Geometric Entropy and confinement/deconfinement transition in d=4 QCD like theories

Mitsutoshi Fujita
(Tokyo U. IPMU)

Collaborator： Tatsuma Nishioka, Hiroshi Ohki,
and Tadashi Takayanagi

Based on JHEP0809:016,2008 [arXiv: 0806.3118[hep-th]]
and JHEP1008:056,2010 [arXiv: 1006.0344[hep-th]]

Contents

- Short introduction to the geometric entropy

- Application for the deconfinement transition in $d=4$ $\mathcal{N}=4$ SYM on $S^1 \times S^3$ (to mimic finite-T QCD)
  - Weak coupling limit
  - Strong coupling limit

    (by using the AdS/CFT correspondence)

- Conclusion

THE GEOMETRIC ENTROPY IN THIS PAPER

- Geometric entropy is related to the entanglement entropy by double Wick rotation. (cf. the condensed matter physics)

  M. Fujita, T. Nishioka, T. Takayanagi ``08

- Geometric entropy is the von-Neumann (information) entropy associated with the coordinate space.

- We want to find the order parameter for the deconfinement transition in the SYM theory on S^3
  (~ YM with Λ_{QCD}) → Geometric entropy on S^3

D=4 $\mathcal{N}$=4 SYM THEORY

- The matter contents are six real scalars Φ, one gauge boson A_μ, four Weyl fermions Ψ.
  SUSY → The degrees of freedom of the bosons (6+2) are equal to those of the fermions (4x2).
  - Superconformal field theory with vanishing beta-function
    Broken by $S^1 * S^3$ compactification to mimic QCD
  - We can analyze the strong coupling $\mathcal{N}$=4 SYM by using **Gauge/Gravity Correspondence.** (large N limit)

D=4 $\mathcal{N}$=4 SYM ON THE ORBIFOLD S^3/Z_N

- $S^1 \times S^3$ compactification of $\mathcal{N}$=4 SYM

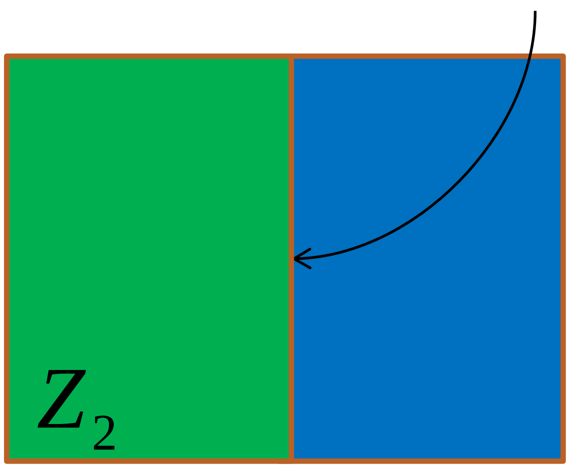
Replica method in terms of n

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta\,(d\psi^2 + \sin^2\psi\, d\phi^2)$$

$$0 \le \phi \le 2\pi \to 0 \le \phi \le \frac{2\pi}{n} \quad \text{(orbifold action; } n \text{ arbitrary)}$$

- Orbifold includes the parity symmetry.
  - Examples

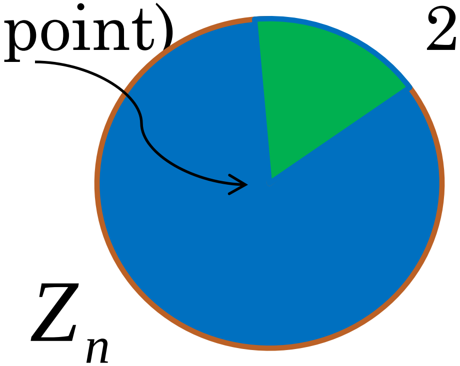
fixed line



The deficit angle

(fixed point)

$2\pi(1\text{-}1/n)$



The geometric entropy: Definition

- $Z(n)$: the partition function of the $\mathcal{N}$=4 SYM on S^3/Z_n
  In particular $Z(1)$ is the partition function on S^3.

- The identification: $$\frac{Z(n)}{Z(1)^{1/n}} = \mathrm{Tr}\, e^{-\frac{2\pi}{n}H} = \mathrm{Tr}\,\rho^{\frac{1}{n}}$$

  ρ: the density matrix
  H: Hamiltonian along ϕ operated by orbifold action

- Define the geometric entropy as follows:

$$S_G = -\mathrm{Tr}\,\rho\log\rho = -\frac{\partial}{\partial n}\log\left[\frac{Z(n)}{Z(1)^{1/n}}\right]\Bigg|_{n=1}$$

Von-Neumann entropy

Deconfinement Transition at Weak Coupling

- Free $\mathcal{N}=4$ SU(N) SYM on S^3 can go through the Confinement/Deconfinement phase transition.

O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas, and M. V. Raamsdonk ``03

- $\mathcal{N}=4$ SU(N) SYM on S^3 (radius R) ~ SU(N) YM with $\Lambda_{QCD} \sim R^{-1}$

- In the small R limit (asymptotic free case), we can see the confinement/deconfinement transition at weak (zero) coupling.

The geometric entropy and Confinement/Deconfinement Transition

- We can integrate out the matter fields and reduce along S^3 to a unitary matrix model.

• Only polyakov loop U=exp($i\, A_0$) is dynamical.

- The unitary matrix model describing the free $\mathcal{N}$=4 SYM on S^1xS^3

$$Z(n) = \int [dU] e^{\sum_{m=1}^{\infty} \frac{1}{m}\left(z_s(x^m)+z_v(x^m)+(-1)^{m+1} z_f(x^m)\right)\mathrm{Tr}(U^m)\mathrm{Tr}(U^{\dagger m})}$$

$$x = e^{-1/TR} \quad (\sim e^{\Lambda_{\mathrm{QCD}}/T})$$

$$z_s(x) = 6\frac{x(1+x^n)}{(1-x)^2(1-x^n)},\ z_v(x) = \frac{2x^2(1+2x^{n-1}-x^n)}{(1-x)^2(1-x^n)},\ z_f = \frac{16x^{\frac{n}{2}+1}}{(1-x)^2(1-x^n)}$$

THE GEOMETRIC ENTROPY AND CONFINEMENT/DECONFINEMENT TRANSITION

- The expectation value of the Polyakov loop $L = \dfrac{\mathrm{Tr}(U)}{N}$

Confinement phase $L = 0$ for $T < T_c$ $(= 0.379\Lambda_{QCD})$

Deconfinement phase $L = 1/2\pi$ for $T \sim T_c$ $(T > T_c)$

Breaking the Z_N symmetry

- The geometric entropy is another order parameter;

$$S_G = O(1) \qquad \text{(for confinement phase)}$$

$$S_G = O(N^2) \qquad \text{(for deconfinement phase)}$$

- High temerature limit

$$S_G = -\frac{\pi^2 N^2}{3} TR \qquad \left(\text{c.f. } S_G^{\text{weak}} = \frac{2}{3} S_G^{\text{strong}}\right)$$

"Deconfinement transition" in a dual description (Hawking-Page transition)

- Deconfinement (Hagedorn) transition = Hawking-Page transition

Low temperature

Thermal AdS

$$ds^2 = \left(\frac{r^2}{b^2}+1\right)dt^2 + r^2 d\Omega_3^2 + \left(\frac{r^2}{b^2}+1\right)^{-1} dr^2$$

High temperature

AdS black hole

$$ds^2 = \left(\frac{r^2}{b^2}+1-\frac{M^2}{r^2}\right)dt^2 + r^2 d\Omega_3^2 + \left(\frac{r^2}{b^2}+1-\frac{M^2}{r^2}\right)^{-1} dr^2$$

Smaller S_{sugra} is chosen

→ First-order phase transition

Deconfinement Transition at Strong Coupling (Gauge/Gravity Correspondence)

- IIB supergravity (dual to strongly coupled SYM)

$$S_{\text{sugra}}(n) = -\frac{1}{16\pi G_N^{(5)}}\int\sqrt{g}R + .. = -\frac{\text{Area}(\gamma)}{4G^{(5)}_N}\left(1-\frac{1}{n}\right)+\ldots$$

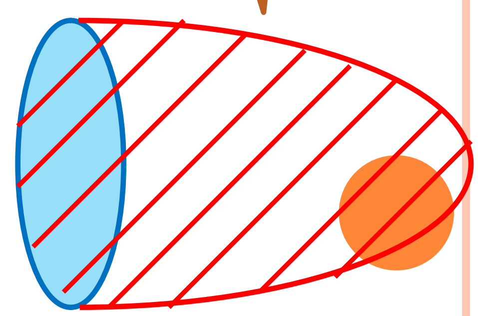
- The geometric entropy is given by

$$S_G = -\frac{\partial}{\partial n}\log\left[\frac{Z(n)}{Z(1)^{1/n}}\right]\Bigg|_{n=1} = -\frac{\partial S_{\text{sugra}}(n)}{\partial n} - S_{\text{sugra}}(1)$$

$$S_G = \frac{\text{Area}(\gamma)}{4G_N^{(5)}} \to -\frac{N^2}{2}\pi^2 Tb \qquad T \sim \frac{\sqrt{bM}}{4\pi b^2}$$

High temperature

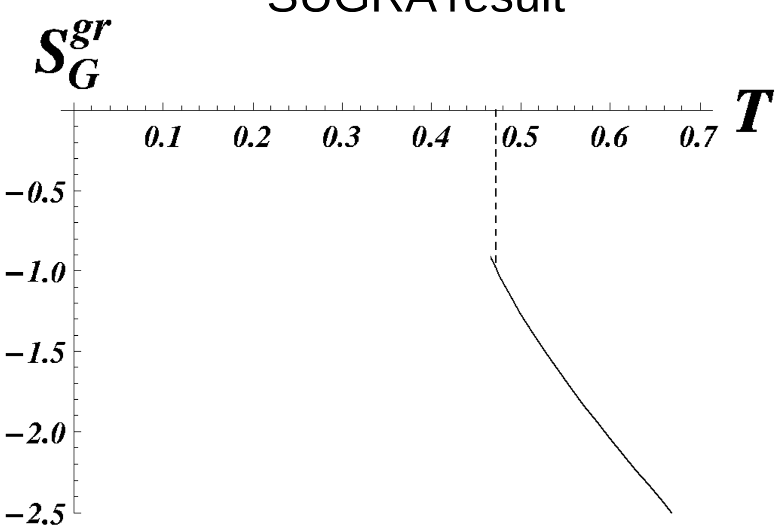
Same formula as
Hawking-Bekenstein entropy!



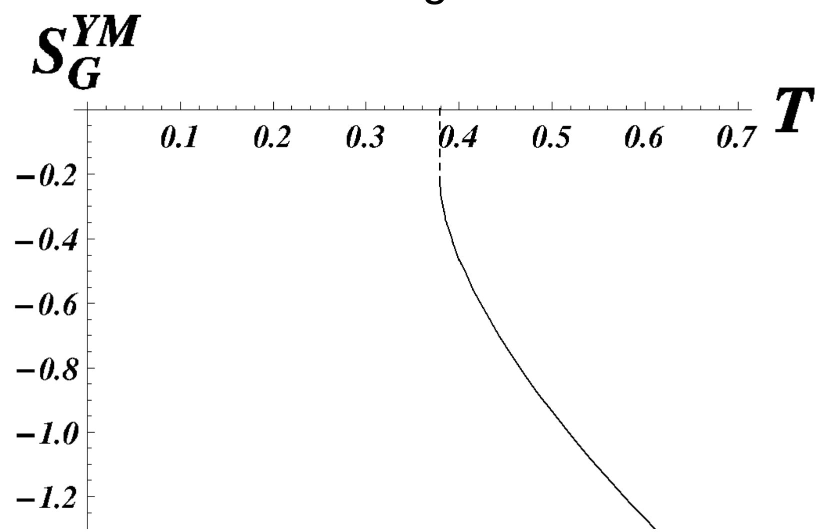
GEOMETRIC ENTROPY AND HAGEDORN/DECONFINEMENT TRANSITION

- Below, we compare the geometric entropy from gravity (*strong-coupling*) with that of the free Yang-Mills (*weak-coupling*)

SUGRA result



Free Yang-Mills result



⟹ Geometric entropy as an order parameter.

MF-Nishioka-Takayanagi (2008)

Conclusion

- In the $\mathcal{N}$=4 SYM, the geometric entropy can be used as an order parameter of the confinement/deconfinement transition at weak coupling.
- In the dual gravity description, the geometric entropy is also the order parameter of the Hagedorn (Hawking-Page) transition, i.e. the confinement/deconfinement transition at strong coupling.
- As future extensions we can introduce the matter field.

Geometric Entropy and confinement/deconfinement transition in d=4 QCD like theories

Mitsutoshi Fujita
(Tokyo U. IPMU)

Collaborator： Tatsuma Nishioka, Hiroshi Ohki, and Tadashi Takayanagi

Based on JHEP0809:016,2008 [arXiv: 0806.3118[hep-th]] and JHEP1008:056,2010 [arXiv: 1006.0344[hep-th]]

Contents

- The geometric entropy in condensed matter physics
- Motivation for the geometric entropy in QCD like theories
- Application for the deconfinement transition in free $\mathcal{N}$=2 SYM with flavor on $S^1 \times S^3$ (to mimic finite-T QCD)
  - Breaking supersymmetry
  - Finite density system
- Conclusion

The geometric entropy in the condensed matter physics

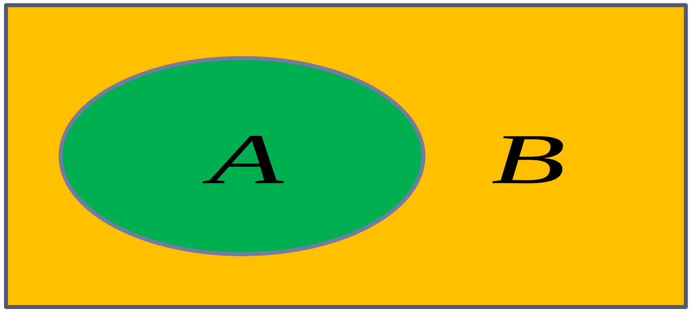
- System whose total Hilbert space is a direct product:

$$H = H_A \otimes H_B$$

- Entanglement Entropy (EE) is defined using the density matrix ρ by

$$S_A = -Tr_A(\rho_A \log \rho_A)$$

$$\rho_A = Tr_B(\rho)$$



- If *A* and *B* are a spatial bipartion

of the system, EE is called geometric entropy!

the geometric entropy in this paper

- Geometric entropy is related to the entanglement entropy by double Wick rotation. (cf. the condensed matter physics)

  M. Fujita, T. Nishioka, T. Takayanagi ``08

- Geometric entropy is the von-Neumann (information) entropy associated with the coordinate space.

- We want to find the order parameter for the deconfinement transition in the SYM theory on S^3 (~YM with Λ_{QCD}) → Geometric entropy on S^3

Motivation for the geometric entropy in QCD like theories

- In QCD, an order parameter is needed.

  Example:
  (a) Polyakov loop, the chiral condensate (in the chiral limit)
  (b) EE is an order parameter for the deconfinement transition in the Yang-Mills theory.

  T. Nishioka, T. Takayanagi, ``06,

  I. Klebanov, D. Kutasov, and A. Murugan,``07

  (c) Geometric entropy in this paper is more convenient to search the finite temperature system.

→ We analyze the phase structure of the guage theory with matter fields on $S^1 \times S^3$ using geometric entropy.

A D=4 $\mathcal{N}$=2 SYM theory with flavor

- $\mathcal{N}$=2 vector multiplet + N_f $\mathcal{N}$=2 hypermultiplet

- The matter contents of $\mathcal{N}$=2 vector multiplet

  Two real scalars Φ, one gauge boson A_μ, 2 Weyl fermions Ψ.

- +N_f Flavor: The matter contents of $\mathcal{N}$=2 hypermultiplet

  Four real scalars, 2 Weyl fermions

- Vanishing beta-function for $2N=N_f$

  Broken by S^1*S^3 compactification to mimic QCD

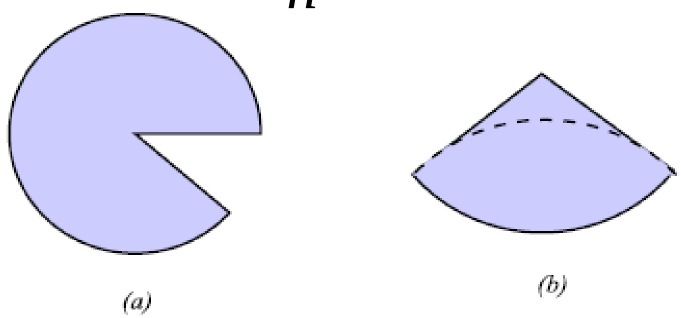
Orbifold gauge theory on S^3/Z_n

- We use the Replica method in terms of *n* in the orbifold.

- We consider the orbifold gauge theory on $S^1 \times S^3/Z_n$.

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta (d\psi^2 + \sin^2\psi \, d\phi^2)$$

$$0 \leq \phi \leq 2\pi \rightarrow 0 \leq \phi \leq \frac{2\pi}{n} \quad \text{(orbifold action; } n \text{ arbitrary)}$$



- $Z(n)$: the partition function of the gauge theory on $S^1 \times S^3/Z_n$

  In particular, Z(1) is the partition function on $S^1 \times S^3$

The geometric entropy Ⅱ

- The identification

$$\frac{Z(n)}{Z(1)^{1/n}} = \mathrm{Tr}\, e^{-\frac{2\pi}{n}H} = \mathrm{Tr}\,\rho^{\frac{1}{n}}$$

H: Hamiltonian along φ
ρ: the density matrix

- Definition of geometric entropy on $S^1 \times S^3$

$$S_G = -\mathrm{Tr}\,\rho \log \rho = -\frac{\partial}{\partial n} \log\left[\frac{Z(n)}{Z(1)^{1/n}}\right]\Bigg|_{n=1}$$

Definition of Von-Neumann entropy

Deconfinement Transition at **Weak Coupling**

- Free $\mathcal{N}$=2 U(N) SYM on $S^1 * S^3$ can go through the Confinement/Deconfinement phase transition as third order phase transition.

H. J. Schnitzer ``04

O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas, and M.V. Raamsdonk ``03

- $\mathcal{N}$=2 U(N) SYM with flavor on S^3 (radius R) ~ U(N) QCD like theories with $\Lambda_{QCD} \sim R^{-1}$

- In the small R limit (asymptotic free case), we can see the confinement/deconfinement transition at weak (zero) coupling.

Partition function of $d=4$ gauge theories with matter on an orbifold S^3/Z_n

- The matter contents of our theory become a $\mathcal{N}=2$ vector multiplet and N_f hypermultiplet

- We can integrate out the matter fields and reduce along S^3 to a unitary matrix model (possible for orbifold S^3/Z_n) → Dynamical field is only Polyakov loop *exp(iβA₀)*.

$$Z(v,f) = \int [dU] \exp\left[\sum_{m=1}^{\infty} \frac{1}{m}\left(v(x^m)\mathrm{Tr}U^m\mathrm{Tr}U^{m\dagger} + \frac{1}{2}Nf(x^m)(\mathrm{Tr}U^m + \mathrm{Tr}U^{m\dagger})\right)\right]$$

$$x = e^{-1/TR} \quad (\sim e^{\Lambda_{\mathrm{QCD}}/T})$$

v(x), f(x): the single particle partition functions for the adjoint fields and the fundamental fields, respectively

Review for the derivation of v(x), f(x)

- The single-particle partition function on S^3/Z_n is given by

$$v(x^m) = v_B(x^m) + (-1)^{m+1} v_F(x^m),$$
$$v_B(x) = 2\,\frac{x^2\,(1+2\,x^{n-1}-x^n)}{(1-x)^2\,(1-x^n)} + 2\,\frac{x\,(1+x^n)}{(1-x)^2\,(1-x^n)},$$
$$v_F(x) = 8\,\frac{x^{n/2+1}}{(1-x)^2\,(1-x^n)},$$

$$f'(x^m) = f'_B(x^m) + (-1)^{m+1} f'_F(x^m),$$
$$f'_B(x) = 4\,\frac{x\,(1+x^n)}{(1-x)^2\,(1-x^n)}, \quad f'_F(x) = 8\,\frac{x^{n/2+1}}{(1-x)^2\,(1-x^n)},$$
$$f = (N_f/N) f'$$

- To derive above formulas, we can consider a conformal transformation:

The states of the field theory on R*S^3 ⟺ local operators on R^4

The energy of the states (∂_r) ⟺ conformal dimension of local operators $(r \cdot \partial_r)$

- Embedding S^3/Z_n in C^2 with the coordinates (z_1,z_2)

⟶ orbifold action

$$z_1 \approx z_1 e^{i\frac{2\pi}{n}}$$

Review for the derivation of v(x), f(x)

- Z_n action on the scalar operator Φ in C^2

$$\phi(z_1,\bar{z}_1,z_2,\bar{z}_2) \sim \phi(e^{i\frac{2\pi}{n}}z_1, e^{-i\frac{2\pi}{n}}\bar{z}_1, z_2, \bar{z}_2),$$

$$\partial_1^2\phi(z_1,\bar{z}_1,z_2,\bar{z}_2) \sim e^{i\frac{4\pi}{n}}\partial_1^2\phi(e^{i\frac{2\pi}{n}}z_1, e^{-i\frac{2\pi}{n}}\bar{z}_1, z_2, \bar{z}_2),$$

$$\partial_i = \partial / \partial z_i \ \ (i = 1,2)$$

- For n=3, the invariant operators are given by

$$\phi,\ \partial_2\phi,\ \bar{\partial}_2\phi,\ \partial_2\partial_2\phi,\ \partial_2\bar{\partial}_2\phi,\ \bar{\partial}_2\bar{\partial}_2\phi,\ \partial_1\partial_1\partial_1\phi,\ \bar{\partial}_1\bar{\partial}_1\bar{\partial}_1\phi, \ldots$$

- The single-particle partition function is computed as follows:

$$z(x) = \sum_{local\, op.} x^{\Delta} = \sum_{k=1}^{\infty} k x^k (1 + 2\sum_{l=1}^{\infty} x^{nl})$$

$$= \frac{x(1+x^n)}{(1-x)^2(1-x^n)}$$

$\partial_1{}^n,\ \bar{\partial}_1{}^n$
depending the orbifold

Review for the derivation of v(x), f(x)

- For the gauge fields and Weyl fermions, the computation is similar to the scalar field:

- For gauge field,

$$z_v(x) = 2x \sum_{k=1} kx^k (1 + 2\sum_{l=1} x^{nl-1}) = \frac{2x^2(1+2x^{n-1}-x^n)}{(1-x)^2(1-x^n)}.$$

Imposing the Gauss law constraint, $x^\mu A_\mu = 0$

- For Weyl fermion,

$$z_f = 4x^{\frac{n}{2}} \sum_{k=1} kx^k (1 + \sum_{l=1} x^{nl}) = \frac{4x^{1+\frac{n}{2}}}{(1-x)^2(1-x^n)},$$

- Then, $v(x) = 2z_v + 2z_B - 2z_F$ and $f' = 4z_B - 2z_F$

Free energy

- Approximation: only the first winding state in the time direction

$v(x^m) = f(x^m) = 0 \ (m \geq 2)$ valid for not sufficiently high temperature region

- Rewriting the partition function:

$$
\begin{aligned}
Z(v,f) &= \frac{N^2}{8\pi v}\int [dU] d\lambda d\bar{\lambda} \exp\left[-\frac{N^2}{4v}(\lambda - f)(\bar{\lambda} - f) + \frac{N}{2}(\lambda \mathrm{Tr} U + \bar{\lambda}\mathrm{Tr} U^\dagger)\right] \\
&= \frac{N^2}{4\pi v}\int_0^\infty g dg \int_{-\pi}^{\pi} d\theta \exp\left[-\frac{N^2}{4v}(g^2 - 2gf\cos\theta + f^2)\right] \cdot \\
&\quad \cdot \int [dU] \exp\left(\frac{Ng}{2}(\mathrm{Tr}U + \mathrm{Tr}U^\dagger)\right) \\
&= \frac{N^2}{2v}\int_0^\infty g dg e^{-N^2 \beta F(v,f,g)},
\end{aligned}
$$

$$
N^2 \beta F(v,f,g) = -\log I_0\left(\frac{N^2 g f}{2v}\right) + \frac{N^2}{4v}(f^2 + g^2) - N^2 K(g) \qquad e^{N^2 K(g)} = \int [dU] \exp \frac{1}{2} N g \left(\mathrm{Tr}U + \mathrm{Tr}U^\dagger\right).
$$

Free energy and third order phase transition

- Asymptotic expansion of *K(g)* in the large *N* limit:

$$K(g) = \begin{cases} \frac{g^2}{4} + O(1/N^3) & (g < 1) \\ g - \frac{1}{2}\log g - \frac{3}{4} + O(1/N^2) & (g > 1) \end{cases}$$

H. Liu, ``04

- We search the saddle point in *Z(v,f)*

When $v<1$ and $f < f_0 = 1-v$,

$$g_0 = \frac{f}{f_0} < 1, \quad \beta F(v, f, g_0) = -\frac{f^2}{4(1-v)}.$$

D. Gross, E. Witten, ``80

Third order phase transition

When $v<1$ and $f > f_0$ or $v>1$,

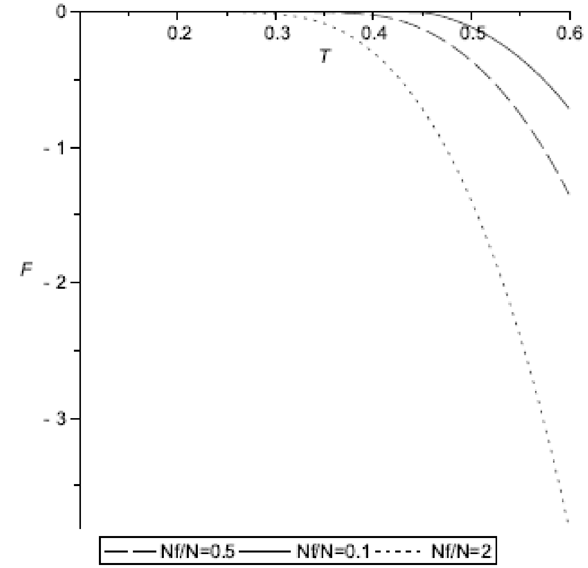
$$g_0 = v + \frac{f}{2} + \sqrt{\left(v + \frac{f}{2}\right)^2 - v}, \quad \beta F(v, f, g_0) = -\frac{g_0}{2} - \frac{f g_0}{4v} + \frac{1}{2} + \frac{1}{2}\log g_0 + \frac{f^2}{4v}.$$

- The critical temperature is determined from the formula

$$v(x) + f(x) = 1, \quad g_0 = 1.$$

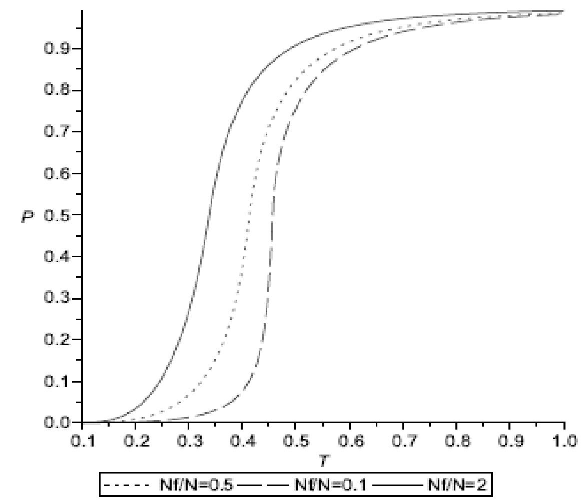
Free energy and Polyakov loop

- The plot of free energy $N^2F = -\log Z / \beta$



- Polyakov loop vev

$$L = \left\langle \frac{\mathrm{Tr}(U)}{N} \right\rangle = \partial(NK(g)) / \partial g$$



- It will be interesting if we compare our result with results of Lattice and soft wall AdS/QCD.

Comment on related works: Polyakov loop in Soft-wall AdS/QCD

- It is possible to compute the expectation value of the Polyakov loop by using the gauge/gravity correspondence. (Andreev 2009).

- The soft-wall model can holographically describe strongly coupled SU(N) QCD.

- The metric of the soft-wall model :

$$ds^2 = G_{mn} dx^m dx^n = \frac{e^{\frac{4}{3}cz^2}}{z^2} \left(f dt^2 + dx^2 + \frac{1}{f} dz^2 \right)$$

$$f(z) = 1 - (z / z_T)^4$$

c ~ typical scale of QCD (Λ_{QCD})
z_T ~ temperature

Expectation value of the Polyakov loop

- Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'}\int dtdz\sqrt{\det G_{mn}\partial_\alpha X^n \partial_\beta X^m}$$

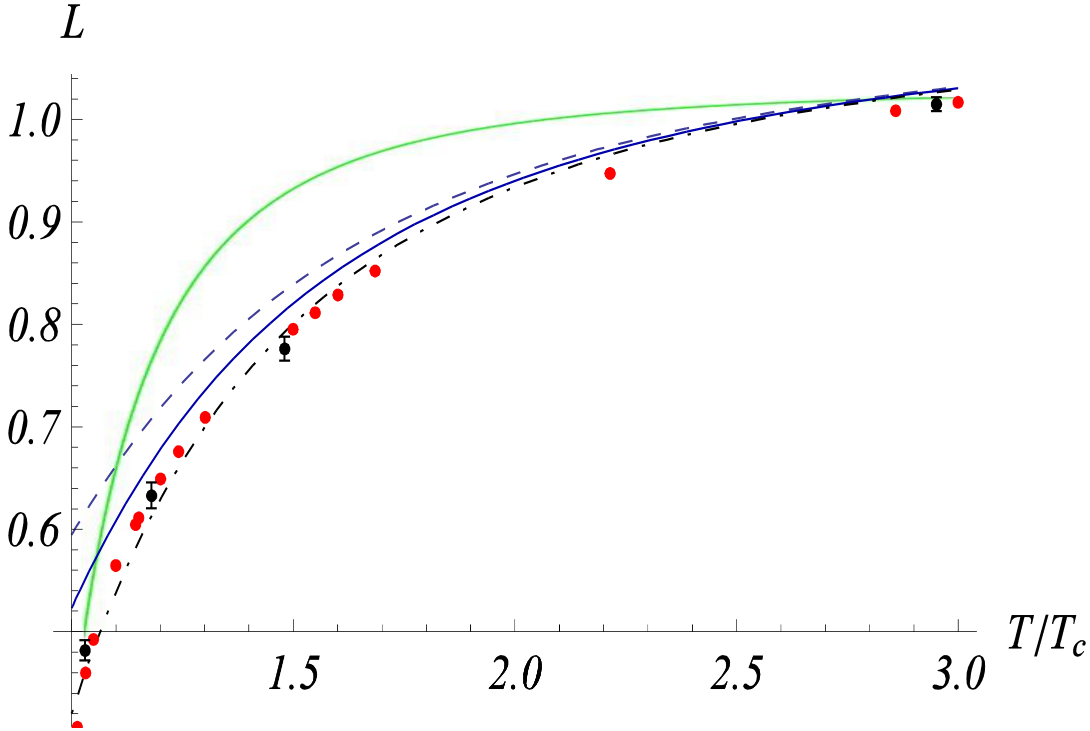
- After subtracting divergent parts

$$S = \left(\sqrt{\pi}\frac{T_c}{T}Erfi\left(\frac{T_c}{T}\right) + 1 - e^{(T_c/T)^2}\right) + \text{const.} \qquad T_c \sim \frac{\sqrt{c}}{\pi}$$

- Polyakov loop

$$L(T) = \exp(-S)$$

Numerical results of the matrix model (ours) and the soft-wall AdS/QCD



- Solid blue curve: soft-wall AdS/QCD results
- Dots: lattice simulations (SU(3) QCD)
- Solid green curve: Our free SYM results

Andreev (2009)
overlaid with our plot

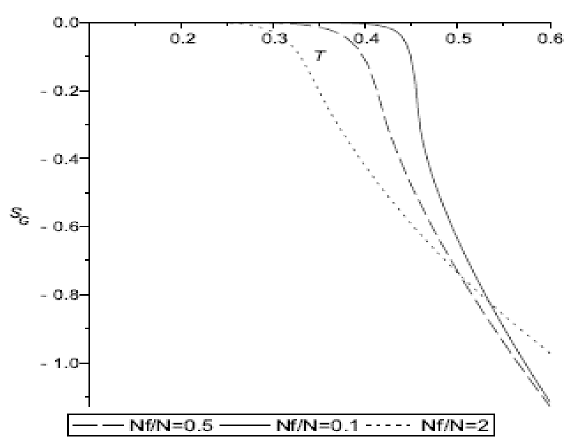
QCD is more like strong coupling limit (AdS/QCD) !

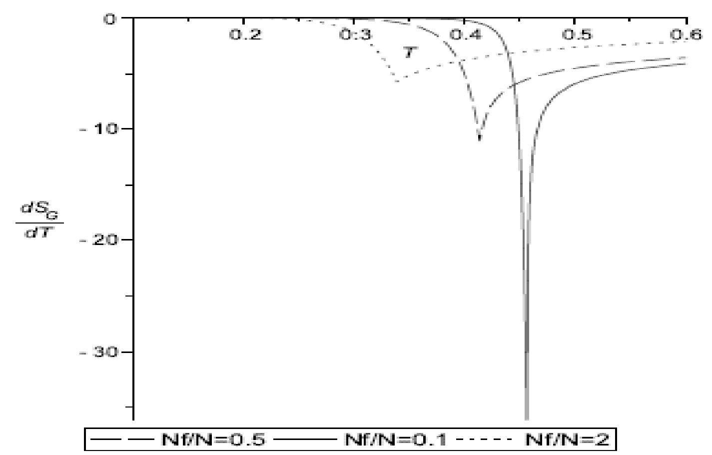
Geometric entropy and Third order Transition

- Geometric entropy in terms of the free energy *F*

$$S_G = -\frac{\partial}{\partial(1/n)}\left(\log Z(n) - \frac{1}{n}\log Z(1)\right)\Bigg|_{n=1} = -\frac{\partial}{\partial n}\left((\beta F)(n) - \frac{1}{n}(\beta F(1))\right).$$

Plot of Geometric entropy S^G and dS^G/dT





- Geometric entropy can capture the third order phase transition of the Gross-Witten model.

High temperature limit

- The behavior of single particle partition function at high temperature limit x → 0 (β → 0)

$$z_c(x^m) = \frac{4}{m^3 n \beta^3} + \frac{n^2-1}{3mn\beta} + O(\beta),$$
$$z_v(x^m) = \frac{4}{m^3 n \beta^3} + \frac{n^2-6n-1}{3mn\beta} + O(1),$$
$$z_f(x^m) = \frac{4}{m^3 n \beta^3} - \frac{2+n^2}{6mn\beta} + O(\beta).$$

- We define the geometric entropy S^P_G with fermions obeying the periodic b.c. in the time direction.

The behavior related with the usual entropy

- The free energy and $\Delta S_G = S_G - S^P_G$ at high temperature limit

$$F = -\frac{1}{12}\pi^2 N^2 T^4 \left(1 + \frac{N_f}{N}\right) V_{S^3}, \qquad \Delta S_G = -\frac{\pi^2 N^2}{6\beta}\left(\frac{N_f}{N} + 1\right).$$

Introducing chemical potential

- The Lagrangian of the hypermultiplet

$$\mathcal{L} = \int (Q_a^\dagger e^{-2V} Q_a + \tilde{Q}_a e^{2V} \tilde{Q}_a^\dagger) + \int d^2\theta (\tilde{Q}_a \Phi Q_a) + h.c.$$

- We introduce the chemical potential conjugate to the following global symmetry
- Subgroup of R-symmetry $U(1)_J \subset SU(2)_R$

$$U(1)_J: \quad \Phi \to \Phi(e^{-i\alpha}\theta), \quad V \to V(e^{-i\alpha}\theta),$$
$$Q \to e^{i\alpha} Q(e^{-i\alpha}\theta), \quad \tilde{Q} \to e^{i\alpha}\tilde{Q}(e^{-i\alpha}\theta),$$

- Baryonic $U(1)_F$ subgroup in $U(N_f)$ flavor symmetry

$$(Q_a, \tilde{Q}_a^\dagger) \to e^{i\alpha}(Q_a, \tilde{Q}_a^\dagger)$$

Geometric entropy at finite density

- We use the approximation neglecting the higher order of winding states (m>1)

- For $U(1)_J$ case, single particle partition function is replaced by

$$v(x^m) = v_B(x^m) + (-1)^{m+1} v_F(x^m),$$

$$v_B(x) = 2\,\frac{x^2\,(1 + 2\,x^{n-1} - x^n)}{(1-x)^2\,(1-x^n)} + 2\,\frac{x\,(1+x^n)}{(1-x)^2\,(1-x^n)},$$

$$v_F(x) = 4\,\frac{x^{n/2+1+\mu}}{(1-x)^2\,(1-x^n)} + 4\,\frac{x^{n/2+1-\mu}}{(1-x)^2\,(1-x^n)},$$

$$f'(x^m) = f'_B(x^m) + (-1)^{m+1} f'_F(x^m),$$

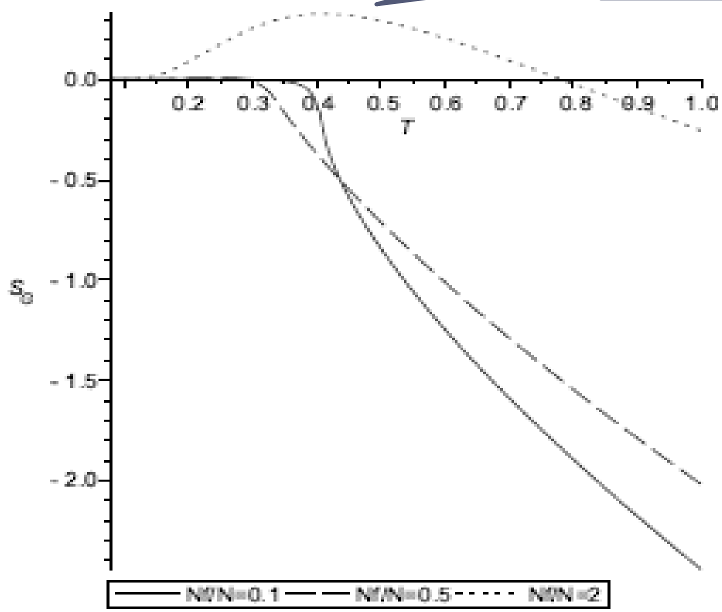
$$f'_B(x) = 4\,\frac{x^{1-\mu}\,(1+x^n)}{(1-x)^2\,(1-x^n)}, \quad f'_F(x) = 8\,\frac{x^{n/2+1}}{(1-x)^2\,(1-x^n)}.$$

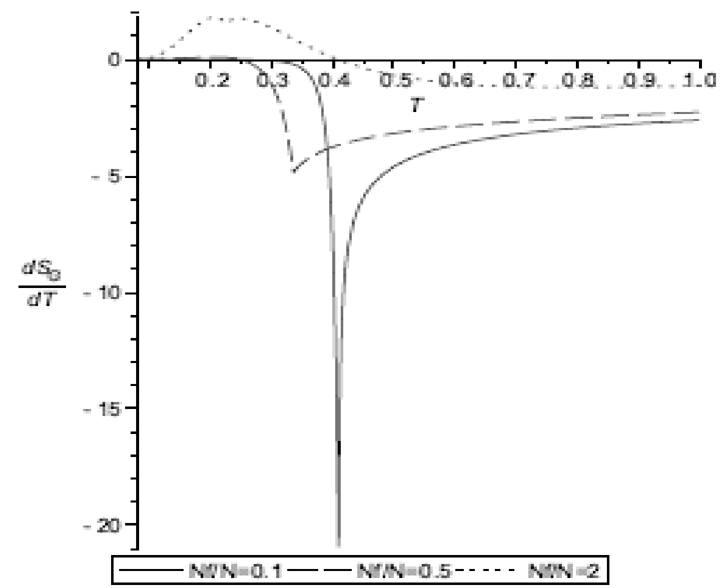
- Stability bounds $\Delta > \mu Q$ ($Q=1$) mean $\mu < 1$

Geometric entropy at finite density

Plots of Geometric entropy S^G and dS^G/dT at μ=1/2

S^G becomes positive value if the temperature is low and Nf is large.

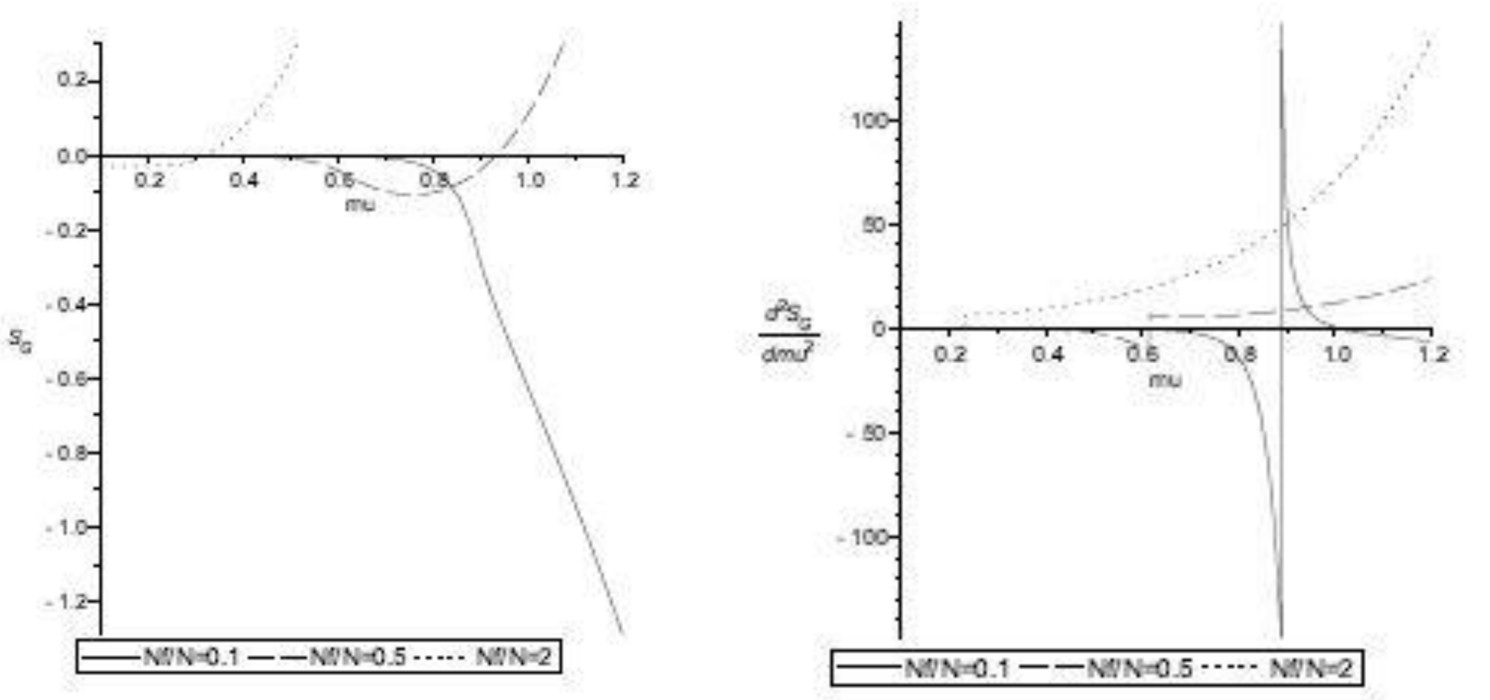




- Geometric entropy can capture the third order phase transition.

Geometric entropy at finite density

- Plots of geometric entropy at *T=0.3* as a function of μ



Mass deformation for flavor

- single particle partition functions of bosons/fermions must be replaced by

$$f_B(x) = \sum_{l=0}^{\infty} l^2 e^{-\sqrt{l^2+m^2}/T}, \quad f_F(x) = \sum_{l=0}^{\infty} l(l+1) e^{-\sqrt{(l+1/2)^2+m^2}/T}$$

- An useful expression, Abel-Plana formula

$$\sum_{l=0}^{\infty} f(l) = \frac{1}{2} f(0) + \int_0^{\infty} f(x)dx + i\int_0^{\infty} \frac{f(it) - f(-it)}{e^{2\pi t} - 1} dt$$

then,

$$f_B(x) = m^2 T K_2(m/T) \ldots,$$

$$f_F(x) = m^2 T K_2(m/T) - \frac{m}{4} K_1(m/T) \ldots$$

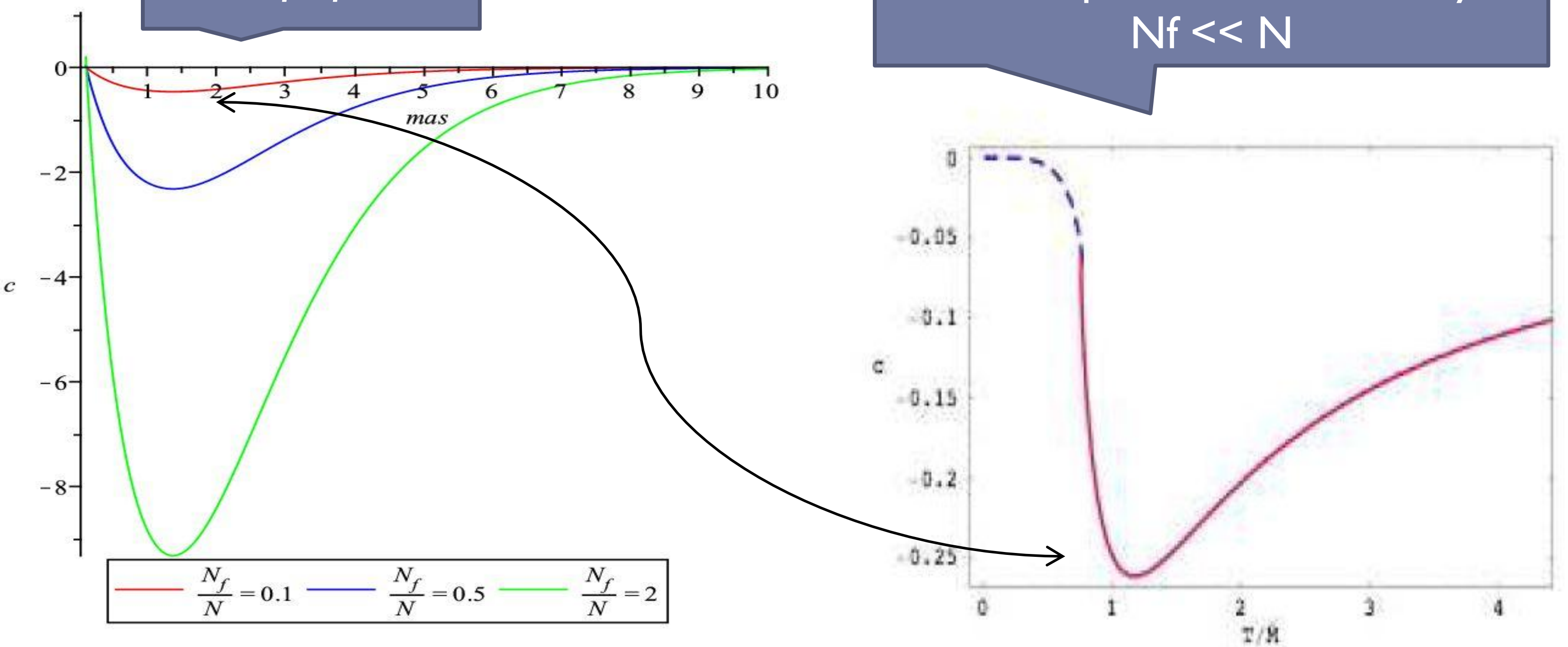
Chiral condensate for N=2 SYM with flavor

$$\left\langle \bar{\psi}\psi \right\rangle_{1-loop}(m) = c = \frac{\partial}{\partial m}\log Z(m) = -\frac{\partial}{\partial m}\beta F$$

The chiral symmetry is restored in the massless limit or in $m \to \infty$ limit ($T \to 0$ limit)

T=1

Reslut of probe D7brane analysis
Nf << N



Conclusion

- We analyzed the phase structure $d=4$ large N QCD like theory using the geometric entropy.
- We can capture the deconfinement phase transition as the third order phase transition using the geometric entropy.
- We enlarged our analysis for the finite density system. For the case $2N \sim N_F$, geometric entropy has interesting behavior.