Inflation, reheating and flat direction preheating

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Outline

1. Inflation, Reheating and Role of Flat Directions
   - Theoretical uncertainties
   - Example: Gravitational decay of inflaton and reheating
   - Reheating in the presence of long lived flat directions

2. Nonperturbative Flat Direction Decay
   - Simple toy models for F.D. decay
   - MSSM example

3. Implications of the Nonperturbative Decay
Flat directions: directions in field space along which \( V = 0 \). ⇒ generic feature of supersymmetric theories.

**Example**

\[ V = e^2 \left( |\phi_1|^2 - |\phi_2|^2 \right)^2 \]

- Potential flat along \( |\phi_1| = |\phi_2| = \Phi \).
- + 2 phases: \( U(1) \) gauge ⇒ Only one combination physical
- Flat direction: \( \Phi e^{i\Sigma} \) ⇒ Two real parameters

- May acquire large VEVs during inflation ⇒ Interesting for cosmology: Generating baryon asymmetry Affleck, Dine 1985 ⋯
  - Inflaton Dvali 1996⋯
  - Curvaton Enqvist, Kasuya, Mazumdar 2002⋯
  - Delayed reheating Allahverdi, Mazumdar 2005

- In this talk, inflaton → external field. Concentrate on reheating and the fate of the flat directions.
Inflation, Reheating and Role of Flat Directions
Nonperturbative Flat Direction Decay
Implications of the Nonperturbative Decay

Theoretical uncertainties
Example: Gravitational decay of inflaton and reheating
Reheating in the presence of long lived flat directions

Thermal history and inflation

- Clear knowledge from BBN on \((z_{BBN} \sim 10^{10})\).
- When radiation dominated era starts? \((z_{RD} \gtrsim z_{BBN})\)
- Before radiation: inflation. Both theoretical control and data
- Slow roll approximation \(\Rightarrow\) Strict predictions within given model.

\[
\epsilon = \frac{M_p^2}{16 \pi} \left( \frac{V'}{V} \right)^2 \ll 1 , \quad \eta = \frac{M_p^2}{8 \pi} \left( \frac{V''}{V} \right) \ll 1
\]

- e.g. Chaotic inflation
  \(V = \phi^{\alpha}\)
  \[n_s - 1 = -\frac{2 + \alpha}{2N}\]
  \[r = \frac{4 \alpha}{N}\]

Gümrukçüoğlu, A.E. - IPMU, University of Tokyo
Predictions of inflation strongly depend on $N$.

Physics at reheating $\Rightarrow$ Uncertainty on $N$

Matching to when R.D. starts gives range of uncertainty on $N$: $\Delta N \sim 14$
**Reheating**

We do not know:
- Energy scale of inflation
- What is inflaton
- Coupling to SM fields

**Bounds on RH**
- BBN: $T_{\text{rh}} \gtrsim \text{MeV}$
- Single field slow roll inflation: $T_{\text{rh}} \lesssim 10^{16}$ GeV
- Gravitino bound: $T_{\text{rh}} \lesssim 10^5$–$10^9$ GeV

Kawasaki et al. 2008
Example of inflaton decay

- Chaotic inflation with massive inflaton $\psi$. Assume preheating negligible $\Rightarrow$ Perturbative decay.

- Inflaton oscillations after inflation:
  $\rho_\psi \sim a^{-3}$

- Gravitational decay:
  when $H = \Gamma \propto \frac{m_\psi^3}{M_p} \sim 10 \text{ TeV}$, into relativistic particles, after $\sim 10^{13}$ oscillations.

- Gauge mediated $2 \rightarrow 3$ interactions lead to very rapid thermalization.
  Davidson, Sarkar '00
  At the time of inflaton decay: $\Gamma_{2\rightarrow3} \sim \alpha^3 \frac{M_p}{m_\psi} H$

  - If $\alpha \gtrsim \left( \frac{m_\psi}{M_p} \right)^{1/3} \sim 10^{-2} \Rightarrow$ Immediate thermalization

  - $T_{rh} \sim \left( \frac{m_\psi^3}{M_p} \right)^{1/2} \sim 10^8 \text{ GeV}$
MSSM Flat Directions

MSSM Potential: \[ V = \sum_{i} |F_i|^2 + \frac{1}{2} \sum_a g_a^2 (\phi^\dagger T^a \phi)^2 \]
\[ F_i \equiv \frac{\partial W_{\text{MSSM}}}{\partial \phi_i}, \quad W_{\text{MSSM}} = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu H_u H_d. \]

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Flat directions catalogued by gauge invariant monomials.

Plethora of flat directions along which $V = 0$.

However, in MSSM, SUSY is broken, so these directions are only approximately flat.

Dine, Randall, Thomas '95
Gherghetta, Kolda, Martin '95
Evolution of Flat Directions

\[ V = (m_\phi^2 - c H^2) |\phi|^2 + \frac{|\lambda|^2 |\phi|^{2d-2}}{M^2 d^{-6}} + \left( A \frac{\lambda \phi^d}{M^{d-3}} + h.c \right), \quad d \geq 4 \]

**During inflation**

- \( c > 0 \Rightarrow \) VEV generation due to instability, up to global minimum
- \( c < 0 \Rightarrow \) No growth.
- \( c \) model dependent. (\( c < 0 \) for minimal Kähler potential)

**After inflation**

- After \( H \sim m_\phi \Rightarrow \) F.D. starts oscillating with initial magnitude
  \[ \langle \phi_0 \rangle \sim \left( \frac{m_\phi M^{d-3}}{|\lambda|} \right)^{1/(d-2)} \]
  and \( \langle \phi \rangle \sim a^{-3/2} \)

- \( A \) terms provide initial angular momentum: Spiral motion towards origin

Dine, Randall, Thomas 1995

Dine, Randall, Thomas 1995
Role in reheating

- F.D. starts rotating before inflaton decays: \( \frac{R_d \psi}{R_\phi} = \frac{M_p^{4/3}}{m_\phi^{2/3}} > 1 \)
- In the presence of F.D., gauge fields acquire masses \( \propto \text{VEV} \), scatterings they mediate slow down.
  
  Allahverdi, Mazumdar '05

At the time of inflaton decay, \( \Gamma_{2 \rightarrow 3} \sim \frac{\alpha^3 M_p^5 m_\phi^2}{m_\psi^5 \phi_0^2} H \)

- Thermalization delayed if \( \phi_0 \gtrsim \frac{\alpha^3/2 M_p^{5/2} m_\phi}{m_\psi^{5/2}} \sim 10^{-2} M_p \)

- F.D. dominates if

\[
\phi_0 \gtrsim \frac{M_p^{4/3}}{m_\psi^{3/4}} \sim 10^{-2} M_p
\]

\[\Rightarrow T_{rh} \sim M_p^{1/6} m_\phi^{5/6} \sim 10^6 \text{GeV}\]

Olive, Peloso 2006
Perturbative or nonperturbative decay?

- These consequences are valid only if flat directions are long lived.
- Standard lore: The fields coupled to flat direction acquire large mass
  \[ \Gamma \sim \frac{m^3}{\phi^2} . \]  
  Affleck, Dine 1985
- Flat direction decays after \( \sim 10^{11} \) rotations.

How about nonperturbative decay?

- Complex scalar field \( \chi \) coupled to flat directions: \( \Delta V = g^2 |\phi|^2 |\chi|^2 \)
- Eq. of motion of fluctuations:  
  \[ \delta \chi'' + [p^2 + m^2 + g^2 |\phi(t)|^2] \delta \chi = 0 . \]

rate of change in frequency

- Maximum \( \omega'/\omega^2 \) obtained when amplitude is minimum:
  \[ \left. \frac{\omega'}{\omega^2} \right|_{\text{max}} \approx \frac{m_\phi}{g \epsilon \phi_0} \sim 10^{-14} \]
- Typically, \( 10^{-3} \lesssim \epsilon \lesssim 10^{-1} \Rightarrow \text{only perturbative decay?} \)
Realistic example: $H_uH_d$ Flat Direction

- **MSSM, with $H_uH_d$ F.D.:**
  \[ H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} h_u \\ \phi + \xi_u \end{pmatrix}, \quad H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi + \xi_d \\ h_d \end{pmatrix}, \quad VEV \rightarrow \phi = |\phi| e^{i\sigma} \]

- **Potential, quadratic in fluctuations:**
  \[ V = \frac{y_u^2}{2} |\phi|^2 \left( |Q_u|^2 + |u|^2 \right) + \frac{y_d^2}{2} |\phi|^2 \left( |Q_d|^2 + |d|^2 \right) + \frac{y_e^2}{2} |\phi|^2 \left( |L_d|^2 + |e|^2 \right) \]
  \[ + \frac{g^2 + g'^2}{16} |\phi|^2 \left( \text{Re}[\xi_u - \xi_d], \text{Im}[\xi_u - \xi_d] \right) \mathcal{M}^2 \left( \begin{array}{c} \text{Re}[\xi_u - \xi_d] \\ \text{Im}[\xi_u - \xi_d] \end{array} \right) \]
  \[ + \frac{g^2}{8} |\phi|^2 \left( \text{Re}[h_u + h_d], \text{Im}[h_u + h_d] \right) \mathcal{M}^2 \left( \begin{array}{c} \text{Re}[h_u + h_d] \\ \text{Im}[h_u + h_d] \end{array} \right) \]
  \[ + \frac{g^2}{8} |\phi|^2 \left( \text{Im}[-h_u + h_d], \text{Re}[h_u - h_d] \right) \mathcal{M}^2 \left( \begin{array}{c} \text{Im}[-h_u + h_d] \\ \text{Re}[h_u - h_d] \end{array} \right) \]

- \[ \mathcal{M}^2 = \begin{pmatrix} \cos^2 \sigma & \cos \sigma \sin \sigma \\ \cos \sigma \sin \sigma & \sin^2 \sigma \end{pmatrix} \Rightarrow \text{Eigenmasses} \left( \frac{g}{2} |\phi|, 0 \right), \text{but eigenvectors have } t \text{ dependence through } \sigma(t). \]
Quantization of coupled bosons

\[ S = \frac{1}{2} \int d^3 k \, d\eta \left( \phi'^\dagger \phi' - \phi^\dagger \Omega^2 \phi \right) \]

- **Nondiagonal, time dependent frequency matrix:**
  \[ \phi^\dagger \Omega^2 \phi = (\phi^\dagger C) (C^T \Omega^2 C) (C^T \phi) \]

- **Kinetic Mixing:**
  \[ \phi'^\dagger \phi' = \tilde{\phi}'^\dagger \tilde{\phi}' + \phi'^T \Gamma \tilde{\phi} + \tilde{\phi}' \Gamma^T \phi'^\dagger + \phi^\dagger C^T C' \tilde{\phi} \]
  \[ (\Gamma \equiv C^T C') \]

- **\( \tilde{\phi}_i \):**
  \[ \tilde{\phi}_i = \left[ \frac{1}{\sqrt{2} \omega} \begin{pmatrix} e^{-i \int^t \omega dt} A + e^{i \int^t \omega dt} B \end{pmatrix} \right]_{ij} \hat{a}_j(\vec{k}) + [\cdots]_{ij}^* \hat{a}_j^\dagger(-\vec{k}) \]

(Bogolyubov Matrices)

\[ \alpha \alpha^\dagger - \beta^* \beta^T = 1, \quad \alpha \beta^\dagger - \beta^* \alpha^T = 0 \]
Quantization of coupled bosons

\[ \mathcal{H} = \frac{1}{2} (\hat{a}^\dagger, \hat{a}) \begin{pmatrix} \alpha^\dagger & \beta^\dagger \\ \beta^T & \alpha^T \end{pmatrix} \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix} \begin{pmatrix} \alpha & \beta^* \\ \beta & \alpha^* \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{a}^\dagger \end{pmatrix} \]

- Time dependent annihilation/creation operators:
  \[ : \mathcal{H} := \omega_i \hat{b}_i^\dagger \hat{b}_i. \]

- Occupation numbers: \( N_i(t) = \langle \hat{b}_i^\dagger \hat{b}_i \rangle = (\beta^* \beta^T)_{ii} \)

- Equations of motion:
  \[
  \begin{align*}
  \alpha' &= (-i \omega - I)\alpha + \left( \frac{\omega'}{2\omega} - J \right) \beta \\
  \beta' &= (i \omega - I)\beta + \left( \frac{\omega'}{2\omega} - J \right) \alpha
  \end{align*}
  \]

Anti-Hermitian: Rotate produced states. Preserves total \( N(t) \)

Hermitian: Particle production.

- Adiabaticity condition
  \[
  \left[ \frac{\omega'}{\omega^2} - 2 \frac{1}{\sqrt{\omega}} J \frac{1}{\sqrt{\omega}} \right]_{ij} = \left[ \frac{\omega'}{\omega^2} - \left( \frac{1}{\sqrt{\omega}} \right) \right]_{ij} \ll 1
  \]
Inflation, Reheating and Role of Flat Directions
Nonperturbative Flat Direction Decay
Implications of the Nonperturbative Decay

Simple toy models for F.D. decay
MSSM example

Nonperturbative Decay of Flat Directions
Toy Model (without gauge field)

- Interaction that mimics the D-term potential: $\Delta V = 2 g^2 \langle \text{Re}\phi \chi^* \rangle^2$

- Coupling between Re and Im parts of $\chi$ through

$$\mathcal{M}^2 = 2 g^2 |\phi|^2 \begin{pmatrix} \cos^2 \sigma & \cos \sigma \sin \sigma \\ \cos \sigma \sin \sigma & \sin^2 \sigma \end{pmatrix} + \begin{pmatrix} m^2_\chi & 0 \\ 0 & m^2_\chi \end{pmatrix}$$

- Frequencies of physical states:
  $$\omega_1 = \sqrt{k^2 + m^2_\chi + 2 g^2 |\phi|^2},$$
  $$\omega_2 = \sqrt{k^2 + m^2_\chi}.$$  

- Eigenfrequencies are adiabatically evolving, but the physical eigenstates are rotating nonadiabatically.

- Exponential particle production due to rotation of eigenstates, if
  $$k < \sqrt{m^2_\phi - m^2_\chi}.$$  

(Similar observation in Kawasaki, Takahashi 2004)
No gauge field in previous toy model

It actually applies to a global $U(1)$ broken by F.D. VEV. The two modes are Higgs+Goldstone

In a local theory, the Goldstone boson removed $\Rightarrow$ No rotation $\Rightarrow$ No decay

Olive-Peloso 2006 argued: “The quick rotation of the mass matrix in field space, and the corresponding preheating effect, takes place if two or more flat directions are excited”.

Flat directions that are mutually exclusive exist. But in general, expect a set of compatible flat directions to be excited during inflation.

No production if hierarchical VEVs. Allahverdi, Mazumdar ’07

“How much hierarchy for production?” $\Rightarrow$ Calculation
Gauged toy model with two FD

- Computation in a model with 2 FD + U(1) gauge field
  \[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi_i|^2 - V , \]
  \[ V = m^2 \left( |\Phi_1|^2 + |\Phi_2|^2 \right) + \tilde{m}^2 \left( |\Phi_3|^2 + |\Phi_4|^2 \right) + \lambda \left( \Phi_1^2 \Phi_2^2 + \text{h.c.} \right) + \tilde{\lambda} \left( \Phi_3^2 \Phi_4^2 + \text{h.c.} \right) + \frac{g^2}{8} \left( q |\Phi_1|^2 - q |\Phi_2|^2 + q' |\Phi_3|^2 - q' |\Phi_4|^2 \right)^2 \]

with \( \langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \frac{F}{a} e^{i \Sigma} \), \( \langle \Phi_3 \rangle = \langle \Phi_4 \rangle = \frac{G}{a} e^{i \tilde{\Sigma}} \)

- Nonrelativistic matter (oscillating massive inflaton) dominates expansion.
- Degrees of freedom in unitary gauge:
  \[ \{ \Phi_i , A_\mu \} \rightarrow \begin{cases} A_\mu^T , & \delta \Phi_1 + \delta \Phi_2 \\ \delta \Phi_3 + \delta \Phi_4 \\ \text{Higgs} A_\mu^L \end{cases} \]
  \[ 8 + 2 = 2 + 4 + 4 \]
  2 light fields

- In contrast, for the 1 FD + U(1) counterpart
  \[ \Rightarrow \text{no extra light degrees} \Rightarrow \text{No nonadiabatic mixing.} \]
Gauged toy model with two FD : Spectrum

Squared-Eigenmasses:

\[ m_1^2 = \frac{g^2}{4a^2} \left( F^2 + G^2 \right), \]
\[ m_2^2 = \frac{g^2}{4a^2} \left( F^2 + G^2 \right) + \frac{(F^2 m^2 + G^2 \bar{m}^2)}{F^2 + G^2} + \frac{3 \left( F^2 \Sigma' + G^2 \tilde{\Sigma}' \right)^2}{a^2 (F^2 + G^2)^2}, \]
\[ m_3^2 = \frac{(F^2 \bar{m}^2 + G^2 m^2)}{F^2 + G^2} + \frac{3 \left( F G' - G F' \right)^2}{a^2 (F^2 + G^2)^2} + \frac{3 F^2 G^2 \left( \Sigma' - \tilde{\Sigma}' \right)^2}{a^2 (F^2 + G^2)^2}, \]
\[ m_4^2 = \frac{(F^2 \bar{m}^2 + G^2 m^2)}{F^2 + G^2}. \]

- \( m_1, m_2 \sim \) VEV (Higgs + Longitudinal vector field)
- \( m_3, m_4 \sim \) TeV (Light fields)
- Quick rotation of eigenstates present. (Light/Light, Light/Higgs)
- \( |m'_{\text{light}}| > m^2_{\text{light}} \rightarrow \) Also have nonadiabatic eigenvalue evolution.
  - Light modes produced
  - Produced light modes rotated into Higgs.
Production when instantaneous vevs comparable, not the initial vevs. \( \Leftarrow \) Elliptic orbits.

\[ \tilde{m}/m = 3.7, \quad G_0/F_0 = 30, \quad k/m = 1/10 \]
Ellipticity

- Degree of ellipticity determines how much initial hierarchy between VEVs can be allowed for production. In this toy model it is provided by quartic terms:

\[ \lambda \left( \Phi_1^2 \Phi_2^2 + \text{h.c.} \right), \quad \tilde{\lambda} \left( \Phi_3^2 \Phi_4^2 + \text{h.c.} \right) \]

- Choice of \( \lambda \) is highly model dependent. We chose \( \lambda = \frac{m^2}{10 |\Phi_0|^2} \Rightarrow \epsilon \sim \mathcal{O}(10^{-2}) \), compatible with Affleck-Dine.

- eg. if \( W = 0 \), no CP violating term \( \Rightarrow \) Radial motion.

\( \text{(Dine, Randall, Thomas '95 ; Giudice, Mether, Riotto, Riva '08)} \) \( \Rightarrow \) Maximal hierarchy allowed : Single F.D. can also decay

- Circular motion \( \Rightarrow \) Requires comparable VEVs for production
Gaused toy model with two FD : Final result

- Hierarchy between heavy and light modes: VEV/TeV $\sim 10^{14}$. Need to control both scales in numerical analysis. Fortunately, the ratio of energy densities have the scaling property
\[
\frac{\rho_{\text{prod}}}{\rho_{\text{flat}}} \sim C \frac{m \tilde{m}}{F_0 G_0} 10^\sigma N_{\text{rot}}
\]

- $C, \sigma = f\left(\frac{G_0}{F_0}, \frac{\tilde{m}}{m}\right)$

- “Decay”, when $\rho_{\text{prod}}/\rho_{\text{flat}} \sim 1$

$m = \text{TeV}, \quad \sqrt{F_0 G_0} = 10^{-2} M_p$
### Realistic MSSM example with 2 FD: $udd + QLd$

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<td>$\phi_6$</td>
<td>$Q_c/s$</td>
<td>$\frac{1}{3}$</td>
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#### VEV configuration:

$$\langle u^c_1 \rangle = \langle s^c_2 \rangle = \langle b^c_3 \rangle = \Phi$$

$$\langle d^c_1 \rangle = \langle \nu_e \rangle = \langle s_1 \rangle = \tilde{\Phi}$$

#### Breaks all SM symmetries

The two flat directions decoupled at background level.

#### Potential:

$$V = |F|^2 + \frac{1}{2} D^2 + V_{\text{soft}}$$

$$|F|^2 = |y_d \phi_6 \phi_2|^2,$$

$$\frac{1}{2} D^2 = \frac{1}{8} g_1^2 \left| \sum_i \phi_i^\dagger Y \phi_i \right|^2 + \frac{1}{8} g_2^2 \left| \sum_{a=1}^3 \phi_i^\dagger \sigma_a \phi_i \right|^2 + \frac{1}{8} g_3^2 \left| \sum_{a=1}^8 \phi_i^\dagger \lambda_a \phi_i \right|^2,$$

$$V_{\text{soft}} = m^2 \sum_{i=1}^3 \phi_i^\dagger \phi_i + \tilde{m}^2 \sum_{i=1}^3 \phi_i^\dagger \phi_i,$$
Quadratic action can be separated into 9 decoupled subsystems

|   | ⊥ Vector | || Vector | Higgs | Flat Dir. | Other heavy | Other light | Total |
|---|---------|--------|--------|--------|-----------|-------------|-----------|-------|
| $S_1$ | 16 |  |  |  |  |  |  | 16 |
| $S_2$ | 8 |  |  |  |  |  |  | 8 |
| $S_3$ |  |  |  |  |  | 2 |  | 2 |
| $S_4$ |  |  |  |  |  |  | 2 | 2 |
| $S_5$ |  | 2 | 2 |  |  |  |  | 4 |
| $S_6$ |  | 2 | 2 |  |  |  |  | 4 |
| $S_7$ |  | 4 | 4 | 4 |  |  |  | 12 |
| $S_8$ |  | 2 | 2 |  |  | 2 | 2 | 8 |
| $S_9$ |  | 2 | 2 |  |  |  | 4 | 8 |
| Total | 24 | 12 | 12 | 4 | 4 | 8 | 64 |

- Only $S_8$ and $S_9$ may contribute to nonperturbative production.
- $S_8$ gives a system where nonadiabatic rotation of eigenvectors may occur.
- $S_9$ gives two copies of the coupled system from $U(1)$ toy model with 2 FD.
  $\Rightarrow$ The numerical analysis is also valid here. F.D. decay after $O(10)$ rotations.
What do we know now?

We know, through semi-analytical studies in a linearized setup, the flat directions start decaying nonperturbatively through the D term, in the following models:

1. Toy model with D–term like potential, 1 FD
   Olive, Peloso ’06
2. 2 FD + $SU(N)$ toy model
   AEG, Olive, Peloso, Sexton ’08
3. Simultaneous excitation of $udd + Qld$ directions in MSSM
   AEG ’09

All these cases involve “independent” flat directions, where the two condensates are decoupled from each other at leading order.

Similar works show nonadiabatic rotation of eigenstates for “overlapping” flat directions

Basbøll, Maybury, Riva, West ’07 --Basbøll ’08

Non-perturbative decay starts for an initial VEV ratio range of 3 orders
⇒ decay in $O(10)$ rotations. (cf. $10^{11}$ rotations in perturbative decay)
The effect of reheating?

The nonperturbative decay necessarily implies that the thermalization is no longer delayed?

- All the previous examples involve actions up to quadratic in perturbations. Particle production inevitably gives rise to breakdown of the linear approximation.
- Is the condensate completely depleted or do high order terms oppose the resonant decay?
- Produced particles are nonrelativistic \( (k \leq m_\phi) \), variances high \( \Rightarrow \) gauge fields still massive?
- Showed: Condensate \( \rightarrow \) Non thermal distribution \( \Rightarrow O(10) \) rotations
- Non thermal distribution \( \rightarrow [E \sim T , N \sim E^3] \) \( \Rightarrow \) How slow/fast?
- “How fast produced quanta thermalize?”: Maybe not in 10 rotations, but we expect \( \ll 10^{11} \) rotations.
Nonlinear study of the decay

Toy model \( V = \frac{1}{2} m_\phi^2 |\phi|^2 + \frac{1}{2} m_\chi^2 |\chi|^2 + g^2 (\phi \chi^* + \phi^* \chi)^2 \), with \( \langle \phi \rangle = \phi_0 e^{i \sigma} \).

- Occupation numbers for \( \text{Re}[\chi] \) for 30 rotations. Thermalization already proceeding. In this toy model, no gauge interactions, no expansion, initially circular orbits.

- Same model, with expansion, elliptical orbits, shows the depletion of the background condensate (Axes multiplied by \( R^{3/2} \) (Dufaux 2009))
The resonant decay of flat directions also gives rise to gravity wave production. Present-day peak frequency and amplitude of the emergent gravity waves:

\[ f_* \sim \left( \frac{a_i}{a_r} \right)^{1/4} \sqrt{\frac{m}{\text{TeV}}} \left( 5 \times 10^2 \text{Hz} \right), \quad h^2 \Omega_{gw}^* \sim 10^{-4} \left( \frac{\Phi_i}{M_p} \right)^4 \left( \frac{a_i}{a_r} \right) \]

- At \( a = a_i \), FD oscillates.
- At \( a = a_r \), RD.

\[ m = 100 \text{ GeV (left)}, m = 10 \text{ TeV (right)} \]

c.f. Inflaton preheating: GW observable only if coupling constants very small.
Nonlinear study of gauged 2 FD model

- Ongoing project: Solving the nonlinear (classical) equations of motion for 4 complex scalar + U(1) gauge field model (with Dufaux and Peloso)
- U(1) gauge fields implemented to ClusterEasy (Felder 2007) complete for \( \dot{R} = 0 \).
- Preliminary runs show that the condensates decay almost completely.

\[
\tilde{\phi}_0 / \phi_0 = 2, \quad \tilde{m} / m = 3.72, \quad \frac{m}{e \Phi_0} = 0.25
\]
Nonlinear study of gauged 2 FD model