Kink dynamics and quantum simulation of supersymmetric lattice Hamiltonians

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Research Center for Quantum Software

Minář - van Voorden – KjS, arXiv:2005.00607, PRL 2021

Main messages

supersymmetry as a tool for analyzing strongly correlated lattice fermions

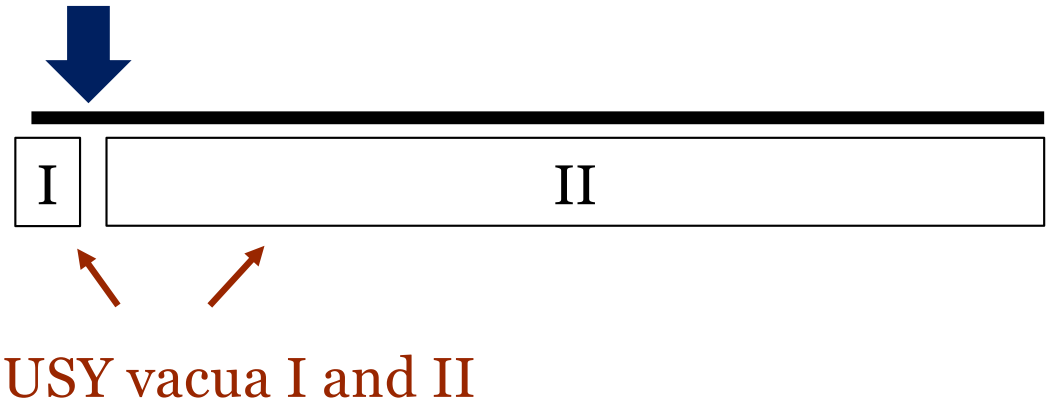
- (staggered) M_1 model in 1D as concrete example
- two SUSY vacua: kinks & skinks as elementary excitations
- kink dynamics – focus on arrival profiles

quantum simulation: atoms in optical lattice interacting via Rydberg-dressed potential

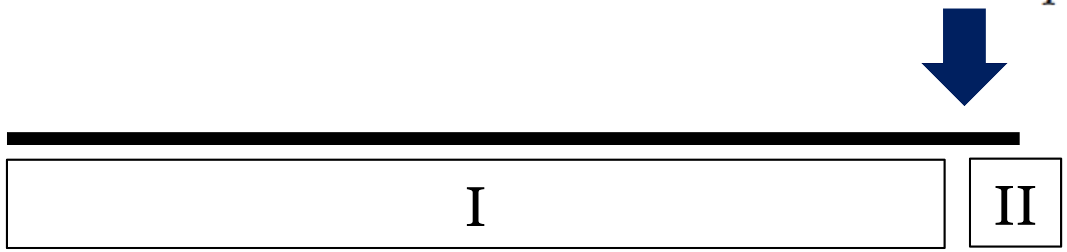
- protocol for observing kink and skink arrival profiles: manifestation of lattice supersymmetry

Proposed (experimental) protocol

t=0 : inject kink $|K_1>$
at left end of system



$$p(t) \equiv \langle K_{l+1} | K_1(t) \rangle$$

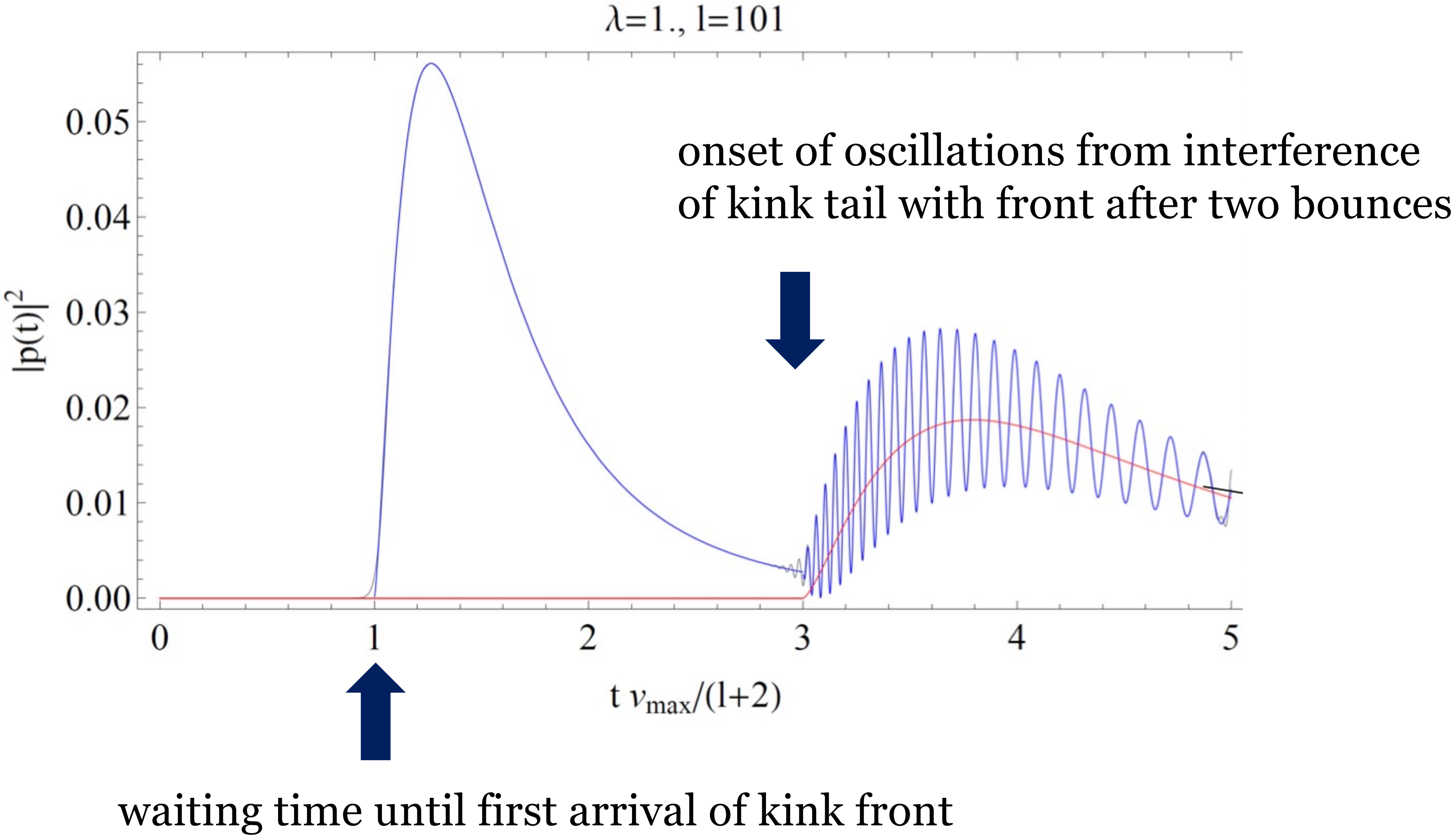


observable *p(t)* at time *t*:
overlap with kink at right end

Kink arrival profile: theory

theory curve (blue)

$$p(t) \equiv \langle K_{l+1} | K_1(t) \rangle$$

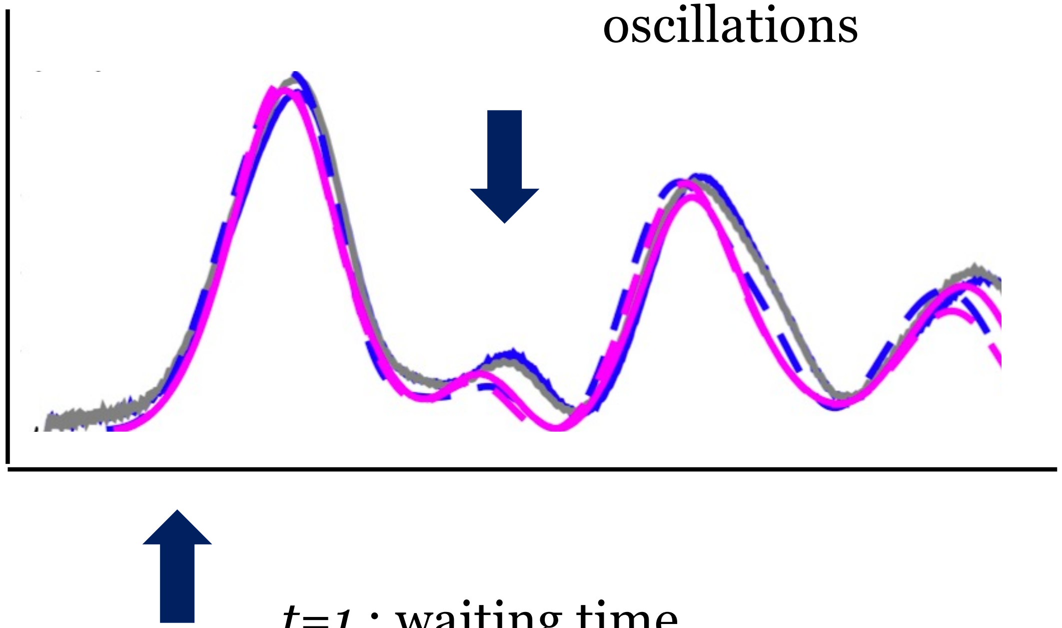


Kink arrival profile: simulation

simulation

- 10-site chain
- plot of observables tracking kink arrival profile $|p(t)|^2$

t=3 : onset of oscillations



t=1 : waiting time

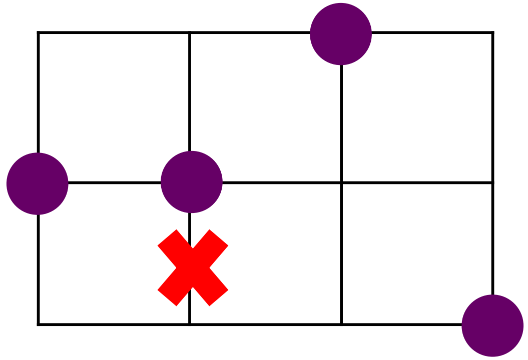
blue: kink dynamics
magenta: skink dynamics

→ Supersymmetry in action

dashed lines:
supersymmetric Hamiltonian
solid lines:
Rydberg atom simulator

→ Quantum simulation in action

Plan for rest of talk



- What are these supersymmetric lattice Hamiltonians?
- M_1 model in 1D as concrete example: vacua, staggering, kinks & dynamics
- Quantum simulation of 1D M_1 model: atoms in optical lattice interacting via Rydberg-dressed potential

Lattice models with *N=2* supersymmetry

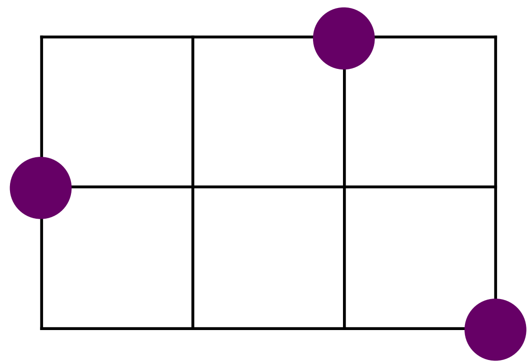
QM with *N*=2 supersymmetry

$$Q^2 = 0, \qquad (Q^\dagger)^2 = 0$$

$$[Q,H] = 0, \qquad H = \{Q,Q^\dagger\}, \qquad [Q^\dagger,H] = 0$$

Lattice fermions (spin-less)

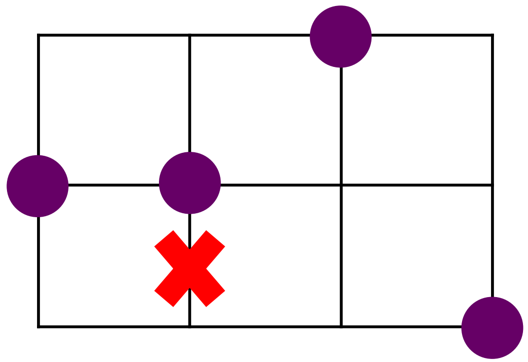
$$\{c_i, c_j^\dagger\} = \delta_{ij}, \qquad i,j \in \Lambda$$



M_1 model

configurations:

lattice fermions with nearest neighbor exclusion



supercharge

takes out particle where possible

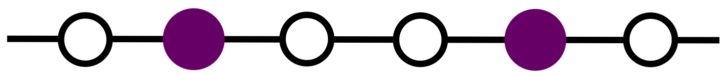
$$Q^{\mathrm{M}_1} = \sum_i c_i P_i, \qquad P_i = \prod_{<ij>} (1 - c_j^\dagger c_j)$$

Fendley-KjS-de Boer 2003

M_1 model on 1D lattice

configurations

lattice fermions with nearest neighbor exclusion



supercharge and Hamiltonian

$$Q^{\mathrm{M}_1} = \sum_i (1 - n_{i-1}) c_i (1 - n_{i+1}), \qquad n_i = c_i^\dagger c_i$$

n.n. exclusion

$$H^{\mathrm{M}_1} = \sum_i \left[(1 - n_{i-1}) c_i^\dagger c_{i+1} (1 - n_{i+2}) + \text{h.c.} \right] + \sum_i n_{i-1} n_{i+1} - 2F + L$$

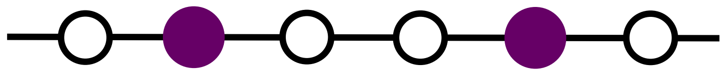
hopping

n.n.n. repulsion

M_1 model on 1D lattice

configurations

lattice fermions with nearest neighbor exclusion



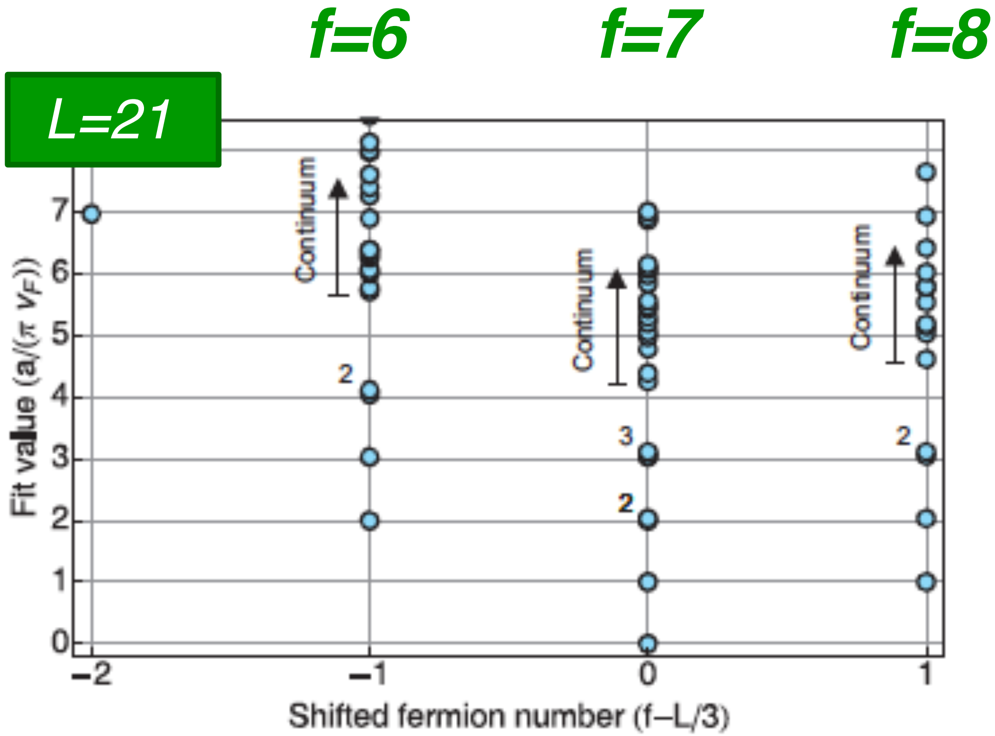
$$H^{M_1} = \sum_i \left[(1 - n_{i-1}) c_i^\dagger c_{i+1} (1 - n_{i+2}) + \text{h.c.} \right] + \sum_i n_{i-1} n_{i+1} - 2F + L$$

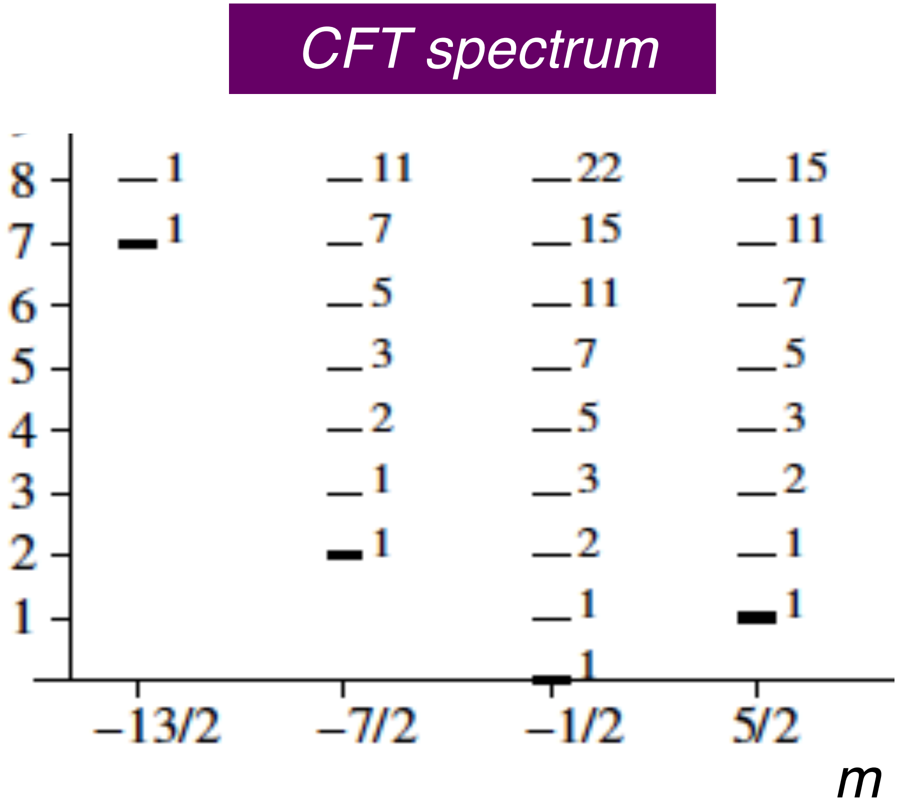
Analysis (Bethe Ansatz, mapping to XXZ model, numerics) shows that M_1 model is **critical**, with universal low-energy behavior described by $k=1$ minimal model of $N=2$ SCFT

Fendley-KjS-de Boer 2003

M_1 model on 1D lattice

Huijse, 2010





Ramond-sector affine *U(1)* modules built on charge *m* vertex operator V_m

$m = 3f - L - ½$

M_1 model: probing the supersymmetry

Challenges

- Probe the M_1 model in a way that explicitly shows the supersymmetry

→ key is to probe the dynamics of the kinks and their superpartners (skinks)

Basic structure of SUSY spectra

- $\boldsymbol{E \geq 0}$ for all states

- $\boldsymbol{E > 0}$ states are paired into **doublets**

$$\{|\psi\rangle, Q^{\dagger}|\psi\rangle\}, \quad Q|\psi\rangle = 0$$

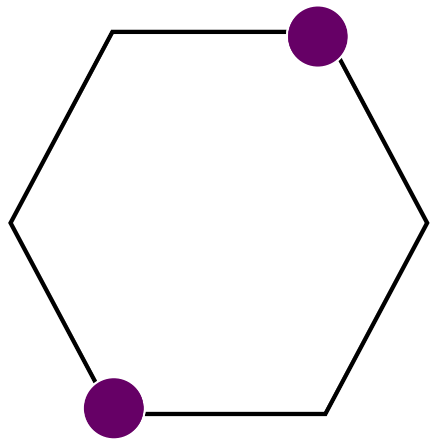
- $\boldsymbol{E = 0}$ iff a state is a **singlet** under supersymmetry

$$Q|\psi_{\mathrm{gs}}\rangle = 0, \quad Q^{\dagger}|\psi_{\mathrm{gs}}\rangle = 0$$

- # of $\boldsymbol{E=0}$ groundstates lower bounded by Witten index W

M_1 model on 6 site chain

$$W = \mathrm{Tr}\left[(-1)^F\right]$$



$F = 0$: 1 state

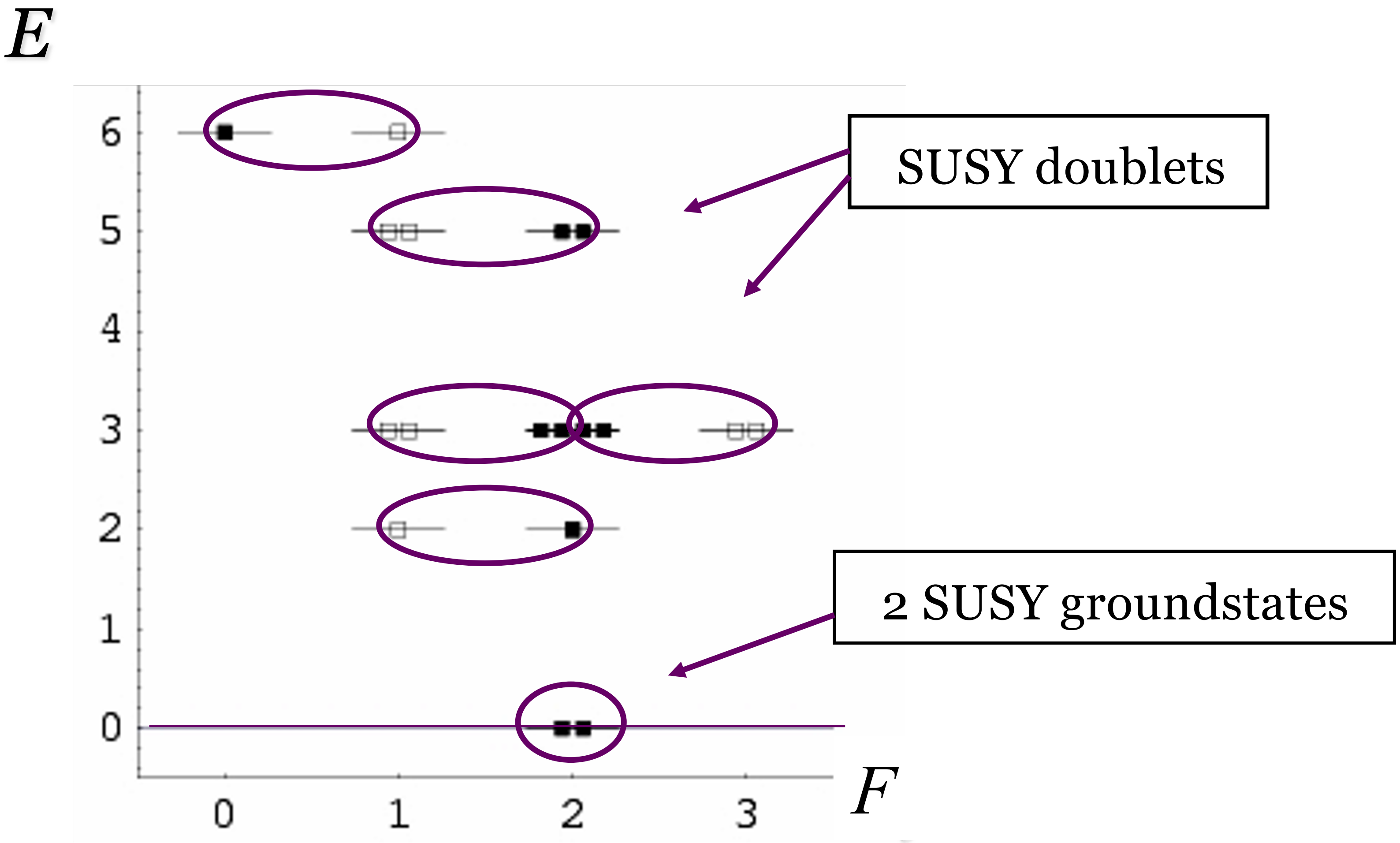
$F = 1$: 6 states

$F = 2$: 9 states

$F = 3$: 2 states

$$\Rightarrow W = 1 - 6 + 9 - 2 = 2$$

M_1 model, 6 site chain, W=2



M_1 model on 1D lattice

- For closed chain, $L=3l$ sites, the Witten index is ± 2; there are two groundstates I and II, both have particle number $F=l$ (1/3 filling)

- For open chain, $L=3l+1$, the Witten index is 0; there are no supersymmetric groundstates; supersymmetry is broken

M_1 model with staggering

- to break criticality, one can **stagger** the amplitudes in the supercharge with position dependent factors λ_i
- staggering **does not** affect the Witten index
- for λ_i periodic with period 3 the staggered model is still integrable, we choose

$$\ldots\ \lambda\ 1\ 1\ \lambda\ 1\ 1\ \lambda\ 1\ 1\ \ldots$$

M_1 model: staggering ... λ 1 1 λ 1 1 λ ...

- For extreme staggering, $\lambda \rightarrow 0$, the susy groundstates are

  I : ... 0( )0( )0 ... with ( )=10-01

  II : ... 1001001 ...

- The lowest energy state for $L=3l+1$, staggering [1 1 λ ... 1 1 λ1] with $F=l$ is of the form

  $|K_i>$: [( )0( )0 ... ( )0001 ... 0010]

  kink at location $3i$-2

M_1 model: staggering ... λ 1 1 λ 1 1 λ ...

- The lowest energy states for $L=3l+1$, staggering [1 1 λ ... 1 1 λ1] with $F=l$, $F=l+1$ are of the form

$|K_i>$: [( )0( )0 ... ( )0001 ... 0010]

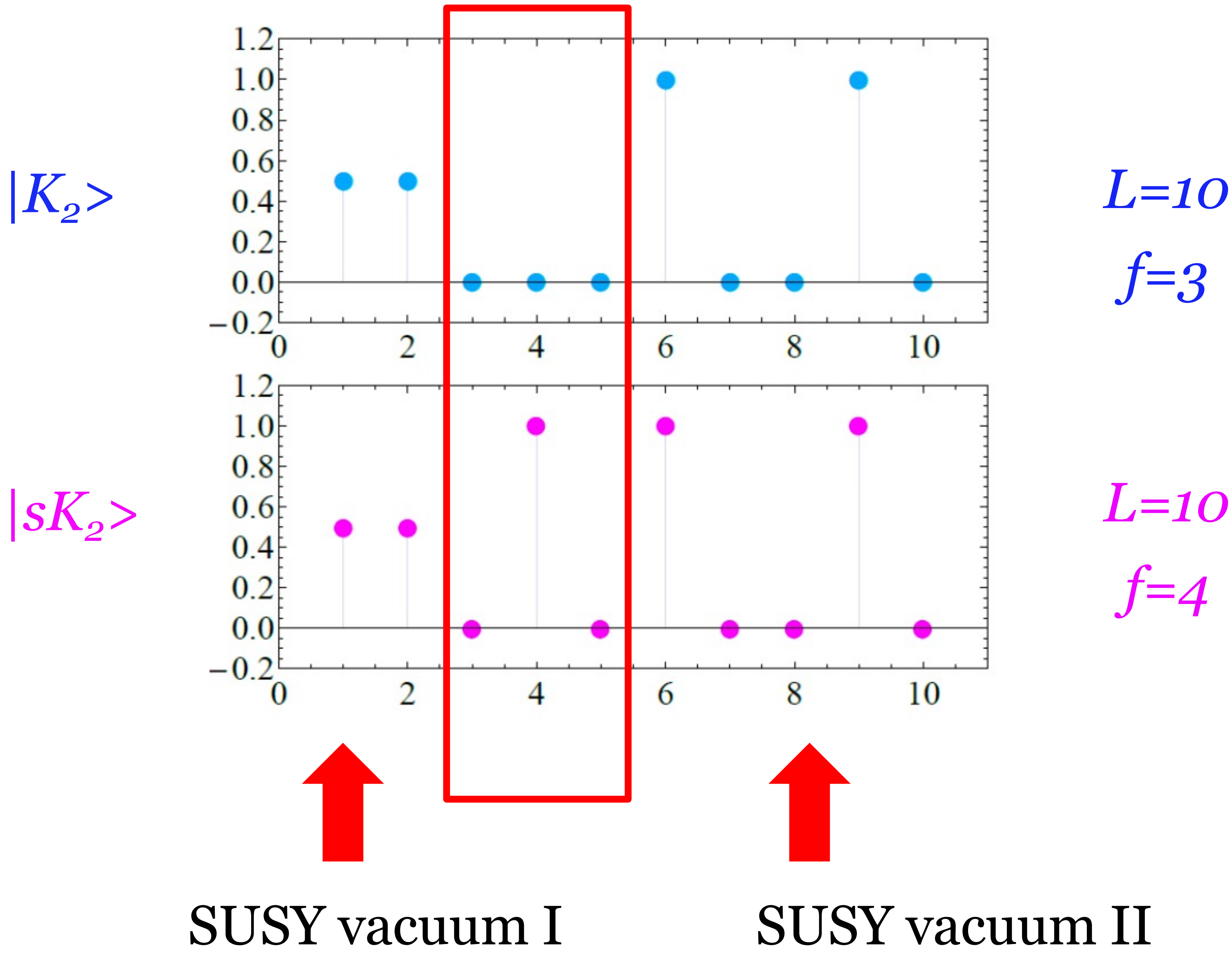
kink at location $3i$-2

$|sK_i>$: [( )0( )0 ... ( )0101 ... 0010]

skink at location $3i$-2

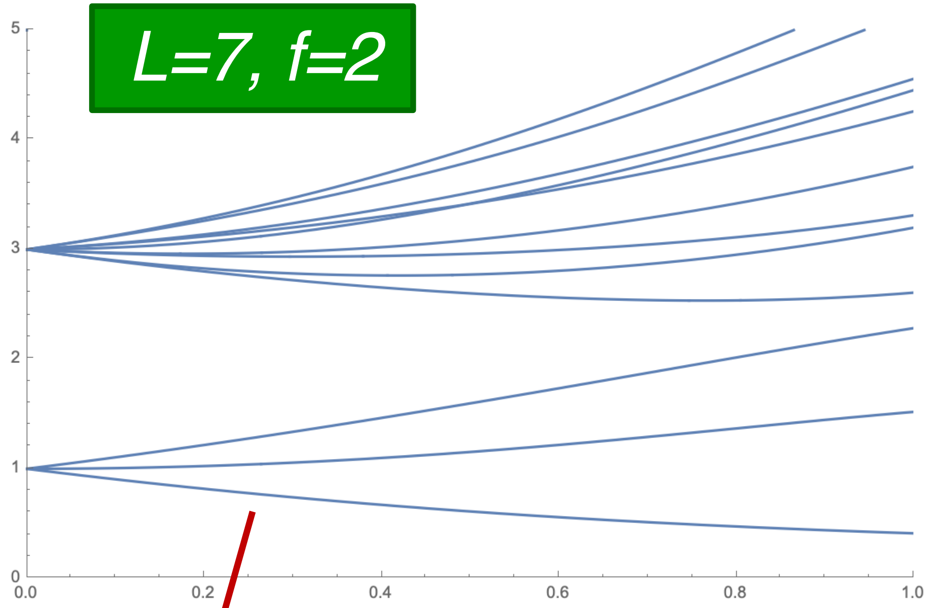
- Both the kink and skink states have energy $E=1$

kinks & skinks at λ=0: particle densities

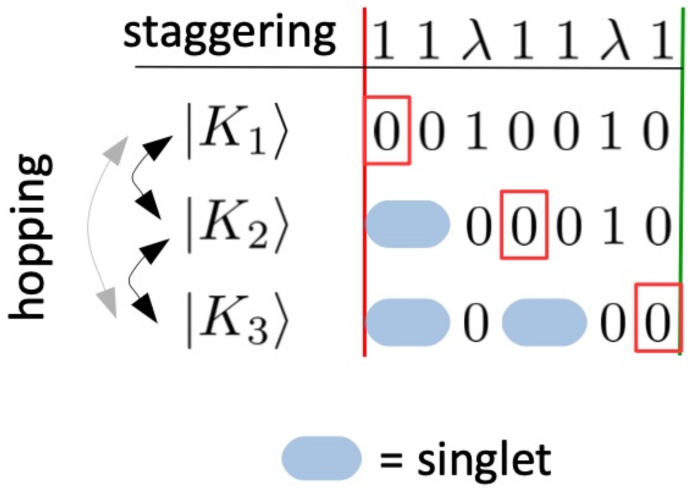


M_1 model with staggering: kinks

$\lambda = 0$ $\lambda = 1$



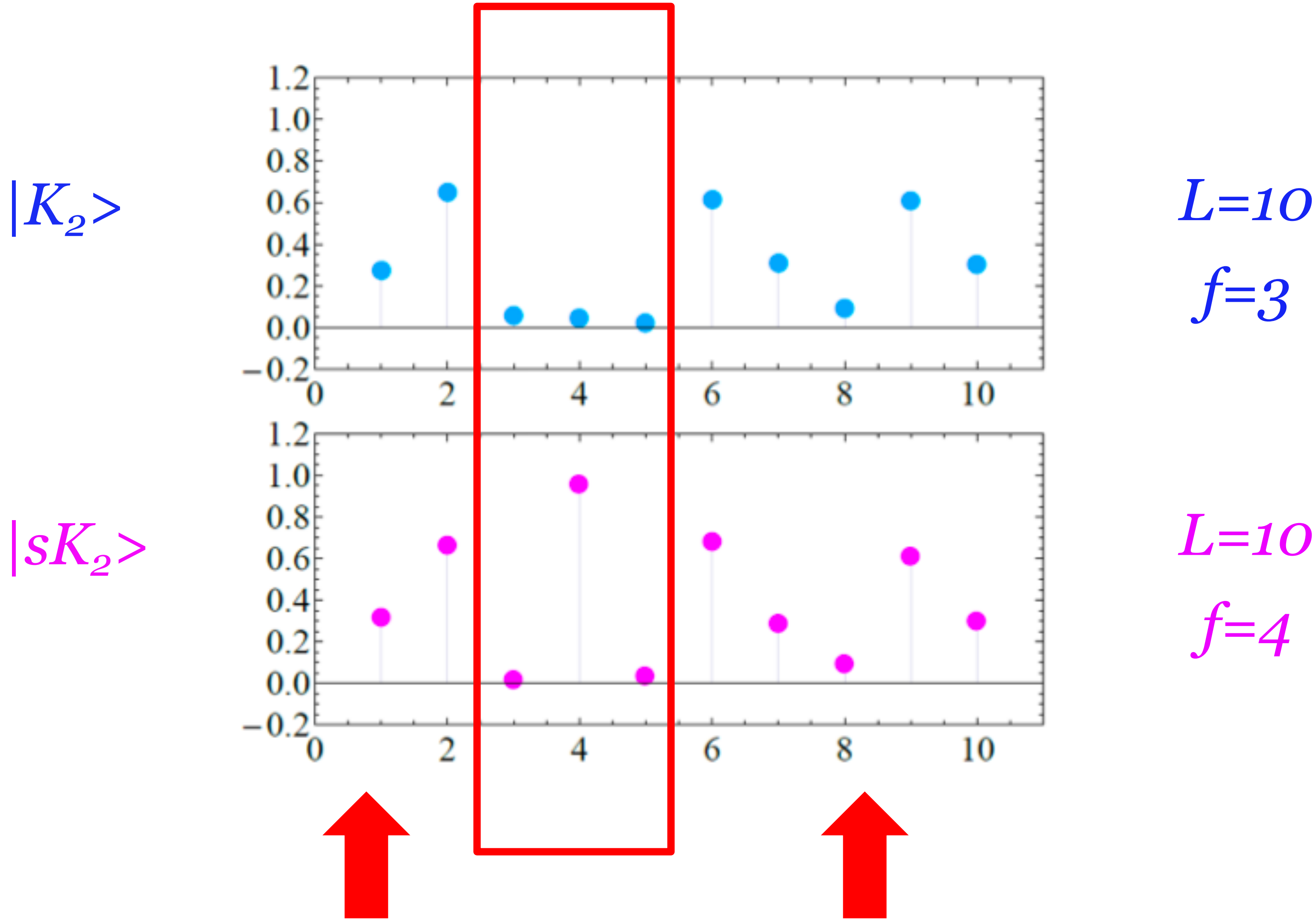
$\lambda << 1$



dressed kinks

$$|v_k\rangle = \frac{2}{l+2}\sum_{j=1}^{l+1}\sin\left(\frac{\pi}{l+2}jk\right)|K_j\rangle \Leftrightarrow |K_j\rangle = \frac{2}{l+2}\sum_{k=1}^{l+1}\sin\left(\frac{\pi}{l+2}jk\right)|v_k\rangle$$

kinks & skinks at λ=1: particle densities

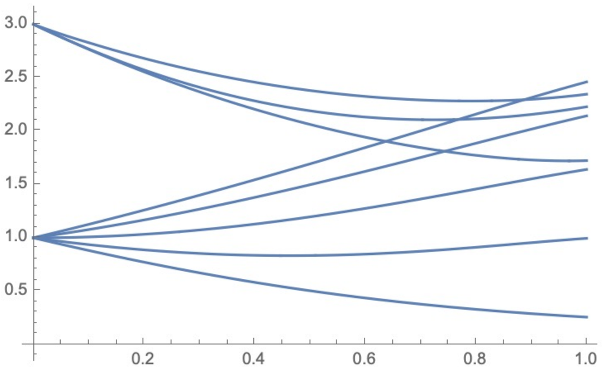


SUSY vacuum I

SUSY vacuum II

M_1 model with staggering

$\lambda = 0$ $\lambda = 1$



static (s)kink

states, $E=1$

$k=1$ $N=2$ SCFT

mobile (s)kinks,

eigenstates $|v_k>$, $|sv_k>$

with energy $E(k)$

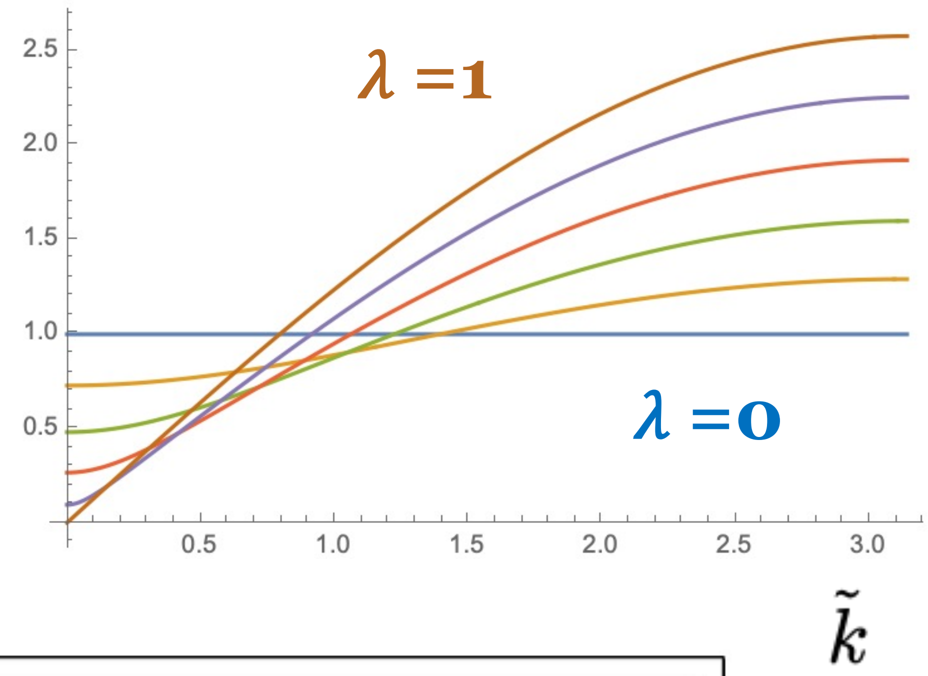
scaling limit for $\lambda \uparrow 1$:

supersymmetric $N=2$ QFT:

sine-Gordon theory at $\beta^2=4/3$

M_1 model with staggering: dispersion

Invoking a relation to the sXYZ model we determined the $L \to \infty$ 1-kink dispersion for general λ



$$\boxed{E(\tilde{k}) = \frac{(3\lambda+s)^{3/2}\sqrt{1-\left(1-\frac{(-3\lambda+s)^3(\lambda+s)}{(-\lambda+s)(3\lambda+s)^3}\right)\cos^2\left(\frac{\tilde{k}}{2}\right)}}{2\sqrt{2}\sqrt{\lambda+s}}}$$

$$s = \sqrt{8+\lambda^2} \qquad 0 \leq \tilde{k} \leq \pi$$

M_1 model: kink dynamics

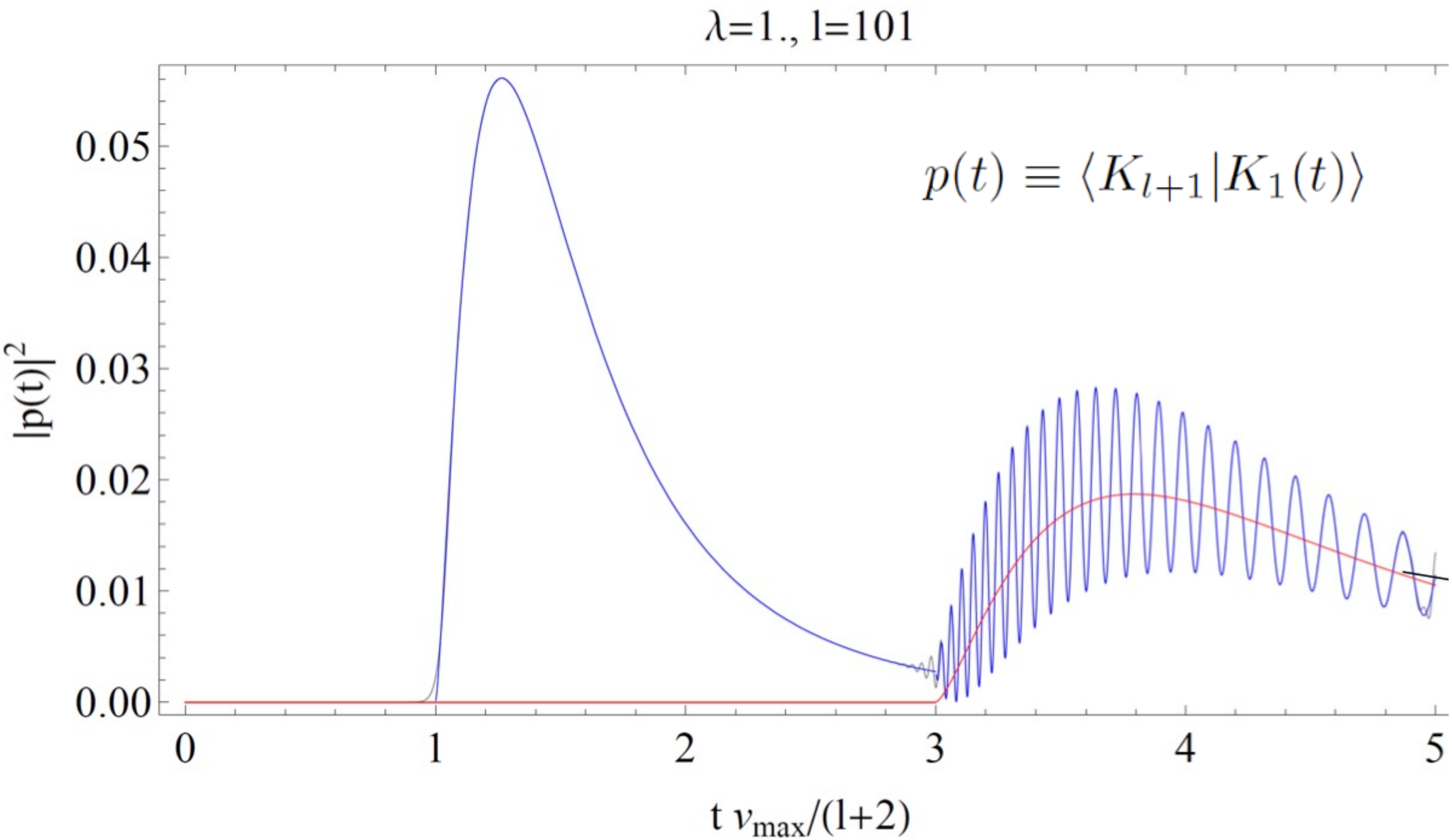
Proposed set-up

- prepare system in leftmost (dressed) 1-kink state and time evolve, detect overlap with right-most kink after time t

$$p(t) \equiv \langle K_{l+1} | K_1(t) \rangle$$

- the dynamics is within the 1-kink sector and is easily computed using the 1-kink dispersion

M_1 model: kink dynamics



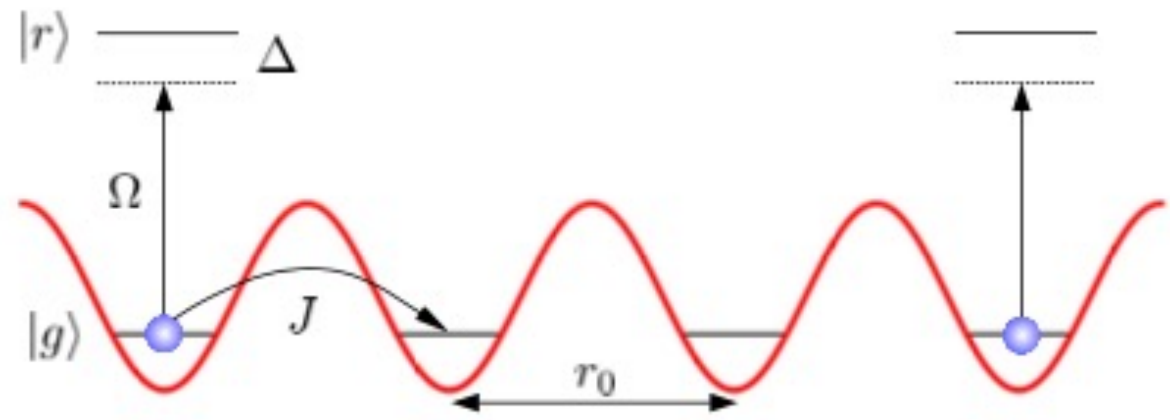
M_1 model: quantum simulation

Challenges

- find experimental system that realizes the M_1 model Hamiltonian in 1D
- find protocols for preparing $|K_1>$ and measuring the arrival amplitude $p(t)$ for kinks
- find protocols for preparing $|sK_1>$ and measuring the arrival amplitude $p(t)$ for skinks

Rydberg atom simulator for M_1 model

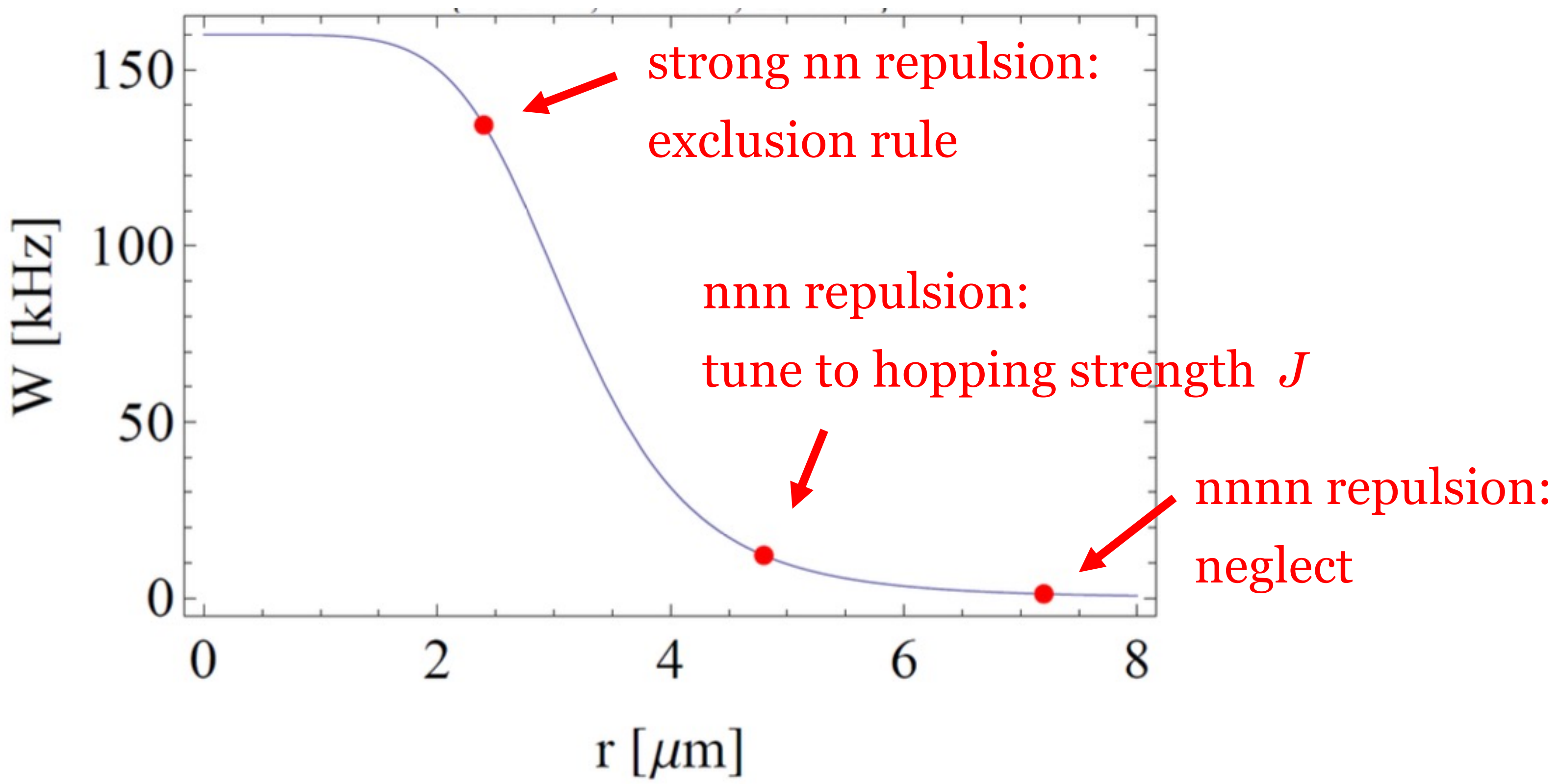
- system of choice: Rydberg-dressed fermionic atoms in optical lattice



$$H_{\mathrm{Ry}} = -J\sum_j \left(c_j^\dagger c_{j+1} + \mathrm{H.c.}\right) + \sum_j \frac{\Omega}{2}\sigma_j^x + \Delta n_j^{(r)} + \sum_{l>j} V(\mathbf{r}_j - \mathbf{r}_l) n_j^{(r)} n_l^{(r)}$$

Rydberg-dressed potential

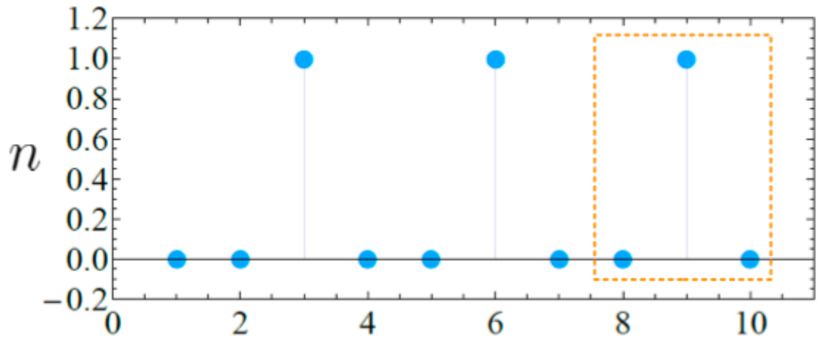
- use Rydberg-dressing to engineer flat-top potential
  → simulation of M_1 model Hamiltonian at $\lambda=1$



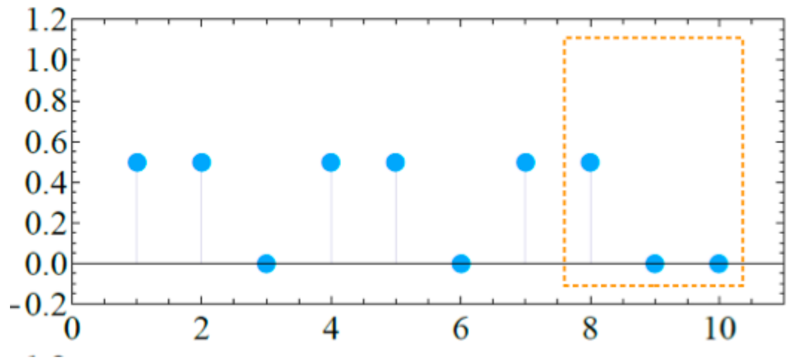
Kink preparation and detection

- Use characteristic density profiles to **create** and **detect** kinks and skinks

$|K_1>$



$|K_4>$

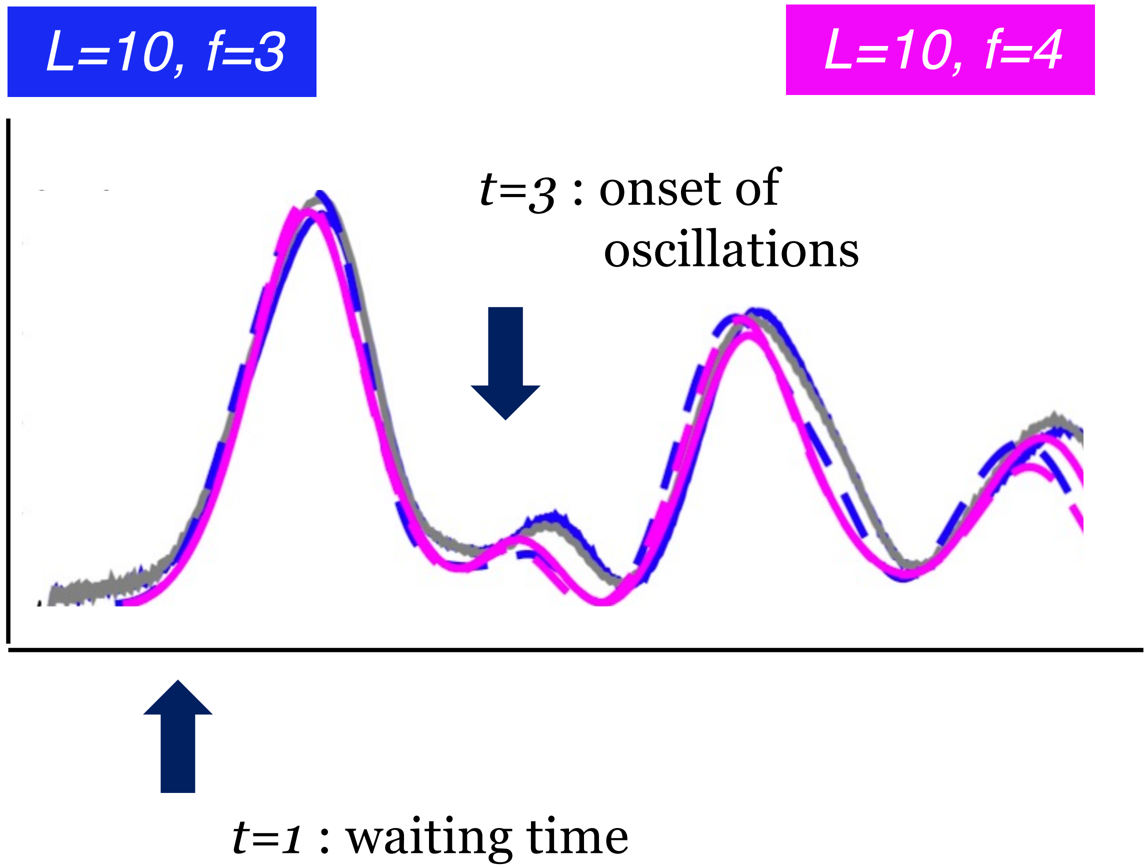


example:

$n_1=n_2=0$ for $|K_1>$, $n_1=n_2=1/2$ for all other $|K_i>$

→ prepare $|K_1>$ by imposing $n_1=n_2=0$ and adiabatically preparing the lowest energy state

SUSY profiles from quantum simulator



blue: kink dynamics

magenta: skink dynamics

→ Supersymmetry in action

dashed lines:

supersymmetric Hamiltonian

solid lines:

Rydberg atom simulator

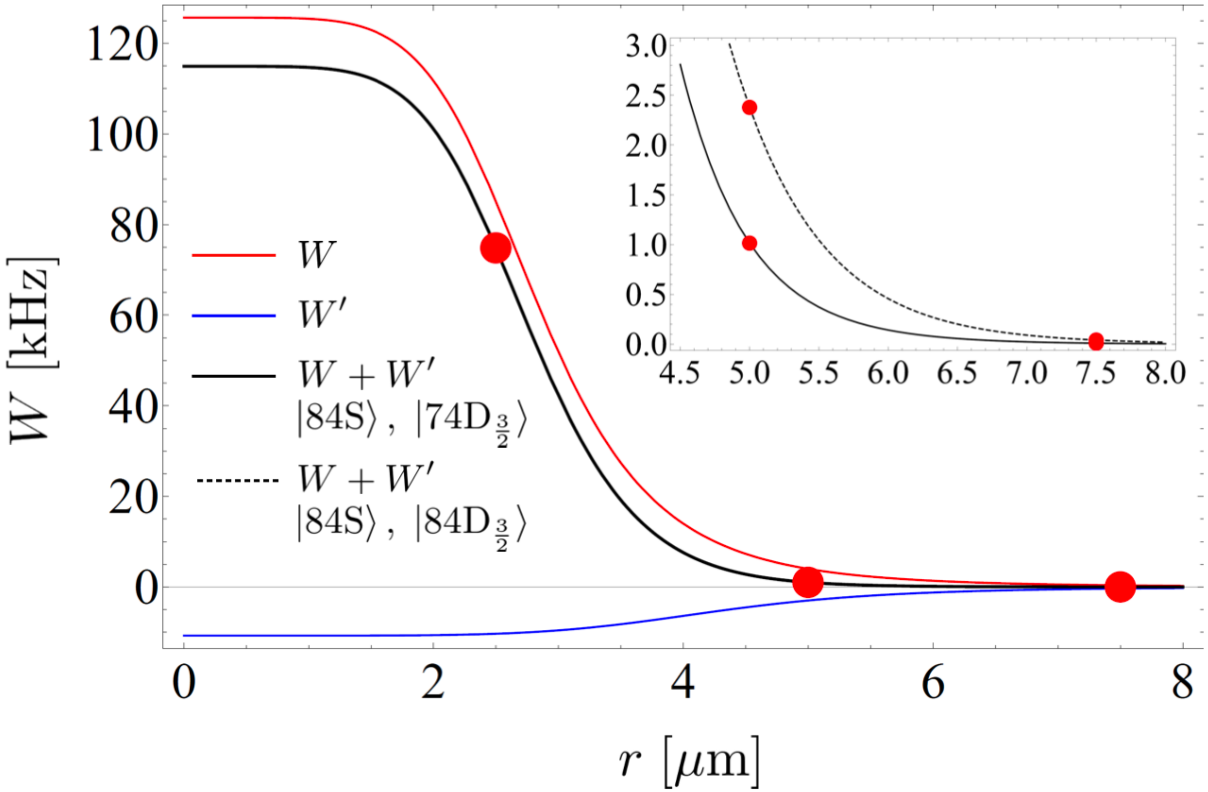
→ Quantum simulation in action

Conclusion: Rydberg atom quantum simulator can accurately track dynamics of supersymmetric lattice Hamiltonian

extra slides

Rydberg atom simulator for M_1 model

- accuracy of Rydberg-dressed potential can be improved by **double-dressing** scheme



Rydberg atom simulator for M_1 model

- extension to $\boldsymbol{\lambda<1}$