

Searching for Walking Technicolor theory on the Lattice

Based on arXiv:1011.2577 [hep-lat/ph]

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in collaboration with

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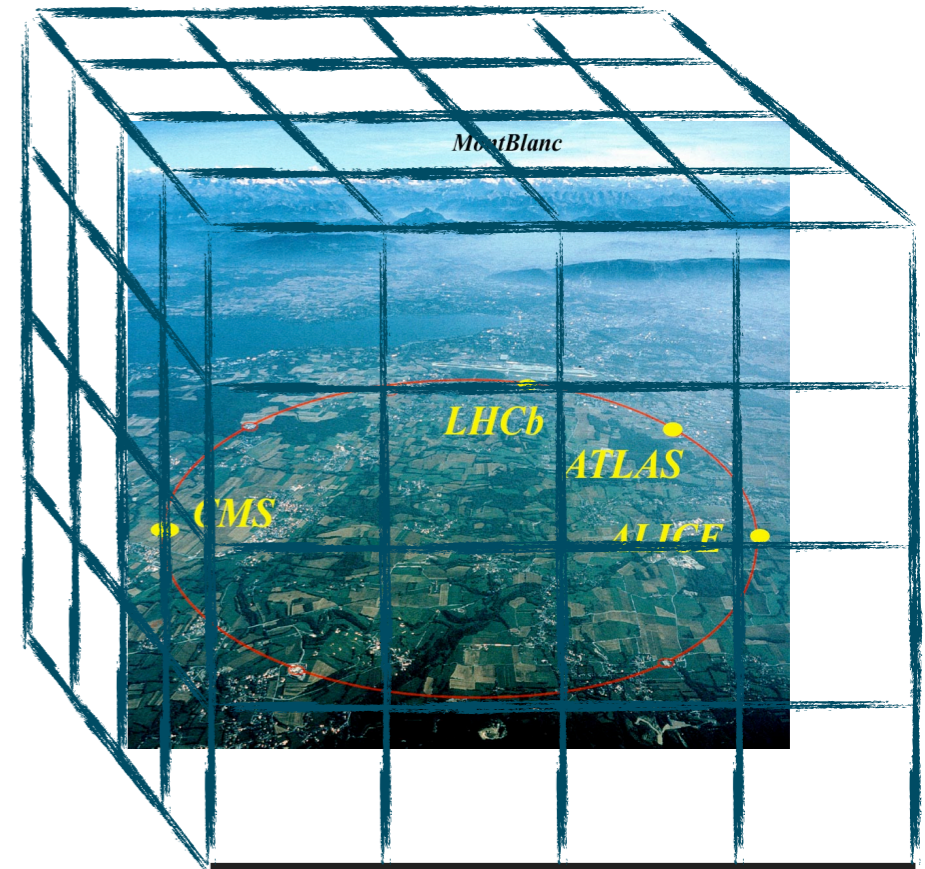
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- Physics background
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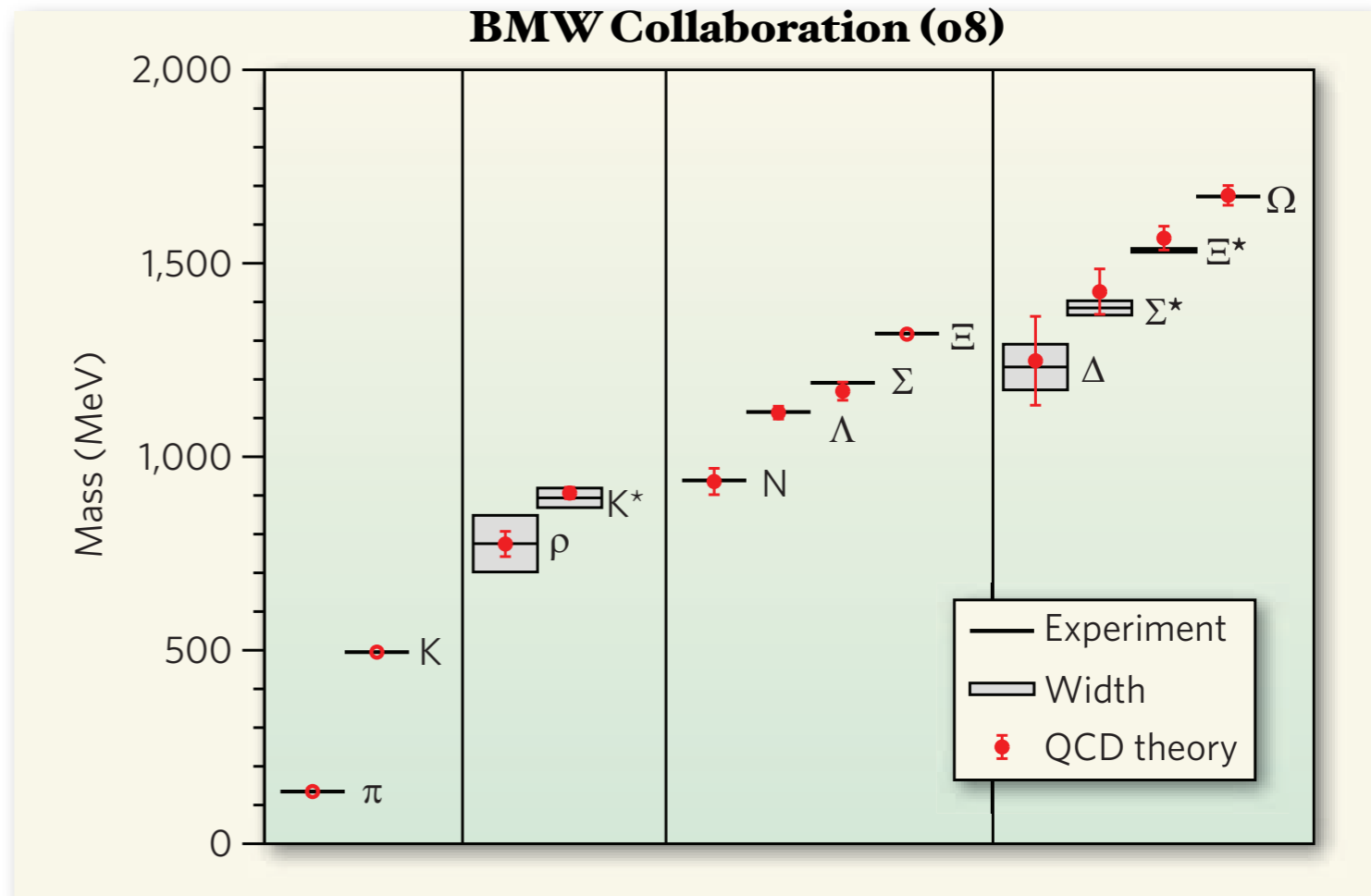
I. Introduction

Successes of Lattice QCD

- ▶ Hadron masses and their interactions
- ▶ Physics@ $T \neq 0$
- ▶ The SM parameters
- ▶ Weak matrix elements
- ▶ ...

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Successes of Lattice QCD

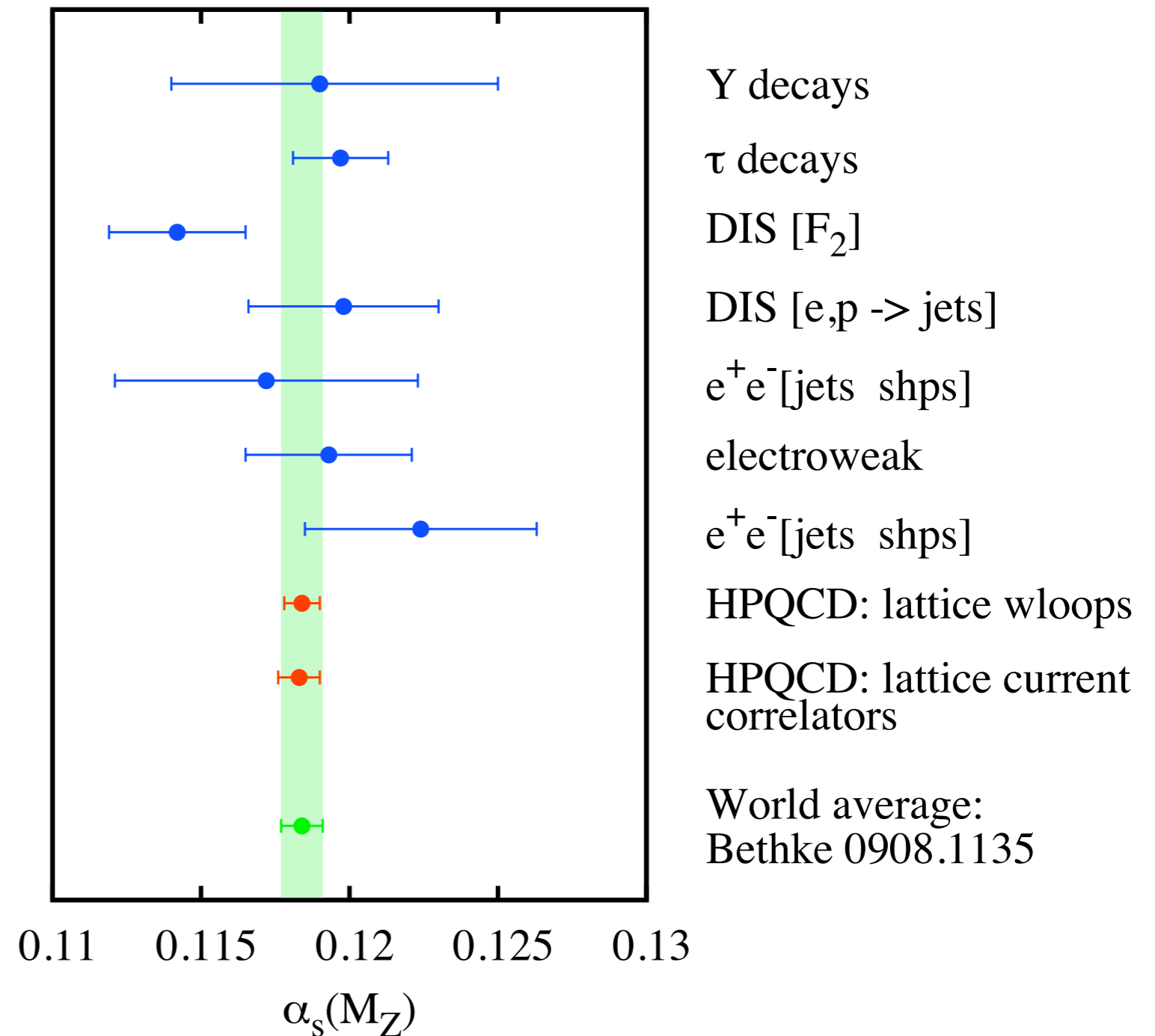
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► The SM parameters

► Weak matrix elements

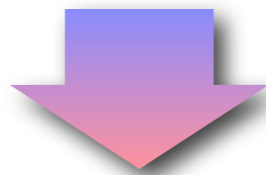
► ...



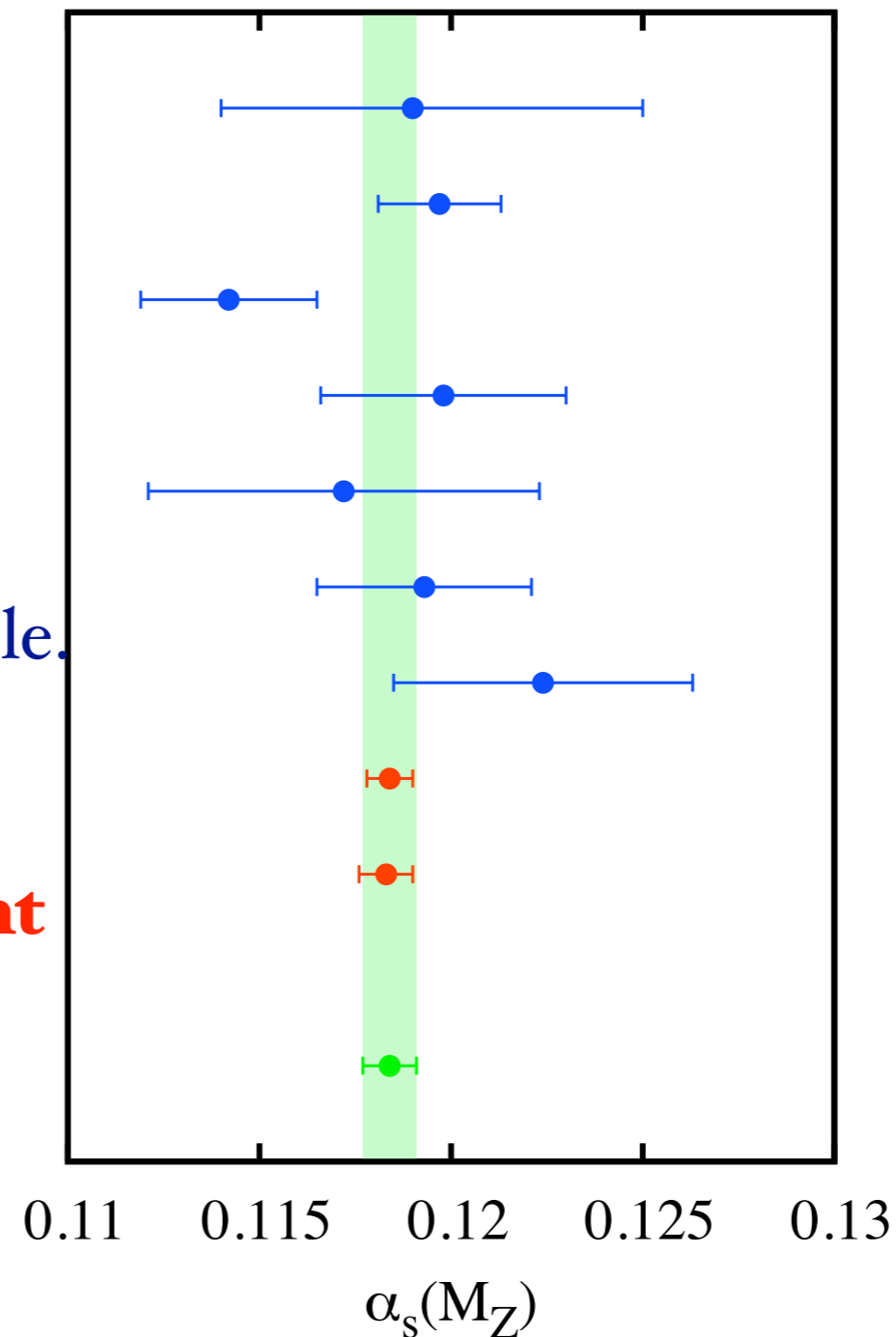
Successes of Lattice QCD

- ▶ Hadron masses and their interactions
- ▶ Physics@T≠0
- ▶ The SM parameters
- ▶ Weak matrix elements
- ▶ ...

Lattice calculations truly reliable.



Apply to **Something different**



Y decays

τ decays

DIS [F_2]

DIS [e,p \rightarrow jets]

e^+e^- [jets shps]

electroweak

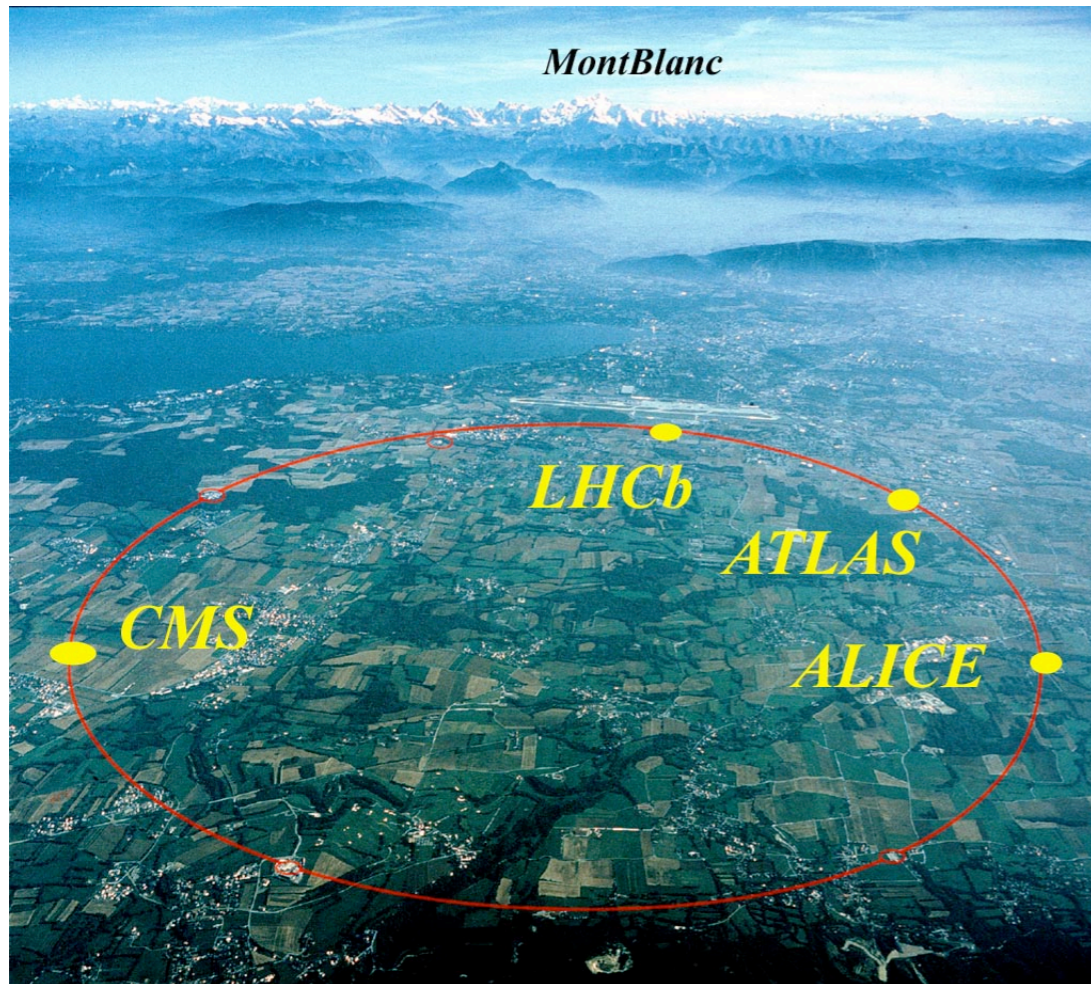
e^+e^- [jets shps]

HPQCD: lattice wloops

HPQCD: lattice current correlators

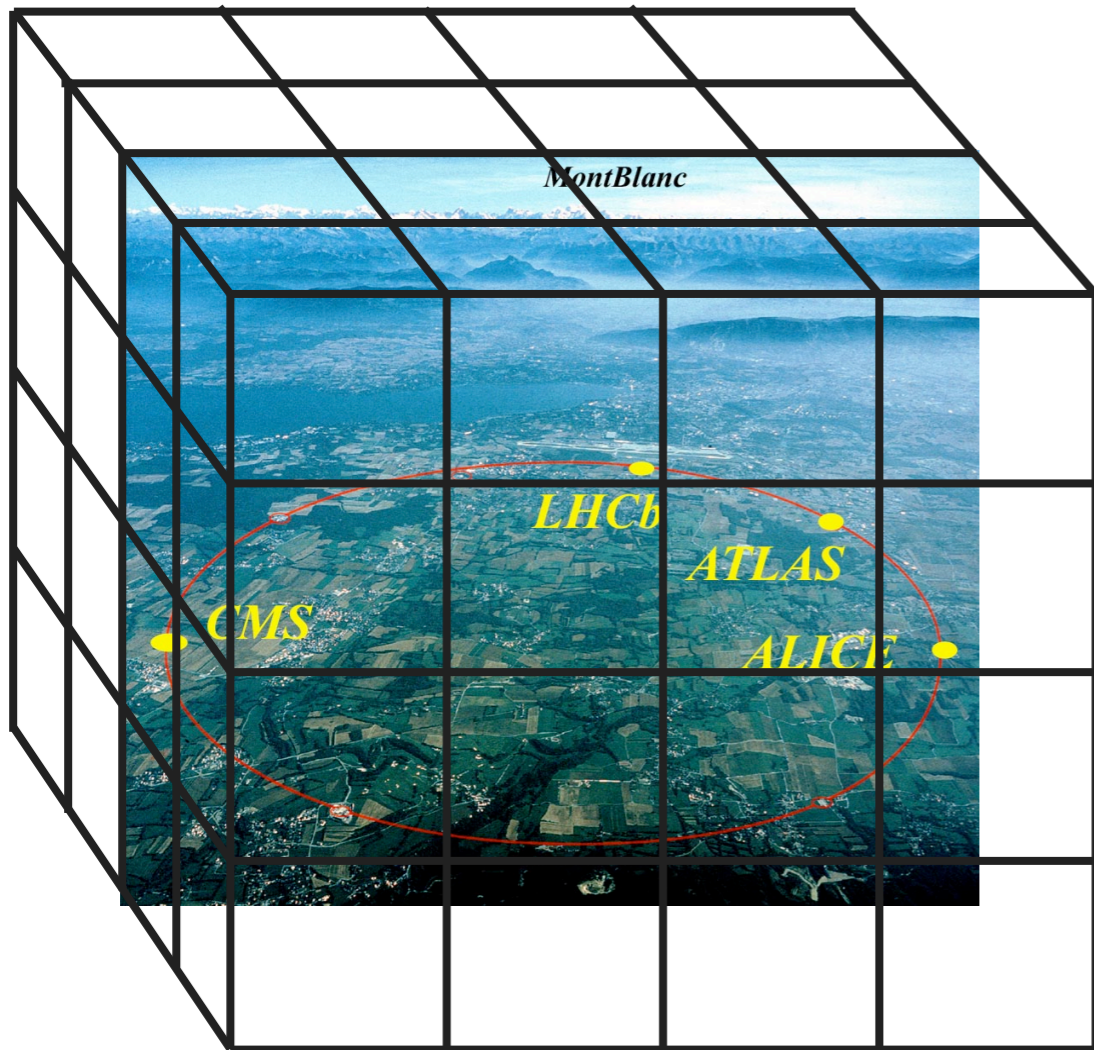
World average:
Bethke 0908.1135

LHC era



- ▶ Higgs mechanism
- ▶ Physics above the EW scale
- ▶ Among many NP candidates, **Technicolor** is attractive and best suited for Lattice Simulation.

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Use Lattice to explore LHC physics

Technicolor (TC) [Weinberg(1979), Susskind(1979)]

Alternative to the Higgs sector in the SM

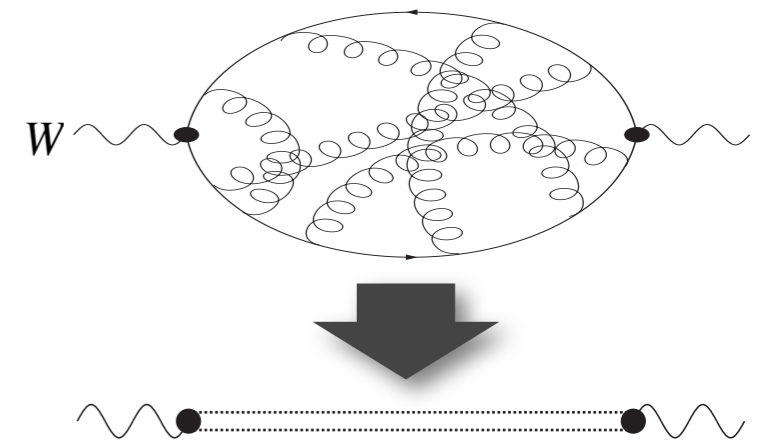
- ▶ QCD-like strong coupling vector-like Gauge theory inducing $S\chi SB$
 $\langle \bar{\psi}_L \psi_R \rangle \neq 0 @ \Lambda_{TC} \Rightarrow$ EWSB (NG boson = techni-pion)

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 $\Lambda_{TC} \sim (F_{TC} / f_\pi) \times \Lambda_{QCD} \sim 2600 \times \Lambda_{QCD} \sim O(1) \text{ TeV}$

$$\frac{g^{\mu\nu} - q^\mu q^\nu / q^2}{q^2 \left[1 - g_2^2 \Pi(q^2) \right]} \xrightarrow{S\chi SB} \frac{g^{\mu\nu} - q^\mu q^\nu / q^2}{q^2 \left[1 - g_2^2 \left(\frac{F_{TC}^2}{4q^2} \right) \right]} = \frac{g^{\mu\nu} - q^\mu q^\nu / q^2}{q^2 - \left(\frac{g_2 F_{TC}}{2} \right)^2}$$

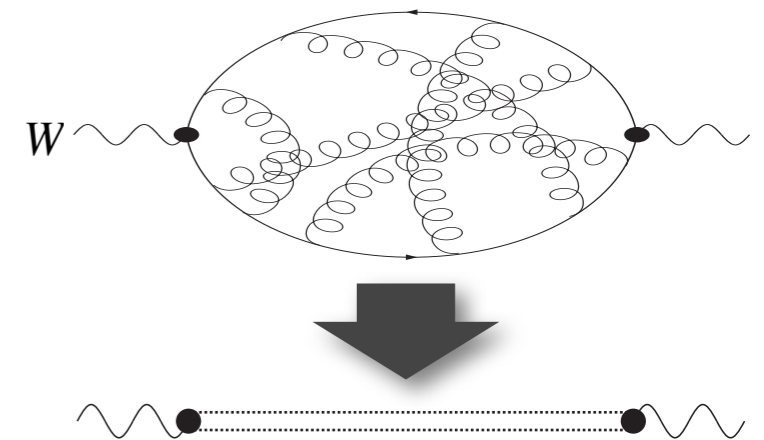


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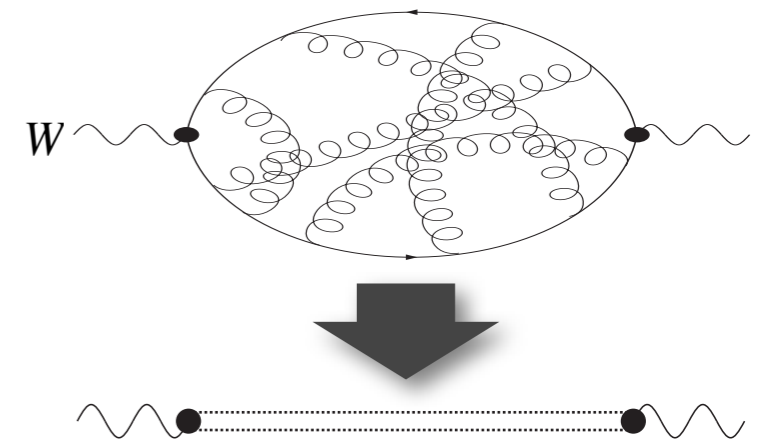


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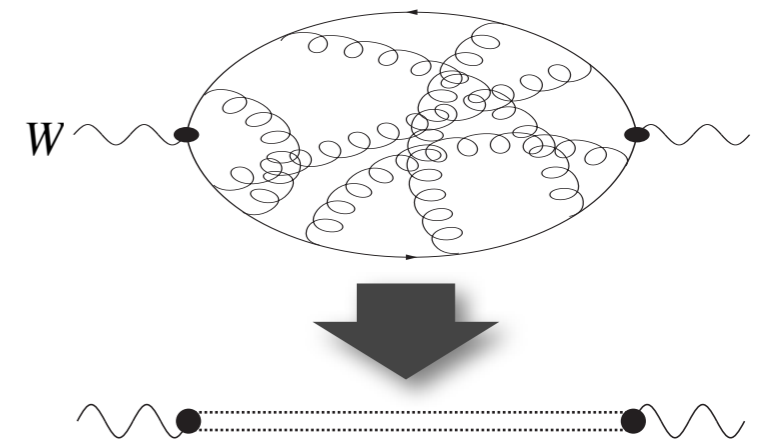


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However, uncalculable ...

Non-Perturbative Dynamics \Rightarrow quantitative treatment difficult

Extended TC and its Breaking

TC must be extended to give mass to the SM fermions.

⇒ New gauge interaction **ETC (unspecified)**

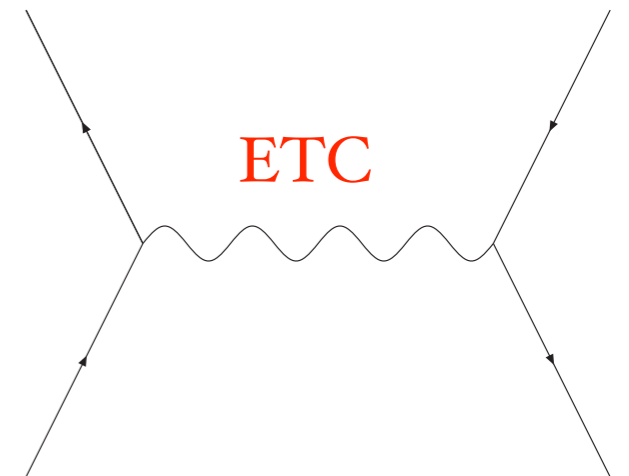
Lane and Eichten ('80). Dimopoulos and Susskind ('79)

ETC breaking @ M_{ETC} ⇒ various four-ferm ops. induced

ETC fermion multiplet: $F^{ETC} = (F_1, \dots, F_{N_{TC}}, f_{SM3}, f_{SM2}, f_{SM1})$

At $\mu < M_{ETC}$, $\frac{g_{ETC}^2}{M_{ETC}^2} C(\mu) (\bar{F} F) (\bar{f} f)$ & $\frac{g_{ETC}^2}{M_{ETC}^2} (\bar{f}' \Gamma_\mu f) (\bar{f}' \Gamma_\mu f)$
 quark/lepton mass & FCNC

$$C(\mu) = \exp \left(\int_\mu^{M_{ETC}} d\mu' \frac{\gamma(\mu')}{\mu'} \right) = \exp \left(\int_{g^2(\mu)}^{g^2(M_{ETC})} dg'^2 \frac{\gamma(g'^2)}{\beta(g'^2)} \right)$$



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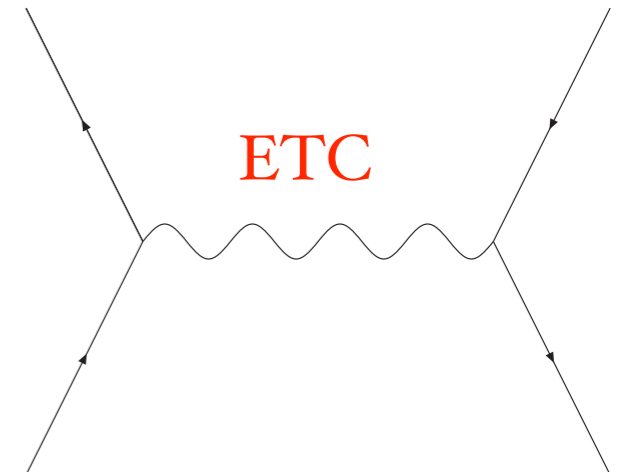
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quark/lepton mass FCNC

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M_{ETC} constrained by

$$m_q \sim g_{ETC}^2 \frac{\langle 0 | \bar{F}F | 0 \rangle}{2 M_{ETC}^2} \sim g_{ETC}^2 \frac{M_{TC}^3}{2 M_{ETC}^2}$$

and

$$\Delta m \text{ in } K^0 - \bar{K}^0, D^0 - \bar{D}^0, \dots$$

Problem I: FCNC & m_q

At $\mu < M_{ETC}$, $\frac{g_{ETC}^2}{M_{ETC}^2} C(\mu) (\bar{F}F)(\bar{f}f)$ & $\frac{g_{ETC}^2}{M_{ETC}^2} (\bar{f}' \Gamma_\mu f) (\bar{f}' \Gamma_\mu f)$
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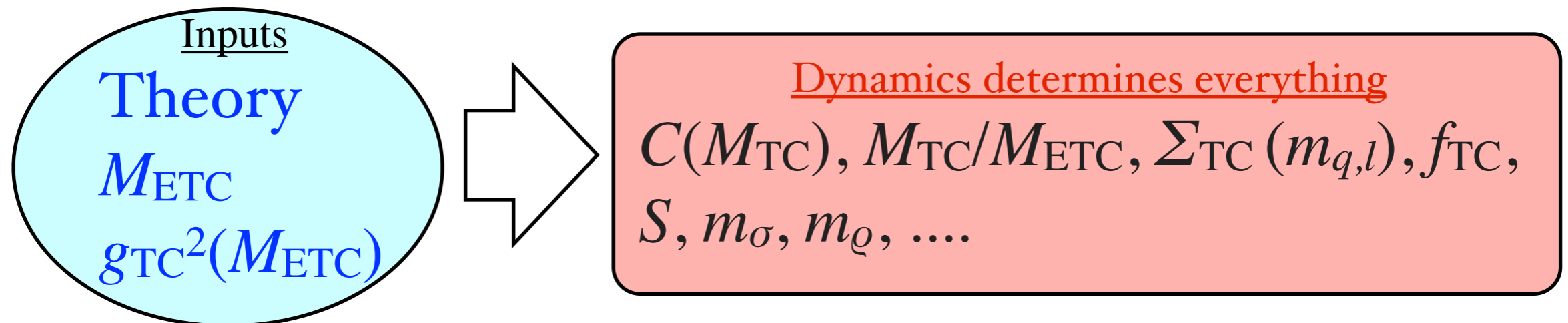
conflicts with

$$\Delta m \text{ in } K^0-\bar{K}^0, D^0-\bar{D}^0, \dots$$

Walking Technicolor (WTC)

Holdom(1981); Yamawaki, Bando, Matsumoto(1986); Appelquist, Karabali, Wijewardhana(1986); Akiba, Yanagida(1986); Bando, Morozumi, So, Yamawaki(1987)

$$C(M_{\text{TC}}) \sim \left(\frac{M_{\text{ETC}}}{M_{\text{TC}}} \right)^\gamma \gg 1 \text{ \& } M_{\text{ETC}} \gg M_{\text{TC}} \text{ must be satisfied.}$$



No tunable parameter!

e.g.) SU(3) gauge theory with 2 or 3-flavors of techni-quarks
(*i.e.* scaled-up QCD) does not respect the condition.

→ Such a self-serving GT really exists?

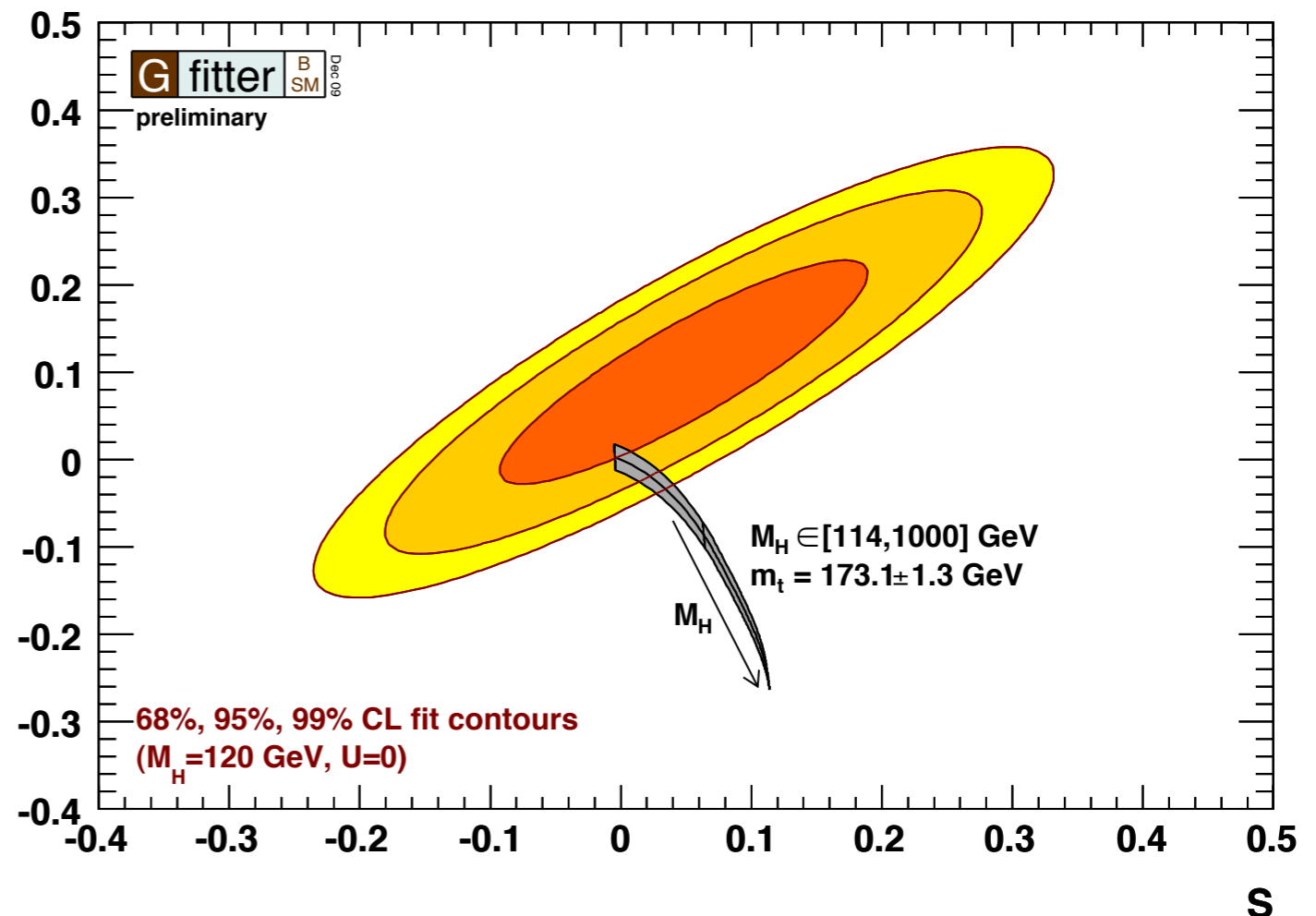
Problem 2: S-parameter

Peskin & Takeuchi (90,92)

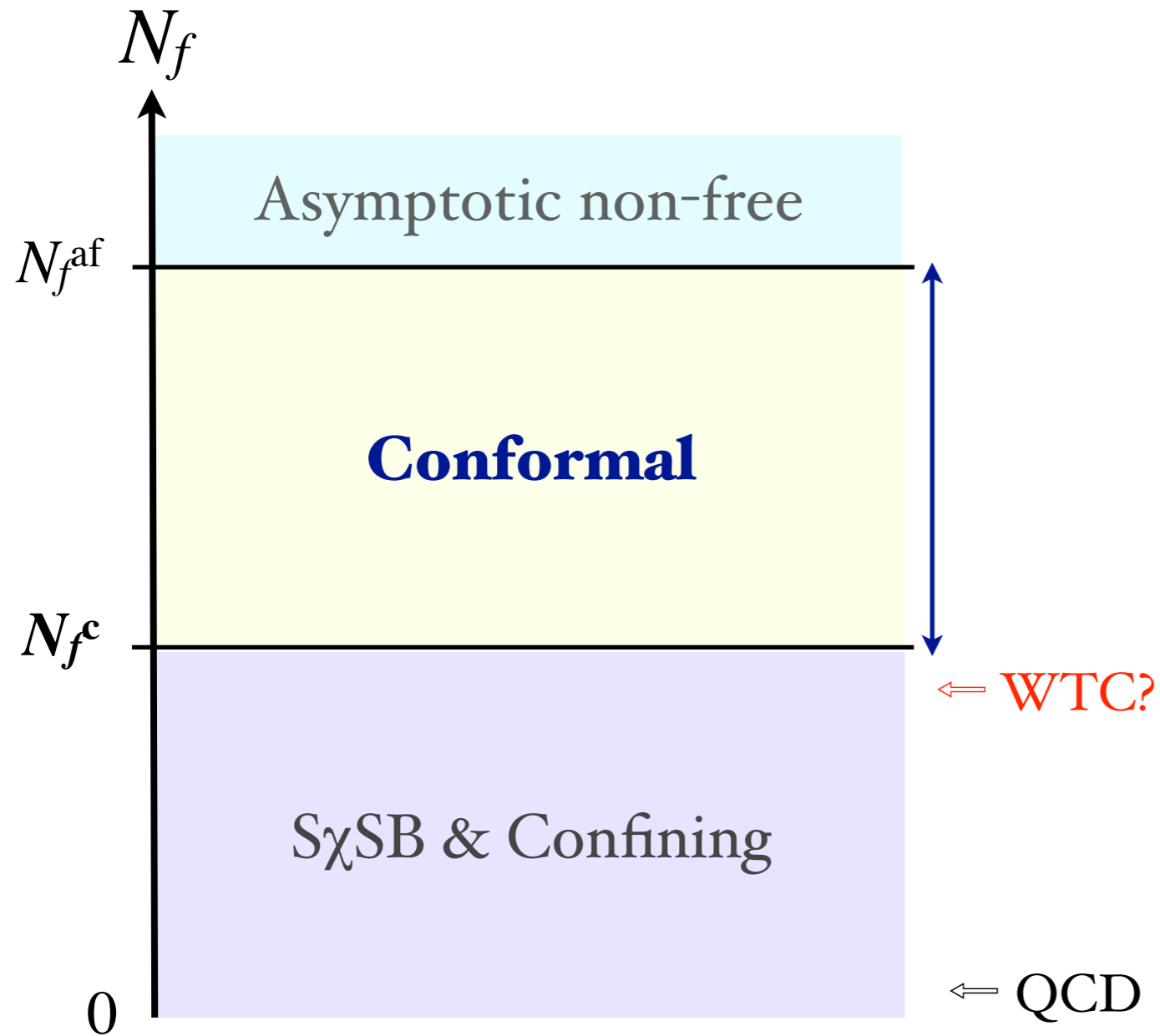
J. Haller [the Gfitter group]
(www.cern.ch/gfitter), hep-ph/1006.0003

- ▶ Rescaled QCD ($S \geq 0.45$) excluded
- ▶ Walking dynamics \Rightarrow S-parameter different from QCD (due to parity doubling?)
- ▶ Non-perturbative method is necessary.
- ▶ We know how to calculate S on the Lattice!

JLQCD (08):two-flavor QCD
c.f. RBC & UKQCD(10):three-flavor QCD
LSD(10): six-flavor QCD
T. DeGrand (10):sextet QCD



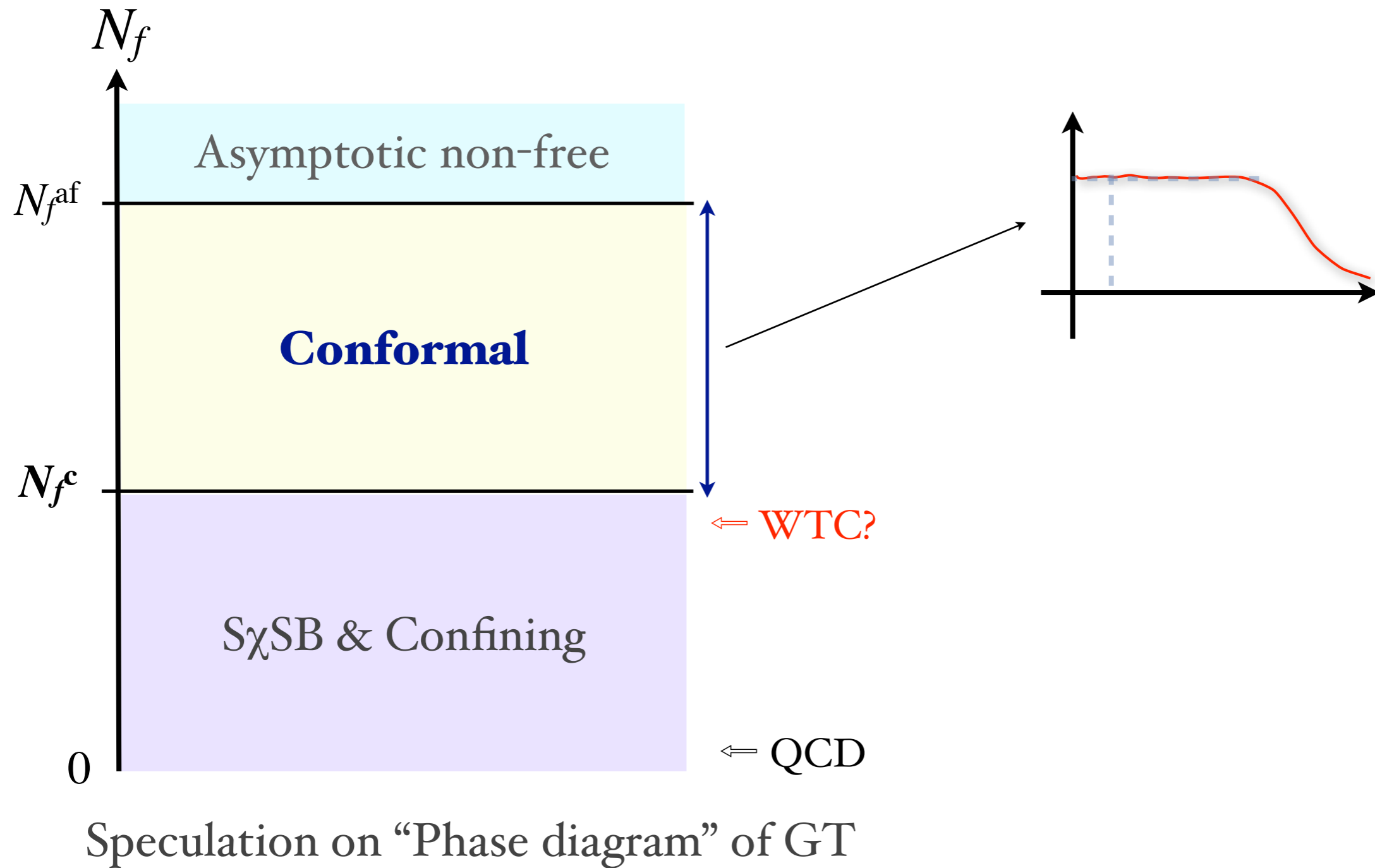
Conformal Window



Speculation on “Phase diagram” of GT

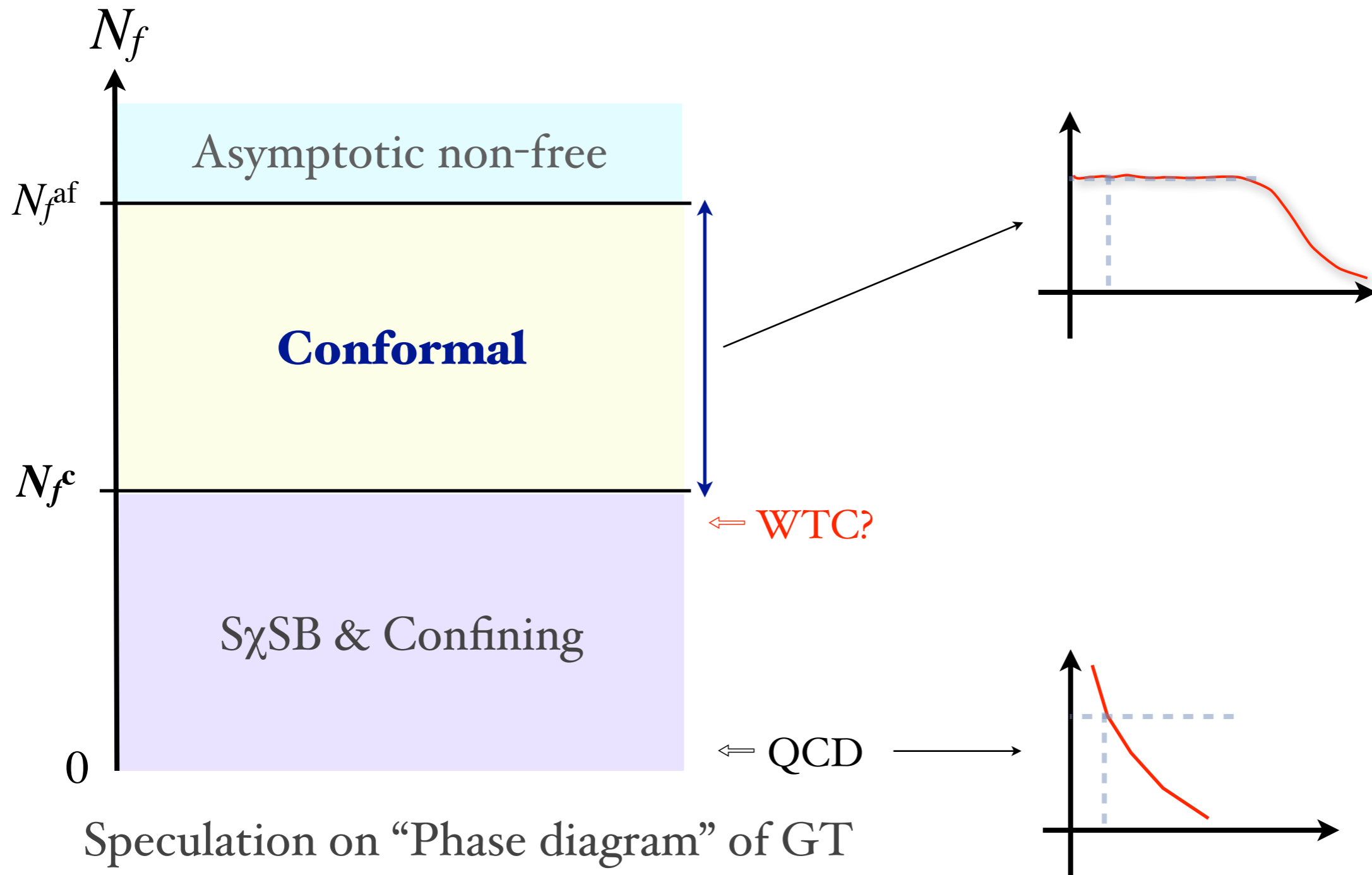
Find the location of N_f^c in various GT.

Conformal Window



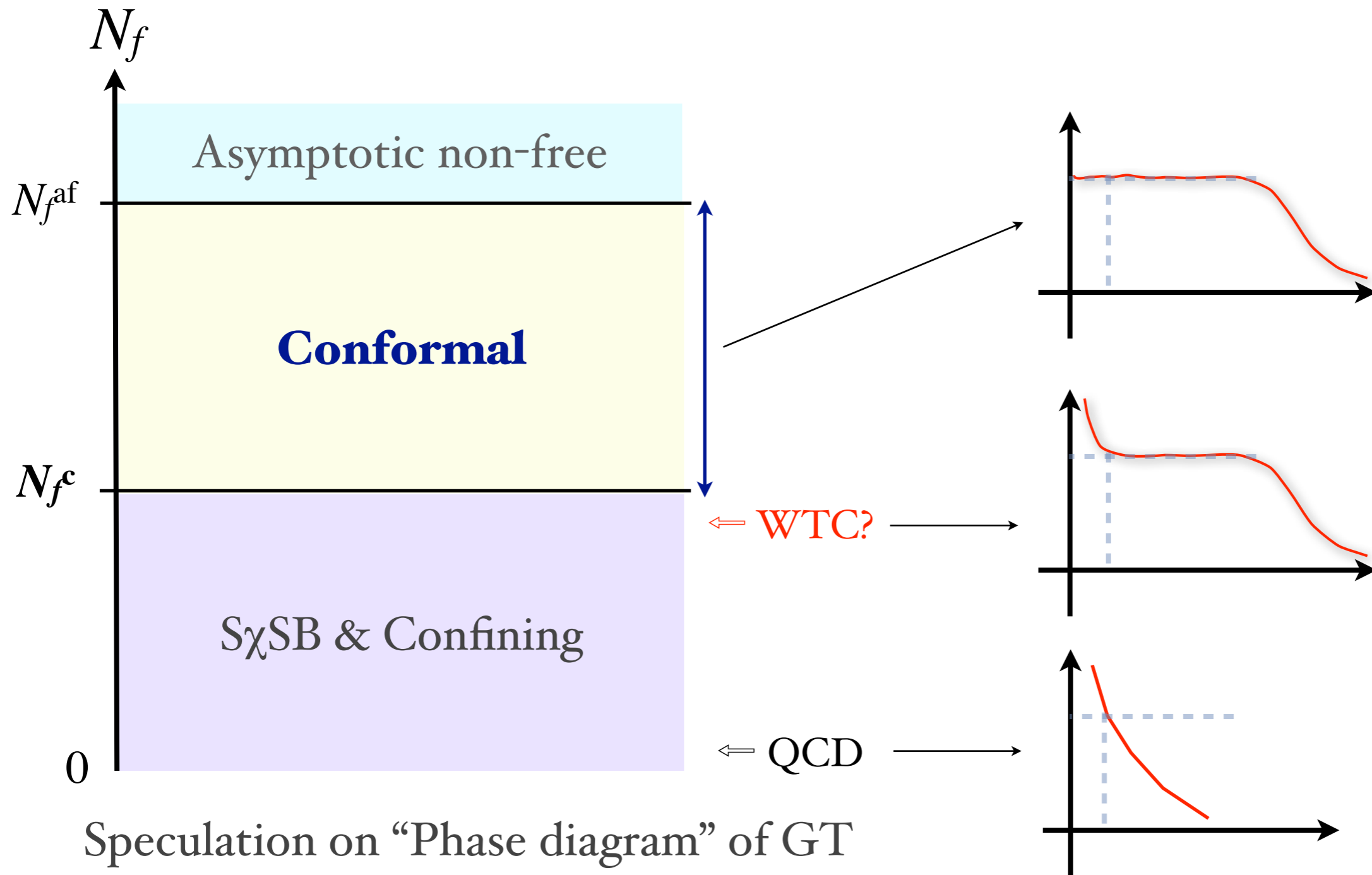
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Conformal Window



Find the location of N_f^c in various GT.

Conformal Window



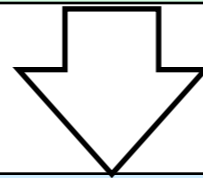
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Strategy on the lattice

Searching phase (-2012?)

Calculate **running coupling** and **anomalous dimension** directly on the lattice.

Calculate **hadron spectrum** to see whether $S\chi SB$ takes place or not.



Prediction phase (2013?-)

Perform large-scale lattice simulation of candidate theories to find the precise values for f_π , m_ρ , (m_σ) , Σ , S-parameter, ...

Now is in **Searching phase**.

Prediction phase on the Next-Generation supercomputer?

Candidates for WTC

So far, the following $SU(N_c)$ gauge theories have been intensively studied.

	N_c	N_f	Rep.	Running g^2	spectroscopy
Large N_f QCD	3	6~16	fund.	$8 < N_{c_f} < 12$	$N_{c_f} > 12$ $N_{c_f} < 12$
Large N_f two-color QCD	2	6, 8	fund.	$N_{c_f} < 6$	-
Sextet QCD	3	2	sextet	conformal	conformal confinement
Two-color adjoint QCD	2	2	adjoint	conformal	conformal

Currently, **many contradictions** and **little consensus**

II. Lattice calculation of running coupling

Machines used

- Supercomputer@KEK (SR11K, BG/L)
- GPGPU & CPU servers@KEK
- INSAM GPU cluster@Hiroshima
- GPGPU, GCOE cluster system@Nagoya
- B-factory computer system



Perturbation

$SU(3)$ gauge theory with N_f fundamental Dirac fermions:

$$\begin{aligned}\beta(g^2(L)) &= L \frac{\partial g^2(L)}{\partial L} \\ &= b_1 g^4(L) + b_2 g^6(L) + b_3 g^8(L) + b_4 g^{10}(L) + \dots,\end{aligned}$$

$$b_1 = \frac{2}{(4\pi)^2} \left[11 - \frac{2}{3} N_f \right], \quad \leftarrow \text{universal and positive for } N_f \leq 16$$

$$b_2 = \frac{2}{(4\pi)^4} \left[102 - \frac{38}{3} N_f \right]. \quad \leftarrow \text{universal}$$

$$b_3^{\overline{\text{MS}}} = \frac{2}{(4\pi)^6} \left[\frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2 \right],$$

$$b_4^{\overline{\text{MS}}} = \frac{2}{(4\pi)^8} \left[29243.0 - 6946.30 N_f + 405.089 N_f^2 + 1.49931 N_f^3 \right],$$

$$b_3^{\text{SF}} = b_3^{\overline{\text{MS}}} + \frac{b_2 c_2^\theta}{2\pi} - \frac{b_1 (c_3^\theta - c_2^{\theta^2})}{8\pi^2}. \quad (c_2^\theta, c_3^\theta: \text{coefficients depending on SF setup})$$

Perturbation

$SU(3)$ gauge theory with N_f fundamental Dirac fermions:

Perturbation

$SU(3)$ gauge theory with N_f fundamental Dirac fermions:

Perturbative IRFP for $SU(3)$ gauge theory with fermions in fund. rep.

N_f	4	6	8	10	12	14	16
2-loop universal				27.74	9.47	3.49	0.52
3-loop SF	43.36	23.75	15.52	9.45	5.18	2.43	0.47
3-loop \overline{MS}		159.92	18.40	9.60	5.46	2.70	0.50
4-loop \overline{MS}			19.47	10.24	5.91	2.81	0.50

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- ▶ N_f^c depends on scheme in perturbation theory
- ▶ Schwinger–Dyson analysis suggests that $S\chi SB$ occurs at $g^2 \sim \pi^2$.
- ▶ PT analysis suggests that $N_f = 8 \sim 12$ are interesting!

Perturbation

$SU(3)$ gauge theory with N_f fundamental Dirac fermions:

Ex) anomalous dimension in SF scheme:

$$\gamma^{\text{SF}} = \frac{8}{(4\pi)^2} g^2 \{1 + (0.1251 + 0.0046 N_f) g^2\}$$

With g_{FP}^2 for 3-loop β -function in SF scheme,

$$\gamma_{\text{FP}}^{\text{SF}} = \begin{cases} 2.76183 & \text{for } N_f = 8 \\ 1.25265 & \text{for } N_f = 10 \\ 0.50772 & \text{for } N_f = 12 \end{cases} \sim O(1)$$

Perturbation is not reliable.

Use Lattice method!

Schrödinger functional scheme

Luscher, Weisz, Wolff, NPB(1991), and subsequent many papers

- ▶ Consider $SU(N_c)$ GT in a cylinder with L^4
- ▶ Boundary conditions in time are fixed such that bulk fields have a non-vanishing color-electric background.

- ▶ Then effective action:

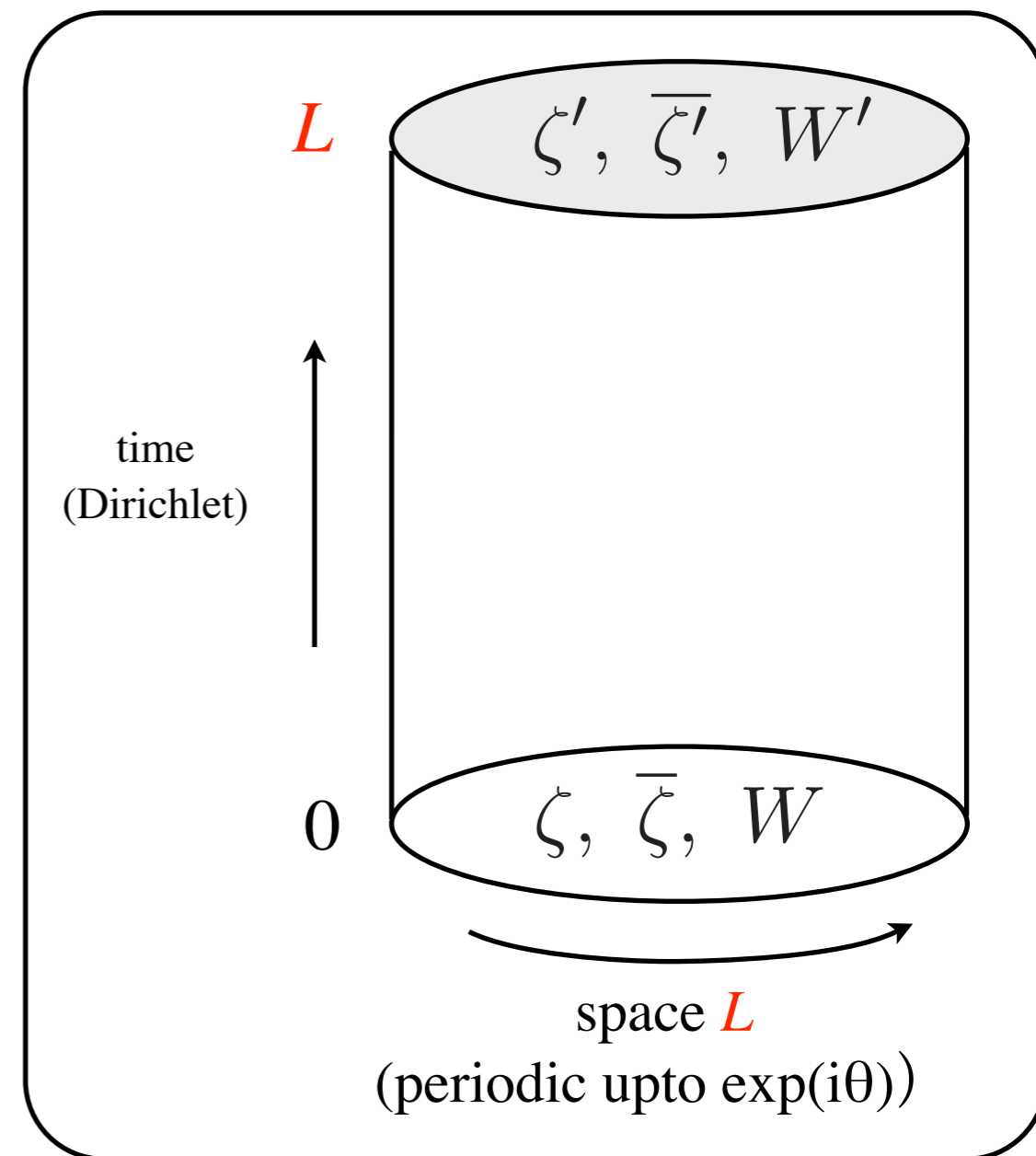
$$\Gamma = g(L)^{-2} S_{YM}^{cl}$$

where $S_{YM}^{cl} = \int d^4x F_{\mu\nu}^2$

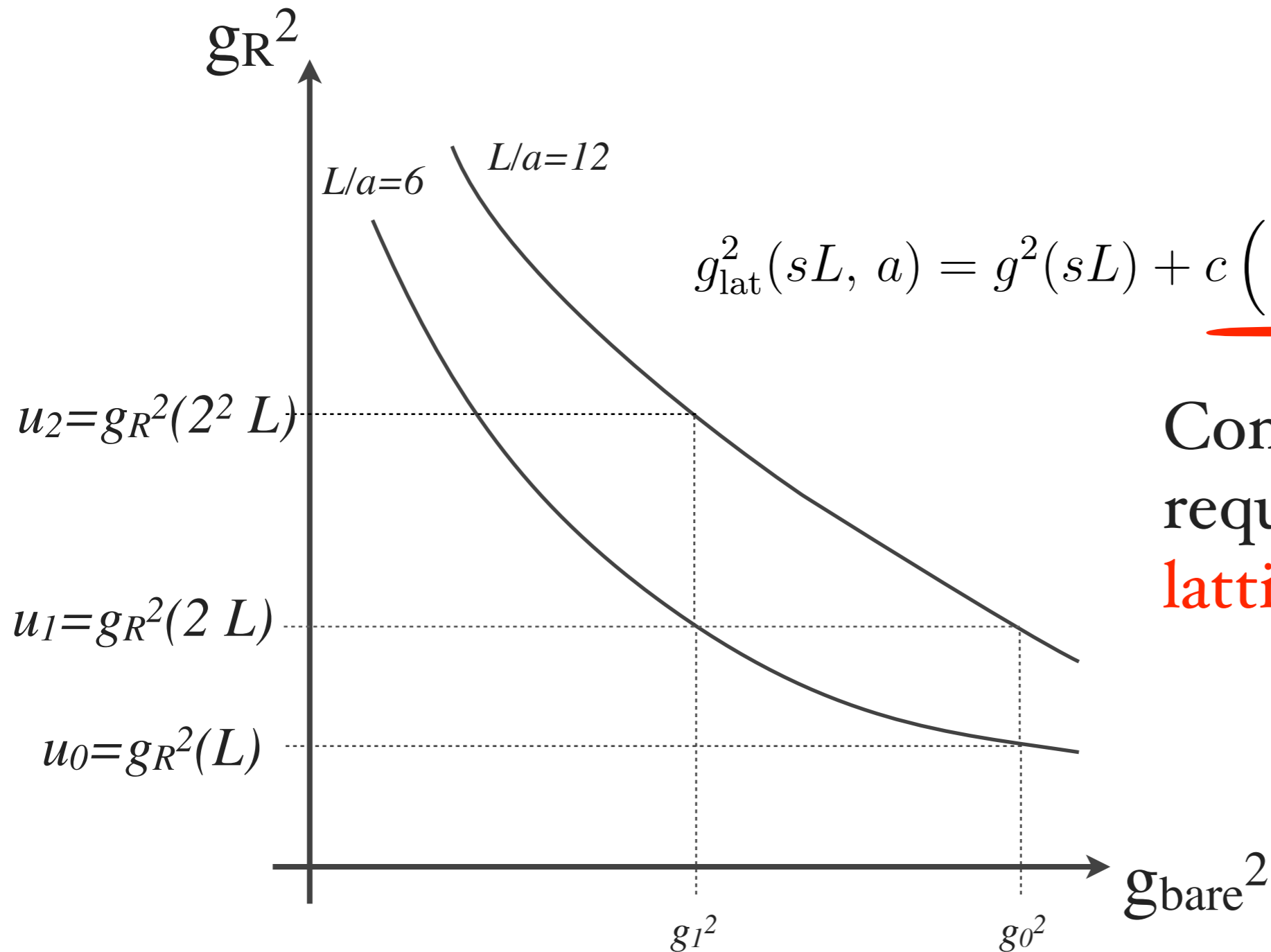
- ▶ $g(L)^2$: **non-perturbative** & gauge-invariant

- ▶ Scale of g^2 is given by L .

Various $L \Rightarrow$ Running of $g^2(L)$



How to calculate “running”?

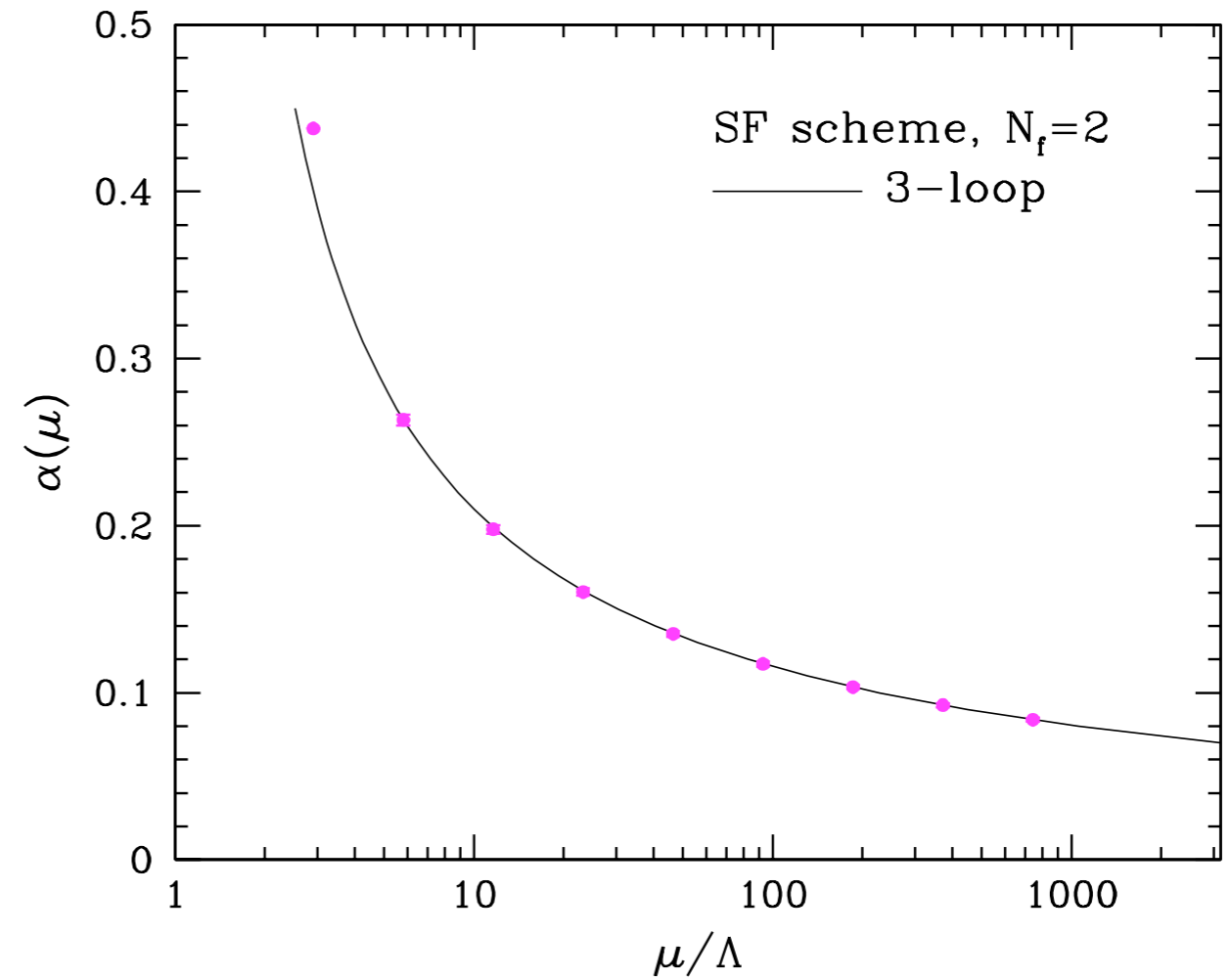
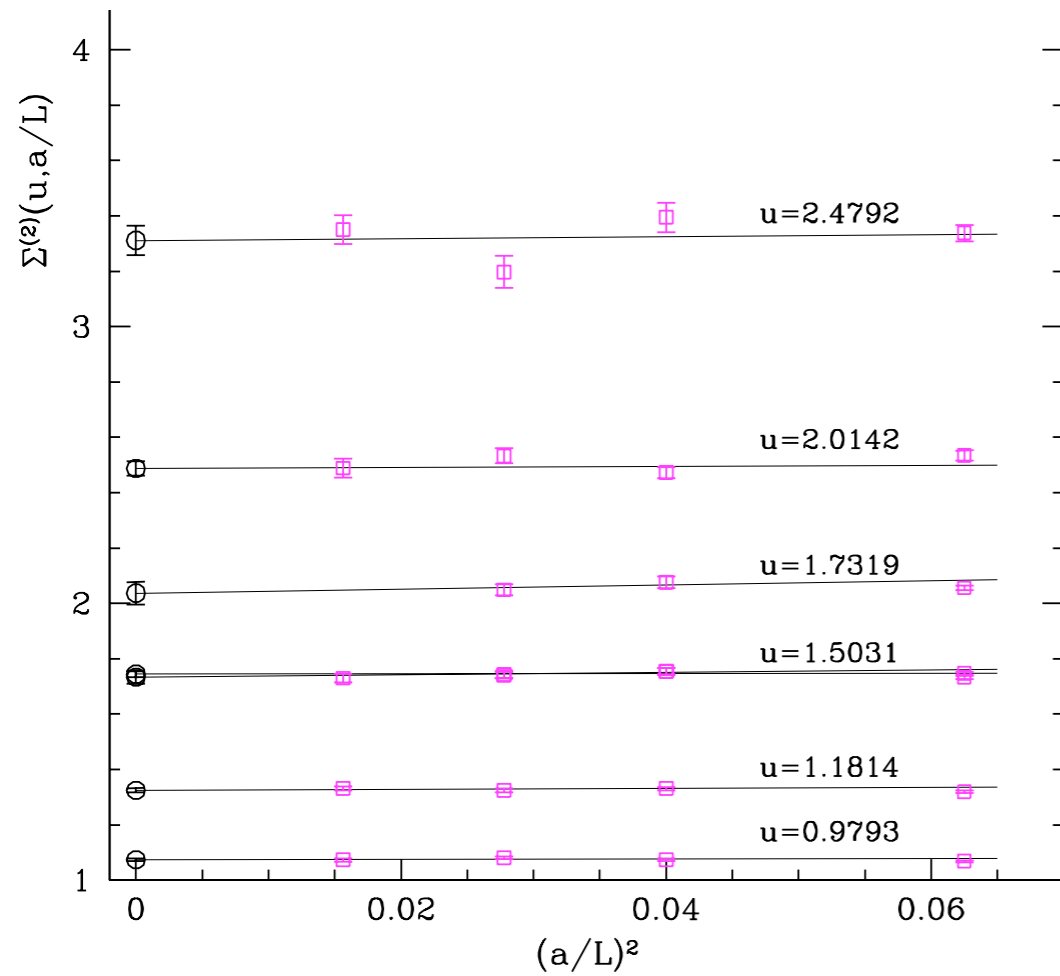


$$g_{\text{lat}}^2(sL, a) = g^2(sL) + c \left(\frac{a}{sL} - \frac{a}{L} \right) + O(a^2/L^2)$$

Continuum limit
required to remove
lattice artifacts.

Example: 2-flavor QCD

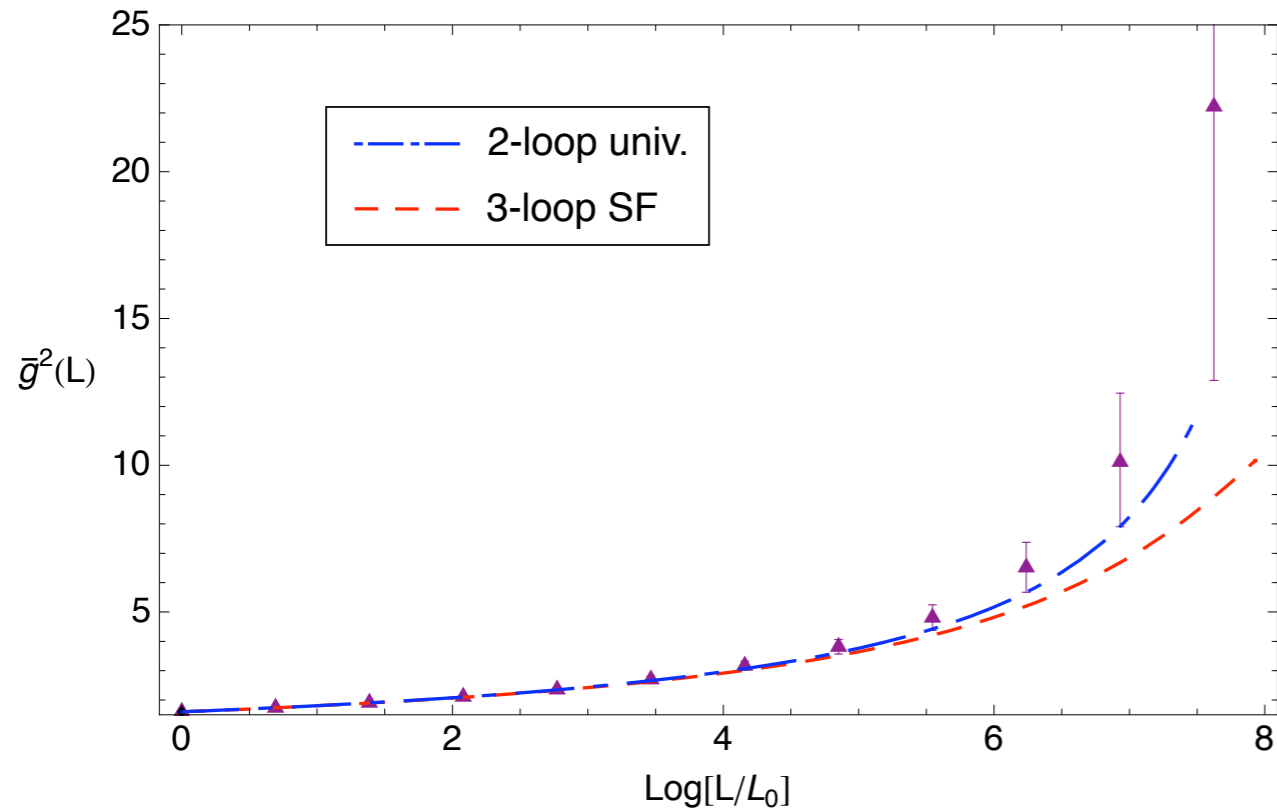
ALPHA Collaboration(2005)



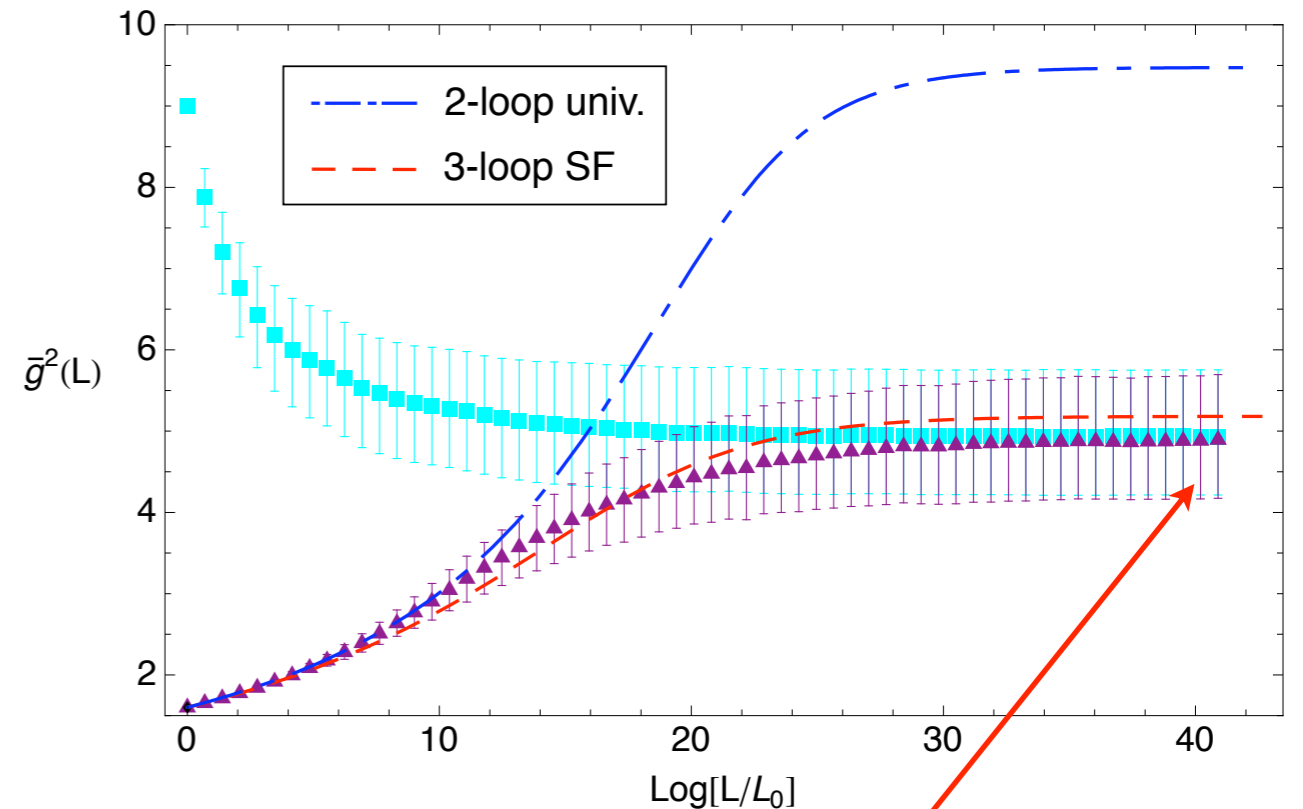
8 & 12 flavor QCD

Appelquist, Fleming, Neil, PRL100:171607, 2008; PRD79:076010, 2009

$N_f = 8$



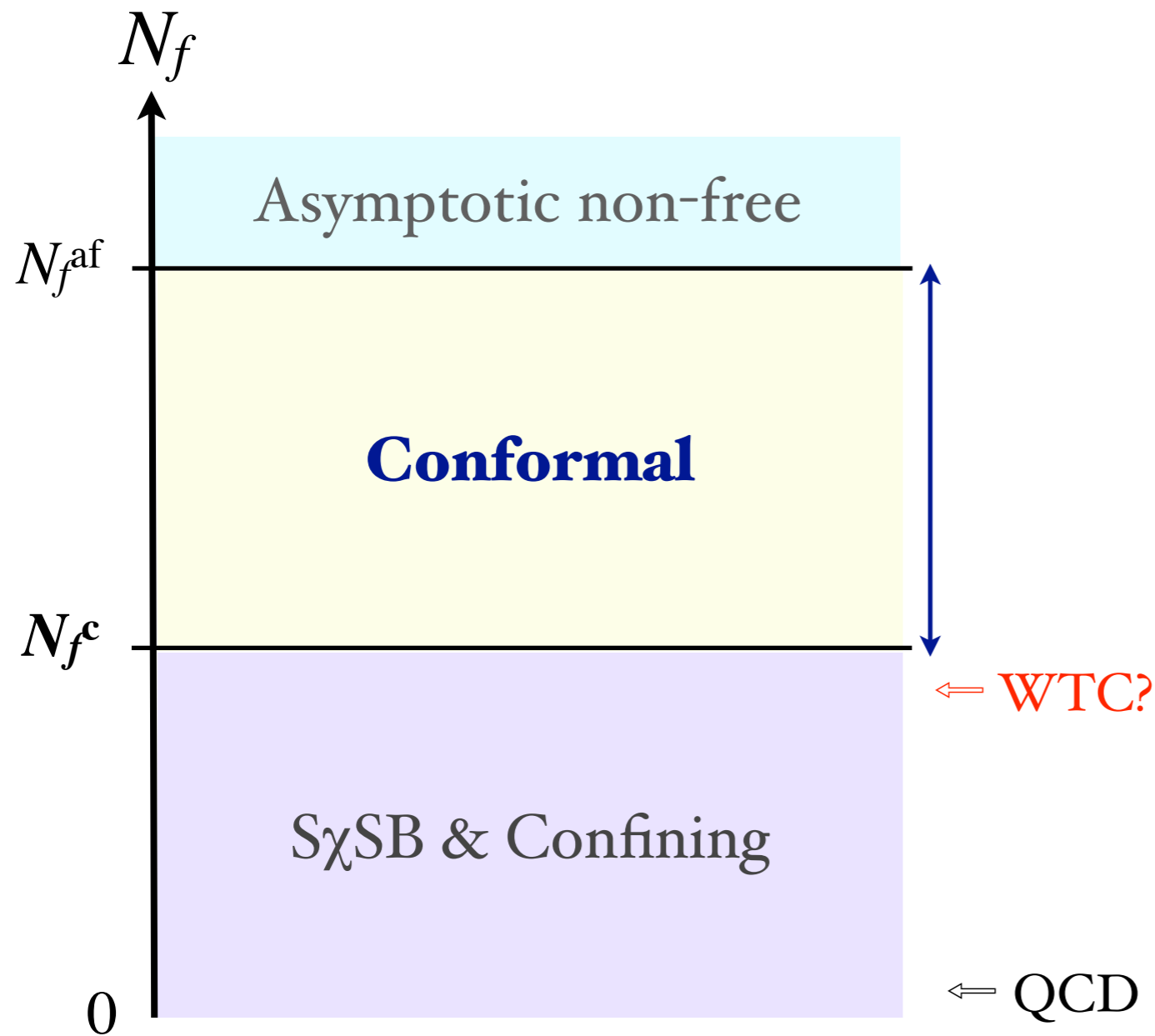
$N_f = 12$



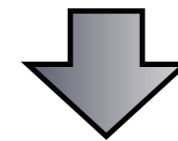
$g^2_{\text{IRFP}} \sim 5$ consistent with PT prediction

Conclusion: $N_f=12$ is too large while $N_f=8$ is too small.
(12-flavor QCD is still under debate.)

Conformal Window



For SU(3) GT,
 $8 < N_f^c < 12$



$N_f = 10$

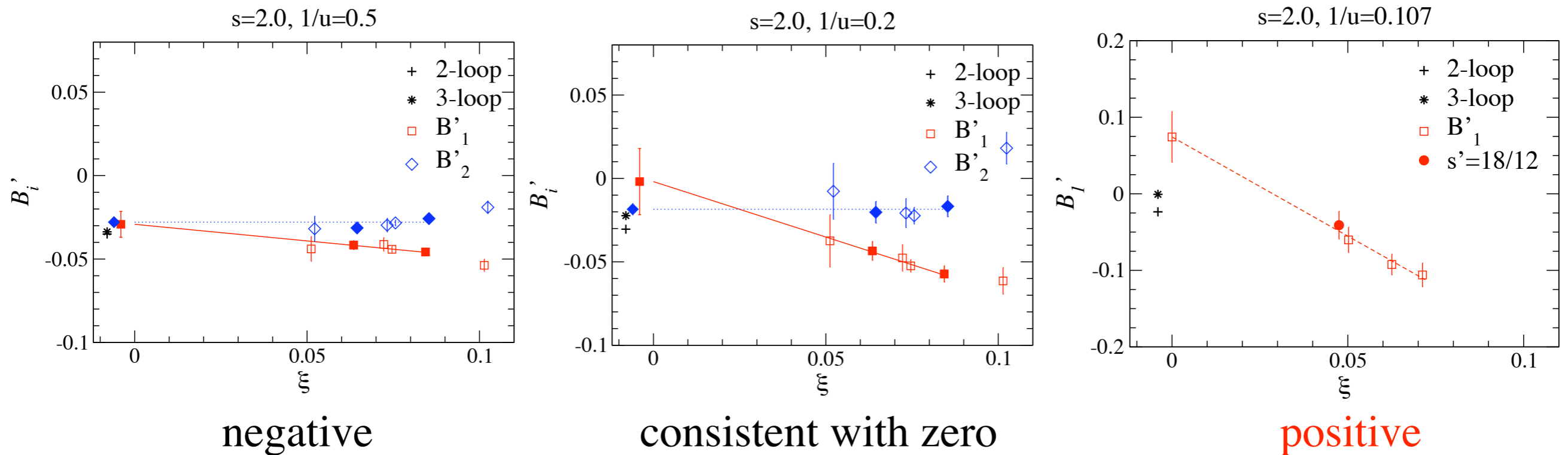
Speculation on "Phase diagram" of GT

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10 flavor QCD (This work)

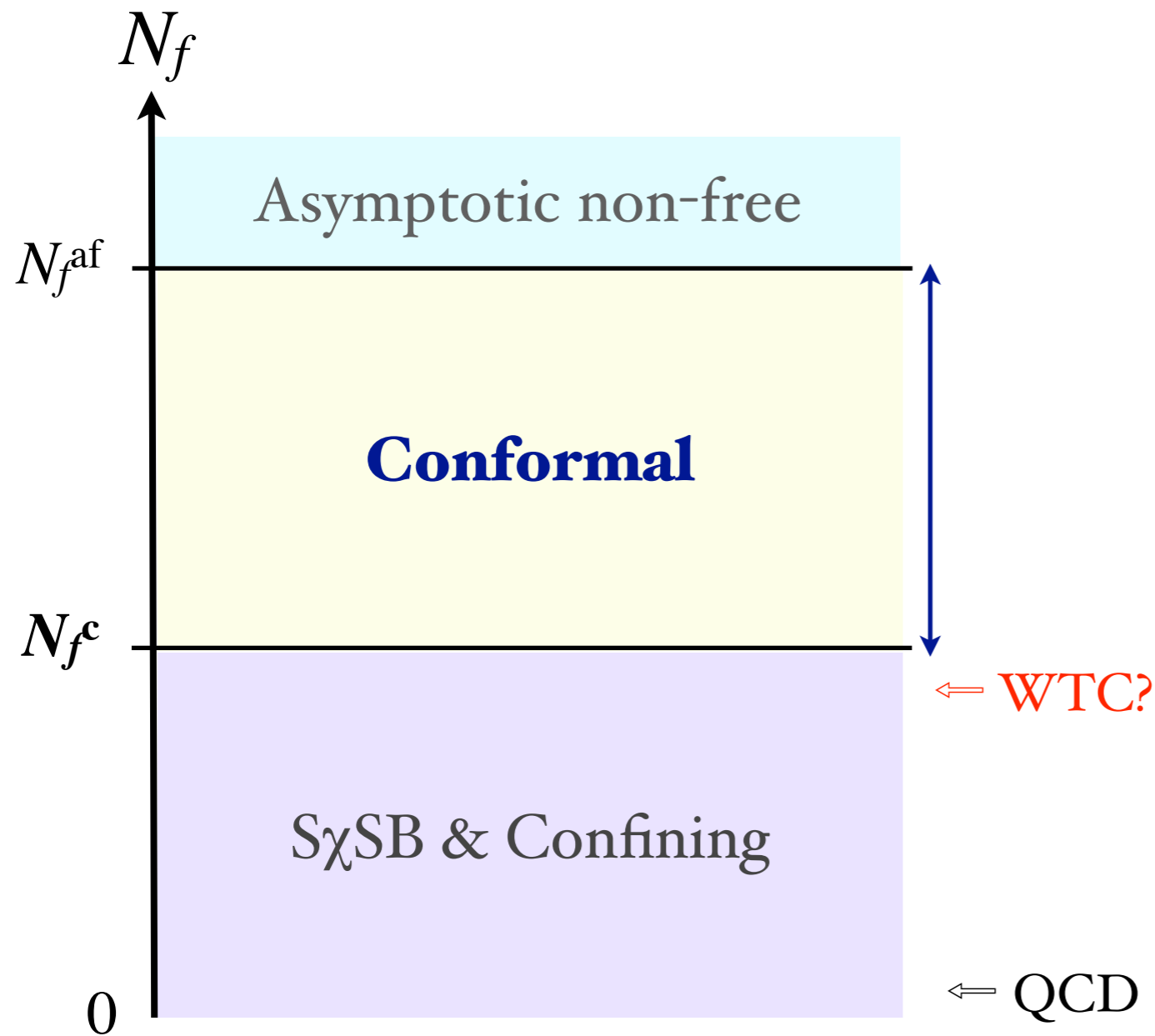
Discrete beta function: $B(u_i, s) = \frac{1}{u_{i+1}} - \frac{1}{u_i}$ where $u_i = g_R^2(s^i L)$

Y. Shamir, B. Svetitsky and T. DeGrand,
PRD78(2008)031502



Extrapolation to the continuum limit shows sign-flip before g_{SF}^2 reaches about 10. $g_{FP}^2 = 3.3 \sim 9.35$

Conformal Window

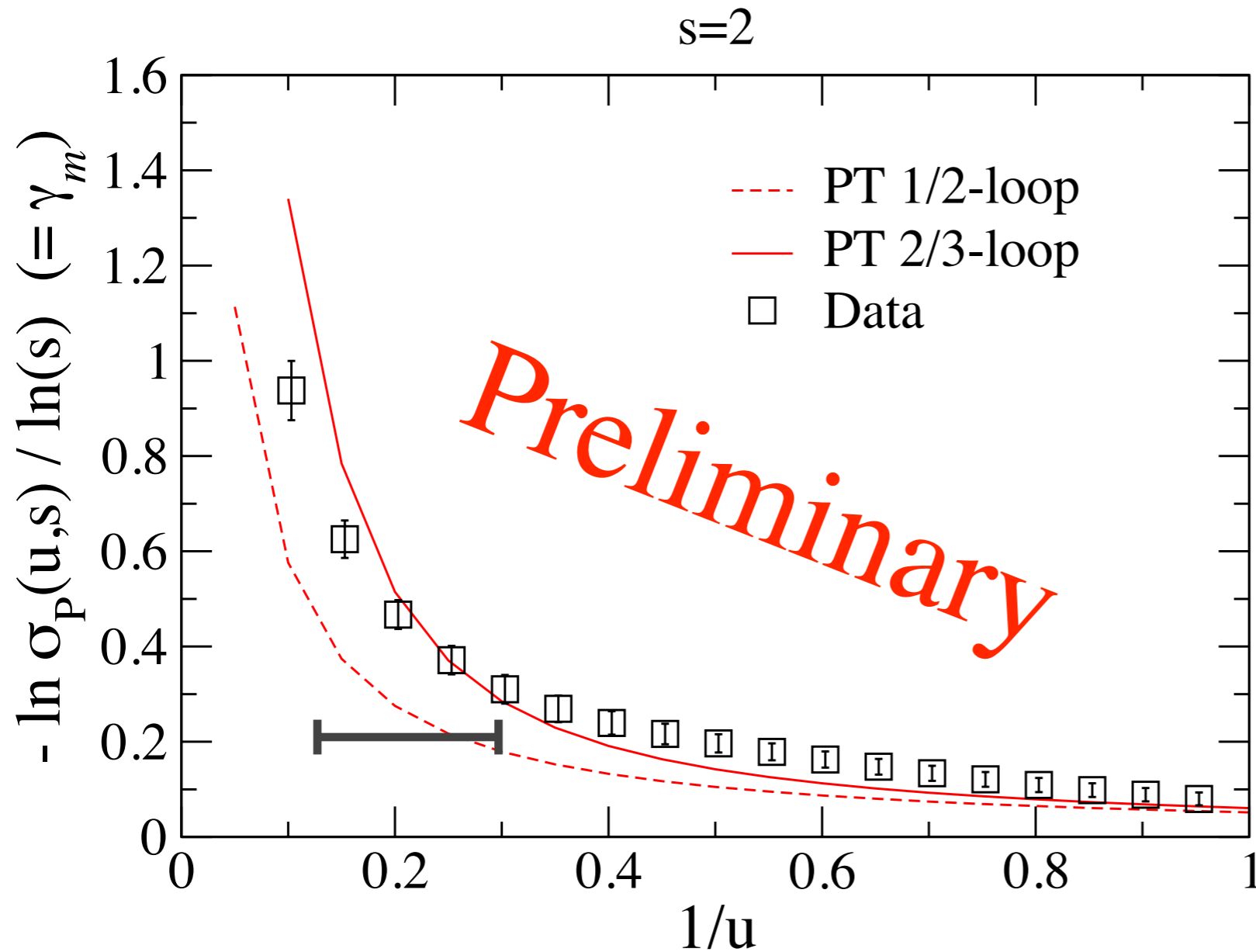


For SU(3) GT,
 $8 < N_f^c < 10$

Speculation on "Phase diagram" of GT

Anomalous dimension

Calculation of γ_m is possible and in progress!



- ▶ $\sigma_P = Z_P(s L) / Z_P(L)$
- ▶ With $3.3 < g^2_{FP} < 9.35$,
 $0.28 < \gamma_m < 1.0$!
- ▶ Precise value of g^2_{FP} is necessary.

Comments

- ▶ Lattice can also calculate γ . In progress!

$\gamma^{\text{SF}} < 0.6$	Sextet QCD with 2-f	[DeGrand et al., arXiv:1006.0707v1]
$0.05 < \gamma^{\text{SF}} < 0.56$	2-color adj. QCD with 2-f	[Bursa et al., arXiv:0910.4535v1]
$0.135 < \gamma^{\text{SF}} < 1.03$	2-color QCD with 6-f	[Bursa et al., arXiv:1007.3067v1]

Not precise yet.

- ▶ If theory is in conformal window, the existence of IRFP is scheme-independent. Calculations from different schemes may give independent check.

Itoh et al. and Holland et al. : 12-flavor QCD

III. Summary and outlook

Summary and outlook

- ✓ Lattice technique can be used to search for realistic WTC models and to see whether the long-standing (~30 yrs) problems in TC are really resolved by WTC.
- ✓ As a first step, we started with the study of running coupling of 10-flavor QCD to identify *conformal window* in SU(3) GT.
- ✓ The result shows evidence of IRFP in $3.3 < g^2_{\text{FP}} < 9.4$.
 $\Rightarrow 8 < N_f^c < 10$
- ✓ $0.28 < \gamma_m < 1.0$ is obtained from preliminary analysis.
Pinning down γ_m requires precise value of the IRFP.
- ✓ Next important task is to calculate S-parameter.