The Landscape of Intersecting Brane Models

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Nikhef, Amsterdam
IPMU, Apr 21, 2009
Based on work in collaboration with Gabriele Honecker:

- *Complete Threshold Corrections for Intersecting Fractional D6-Branes*, [0905.xxxx].
Outline

Introduction

Overview
  Intersecting Brane Models
  Statistics

Details
  More on IBMs
  $T^6/\mathbb{Z}_6'$
  Spectrum
  Statistics of $T^6/\mathbb{Z}_6'$
  Explicit Example
  Correlations

Conclusions
One of the most important tasks of string theorists today is to make contact with particle physics.

In particular in view of the LHC we have to try our best to explain signatures beyond the Standard model.

Up to now there does not exist even one explicit construction that realizes the Standard Model (or one of the obvious extensions like the MSSM or simple GUTs) from a compactification of string theory.

Many different approaches:

- Heterotic compactifications (orbifolds, bundle constructions)  
  \[\text{[Nilles et al.; Ovrut et al.; Lukas et al.]}\]

- Gepner models
  \[\text{[Gepner; Schellekens et al.]}\]

- Type II Intersecting Brane Models
  \[\text{[Berkooz, Douglas, Leigh; Ibañez, Uranga et al.; Blumenhagen, Lüst et al.; Cvetić et al.]}\]

- F-theory
  \[\text{[Vafa et al.]}\]
The Landscape

- Up to now there is quite some evidence that (even after moduli stabilisation) string theory can lead to a huge\(^1\) amount of vacua.
  [Susskind; Schellekens; Busso, Polchinski; Douglas et. al.]

- Information about the structure and actual content (of theories) of this landscape has been obtained only in particular cases (easy to calculate).
  [Dijkstra, Huiszoon, Schellekens; Blumenhagen et. al.; Dienes, Lennek; Douglas, Taylor; Honecker, FG]

- New techniques to analyse the large amount of vacua are needed, in particular a statistical approach has been advocated.
  [Denef, Douglas]

\(^1\)A popular number is \(10^{500}\), but the actual number is not even known to be finite (although there are arguments for that).
The Landscape

- String theory is unique...
  in 10 dimensions.

- Going from 10 to 4 dimensions introduces a lot of possibilities, due to "sizes" and "shapes" of the six-dimensional compact space.
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- Going from 10 to 4 dimensions introduces a lot of possibilities, due to “sizes” and “shapes” of the six-dimensional compact space.
Questions

▶ Does the Landscape include the Standard Model?
▶ How does this Landscape look like? Which types of low energy effective theories are typical, which are rare?
▶ Are there common features and/or correlations between the properties of the low energy models?
▶ How can we make predictions for particle physics experiments?

Strategies

▶ Compute as many solutions as possible of low energy theories and look for common patterns.
▶ Look for correlations between properties of the low energy theories. This could even lead to predictions.
▶ Compare results of analyses at different “corners” of the landscape.
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Intersecting Brane Models

- Type IIA string theory
- Use intersecting D6-branes to generate gauge groups and matter content.
- Branes fill space-time and wrap three–cycles in the internal compact space.
- Compactification on $\mathbb{R}^{3,1} \times M$ to $\mathcal{N} = 1$ supersymmetric solutions in four dimensions.
- $M$ here toroidal orbifold $T^6/G$ with $G = \mathbb{Z}_2 \times \mathbb{Z}_2$, $G = \mathbb{Z}_6$, $G = \mathbb{Z}_6'$. 
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Standard model embedding

\[ U(3)_a \times U(2)_b/Sp(2)_b \times U(1)_c \times U(1)_d \]

\[ U(3)_a = SU(3)_{QCD} \times U(1)_a \]

\[ U(2)_b = SU(2)_w \times U(1)_b \]

\[ U(1)_Y: \text{appropriate (massless) combination} \quad Q_Y = \sum x_i Q_i \]
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Methods to do statistics
Obtain statistical results about 4d properties in large sets of models by
- complete computation of all possible solutions (impossible) or
- choosing subsets in parameterspace, preferably completely at random. Due to computational complexity a random choice is not always possible.

Caveat
The choice of subsets (bias) could influence the result. These unwanted correlations have to be avoided.
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Rank of the gauge group (visible+hidden sector)

Frequency distribution of the total rank $r$ of all models.
Rank of the gauge group (visible + hidden sector)

Frequency distribution of the total rank $\tau$ of all models.
Gauge group factors

Frequency distribution of the rank of gauge group factors.
Gauge group factors

Frequency distribution of the rank of gauge group factors.
Number of generations

Frequency distribution of standard models with $g$ generations.
Exotic matter vs. Higgs multiplets

Correlation between exotic matter and Higgs multiplets.
to be continued...
IBMs, part II

- Type IIA string theory on an orbifold background $\mathbb{R}^{3,1} \times T^6/(\Omega R \times G)$, $G$ being a discrete group.

- Orientifold projection $\mathcal{R}$ leads to $O6$-planes, wrapping 3-cycles $\Pi_{O6}$, RR charged.

- Introduce stacks of $N_i$ $D6$-branes wrapping cycles $\Pi_i$ to cancel RR tadpoles.

- Matter arises at intersections of $\Pi_i, \Pi'_i, \Pi_{O6}$.
Constraints

- **Supersymmetry**
  \[ \leadsto \text{Branes have to wrap calibrated cycles.} \]

- **Tadpole cancellation**
  \[ \sum_i N_i (\Pi_i + \Pi_i') = L \Pi_{O6}. \]

- **K-theory**
  \[ \sum_i N_i \Pi_i \circ \Pi_{Sp(2)} \equiv 0 \pmod{2}. \]

Spectrum

- **Closed strings**: \( \mathcal{N} = 1 \) sugra, axion-dilaton, \( h_{1,1}^- \) Kähler + \( h_{2,1} \) compl.str. moduli, \( h_{1,1}^+ \) vector multiplets

- **Open strings**: \( U(N) / SO(2N) / Sp(2N) \) gauge groups + charged matter
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Three-cycles

- **Fractional cycles** on $T^6/\mathbb{Z}_{2N}$: at $\mathbb{Z}_2$ fixed points on $T^4$. Continuous displacement and Wilson line on the remaining $T^2$ is described by chiral adjoints.

$$\Pi_{frac} = \frac{1}{2} (\Pi_{torus} + \Pi_{exc})$$

- **Rigid cycles** on $T^6/(\mathbb{Z}_{2N} \times \mathbb{Z}_{2M})$: no displacement, only discrete Wilson lines, no adjoints.

$$\Pi_{rigid} = \frac{1}{4} (\Pi_{torus} + \Pi_{exc})$$
Intersection numbers

The torus part of intersection numbers between two branes on $T^6/\mathbb{Z}_{2N}$ can be split into contributions from orientifold images:

$$
\Pi_{torus}^a \circ \Pi_{torus}^b = 2 \sum_{k=0}^{N-1} I^a(\theta^k b).
$$

The $\mathbb{Z}_2$ invariant intersections of the exceptional branes are given by

$$
\Pi_{exc}^a \circ \Pi_{exc}^b = 2 \sum_{k=0}^{N-1} I_{\mathbb{Z}_2}^a(\theta^k b).
$$

This allows to write down the full spectrum, including non-chiral matter.
Geometry of $T^6/\mathbb{Z}_6'$

- Orbifold action $\theta: z^i \rightarrow e^{2\pi iv_i} z^i$ with $v_i = \{1/6, 1/3, -1/2\}$.
- Two shapes of tori compatible with $\mathcal{R}$:
3-cycles

$$\Pi = \frac{1}{2} (\Pi_{torus} + \Pi_{exc}).$$

- 4-dim. basis of torus-cycles:

$$\rho_1 = \sum_{k=0}^{5} \theta^k \pi_{135}, \quad \rho_2 = \sum_{k=0}^{5} \theta^k \pi_{235}, \quad \rho_3 = \sum_{k=0}^{5} \theta^k \pi_{136}, \quad \rho_4 = \sum_{k=0}^{5} \theta^k \pi_{236},$$

which allows to expand

$$\Pi_{torus} = P\rho_1 + Q\rho_2 + U\rho_3 + V\rho_4.$$
3-cycles

\[ \Pi = \frac{1}{2} (\Pi_{\text{torus}} + \Pi_{\text{exc}}). \]

- 8-dim. basis of exceptional cycles combined from two-cycles wrapping \( \theta^3 \) fixed-points on \( T_1 \times T_3 \) and 1-cycles on \( T_2 \):

\[
\delta_j = \sum_{k=0}^{2} \theta^k (e_{4j} \otimes \pi_3), \quad \tilde{\delta}_j = \sum_{k=2}^{2} \theta^k (e_{4j} \otimes \pi_4),
\]

so a generic exceptional cycle is given by

\[
\Pi_{\text{exc}} = \sum_{j=1}^{4} \left( d_j \delta_j + e_j \tilde{\delta}_j \right).
\]
Tadpole conditions

- The conditions for torus and exceptional cycles factorise.
- The $O6$ planes contribute only to the torus part, which is geometry dependend. For $ABa$ e.g.

$$\sum_a N_a (P_a + Q_a) = 8, \quad \sum_a N_a (U_a - V_a) = 24.$$ 

- All branes contribute $\geq 0 \iff$ finiteness of solutions.
- Exceptional part amounts to algebraic equations for $d_a$ and $e_a$ coefficients of all exceptional branes.
Supersymmetry conditions

▶ Toroidal part

\[ \frac{R_1}{\sqrt{3}R_2} (P - Q) - (U + V) = 0, \quad (P + Q) - \frac{R_2}{\sqrt{3}R_1} (V - U) > 0. \]

▶ Exceptional part gives restriction on $\mathbb{Z}_2$ eigenvalues and Wilson lines on $T_1 \times T_3$.

K-theory conditions
can be shown to be always fulfilled for supersymmetric branes – no additional constraints – as in $\mathbb{Z}_6$ case.
Supersymmetry conditions

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**K-theory conditions**

can be shown to be always fullfilled for supersymmetric branes – no additional constraints – as in \(\mathbb{Z}_6\) case.
Spectrum

Chiral matter spectrum

| (Anti$_a$) | $\frac{1}{2}(I_{aa'} + I_{aO6})$ |
| (Sym$_a$) | $\frac{1}{2}(I_{aa'} - I_{aO6})$ |
| (N$_a$, N$_b$) | $I_{ab}$ |
| (N$_a$, N$_b$) | $I_{ab'}$ |

Full matter spectrum

| (Adj$_a$) | $1 + \frac{1}{4} \sum_{k=0}^{N-1} I_a(\theta^k a) + \sum_{k=1}^{N-1} I_a(\theta^k a) + I_{Z2}^{a(\theta^k a)} + I_{\Omega R \theta^{-k}} + I_{\Omega R \theta^{-k+N}}$ |
| (Anti$_a$) | $\frac{1}{4} \sum_{k=0}^{N-1} I_a(\theta^k a') + I_{Z2}^{a(\theta^k a')} + I_{\Omega R \theta^{-k}} + I_{\Omega R \theta^{-k+N}}$ |
| (Sym$_a$) | $\frac{1}{4} \sum_{k=0}^{N-1} I_a(\theta^k a') + I_{Z2}^{a(\theta^k a')} - I_{\Omega R \theta^{-k}} - I_{\Omega R \theta^{-k+N}}$ |
| (N$_a$, N$_b$) | $\frac{1}{2} \sum_{k=0}^{N-1} I_a(\theta^k b) + I_{Z2}^{a(\theta^k b)}$ |
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### Spectrum

#### Chiral matter spectrum

| \((\text{Anti}_a)\) | \(\frac{1}{2}(I_{aa'} + I_{aO6})\) |
| \((\text{Sym}_a)\) | \(\frac{1}{2}(I_{aa'} - I_{aO6})\) |
| \((N_a, \overline{N}_b)\) | \(I_{ab}\) |
| \((N_a, \overline{N}_b)\) | \(I_{ab'}\) |

#### Full matter spectrum

| \((\text{Adj}_a)\) | \(1 + \frac{1}{4} \sum_{k=1}^{N-1} I_a(\theta^k a) + I_{a}^{\mathbb{Z}_2}\theta^{-k} + I_{a}^{\mathbb{Z}_2}\theta^{-k+N}\) |
| \((\text{Anti}_a)\) | \(\frac{1}{4} \sum_{k=0}^{N-1} I_a(\theta^k a') + I_{a}^{\mathbb{Z}_2}\theta^{-k} + I_{a}^{\mathbb{Z}_2}\theta^{-k+N}\) |
| \((\text{Sym}_a)\) | \(\frac{1}{4} \sum_{k=0}^{N-1} I_a(\theta^k a') - I_{a}^{\mathbb{Z}_2}\theta^{-k} - I_{a}^{\mathbb{Z}_2}\theta^{-k+N}\) |
| \((N_a, \overline{N}_b)\) | \(\frac{1}{2} \sum_{k=0}^{N-1} I_a(\theta^k b) + I_{a}^{\mathbb{Z}_2}\theta^{-k}\) |
| \((N_a, N_b)\) | \(\frac{1}{2} \sum_{k=0}^{N-1} I_a(\theta^k b') + I_{a}^{\mathbb{Z}_2}\theta^{-k'}\) |
Number of solutions to constraining equations depends on torus shapes (first two tori on horizontal axis, third torus color-coded: blue/red = $a/b$).

Inclusion of exceptional cycles increases the number solutions exponentially – as in $\mathbb{Z}_6$ case.

$\mathbb{A}/\mathbb{B}$ and $\mathbb{A}/\mathbb{B}$ are equivalent.

$O(10^{23})$ inequivalent (?) solutions.
Distribution shows same behaviour for torus cycles as $\mathbb{Z}_2 \times \mathbb{Z}_2$ – can be fitted to a Gaussian with maximum at $\approx \sum L_i/2$.

Exceptional cycles enhance large ranks – due to the fact that large rank $\sim$ large number of branes – exponential enhancement.
Single gauge group factors

Distribution scales for bulk models $\sim (L + 1 - N) \frac{L^4}{N^2}$ - as found for $\mathbb{Z}_2 \times \mathbb{Z}_2$ by Douglas, Taylor.

Inclusion of exceptional cycles gives $\sim n_{\text{e}}^{L+1-N} \frac{L^4}{N^2}$ - exponential fall-off.
Standard models

- Blue/red bars: massive/massless hypercharge.
Standard models

- Comparison with $\mathbb{Z}_6$: two- and three-generation models with massless hypercharge exist.
- Fundamentally different spectra.
Complex structure dependence

- Complex structure parameter: $\rho = \frac{\sqrt{3} R_2}{2 R_1}$.
- One/two/three generation models = blue/red/yellow.
Absolute number of chiral exotics

\[ \xi = \sum_{v, h} \left| \chi^{vh} - \chi^{v'h} \right|. \]

\(\mathcal{O}(10^7)\) three generation models without chiral exotics.
This gives an upper limit on Higgs families, it could also be non-chiral lepton pairs (can be differentiated by B-L charge, if $U(1)_{B-L}$ is massless).

Correlation between number of exotics and number of Higgs.

Example with 9 ($H_u + H_d$).
Gauge couplings

The coupling $g_a$ for a gauge group factor $G_a$ at $\mu < M_{string}$ is given by

$$\frac{8\pi^2}{g_a^2(\mu)} = \frac{8\pi^2}{g_{a,string}^2} + \frac{b_a}{2}\ln\left(\frac{M_{string}^2}{\mu^2}\right) + \frac{\Delta_a}{2}.$$ 

Contributions:

- **Tree level** ($\kappa_a = 1$ for $SU(N)$, 2 for $SO/Sp(2N)$)

  $$\frac{1}{\alpha_{a,string}} = \frac{4\pi}{g_{a,string}^2} = \frac{M_{Planck}}{2\sqrt{2}\kappa_a M_{string}} \frac{V_a}{\sqrt{V_6}},$$

At the orbifold point the volumes of the exceptional cycles are zero, so only toroidal contribution: $V_a = c L_1^a \cdot L_2^a \cdot L_3^a$.  

Running

Due to **massless** string modes charged under $G_a = SU(N_a)$ is encoded in the beta function coefficient $b_a$ with

$$b_{SU(N_a)} = -N_a \left( 3 - \varphi_{\text{Adj}_a} \right) + \sum_{b \neq a} \frac{N_b}{2} \left( \varphi^{ab} + \varphi^{ab'} \right)$$

$$+ \frac{N_a - 2}{2} \varphi_{\text{Anti}_a} + \frac{N_a + 2}{2} \varphi_{\text{Sym}_a}.$$ 

Threshold corrections

- $\Delta_a$ due to charged massive string modes.
- Can be computed using the **background field method**: Computation of one-loop vacuum energy of a string quantized in magnetic background $B$. The thresholds can then be obtained from $B^2$-term in expansion.
- Is expected to be small, explicit calculation: work in progress...
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Gauge couplings

- Tree level coupling ratios $\alpha_s / \alpha_w$ independent of scales.
- Very few different cases occur, always $\alpha_s \neq \alpha_w$. 
Models without hidden sector exist with 18 or 21 Higgs families.

All of them have a massless $B - L$, chiral spectra look identical - are these really independent models?
Example

Geometric setup of example model with MSSM spectrum.

\[ U(1)_Y = \frac{1}{6} U(1)_B + \frac{1}{2} (U(1)_S + U(1)_A) \]
Example

Chiral matter

\[ [C] = 3 \times \left( \begin{array}{c}
(3, 2)_{1/6, 1/3}^{(0, 0)} + (\overline{3}, 1)_{1/3, -1/3}^{(1, 0)} + (\overline{3}, 1)_{-2/3, -1/3}^{(-1, 0)} + (1, 1)_{1, 1}^{(1, 1)} + (1, 1)_{0, 1}^{(-1, 1)} \\
+ 2 \times (1, 2)_{-1/2, -1}^{(0, -1)} + (1, 2)_{1/2, 1}^{(0, 1)} + 6 \times (1, 2)_{-1/2, 0}^{(-1, 0)} + 6 \times (1, 2)_{1/2, 0}^{(1, 0)} + 3 \times (1, 1^A)_{0, 0}^{(0, 0)}
\end{array} \right) \]

\[ \equiv 3 \times \left( Q_L + d_R + u_R + e_R + \nu_R + 2 \times L + \overline{L} \right) + 18 \times \left( H_d + H_u \right) + 9 \times S, \]

Non-chiral matter

\[ [V] = 2 \times (8, 1)_{0, 0}^{(0, 0)} + 10 \times (1, 3)_{0, 0}^{(0, 0)} + 26 \times (1, 1)_{0, 0}^{(0, 0)} + \left( \begin{array}{c}
(3, 2)_{1/6, 1/3}^{(0, 0)} \\
+ 3 \times (\overline{3}, 1)_{1/3, 2/3}^{(0, 1)} + 3 \times (\overline{3}, 1)_{-2/3, -4/3}^{(0, -1)} + (3 - x + 1_m) \times (1, 1)_{1, 0}^{(2, 0)} + (1 + 2_m) \times (\overline{3}_A, 1)_{1/3, 2/3}^{(0, 0)} \\
+ (9 + 1_m) \times (1, 3)_{0, 0}^{(0, 0)} + 2_m \times (1, 2)_{-1/2, 0}^{(-1, 0)} + 2_m \times (1, 2)_{1/2, 0}^{(1, 0)} + 2_m \times (1, 2)_{-1/2, -1}^{(0, -1)} \\
+ 1_m \times (1, 2)_{1/2, 1}^{(0, 1)} + 1_m \times (1, 1^A)_{0, 0}^{(0, 0)} + 1_m \times (1, 1)^{(1, -1)}_{0, 1} + 1_m \times (1, 1)^{(1, 1)}_{1, 1} + c.c.
\end{array} \right). \]

Remarks

- Massless \( U(1)_Y \) and \( U(1)_{B-L} = \frac{1}{3} U(1)_a + U(1)_d \).
- "\( m \)" reps. become massive after brane displacement.
- Since \( U(1)_b \) acquires a mass absorbing a neutral closed string field, \((H_u + H_d), (L + \overline{L})\) and \( S \) are vector-like.
- \( \mu \)-term perturbatively forbidden, as well as \( \nu_R^2 \) and \( L^2 H_u^2 \).
Example

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\[ [C] = 3 \times \left[ (3, 2)^{(0,0)}_{1/6,1/3} + (\bar{3}, 1)^{(1,0)}_{1/3,-1/3} + (\bar{3}, 1)^{(-1,0)}_{-2/3,-1/3} + (1, 1)^{(1,1)}_{1,1} + (1, 1)^{(-1,1)}_{0,1} \right. \\
\left. + 2 \times (1, 2)^{(0,-1)}_{-1/2,-1} + (1, 2)^{(0,1)}_{1/2,1} + 6 \times (1, \bar{2})^{(-1,0)}_{-1/2,0} + 6 \times (1, \bar{2})^{(1,0)}_{1/2,0} + 3 \times (1, 1_A)^{(0,0)}_{0,0} \right] \]
\[ \equiv 3 \times \left[ Q_L + d_R + u_R + e_R + \nu_R + 2 \times L + \bar{L} \right] + 18 \times \left[ H_d + H_u \right] + 9 \times S. \]

Non-chiral matter
\[ [V] = 2 \times (8, 1)^{(0,0)}_{0,0} + 10 \times (1, 3)^{(0,0)}_{0,0} + 26 \times (1, 1)^{(0,0)}_{0,0} + \left[ (3, 2)^{(0,0)}_{1/6,1/3} \right. \\
\left. + 3 \times (\bar{3}, 1)^{(0,1)}_{1/3,2/3} + 3 \times (\bar{3}, 1)^{(0,-1)}_{-2/3,-4/3} + (3 - x + 1_m) \times (1, 1)^{(2,0)}_{1,0} + (1 + 2_m) \times (\bar{3}A, 1)^{(0,0)}_{1/3,2/3} \right. \\
\left. + (9 + 1_m) \times (1, 3)^{(0,0)}_{0,0} + 2_m \times (1, \bar{2})^{(-1,0)}_{-1/2,0} + 2_m \times (1, \bar{2})^{(1,0)}_{1/2,0} + 2_m \times (1, 2)^{(0,-1)}_{-1/2,-1} \right. \\
\left. + 1_m \times (1, 2)^{(0,1)}_{1/2,1} + m \times (1, 1_A)^{(0,0)}_{0,0} + 1_m \times (1, 1)^{(1,-1)}_{0,-1} + 1_m \times (1, 1)^{(1,1)}_{1,1} + c.c. \right]. \]

Remarks

- Massless \( U(1)_Y \) and \( U(1)_{B-L} = \frac{1}{3} U(1)_a + U(1)_d \).

- "\( m \)" reps. become massive after brane displacement.

- Since \( U(1)_b \) acquires a mass absorbing a neutral closed string field, \( (H_u + H_d) \), \( (L + \bar{L}) \) and \( S \) are vector-like.

- \( \mu \)-term perturbatively forbidden, as well as \( \nu_R^2 \) and \( L^2 H_u^2 \).
Observables for correlations
Number of bifundamental representations

\[ \Delta^\pm := \#(N_a, \overline{N}_b) \pm \#(N_a, N_b) \]

and number of (Anti-)Symmetric representations \( \chi^{Sym/Anti} \) for different constructions.

Choice of samples
Different strategies to obtain statistical results are used:

- \( T^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \) (\( O(10^{10}) \) models): explicit cutoff in the parameter space.
- \( T^6/\mathbb{Z}_6 \) (\( O(10^{28}) \)) and \( T^6/\mathbb{Z}_6' \) (\( O(10^{23}) \) models): random samples of different sizes.
- Gepner models: subset of models containing a realisation of the standard model without tadpole cancellation (biased subset).
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[Dijkstra]
Correlation between number of bifundamental matter representations on.
Top left to down right: $T^6 / \mathbb{Z}_2 \times \mathbb{Z}_2$, $T^6 / \mathbb{Z}_6$, $T^6 / \mathbb{Z}'_6$, Gepner models.
Correlation between number of symmetric and antisymmetric representations.
Top left to down right: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, $T^6/\mathbb{Z}_6$, $T^6/\mathbb{Z}'_6$, Gepner models.
Conclusions

- Within this (very limited) study an explicit realisation of just the MSSM is very unlikely.
- Including a hidden sector (interesting for susy breaking) allows for MSSM constructions, however all with large number of Higgs multiplets.
- Many unsolved issues: Complete moduli stabilization, SUSY breaking, Yukawa couplings, ...
- Statistically three generations are less likely than one - exponential falloff in the number of generations that has also been observed in other constructions..
- Correlations in the matter content and couplings do occur, but how generic these are is unclear - better comparison to other constructions is needed.