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# Fluxes & BPS Vortices in ABJM model

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# Contents

# I. Motivation

- The world-volume action of N stacked M2-branes is understood in the low-energy limit.
  [N=8 BLG model, N=6 ABJM model:
  Superconformal Chern-Simons matter theories of U(N)XU(N) ]

- Various mass deformations
  [SUSY preserving, D-term, F-term: equivalence]
  [constant 3- & 6-form fluxes as source of mass deformation]

- Fluxes from the background bulk (3- & 6-form) fields
  [multiple M2-branes in the presence of (constant) fluxes]

- BPS equations & solitonic objects in Chern-Simons gauge theory
  [N=3,2,1+5/2,3/2,1/2 BPS equations,
  singular and regular vortex-type solitons]

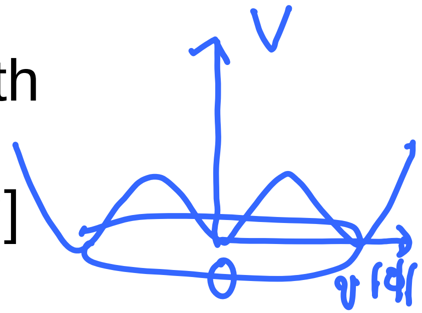
- Interpretation of BPS solitons in M-theory & type II string theory
  [???]

# II. Field Theoretic Construction of ABJM Model and BPS Chern-Simons Solitons

**Brief History of BPS Chern-Simons solitons**

1. Chern-Simons gauge theories
   - anyon superconductivity for high T_c superconductivity [Chen-Wilczek-Witten-Halperin]

2. BPS bound for multi-vortices in Chern-Simons-Higgs model with sextic scalar potential
   - type I & II superconductors [Hong-Kim-Pac, Jackiw-Weinberg]




3. BPS equations from N=2 SUSY Chern-Simons-Higgs model [Lee-Lee-Weinberg]

$V \sim |\phi|^2 (|\phi|^2 - v^2)^2$

4. Various BPS Chern-Simons solitons
   - topological vortex, Q-ball, nontopological Q-vortex [Lee-Jackiw-Weinberg]

$0 \to v$ $\quad 0 \leftarrow v$ $\quad 0 \to v \to 0$

5. Various models
   - Maxwell-Chern-Simons-Higgs, nonabelian, U(1)^N, nonrelativistic limit,
...

6. Mathematical subject: existence & uniqueness of solutions

7. Low-energy world-volume theory of stacked flat M2-branes
   - BLG model of SU(2)XSU(2) & ABJM model of U(N)XU(N) gauge group
     : conformal limit of Chern-Simons-Higgs model
   - BLG & ABJM model with mass deformation
     (some nonvanishing (constant) 4-form and 7-form fluxes)

$$|\phi|^2(|\phi|^2 - v^2)^2 \underset{\text{mass deformation}}{\overset{v\to 0}{\rightleftarrows}} |\phi|^6$$

# Aharony-Bergman-Jefferis-Maldacena (ABJM)

⊙ action :

$$S_{\rm ABJM} = \int d^3x \left\{ \frac{k}{4\pi}\epsilon^{\mu\nu\lambda}{\rm tr}\left( A_\mu\partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda - \hat{A}_\mu\partial_\nu\hat{A}_\lambda - \frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda \right) - {\rm tr}\big(D_\mu Y_A^\dagger D^\mu Y^A\big) + {\rm tr}\big(\psi^{A\dagger} i\gamma^\mu D_\mu \psi_A\big) - V_{\rm ferm} - V_0 \right\},$$

- covariant derivative $D_\mu Y^A = \partial_\mu Y^A + iA_\mu Y^A - iY^A\hat{A}_\mu.$

- Yukawa-type quartic interaction :

$$V_{\rm ferm} = \frac{2i\pi}{k}{\rm tr}\big(Y_A^\dagger Y^A\psi^{B\dagger}\psi_B - Y^A Y_A^\dagger \psi_B\psi^{B\dagger} + 2Y^A Y_B^\dagger\psi_A\psi^{B\dagger} - 2Y_A^\dagger Y^B\psi^{A\dagger}\psi_B - \epsilon^{ABCD}Y_A^\dagger\psi_B Y_C^\dagger\psi_D + \epsilon_{ABCD}Y^A\psi^{B\dagger}Y^C\psi^{D\dagger}\big),$$

- sextic scalar potential :

$$V_0 = -\frac{4\pi^2}{3k^2}{\rm tr}\big(Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\big)$$

$$= \frac{2}{3}\left|\beta_A^{BC} + \delta_A^{[B}\beta_D^{C]D}\right|^2$$ : manifestly positive-definite!

$$\boxed{\beta_C^{AB} = \frac{4\pi}{k}Y^{[A}Y_C^\dagger Y^{B]}}$$

- Symmetries :

  - gauge symmetry : $A_\mu$ & $\hat{A}_\mu$ : U(N)XU(N) or SU(N)XSU

  - parity : $(k, -k)$

  - N=6 superconformal symmetry :

$$
\begin{cases}
\delta Y^A = i\omega^{AB}\psi_B, \\
\delta\psi_A = -\gamma_\mu\omega_{AB}D_\mu Y^B + \frac{2\pi}{k}\left[-\omega_{AB}\left(Y^C Y^\dagger_C Y^B - Y^B Y^\dagger_C Y^C\right) + 2\omega_{BC}Y^B Y^\dagger_A Y^C\right] \\
\qquad = -\gamma^\mu\omega_{AB}D_\mu Y^B + \omega_{BC}\left(\beta^{BC}_A + \delta^{[B}_A \beta^{C]D}_D\right), \\
\delta A_\mu = -\frac{2\pi}{k}\left(Y^A\psi^{B\dagger}\gamma_\mu\omega_{AB} + \omega^{AB}\gamma_\mu\psi_A Y^\dagger_B\right), \\
\delta\hat{A}_\mu = \frac{2\pi}{k}\left(\psi^{A\dagger}Y^B\gamma_\mu\omega_{AB} + \omega^{AB}\gamma_\mu Y^\dagger_A\psi_B\right),
\end{cases}
$$

SUSY parameters $\omega^{AB} = (\omega_{AB})^* = -\frac{1}{2}\epsilon^{ABCD}\omega_{CD}$

real gamma matrices $\gamma^0 = i\sigma^2, \quad \gamma^1 = \sigma^1, \quad \gamma^2 = \sigma^3$

# III. Mass Deformations in ABJM Model

- Various mass deformations in ABJM model
  - single mass parameter $\mu$
  - three kinds of mass deformations: SUSY preserving, D-term, F-

- Mass deformation respecting full N=6 SUSY [Hosomichi-Lee-Lee]
  - In the SUSY transformations

$$\delta_{\rm m}\psi_A = \mu M_A{}^B \omega_{BC} Y^C$$

: unique

$$M_A^B = \mathrm{diag}(1,1,-1,-1)$$

)

  - In the Lagrangian

$$\Delta V_{\rm ferm} = \mathrm{tr}\, \mu \psi^{\dagger A} M_A{}^B \psi_B ,$$

$$\Delta V_0 = \mathrm{tr}\left( \frac{4\pi\mu}{k} Y^A Y_A^\dagger Y^B M_B{}^C Y_C^\dagger - \frac{4\pi\mu}{k} Y_A^\dagger Y^A Y_B^\dagger M_C{}^B Y^C + \mu^2 Y_A^\dagger Y^A \right)$$

: R-symmetry SU(4) -> SU(2)XSU(2)XU(1)

  - combined with the undeformed potential

$$\boxed{V_{\rm m} = V_0 + \Delta V_0 = \frac{2}{3}\left| \beta_A^{BC} + \delta_A^{[B}\beta_D^{C]D} + \mu M_A^{[B} Y^{C]} \right|^2}$$

: manifestly positive-definite
: suitable for half-BPS

- N=1 superfield formalism [Hosomichi-Lee^3-Park]
  - notation : $Y^A = (Z^1, Z^2, W^{\dagger 1}, W^{\dagger 2})$

(mass deformation)

  - N=1 superpotential :

$$\Delta\mathcal{W}_{\mathcal{N}=1} = -\mu\,\mathrm{tr}\big(Z_a^\dagger Z^a - W^{\dagger a}W_a\big)$$

$$\mathcal{W}_{\mathcal{N}=1} = \frac{2\pi}{k}\mathrm{tr}\Big(\frac{1}{2}Z_a^\dagger Z^a Z_b^\dagger Z^b - \frac{1}{2}Z^a Z_a^\dagger Z^b Z_b^\dagger + \frac{1}{2}W_a W^{\dagger a} W_b W^{\dagger b} - \frac{1}{2}W^{\dagger a} W_a W^{\dagger b} W_b$$
$$+ Z^a Z_a^\dagger W^{\dagger b} W_b - Z_a^\dagger Z^a W_b W^{\dagger b} + 2 Z_a^\dagger Z^b W_b W^{\dagger a} - 2 Z^a Z_b^\dagger W^{\dagger b} W_a\Big),$$

  - calculation :

$$\hat{N}^a = -\frac{\partial \mathcal{W}_{\mathcal{N}=1}}{\partial Z_a^\dagger}$$
$$= \frac{2\pi}{k}\big(Z^b Z_b^\dagger Z^a - Z^a Z_b^\dagger Z^b - W^{\dagger b} W_b Z^a + Z^a W_b W^{\dagger b} - 2Z^b W_b W^{\dagger a} + 2W^{\dagger a} W_b Z^b\big)$$
$$\hat{M}_a = \frac{\partial \mathcal{W}_{\mathcal{N}=1}}{\partial W^{\dagger a}}$$
$$= \frac{2\pi}{k}\big(W_b W^{\dagger b} W_a - W_a W^{\dagger b} W_b + W_a Z^b Z_b^\dagger - Z_b^\dagger Z^b W_a + 2Z_a^\dagger Z^b W_b - 2W_b Z^b Z_a^\dagger\big).$$

$$\hat{N}^a \to \hat{N}^a + \mu Z^a, \quad \hat{M}_a \to \hat{M}_a + \mu W_a.$$

  - Bosonic potential : perfect square form : coincides with N=6

$$\boxed{V_0 = \mathrm{tr}\big(\hat{N}_a^\dagger \hat{N}^a + \hat{M}^{\dagger a}\hat{M}_a\big)} \longrightarrow \boxed{V_{\mathrm{m}} = |\hat{N}^a + \mu Z^a|^2 + |\hat{M}_a + \mu W_a|^2}$$

- N=2 superfield formalism [Gomis-Rodriguez-Gomez-Raamsdonk-

$V_0 = V_D + V_F$ : (bosonic potential) = (D-term potential) + (F-term

- D-term potential $V_D = \mathrm{tr}\big(N_a^\dagger N^a + M^{\dagger a} M_a\big)$

- From N=2 superpotential :

$$N^a = \frac{2\pi}{k}\big(Z^b Z_b^\dagger Z^a - Z^a Z_b^\dagger Z^b - W^{\dagger b} W_b Z^a + Z^a W_b W^{\dagger b}\big),$$
$$M_a = \frac{2\pi}{k}\big(W_b W^{\dagger b} W_a - W_a W^{\dagger b} W_b + W_a Z^b Z_b^\dagger - Z_b^\dagger Z^b W_a\big)$$

(mass

$$N^a \to N^a + \mu Z^a, \quad M_a \to M_a + \mu W_a$$

$$\boxed{|N^a|^2 + |M_a|^2 + |F^a|^2 + |G_a|^2} \to \boxed{|N^a + \mu Z^a|^2 + |M_a + \mu W_a|^2 + |F^a|^2 + |G_a|^2}$$

: same as N=1 case

- F-term potential : $V_F = \mathrm{tr}\left(F_a^\dagger F^a + G^{\dagger a} G_a\right)$

- N=2 superpotential :

$$\mathcal{W}_{\mathcal{N}=2} = \frac{2\pi}{k}\epsilon_{ac}\epsilon^{bd}\mathrm{tr}\left(Z^a W_b Z^c W_d\right)$$

$$F^a = \frac{\partial \mathcal{W}^\dagger_{\mathcal{N}=2}}{\partial Z_a^\dagger} = \frac{4\pi}{k}\epsilon^{ac}\epsilon_{bd} W^{\dagger b} Z_c^\dagger W^{\dagger d}$$

$$G_a = \frac{\partial \mathcal{W}^\dagger_{\mathcal{N}=2}}{\partial W^{\dagger a}} = -\frac{4\pi}{k}\epsilon_{ac}\epsilon^{bd} Z_b^\dagger W^{\dagger c} Z_d^\dagger$$

(mass deformation)

$$\Delta\mathcal{W}_{\mathcal{N}=2} = \mu\,\mathrm{tr}\left(Z^a W_a\right)$$

$$M_A{}^B = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

- diagonalize off-diagonal mass matrix
  : equivalent to D-term deformation

D-term deformation and F-term deformation
are the same as
maximal(N=6) SUSY preserving mass

Possible origin of SUSY-preserving mass deformation

Step 1. $\boxed{\text{Construction of Wess-Zumino type interaction between the bulk form fields \& the world-volume fields of D2's (IIA theory)}}$


M. Li, *Boundary States of D-branes and Dy-Strings*, *Nucl. Phys.* **B 460** (1996) 351 [hep-th/9510161] [SPIRES].

M.R. Douglas, *Branes within branes*, hep-th/9512077 [SPIRES].

M.B. Green, J.A. Harvey and G.W. Moore, *I-brane inflow and anomalous couplings on D-branes*, *Class. Quant. Grav.* **14** (1997) 47 [hep-th/9605033] [SPIRES].

R.C. Myers, *Dielectric-branes*, *JHEP* **12** (1999) 022 [hep-th/9910053] [SPIRES].


R-R charge of Dp-brane

interior product with $\tilde{X}^i$

NS-NS 2-form

gauge field (strength) $\tilde{F} = d\tilde{A} + \tilde{A}^2$

$$S_{\tilde{C}} = \mu_p \int_{p+1} \mathrm{Tr}\left(P\left[e^{i\tilde{\lambda}\mathrm{i}^2_{\tilde{X}}} \sum \tilde{C}_{(n)} e^{\tilde{B}}\right] e^{\tilde{\lambda}\tilde{F}}\right)$$

pull-back on to the WV of D-brane

$\tilde{\lambda} = 2\pi l_s^2$

For a single M2-brane,

$$S_{11} = -\mu_2 \int d^3\sigma \sqrt{-\det(\partial_\mu x^m \partial_\nu x^n g_{mn})} + \frac{\mu_2}{3!}\int d^3\sigma \epsilon^{\mu\nu\rho} \hat{C}_{mnp} \partial_\mu x^m \partial_\nu x^n \partial_\rho x^p$$

$0, 1, 2$

$\sigma^\mu = x^\mu$

$0, 1, \cdots, 10$

$g_{mn} = \eta_{mn}$

$3, \cdots, 10$

$x^I = \lambda X^I$

Step 2. Construction of Wess-Zumino type interaction between the bulk form fields & the world-volume fields of multiple M2's

M. Li and T. Wang, *M2-branes Coupled to Antisymmetric Fluxes*, JHEP **07** (2008) 093 [arXiv:0805.3427] [SPIRES].

M.A. Ganjali, *On Dielectric Membranes*, JHEP **05** (2009) 047 [arXiv:0901.2642] [SPIRES].

Y. Kim, O.-K. Kwon, H. Nakajima and D.D. Tolla, *Coupling between M2-branes and Form Fields*, JHEP **10** (2009) 022 [arXiv:0905.4840] [SPIRES]. + JHEP 11 (2010) 069 [1009.5209]

N. Lambert and P. Richmond, *M2-Branes and Background Fields*, JHEP **10** (2009) 084 [arXiv:0908.2896] [SPIRES].

S. Sasaki, *On Non-linear Action for Gauged M2-brane*, JHEP **02** (2010) 039 [arXiv:0912.0903] [SPIRES].

Primitive : A construction was based on

1. gauge symmetry
2. dimensional reduction to multiple D2's
3. ~~SUSY~~ : complicated

→ term by term

3-form
$$S_C^{(3)} = \mu_2 \int d^3x \frac{1}{3!}\epsilon^{\mu\nu\rho}\{\mathrm{Tr}\}\Big[C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + 3\lambda^2 (C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B) + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_C) + (\mathrm{c.c.})\Big]$$

6-form

$$
\begin{aligned}
S_C^{(6)} = \mu_2' \int d^3x\, \frac{1}{3!}\epsilon^{\mu\nu\rho}\, \{\mathrm{Tr}\}\Big( & C_{\mu\nu\rho AB\bar{C}}\beta^{AB}_{\ C} + 3\lambda\big(C_{\mu\nu ABC\bar{D}} D_\rho Y^A \beta^{BC}_{\ D} + C_{\mu\nu AB\bar{C}\bar{D}} D_\rho Y^\dagger_C \beta^{AB}_{\ D}\big) \\
& + 3\lambda^2\big(C_{\mu ABCD\bar{E}} D_\nu Y^A D_\rho Y^B \beta^{CD}_{\ E} + C_{\mu ABC\bar{D}\bar{E}} D_\nu Y^A D_\rho Y^\dagger_D \beta^{BC}_{\ E} \\
& + C_{\mu AB\bar{C}\bar{D}\bar{E}} D_\nu Y^\dagger_C D_\rho Y^\dagger_D \beta^{AB}_{\ E}\big) \\
& + \lambda^3\big(C_{ABCDE\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^C \beta^{DE}_{\ F} + C_{ABCD\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_E \beta^{CD}_{\ F} \\
& + C_{ABC\bar{D}\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{BC}_{\ F} + C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y^\dagger_C D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{AB}_{\ F}\big) + (\mathrm{c.c.})\Big),
\end{aligned}
\tag{2.8}
$$

① a la Muhki-Papageogakis

② reshuffling the fields

$$\xrightarrow{X^8:\ \text{compactify}} \quad (\qquad) \quad \overset{\text{compare}}{\Longleftrightarrow} \quad \left(\begin{array}{c}\text{10D R-R} \\ \text{interaction}\end{array}\right)$$

$C_{012}$ in 11D $\neq$ $\tilde{C}_{012}$ in 10D

R-R

NS-NS

Note. Non-abelian structure ( U(N)× U(N), bifundamental)

→ gauge invariance: nontrivial

# Identification of SUSY-preserving mass deformation as nontrivial constant flux & its backreaction

In ABJM model

→ SUSY-preserving mass deformation

$$\boxed{S_\mu = \mu^2 \int d^3x \,\mathrm{Tr}\big(Y^A Y_A^\dagger\big) - \frac{2\pi\mu}{k}\int d^3x\,\mathrm{Tr}\big(T_{AB\bar{C}\bar{D}} Y_D^\dagger \beta^{AB}_{C}\big) + (\mathrm{c.c.}),}$$

compare

In WZ type interaction

→ ∃ an interaction term of 6-form field

$$\boxed{S_\mu^{(6)} = \mu_2' \int d^3x \frac{1}{3!}\epsilon^{\mu\nu\rho}\mathrm{Tr}\big(C_{\mu\nu\rho AB\bar{C}}\beta^{AB}_{C}\big) + (\mathrm{c.c.})}$$

$$\boxed{C_{\mu\nu\rho AB\bar{C}} = -\frac{2\mu}{\lambda\mu_2}\epsilon_{\mu\nu\rho}T_{AB\bar{C}\bar{D}}Y_D^\dagger, \quad C^\dagger_{\mu\nu\rho AB\bar{C}} = -\frac{2\mu}{\lambda\mu_2}\epsilon_{\mu\nu\rho}T^\dagger_{AB\bar{C}\bar{D}}Y^D}$$ : linear in X

symmetric gauge

$$\boxed{F_{\mu\nu\rho AB\bar{C}\bar{D}} = F^\dagger_{\mu\nu\rho AB\bar{C}\bar{D}} = -\frac{2\mu}{\lambda^2\mu_2}\epsilon_{\mu\nu\rho}T_{AB\bar{C}\bar{D}}}$$ : constant 7-form flux

Note. quadratic mass deformation term

$\xleftarrow[\text{from}]{\text{originated}}$ the backreaction of the bg metric in the presence of form-flux

$G \sim (F^{(4)})^2$ "unsatisfactory"

# IV. Vortex-type BPS Solitons

## Half-BPS equations (N=3)

- From SUSY variation of fermions :
  - Impose supersymmetric $\gamma^0 \omega_{AB} = i s_{AB} \omega_{AB}, \qquad s_{AB} = s_{BA} = \pm 1$
  - some analysis -> half-BPS

$$
\begin{cases}
(D_1 - isD_2)Y^1 = 0 & D_i Y^A = 0, \qquad (A \neq 1). \\
D_0 Y^1 + is(\beta_2^{21} + \mu Y^1) = 0, & D_0 Y^2 - is(\beta_1^{12} + \mu Y^2) = 0, \\
D_0 Y^3 - is\beta_1^{13} = 0, & D_0 Y^4 - is\beta_1^{14} = 0, \\
\beta_3^{31} = \beta_4^{41} = \beta_2^{21} + \mu Y^1, & \beta_4^{43} = \mu Y^3, \qquad \beta_3^{34} = \mu Y^4, \\
\beta_3^{32} = \beta_4^{42} = \beta_2^{23} = \beta_2^{24} = 0, & \\
\beta_A^{BC} = 0 \qquad (A \neq B \neq C \neq A). &
\end{cases}
$$

- Gauss laws : $\frac{k}{2\pi} B = \frac{k}{2\pi} F_{12} = j^0, \qquad -\frac{k}{2\pi}\hat{B} = -\frac{k}{2\pi}\hat{F}_{12} = \hat{j}^0$

- From bosonic part of energy

$$E = \int d^2x (|D^0 Y_A|^2 + |D_i Y^A|^2 + V_{\rm m})$$

$$= \frac{1}{3}\int d^2x \left\{ 2\sum_{A,B,C} \left| \delta_A^{[B} D_0 Y^{C]} + i s_{BC}\left(\beta_A^{BC} + \delta_A^{[B}\beta_D^{C]D} + \mu M_A^{[B} Y^{C]}\right)\right|^2 + \sum_{A\neq B} |(D_1 - i s_{AB} D_2) Y^A|^2 \right\}$$

$$+ is\,\mathrm{tr}\int d^2x\, \epsilon_{ij}\partial_i \left( Y_1^\dagger D_j Y^1 - \frac{1}{3}\sum_{A=2}^{4} Y_A^\dagger D_j Y^A \right) - \frac{s}{3}\mu\,\mathrm{tr}\int d^2x (j^0 + 2J_{12}^0)$$

$$\geq \frac{1}{3}|\mu(Q + 2R_{12})|$$

: energy is bounded by U(1) charge $\quad Q = \mathrm{tr}\int d^2x\, j^0$

and by R-charge $\quad R_{12} = \mathrm{tr}\int d^2x\, J_{12}^0$

$$J_{12}^0 = i(Y^1 D_0 Y_1^\dagger - D_0 Y^1 Y_1^\dagger) - i(Y^2 D_0 Y_2^\dagger - D_0 Y^2 Y_2^\dagger)$$

: proportional to mass-deformation $\quad \mu$

Vanishing (spatial) stress components:

$$T_{ij} = \frac{1}{3}\eta_{ij}\,\mathrm{tr}\Bigg\{ \sum_{A,B,C} \Big[ \Big(\delta_A^{[B} D_0 Y^{C]} + i s_{BC}(\beta_A^{BC} + \delta_A^{[B}\beta_D^{C]D} + \mu M_A^{[B} Y^{C]})\Big)^\dagger \times \Big(\delta_A^{[B} D_0 Y^{C]} - i s_{BC}(\beta_A^{BC} + \delta_A^{[B}\beta_D^{C]D} + \mu M_A^{[B} Y^{C]})\Big) + \Big(\delta_A^{[B} D_0 Y^{C]} - i s_{BC}(\beta_A^{BC} + \delta_A^{[B}\beta_D^{C]D} + \mu M_A^{[B} Y^{C]})\Big)^\dagger \times \Big(\delta_A^{[B} D_0 Y^{C]} + i s_{BC}(\beta_A^{BC} + \delta_A^{[B}\beta_D^{C]D} + \mu M_A^{[B} Y^{C]})\Big) \Big]$$
$$+ \mathrm{tr}\Bigg\{ \frac{1}{4}\Big[ ((D_i + i s\epsilon_{ik} D_k) Y^A)^\dagger (D_j - i s\epsilon_{jl} D_l) Y^A + ((D_j - i s\epsilon_{jl} D_l) Y^A)^\dagger (D_i + i s\epsilon_{ik} D_k) Y^A + ((D_i - i s\epsilon_{ik} D_k) Y^A)^\dagger (D_j + i s\epsilon_{jl} D_l) Y^A + ((D_j + i s\epsilon_{jl} D_l) Y^A)^\dagger (D_i - i s\epsilon_{ik} D_k) Y^A \Big] \Bigg\}$$

$$\xrightarrow[\text{for}]{\text{sufficient}} \quad \mathcal{F}^i = \frac{\partial}{\partial t} T^{i0} = \nabla_j T^{ij}$$

: vanishing force everywhere!

: noninteracting everywhere $\leftarrow$ BPS

## ○ Solving half-BPS equations without mass deformation

1. All the constraint equations are completely solved
2. Half-BPS equations and Gauss' laws reduce to

$$\boxed{\begin{aligned}&(D_1 - isD_2)Y^1 = 0,\\ &B = \hat{B} = -\frac{s}{2}\left(\frac{2\pi v}{k}\right)^2 [Y^1, Y_1^\dagger]\end{aligned}}$$

$v^2 = \sum_{A=2}^{4} |v^A|^2$

: BPS equations from super Yang-Mills theory
without symmetry breaking potential

3. -> some simple cases reduce to

$$\boxed{\begin{aligned}&\partial\bar{\partial}\ln|y_a|^2 = 4v\left(\frac{2\pi}{k}\right)^2 \sum_{b=1}^{N-1} K_{ab}\left(|y_b|^2 - \frac{|G(z)|^2}{|c_b|^2 \prod_{c=1}^{N-1}|y_c|^2}\right)\\ &\qquad y_M = \frac{G(z)}{\prod_{a=1}^{N-1} y_a},\end{aligned}}$$

: (affine-) Toda-type equation

: SU(2) case to Liouville-type equation (G=0) or Sinh-Gordon-

## Solving half-BPS equations in mass-deformation

– U(2)XU(2) case:
BPS equations reduce to

$$\boxed{\partial\bar{\partial}\ln|f|^2 + i(\partial\bar{\partial} - \bar{\partial}\partial)\Omega = \mu^2\left[(2a^2+1)|f|^2 - 1\right]}$$

: Maxwell-Higgs theory

: a=1 -> finite-energy topological vortex
a=\ 1 -> topological vortex in constant background of magnetic

- U(N)XU(X) case :
For some reducible cases, BPS equations reduce

$$\boxed{\partial\bar{\partial}\ln|f_n|^2 = -\mu^2[a_n^2|f_{n-1}|^2 - (a_n^2 + a_{n+1}^2)|f_n|^2 + a_{n+1}^2 + 1]}$$

: U(1)^{N-1} gauge theories with N-1 Higgs fields

$\boxed{N=2}$

$\mu \neq 0$: two inequivalent cases

$$\left\{\begin{array}{ll}
(D_1 - isD_2)Y^1 = 0, & (D_1 + isD_2)Y^2 = 0, \\
D_1Y^A = D_2Y^A = 0, \quad (A = 3, 4), & \\
D_0Y^1 + is(\beta_2^{21} + \mu Y^1) = 0, & D_0Y^2 - is(\beta_1^{12} + \mu Y^2) = 0, \\
D_0Y^A + is(\beta_2^{2A} - \beta_1^{1A}) = 0, \quad (A = 3, 4), & \\
\beta_3^{3B} = \beta_4^{4B} \quad (B = 1, 2), & \beta_4^{43} - \mu Y^3 = \beta_3^{34} - \mu Y^4 = 0, \\
\beta_1^{23} = \beta_1^{24} = \beta_2^{13} = \beta_2^{14} = \beta_3^{14} = \beta_3^{24} = \beta_4^{13} = \beta_4^{23} = 0. &
\end{array}\right.$$

$$\left\{\begin{array}{ll}
(D_1 - isD_2)Y^1 = 0, & (D_1 + isD_2)Y^3 = 0, \\
D_1Y^A = D_2Y^A = 0, \quad (A = 2, 4), & \\
D_0Y^1 + is\beta_3^{31} = 0, & D_0Y^2 - is(\beta_1^{12} - \beta_3^{32} + \mu Y^2) = 0, \\
D_0Y^3 - is\beta_1^{13} = 0, & D_0Y^4 - is(\beta_1^{14} - \beta_3^{34} + \mu Y^4) = 0, \\
\beta_4^{42} = \beta_2^{24} = 0, & \beta_2^{21} - \beta_4^{41} = -\mu Y^1, \qquad \beta_2^{23} - \beta_4^{43} = -\mu Y^3, \\
\beta_1^{23} = \beta_1^{34} = \beta_2^{34} = \beta_2^{14} = \beta_3^{12} = \beta_3^{14} = \beta_4^{12} = \beta_4^{23} = 0. &
\end{array}\right.$$

Energy bound:

$$
\begin{aligned}
E = \frac{1}{2}\int d^2x \,\mathrm{tr}\Bigg[ & 2\sum_{p,q=3,4}\sum_{a=1,2}\left|\delta_p^{[q} D_0 Y^{a]} - i s_{qa}\left(\beta_p^{qa} + \delta_p^{[q}\beta_A^{a]A} + \mu M_p^{[q} Y^{a]}\right)\right|^2 \\
& + 2\sum_{a,b=1,2}\sum_{p=3,4}\left|\delta_a^{[b} D_0 Y^{p]} - i s_{bp}\left(\beta_a^{bp} + \delta_a^{[b}\beta_A^{p]A} + \mu M_a^{[b} Y^{p]}\right)\right|^2\Bigg] \\
+ \int d^2x \,\mathrm{tr}\Bigg[ & \left|(D_1 - isD_2)Y^1\right|^2 + \left|(D_1 + isD_2)Y^2\right|^2 \\
& + \frac{1}{2}\sum_{p=3,4}\left(|(D_1 - isD_2)Y^p|^2 + |(D_1 + isD_2)Y^p|^2\right)\Bigg] \\
+ is\,\mathrm{tr} & \int d^2x\,\epsilon_{ij}\partial_i(Y_1^\dagger D_j Y^1 - Y_2^\dagger D_j Y^2) \underbrace{- s\mu\,\mathrm{tr}\int d^2x\, J_{12}^0}_{},
\end{aligned}
$$

$\mu = 0$ case:

$$(D_1 - isD_2)Y^1 = 0, \qquad (D_1 + isD_2)Y^2 = 0,$$
$$B = -2s\left(\frac{2\pi}{k}\right)^2 \left\{[Y^1Y_2^\dagger, Y^2Y_1^\dagger] + v^2\left([Y^1, Y_1^\dagger] - [Y^2, Y_2^\dagger]\right)\right\}$$
$$\hat{B} = -2s\left(\frac{2\pi}{k}\right)^2 \left\{[Y_2^\dagger Y^1, Y_1^\dagger Y^2] + v^2\left([Y^1, Y_1^\dagger] - [Y^2, Y_2^\dagger]\right)\right\}$$

; seems $\nexists$ regular solution (almost proved)

2X2 case: master case

$$\boxed{\partial\bar\partial X_1 = \mu^2 e^{X_2}(e^{X_1}-1), \qquad \partial\bar\partial X_2 = \mu^2 e^{X_1}(e^{X_2}-1).} \quad (*)$$

$X_2 = 0$

$X_1 = X_2$

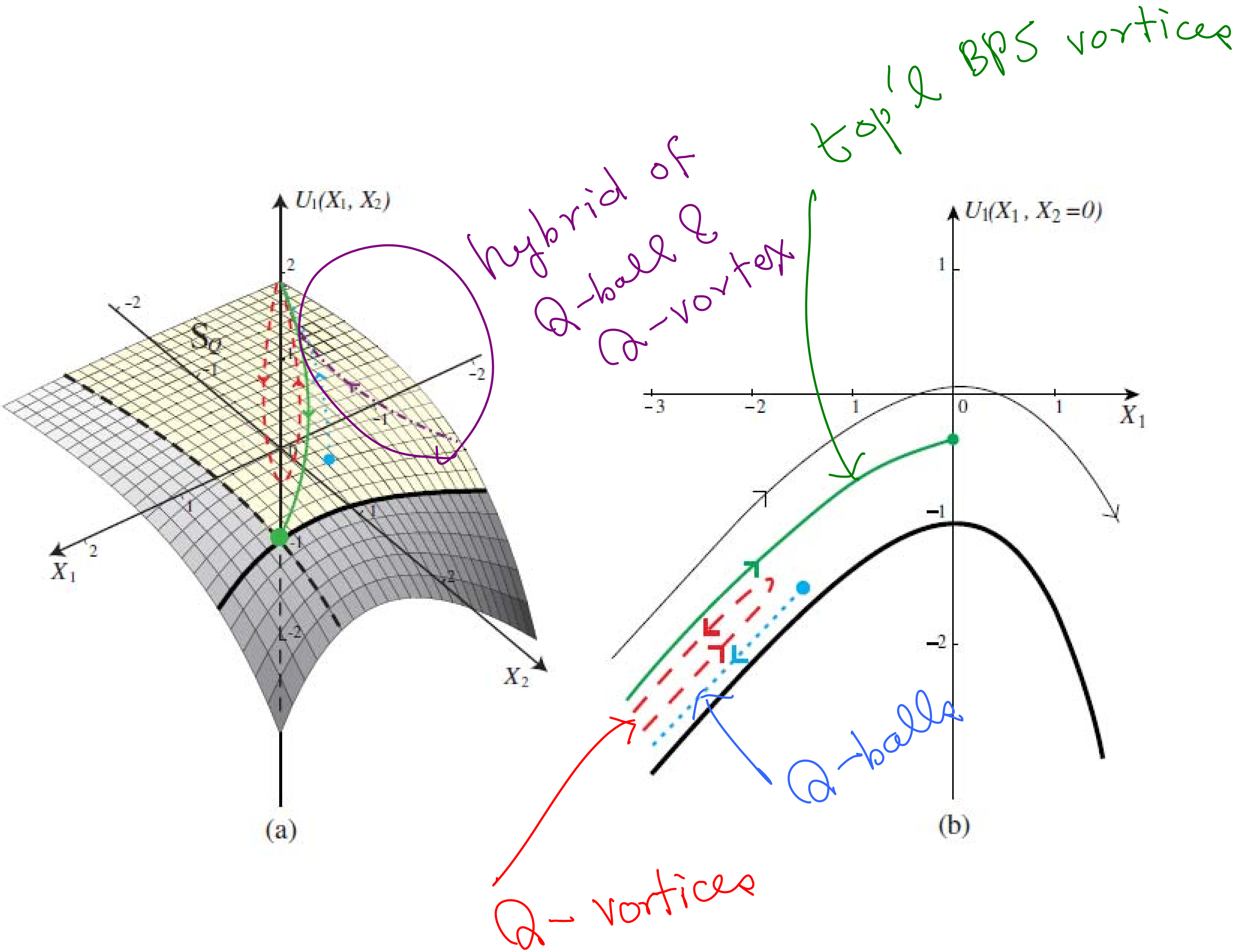
$$\partial\bar\partial X_1 = \mu^2 (e^{X_1}-1)$$

: Maxwell-Higgs

$$\partial\bar\partial X_1 = \mu^2 e^{X_1}(e^{X_1}-1)$$

: Chern-Simons Higgs

(Arai - ---)




2 → N : complicated but (*) seems "representative"!

$N=1$

complicated!

# V. Concluding Remarks

We consider ABJM model without and with mass deformation:

- Equivalence among single parameter mass deformations :
  maxmal SUSY preserving, D-term, F-term

- SUSY-preserving mass deformation is identified as
  (1) quartic term <= a constant 7-form field strength in WZ type interaction
  between WV fields and bulk form fields
  (2) quadratic term <= from a backreaction in the presence of constant
  form field strength

- All the vortex-type BPS equations from
  Killing spinor condition & bosonic energy bound
  for N=3, 5/2, 2, 3/2, 1, 1/2 SUSY

## N=3 : energy is bounded by both U(1) and R-charges

- In undeformed ABJM model,
  - (i) all the constraints are solved
  - (ii) resulting equation is half-BPS equation in SUSY Yang-Mills theory
  - (iii) no finite energy regular solution

- In mass-deformed ABJM model,
  - (i) U(2)XU(2) case -> vortex equation in Maxwell-Higgs theory -> static multi-BPS vortices in constant background of magnetic field
  - (ii) U(N)XU(N) (N>2) -> nonabelian vortex equation of Yang-Mills-Higgs

## N=2 : energy is bounded only by R-charge

- In undeformed ABJM model,
  - (i) all the constraints are almost solved
  - (ii) resulting equation is BPS equation connecting those in Chern-Simons and Yang-Mills theory
  - (iii) seems no finite energy regular solution

- In mass-deformed ABJM model,
  - (i) U(2)XU(2) case -> vortex equation in a hybrid theory of Maxwell-Higgs and Chern-Simons theory
    -> topological BPS vortices
    + nontopological Q-balls and Q-vortices
    & their hybrids
  - (ii) U(N)XU(N) -> N=2 case seems representing the general case

- N=1: energy is bounded by U(1) charge
  : seems similar to N=2 case (complicated)

- N=5/2, 3/2, 1/2 cases are respectively equivalent to
  N=3, 2, 1 for the spectra of BPS vortices

Questions include

- proof of nonexistence of regular finite-energy soliton solutions in the undeformed theory

- possibility of configurations with fractional vorticity

- interpretation of solutions in the context of M-theory