Gauge invariant critical exponents at large charge

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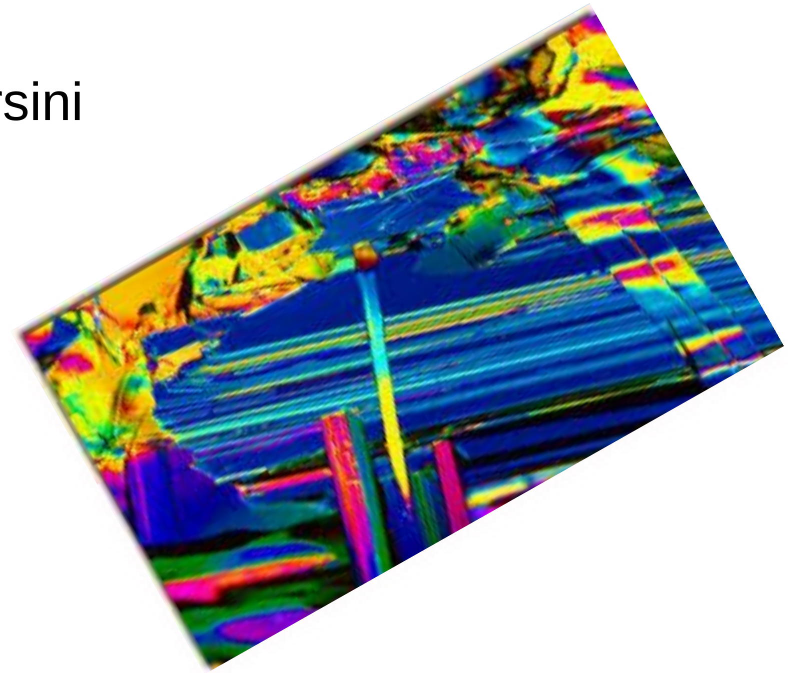

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Based on: O. Antipin, A. Bednyakov, JB, P. Panopoulos, A. Pikelner, 2210.10685 [hep-th].

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Superconductors

SUPERCONDUCTORS = materials with spontaneously broken U(1) local symmetry.

Described by the Euclidean Abelian Higgs model in d=3:

$$S = \int d^d x \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger D_\mu \phi + a(T - T_c) \bar{\phi}\phi + \frac{\lambda (4\pi)^2}{6} (\bar{\phi}\phi)^2 \right)$$

$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

EFT for the complex order parameter Φ of the superconducting phase transition.

Temperature T, Gauge coupling e, Quartic coupling λ.

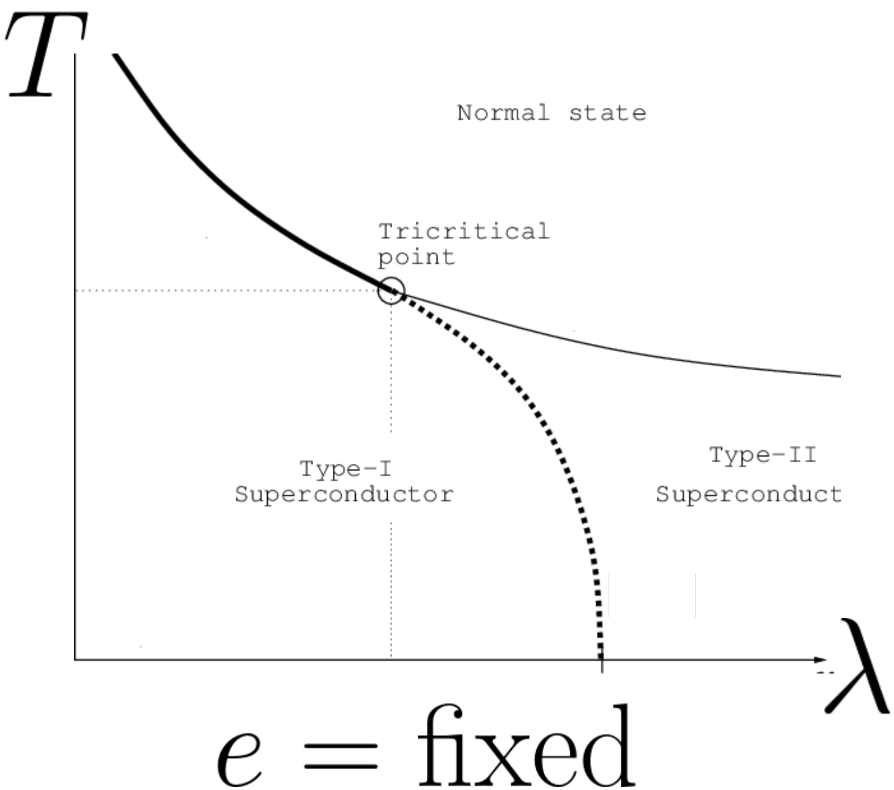
Approaches:

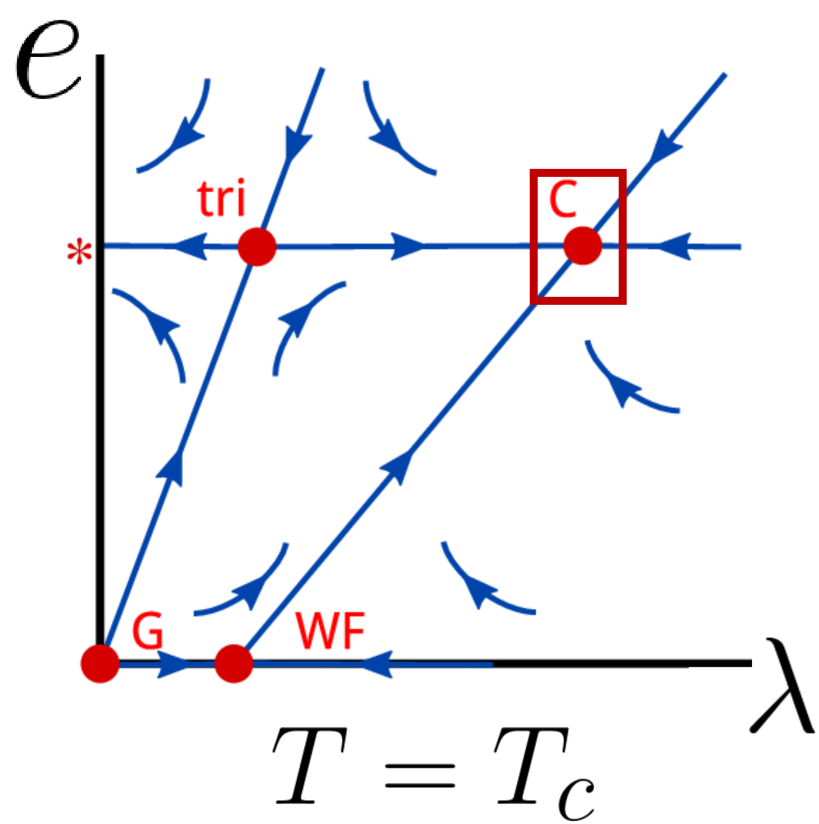
1) Lattice field theory in d=3.

2) Large N expansion in d=3.

3) ε-expansion in d=4-ε.

Phases and RG flow





Type II superconductors: second-order phase transition described by the conformal field theory (CFT) defined at the fixed point "C" of the renormalization group flow.

We study this CFT in d=4-ε, i.e. we take $T=T_c$, $\lambda=\lambda_c^*(\varepsilon)$, $e=e_c^*(\varepsilon)$.

Critical exponents

The exponents usually considered for the superconducting phase transition are:

ν	**ν'**	**α**
Correlation length ξ	London penetration depth Λ	Specific heat C

$$\xi, \Lambda, C = |T - T_c|^{-i}, \qquad i = \nu,\ \nu',\ \alpha$$

Related to the scaling dimension of the mass operator: $\Delta_{\phi\bar{\phi}}$

However, there is another important critical exponent **η** related to the scaling of the two-point function of the order parameter:

$$G(x_f - x_i) = \langle \bar{\phi}(x_f)\phi(x_i) \rangle = \frac{1}{|x_f - x_i|^{d-2+2\eta}}$$

However, this correlator is not gauge-invariant and vanishes due to the Elitzur's theorem (S. Elitzur 1975; no SSB of local symmetries).

<u>We want to define a gauge-invariant non-local order parameter.</u>

However the choice is not unique.

Non-local order parameter

SCHWINGER TYPE

$$G_S(x_f - x_i) = \langle \bar{\phi}(x_f) \exp\left(-ie\int dx^\mu A_\mu(x)\right)\phi(x_i)\rangle = \frac{1}{|x_f - x_i|^{d-2+2\eta_S}}$$

Insertion of a Wilson line on the shortest path connecting x_i to x_f.

DIRAC TYPE

$$G_D(x_f - x_i) = \langle \bar{\phi}(x_f) \exp\left(-i\, e\int d^d x J^\mu(x) A_\mu(x)\right)\phi(x_i)\rangle = \frac{1}{|x_f - x_i|^{d-2+2\eta_D}}$$

where

$$J_\mu = J'_\mu(z - x_f) - J'_\mu(z - x_i)\,, \qquad J'_\mu(z) = -\frac{\Gamma(d/2-1)}{4\pi^{d/2}}\partial_\mu \frac{1}{z^{d-2}}$$

From G_D I can define a non-local order parameter Φ_{NL} as

$$\phi_{NL}(x) \equiv e^{-ie\int d^d z J'_\mu(z-x) A^\mu(z)}\phi(x)$$

Φ_{NL} reduces to Φ in the Landau gauge $\partial^\mu A_\mu = 0$. (that is $\eta_D = \eta$ in the Landau gauge.)

Physical meaning: creation operator of a charged particle dressed with a coherent state of photons describing its Coulomb field.

The large-charge expansion

We want to study the issue of defining a gauge-invariant order parameter (and compute the associated critical exponent) from a new perspective.

- CFT (QFT) simplifies in certain limits when a small/large parameter exists.
- Our large parameter(s): conserved charge(s) of the symmetry group of the CFT:

LARGE-CHARGE EXPANSION FOR CFT OBSERVABLES
(e.g. critical exponents)

Initially developed for global symmetries.

[S. Hellerman, D. Orlando, S. Reffert, M. Watanabe (2015)]

Here applied to gauge symmetries.

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Diagrammatics

Conventional Feynman diagram expansion (in the number of loops):

$$\mathcal{O} = \sum c_i(Q, N, N_f, \ldots) g^i$$

Tree-level diagrams dominates

Large-N (number of colors) expansion in gauge theories

$$\mathcal{O} = \sum d_i(Q, N_f, \mathcal{A}, \ldots) \frac{1}{N^i} , \qquad \mathcal{A} \equiv gN$$

Planar diagrams dominates

Large-Nf (number of flavors) expansion

$$\mathcal{O} = \sum b_i(Q, N, \mathcal{A}, \ldots) \frac{1}{N_f^i} , \qquad \mathcal{A} \equiv gN_f$$

Bubble diagrams dominates

Large-charge expansion

$$\mathcal{O} = \sum a_i(N, N_f, \mathcal{A}, \ldots) \frac{1}{Q^i} , \qquad \mathcal{A} \equiv gQ$$

Fixed " 't Hooft-like coupling "

Quantum VS classical

Quantum physics *“classicalizes”* in the presence of large quantum numbers.

Hydrogen atom with infinite mass of the proton at fixed magnetic quantum number m:

QUANTUM ground state energy:

$$E_0^{\mathrm{QM}}(m) = -\frac{M_e \alpha^2}{2(m+1)^2}$$

CLASSICAL ground state energy:

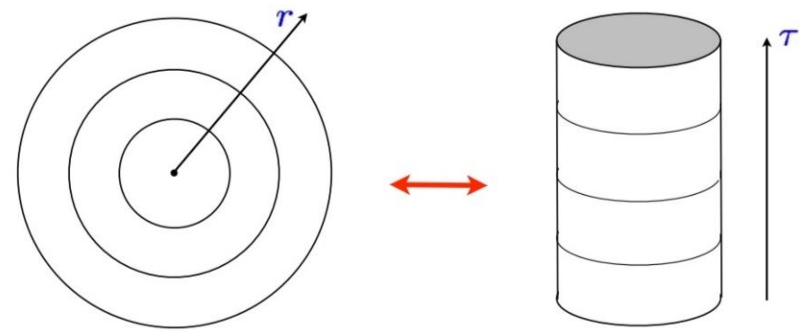
$$E_0^{cl}(m) = -\frac{M_e \alpha^2}{2m^2}$$

$$\lim_{m\to\infty} \left(E_0^{\mathrm{QM}}(m) - E_0^{\mathrm{cl}}(m)\right) = 0$$

LARGE-CHARGE EXPANSION =
SEMICLASSICAL EXPANSION

Map to the cylinder

$$\mathbb{R}^d \to \mathbb{R}\times S^{d-1}$$



The eigenvalues of the dilation charge (the scaling dimensions) become the energy spectrum on the unit r cylinder (state-operator correspondence)

$$\Delta = E$$

We compute the scaling dimension of operators with total charge Q and the minimal scaling dimension.

i.e. we compute the ground state energy on the cylinder.

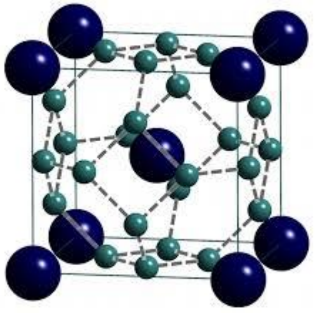
<u>LARGE-CHARGE EXPANSION = FINITE DENSITY QFT</u>

Selecting the order parameter

The approach automatically selects the scaling dimension Δ_Q of the lowest-lying operator with U(1) charge Q.
The Q=1 case corresponds to the scaling dimension of the non-local order parameter, i.e. the associated critical exponent.

OUR STRATEGY: we compare our results for Δ_Q with perturbative computations of the critical exponent associated with the various proposals (e.g. Schwinger, Dirac, …) for the order parameter and look for an agreement.

NB: For Q>1, Δ_Q defines a set of crossover (critical) exponents measuring the stability of the system (e.g. a superconductor) against anisotropic perturbations (e.g. their crystal structure).



Computation

To get the ground state energy on the cylinder we consider the matrix element of the evolution operator between arbitrary charge-Q states.

$$\langle Q|e^{-H(\tau_f-\tau_i)}|Q\rangle = \frac{1}{\mathcal{Z}}\int D\phi D\bar{\phi} DA e^{-QS_{\text{eff}}[\phi,\bar{\phi},A_\mu,\lambda Q,eQ]} \underset{\tau_f-\tau_i\to\infty}{=} e^{-\Delta_Q(\tau_f-\tau_i)}$$

$$\mathcal{S}_{\text{eff}} = \mathcal{S} + \mu Q + \frac{1}{8}\int d^d x\, (d-2)^2 \phi\bar{\phi}$$

Charge-fixing *Conformal coupling*

Q counts loops.
Computing the path integral semiclassically, we have

$$\Delta_Q = \sum_{k=-1} \frac{1}{Q^k}\Delta_k(Qe, Q\lambda)$$

Every Δ_k resums an infinite series of Feynman diagrams.

Leading order: Δ_{-1}

$$\mathcal{S} = \mathcal{S}(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 \mathcal{S}''(\phi_0) + \dots$$

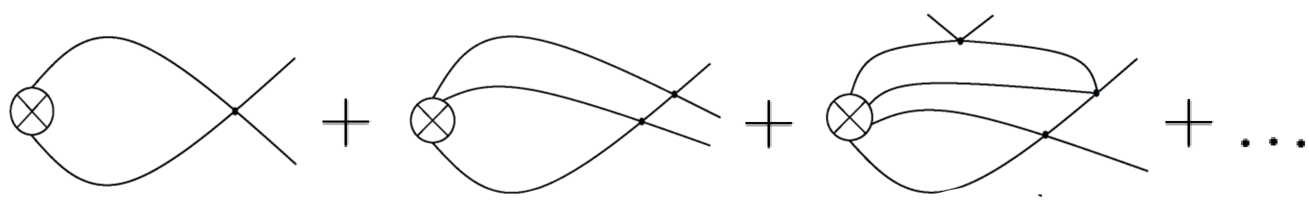
$$\Delta_Q = \sum_{k=-1} \frac{1}{Q^k} \Delta_k(Qe, Q\lambda)$$

Given by the effective action evaluated on the classical solution of the EOM

$$4\,\Delta_{-1} = \frac{3^{2/3}\left(x+\sqrt{-3+x^2}\right)^{1/3}}{3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}} + \frac{3^{1/3}\left(3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}\right)}{\left(x+\sqrt{-3+x^2}\right)^{1/3}} \qquad x \equiv 6\lambda Q$$

This classical result resums an infinite number of Feynman diagrams!

$$Q\Delta_{-1} =$$



$$\sim \lambda Q^2 \qquad \sim \lambda^2 Q^3 \qquad \sim \lambda^3 Q^4$$

Q counts the number of external legs.

λ counts the number of quartic vertices.

Next-to-leading order: Δ_0

$$\mathcal{S} = \mathcal{S}(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 \mathcal{S}''(\phi_0) + \dots$$

At NLO we have to compute a quadratic (Gaussian) path integral.

Δ_0 is given by the fluctuation determinant around the classical trajectory, i.e. by a sum of zero-point energies:

$$\Delta_0 = \frac{1}{2} \sum_{\ell=\ell_0}^{\infty} \sum_i d_\ell \, \omega_i(\ell) \ ,$$

ℓ labels the eigenvalues of the momentum which have degeneracy d_ℓ.
$\omega_i(\ell)$ are the dispersion relations of the spectrum.

Field	d_ℓ	$\omega_i(\ell)$	ℓ_0
B_i	$n_v(\ell)$	$\sqrt{J^2_{\ell(v)} + (D-2) + e^2 f^2}$	1
C_i	$n_s(\ell)$	$\sqrt{J^2_{\ell(s)} + e^2 f^2}$	1
$(c, \bar{c})$	$-2n_s(\ell)$	$\sqrt{J^2_{\ell(s)} + e^2 f^2}$	0
A_0	$n_s(\ell)$	$\sqrt{J^2_{\ell(s)} + e^2 f^2}$	0
ϕ	$n_s(\ell)$	$\sqrt{J^2_{\ell(s)} + 3\mu^2 - m^2 + \frac{1}{2}e^2 f^2 \pm \sqrt{\left(3\mu^2 - m^2 - \frac{1}{2}e^2 f^2\right)^2 + 4J^2_{\ell(s)}\mu^2}}$	0

Comparing to diagrammatics

By expanding the Δ_k's in the limit of small 't Hooft-like couplings (λQ, eQ) we obtain the conventional perturbative expansion. We independently computed the scaling dimension of Φ at the three-loop level and found an agreement for Q=1 in the Landau gauge.

$$\Delta_Q = Q\frac{d-2}{2} + \sum_{j=1}^{\infty} \gamma_Q^{(j-\text{loop})}(\lambda,\alpha) \quad \alpha \equiv \frac{e^2}{(4\pi)^2}$$

Red terms: Δ_{-1}

Blue terms: Δ_0

$$\gamma_Q^{(1)}(\lambda,\alpha) = \frac{\lambda}{3}Q^2 - Q\left(3\alpha + \frac{\lambda}{3}\right)$$

$$\gamma_Q^{(2)}(\lambda,\alpha) = -\frac{2\lambda^2}{9}Q^3 + \left(\alpha^2 - \frac{4\alpha\lambda}{3} + \frac{2\lambda^2}{9}\right)Q^2 + \left(\frac{7\alpha^2}{3} + \frac{4\alpha\lambda}{3} + \frac{\lambda^2}{9}\right)Q$$

$$\gamma_Q^{(3)}(\lambda,\alpha) = \ldots$$

Therefore, our computation selects the non-local order parameter of the Dirac type as the relevant order parameter for superconductors and generalizes the construction to arbitrary Q, i.e. Δ_Q is the scaling dimension of the non-local operators Φ_{NL}^Q.

Conclusions

1. We showed that the large-charge expansion can be applied also to gauge theories where the relevant gauge-invariant observables are in general non-local.

2. We explicitly showed that the non-local operators $\Phi_{NL}{}^{Q}$ are the lowest-lying operators with charge Q well-defined at criticality. In particular, this signals that Φ_{NL} is the relevant order parameter for superconductors.

3. As a byproduct, we provided novel results for the associated scaling dimensions (crossover critical exponents) Δ_Q .

Semiclassical expansion

$$\mathcal{L} = \partial\bar{\phi}\partial\phi + \lambda_0\left(\bar{\phi}\phi\right)^2$$

The operator Φ^Q carries U(1) charge Q.

$$< \bar{\phi}^Q(x_f)\phi^Q(x_i) > = Q^Q\frac{1}{\mathcal{Z}}\int D\phi D\bar{\phi}\,\bar{\phi}^Q(x_f)\phi^Q(x_i)e^{-Q\mathcal{S}}$$

$$\phi \to \phi\sqrt{Q}$$

We bring the field insertions into the exponent, obtaining

$$< \bar{\phi}^Q(x_f)\phi^Q(x_i) > = Q^Q\frac{1}{\mathcal{Z}}\int D\phi D\bar{\phi}\; e^{-Q\left[\int \partial\bar{\phi}\partial\phi + \frac{Q\lambda_0}{4}\left(\bar{\phi}\phi\right)^2 - \ln\bar{\phi}(x_f) - \ln\phi(x_i)\right]}$$

For large Q the path integral is dominated by the extrema of

$$\mathcal{S}_{eff} \equiv \int d^dx\left[\partial\bar{\phi}\partial\phi + \frac{Q\lambda_0}{4}\left(\bar{\phi}\phi\right)^2 - \ln\bar{\phi}(x_f) - \ln\phi(x_i)\right]$$

We can evaluate the integral via a saddle-point expansion
1/Q counts loops and is our expansion parameter.