Planckian dissipation

Jan Zaanen

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Nederlandse Organisatie voor
Wetenschappelijk Onderzoek

The dissipative world of apes ...

QuickTime™ and a
decompressor
are needed to see this picture.

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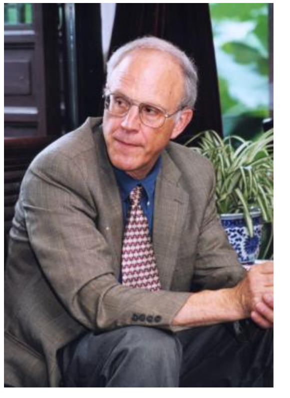
decompressor
are needed to see this picture.

The Gross list: the 14 Big Questions

0. The origin of temperature, dissipation and probability?
1. The origin of the universe?
2. What is dark matter?
   •
   •
11. What is space-time?
   •



2004

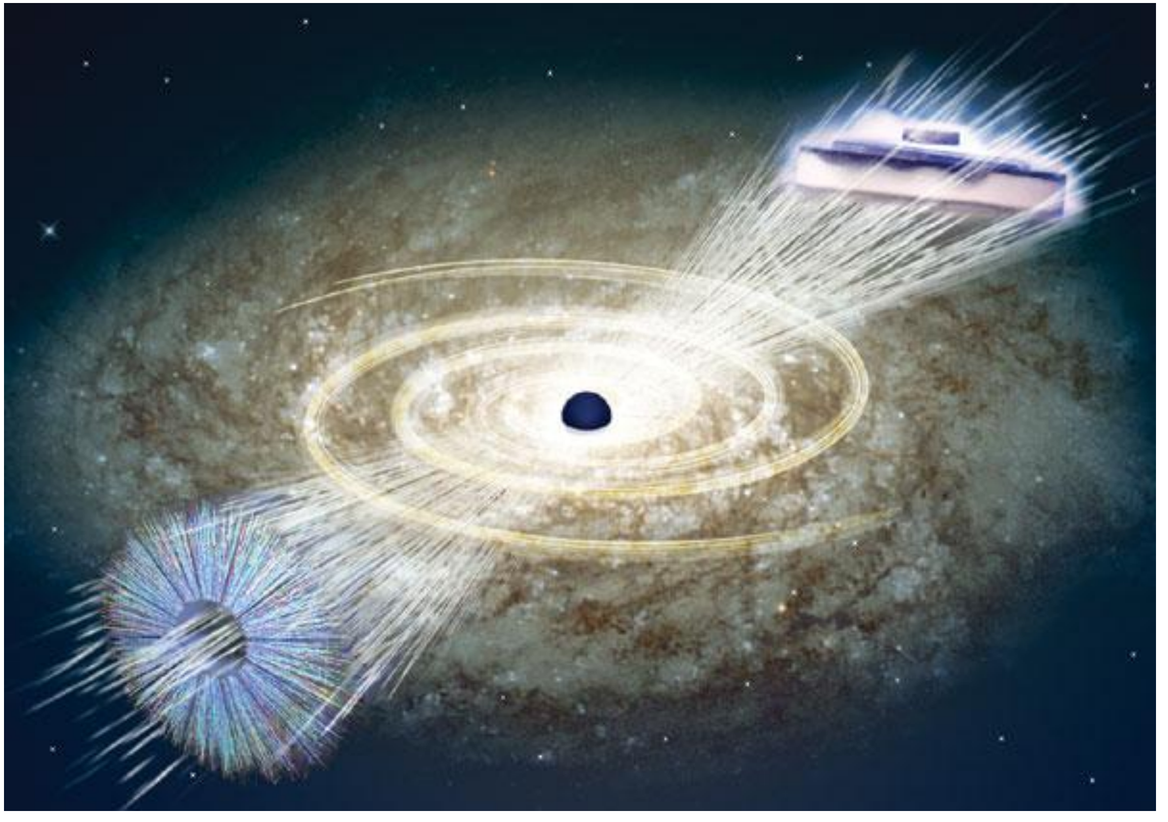
THEORETICAL PHYSICS

A black hole full of answers

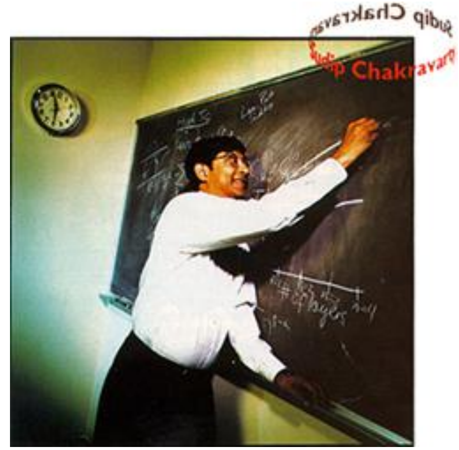
Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007



Quantum criticality



Sudip Chakravarty (UCLA)

QuickTime™ and a decompressor are needed to see this picture.

Subir Sachdev (Harvard)

Plan

1. The dissipative world of apes ....

2. The high Tc superconductivity saga: the past.

3. Viscosity, dissipation and black holes: the AdS/CFT magic

4. The high Tc superconductivity saga: the future is critical.

5. Conclusions

The second law ....

Dissipation: useful work has to turn into worthless heat ...

$$\frac{dS_{Cl}}{dt} > 0$$

Let's go quantum:

Von Neumann

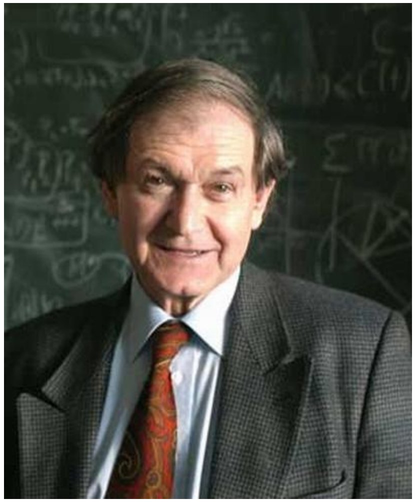
$$S_{vN} = Tr[\rho \ln \rho]$$

Thermal density matrix: $\rho_{ij} = \delta_{ij} \frac{e^{-\beta E_i}}{\sum_i e^{-\beta E_i}}$ $\Rightarrow S_{vN} = S_{Cl}$

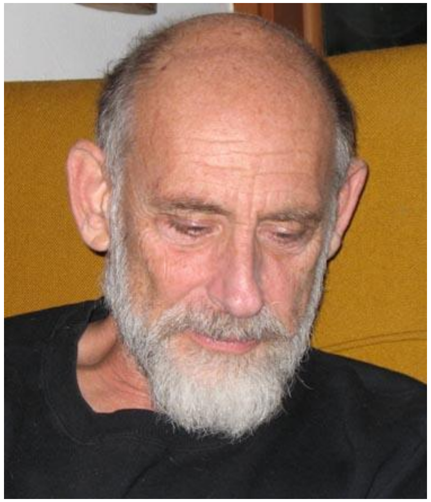
Unitary time evolution: $i\hbar \partial_t \rho = -[\rho, H]$ $\Rightarrow \frac{dS_{vN}}{dt} = 0$

The second law is beyond quantum physics !!

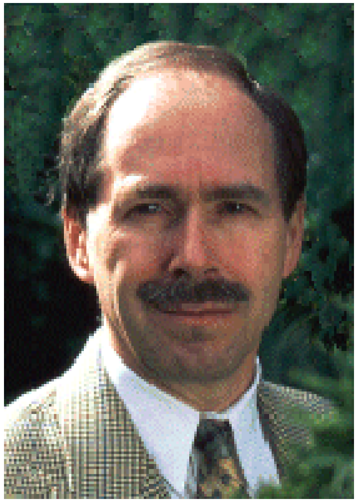
Unitarity versus general covariance



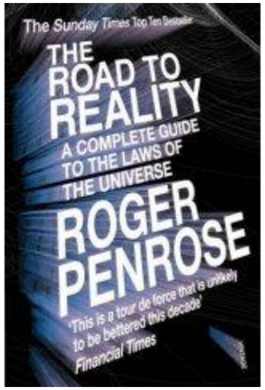
Penrose

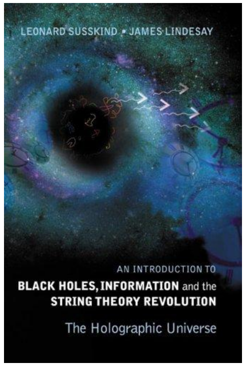


Susskind



't Hooft





See also: Van Wezel, Oosterkamp, JZ, cond-mat/0706.3976

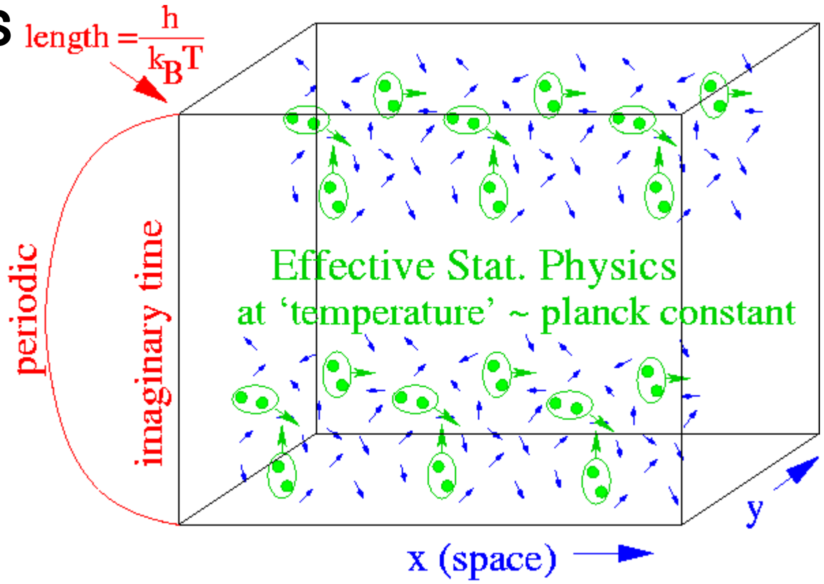
Quantum dissipation in equilibrium worlds

Fluctuation-dissipation theorem:

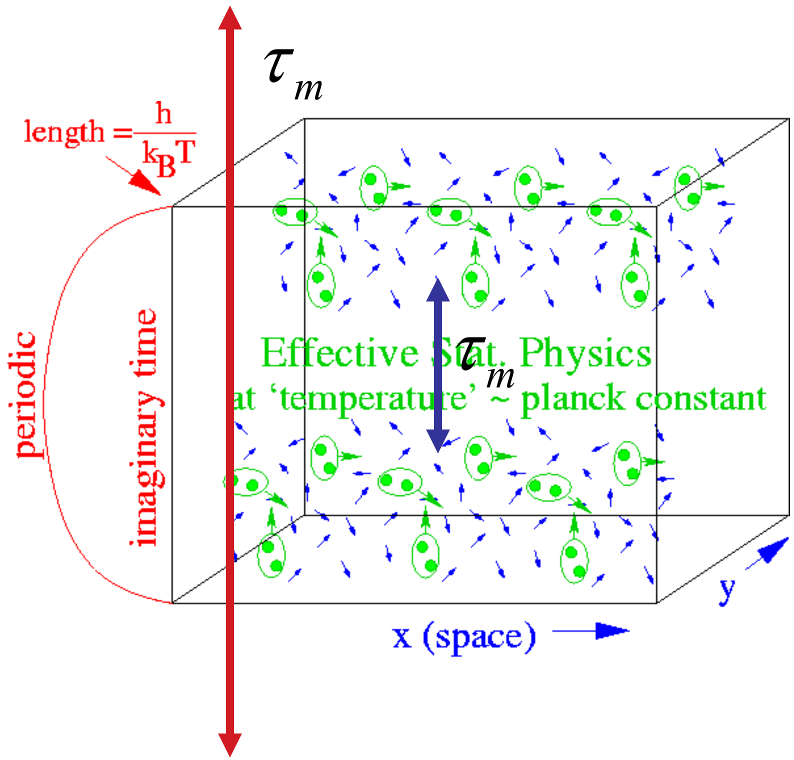
$$\chi''(\vec{k},\omega) = \frac{1}{2\hbar}\left(1 - e^{-\hbar\omega\beta}\right) S\left(\vec{k},\omega\right)$$

Green-Kubo-Matsubara

Thermal quantum field theory = Euclidean path integrals



Wick rotation: time versus temperature



Two point Euclidean correlators:

$$\Psi(\tau,\vec{r}) = \langle \phi(\tau,\vec{r})\,\phi(0,0)\rangle$$

Analytically continue to 'our' Minkowski time => susceptibilities = observables

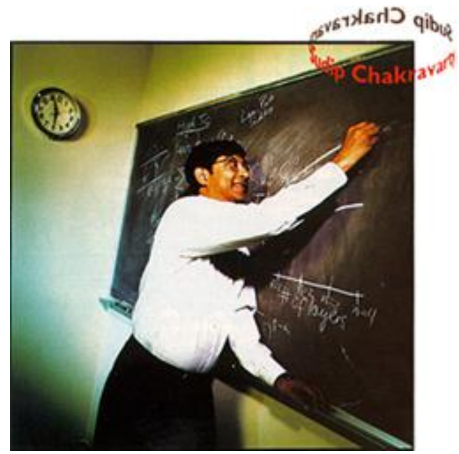
$$\chi(t,\vec{r}) = \Psi(i\tau,\vec{r})$$

$$\tau_\hbar = \frac{\hbar}{k_B T}$$

Measurement time **short** compared to $\tau_\hbar$: **'coherent' = unitary dynamics**

Measurement time **long** compared to $\tau_\hbar$: **'overdamped' = dissipative dynamics**

Quantum criticality



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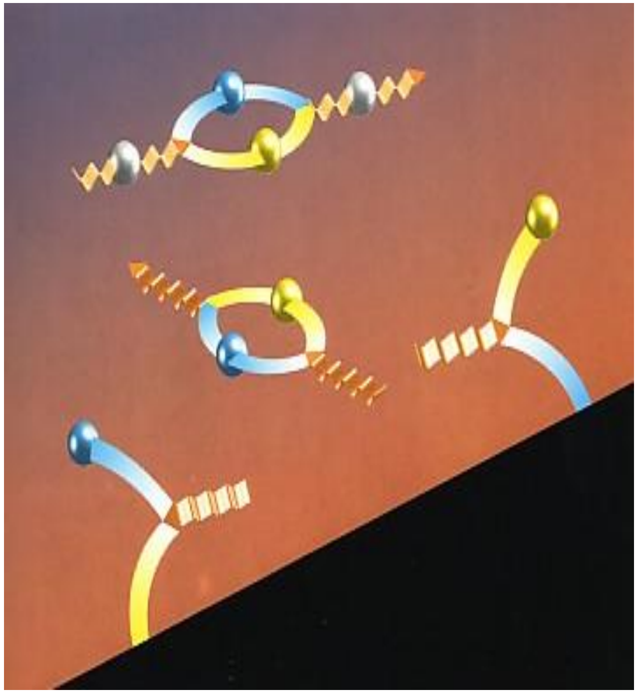
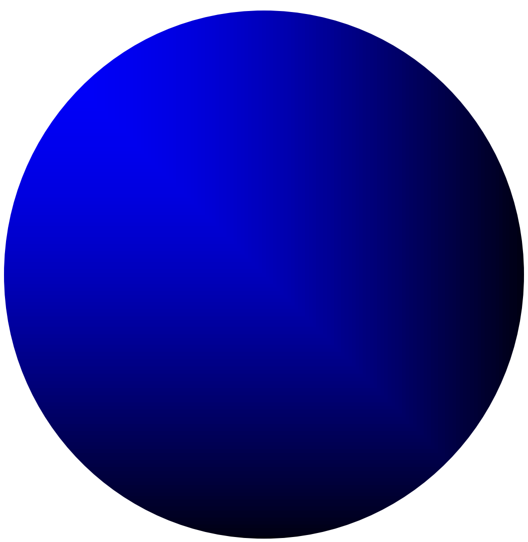
Hawking Temperature & Entropy



$$T = \frac{\hbar g}{2\pi k c}$$

g = acceleration at horizon

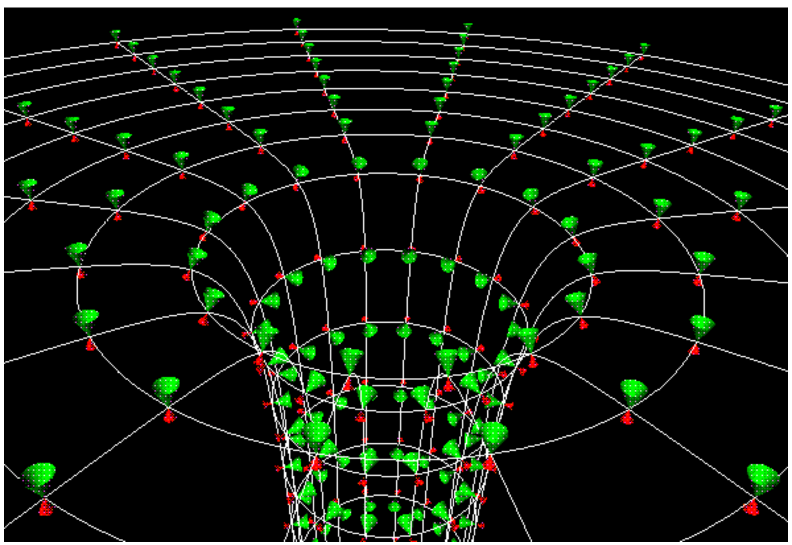


$$S = \frac{k c^3 A}{4 \hbar G}$$

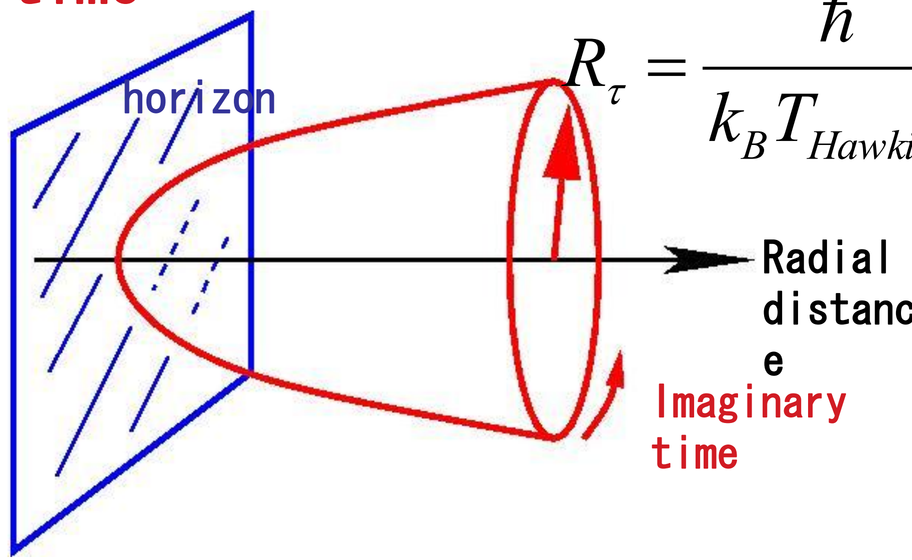
A = area of horizon

Minkowski versus Euclidean black holes

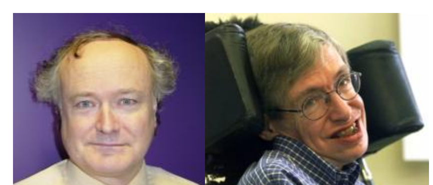
Schwarzschild in real time



Schwarzschild in imaginary time



$$R_\tau = \frac{\hbar}{k_B T_{Hawking}}$$



Gibbons-Hawking

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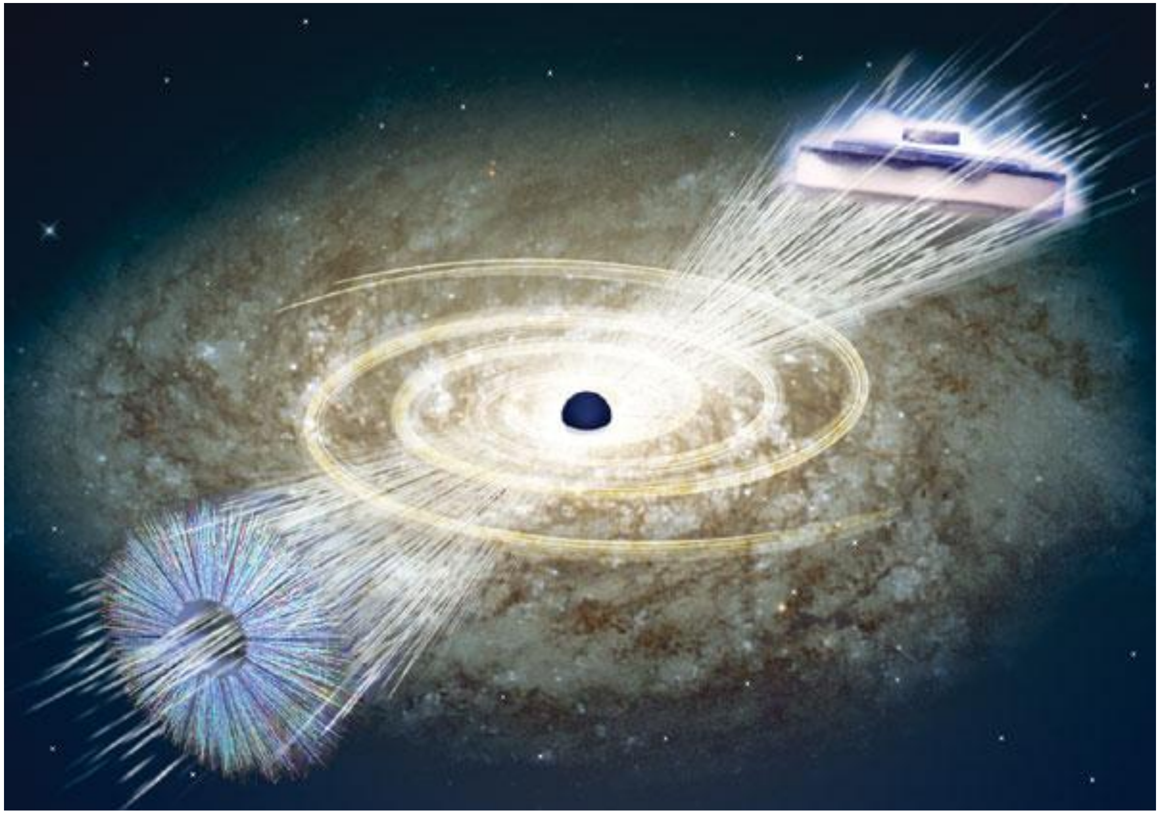

Planck scales

Phenomenon: gravity, matter warps space-time

Characteristic dimensions: Newton's constant G, light velocity c.

Planck's constant carries dimension energy-seconds: Einstein comes to an end at the Planck scale $\tau_G = \sqrt{G\hbar/c^5} \approx 10^{-42}\,\mathrm{sec}$

Phenomenon: dissipation, work turns irreversibly into heat, takes a characteristic (relaxation) time

Characteristic dimension: temperature $k_B T$

Given hbar, the shortest possible relaxation time is

$$\tau_{Planck} = \frac{\hbar}{k_B T}$$

'Planckian dissipation': requires that the quantum dynamics is scale invariant (quantum critical)!!!!

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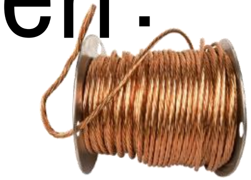
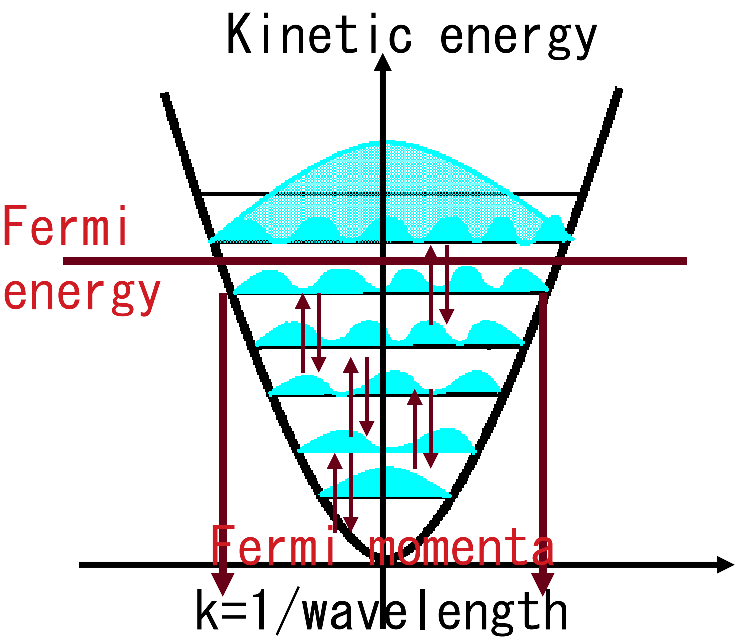
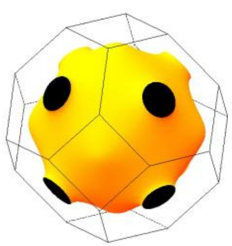
The quantum in the kitchen: Landau's miracle







Fermi surface of copper

Electrons are waves

Pauli exclusion principle: every state occupied by one electron

Unreasonable: electrons strongly interact !!

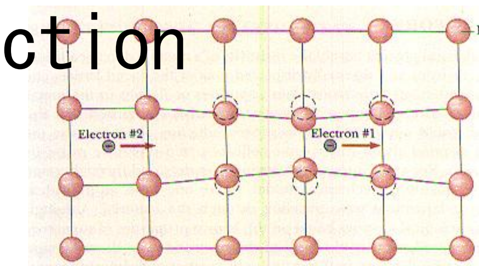
Landau's Fermi-liquid: the highly collective low energy quantum excitations are like electrons that do not interact.

BCS theory: fermions turning into bosons



Bardeen Cooper Schrieffer

Fermi-liquid + attractive interaction

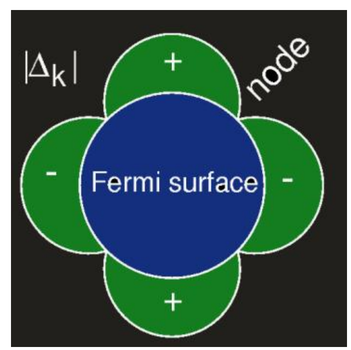


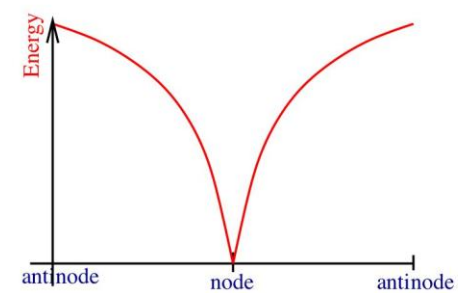
Quasiparticles pair and Bose condense:

Ground state

$$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow}\right)|vac.\rangle$$

D-wave SC: Dirac spectrum

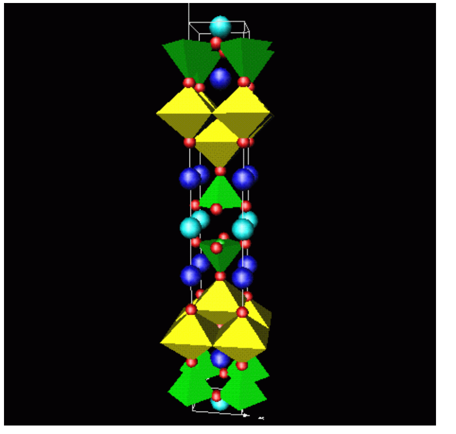


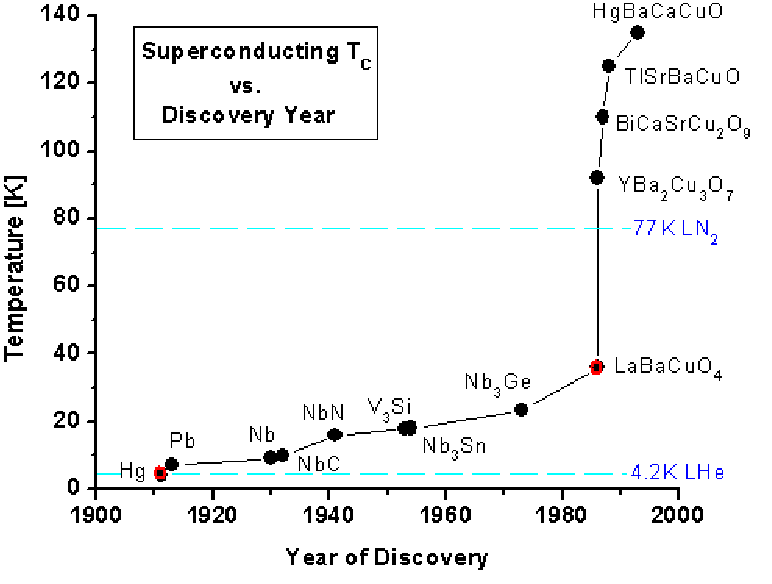


Twenty two years ago …

Mueller Bednorz

Ceramic CuO's, likeYBa2Cu3O7





Superconductivity jumps to 'high' temperatures

Graveyard of Theories

Mott

Laughlin

Muelle
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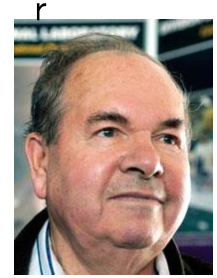
Schrieff
er

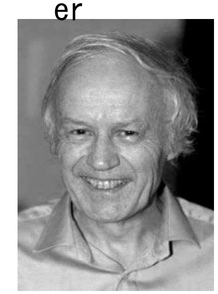
De Gennes

Bednorz

Anderson



Abrikoso
v



Legget
t

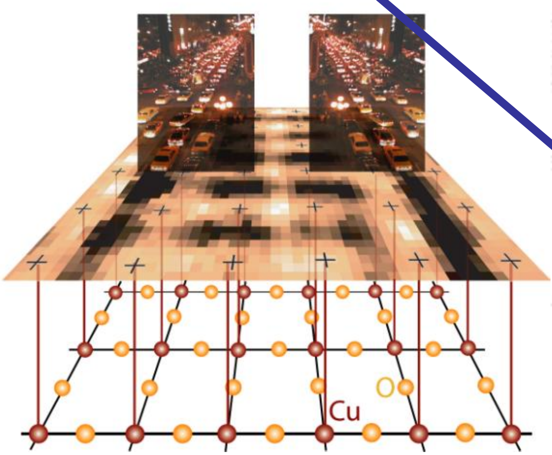
Wilczek

Ginzburg

Yang

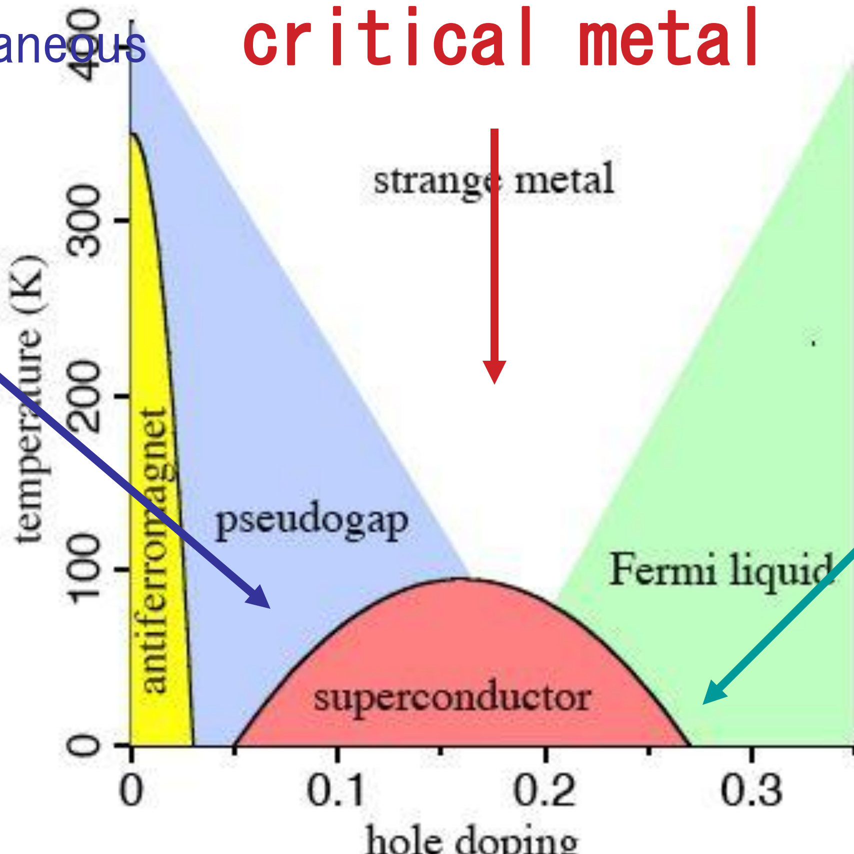
Phase diagram high Tc superconductors

'Stripy stuff', spontaneous currents, phase fluctuations ..

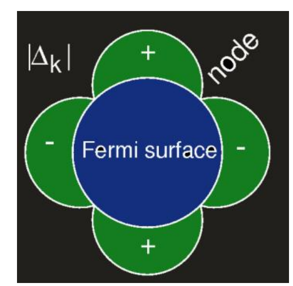


JZ, Science **315**, 1372 (2007)

Mystery quantum critical metal

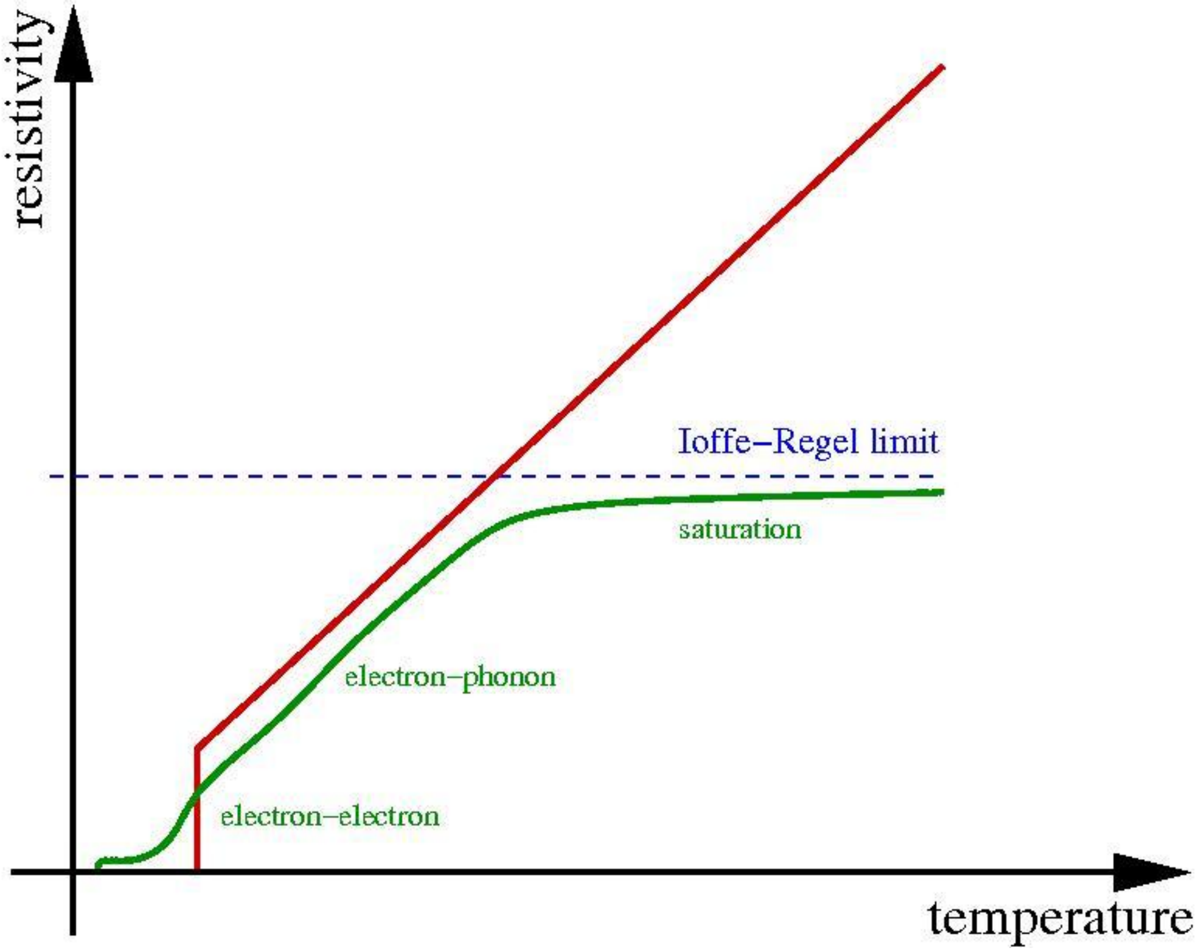


The return of normalcy



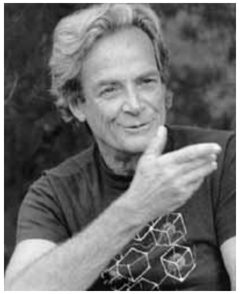
$$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) |vac.\rangle$$

Divine resistivity



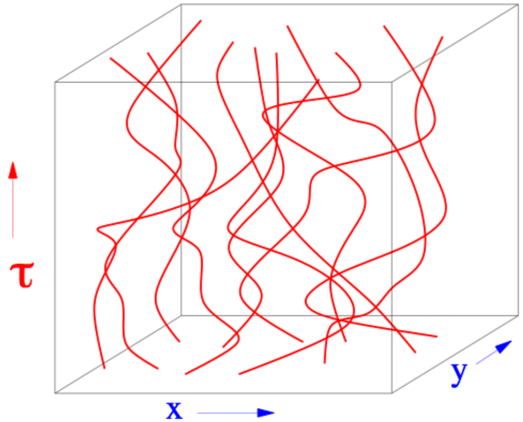
Fermion sign problem

Imaginary time path-integral formulation



$$\begin{aligned} \mathcal{Z} &= \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) \\ &= \int \mathrm{d}\mathbf{R}\rho(\mathbf{R},\mathbf{R};\beta) \end{aligned}$$

$$\mathbf{R} = (\mathbf{r}_1,\ldots,\mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\begin{aligned} \rho_{B/F}(\mathbf{R},\mathbf{R};\beta) &= \frac{1}{N!}\sum_{\mathcal{P}}(\pm1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta) \\ &= \frac{1}{N!}\sum_{\mathcal{P}}(\pm1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\} \end{aligned}$$

Boltzmannons or Bosons:

- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

Fermions:

- negative Boltzmann weights
- non probablistic: NP-hard problem (Troyer, Wiese)!!!

Planck scales

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Characteristic dimension: temperature $k_B T$

Given hbar, the shortest possible relaxation time is

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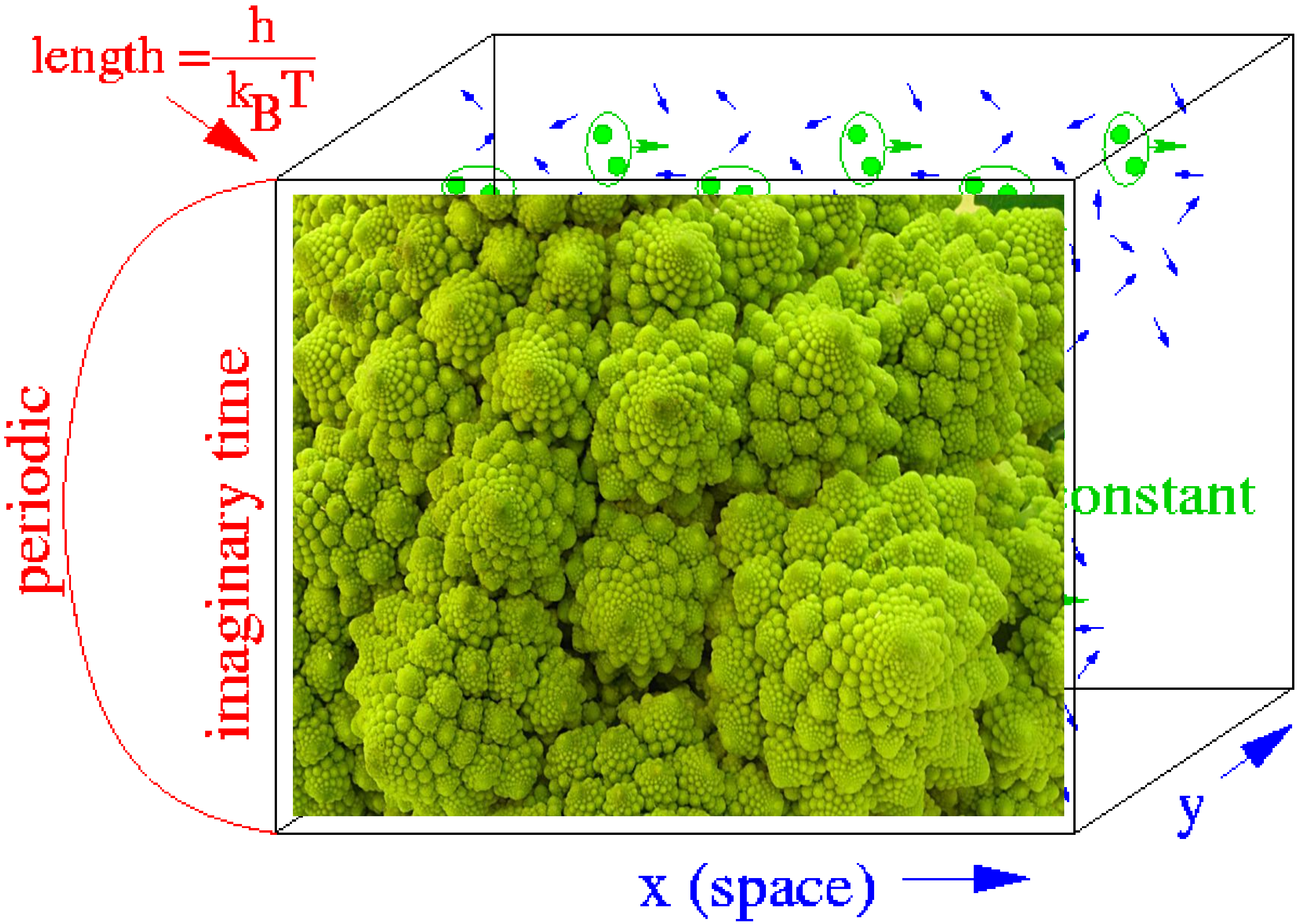
'Planckian dissipation': requires that the quantum dynamics is scale invariant (quantum critical)!!!!

Quantum criticality or 'conformal fields'



Fractal Cauliflower (romanesco)

The quantum critical response

Scaling form dynamical susceptibility:

$$\chi(\omega) \propto \frac{1}{T^{2-\eta}} \Psi\left(\frac{\hbar\omega}{k_B T}\right)$$

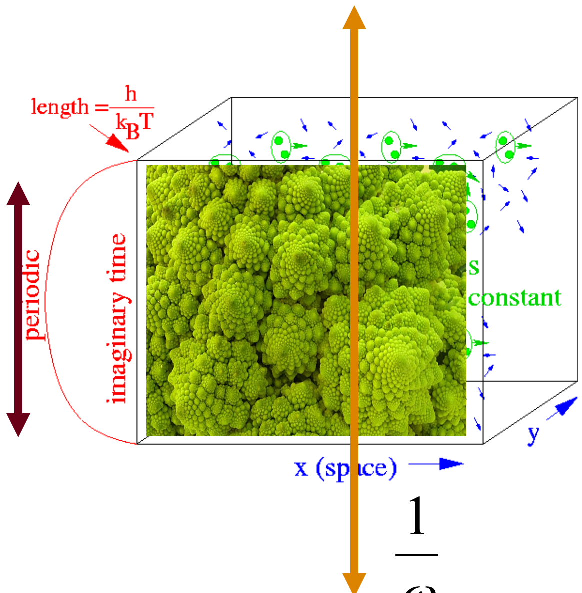
Quantum critical regime

$$\hbar\omega << k_B T ??$$

$$\chi(\omega) \propto \frac{1}{T^{2-\eta}} \frac{1}{1 - i\omega\tau_\hbar}$$

Sachdev

$$\frac{\hbar}{k_B T}$$



$$\frac{1}{\omega}$$

Planckian dissipation:

$$\tau_\hbar = const. \frac{\hbar}{k_B T}, \quad const. = O(1)$$

Dissipation in scale-full quantum systems

Example: dissipation in the Fermi-liquid (electron-electron scattering)

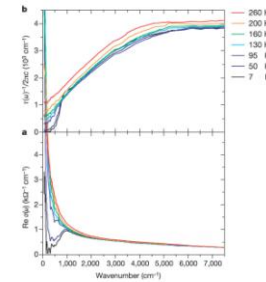
$$\sigma = \frac{n_e e^2}{m} \tau_{inel.} \qquad \tau_{inel.} = \frac{\hbar E_F}{(k_B T)^2} = \left(\frac{E_F}{k_B T} \right) \tau_{\hbar}$$

It takes a time **longer** by $E_F / k_B T$ than the **Planck time**

Critical Cuprates are Planckian Dissipators

van der Marel, JZ, ... Nature 2004:

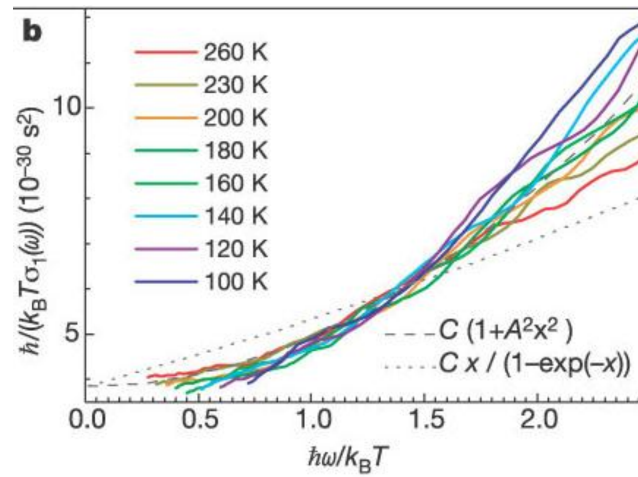
Optical conductivity QC cuprates



Frequency less than temperature:

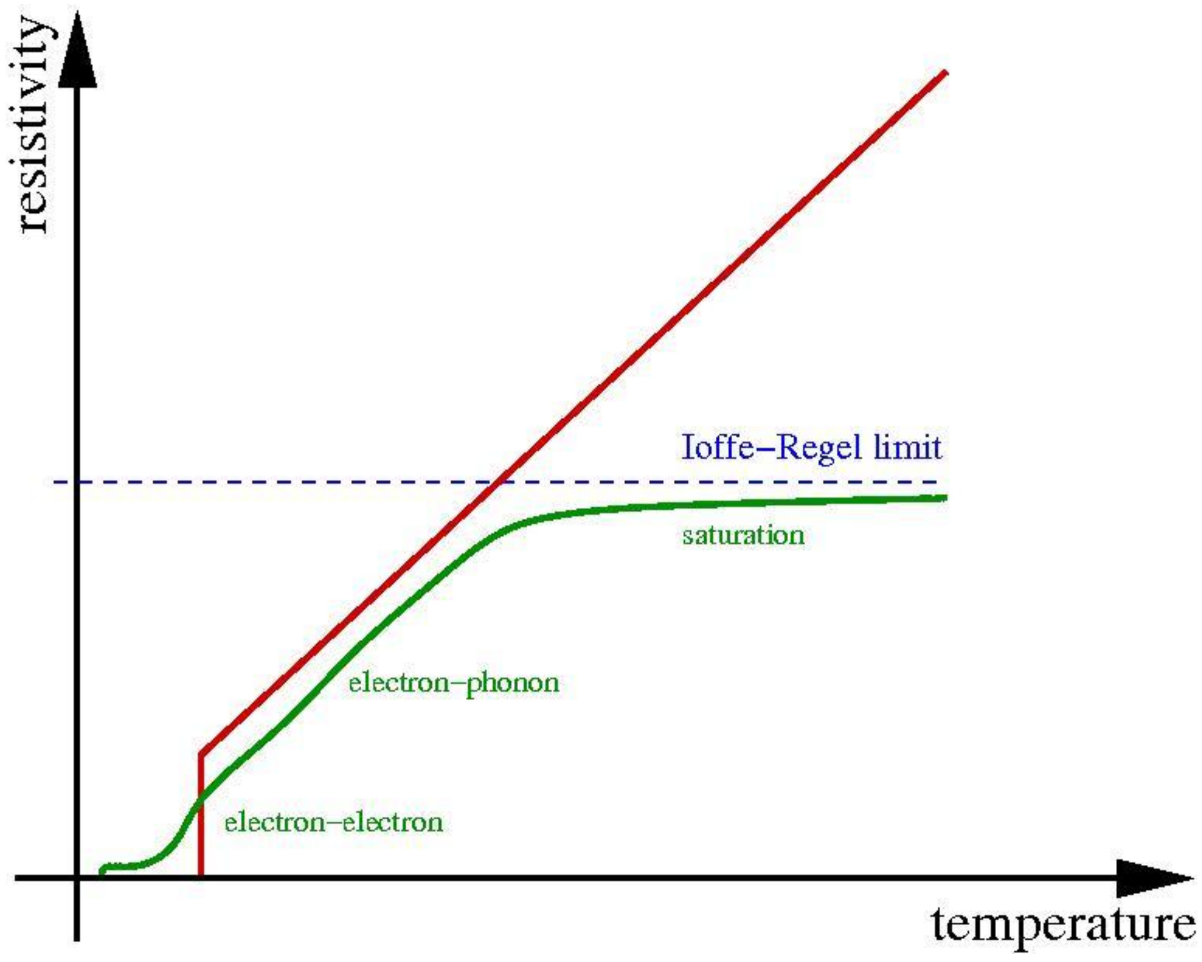
$$\sigma_1(\omega,T) = \frac{1}{4\pi}\frac{\omega_{pr}^2\tau_r}{1+\omega^2\tau_r^2}, \quad \tau_r = A\frac{\hbar}{k_B T}$$

$$\Rightarrow \quad [\frac{\hbar}{k_B T\sigma_1}] = const.(1+A^2[\frac{\hbar\omega}{k_B T}]^2)$$



A= 0.7: the normal state of optimallly doped cuprates is a Planckian dissipator!

Divine resistivity



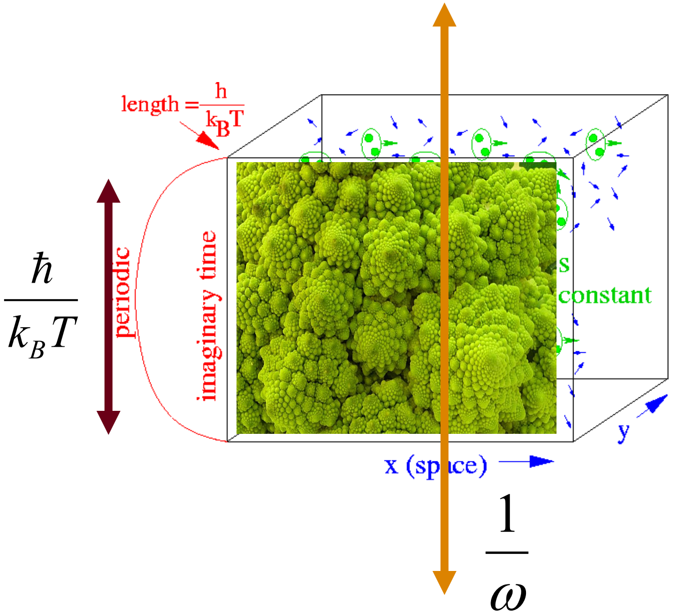
'Real' hydrodynamics: Planckian viscosity

Viscosity, entropy density:

$$\eta \approx (\varepsilon + p)\tau$$

$$s = \frac{\varepsilon + p}{T}$$

$$\frac{\eta}{s} \approx T\tau$$



Planckian dissipation:

$$\tau = \tau_\hbar \approx \frac{\hbar}{k_B T}$$

Planckian viscosity:

$$\frac{\eta}{s} \approx \frac{\hbar}{k_B}$$

Plan

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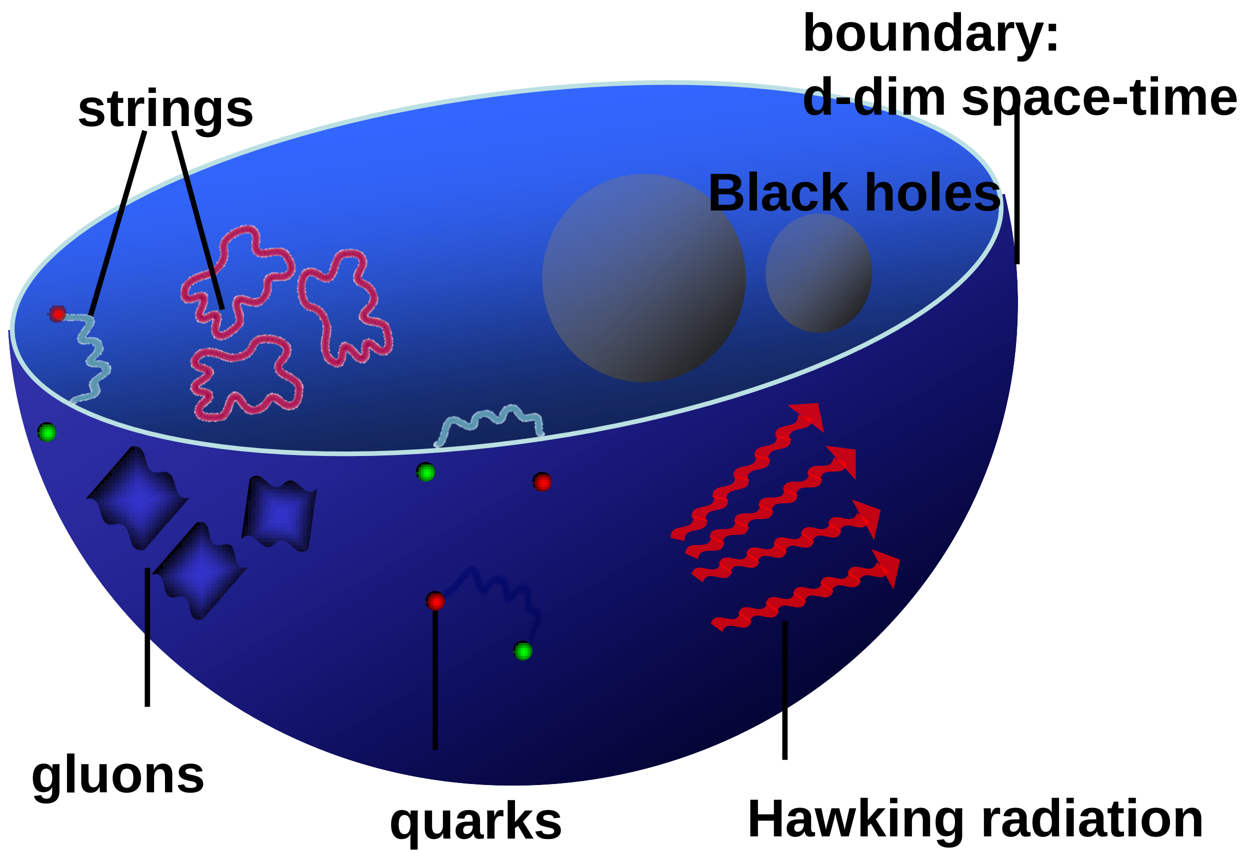
AdS/CFT correspondence: String theory Magic!

d-dim. gauge theory / conformal field theory ⟷ **(d+1)-dim string theory / gravity theory**

Maldacena

Witten,
Gubser, Klebanov, Polyakov

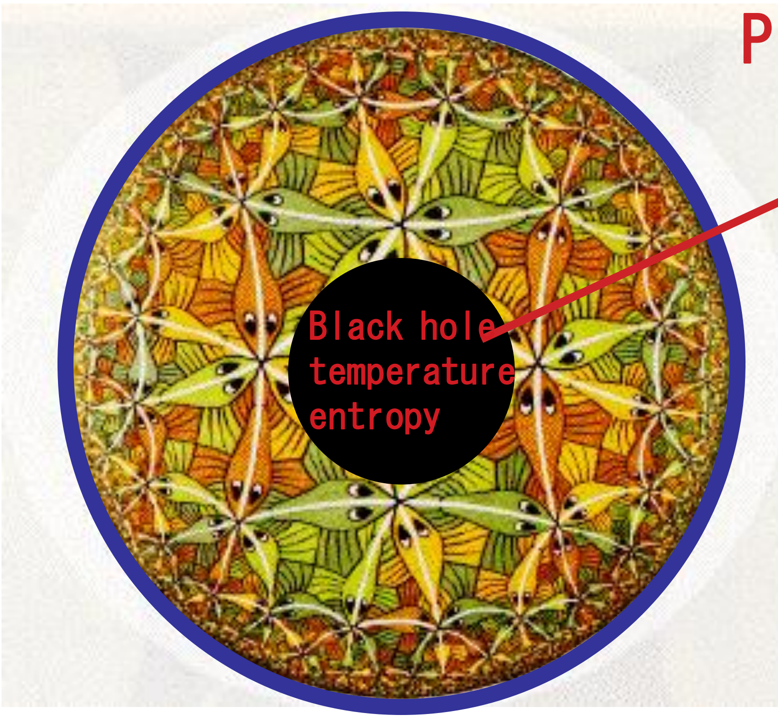


The bulk: Anti-de Sitter space

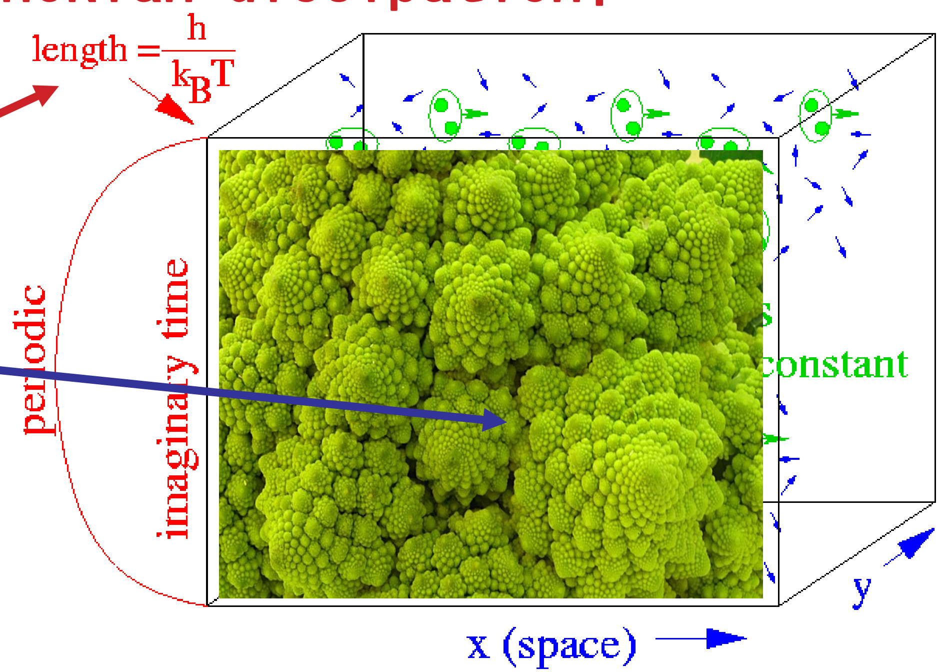
$$d\overset{s}{r}^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$F(r) = -\Lambda r^2 + 1, \qquad \Lambda < 0$$

The boundary: conformal fields or quantum criticality!



Planckian dissipation!



GR in Anti de Sitter

$$dr^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$F(r) = -\Lambda r^2 + 1 - \frac{GM}{r}$$

Quantum-critical fields on the boundary

Quantum critical dynamics: classical waves in AdS

$$W_{CFT}(J) = S_{AdS}(\phi)_{\phi_{x0} \to 0 = J}$$

$$g_{YM}^2 N = \frac{R^4}{\alpha}$$

$$g_{YM}^2 = g_s$$

The AdS/CFT dictionary

SUSY Einstein-Maxwell in AdS <==> SUSY Yang-Mills CFT

E-field

transverse E-field <=> 3d electric field

radial E-field <=> 3d charge density

B-field

radial B-field <=> 3d magnetic field

transverse B-field <=> 3d current density

spatial metric perturb.

transverse gradient <=> 3d distortion

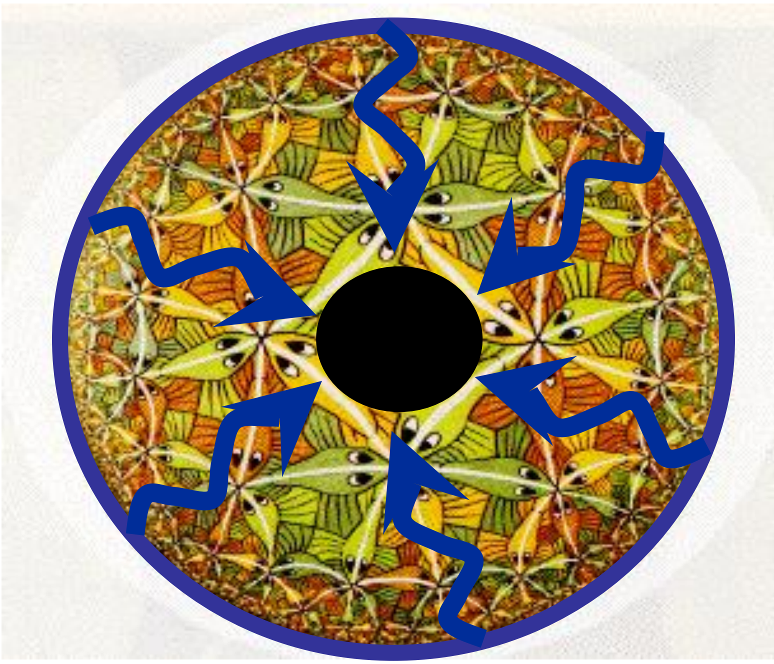
radial gradient <=> 3d stress tensor

temporal metric perturb.

transverse gradient <=> temperature gradient

radial gradient <=> heat flow

Dissipation = absorption of classical waves by Black hole!



Hartnoll-Son-Starinets (2002):

Viscosity: absorption cross section of gravitons by black hole

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

= area of horizon (GR theorems)

Entropy density s: Bekenstein-Hawking BH entropy = area of horizon

Universal viscosity-entropy ratio for CFT's with gravitational dual limited in large N by:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

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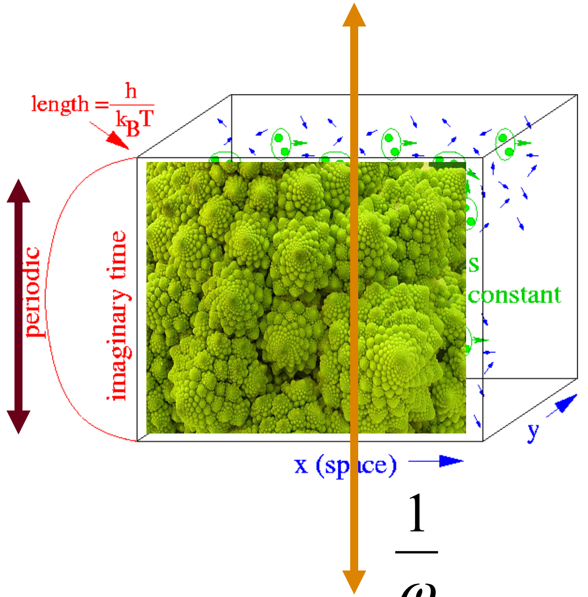
$$\frac{\eta}{s} \approx T\tau$$



$\frac{\hbar}{k_B T}$

$\frac{1}{\omega}$

Planckian dissipation:

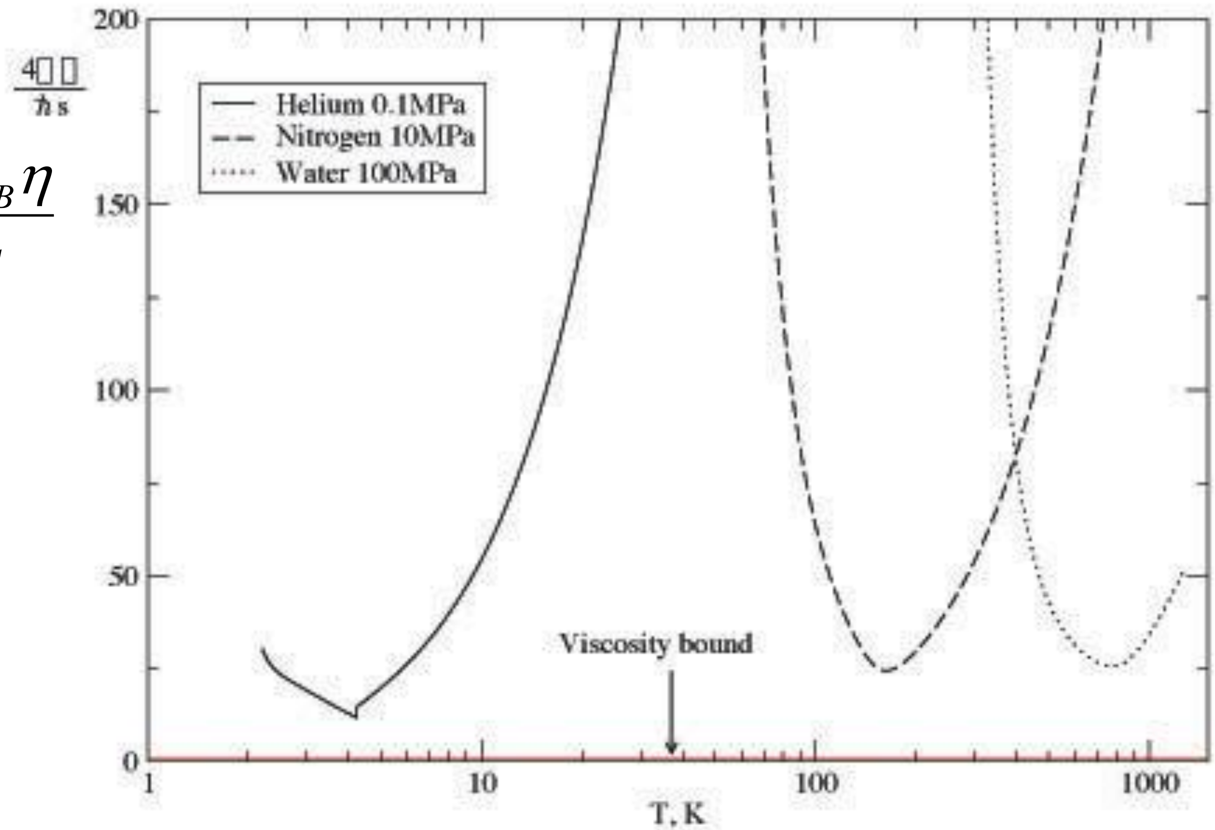
$$\tau = \tau_\hbar \approx \frac{\hbar}{k_B T}$$

Planckian viscosity:

$$\frac{\eta}{s} \approx \frac{\hbar}{k_B}$$

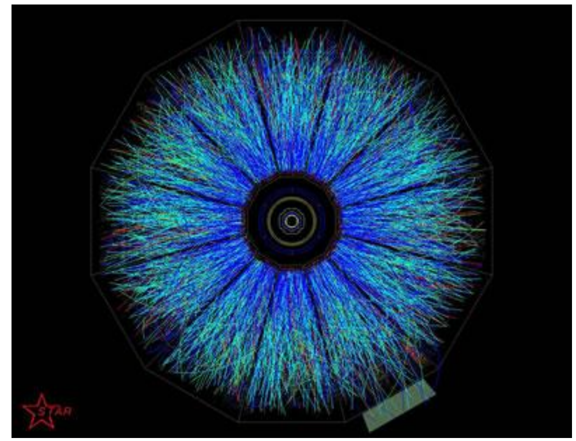
AdS/CFT viscosity
Kovtun-Son-Starinets (2005)

$$\frac{4\pi k_B \eta}{\hbar s}$$

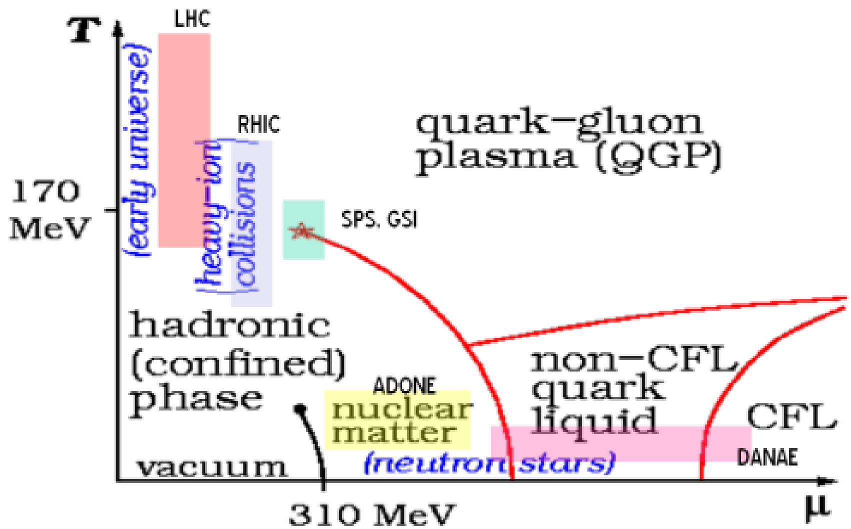


The quark-gluon plasma

Relativistic Heavy Ion Collider

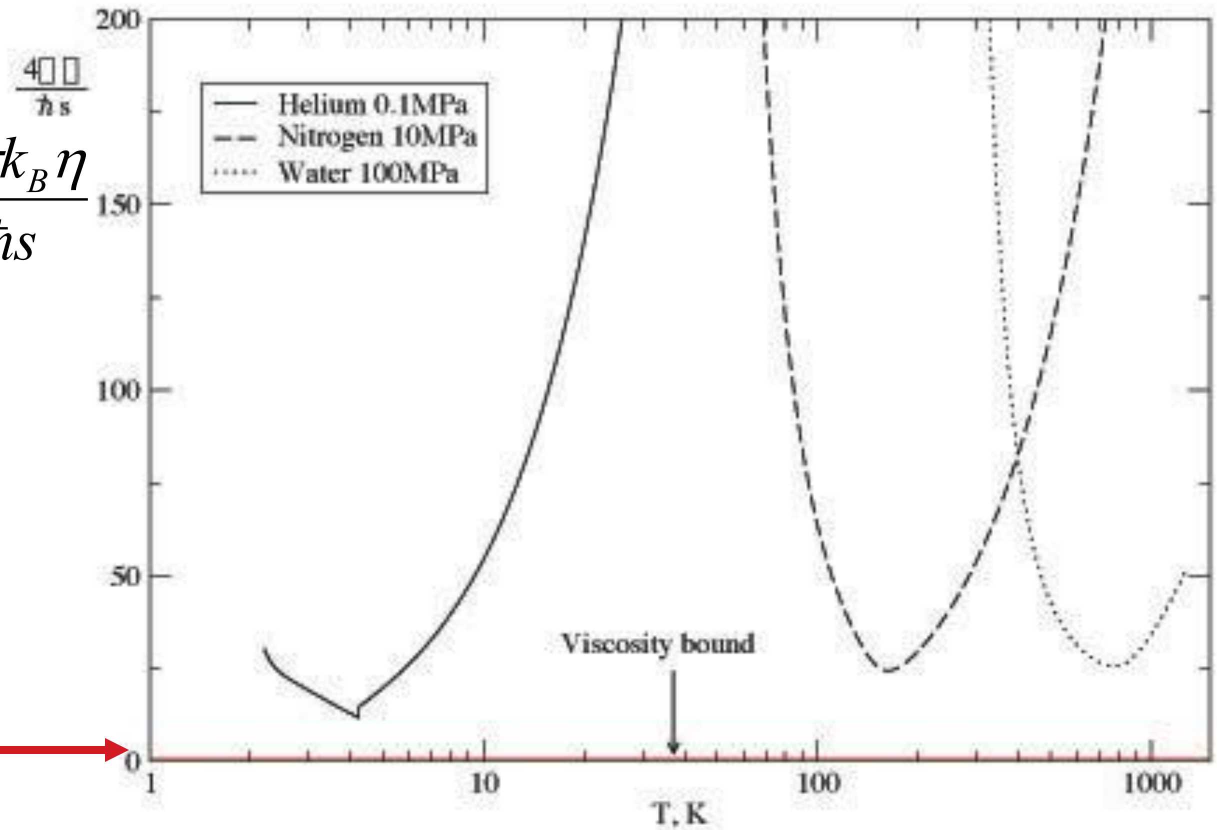


Quark-gluon
all’



The tiny viscosity of the Quark-Gluon plasma

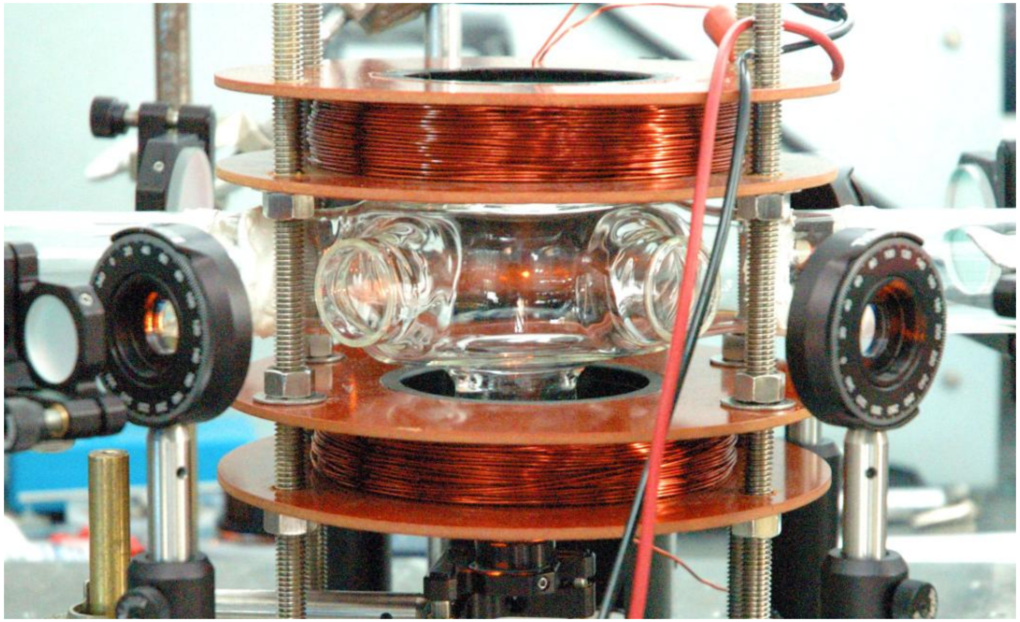
$$\frac{4\pi k_B \eta}{\hbar s}$$



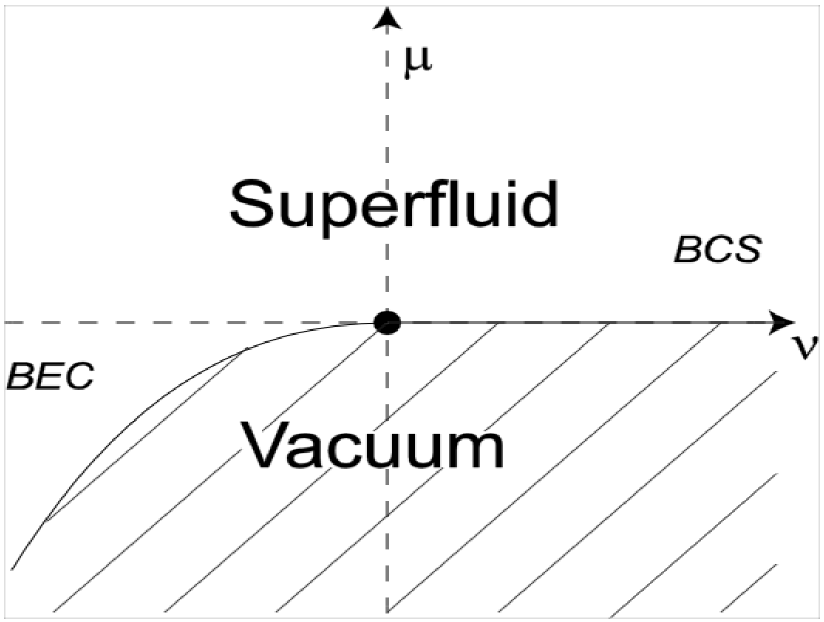
QG plasma: within 20% of the AdS/CFT viscosity!

Very cold fermionic atoms

S=1/2 Fermi gas at a Feshbach resonance



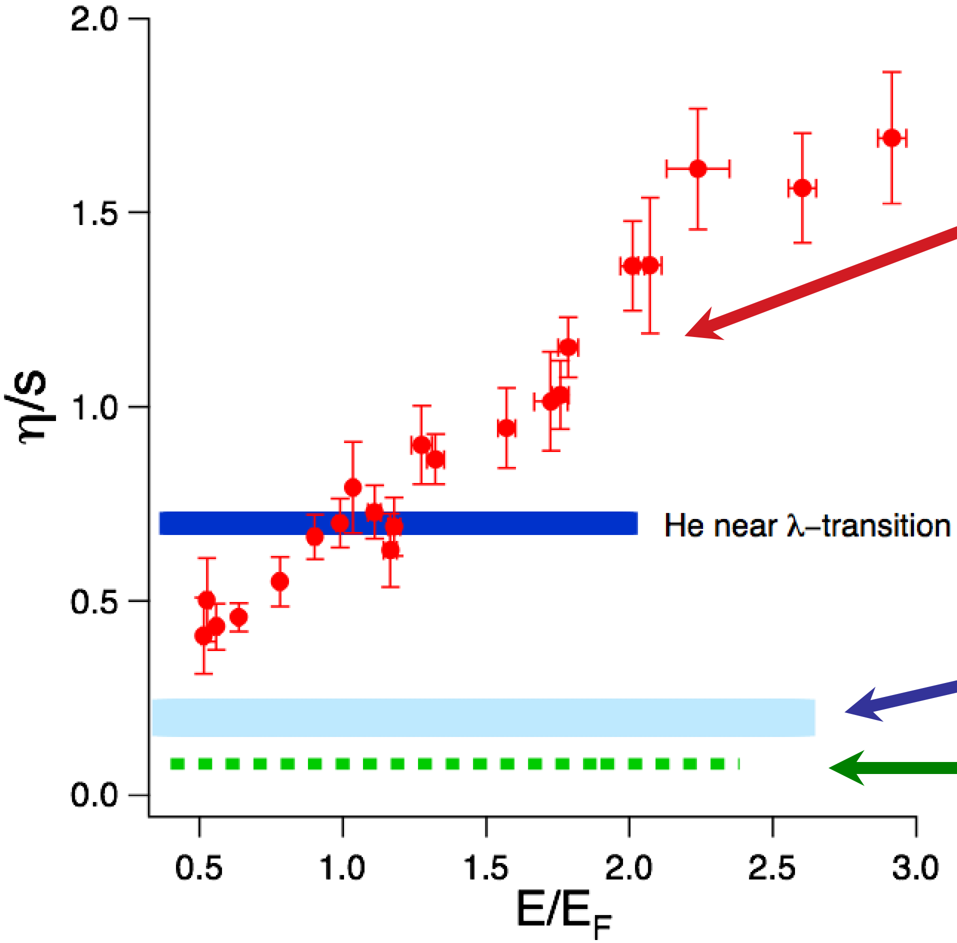
Quantum criticality at the Feshbach resonance



RG fixed point described by a "non-relativistic" CFT: special gravitational AdS dual

See arXiv:0804.3972,0804.4053,0806.2867,0806.3244,0807.1100,0807.1111

Planckian dissipation in the atom trap



Ultracold 6-Li gas at Feshbach resonance

Quark Gluon plasma

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

T. Schafer, Phys. Rev. A 76, 063618 (2007); A. Turlapov et. al., J. Low Temp. Physics 150, 567 (2008)

Plan

1. The dissipative world of apes ....

2. The high Tc superconductivity saga: the past.

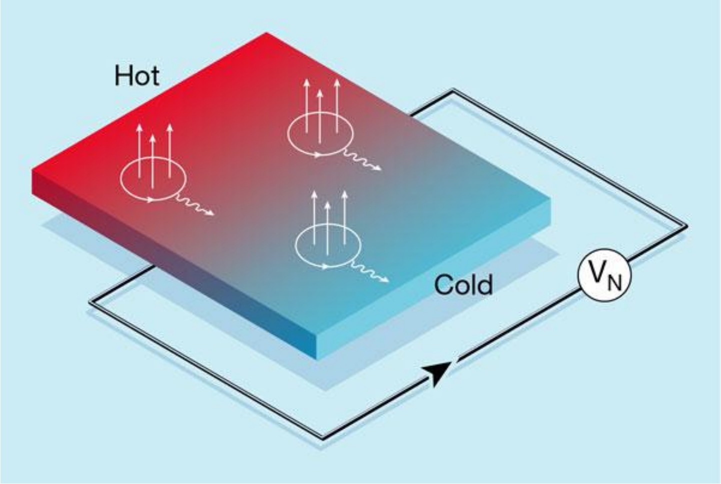
3. Viscosity, dissipation and black holes: the AdS/CFT magic

4. The high Tc superconductivity saga: the future is critical.

5. Conclusions

Thermo-electric transport at the insulator-superfluid QPT.

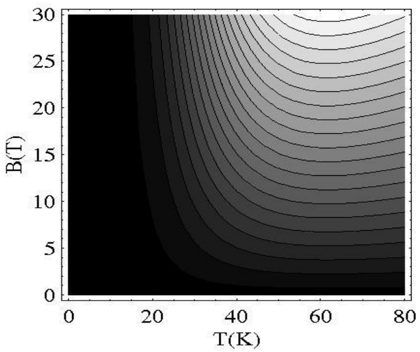
E.g. Nernst effect:

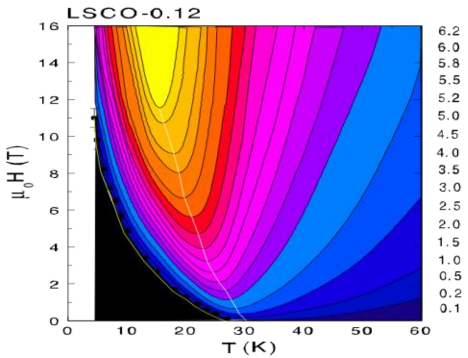


$$J_y = -\alpha_{xy} \frac{dT}{dx}$$

AdS: graviton-photon cross correlators in the presence of a dyonic black hole!

Hartnoll, Kovtun, Müller, Sachdev, *Phys. Rev.* B **76** 144502 (2007)

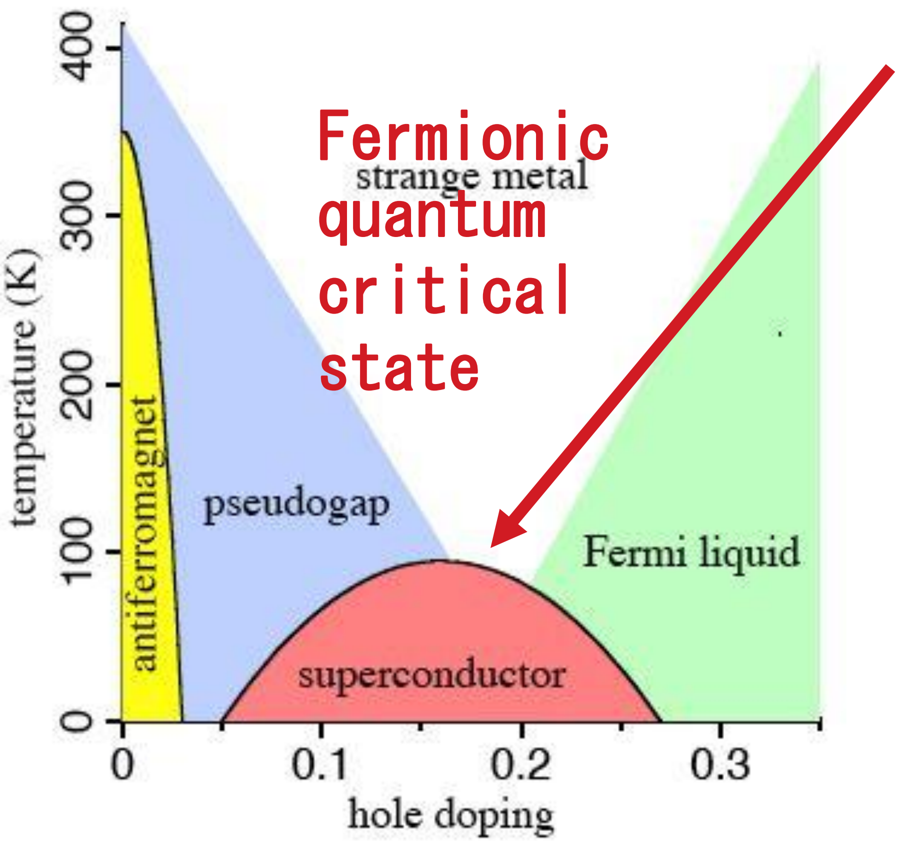




Stripy cuprates: Ong group

Why Tc is high …

JZ, Nature 430, 512(2004)



BCS type transition: pairs form at Tc

But BCS wisdoms like:

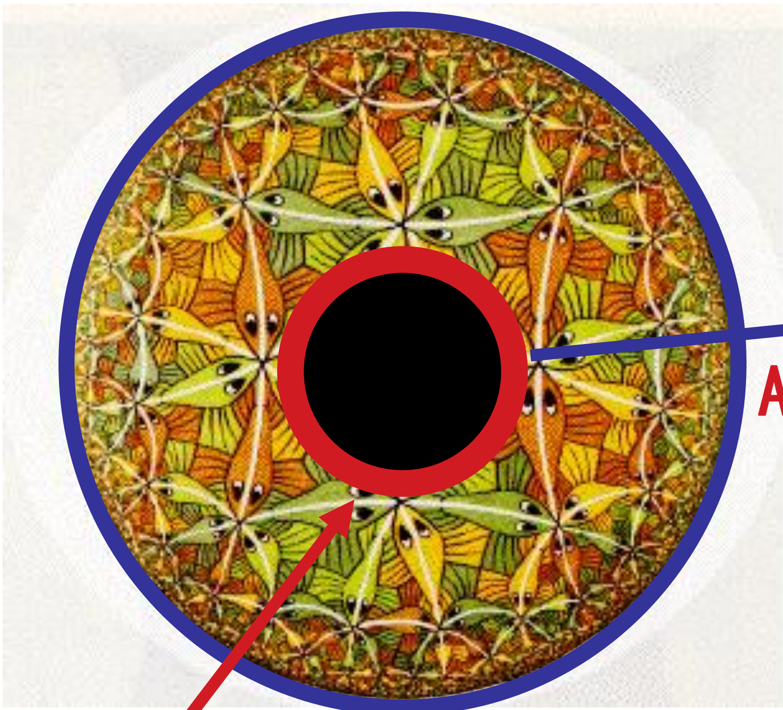
$$2\Delta = \hbar\omega_{boson} e^{-1/\lambda}$$

$$2\Delta \approx 3.5 k_B T_c$$

Need the Fermi energy!

The hairy black hole

Hartnoll, Herzog, Horowitz, arXiv:0803.3295

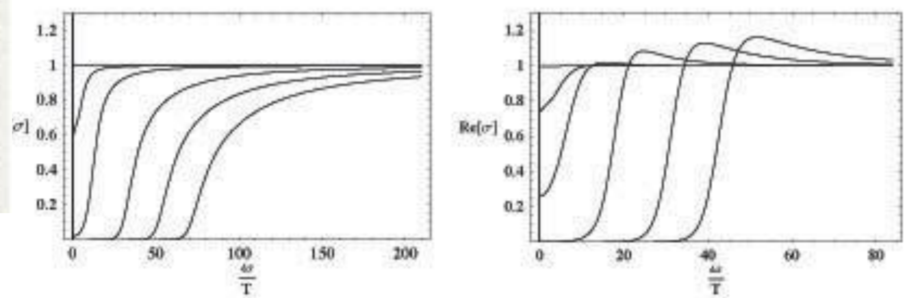


AdS–CFT

Condensate ‘atmosphere’

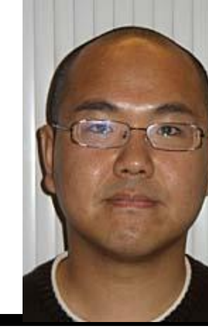
BCS-like Superconductor on the boundary!

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.



$$\frac{2\Delta_0}{k_B T_c} \approx 8$$

Scale invariance versus BCS



J.-H. She

Gap equation: $1-\lambda\chi_{pp}(k_B T,\Delta,\hbar\omega_B)=0$

Glue strength — SC gap — Glue frequency

Fermi-liquid:

$$\chi_{pp}(k_B T,\cdots)=\ln\left(\frac{E_F\to\hbar\omega_B}{k_B T},\cdots\right)\Rightarrow k_B T_c=\hbar\omega_B e^{-1/\lambda},\ 2\Delta\approx 3.5 k_B T_c$$

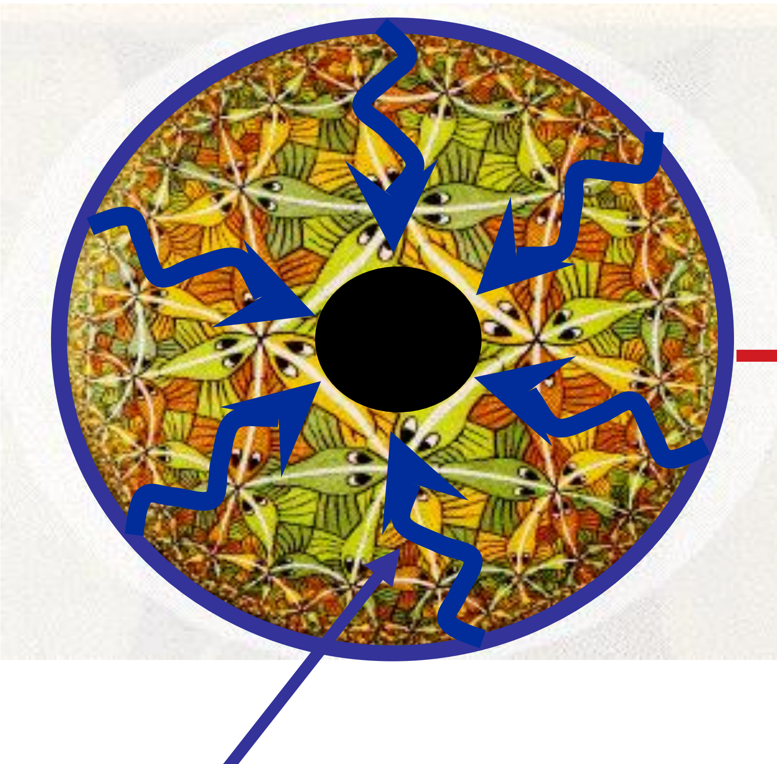
Fermionic qu. critical:

$$\chi_{pp}(k_B T,\cdots)\propto\left(\frac{1}{k_B T}\right)^{\frac{d-2+\eta_{pp}}{z}},\quad\left(\frac{1}{i\Delta}\right)^{\frac{d-2+\eta_{pp}}{z}}$$

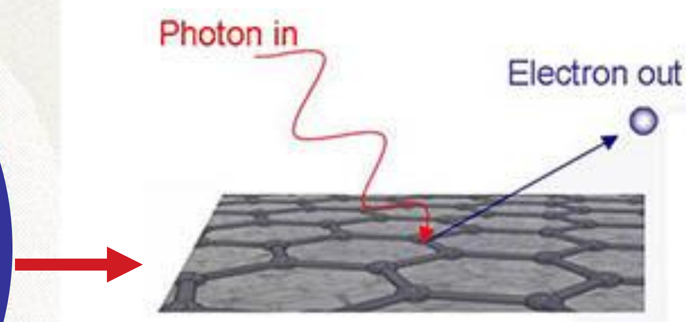
$$\Rightarrow k_B T_c=\hbar\omega_B\left(\lambda\Big/\left(\lambda+\left(\frac{2\omega_B}{\omega_c}\right)^{\frac{2-z-\eta}{z}}\right)\right)^{\frac{z}{2-z-\eta}},\ 2\Delta\approx A\,k_B T_c$$

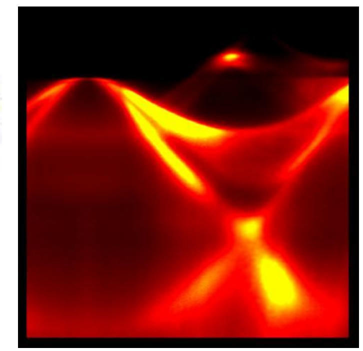
Tc is large for moderate glue, gap to Tc ratio "universal" number!

Cracking the Fermion signs: AdS-to-ARPES



Classical 'Dirac waves'





In progress ....

Schalm Sadri

In conclusion ...

Planckian dissipation: the beautiful hydrodynamic behaviors of quantum critical states of matter.

With help of string theory: this is in literal, but dual correspondence with the physics of black holes.

This sheds unexpected light on real life physics: the quark-gluon plasma and cold atoms.

AdS/CFT might well be the mathematics behind the enigma called High Tc superconductivity !

Further reading: Nature 430, 512 (2004), Science 315, 1372 (2007), Nature 448, 1000 (2007), Science 319, 1205 (2008); Hartnoll, Science 322, 1639 (2008)

The dissipative world of apes …

QuickTime™ and a
decompressor
are needed to see this picture.

QuickTime™ and a
decompressor
are needed to see this picture.

The fundamental constants

$$\hbar, c, G, e, k_B, \cdots \,?$$

Boltzmann's constant is the conversion factor between time and probability !?

In conclusion ...

Planckian dissipation: the beautiful hydrodynamic behaviors of quantum critical states of matter.

With help of string theory: this is in literal, but dual correspondence with the physics of black holes.

This sheds unexpected light on real life physics: the quark-gluon plasma and cold atoms.

AdS/CFT might well be the mathematics behind the enigma called High Tc superconductivity !

Further reading: Nature 430, 512 (2004), Science 315, 1372 (2007), Nature 448, 1000 (2007), Science 319, 1205 (2008); Hartnoll, Science 322, 1639 (2008).

empty

Empty

Empty

Penrosian gravitational wave function collapse

Unitary/entangled/microscopic: $|\,Live\,\rangle + |\,Dead\,\rangle$

Dissipative/collapsed/macroscopic: $|\,Live\,\rangle$ or $|\,Dead\,\rangle$

Condition for unitarity: global time like Killing vector

$$i\hbar \frac{d}{dt_{Live}} \left|Live\right\rangle = H \left|Live\right\rangle \qquad i\hbar \frac{d}{dt_{Dead}} \left|Dead\right\rangle = H \left|Dead\right\rangle$$

$$\frac{d}{dt_{Live}} = \frac{d}{dt_{dead}}$$

Einstein gravity = 'gauge theory of diffeomorphism': global time like killing vector does not exist in universes with different mass distributions!

Conflict becomes manifest at 'collapse time':

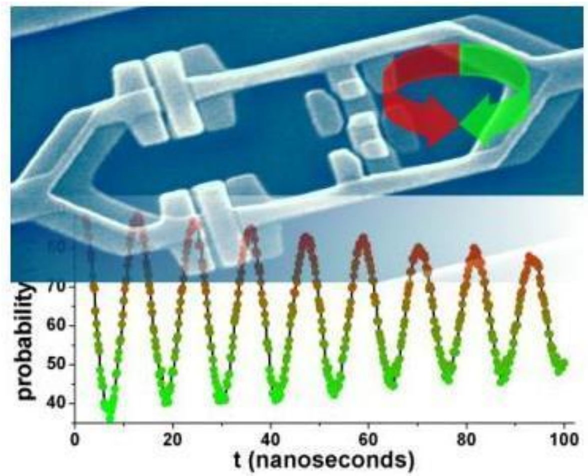
$$\tau_{coll.} \approx \frac{\hbar}{\Sigma_G} \approx \frac{\hbar L}{GM^2}$$

Measuring quantum gravity

Penrosian collapse: 'it takes a Schroedinger state made from e-coli's one micrometer apart one second to collapse.'

Flux qubits closest approach with available technology!

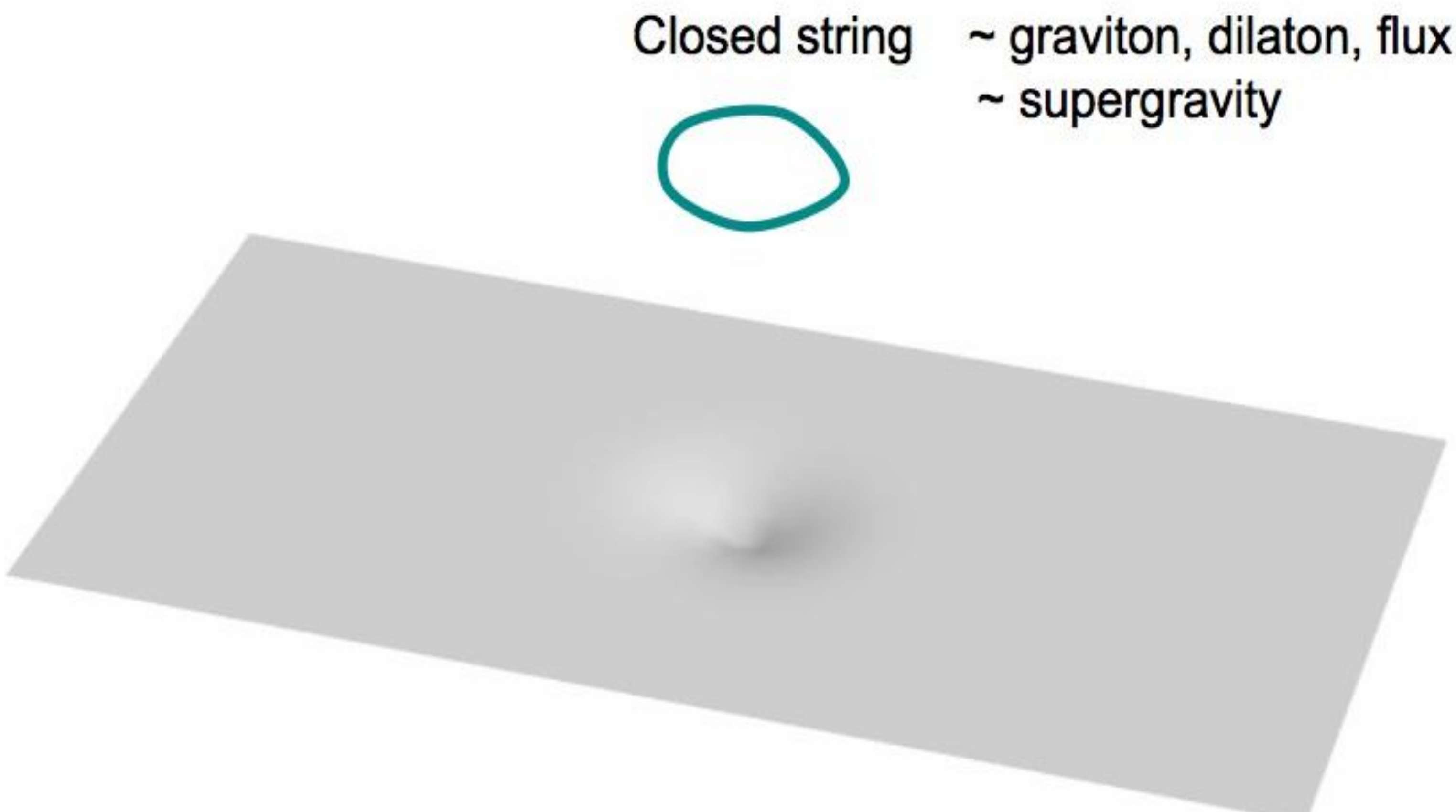
Van Wezel, Oosterkamp, JZ (cond-mat/0706.3976, Phil. Mag. B)



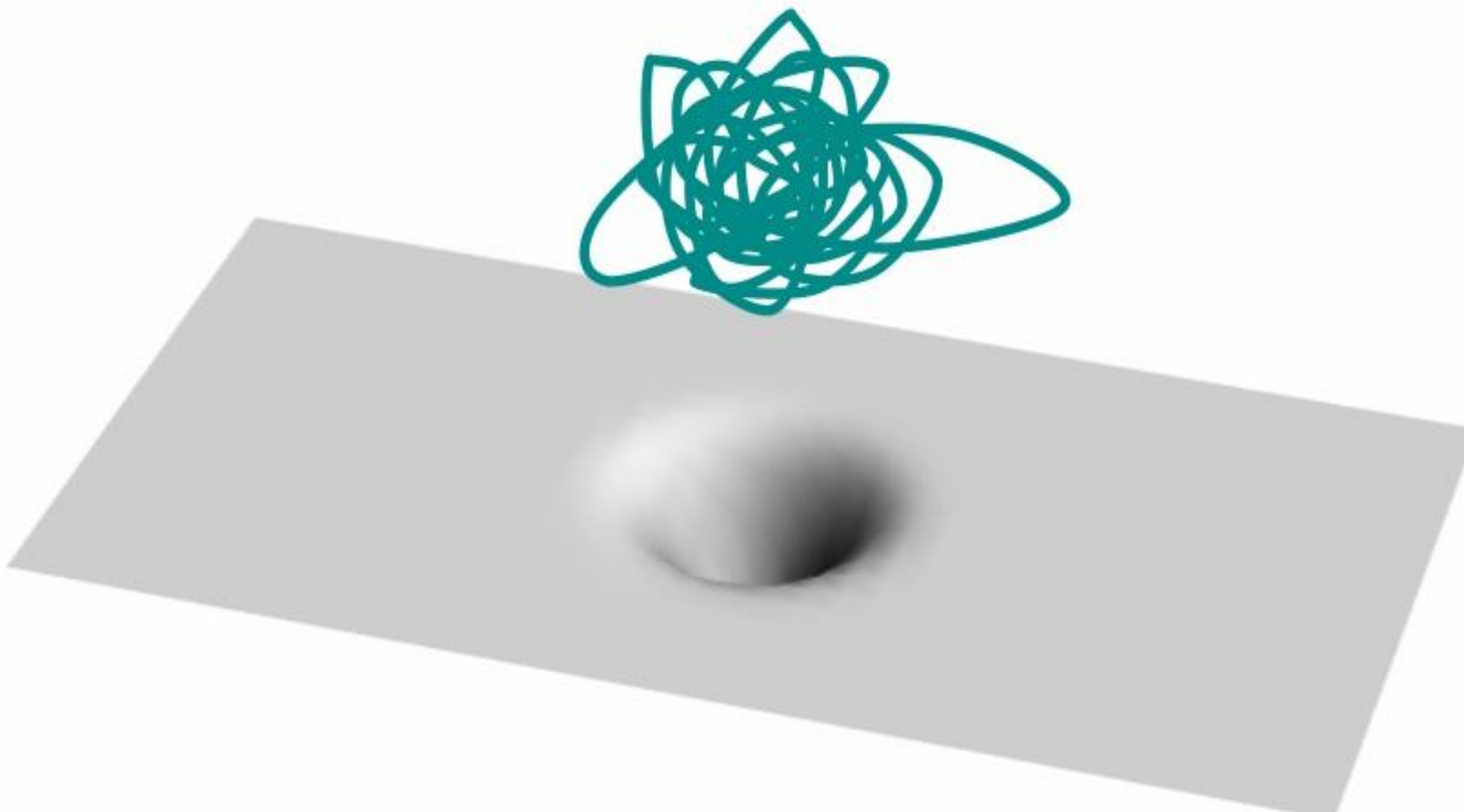
MicroAmp currents, cross section 1 micron, height 0.1 mm:

Collapse time of order of seconds!

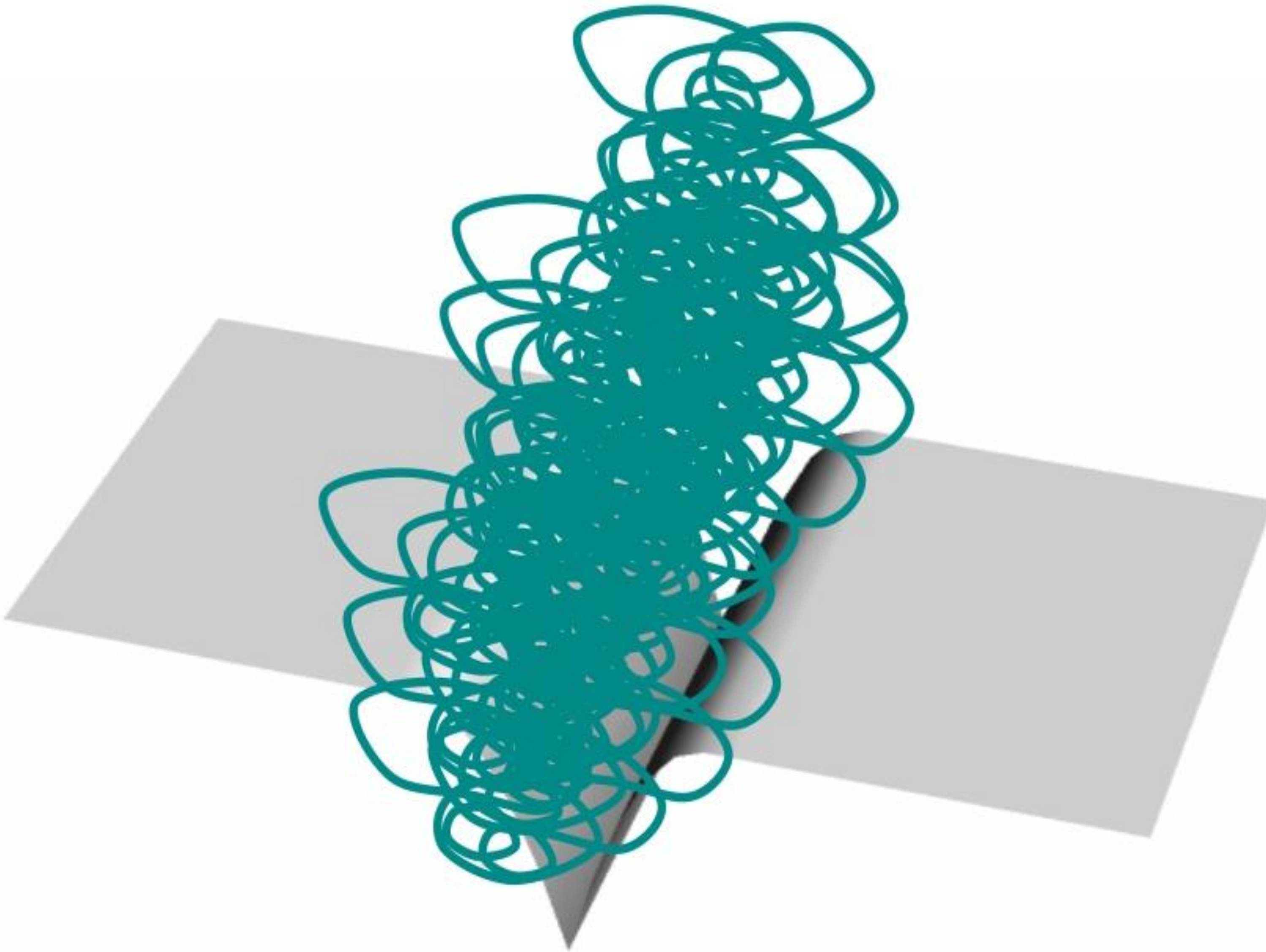
Closed string



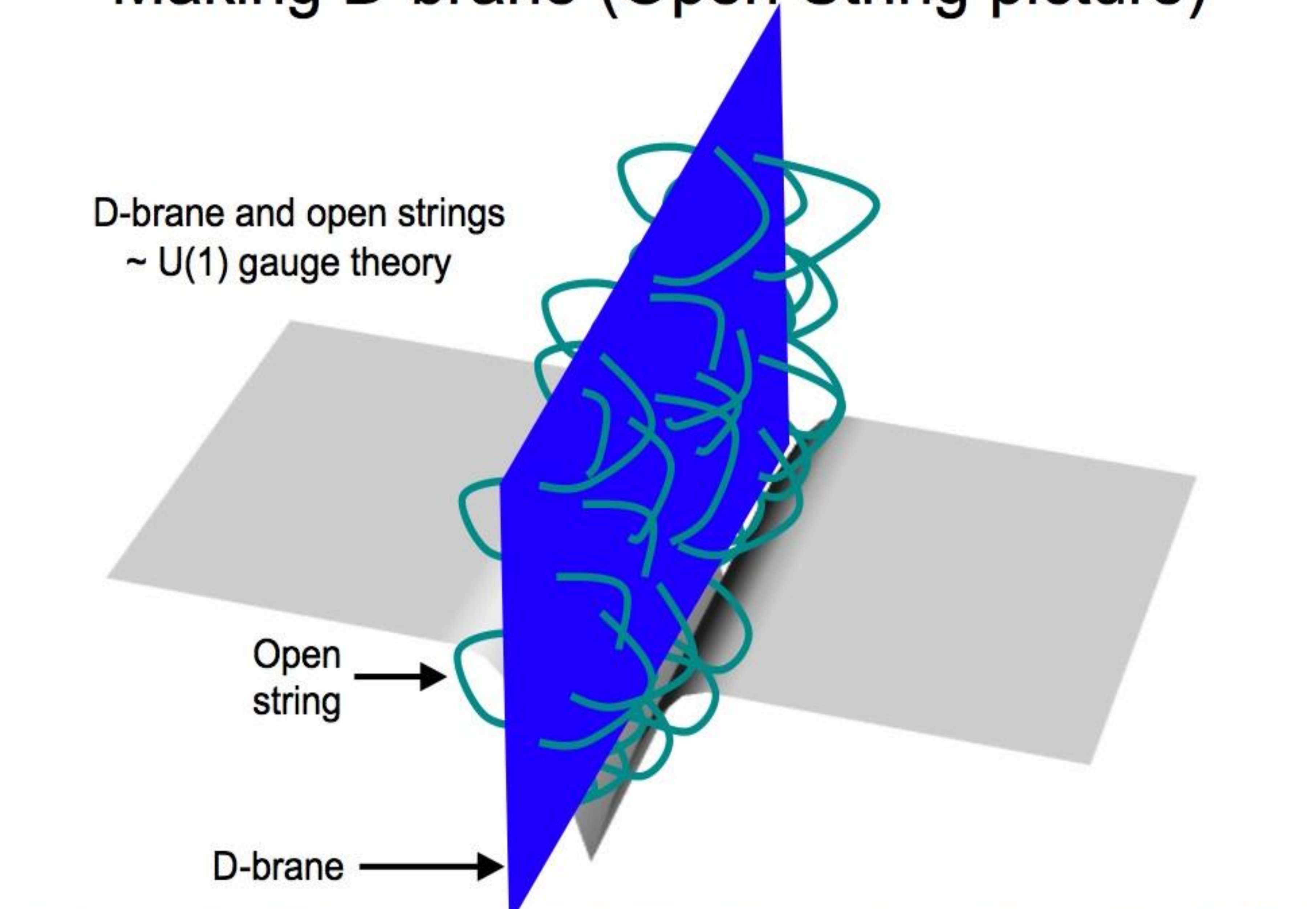
Making Black Hole



Making D-brane (Closed string picture)



Making D-brane (Open String picture)

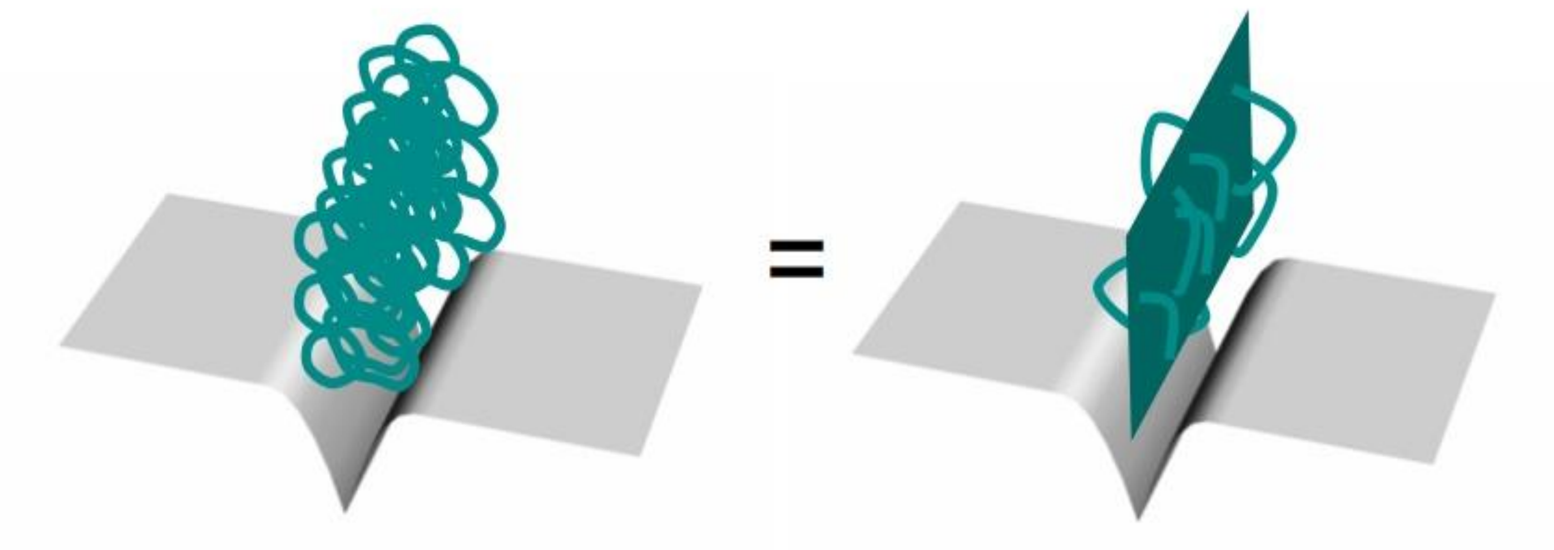


AdS/CFT correspondence

We have two different descriptions for same object!

Closed string description

Open string description



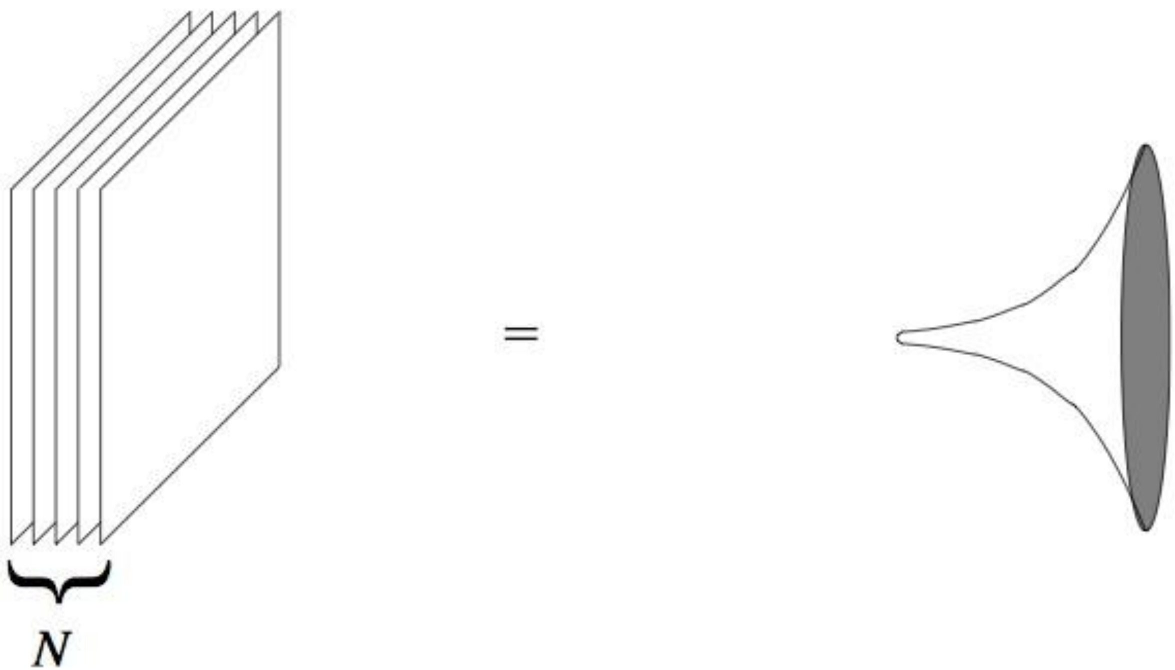
Especially, in the case of D3-brane, at low energy these two description will be approximated by ….

The Gauge/Gravity Duality

Stack of N D3-branes in type IIB string theory: described in two different pictures:

As a quantum field theory of degrees of freedom on the branes: $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

As string theory on a the curved spacetime (induced by the matter density on the branes)



The limit of infinitely strong coupling in gauge theory is the limit when string theory becomes Einstein's general relativity

The high Tc enigma

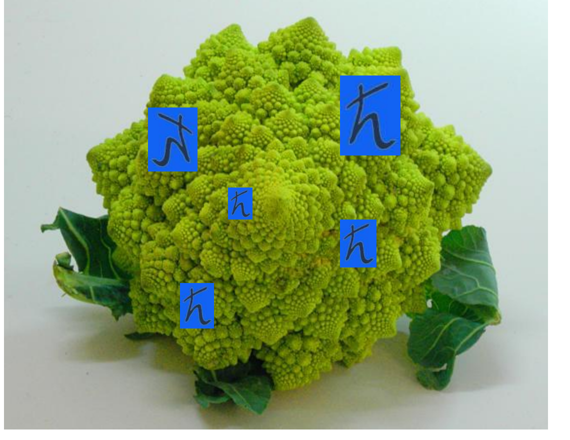
temperature

Strange metal

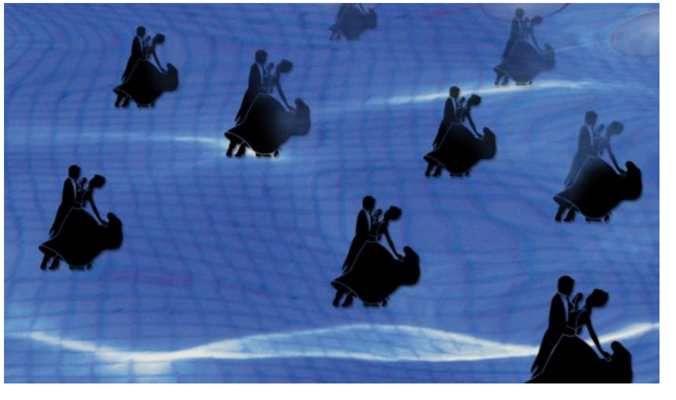
$-200^o C$

Electron correlations

Quantum critical fermion matter

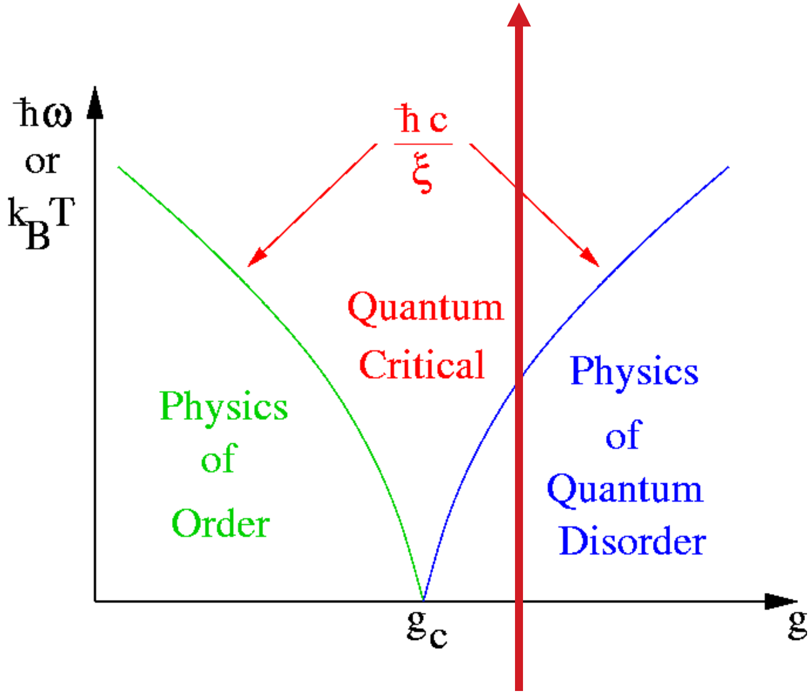


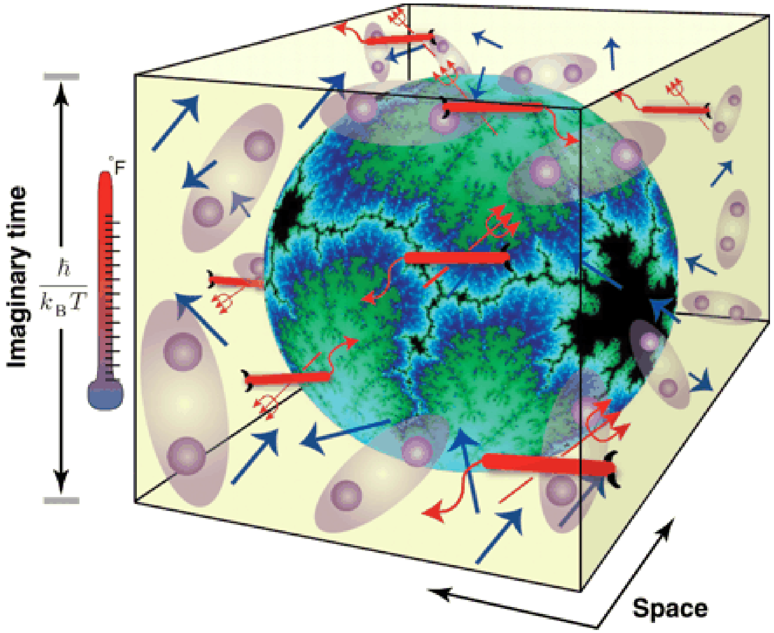
Super-conductor



Quantum Phase transitions

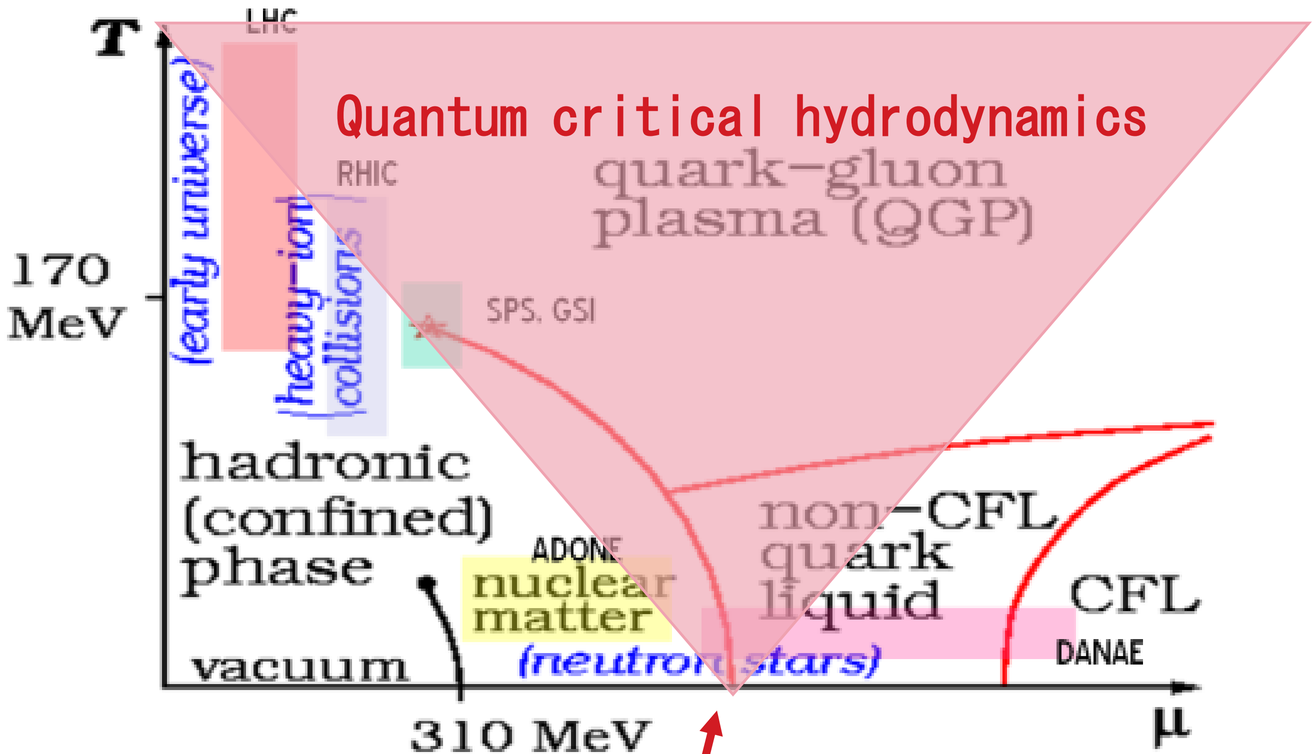
Quantum scale invariance emerges naturally at a zero temperature continuous phase transition driven by quantum fluctuations:





JZ, Science 319, 1205 (2008)

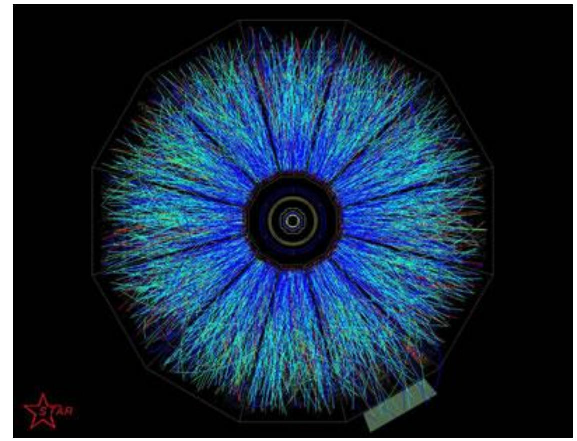
Quantum critical QCD ??



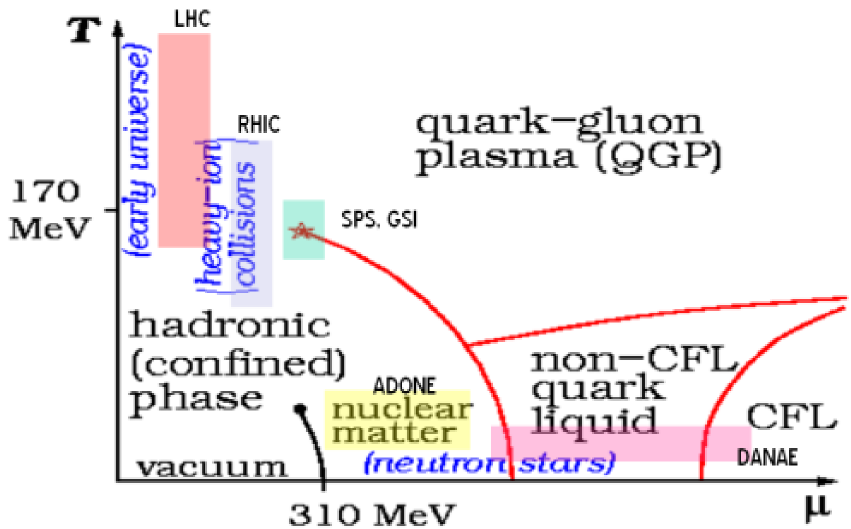
Quantum phase transition
????

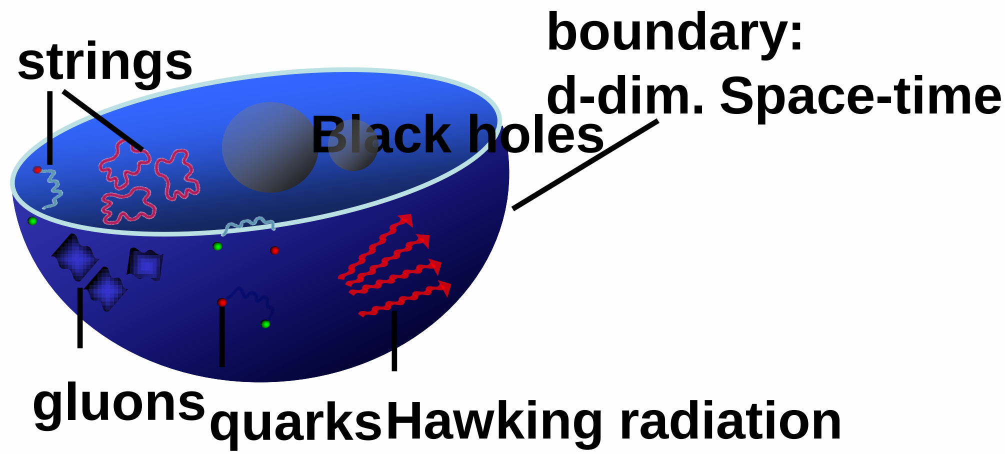
The quark-gluon plasma

Relativistic Heavy Ion Collider



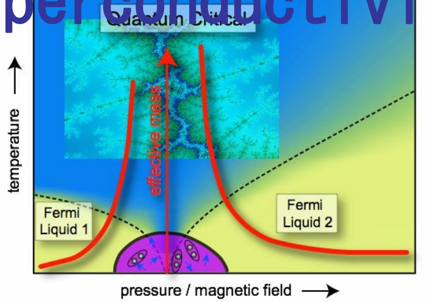
Quark-gluon ball’





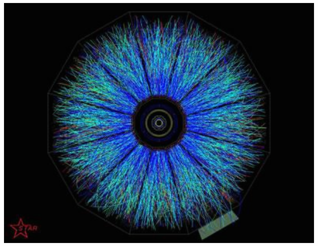
String theory: ‘AdS/CFT correspondence’

High Tc superconductivity



QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

Quark-gluon plasma



SUSY Einstein-Maxwell in AdS <==> SUSY Yang-Mills CFT

AdS/CFT dictionary:

E-field

D transverse E-field <=> D-1 electric field
D radial E-field <=> D-1 charge density

B-field

D radial B-field <=> D-1 magnetic field
D transverse B-field <=> D-1 current density

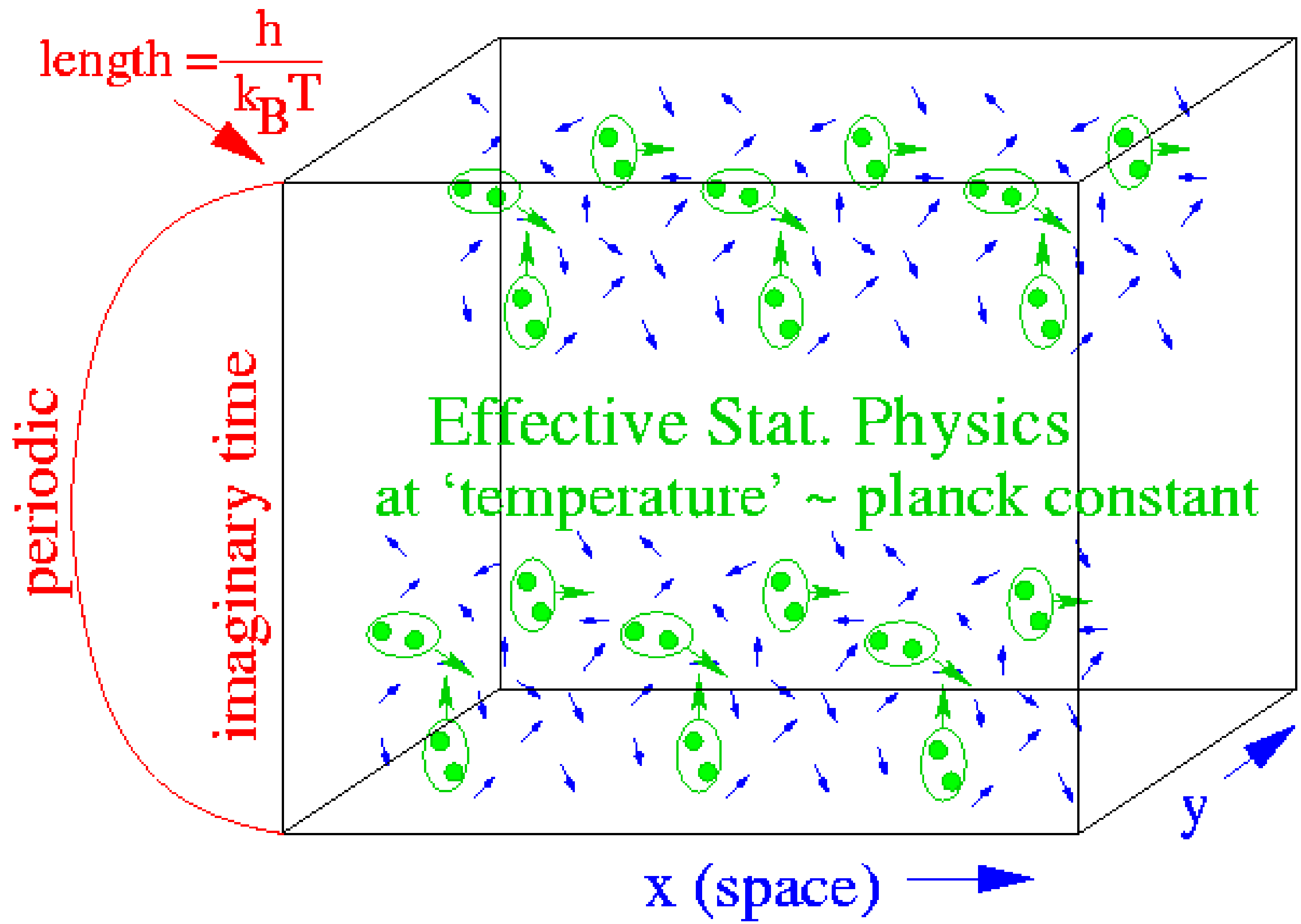
spatial metric perturb.

D transverse gradient <=> D-1 distortion
D radial gradient <=> D-1 stress tensor

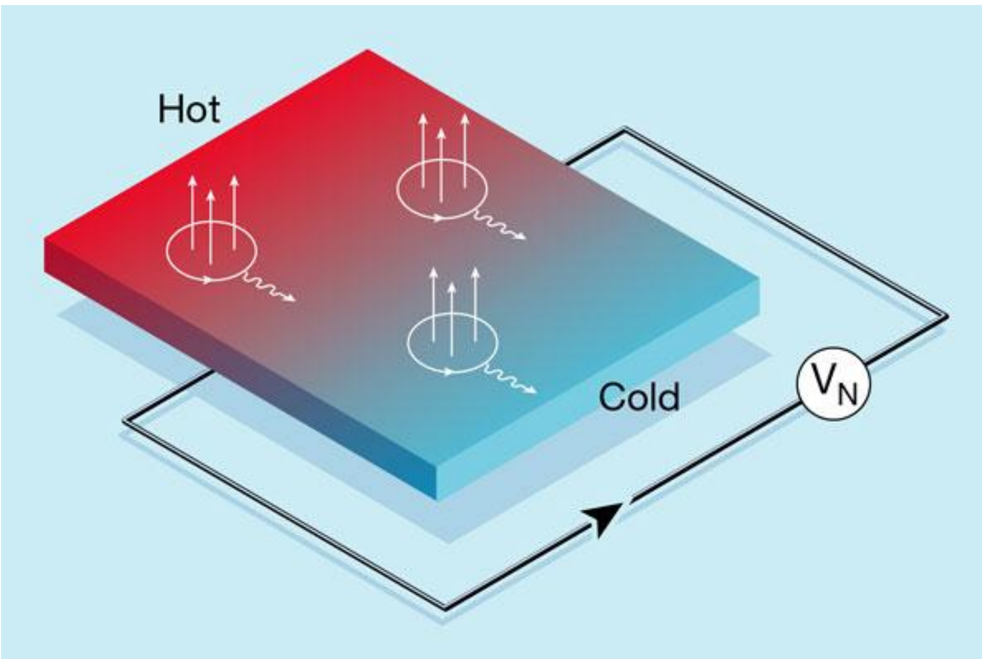
temporal metric perturb.

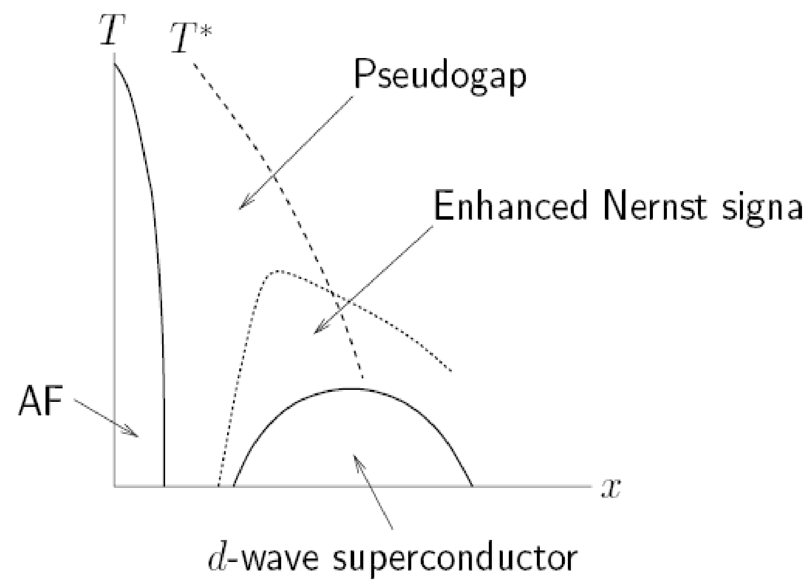
D transverse gradient <=> D-1 temperature gradient
D radial gradient <=> D-1 heat flow

Finite temperature fields: Euclidean Path Integrals



Nernst effect: experiment





$$J_y = -\alpha_{xy} \frac{dT}{dx}$$

SUSY Einstein-Maxwell in AdS <==> SUSY Yang-Mills CFT

AdS/CFT dictionary:

E-field

transverse E-field <=> 3d electric field
radial E-field <=> 3d charge density

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transverse B-field <=> 3d current density

spatial metric perturb.

transverse gradient <=> 3d distortion
radial gradient <=> 3d stress tensor

temporal metric perturb.

transverse gradient <=> temperature gradient
radial gradient <=> heat flow

Nernst effect: outcomes

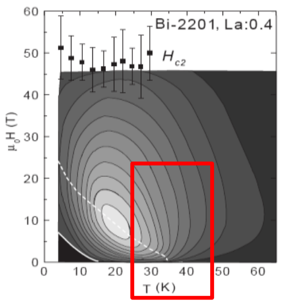
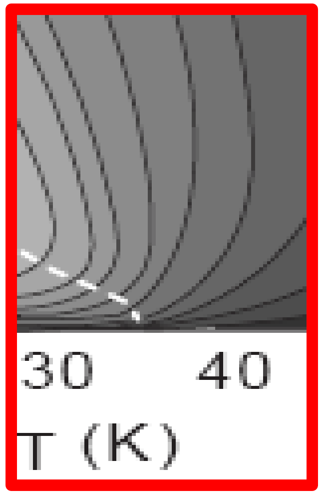


FIG. 13: Contour plot of $e_N(T, H)$ in OP Bi 2201 (y_{La} = 0.4). The value of e_N is highest in the light-gray region and zero in black regions. The white curve terminating at 15 K is $H_m(T)$. The dashed curve is the ridge-line joining points of maxima of e_N vs. H. Solid squares are values of H_{c2} estimated from extrapolation of the curves in Fig. 11. The plot emphasizes the smooth continuity of the vortex signal to temperatures high above T_c (28 K).



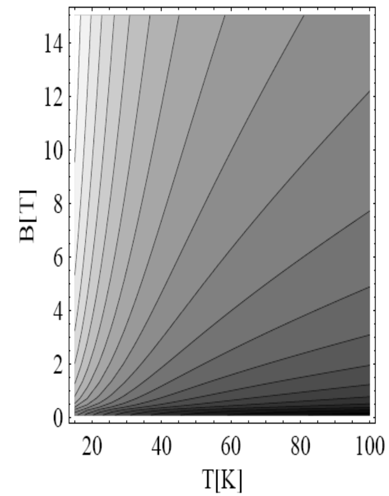


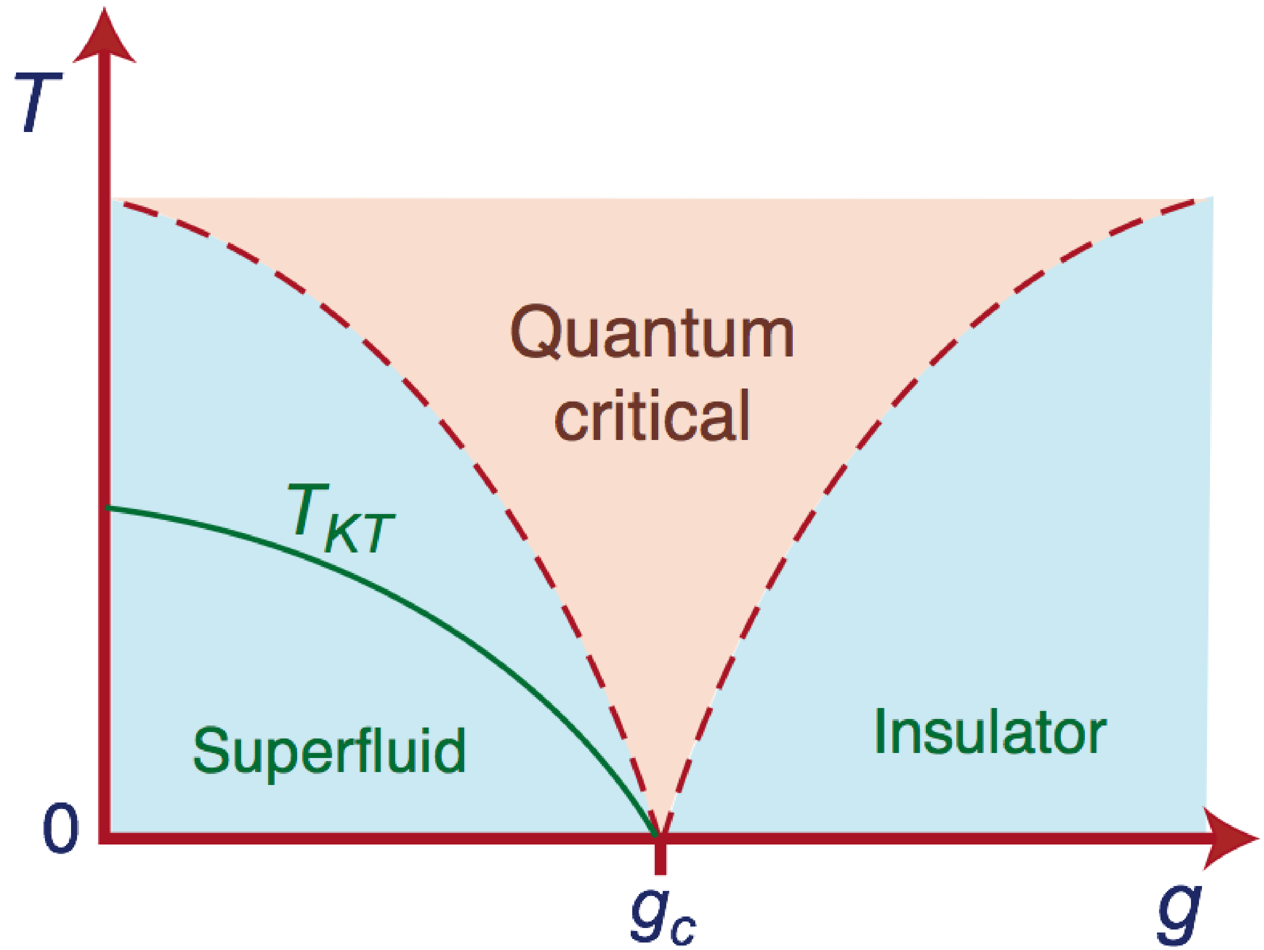
FIG. 3: Contour plot (with logarithmic spacing) of the thermoelectric conductivity α_{xy} (Eq. 2.3) as a function of temperature T and magnetic field B, for parameters $\hbar v = 47$ meV Å, $\delta - \delta_I = 0.025$ and $\tau_{\text{imp}} = 10^{-12}$s estimated for LSCO. In the ordered low temperature regime $T < T_c \approx 30$K, Eq. (2.3) will receive modifications.

QuickTime™ and a
decompressor
are needed to see this picture.

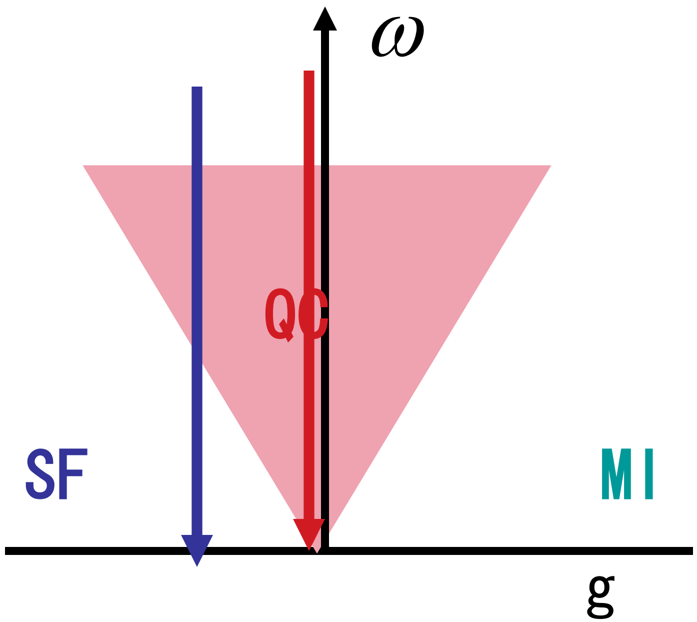
Experiment
Phuan Ong et al.

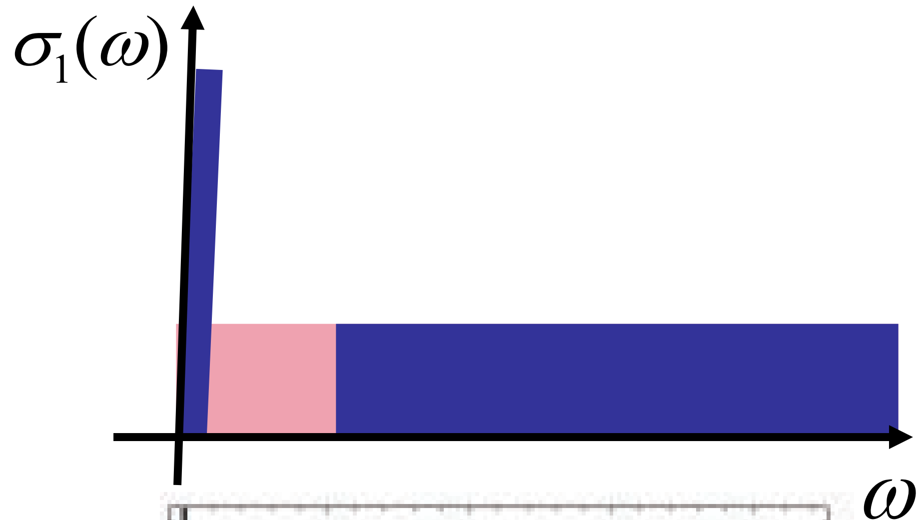
AdS-CFT computation:
Hartnoll, Kovtun, Mueller, Sachdev

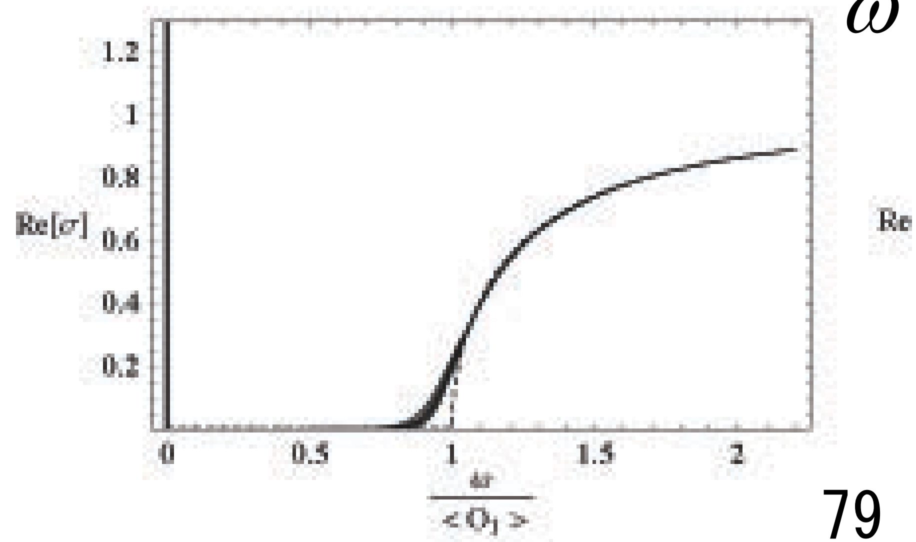
Superfluid–Mott insulator Quantum phase transition



Optical conductivity near 2+1D superfluid-Mott insulator QPT



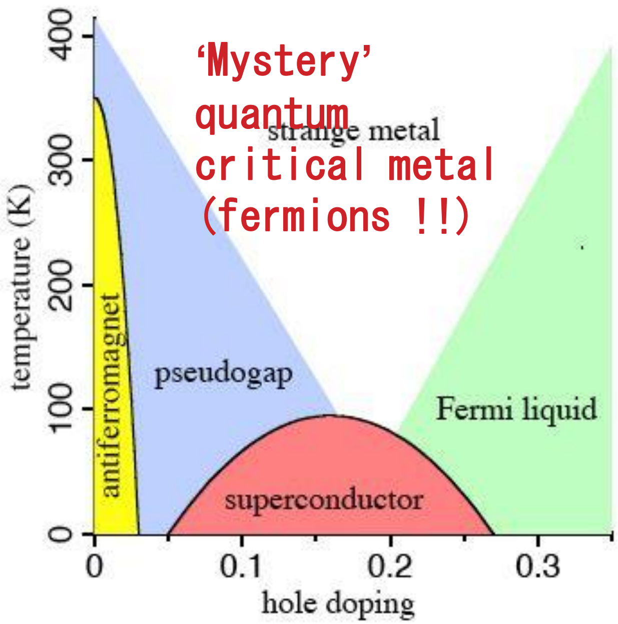




Hartnoll,
Herzog, Horowitz,
arXiv:0803.3295

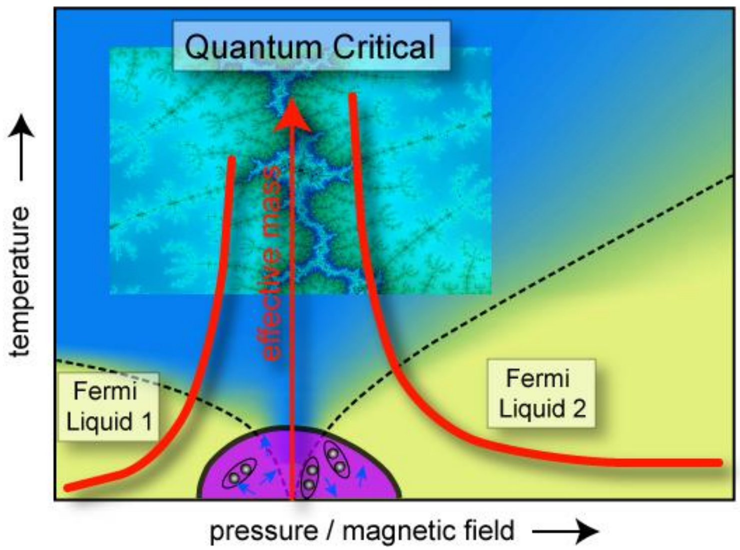
Fermionic quantum phase transitions

High Tc superconductors



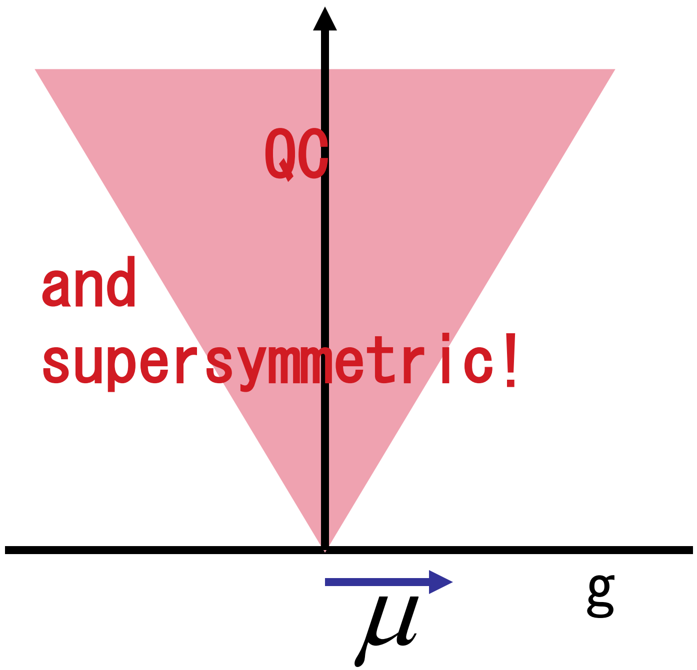
JZ, Nature 430, 512 (2004)

Heavy fermion metals



JZ, Science 319, 1205 (2008)

Breaking SUSY CFT by chemical potentials ...



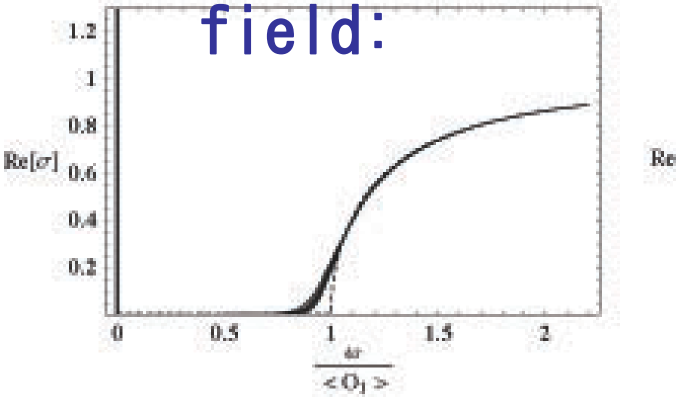
Driving away from criticality by imposing finite fermion and boson charge density.
Breaks scale invariance ...
and supersymmetry!

Like fermion-boson model (?):

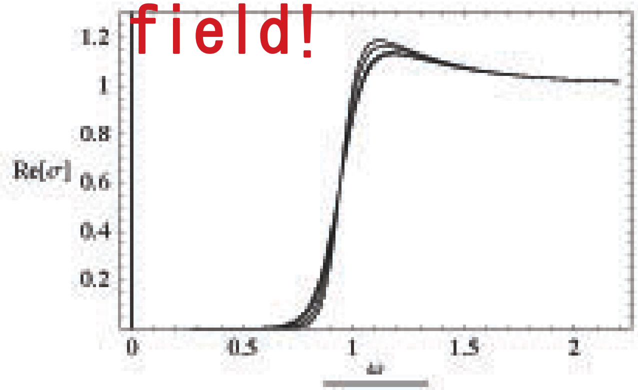
$$H = b^{+}b + f^{+}f + (b\ f^{+}f^{+} + f\ f\ b^{+})$$

Boson and Fermion pair optical conductivity

Bose field:

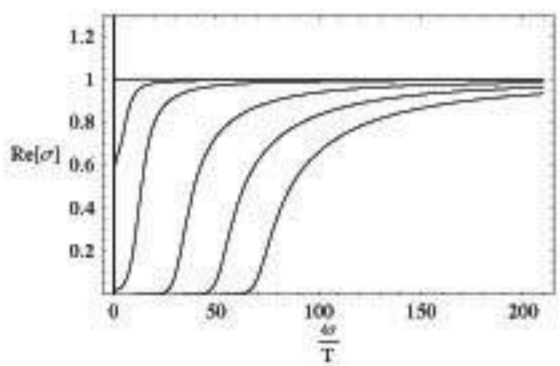


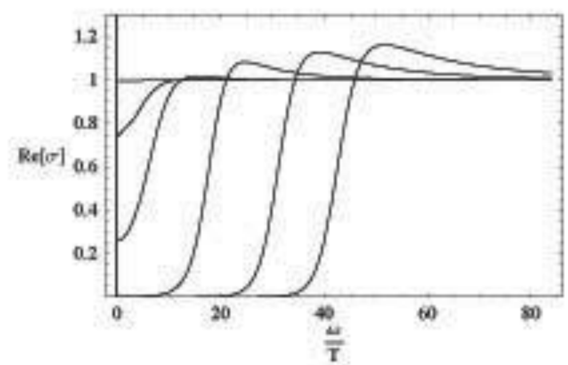
Fermion pair field!



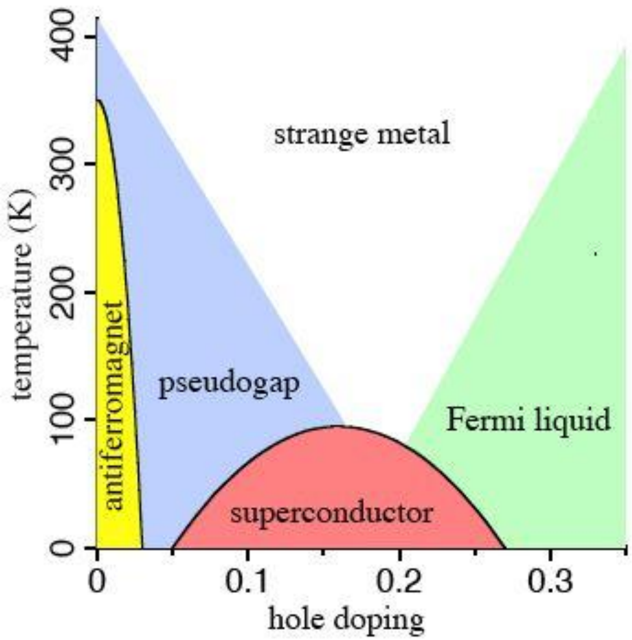
BCS like temperature dependence with

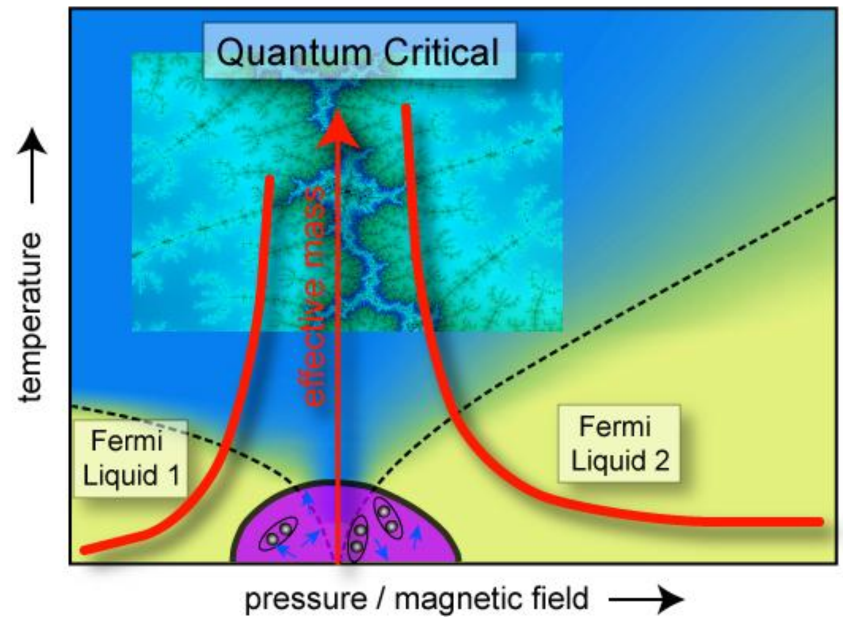
$$\frac{2\Delta_0}{k_B T_c} \approx 8$$



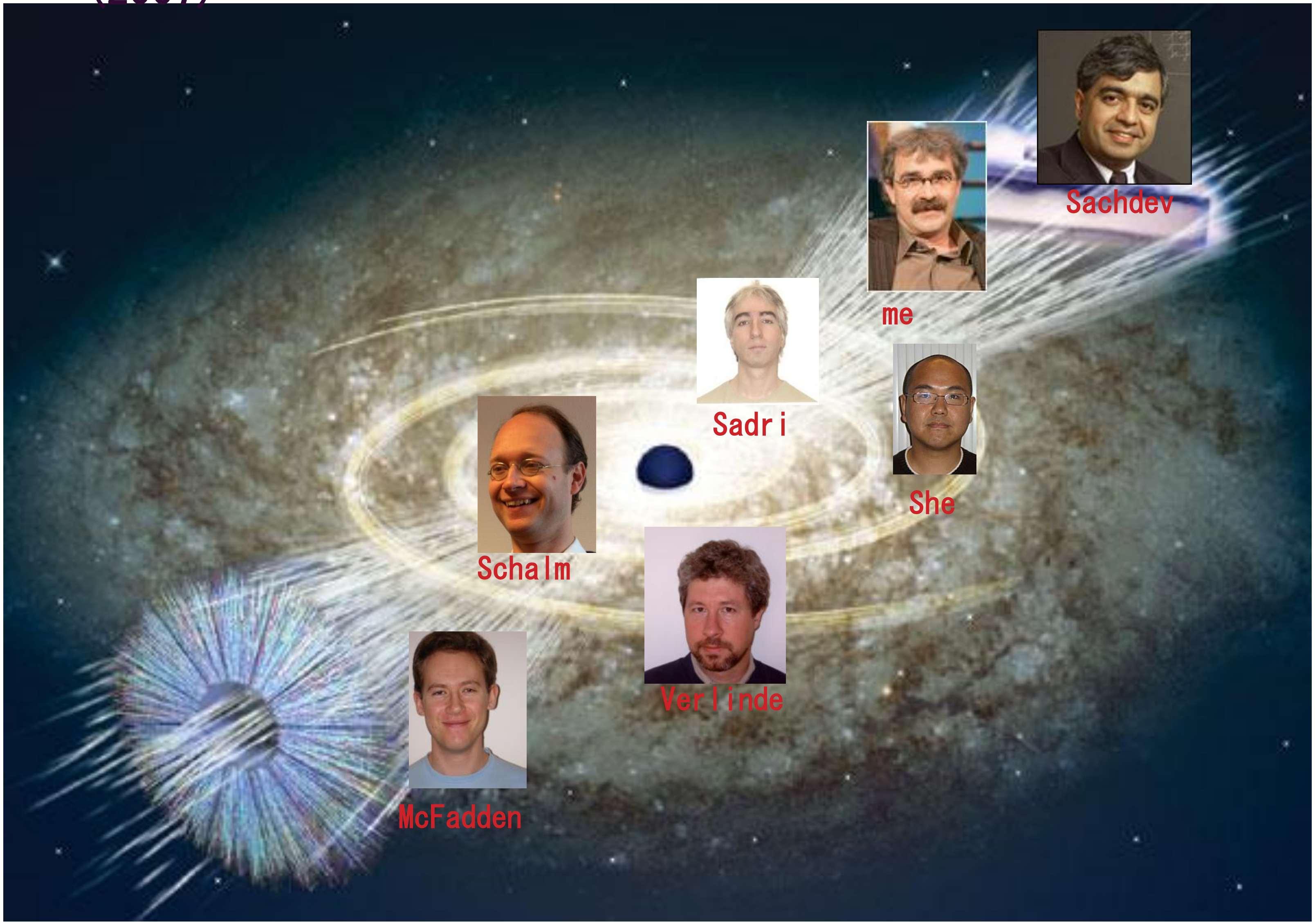


Fermionic quantum criticality??





JZ: *‘A black hole full of answers’*, Nature 448, 1001 (2007)



The Viscosity-Entropy ratio

Hartnoll-Son-Starinets (2002)

CFT viscosity, Kubo formula:

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dt\,d\vec{x}\,e^{i\omega t}\left\langle\left[T_{xy}(t,\vec{x}),T_{xy}(0,0)\right]\right\rangle = \lim_{\omega\to 0}\lim_{q\to 0}\operatorname{Im}G^{R}_{xy,xy}(\omega,\vec{q})$$

AdS correspondent: graviton absorption cross section by black hole:

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G} \qquad \sigma_{abs} = -\frac{16\pi G}{\omega}\operatorname{Im}G^{R}(\omega)$$

= area of horizon (GR theorems)

CFT entropy density: AdS correspondent = BH entropy = **area of horizon**

Universal viscosity-entropy ratio for CFT's with gravitational dual:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Conformal symmetry = quantum criticality

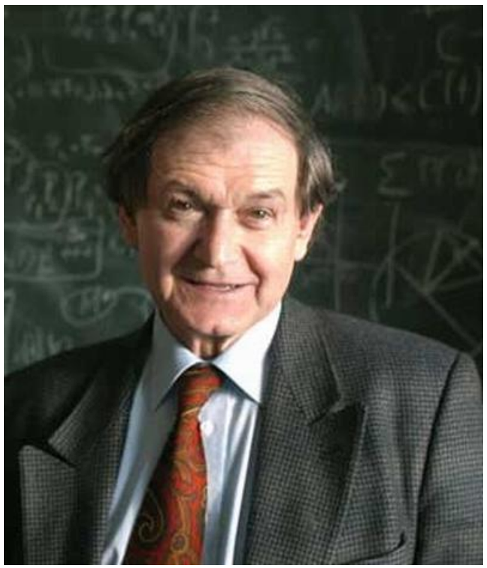
Supersymmetry <==> non-renormalization theorems 'Planes of strongly interacting unstable fixed points'

AdS is the 'geometrical dual' of conformal invariance

Black holes, gauge background sources, etcetera, break conformal invariance

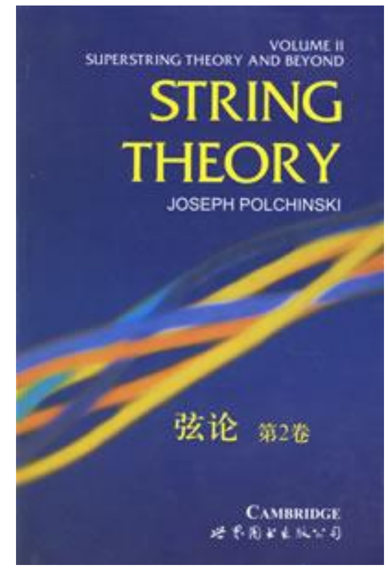
AdS/CFT is merely a 'generating functional' of fanciful scaling analysis ????

Why do I care?



Roger Penrose

Darius Sadri



AdS/CFT for finite temperature hydrodynamics

Finite temperature fields: imaginary time compactification radius

$$R_\tau = \frac{\hbar}{k_B T}$$

Are there AdS spaces that are periodic in Euclidean time?

Yes, two (Hawking-Page, Witten):

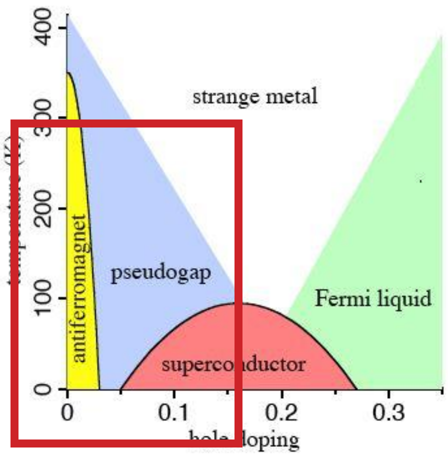
(1) EAdS (low T confined phase, no entropy)

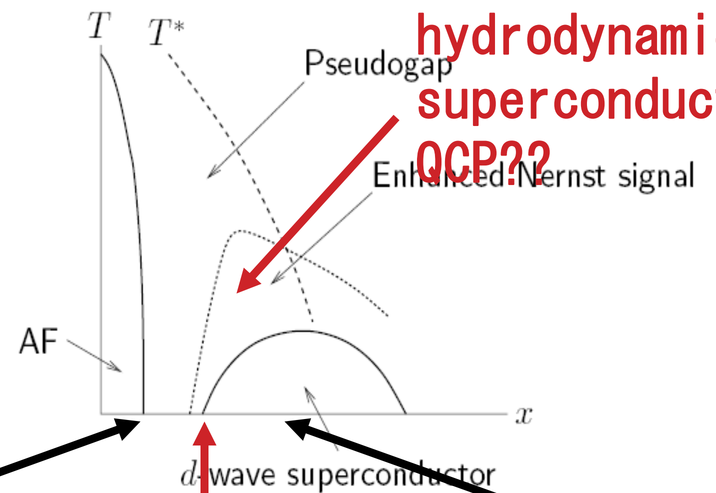
(2) Black hole in AdS (high T deconfined

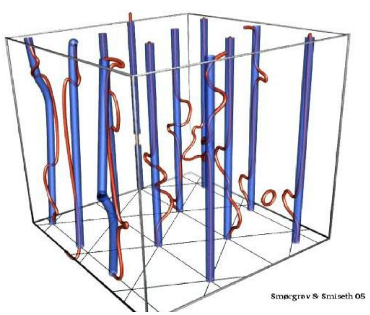
$$ds^2_{AdS-BH} = \frac{R^2}{x_0^2}\left((1-x_0^4T^4)dx_0^2 - \frac{dt^2}{(1-x_0^4T^4)} + dr^2 + r^2 d\Omega^2\right)$$

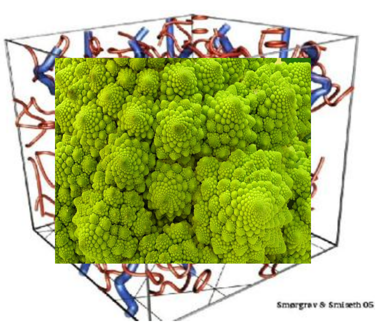
‘Nernst regime’: fluctuating superconducivity

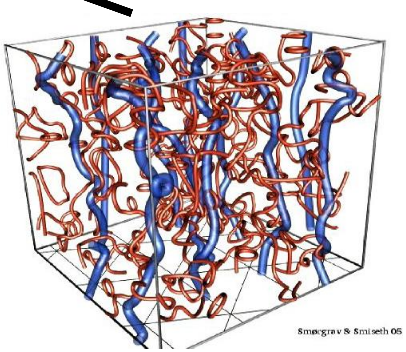
Finite temperature hydrodynamics of superconductor-insulator QCP??







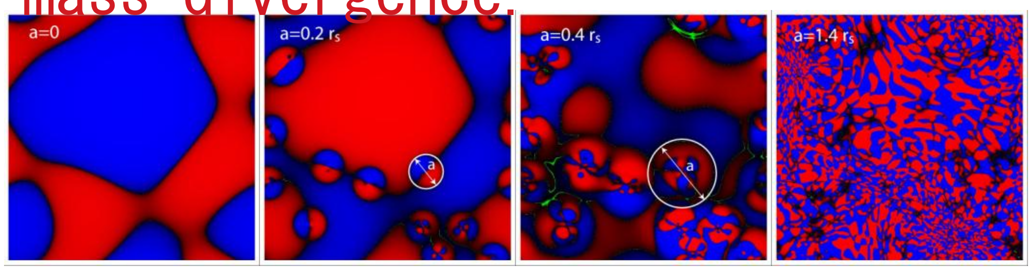


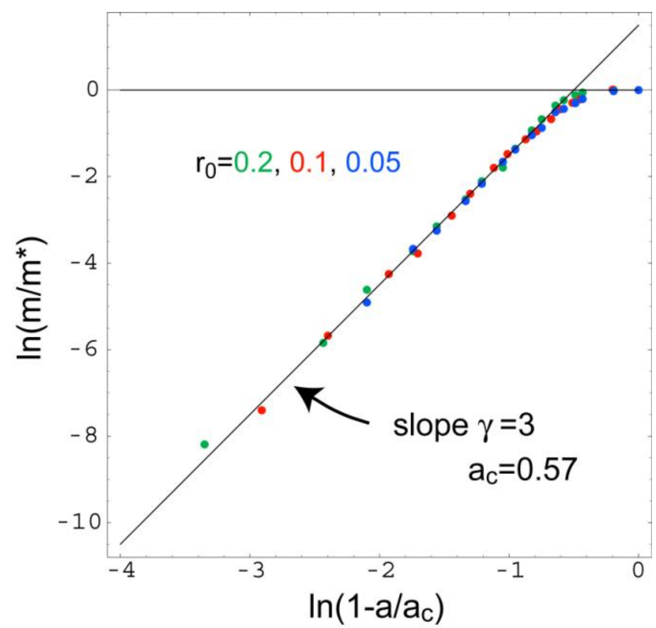


Ceperley path integrals and fermionic quantum criticality

Fermi-Dirac statistics encoded in 'nodal surface constraints' : probablistic theory.

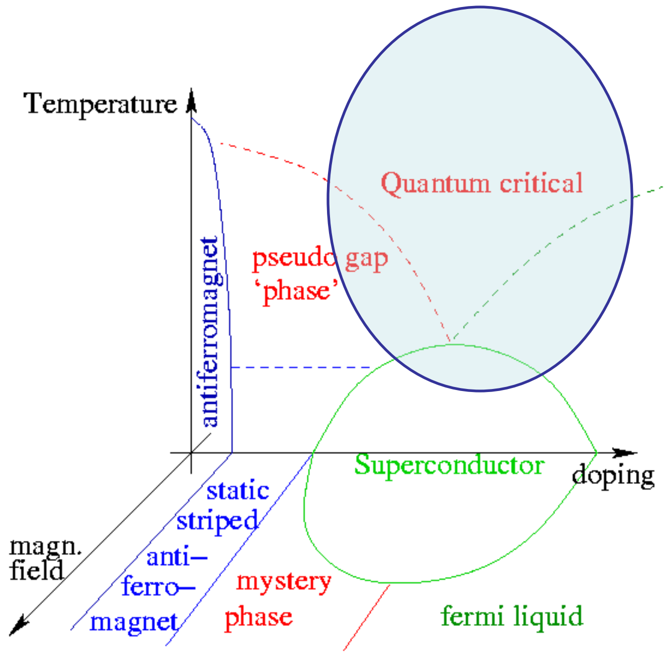
Explicit example: Feynmannian Fermionic back flow, fractal nodal surface and quasiparticle effective mass divergence.

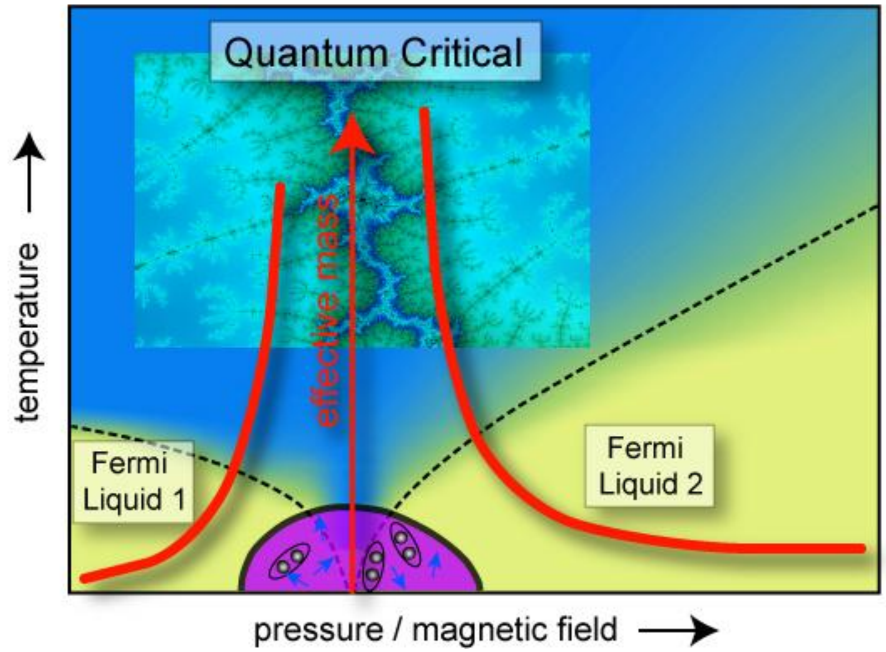




Krueger, JZ,
arXiv:0804.2161

The AdS/CFT challenge: fermionic quantum criticality





High Tc Superconductors

JZ, Nature 430, 512 (2004)

Heavy Fermion metals

JZ, Science 319, 1205 (2008)

NSF-KITP-08-75
arXiv:0805.3898 [hep-th]

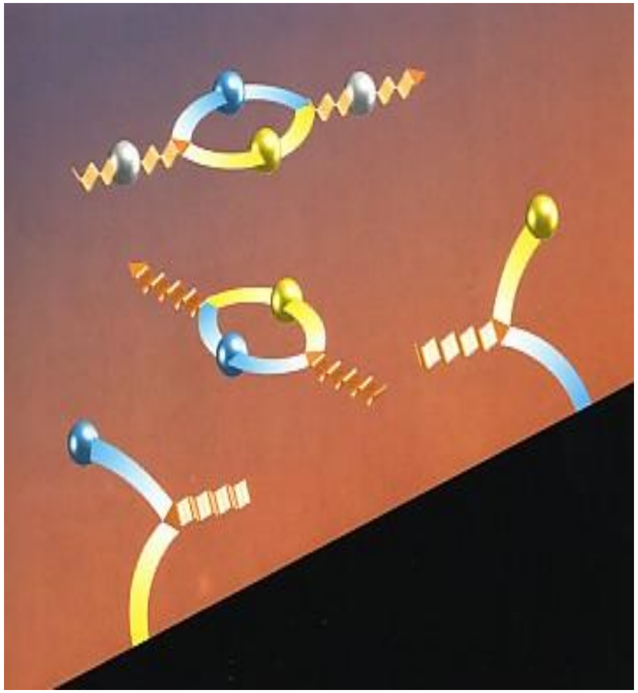
Pseudogap and time reversal breaking in a holographic superconductor

Matthew M. Roberts[♮] and Sean A. Hartnoll[♭]

It has been appreciated for some time that materials of significant theoretical and practical interest, such as the heavy fermion compounds [3, 4] or the high T_c cuprates [5], require new theoretical input. For these materials, neither the pairing mechanism, leading to the charged condensate, nor the properties of the superconducting state itself are those of BCS theory. Furthermore, there are indications that the relevant new physics is strongly coupled, requiring a departure from the quasiparticle paradigm of Fermi liquid theory [3, 5].

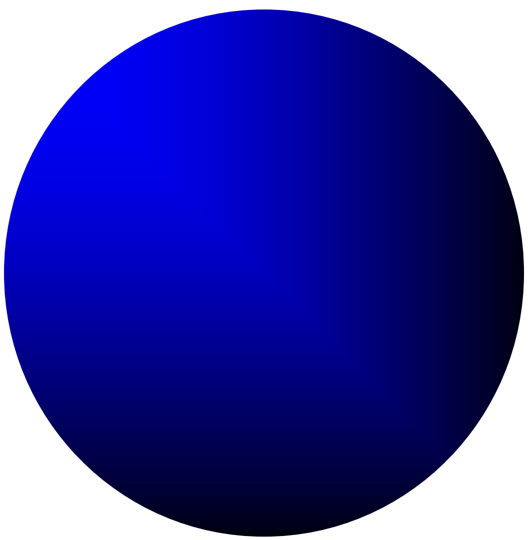
Our hope is that a solvable model of a strongly coupled system undergoing a superconducting phase transition might help the development of new theories of superconductivity. It has recently been shown that the AdS/CFT correspondence [6] can indeed provide models of strongly interacting superconductors in which calculations can be performed from first principles [7, 8, 9, 10]. These recent works are part of a wider program of applying the AdS/CFT correspondence to condensed matter systems [11, 12, 13, 14, 15, 16]. The philosophy is that even if the underlying microscopic descriptions of theories with AdS duals are likely quite different to those arising in materials of experimental interest, aspects of the strongly coupled dynamics and kinematics may be universal. Kinematically speaking, theories with AdS duals are quantum critical [17]. The superconductors described to date within the AdS/CFT framework are quantum critical systems that undergo a superconducting phase transition as a function of temperature over chemical potential.

Hawking Temperature & Entropy



$$T = \frac{\hbar g}{2\pi k c}$$

g = acceleration at horizon



$$S = \frac{k c^3 A}{4\hbar G}$$

A = area of horizon

Strings and Black Holes

Universal physics of 0+1D Black hole <=> 1+1D CFT

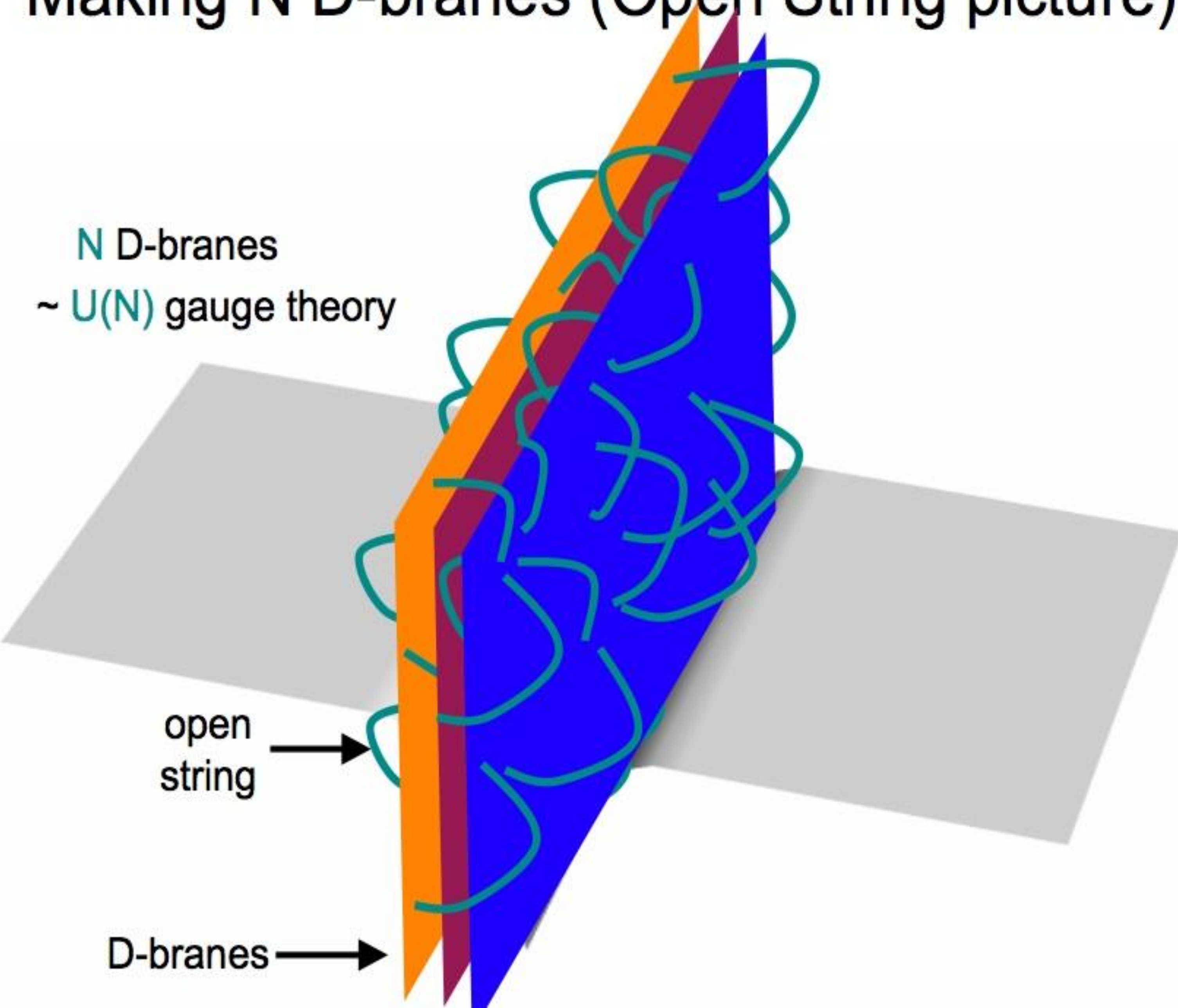
A classical = cosmic string is a Black string
String theory has non-perturbative extended objects (like strings/vortices) called (mem)branes with p spatial dimensions

CLAIM:
For certain (extremal) Black p-branes the universal (near horizon) brane physics is described by a p+1 dimensional CFT

[Maldacena]

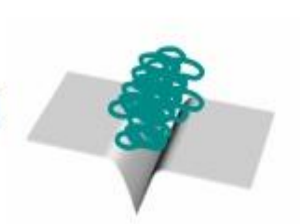
The near-horizon of an extremal black brane is an Anti-de-Sitter space (homogeneous negative curvature)

Making N D-branes (Open String picture)



AdS/CFT correspondence

Closed string description



=

Open string description

Low energy limit with large N

Supergravity on $AdS_5 \times S^5$

dual

D=4 $\mathcal{N}$=4 U(N) Super Yang-Mills

Good approximation for strong coupling

$\lambda = g_{\mathrm{YM}}^2 N \gg 1$

Good approximation for weak coupling

$\lambda = g_{\mathrm{YM}}^2 N \ll 1$

Viscosity/Entropy ratio

- Kubo formula = Zero frequency absorption of graviton by BH

- Zero frequency absorption of a graviton by BH = Area

- BH entropy = Area [Hawking]

$\Longrightarrow$

Viscosity/Entropy ratio is order (1)!

[In pQCD long calculation Visc/entropy ~ $1/g^2 \ln(g)$ [Arnold, Moore, Yaffe]]

[Policastro, Son, Starinets]

More formally

- Viscosity is given by Kubo's formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\vec{x}\, e^{i\omega t} \langle [T_{xy}(t,\vec{x}),\, T_{xy}(0,0)] \rangle$$
$$= \lim_{\omega \to 0} \lim_{\vec{q} \to 0} \mathrm{Im}\, G^R_{xy,xy}(\omega, \vec{q})$$

- Via AdS/CFT correspondence, the imaginary part of the retarded Green's function is mapped to the graviton absorption cross section.

$$\sigma_{\rm abs} = -\frac{16\pi G}{\omega} \mathrm{Im}\, G^R(\omega)$$

- viscosity $\sim$ absorption cross section for low-energy gravitons

$$\eta = \frac{\sigma_{\rm abs}(0)}{16\pi G}$$

Universality of viscosity/entropy density ratio

- Absorption cross section = area of horizon (follows from a couple of theorems in general relativity)
- Entropy is also proportional to area of horizon: $S = A/(4G)$

$\Rightarrow$ in *all* theories with gravity duals:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi\, k_B}$$

where η is the shear viscosity, s is the entropy per unit volume.

This is valid in a large, but restricted, class of strongly coupled quantum field theories, which are in a sense infinitely strongly coupled

Boltzmann equation is never used

Conformal symmetry = quantum criticality

Supersymmetry <==> non-renormalization theorems

'Planes of unstable (strongly interacting) quantum critical points'

AdS is the 'geometrical dual' of conformal invariance
Black holes, gauge sources break conformal invariance

AdS-CFT is 'generating functional' of fanciful scaling analysis ????

Thermo-electric response coefficients

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \sigma & \alpha \\ T\alpha & \kappa \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix}$$

$$\vec{Q} = \text{ heat flow}$$

Kubo formula. e.g.

$$Q_i = T_{0i}$$

$$\alpha = \lim_{\omega \to 0} \int dt \, \frac{e^{i\omega t}}{\omega} \langle \vec{Q}(t) \vec{J}(0) \rangle$$

Hartnol, Kovtun
Muller, Sachdev

(d+1)-dim Dyonic AdS Black Hole

$$Q^2 = q_e^2 + q_m^2$$

metric + B-field fluctuations

$$F(r) = -\Lambda r^2 - \frac{GM}{r} + \frac{Q^2}{r^2}$$

$$ds^2 = -F(r)(dt + h^{0i}dx_i)^2 + \frac{dr^2}{F(r)} + r^2 dx_i^2$$

$$B_r = \frac{q_m}{r^2}, \qquad B_i = \epsilon_{ijk}\partial_j A_k$$

linearized Einstein-Maxwell equation =>
two point functions of heat flow and electric currents.

$$\left\langle T_{0i}\, J_j \right\rangle = \frac{\partial^2}{\partial h^{0i}\partial A^j} S_{Einst-Maxw}(h, A)$$

⇒thermo-electric coefficients follow from Kubo formula

⇒ dyonic AdS black holes exhibit Nernst effect!

Hartnol, Kovtun
Muller, Sachdev

(d+1)-dim AdS Black Hole = d-dim Thermal CFT

Black hole + metric fluctuations $F(r) = -\Lambda r^2 - \frac{GM}{r} + \frac{Q}{r^2}$

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(dx_i^2 + h_{ij}dx^i dx^j)$$

linearized Einstein equation => quadratic Einstein action

= sufficient to compute two point functions of stress tensor.

$$\langle T_{ij} T_{kl} \rangle = \frac{\partial^2}{\partial h^{ij} \partial h^{kl}} S_{Einst}(h)$$

Two point functions follow from free wave propagation

AdS/CFT

Map of theories:

map of 5d to 4d correlation functions

[Follows from open-closed string duality]

$$
\begin{aligned}
W_{CFT}(J) &= S_{AdS}(\phi)_{\phi_{x_0 \to 0} = J} \\
g_{YM}^2 N_c &= R^4/\alpha' \\
g_{YM}^2 &= g_s
\end{aligned}
$$

A Duality

[Note: CFT sources are AdS fields] [Maldacena, Witten
Gubser, Klebanov,
Polyakov]

Entropy at strong coupling

Black 3-brane background:

$$ds^2 = \frac{r^2}{R^2}[-f(r)dt^2 + d\vec{x}^2] + \frac{R^2}{r^2 f(r)}dr^2 + R^2 d\Omega_5^2, \qquad f(r) = 1 - \frac{r_0^4}{r^4}$$

Hawking temperature;

$$T_H = \frac{r_0}{\pi R^2}$$

Entropy is computed from area of the horizon, and the result is $S = \pi^6 R^8 T^3 V_{3\mathrm{D}}$.
Using AdS/CFT mapping:

$$s = \frac{S}{V_{3\mathrm{D}}} = \frac{\pi^2}{2} N_c^2 T^3$$

At zero 't Hooft coupling: $s = \frac{2\pi^2}{3} N_c^2 T^3$

$$s(g^2 N_c = \infty) = \frac{3}{4} s(g^2 N_c = 0)$$

String Theory for RHIC physics

meson melting at high J
Mach cones due to heavy quarks
Photon Production Rates
J/Psi screening
...

Conceptually very simple calculations
compared to (strong coupling) QCD
with answers very close to experiment (5-30%)

Benefits from a ~~natural~~ universal point of view

AdS/CFT:

CFTs are scale invariant

CFTs: **universal** dynamics of continuous phase transitions

...String theory for the real world?

Viscosity of the thermal conformal field theory is computed from scattering of gravity waves off an AdS black hole background

Viscosity:

$$\langle T_{ij} \rangle = \eta \dot{h}_{ij} \ , \qquad i \neq j$$

Kubo formula

$$\eta = \lim_{\omega \to 0} \int dt \, \frac{e^{i\omega t}}{\omega} \langle T_{ij}(t) T_{ij}(0) \rangle$$

Gauge/String Duality