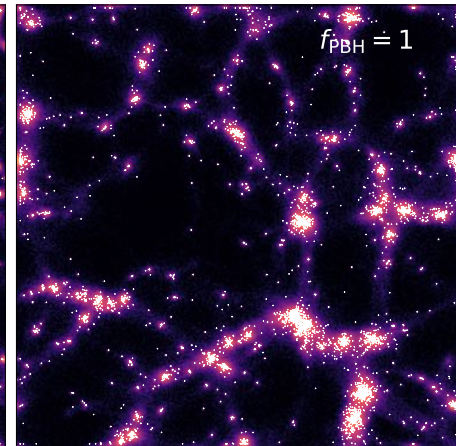
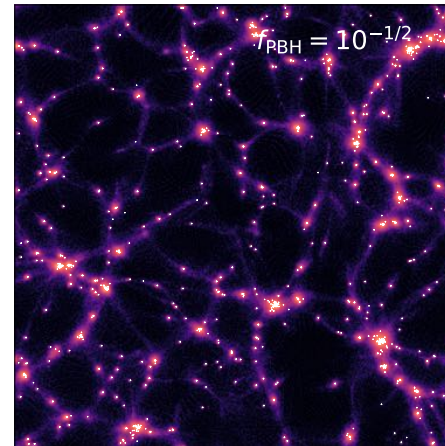
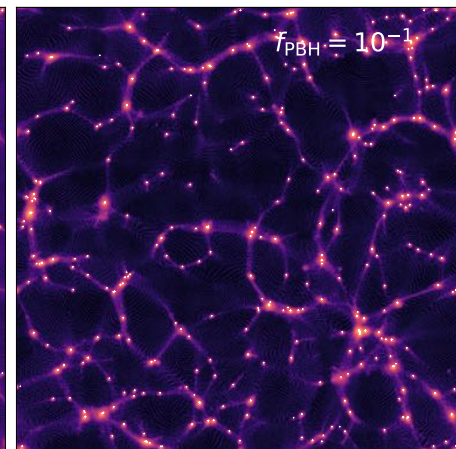
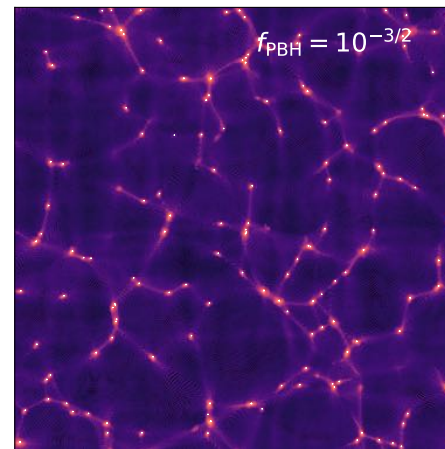
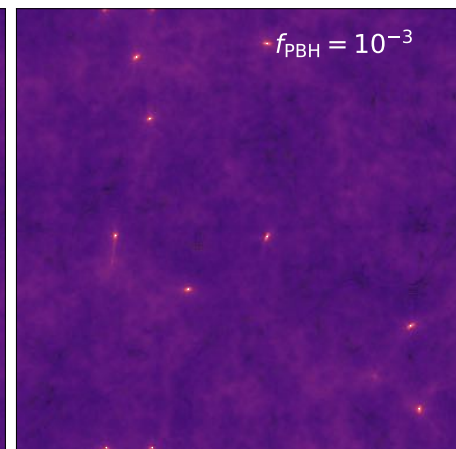
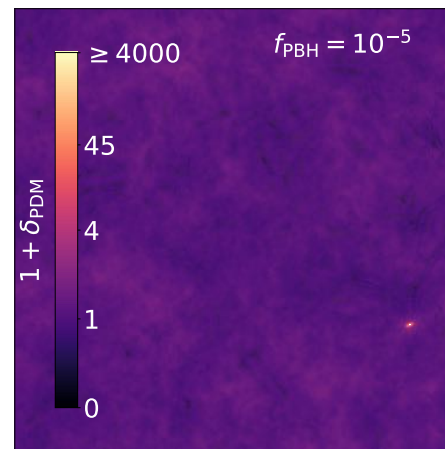
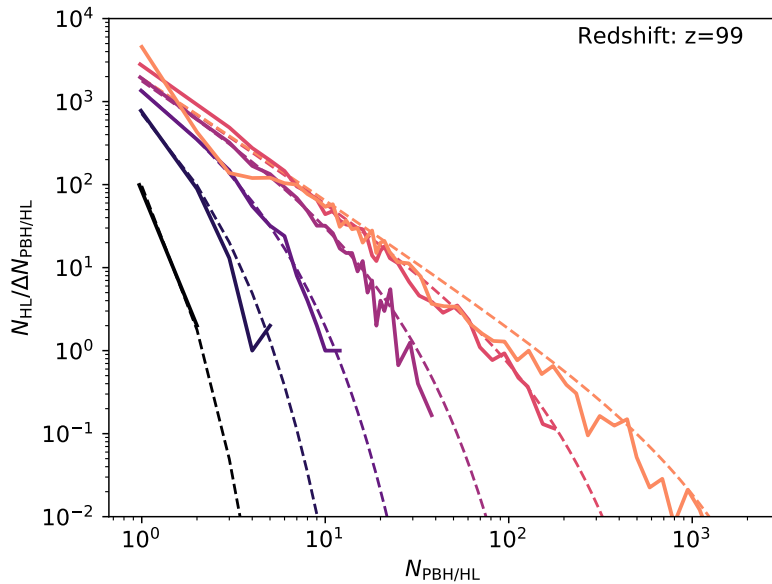


Primordial Black Holes



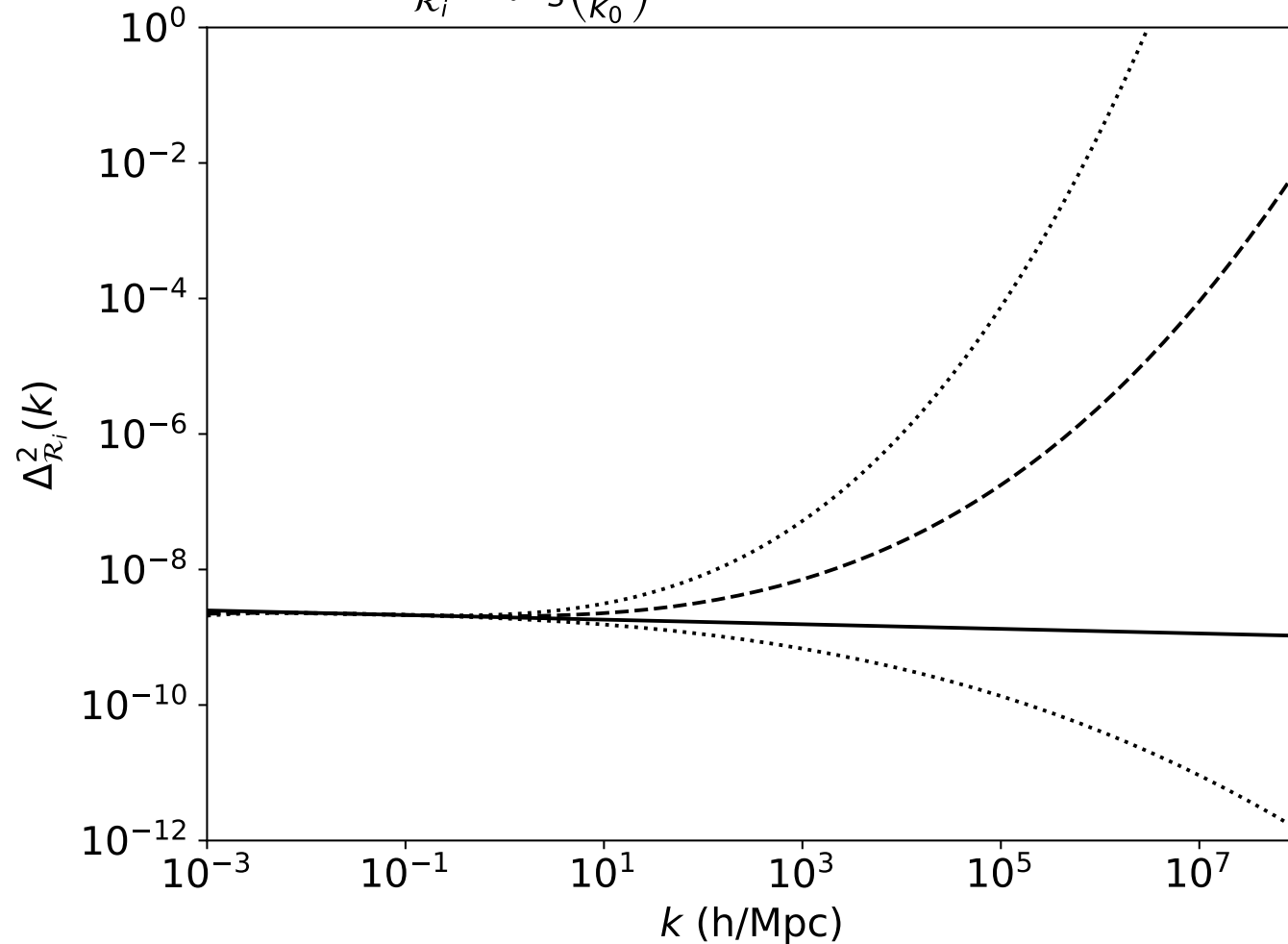
Primordial Black Holes

- PBHs are initially Poisson distributed
- After matter-radiation equality PBHs begin to cluster \rightarrow Halos!



Enhanced power spectrum without PBHs

$$\Delta_{\mathcal{R}_i}^2 = \mathcal{A}_s \left(\frac{k}{k_0} \right)^{n_s - 1 + \frac{\alpha_s}{2} \log\left(\frac{k}{k_0}\right) + \frac{\beta_s}{6} \log^2\left(\frac{k}{k_0}\right)}$$



Weakly Interacting Massive Particles

- WIMPs (e.g. Bringmann 2009)
 - Mass $m_\chi > \text{GeV}$, freeze out when $T < m_\chi \rightarrow$ abundances set at GeV scales
 - Today: assume that WIMPs make up all of the dark matter
 - Elastic scattering with electrons/neutrinos \rightarrow perturbations set at MeV scales
 - Power spectrum is truncated at $10^6 h/\text{Mpc}$ by diffusive/friction processes
 - WIMPs then free-stream until gravitational collapse \rightarrow halos set at eV scales
 - Minimum halo mass around the Earth mass

WIMP Dynamics

- Boltzmann-Fokker-Planck Equation:

$$\dot{f} + \frac{\vec{v}}{a} \cdot \vec{\nabla}_x f + \left[\vec{v} \dot{\phi} - a \vec{\nabla}_x \psi \right] \cdot \vec{\nabla}_v f = a\gamma(1 + \psi) \vec{\nabla}_v \cdot \left[(\vec{v} - a\vec{V}_R) f + \frac{a^2 T_R}{m_\chi} \vec{\nabla}_v f \right]$$

Bertschinger (2006), Binder et al. (2016)

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Momentum Exchange

Diffusion

Bertschinger (2006), Binder et al. (2016)

Free-streaming

Gravitational Collapse

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Bertschinger (2006), Binder et al. (2016)

Free-streaming

Gravitational Collapse

- Background & Linear solutions:

$$f_0 = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp \left[-\frac{1}{2} \frac{v^2}{\sigma^2} \right]$$

$$\delta_\chi(\eta) = \delta_\chi(\eta \rightarrow 0) G_\eta(\eta \rightarrow 0) + \int_0^\eta d\eta' S_\eta(\eta') G_\eta(\eta')$$

WIMP Dynamics

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$$\dot{f} + \frac{\vec{v}}{a} \cdot \vec{\nabla}_x f + \left[\vec{v} \dot{\phi} - a \vec{\nabla}_x \psi \right] \cdot \vec{\nabla}_v f = a\gamma(1 + \psi) \vec{\nabla}_v \cdot \left[(\vec{v} - a\vec{V}_R) f + \frac{a^2 T_R}{m_\chi} \vec{\nabla}_v f \right]$$

Momentum Exchange

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Bertschinger (2006), Binder et al. (2016)

Free-streaming Gravitational Collapse

- Approximate solution scheme:

1. Solve decoupling assuming linear and deep in radiation era
2. Propagate towards matter era assuming linear and collisionless
3. Use N-body simulations to obtain nonlinear results

1. Decoupling in the radiation era

- Background is Gaussian with dispersion:

$$\frac{\sigma^2}{\sigma_d^2} = \exp \left[\frac{1}{y^{n_\gamma}} \right] \Gamma \left[\frac{n_\gamma - 1}{n_\gamma}, \frac{1}{y^{n_\gamma}} \right]$$

$$\sigma_d^2 = \frac{a_d^2 T_d}{m_\chi}$$

$$y = \frac{a}{a_d} = \frac{T_d}{T}$$

$$\gamma \propto \left(\frac{T}{T_d} \right)^{2+n_\gamma}$$

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 1. Solve equations assuming no diffusion:

$$\frac{d\delta_{\chi 0}}{dy} + \eta_d \theta_{\chi 0} = 3 \frac{d\phi}{dy}$$

$$\frac{d\eta_d \theta_{\chi 0}}{dy} + \frac{1}{y} \eta_d \theta_{\chi 0} = (k\eta_d)^2 \phi + \frac{n_\gamma}{2} \frac{\eta_d \theta_R - \eta_d \theta_{\chi 0}}{y^{1+n_\gamma}}$$

$$\therefore \delta_{\chi 0} = \delta_\chi(\eta \rightarrow 0) + \int_0^y dx \left[3 \frac{d\phi}{dx} - u_y(x) \left(\frac{n_\gamma}{2} \frac{\eta_d \theta_R}{x^{n_\gamma}} + x(k\eta_d)^2 \phi \right) \right]$$

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 1. Solve equations assuming no diffusion:
 2. Multiply by a diffusion damping term:

$$\delta_\chi = \delta_{\chi 0} G_\eta(\eta \rightarrow 0)$$

$$G_\eta(\eta \rightarrow 0) = \exp \left[-\frac{T_d}{m_\chi} (k\eta_d)^2 \int_{\eta'}^{\eta} a\gamma_0 \frac{a^2}{a_d^2} \frac{T_0}{T_d} u_\eta^2(\eta'') d\eta'' \right]$$

$$u_\eta(\eta') = \int_{\eta'}^{\eta} \frac{d\eta''}{\eta_d} \frac{a_d}{a} \exp \left[-\int_{\eta'}^{\eta''} a\gamma_0 d\eta''' \right]$$

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- Background is Gaussian with dispersion:

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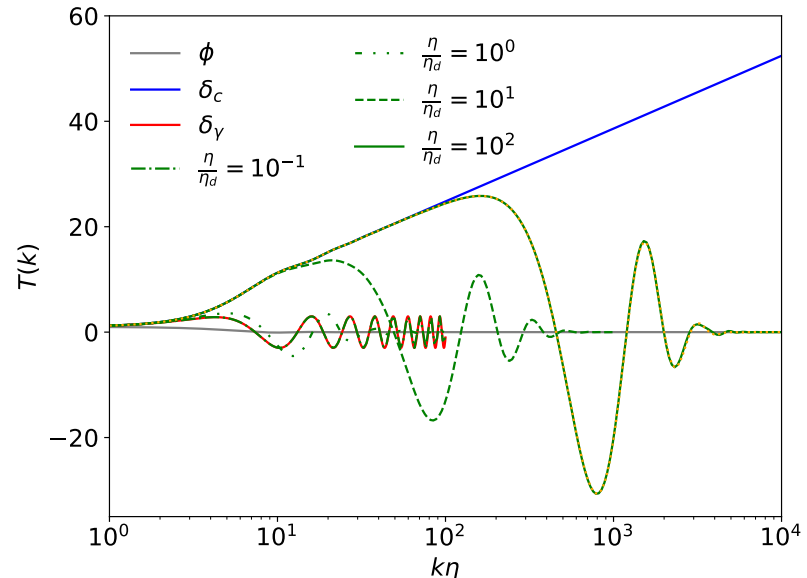
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2. Collisionless evolution

- After decoupling:
 - Collisionless evolution
 - WIMPs are also gravitationally decoupled (Vorz et al 2014) → Self-gravity

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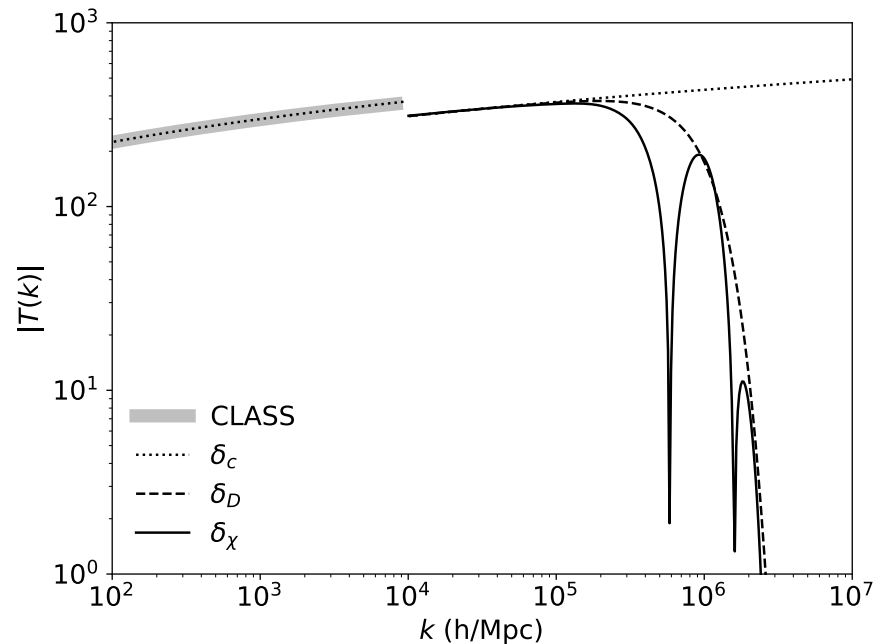
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$$\delta_\chi(\tau) \simeq \delta_\chi^r(\tau) + \frac{3}{2} f_c H_r^2 \int_{\gg \tau_d}^{\tau} d\tau' (\tau - \tau') \frac{a}{a_{\text{eq}}} \delta_\chi(\tau') \exp \left[-\frac{1}{2} (k\sigma(\tau - \tau'))^2 \right]$$

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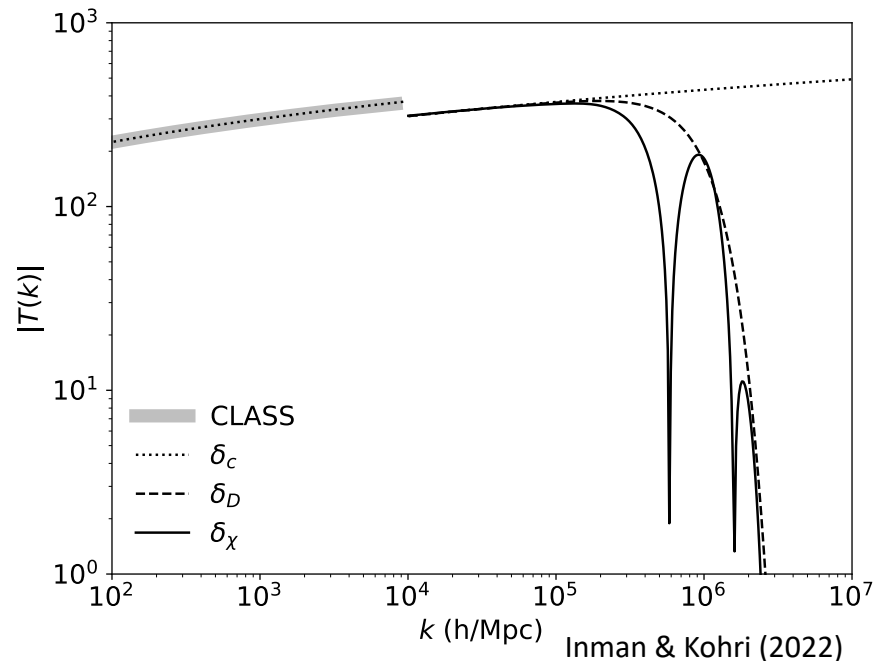


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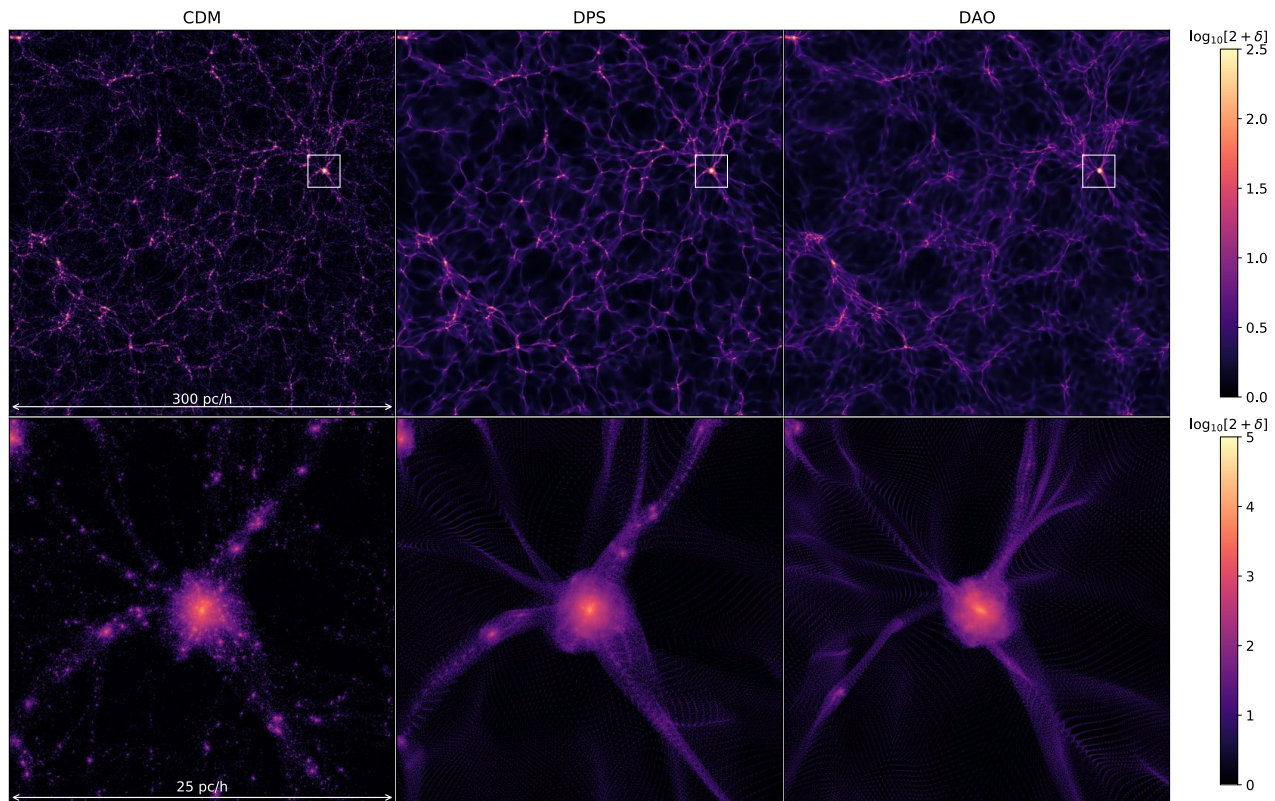
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- Not included in our calculation
 - Neutrino free-streaming
 - Electron/positron annihilation
- Use Hu & Sugiyama (1996) approximation on larger scales

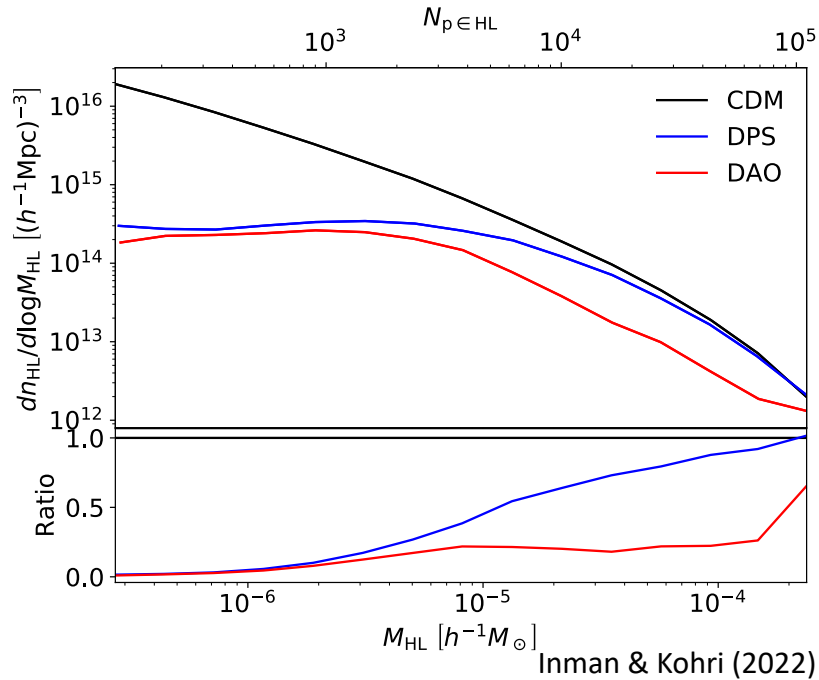


3. N-body calculation

- Cosmological simulations starting in the radiation era ($z=10^5$) with 2×768^3 particles in a $(300 \text{ pc}/h)^3$ volume using transfer functions computed from BFP equation and evolved to $z=300$.



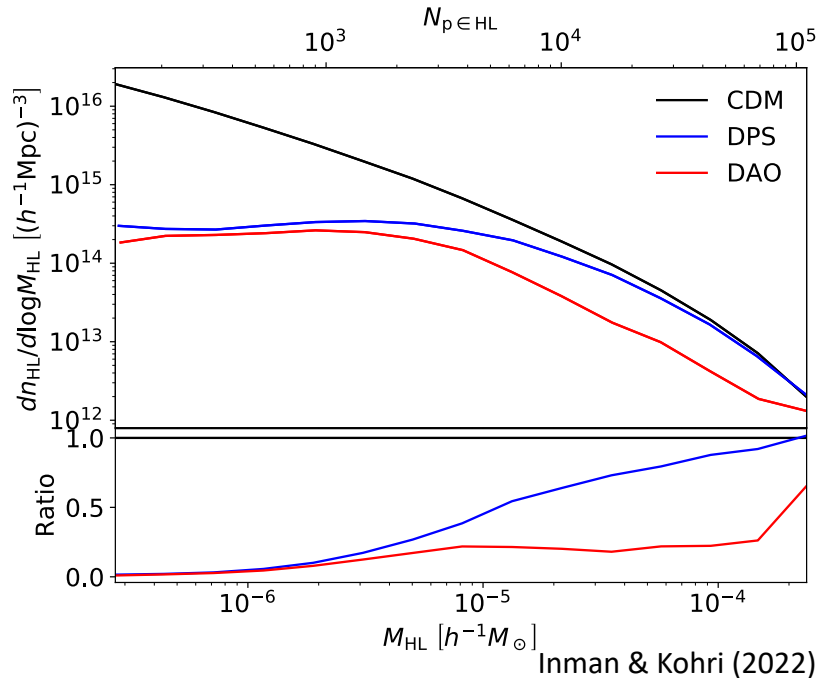
Halos at $z > 100$!



Halo mass function:

- Truncated at a scale characteristic of WIMP decoupling
- Evidence for spurious halos due to artificial fragmentation (Wang & White 2007)

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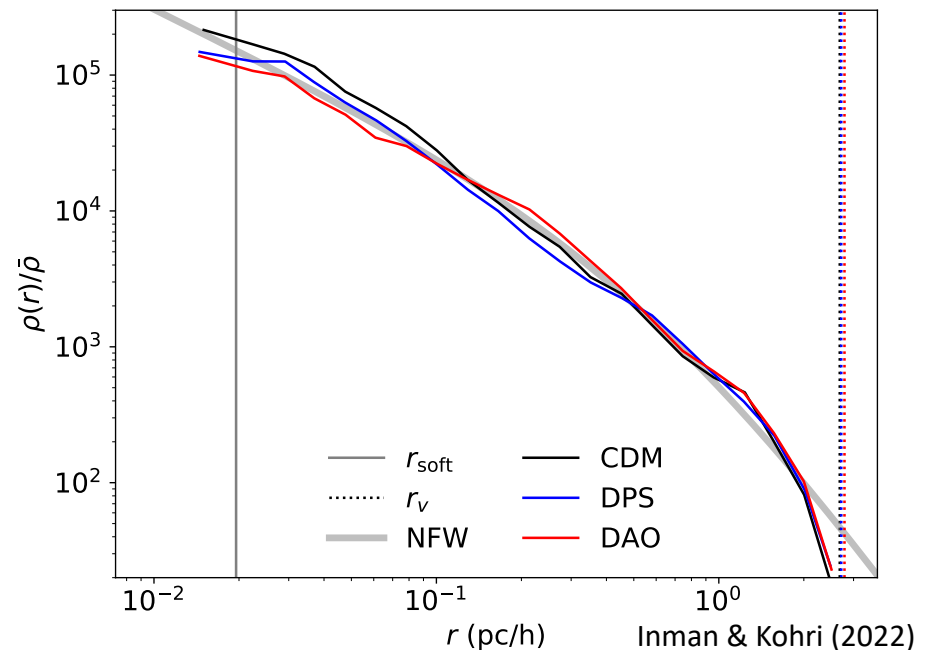


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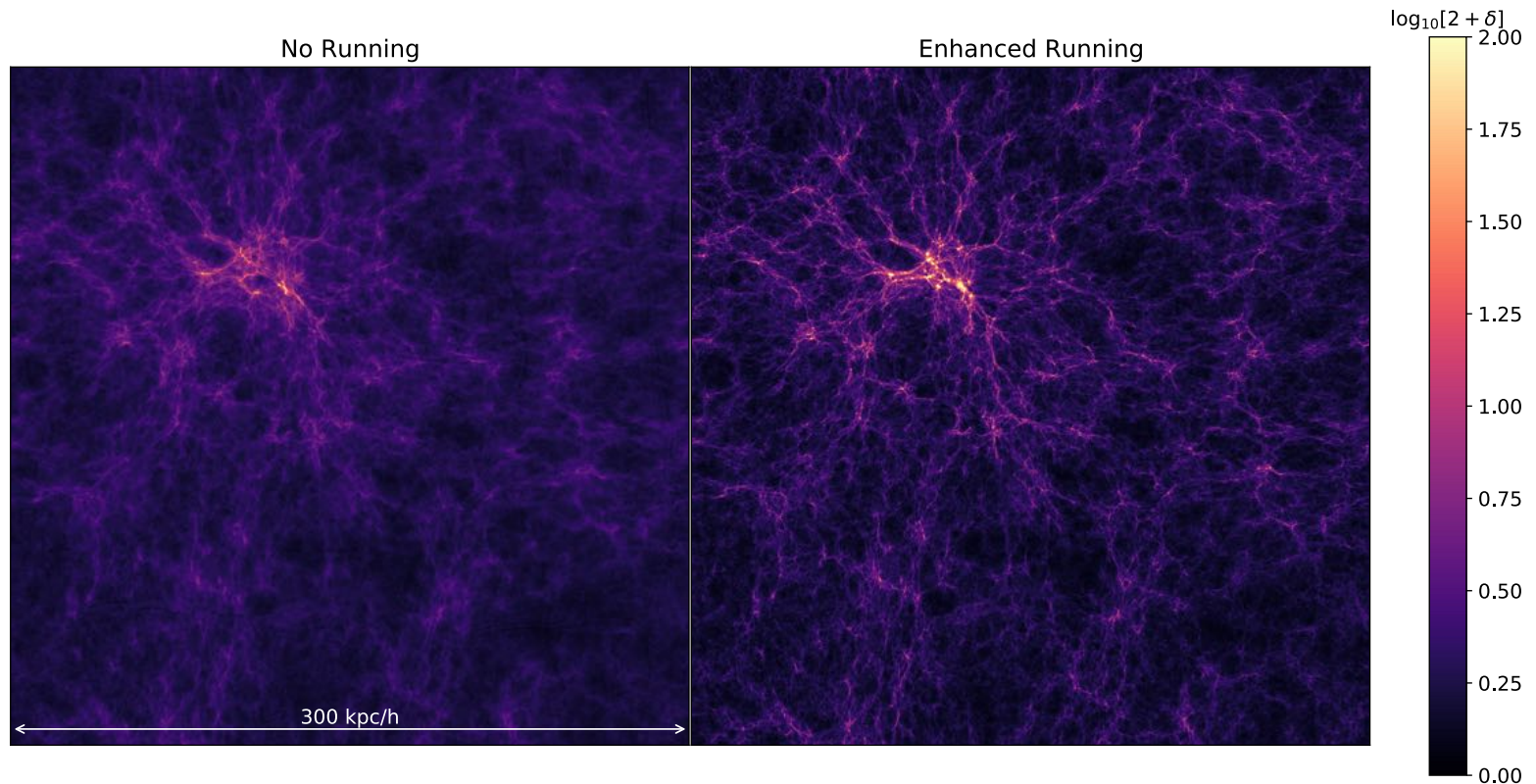
Halo density profile

- Most massive halo has NFW profile
 - Halos near minimum mass may not (e.g. Delos & White 2022)



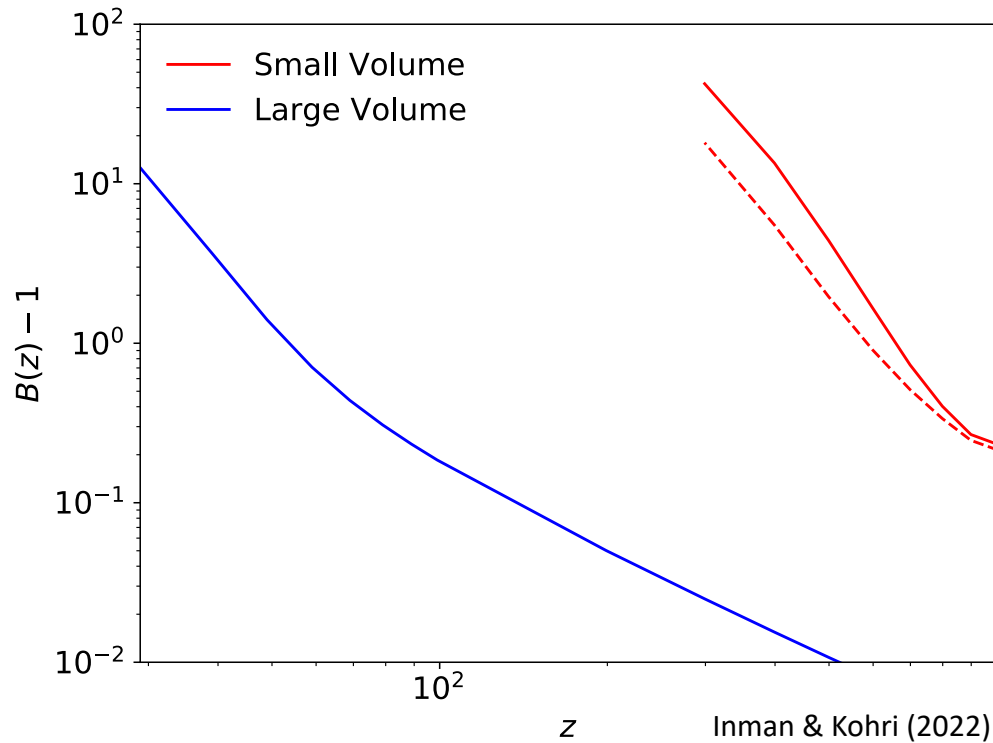
Halos at $z < 100$!

- Can't evolve small volumes to very low redshifts
- BUT can evolve larger volumes: $(300 \text{ kpc}/h)^3$ at $z=30$



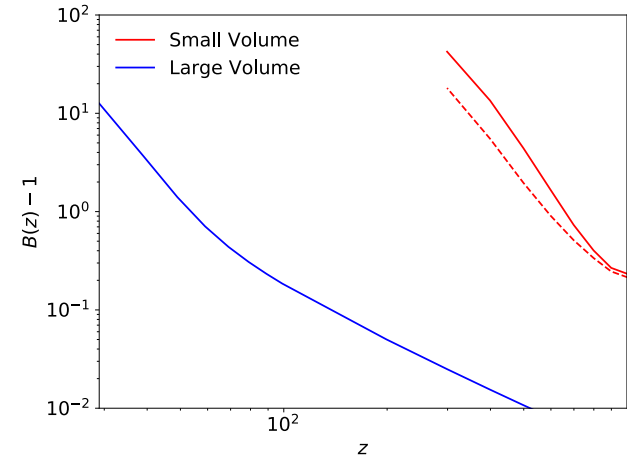
Potential consequences

- WIMP Annihilation:
 - (density)² process
 - Boost factor is large after matter-radiation equality



Potential consequences

- WIMP Annihilation:
 - (density)² process
 - Boost factor is large after matter-radiation equality



Inman & Kohri (2022)

- First galaxies and stars
 - Halos that host the first stars form earlier (see also Hirano et al. 2015)

	$M > 10^4 M_\odot h^{-1}$	$M > 10^5 M_\odot h^{-1}$	$M_{\max} (10^5 M_\odot h^{-1})$
Enhanced	436	8	3.2
Standard	8	0	0.4

Conclusion

- Halos can form throughout the cosmic dark ages!
 - Minimum halo mass set by particle microphysics
 - If enhancement is over a broad range of scales, can affect both lightest halos and halos that start cosmic dawn
- Next steps:
 - Numerical improvements:
 - Transfer functions → neutrinos free-streaming, electron/positron annihilation
 - N-body simulations → implement thermal velocities, deal with artificial fragmentation
 - Phenomenology & Analysis
 - Different primordial power spectra affect and associated phenomenology
 - When do baryons start being affected?
 - Constraining this scenario with CMB and first stars

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