Exploring the Phases of Gauge Theories via AMSB

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with
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KIPMU APEC Seminar
April 19 2023

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Outline

- Introduction
- Brief review of Seibergology and use
- Review of AMSB
- AMSB applied to SUSY QCD
- Chiral Lagrangian, η' potential, large N limit…
- Exact results in chiral theories
- Confinement in the SO(N) series
- Summary

Introduction

- One of the most important unsolved questions - find the phases of strongly coupled gauge theories

- Nail down the mechanism of confinement in QCD

- Find for how many flavors do we expect chiral symmetry breaking, does confinement persist for larger number of flavors, …?

- Are there other phases realized? Expect at least conformal phase a la Banks-Zaks

Introduction

- Not that many tools available for studying this question

- Lattice gauge theories (not yet applicable for chiral theories)

- Anomaly matching conditions

- ``Tumbling” for chiral theories - will discuss

- Extrapolation from SUSY theories - will focus on this

Exact results in SUSY gauge theories

- SUSY gives powerful constraints on strong dynamics

- Seiberg was able to nail down phase structure of SUSY QCD in 1994 using
  - Holomorphy
  - ’t Hooft anomaly matching
  - Instanton calculations
  - Integrating out/Higgsing

- Obtained many different phases depending on F vs N

SUSY QCD

- N=1 SUSY SU(N) gauge theory with F flavors

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Φ, Q	$\square$	$\square$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\Phi}$, $\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{F-N}{F}$

- Moduli space (D-flat directions) parametrized by holomorphic gauge invariants, generically mesons $M_{ij} = Q_i\bar{Q}_j$ and baryons $B_{ij\ldots k} = Q_iQ_j\ldots Q_k$ where the baryons are totally antisymmetric in the flavor indices, and only exist for F≥N

The phases of SUSY QCD

$F > 3N$ Theory IR free

$F = 3N$

Conformal - non-abelian Coulomb phase

$F = \frac{3}{2}N$

Free magnetic phase

$F = N + 1$ s-confinement

$F = N$ Quantum modified constraint

$F = N - 1$

ADS superpotential, runaway vacuum

$F = 0$ Pure SYM - gaugino condensation

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F=0 - Pure SYM

- No matter fields, no continuous flavor symmetry
- Z_{2N} discrete R-symmetry rotating gauginos
- Dynamics: gaugino condensation
- $W = N\Lambda^3$ $\langle\lambda\lambda\rangle = -32\pi^2\omega_k\Lambda^3$
- Should be truly confining $V(R) \sim \sigma R$

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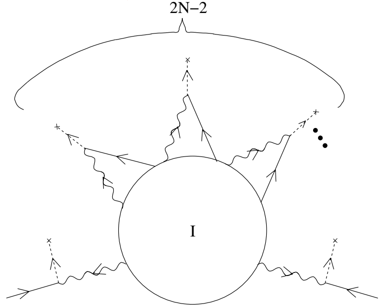
0<F<N: ADS superpotential

- First obtained by Affleck, Dine, Seiberg 1984

- Dynamics generates a non-perturbative superpotential

$$W_{\rm ADS} = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)}$$

- For F=N-1 actually generated by instanton, calculable



- Gauge group (partially) Higgsed

- $V(R) \sim \mathrm{constant}$ (at least for F=N-1)

- For F<N-1 gaugino condensation in unbroken group

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F=N,N+1: special cases

- Both have description in terms of gauge singlet mesons and baryons

- F=N: Quantum modified constraint

$$W_{\text{constraint}} = X\left(\det M - \overline{B}B - \Lambda^{2N}\right)$$

	$U(1)_A$	$U(1)$	$U(1)_R$
$\det M$	$2N$	0	0
B	N	N	0
$\overline{B}$	N	$-N$	0
Λ^{2N}	$2N$	0	0

- ’t Hooft anomalies all matched (as long as the constraint is satisfied)

F=N,N+1: special cases

- F=N+1 s-confinement - all `t Hooft anomalies matched by meson+baryons

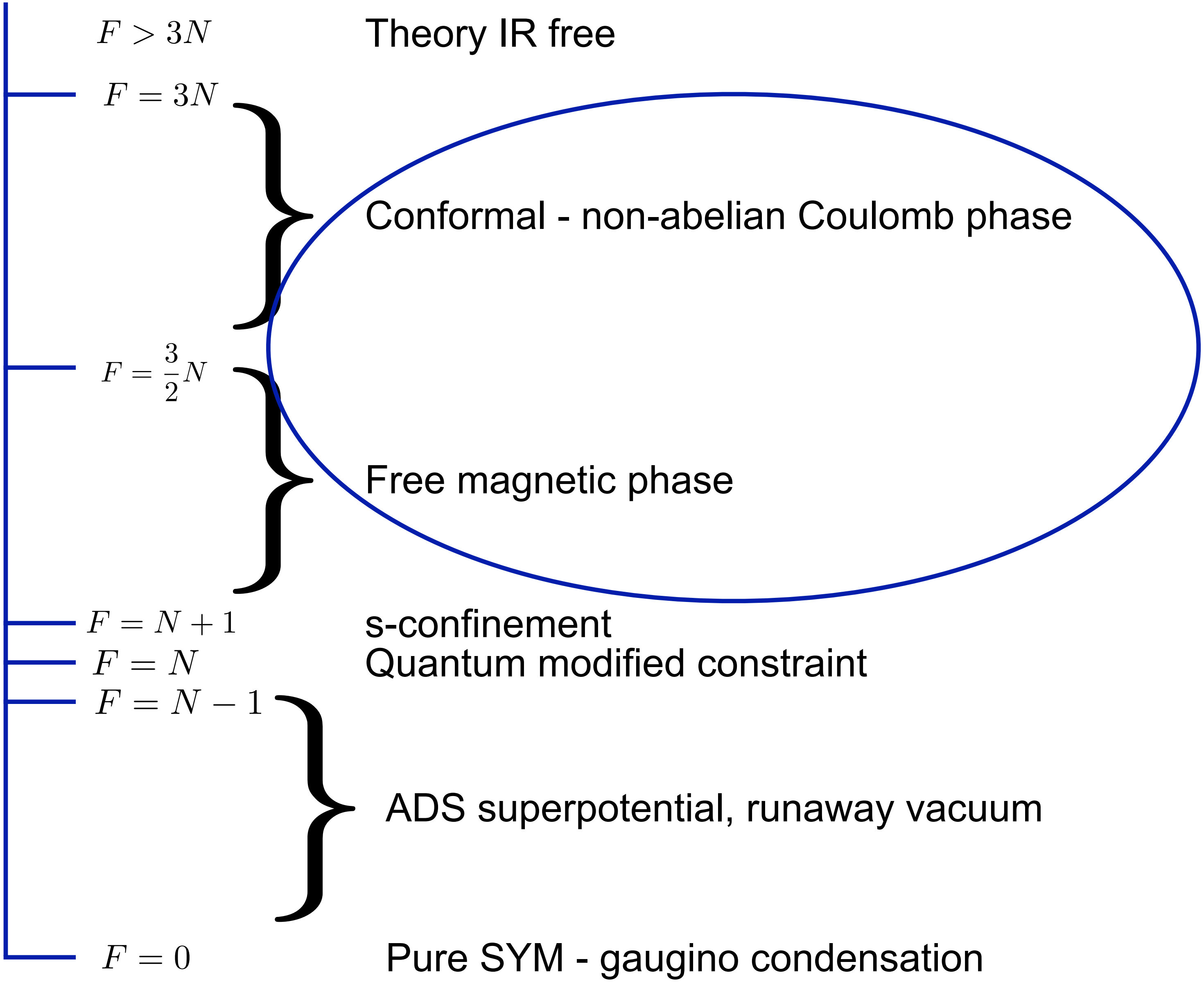
	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
M	$\Box$	$\overline{\Box}$	0	$\frac{2}{F}$
B	$\overline{\Box}$	$\mathbf{1}$	N	$\frac{N}{F}$
$\overline{B}$	$\mathbf{1}$	$\Box$	$-N$	$\frac{N}{F}$

- Dynamical superpotential implements classical constraints

$$W = \frac{1}{\Lambda^{2N-1}}\left[B^i M_i^j \overline{B}_j - \det M\right]$$

- Both F=N,N+1 ``screened phase" - complementarity no phase boundary between Higgs and screened phase

The phases of SUSY QCD



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N+1<F<3N: Seiberg duality

- In this range there is a magnetic dual (Seiberg 1994)

- Electric theory

	$SU(N_c)$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_B$	$U(1)_R$
Q^i	$\square$	$\square$	$\mathbf{1}$	1	$\frac{N_F-N_c}{N_F}$
$\widetilde{Q}_i$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{N_F-N_c}{N_F}$

- Magnetic theory

	$SU(N_F-N_c)$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_B$	$U(1)_R$
q_i	$\overline{\square}$	$\overline{\square}$	$\mathbf{1}$	$\frac{N_c}{N_F-N_c}$	$\frac{N_c}{N_F}$
$\widetilde{q}^i$	$\square$	$\mathbf{1}$	$\square$	$-\frac{N_c}{N_F-N_c}$	$\frac{N_c}{N_F}$
M^i_j	$\mathbf{1}$	$\square$	$\overline{\square}$	0	$\frac{2N_F-2N_c}{N_F}$

- Flow to the same IR theory - describe the same low-energy physics

- All anomalies matched, same flat directions, can move up and down by integrating out, Higgsing…

N+1<F<3/2 N: Free magnetic theory

- The magnetic SU(F-N) group is actually IR free

- In this range the theory will have free dual gluons, quarks and meson in the IR

- An emergent gauge symmetry! In the UV start out with SU(N) and in the IR end up with completely different group. ``Massless composite gauge bosons” - could be used for composite model building etc.

- $$V(R) \sim \frac{\ln(R\Lambda)}{R}$$

3/2N < F < 3N:

- The electric and magnetic groups flow to the same IR fixed point

- Conformal phase, ``non-Abelian Coulomb phase"

- Close to the edges of the boundary could be perturbative - electric or magnetic Banks-Zaks fixed points

- $V(R) \sim \frac{1}{R}$

The phases of SUSY QCD

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The phases of SUSY QCD

- A beautiful picture, BUT very different from what we expect in non-SUSY QCD

- Lattice simulations suggest only 2 phases
  - Chiral symmetry breaking
  - For large number of flavors (perhaps as high as F>3N) conformal phase

- Would like to start making connection between SUSY and non-SUSY theories

How to add SUSY breaking?

- Clearly one of the most important questions - started very early on

- Aharony, Sonnenschein, Peskin, Yankielowicz ’95: add soft SUSY breaking on electric side

$$\Delta\mathcal{L}_{UV}^{soft} = -m_Q^2 \left(|Q|^2 + |\bar{Q}|^2\right) + (m_g S + \text{ h.c. })$$

- Guess effect on magnetic side

$$\Delta\mathcal{L}_{IR}^{soft} = -m_Q^2 \left[B_M |M|^2 + B_M \left(|B|^2 + |\widetilde{B}|^2\right)\right] + (m_g \langle S \rangle + \text{ h.c. })$$

- Assumed positive soft breaking masses for composites

- For F<N gave ``right” symmetry breaking pattern, but for F=N unpredictive, no χSB for F>N

How to add SUSY breaking?

- Around same time: Evans, Hsu, Schwetz ‘95

- Couple to spurions and use holomorphy and broken global symmetries to restrict the allowed SUSY breaking terms

- In their analysis they still found runaway direction for ADS case

How to add SUSY breaking?

- Another more systematic approach: Cheng & Shadmi 1998

- Try to find the mapping of SUSY breaking by turning it into a gauge mediated model

- Add extra massive quark flavor and couple that directly to SUSY breaking spurion X

- Map $XQ_{F+1}\bar{Q}_{F+1} \to XM_{F+1,F+1}$

- Will be essentially messenger in UV theory, calculate effect in IR

How to add SUSY breaking?

- Calculated resulting soft breaking terms using RGE's

- Result - free magnetic:

$$\tilde{m}_q^2 = \tilde{m}_{\tilde{q}}^2 = -\frac{\tilde{m}_M^2}{2} = \frac{1}{2N_f + \bar{N}} \left[\left(\bar{N}\tilde{m}_q^2(v) - N_f \tilde{m}_M^2(v) \right) + \frac{\bar{N}^2 - 1}{N_f - 3\bar{N}} \tilde{M}_{\tilde{g}}^2(v) \right]$$

- They find runaway direction either in squark or meson field

- Symmetry breaking pattern is not as expected in QCD

How to add SUSY breaking?

- Arkani-Hamed & Rattazzi ‘98; Luty & Rattazzi ‘99
- Map of soft breaking masses through the duality by coupling to background anomalous gauge fields, gauged $U(1)_R$ and SUGRA
- However ignored pure AMSB effects
- Magnetic squark runaway, results not clear for F=N

How to add SUSY breaking?

- Most systematic previous approach:
  Abel, Buican, Komargodsky ‘11

- Idea: Conserved currents are easy to exactly map through the duality

- Relate SUSY breaking terms in UV to conserved (or anomalous) currents and find the corresponding Noether currents in the IR

- For F>N+1 result

$$\delta\mathcal{L}_{\rm el} = -m^2\left(QQ^\dagger + \widetilde{Q}\widetilde{Q}^\dagger\right) + m_\lambda(\lambda_{\rm el}^2 + c.c.)$$

$$\Big\downarrow$$

$$\delta\mathcal{L}_{\rm mag} = -m^2\cdot\frac{2N_f - 3N_c}{3N_c - N_f}\left(qq^\dagger + \widetilde{q}\widetilde{q}^\dagger - 2MM^\dagger\right) + m_\lambda\cdot\frac{2N_f - 3N_c}{3N_c - N_f}(\lambda_{\rm mag}^2 + c.c.)$$

How to add SUSY breaking?

- Baryonic runaway direction, breaking pattern not like in QCD again

- Consistent with Cheng/Shadmi

- Would like to find a different method where we have full control over all aspects of SUSY breaking

- Ideally should produce symmetry breaking pattern consistent with QCD at least as a local minimum

The use of AMSB

- Recent proposal of Murayama ’21: use anomaly mediated SUSY breaking for perturbing the Seiberg exact results

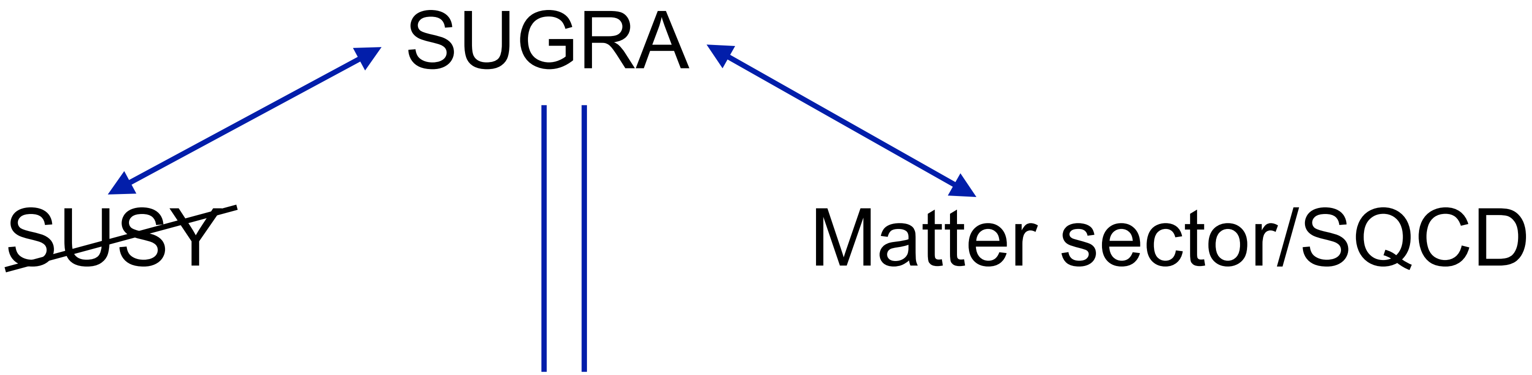
- AMSB: originally ``designed” to provide a specific implementation for MSSM with predictive soft breaking patterns

- Here we will simply use it only to study phases of gauge theories, not as a BSM model

- Assumption of AMSB: SUSY breaking mediated purely by supergravity, no direct interaction between SUSY breaking sector and matter sector

AMSB

Randall, Sundrum ‘98
Giudice, Luty, Murayama, Rattazzi ’98
see also Arkani-Hamed, Rattazzi ‘98



SUGRA

~~SUSY~~

Matter sector/SQCD

- Assume matter sector sequestered - no direct interactions with SUSY breaking generated

- Only source of ~~SUSY~~ the auxiliary field of supergravity multiplet

AMSB

- Best way to describe effect of AMSB is via the introduction of the Weyl compensator Φ

Pomarol, Rattazzi ‘99

- This conformal compensator is a spurion for super-Weyl transformations (SUSY rescaling + U(1) rotations) with weight 1

- The effects of ~~SUSY~~ will show up through the coupling

$$\mathcal{L} = \int d^4\theta \Phi^* \Phi K + \int d^2\theta \Phi^3 W + c.c.$$

- With the spurion $\Phi = 1 + \theta^2 m$

AMSB

- If the matter sector is conformal: can scale out $\mathbf{\Phi}$ by rescaling the fields $\phi_i \rightarrow \Phi^{-1}\phi_i$
- For example if $K = \Phi^*\Phi\phi^+\phi$ and $W = \Phi^3\phi^3$
- $\phi_i \rightarrow \Phi^{-1}\phi_i$ rescaling will completely remove $\mathbf{\Phi}$ from the theory - no SUSY breaking
- SUSY breaking will be tied to violations of conformality! UV insensitive process!

Loop induced AMSB effects

- If scale invariance broken via RGE running:

$$\beta(g^2) = \frac{g^3}{16\pi^2}\left[S(R) - 3C(G)\right] + \ldots \longrightarrow m_\lambda(\mu) = -\frac{\beta(g^2)}{2g^2}(\mu)m$$

$$\begin{tikzcd}
 & m_i^2(\mu) = -\frac{1}{4}\dot{\gamma}_i(\mu)m^2 \\
\frac{d}{dt}Y^{ijk} = Y^{ijp}\left[\frac{1}{16\pi^2}\gamma_p^k + \ldots\right] + (k \leftrightarrow i) + (k \leftrightarrow j) \arrow[ur] \arrow[dr] & \\
 & A_{ijk}(\mu) = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)(\mu)m
\end{tikzcd}$$

- For example in SUSY QCD

$$m_\lambda = \frac{g^2}{16\pi^2}(3N_c - N_f)m$$

$$m_Q^2 = m_{\tilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2}2C_i(3N_c - N_f)m^2$$

Loop induced AMSB effects

- Loop induced breaking terms provide positive squark masses and gaugino mass - massless spectrum that of ordinary QCD

- For AMSB version of MSSM slepton masses were problematic - right handed sleptons were tachyonic. Here only AF gauge group - AMSB gives perfect UV boundary condition

A surprise - tree-level AMSB effects

- If there is a non-scale invariant superpotential: will contribute to AMSB potential

$$\mathcal{L}_{\text{tree}} = m\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right) + c.c.$$

- Vanishes for dim 3 superpotential, but not in general

- Expression for general Kähler potential:

C.C., Gomes, Murayama, Telem ‘21

$$\begin{aligned} V_{\text{tree}} \ =& \partial_i W g^{ij^*} \partial_j^* W^* + m^* m \left(\partial_i K g^{ij^*} \partial_j^* K - K\right) \\ &+ m\left(\partial_i W g^{ij^*} \partial_j^* K - 3W\right) + c.c. \end{aligned}$$

A non-perturbative AMSB potential

(Murayama '21)

- Example: SU(N) for $N_f < N_c$. ADS Superpotential

$$(N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)}$$

- Will lead to induced term from $\int d^2\theta \Phi^3 W_{ADS}$

$$-(3N_c - N_f)m\left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}}\right)^{1/(N_c - N_f)} + c.c.$$

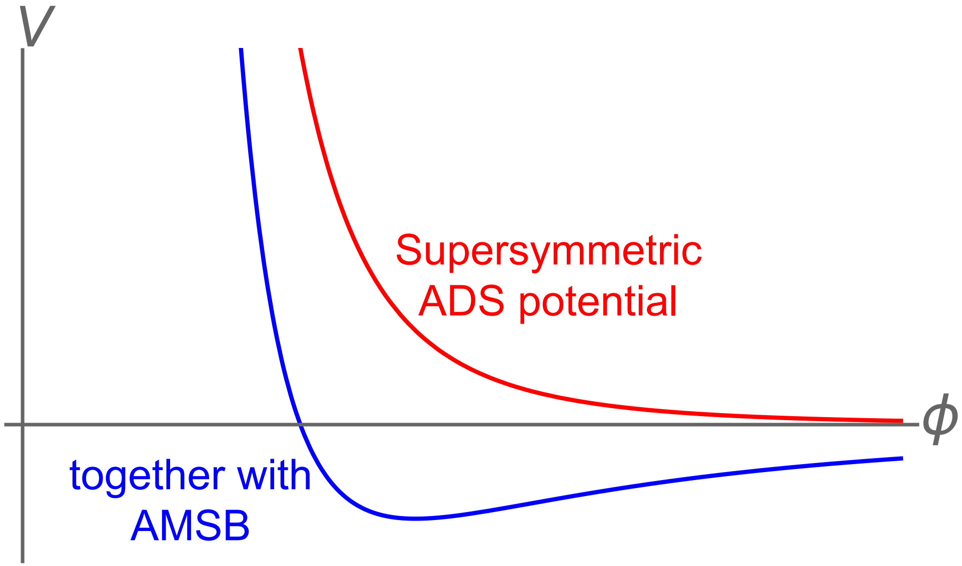
- Along direction

$$Q = \tilde{Q} = \left(\begin{array}{ccc} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ \hline 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{array}\right)\phi, \qquad M = \phi^2.$$

A non-perturbative AMSB potential

- $-(3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} + c.c.$ term is key
- Non-perturbative effect involving SUSY breaking
- AMSB allows us to pin down this term
- Formally tree-level but really must be a non-perturbative effect including SUSY breaking
- Will stabilize ADS superpotential!
- Will give rise to proper symmetry breaking pattern!

Phase for QCD* for N_f<N_c



- Symmetry breaking pattern $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

- As in QCD, massless DOF’s just pions

- Could be continuously connected to actual QCD for m>>Λ

The Phases of AMSB QCD

- We have seen for $N_f < N_c$ get chiral symmetry breaking $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$ QCD-like vacuum

- What happens for higher flavors? More subtle, recent analysis

(Gomes, Murayama, Noether, Ray-Varier, Telem +C.C.)

$N_f=N_c$ - Quantum Modified Constraint

- The first case where baryons show up

- Seiberg: $\det M - B\bar{B} = \Lambda^{2N_c}.$

- Issue: VEVs O(Λ) - higher order corrections in Kahler not suppressed!

$N_f=N_c$ - Quantum Modified Constraint

- Use non-linear analysis
- Meson point (in units where Λ=1)

$$M=(1+B\bar{B})^{1/N_c}e^{\Pi}=\mathbf{1}+\frac{1}{N_c}B\bar{B}+\Pi+\frac{1}{2}\Pi^2+\cdots$$

- Π is a traceless complex matrix
- What is the Kähler potential? $\mathrm{Tr}\,M^\dagger M,\ (\mathrm{Tr}\,M^\dagger M)^2,\ \ \mathrm{Tr}\,M^\dagger M M^\dagger M$
- For example: $\mathrm{Tr}\,M^\dagger M\supset\mathrm{Tr}\,\Pi^\dagger\Pi+\frac{1}{2}\,\mathrm{Tr}\,\Pi^2+\frac{1}{2}\,\mathrm{Tr}\,{\Pi^\dagger}^2$
- Resulting potential: for $K=\varphi^\dagger\varphi+\alpha/2\,(\varphi^2+{\varphi^\dagger}^2)$

$$V_{\mathrm{AMSB}}=(\alpha^2+\alpha)m^2(\mathrm{Re}\,\varphi)^2+(\alpha^2-\alpha)m^2(\mathrm{Im}\,\varphi)^2$$

$N_f = N_c$ - Quantum Modified Constraint

- Mesons are stable at this point! However baryons uncalculable… $K \supset \alpha(B^\dagger B + \bar{B}^\dagger \bar{B}) + \frac{\beta}{2}(B\bar{B} + c.c.)$

- Depending on **α/β** ratio may or may not have baryonic runaways. Theory strongly coupled - simply can not say

- Baryon point:

$$B = (1 - \det M)^{1/2} e^b$$
$$\bar{B} = -(1 - \det M)^{1/2} e^{-b}$$

- Kähler: $B^\dagger B + \bar{B}^\dagger \bar{B} = 2 + (b + b^\dagger)^2 + \cdots$

- Im b Goldstone, Re b positive mass, OK

- Mesons: again uncalculable due to higher order terms

N_f=N_c - Quantum Modified Constraint

- N_f=N_c=2 special case - SU(2)xSU(2) flavor symmetry enhanced to SU(4), no difference between meson and baryon.

- Constraint $M^a M^a = 1$ solution breaks SU(4)→Sp(4), 5 Goldstones.

- Positivity of kinetic term will imply positive masses for all non-Goldstone fluctuations - will give rise to a stable global minimum

N_f=N_c+1 - s-confinement

- Subtle case - previously claimed it has runaway directions, but we found this one is actually predictive and no runaways with expected QCD-like vacuum

- Superpotential: $W = \alpha B M \bar{B} - \beta \det M$

- **α**, **β** O(1) to make Kähler canonical

- $$\begin{aligned} V_{\text{SUSY}} &= \alpha^2 (|(M\bar{B})_a|^2 + |(BM)_a|^2) \\ &\quad + |\alpha \bar{B}_a B_b - \beta \det M (M^{-1})_{ab}|^2 \\ V_{\text{AMSB}} &= -(N_c - 2)\beta m \det M + c.c. \end{aligned}$$

$N_f=N_c+1$ - s-confinement

- Minimize the potential along direction

$$B = \begin{pmatrix} b \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \bar{B} = \begin{pmatrix} \bar{b} \\ 0 \\ \vdots \\ 0 \end{pmatrix}, M = \begin{pmatrix} x & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}$$

- Most general by symmetries. Assume m real - all VEVs can be taken real

$$V = 2\alpha^2 x^2 b^2 + (\alpha b^2 - \beta v^{N_c})^2 + N_c \beta^2 x^2 v^{2(N_c-1)}$$
$$- 2(N_c-2)\beta m x v^{N_c}.$$

$N_f=N_c+1$ - s-confinement

- Baryon number conserving direction b=0

- This is the usual QCD-like vacuum with chiral symmetry breaking

$$v = x = \left(\frac{(N_c-2)m}{N_c\beta}\right)^{\frac{1}{N_c-1}}, V_{\min} = -\mathcal{O}(m^{2N_c/(N_c-1)}).$$

- Along this direction baryons massive - integrate them out. Effect of Yukawa coupling will be two loop meson mass

$$m_M^2 = \frac{(2N_c+3)\alpha(v)^4 m^2}{(16\pi^2)^2}$$

- Leading to potential $V_{2-\text{loop}} = \frac{(N_c+1)(2N_c+3)\alpha(v)^4}{(16\pi^2)^2} m^2 v^2$

$N_f = N_c + 1$ - s-confinement

- At the minimum this is same order $\mathcal{O}(m^{2N_c/(N_c-1)})$ in m but two loop, so will not destabilize.

- Higher order Kähler terms? $(\mathrm{Tr}\, M^\dagger M)^2 \quad \mathrm{Tr}\, M^\dagger M M^\dagger M$

- Higher order in m at the VEV $\sim m^2 v^4$

- This direction is a stable minimum

- Baryon number breaking direction b≠0

- Minima at $$b^2 = \frac{\beta}{\alpha} v^{N_c} - 2x^2$$ $$x = \frac{(N_c-2)m}{2\alpha}.$$ Runaway potential????

$$V|_{b,x} = -\frac{(N_c-2)^2\beta}{2\alpha} m^2 v^{N_c}$$

$N_f = N_c + 1$ - s-confinement

- This led to the conclusion that there is no stable minimum but runaway baryonic direction.

- BUT: loop effects! b gets VEV - bottom N_c components of B and $\bar{B}$ get masses - integrate them out. Gives all but M_{11} two-loop mass as before. At this point remaining $W_{\_} = \alpha B_1 M_{11} \bar{B}_1$. So M_{11} gets a mass at lower scale $\sqrt{2}\alpha b$ - integrate it out will give 2 loop AMSB mass to baryons:

$$m_b^2 = \frac{3\alpha(b)^4 m^2}{(16\pi^2)^2}$$

- So loop induced potential:

$$V_{2-\text{loop}} = \frac{m^2}{(16\pi^2)^2}[N_c(2N_c+3)\alpha(v)^4 v^2 + 6\alpha(b)^4 b^2]$$

$N_f=N_c+1$ - s-confinement

- Runaway vs. loop

$$V|_{b,x} = -\frac{(N_c-2)^2\beta}{2\alpha} m^2 v^{N_c}$$

$$V_{2-\text{loop}} = \boxed{\frac{m^2}{(16\pi^2)^2}[N_c(2N_c+3)\alpha(v)^4 v^2} + 6\alpha(b)^4 b^2]$$

Dominates!

- Even though loop suppressed, lower power in v, will stabilize around the origin! No runaway direction here!

- Loops come in to save the day from a tree-level runaway, quite remarkable.

- Don't know what happens for O(Λ) fields, but origin stable with expected VEV around there.

$3/2N_c$>Nf>Nc+1 Free Magnetic Phase

- Will not show full analysis here. Expected to be beset by baryonic runaway directions - no useful info?

- Analysis very subtle - even more than s-confining. Found: $N_f \lesssim 1.43 N_c$ the baryonic runaways lifted!

- Need to analyze several branches

Baryonic branch - no runaways for $N_f \lesssim 1.43 N_c$

Mesonic branch: stable chiral SB minimum

Mixed branch: check no runaways

$3/2N_c$>Nf>Nc+1 Free Magnetic Phase

- The baryonic branch: $W = \lambda \,\mathrm{Tr}\, q_i M_{ij} \bar{q}_j$

- Both g, λ go to 0 (IR free), BUT $$0 = \frac{d}{d\log\mu}\frac{g^2}{\lambda^2}.$$

so λ can be expressed in terms of g in the deep IR.

- Loop induced AMSB: $$m_q^2 = \frac{(-\widetilde{b})g^4}{(16\pi^2)^2}\frac{N_f^2 - 3N_f\widetilde{N}_c - \widetilde{N}_c^2 + 1}{2N_f + \widetilde{N}_c}m^2$$

$$\widetilde{b} = 3\widetilde{N}_c - N_f \qquad m_M^2 = \frac{(-\widetilde{b})\widetilde{N}_c\lambda^2 g^2}{(16\pi^2)^2}m^2$$

- Until $N_f \lesssim 1.43 N_c$ (almost all free magnetic window) these are positive - no baryonic runaway expected!

3N$_c$>Nf>3/2 Nc Conformal Window

- Three regions

  Lower conformal window: baryonic runaways to uncalculable regions

  Intermediate regime: fully uncalculable

  Upper conformal window: no runaways, stable chiral symmetry breaking minimum

- SCFT destroyed by AMSB in all regions

- Lower conformal window (BZ in dual) $N_f = 3\tilde{N}_c/(1+\epsilon)$

$$x \equiv \frac{\tilde{N}_c}{8\pi^2}\lambda^2, \qquad y \equiv \frac{\tilde{N}_c}{8\pi^2}g^2.$$

$$\beta(x) = x(-2y + 7x),$$
$$\beta(y) = -3y^2(\epsilon - y + 3x).$$

$3N_c$>Nf>3/2 Nc Conformal Window

- Admits BZ fixed point at $(x_0, y_0) = (2\epsilon, 7\epsilon)$

- Along the approach to the FP the AMSB masses:

$$m_M^2 = \frac{3}{2}\epsilon^2 \delta y\, m^2$$

$$m_q^2 = -\frac{3}{4}\epsilon^2 \delta y\, m^2.$$

$$\delta y \sim \mu^{21\epsilon^2}$$

- Negative squark mass - will yield runway baryonic dir.

- Upper conformal window (BZ in electric theory):

$$m_Q^2 = \frac{3}{4}\epsilon^2(-\delta y)m^2$$

$$m_\lambda = \frac{3}{2}(-\delta y)m.$$

$$(-\delta y) \sim \mu^{3\epsilon^2}$$

$3N_c$>Nf>3/2 Nc Conformal Window

- QCD-like chiral symmetry breaking phase exists throughout the window as local minimum

- Along mesonic branch dual quarks massive, can be integrated out, superpotential:

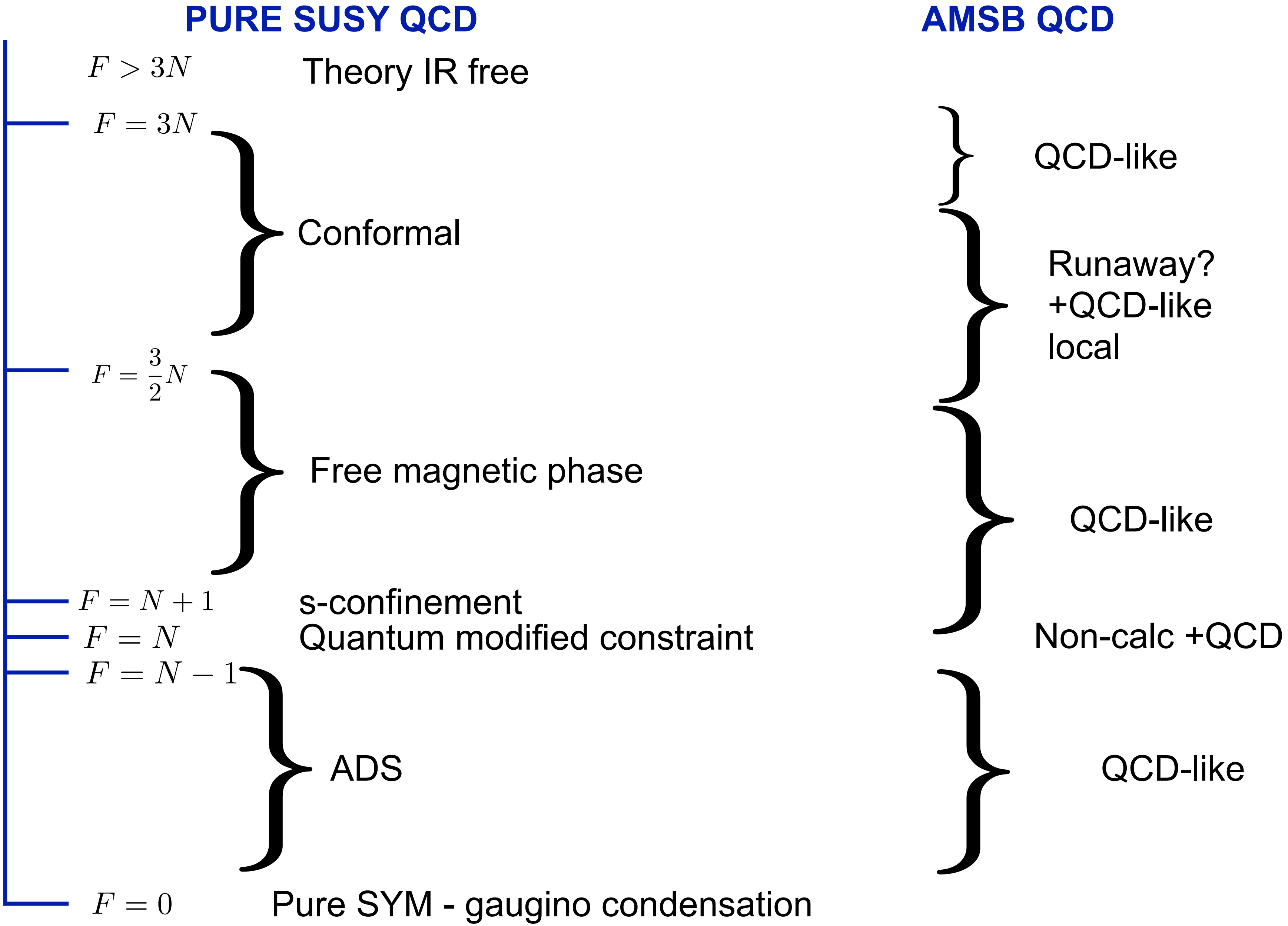
$$W = \widetilde{N}_c \Lambda_L^3 = \widetilde{N}_c (\det M)^{1/\widetilde{N}_c}$$

- In conformal window power-law wave-function renormalization $Z_M(\mu) \sim \mu^{1-3\widetilde{N}_c/N_f}$

- Scaling of local minimum: **σ**: 4→5→4

$$V = -\mathcal{O}(m^\sigma), \quad \sigma = 1 + \frac{N_f^2}{N_f^2 - 3N_f\widetilde{N}_c + 3\widetilde{N}_c^2}.$$

The phases of SUSY QCD/AMSB QCD



PURE SUSY QCD

AMSB QCD

$F > 3N$ Theory IR free

$F = 3N$

Conformal

$F = \frac{3}{2}N$

Free magnetic phase

$F = N + 1$ s-confinement

$F = N$ Quantum modified constraint

$F = N - 1$

ADS

$F = 0$ Pure SYM - gaugino condensation

QCD-like

Runaway?
+QCD-like
local

QCD-like

Non-calc +QCD

QCD-like

The Chiral Lagrangian and η’ potential

R. d’Agnolo, R. Gupta, E. Kuflik, T. Roy,
M. Ruhdorfer and C.C.

- AMSB also a nice tool to find chiral Lagrangian and examine dynamics leading to η’ mass (or axion mass in extensions)

- Naive assumption U(1)$_A$ anomalous, broken by instantons, so instanton effects will give mass to η’?

- Form of chiral Lagrangian would be

$$\mathcal{L} = f_\pi^2 \mathrm{Tr}\left[(\partial_\mu U)^\dagger \partial^\mu U\right] + a\Lambda f_\pi^2 \mathrm{Tr}\, m_Q U + \mathrm{h.c.}$$

$$\mathcal{L}_{inst} = b\Lambda^2 f_\pi^2 e^{i\theta} \det U + \mathrm{h.c.}$$

The Chiral Lagrangian and η' potential

- Would correspond to instanton effect because ~ $e^{i\theta}$

- Would give η' mass ~ Λ

- Consistent with spurion analysis for axial U(1):

$$\theta \to \theta - F\alpha$$

$$\eta'/f_{\eta'} \to \eta'/f_{\eta'} + \alpha$$

- After integrating out η' get low-energy action

$$V_{min} = -2|a|\Lambda f_\pi^2 \sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos\bar{\theta}}$$

The Chiral Lagrangian and η' potential

- Issue: large N limit anomaly vanishes

$$\partial_\mu j^\mu_A \sim F\frac{g^2}{16\pi^2}\mathrm{Tr}G\tilde{G} \sim \frac{\lambda}{16\pi^2}\frac{F}{N}\mathrm{Tr}G\tilde{G} \to 0$$

- η' mass should vanish in this limit

- But from $V_{\eta'} = 2b\Lambda^2\cos(\theta + F\eta')$ does not vanish for large N

- Witten: η' needs to cancel *θ* dependence of pure QCD vacuum energy $E(\theta) = N^2 f(\theta)$

- Form of potential more like $\mathcal{L}_{\eta'} = \Lambda^2 f_\pi^2 (e^{i\theta}\mathrm{detU})^{1/\mathrm{N}}$

The Chiral Lagrangian and η’ potential

- Non-analytic - how is it 2**π** periodic in θ?

- Need to have several branches, potential of the form

$$V(\theta,\eta') = \Lambda^2 f^2 \mathrm{Min}_k \cos(\frac{\theta + F\eta' + 2\pi k}{N}), \quad k = 0, \ldots, N-1$$

- **η’** acts like a (heavy) axion and relaxes to minimum of potential to cancel θ dependence (and wash out branch structure)

$$\langle \eta' \rangle = -\frac{\theta + 2\pi k}{F}$$

- Check this picture in AMSB QCD

The Chiral Lagrangian in AMSB QCD

- Consider first F<N as we did before - with quark mass

$$W = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} + \mathrm{Tr}(M_Q M)$$

- The meson VEV as usual

$$\phi = \Lambda\left(\frac{N+F}{3N-F}\frac{\Lambda}{m}\right)^{(N-F)/(2N)} + \mathcal{O}(m_Q/m)$$

- The meson matrix:

$$Q^a_f = |\phi|\delta^a_f\,, \quad \bar{Q}^a_f = Q^a_{f'}U_{f'f}\,, \quad M = |\phi|^2 U$$

- **η**' part of U matrix, need to make sure we keep the whole phase everywhere

The Chiral Lagrangian in AMSB QCD

- Chiral Lagrangian:

$$V = -\,m\left[(3N-F)\left(\frac{\Lambda^{3N-F}}{|\phi|^{2F}}\right)^{1/(N-F)}\det(U)^{-1/(N-F)} + |\phi|^2\mathrm{Tr}(m_Q U)\right] + c.c.$$
$$-\,2\left(\frac{\Lambda^{3N-F}}{|\phi|^{2F}}\right)^{1/(N-F)}\det(U)^{-1/(N-F)}\mathrm{Tr}(m_Q^\dagger U^\dagger) + c.c\,.$$

- Has the branch structure like Witten predicted, but 1/(N-F) power. η' potential:

$$V = -\,2(3N-F)\left(\frac{N+F}{3N-F}\right)^{-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N} m|\Lambda|^3\cos\left(\frac{F}{N-F}\frac{\eta'}{f_{\eta'}} - \frac{\theta+2\pi k}{N-F}\right)$$
$$-\,2F\left(\frac{N+F}{3N-F}\right)^{1-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N}|m_Q||\Lambda|^3\cos\left(\frac{\eta'}{f_{\eta'}} + \theta_Q\right)$$
$$-\,4F\left(\frac{N+F}{3N-F}\right)^{-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N}|m_Q||\Lambda|^3\cos\left(\frac{N}{N-F}\frac{\eta'}{f_{\eta'}} + \theta_Q - \frac{\theta+2\pi k}{N-F}\right)\,,$$

The Chiral Lagrangian in AMSB QCD

- For N-F>1 NOT an instanton effect
- We know it is actually gaugino condensation
- For F=N-1 it actually IS an instanton effect, and no branches in QCD
- In that case the η' mass does not vanish for large N
- But also anomaly does not vanish, since both F,N →∞
- Which one is QCD? Does QCD with F=N have branches or not?

The Chiral Lagrangian in AMSB QCD

- F~N both large the situation is very different!

- For example F=N-1 and both large

$$V \overset{N\gg 1}{\to} -4N^{5/3}m^2|\Lambda_{\rm phys}|^2\cos\left((N-1)\frac{\eta'}{f_{\eta'}}-\theta\right)-2N^{5/3}|m_Q|m|\Lambda_{\rm phys}|^2\cos\left(\frac{\eta'}{f_{\eta'}}+\theta_Q\right)$$
$$-4N^{5/3}|m_Q|m|\Lambda_{\rm phys}|^2\cos\left(N\frac{\eta'}{f_{\eta'}}+\theta_Q-\theta\right) .$$

- No branches, η' mass does not go to zero

$$m^2_{\eta'} \propto N^{11/3}m^2|\Lambda_{\rm phys}|^2/f^2_{\eta'} \sim N^3$$

- Large F,N qualitatively different from large N, fixed F limits!

The Chiral Lagrangian in AMSB QCD

- The F=N,N+1 special cases

- Only consider mesonic VEV, assume other branches OK

- For example F=N

$$W = X\left(\frac{\det(M)-\bar{B}B}{\Lambda^{2N}}-1\right)+m_Q\mathrm{Tr}(M)$$

$$K = \frac{\mathrm{Tr}(M^\dagger M)}{\alpha|\Lambda|^2}+\frac{X^\dagger X}{\beta|\Lambda|^4}+\frac{\bar{B}^\dagger\bar{B}}{\gamma|\Lambda|^{2N-2}}+\frac{B^\dagger B}{\delta|\Lambda|^{2N-2}}$$

- Resulting η' potential

$$V = -2|\Lambda|^2(|\Lambda|^2+(N-2)m^2)\cos\left(N\frac{\eta'}{f_{\eta'}}-\theta\right)-2Nm|m_Q||\Lambda|^2\cos\left((N-1)\frac{\eta'}{f_{\eta'}}-\theta_Q-\theta\right)$$
$$-4Nm|m_Q||\Lambda|^2\cos\left(\frac{\eta'}{f_{\eta'}}+\theta_Q\right).$$

- No branches - looks like an instanton effect!

The Chiral Lagrangian in AMSB QCD

- After integrating out η' get the usual F branches

$$V_k(\theta) = -6Nm|m_Q||\Lambda|^2 \cos\left(\frac{\theta + N\,\theta_Q + 2\pi k}{N}\right)$$

- Very similar results for F=N+1:

$$\begin{aligned} V = &- 2(N-2)\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{(N+1)/(N-1)} m|\Lambda|^3 \cos\left((N+1)\frac{\eta'}{f_{\eta'}} - \theta\right) \\ &- 2(N+1)\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{N/(N-1)} |m_Q||\Lambda|^3 \cos\left(N\frac{\eta'}{f_{\eta'}} - \theta_Q - \theta\right) \\ &- 4(N+1)\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{1/(N-1)} m|m_Q||\Lambda|^2 \cos\left(\frac{\eta'}{f_{\eta'}} + \theta_Q\right) . \end{aligned}$$

- Again looks like instanton effect - no branches

The Chiral Lagrangian in AMSB QCD

- For F>N+1 - will have a Seiberg dual SU(F-N)
- Along meson direction DUAL quarks get mass - will get gaugino condensate in dual group - analogous to ADS superpotential, will again have F-N branches…
- We will get very similar results as for F<N, with N-F↔F-N

$$\begin{aligned} V = & - 4(3N-2F)\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{F/(2N-F)} m|\Lambda|^3 \cos\left(\frac{F}{F-N}\frac{\eta'}{f_{\eta'}} - \frac{\theta+2\pi k}{F-N}\right) \\ & - 2F\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-F)} |m_Q||\Lambda|^3 \cos\left(\frac{N}{F-N}\frac{\eta'}{f_{\eta'}} - \theta_Q - \frac{\theta+2\pi k}{F-N}\right) \\ & - \frac{4FN}{2F-3N}\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-F)} |m_Q||\Lambda|^3 \cos\left(\frac{\eta'}{f_{\eta'}} + \theta_Q\right) . \end{aligned}$$

New results in chiral gauge theories

(C.C., Murayama, Telem ’21)

- These are the hardest to analyze, currently no technique on the lattice (yet) that could do a reliable serious simulation

- Proposal from 70’s-80’s: ``tumbling”

- Postulate the presence of fermion bilinear condensates that break the gauge group until it is QCD-like

- Usually assume most attractive channel (MAC) is condensing first

New results in chiral gauge theories

- Example of tumbling: SU(N) with anti-symmetric fermion and (N-4) anti-fundamentals

- $\langle A^{ab}\bar{F}^{b}_{i}\rangle = v^3\delta^{a}_{i} \neq 0$ breaking to SU(N) x SU(N-4) to SU(N-4)$_V$ x SU(4) where SU(4) is the remaining gauge symmetry that is QCD-like.

- Resulting theory would have SU(N-4) global symmetry with massless composite $A\bar{F}_{\{i,}\bar{F}_{j\}}$

- ’t Hooft anomaly matching conditions satisfied, but not really clear if this is indeed the correct low-energy phase of the theory

SU(5)

- This is one of the most well-known SUSY theories

- ``The mother” of SUSY breaking

- No flat directions, anomaly-free R-symmetry

- Dynamical ~~SUSY~~ expected w/o AMSB

- Can make theory calculable by adding extra flavor(s)

- Interesting new(?) observation: there is an unbroken $U(1)_5$ in the DSB vacuum

SU(5) with an extra flavor

	$SU(5)$	$SU(2)$	$U(1)_M$	$U(1)_Y$	$U(1)_R$	$U(1)_5$
A	$\begin{array}{c}\square\\[-6pt]\square\end{array}$	1	2	1	0	1
$\bar{F}_i$	$\overline{\square}$	$\square$	-1	-3	-6	2 -3
F	$\square$	1	-4	3	8	-2
$B_1 = AAF$	1	1	0	5	8	0
$H = A\bar{F}_1\bar{F}_2$	1	1	0	-5	-12	0
$M = F\bar{F}$	1	$\square$	-5	0	2	0 -5

- The unbroken U(1) is $U(1)_5 = 5T_3 + \frac{1}{2}Q_M$

- In this theory need tree-level

$$W_{tree} = \lambda_1 B_1 + \lambda_2 H + m_M F\bar{F}_1$$

- A massless fermion will match the 't Hooft anomalies, in $m_M \longrightarrow \infty$ limit will be $A\bar{F}\bar{F}$

The original SU(5) theory

- The U(1)$_5$ symmetry will be unbroken, with a massless fermion $A\bar{F}\bar{F}$ matching the anomalies

- This is in addition to Goldstino (not carrying global charges)

- Adding AMSB: don’t expect the dynamics to be influenced much, since DSB ~ $\Lambda \gg$m. But: Goldstino will pick up mass, while $A\bar{F}\bar{F}$ remains exactly massless.

The original SU(5) theory

- Tumbling picture: SU(5)→SU(4)xU(1)$_D$ via $\epsilon_{abcde}A^{bc}A^{de}$ most attractive channel

- Decomposition of remaining fermions:

$$A \rightarrow \mathbf{6}_0 + \mathbf{4}_{-5/2} \qquad \bar{F} \rightarrow \bar{\mathbf{4}}_{5/2} + \mathbf{1}_{-5}$$

- The vectorlike pieces condense, but $\mathbf{1_{-5}}$ remains

- We will see more general case will require augmenting this by additional condensate $A^{ab}\bar{F}_b$

- Breaking pattern same as first condensate

The SU(N) case for N=2n+1 odd

	$SU(N)$	$U(1)$	$SU(N-4)$	$Sp(N-5)$	$U(1)'$
A	$\begin{array}{c}\square\\\square\end{array}$	$-N+4$	$\mathbf{1}$	$\mathbf{1}$	$-N+4$
$\bar{F}_i$	$\overline{\square}$	$N-2$	$\square$	$\square$ $\mathbf{1}$	$\frac{1}{2}(N-4)$ $\frac{(N+1)(N-4)}{2}$
$A\bar{F}_i\bar{F}_{N-4}$	$\mathbf{1}$	N	$\square$	$\square$ $\mathbf{1}$	$\frac{N(N-4)}{2}$ $N(N-4)$

- Flat directions in the SUSY limit:

$$A = \frac{\varphi}{\sqrt{2}} \left(\begin{array}{c|c} J_{(N-5)} & 0 \\ \hline 0 & 0_{5\times 5} \end{array} \right) \ , \quad \bar{F} = \varphi \left(\begin{array}{c|c} I_{(N-5)} & 0 \\ \hline 0 & 0_{5\times 1} \end{array} \right) \ ,$$

- Break SU(N)xSU(N-4)xU(1) to SU(5)xSp(N-5)xU(1)

- Sp due to fact that $A\bar{F}_i\bar{F}_j \;=\; \frac{1}{\sqrt{2}}\varphi^3 J_{ij},$ along flat direction

The SU(N) case for N=2n+1 odd

- Dynamical scale of the unbroken SU(5) (which has same matter content as previous theory:

$$\Lambda_5^{13} = \frac{\Lambda_N^{2N+3}}{(\mathrm{Pf}' A\bar{F}\bar{F})(\mathrm{Pf}' A)}$$

- After DSB in SU(5) will get potential

$$V \approx \Lambda_5^4 = \left(\frac{\Lambda_N^{2N+3}}{\varphi^{2N-10}}\right)^{4/13}$$

- On its own runaway, in SUSY can stabilize via $\lambda A\bar{F}_i\bar{F}_j J^{ij}$

- With AMSB no stabilization needed, since loop induced soft breaking terms will stabilize

The SU(N) case for N=2n+1 odd

- With AMSB $m^2_{A,\bar{F}_i} = \frac{g^4}{(8\pi^2)^2} 2C_i(2N+3)m^2,$ $C_i = \begin{cases} \frac{(N+1)(N-2)}{N} & \text{for } A, \\ \frac{N^2-1}{2N} & \text{for } \bar{F}_i. \end{cases}$
- Stable ground state at $\varphi \approx \Lambda \left(\frac{4\pi\Lambda}{m}\right)^{13/(4N-7)} \gg \Lambda.$
- Note: runaway potential here from DSB NOT from ADS superpotential - no corresponding tree-level AMSB generated (unlike QCD example, or even case to come)
- The low-energy dynamics: symmetry breaking pattern SU(N-4)xU(1) →Sp(N-5)xU(1)

The SU(N) case for N=2n+1 odd

- Can check all anomalies matched by massless fermions $A\bar{F}_i\bar{F}_{N-4}$ fundamental + singlet under Sp

- Tumbling picture: MAC

$$\langle A^{ab}\bar{F}_{bi}\rangle \sim \Lambda^3\delta_i^a \neq 0, \qquad i, a \leq N-4\,.$$

- Would break SU(N)xSU(N-4)xU(1) → SU(4)xSU(N-4)$_V$xU(1)

- The SU(4) is diagonal QCD-like
- SU(N-4) global symmetry with color-flavor locking.

- To make tumbling agree with AMSB picture:

The SU(N) case for N=2n+1 odd

- Second most attractive channel

$$\langle \bar{F}_{ai}\bar{F}_{bj}\rangle \sim \Lambda^3 J_{ab}J_{ij} \neq 0, \qquad 1 \leq i,j,a,b \leq N-5\,,$$

- This will break the SU(N-4)⟶Sp(N-5)

- Only antisymmetric part is attractive hence the J’s

- AMSB provides an alternative proposal to the actual phase of this theory - should in principle at some point be testable.

- Dynamics can persist to m≫**Λ** - good guess for non-SUSY phase?

The SU(N) case for N even

	$SU(N)$	$U(1)$	$SU(N-4)$	$Sp(N-4)$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$-N+4$	$\mathbf{1}$	$\mathbf{1}$
$\bar{F}_i$	$\overline{\square}$	$N-2$	$\square$	$\square$
$A\bar{F}_i\bar{F}_j$	$\mathbf{1}$	N	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix} \oplus \mathbf{1}$
$\mathrm{Pf}A$	$\mathbf{1}$	$-\frac{1}{2}N(N-4)$	$\mathbf{1}$	$\mathbf{1}$

- D-flat direction breaks theory to Sp(4)=SO(5), gaugino condensate generates

$$W = \left(\frac{\Lambda^{2N+3}}{(\mathrm{Pf}\,A\bar{F}\bar{F})(\mathrm{Pf}\,A)} \right)^{1/3}$$

- This will have a corresponding AMSB term, which will stabilize runaways at $A \sim \bar{F} \sim \Lambda(\Lambda/m)^{3/2N}$

- Remaining global symmetry Sp(N-4), all fermions massive, no ’t Hooft anomalies

The SU(N) case for N even

- Tumbling picture:

$$\langle A^{ab}\bar{F}_{bi}\rangle \sim \Lambda^3 \delta^a_i , \qquad i,j,a,b \leq N-4.$$
$$\langle \bar{F}_{ai}\bar{F}_{bj}\rangle \sim \Lambda^3 J_{ab} J_{ij}$$

- Unbroken Sp(N-4) with no massless fermions

- Picture can survive to m»$\boldsymbol{\Lambda}$ again

SU(N) with a symmetric tensor

(C.C., Murayama, Telem ’21)

- Another example of a chiral gauge theory - more difficult to analyze

	$SU(N-4)$	$SU(N)$	$U(1)$	$U(1)_R$
S	$\square\!\square$	$\mathbf{1}$	$-2N$	$\frac{12}{(N+1)(N-4)}$
$\bar{F}_i$	$\bar{\square}$	$\square$	$2N-4$	$\frac{6(N-5)}{(N+1)(N-4)}$
$M_{ij}=S\bar{F}_i\bar{F}_j$	$\mathbf{1}$	$\square\!\square$	$2N-8$	$\frac{12}{N+1}$
$U=\mathrm{det}S$	$\mathbf{1}$	$\mathbf{1}$	$2N(4-N)$	$\frac{12}{N+1}$

- Magnetic dual found by Pouliot & Strassler

	$Spin(8)$	$SU(N)$	$U(1)$	$U(1)_R$	$SO(N)$
q^i	$\mathbf{8_v}$	$\bar{\square}$	$4-N$	$\frac{N-5}{N+1}$	$\square$
p	$\mathbf{8_s}$	$\mathbf{1}$	$N(N-4)$	$\frac{N-5}{N+1}$	$\mathbf{1}$
M_{ij}	$\mathbf{1}$	$\square\!\square$	$2N-8$	$\frac{12}{N+1}$	$\mathbf{1}+\square\!\square$
U	$\mathbf{1}$	$\mathbf{1}$	$2N(4-N)$	$\frac{12}{N+1}$	$\mathbf{1}$

$$\tilde{W}_{\rm tree} \;=\; \frac{1}{\mu_1^2}M_{ij}q^iq^j+\frac{1}{\mu_2^{N-5}}Upp$$

$$(\Lambda^{2N-11})^2\tilde{\Lambda}^{17-N} \;=\; \mu_1^{2N}\mu_2^{N-5}$$

SU(N) with a symmetric tensor

- Superpotential cubic, so adding AMSB naively loop-level leading to a local minimum

$$V \approx -\left(\frac{\lambda^2}{16\pi^2}\right)^4 m^4$$

- However there is a deeper one - go out on moduli space along M and S, then dual quarks and spinor become massive and integrate them out - gaugino condensate

$$\tilde{\Lambda}_L^{18} = \frac{\det \tilde{M}\, \tilde{U}}{\tilde{\Lambda}^{N-17}}$$

$$W_{\rm dyn} = e^{i\frac{\pi k}{3}} (\tilde{\Lambda}_L^{18})^{1/6} = e^{i\frac{\pi k}{3}} \left(\frac{\det \tilde{M}\, \tilde{U}}{\tilde{\Lambda}^{N-17}}\right)^{1/6}$$

SU(N) with a symmetric tensor

- This will imply a ``tree-level" AMSB term

$$\mathcal{L}_{\rm tree} = m\frac{N-17}{6}W_{\rm dyn} + c.c.$$

- The minimum along this direction (loop induced AMSB is negligible here) is at

$$\tilde{M}_{ij} \approx \delta_{ij} m \left(\frac{\tilde{\Lambda}}{m}\right)^{\frac{N-17}{N-11}}, \quad \tilde{U} \approx m\left(\frac{\tilde{\Lambda}}{m}\right)^{\frac{N-17}{N-11}},$$

$$V \approx -m^4 \left(\frac{\tilde{\Lambda}}{m}\right)^{\frac{2(N-17)}{N-11}}.$$

- Minimum deeper for N>17

SU(N) with a symmetric tensor

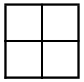
- Symmetry breaking pattern: SU(N)xU(1) → SO(N)

- Note M has maximal rank N - OK in ~~SUSY~~ vacua

- Differs again from old tumbling predictions

- Tumbling interpretation: want a symmetric condensate for SO, but it is not attractive.

- Resolution: two condensates, first:



$$: \qquad S_{ab}S_{cd} - S_{ad}S_{cb} \propto \delta_{ab}\delta_{cd} - \delta_{ad}\delta_{cb}\,.$$

Breaks SU(N-4) gauge to SO(N-4), now symmetric attractive

$$\delta_{ab}\bar{F}_i^a\bar{F}_j^b \propto \delta_{ij}$$

- Breaks SU(N) global to SO(N)

SU(N) with a symmetric tensor

- Discussion valid as long as N>17 (dual theory in free magnetic phase)

- For N≤17 dual theory conformal - AMSB naively vanishes at fixed point.

- Our initial guess was theory flows to conformal fixed point - not quite sure if this is correct (see Hitoshi’s upcoming paper for QCD in the conformal regime)

The SO(N) series

(C.C., Gomes, Murayama, Telem ’21)

- Interesting since this could have ``true confinement”

- For SU(N) with quarks charges can always be screened, don’t expect true area law for Wilson loops

- For SO(N) with matter in vector (N-dim’l rep)

- Note SO(N) could stand for several groups with same Lie algebra but slightly different global structures, Spin(N), $SO(N)_+$ or $SO(N)_-$ for now doesn’t matter (but will make a difference for actual Wilson loop behavior)

SUSY SO(N)

- SO(N) with F vectors

	$SO(N_c)$	$SU(N_F)$	$U(1)_R$
Q^i	□	□	$\frac{N_F-N_c+2}{N_F}$
λ	□ □	$\mathbf{1}$	1
M_{ij}	$\mathbf{1}$	□□	$2\frac{N_F-N_c+2}{N_F}$

- Global symmetries $(SU(N_F) \times U(1)_R \times \mathbb{Z}_{2N_F} \times \mathbb{Z}_2/\mathbb{Z}_{N_F})$

- Flat directions parametrized by mesons and baryons $M^{ij} = Q^i Q^j$ $B^{[i_1,\ldots i_{N_c}]} = Q^{[i_1} \ldots Q^{i_{N_c}]}$

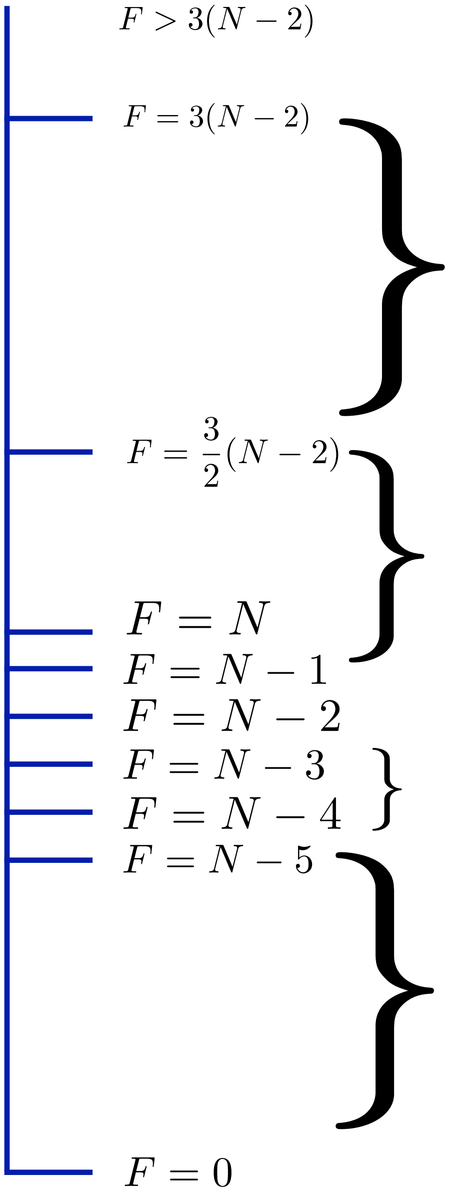
$$M = \left(\begin{array}{c|c} \operatorname{diag}\left(\varphi_1^2, \ldots, \varphi_{N_c}^2\right) & 0 \\ \hline 0 & 0_{N_F-N_c \times N_F-N_c} \end{array}\right)$$

$$B^{1,\ldots N_c} = \varphi_1 \ldots \varphi_{N_c} \, .$$

- For SO(N) baryons NOT independent

The phases of the SUSY SO(N) theories

Intriligator, Seiberg '95



$F > 3(N-2)$

IR free

$F = 3(N-2)$

Conformal - non-abelian Coulomb phase

$F = \frac{3}{2}(N-2)$

Free magnetic phase

$F = N$

$F = N-1$

$F = N-2$

Abelian Coulomb phase

$F = N-3$

$F = N-4$

Multiple branches,
s-confining vs ADS

$F = N-5$

ADS superpotential, runaway vacuum

$F = 0$

Pure SYM - gaugino condensation

Phases of SUSY SO(N)

- Phase structure more rich than for SU(N)
- Most notable difference: for F=N-2 we have an abelian Coulomb phase
- Simple explanation: N-2 vectors generically break SO(N)→SO(2)~U(1)
- This case is essentially a Seiberg-Witten type theory (but for N=1 SUSY, so no pre-potential, only fix the holomorphic gauge kinetic terms)

The SUSY F=N-2 theory

(C.C., Gomes, Murayama, Telem ’21)

- Since R-charge of M is zero, quantum moduli space.

- U=det M is the variable on the moduli space, and gauge kinetic function will depend on that

- The Seiberg-Witten curve is

$$y^2 = x^3 + x^2 \left(8\Lambda^{2N_c-4} - \mathrm{det}M\right) + 16\Lambda^{4N_c-8}x$$

- Has 2 singularities at U=0 and and $U=U_1= 16\ \Lambda^{2F}$

- There are massless monopoles/dyons at those singularities

The theory around the singularities

- Around U=0

	$SO(N_c)$	$SU(N_F)$	$U(1)_R$
Q^i	$\square$	$\square$	0
λ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	1
M_{ij}	$\mathbf{1}$	$\square\square$	0
$q_i^{\pm}$	$-$	$\bar{\square}$	1
λ_{mag}	$-$	$\mathbf{1}$	1

- N Massless dyons and anti-dyons satisfying anomaly matching

- Around this singularity superpotential

$$W_{\text{dyon}} \;=\; \frac{1}{\mu} f(t)\, M^{ij} q_i^+ q_j^-$$

- Where $t = U\Lambda^{4-2N_c}$ and f(t) holomorphic, f(0)=1.

The theory around the singularities

- Around U=U_1

	$SO(N_c)$	$SU(N_F)$	$U(1)_R$	$U(1)_{\rm mag}$	$SO(N_F)$
Q^i	$\square$	$\square$	0	$-$	$\square$
λ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	1	$-$	$\mathbf{1}$
M_{ij}	$\mathbf{1}$	$\square\square$	0	$-$	$\mathbf{1}+\square\square$
$E^{\pm}$	$-$	$\mathbf{1}$	1	± 1	$\mathbf{1}$
$\lambda_{\rm mag}$	$-$	$\mathbf{1}$	1	0	$\mathbf{1}$

- Massless monopole-antimonopole

- Around this singularity $$W_{\rm mon} = \tilde{f}\left(\frac{U-U_1}{\Lambda^{2N_F}}\right) E^+ E^-$$

- Leading expression for canonically normalized fields

$$W_{\rm mon} = \Lambda\left(\frac{\tilde{U}}{\Lambda^{N_F}} - 16\right) \tilde{E}^+ \tilde{E}^-$$

Adding AMSB

- Around U=0

- Tree-level AMSB highly suppressed since essentially cubic superpotential

- Loop induced will dominate + tree-level quartics

- AMSB mass term: gauge contribution negative, but Yukawa induced term positive. Since ratio of couplings flows to fixed number possible to find overall sign - positive.

- So potential: positive loop AMSB + loop AMSB A-terms + tree-level quartic

Adding AMSB

- Will result in local minimum $\mathcal{O}(\frac{m}{16\pi^2})$ from origin

- Symmetry breaking pattern from

$$V = \frac{1}{2}(q^+ \cdot q^{+*})(q^- \cdot q^{-*}) + |Mq^+|^2 + |Mq^-|^2 + V_{\text{AMSB}}$$

- VEV of form

$$q^+ = \begin{pmatrix} 1 \\ 0 \\ \hline 0 \\ \vdots \\ 0 \end{pmatrix} \alpha, \quad q^- = \begin{pmatrix} 0 \\ 1 \\ \hline 0 \\ \vdots \\ 0 \end{pmatrix} \alpha, \qquad M \propto \left(\begin{array}{cc|ccc} 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \hline 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{array}\right)$$

- SU(F)→SU(F-2) global symmetry breaking,

$$V = -\mathcal{O}(\frac{m}{16\pi^2})^4$$

Adding AMSB

- Around U=U$_1$ very different - tree-level AMSB!

$$V_{\tilde U\sim\tilde U_1} = \Lambda^2\left|\left(\frac{\tilde M}{\Lambda}\right)^{N_F} - 16\right|^2 \left(|\tilde E^+|^2+|\tilde E^-|^2\right) + \frac{1}{kN_F}\left|N_F\left(\frac{\tilde M}{\Lambda}\right)^{N_F-1}\right|^2 |\tilde E^+\tilde E^-|^2 +$$

$$m\Lambda\left[16+(N_F-1)\left(\frac{\tilde M}{\Lambda}\right)^{N_F}\right]\tilde E^+\tilde E^- + \text{c.c.}$$

- Will force monopole condensation

- Minimum at $\tilde M = 16^{\frac{1}{N_F}}\Lambda \quad , \quad |\tilde E^+||\tilde E^-| = 16^{\frac{2}{N_F}-1}\, km\Lambda$

$$V_{\min} = -16^{\frac{2}{N_F}} N_F\, km^2\Lambda^2\,.$$

- This is the true global minimum

- SU(F)→SO(F) symmetry breaking pattern

Adding AMSB

- Confinement with chiral symmetry breaking in non-SUSY setting

- Relate it to fermion bilinears:

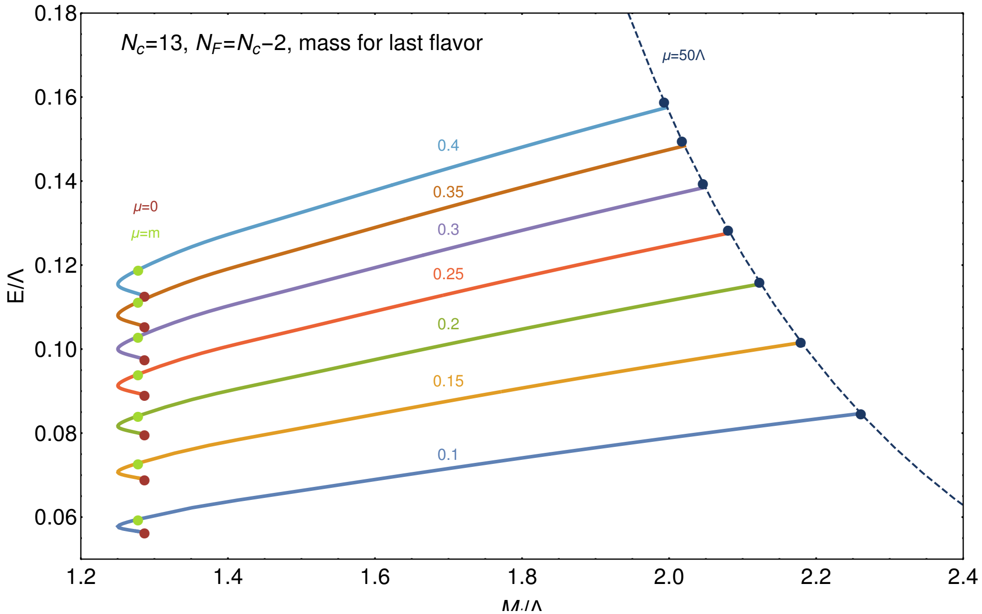
$$\langle \psi_i^* \psi_j^* \rangle = F^*_{M_{ij}} = 16\Lambda^2 M_{ij}^{-1} E^+ E^- \propto \delta_{ij} k m \Lambda^2 \neq 0.$$

F<N-2

- Would like to show that these theories confining as well with same mechanism, but don’t have monopoles there

- Add extra flavors to reach F=N-2 and explicit mass **μ** to additional flavors - want to show that still get monopole condensate and same symmetry breaking pattern

- Monopole condensate persists, M interpolates smoothly to known VEV with lower number of F SU(F)→SO(F) always



Free magnetic phase: N-2<F<3/2(N-2)

- In this case there is an IR free Seiberg dual:

	$SO(N_F - N_c + 4)$	$SU(N_F)$	$U(1)_R$
q^i	$\square$	$\overline{\square}$	$\frac{N_c-2}{N_F}$
M_{ij}	$\mathbf{1}$	$\square\square$	$2\frac{N_F-N_c+2}{N_F}$

$$W_{\rm dual} \;=\; \frac{1}{2\mu} M^{ij} q_i q_j \,.$$

- Near the origin there is a local minimum with

$$V \;=\; -\mathcal{O}(\tfrac{m}{16\pi^2})^4$$

- However far out on moduli space all dual quarks massive, generate superpotential ~$\Lambda_L{}^3$

Free magnetic phase: N-2<F<3/2(N-2)

- There will be a corresponding tree-level AMSB term

$$V_{\mathrm{AMSB}} = \quad -2m\tilde{\Lambda}^3 \, \frac{\frac{3}{2}(N_c-2)-N_F}{N_F-(N_c-2)} \left(\frac{16^{\frac{1}{N_F}}\det(\tilde{M})}{\tilde{\Lambda}^{N_F}}\right)^{\frac{1}{N_F-(N_c-2)}} + c.c.$$

- Minimum at $\tilde{M}^{ij} \sim 4^{\frac{N_F-(N_c-2)-2}{2(N_c-2)-N_F}} \left(f\frac{m}{\Lambda}\right)^{\frac{N_F-(N_c-2)}{2(N_c-2)-N_F}} \Lambda\,\delta^{ij}$

$$V_{\min} \sim \left(f\frac{m}{\Lambda}\right)^{\frac{2(N_c-2)}{2(N_c-2)-N_F}} \Lambda^4$$

- SU(F)→SO(F) breaking again

Summary of SO(N) phases

(C.C., Gomes, Murayama, Telem ’21)

Range	SUSY	+AMSB
$N_F = 1$	run-away	confinement
$1 < N_F < N_c - 4$	run-away	confinement+χSB
$N_F = N_c - 4$	2 branches	confinement+χSB
$N_F = N_c - 3$	2 branches	confinement+χSB
$N_F = N_c - 2$	Coulomb	confinement+χSB
$N_F = N_c - 1$	free magnetic 2 branches	confinement+χSB
$N_F = N_c$	free magnetic 2 branches	confinement+χSB
$N_c + 1 \leq N_F \leq \frac{3}{2}(N_c - 2)$	free magnetic	confinement +χSB
$\frac{3}{2}(N_c - 2) < N_F \leq 3(N_c - 2)$	CFT	CFT
$3(N_c - 2) < N_F$	IR free	run-away

← CFT???

- All of the various exotic phases collapse to one and same confinement +χSB phase with same SU(F)→SO(F) breaking pattern

What happens for m≫Λ?

- Our results pretty clean for m≪Λ - when perturbing around SUSY theories. Already find quite interesting results and breaking patterns.

- What happens for m≫Λ? Possibly phase transition

- We have seen our results can at least in principle be connected to non-SUSY m⟶∞ limit no PT needed

- Can holomorphy help? If PT must be for |m| ~ |Λ|

- But both holomorphic - not possible to write a holomorphic equation for phase boundary of this sort????

Summary

- Use SUSY theories for finding vacuum structure of gauge theories

- AMSB appears to be superior method for perturbing SUSY theories

- UV insensitive, predictive and usually gives results as expected in non-SUSY theories

- Established QCD-like vacua for most of SUSY-QCD based theories

- Established η' potential and branches - for fixed F behaves as guessed but for large F qualitatively new

Summary

- Found novel symmetry breaking patterns for chiral gauge theories (antisymmetric & symmetric)

- Establish confinement & χSB in SO(N) theories via monopole condensation