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Yangian symmetry in deformed WZNW models on squashed spheres

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Based on

I. Kawaguchi, D. Orlando and K.Y., arXiv: 1104.0738.

I. Kawaguchi and K.Y., JHEP 1011 (2010) 032 [arXiv:1008.0776]

Introductory part

Introduction

One of the most studied subjects in string theory.

→ **AdS/CFT correspondence**

= duality between string (gravity) on AdS space and CFT

No rigorous proof but enormous amount of evidence support this conjecture.

Two topics in AdS/CFT (related to my talk)

1. Integrability in AdS/CFT

2. Applications of AdS/CFT to condensed matter physics

1. Integrability in AdS/CFT

AdS/CFT correspondence

IIB string on $AdS_5 \times S^5$ ⟷ 4D N=4 SYM theory [Maldacena,1997]

The current understanding

Integrable spin chain [Beisert-Staudacher, …]

$\lambda \gg 1$ ↙ ↘ $\lambda \ll 1$

IIB string on $AdS_5 \times S^5$ ⟷ 4D N=4 SYM theory

[Frolov-Tseytlin, Bena-Polchinski-Roiban, …] [Minahan-Zarembo, BMSZ, …]

Integrable structure of spin chain ⟶ Spacetime structure of $AdS_5 \times S^5$

$\parallel$

symmetric coset

What is a symmetric coset?

Consider a coset $M = G/H$.

Lie algebra of *H*

Ortho-complement space

Then $\hat{g}$: Lie algebra of *G* $\xrightarrow{\text{decompose}}$ $\hat{g} = \hat{h} \oplus \hat{m}$

M is called symmetric coset, when $\hat{h}$ and $\hat{m}$ satisfy the following relations:

$$\boxed{[\hat{h}, \hat{h}] \subset \hat{h} \ , \quad [\hat{h}, \hat{m}] \subset \hat{m} \ , \quad [\hat{m}, \hat{m}] \subset \hat{h}}$$

AdS background

$$\mathrm{AdS}_5 \times \mathrm{S}^5 = \frac{SO(2,4)}{SO(1,4)} \times \frac{SO(6)}{SO(5)}$$: symmetric coset

G = SO(2,4) x SO(6) (global isometry), *H* = SO(1,4) x SO(5) (local Lorentz)

FACT

2D NLSM is classically integrable, if its target space is given by symmetric coset.

EX. O(3)-inv. NLSM, SU(2) principal chiral model [Lüscher-Pohlmeyer, 1978]

Symmetric coset $\Rightarrow$ classical integrability

= an infinite number of conserved charges

$\Rightarrow$ Yangian symmetry (infinite dim.)

Classification of symmetric cosets applicable to AdS/CFT [K. Zarembo, 1003.0465]

(based on the grading-property, vanishing beta-function, $c \leq 26$)

In particular $c = 26 \Rightarrow \mathrm{AdS}_5 \times \mathrm{S}^5, \ \mathrm{AdS}_4 \times CP^3$

- Integrability plays an important role in AdS/CFT

2. Applications of AdS/CFT to condensed matter physics

Let us consider the gravitational description of condensed matter systems

by assuming the AdS/CFT correspondence.

"Holographic condensed matter physics" (often called AdS/CMP)

e.g. Entanglement entropy, superconductor, (non) Fermi liquid, etc.

Most of condensed matter systems are non-relativistic.

→ Motivation to consider non-relativistic field theories in AdS/CMP

EX. Schrödinger systems, Lifshitz field theories

→ Non-relativistic AdS/CFT

The gravitational background is modified from AdS

EX. Schrödinger spacetime, Lifshitz spacetime

[Son, Balasubramanian-McGreevy] [Kachru-Liu-Mulligan]

Sch. $$ds^2 = r^2\left[-2dx^+dx^- + (dx^i)^2\right] + \frac{dr^2}{r^2} \underline{- r^{2z}(dx^+)^2}$$

Lif. $$ds^2 = \underline{-r^{2z}dt^2} + r^2(dx^i)^2 + \frac{dr^2}{r^2}$$

z: const.

NOTE: these backgrounds are described by non-symmetric cosets

[S.Schafer-Nameki, M. Yamazaki, K.Y., 0903.4245]

There is a motive to consider non-symmetric cosets in AdS/CMP.

No one knows whether the NLSM on them are integrable or not, at least so far.

(Probably, it should be a difficult task)

Motivated by these topics,

Our aim Find out some examples of non-symmetric and integrable backgrounds

$\boxed{\text{squashed spheres, warped AdS spaces (3 dim.)}}$

We discuss an infinite-dimensional symmetry of NLSMs on squashed spheres.

Summary of my talk

1. Yangian symmetry is realized for the squashed S^3 .

2. This is the case even after the Wess-Zumino term has been added.

3. RG flow of the squashed WZNW model

IR fixed point is the same as the SU(2) WZW model

Technical part

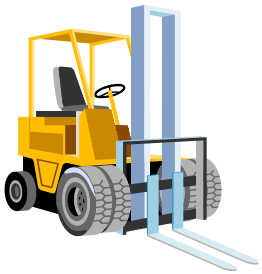
Plan of my talk

1. BIZZ construction

2. Yangian symmetry in squashed sigma model

3. Squashed WZNW model

4. Summary and Discussions



1. BIZZ construction

Let us consider a 2D NLSM on *G* or *M=G/H* (not necessarily symmetric)

FACT

If a 2D NLSM has a conserved current j_μ satisfying the flatness condition,

$$\epsilon^{\mu\nu}(\partial_\mu j_\nu - j_\mu j_\nu) = 0$$

then an infinite number of conserved non-local charges can be constructed and the NLSM is classically integrable.

There are some methods, one of which is the BIZZ construction.

[Brezin,Itzykson,Zinn-Justin,Zuber,1979]

KEY INGREDIENT: flat conserved current

(= a conserved current satisfying the flatness condition)

FACT2: There exists always a flat conserved current, if the target space is *G* itself or *M* is a symmetric coset.

BIZZ construction

[Brezin, Itzykson, Zinn-Justin, Zuber, 1979]

Assume that we have a flat conserved current j_μ

Let's introduce the covariant derivative: $\boxed{D_\mu = \partial_\mu - j_\mu}$

satisfies:

$$\boxed{\begin{array}{c} \partial^\mu D_\mu = D_\mu \partial^\mu \\ \epsilon^{\mu\nu} D_\mu D_\nu = 0 \end{array}} \begin{array}{l} \longleftarrow \quad \partial^\mu j_\mu = 0 \\ \longleftarrow \quad \epsilon^{\mu\nu}(\partial_\mu j_\nu - j_\mu j_\nu) = 0 \end{array}$$

With the covariant derivative, one can construct an infinite number of non-local charges recursively.

NOTE If there is a flat conserved current, then *M* is not needed to be symmetric.

Let's take the Noether current as the 0th current :

$$J_{(0)\mu} = j_\mu = D_\mu \chi_{(0)} \longrightarrow \partial^\mu J_{(0)\mu} = 0$$ Conserved by definition.

$$(\chi_{(0)} = -1)$$

$$\Longrightarrow \quad J_{(0)\mu} = \epsilon_{\mu\nu} \partial^\nu \chi_{(1)}$$

$$\epsilon(x-y) \equiv \theta(x-y) - \theta(y-x)$$

$$\chi_{(1)}(x) = \frac{1}{2} \int dy\, \epsilon(x-y) J_{(0)t}(y)$$

Then the next current is defined as $J_{(1)\mu} \equiv D_\mu \chi_{(1)}$:conserved

---

$$(\because) \quad \partial^\mu J_{(1)\mu} = \partial^\mu D_\mu \chi_{(1)} = D_\mu \partial^\mu \chi_{(1)} = \epsilon^{\mu\nu} D_\mu J_{(0)\nu}$$
$$= \epsilon^{\mu\nu} D_\mu D_\nu \chi_{(0)} = 0$$

Repeat the same step $\Longrightarrow$ Infinite number of non-local charges

Non-local charges: $$Q_{(n)} = \int dx \, J_{(n)t}(x)$$

Explicit expressions of the charges: $$Q_{(n)} = Q^A_{(n)} T^A$$

0-th $$Q^A_{(0)} = \int dx \, j^A_t(x)$$

$$\epsilon(x-y) \equiv \theta(x-y) - \theta(y-x)$$

1-st $$Q^A_{(1)} = \int dx \, j^A_x(x) + \frac{1}{4} \int\int \underbrace{dx dy \, \epsilon(x-y) f_{BC}{}^A j^B_t(x) j^C_t(y)}_{\text{Non-local}}$$

$\vdots$

where T_A 's are the generators of G : $[T_A, T_B] = f_{AB}{}^C T_C$

What is the algebra that the charges satisfy ?

Current algebra

fixed by the classical action

$$\{j_t^A(x), j_t^B(y)\}_{\mathrm{P}} = f^{AB}{}_C\, j_t^C(x)\delta(x-y)$$

$$\{j_t^A(x), j_x^B(y)\}_{\mathrm{P}} = f^{AB}{}_C\, j_x^C(x)\delta(x-y) + \delta^{AB}\partial_x\delta(x-y)$$

$$\{j_x^A(x), j_x^B(y)\}_{\mathrm{P}} = 0$$

[a typical form, e.g. O(3)-inv. NLSM]

Yangian algebra

[Drinfel'd,1988]

$$\{Q^A_{(0)}, Q^B_{(0)}\}_{\mathrm{P}} = f^{AB}{}_C\, Q^C_{(0)}$$

$$\{Q^A_{(0)}, Q^B_{(1)}\}_{\mathrm{P}} = f^{AB}{}_C\, Q^C_{(1)}$$

\+ Serre relations

NOTE: Yangian is generated by $Q_{(0)}$ and $Q_{(1)}$.

Serre relations

S1. $$\{Q^A_{(1)}, \{Q^B_{(1)}, Q^C_{(0)}\}_{\mathrm{P}}\}_{\mathrm{P}} + \text{cyclic} = \alpha^{ABC}{}_{DEF}\{Q^D_{(0)}, Q^E_{(0)}, Q^F_{(0)}\}$$

only for A,B,C

S2. $$\{\{Q^A_{(1)}, Q^B_{(1)}\}_{\mathrm{P}}, \{Q^C_{(0)}, Q^D_{(1)}\}_{\mathrm{P}}\}_{\mathrm{P}} + (A,B) \leftrightarrow (C,D)$$
$$= (\alpha^{ABH}{}_{EFG} f^{CD}{}_{H} + (A,B) \leftrightarrow (C,D))\{Q^E_{(0)}, Q^F_{(0)}, Q^G_{(1)}\}$$

where $$\alpha^{ABC}{}_{DEF} \equiv \frac{1}{24} f^{AI}{}_{D} f^{BJ}{}_{E} f^{CK}{}_{F} f_{IJK}$$

$\{A, B, C\}$:symmetrized product

2. Yangian symmetry in squashed sigma model

I. Kawaguchi and K.Y., JHEP 1011 (2010) 032, [arXiv:1008.0776]

Round S^3 with the radius L

$$ds^2 = \frac{L^2}{4}[\underbrace{d\theta^2 + \cos^2\theta d\phi^2}_{S^2} + \underbrace{(d\psi + \sin\theta d\phi)^2}_{S^1\text{-fibration}}]$$

3 angles

(θ, ϕ, ψ)

Isometry: $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$

a deformation of the round S^3

Squashed S^3

$$ds^2 = \frac{L^2}{4}[d\theta^2 + \cos^2\theta d\phi^2 + (1 + \underline{C})(d\psi + \sin\theta d\phi)^2]$$

squashing parameter

Isometry: $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{R}}$

Warped AdS_3 $=$ a double Wick rotation of squashed S^3

$$S^3 \to AdS_3\,, \quad SU(2) \to SL(2,R)$$

1) space-like warped AdS_3 : $\theta \to i\sigma,\ \phi \to iu,\ \psi \to \tau$

$$ds^2 = \frac{L^2}{4}[-\cosh^2\sigma d\tau^2 + d\sigma^2 + (1+C)(du + \sinh\sigma d\tau)^2]$$

2) time-like warped AdS_3 : $\theta \to i\sigma,\ \phi \to \tau,\ \psi \to iu$

$$ds^2 = \frac{L^2}{4}[-(1+C)(d\tau - \sinh\sigma du)^2] + d\sigma^2 + \cosh^2\sigma du^2$$

The difference between warped AdS_3 and squashed S^3 is just signature at least at classical level.

Hereafter we will consider the case of squashed spheres only.

Group element representation of squashed sphere

Let us introduce the *SU(2)* group element: $g = e^{\phi T_1} e^{\theta T_2} e^{\psi T_3} \in SU(2)$

Here θ, ϕ, ψ are the angles of S^3 and T_A's are the SU(2) generators:

$$[T_A, T_B] = \varepsilon_{AB}{}^{C} T_C \quad , \qquad \mathrm{Tr}(T_A T_B) = -\frac{1}{2}\delta_{AB}$$

Then the left-invariant 1-form is expanded as

$$J = g^{-1}dg = J^1 T_1 + J^2 T_2 + J^3 T_3$$

Finally the metric of squashed S^3 is rewritten as

$$\begin{aligned} ds^2 &= \frac{L^2}{4}[(J^1)^2 + (J^2)^2 + (1+C)(J^3)^2] \\ &= -\frac{L^2}{2}\left[\mathrm{Tr}[(J)^2] - 2C(\mathrm{Tr}(JT_3))^2\right] \end{aligned}$$

Sigma model action on squashed S^3

$$S = \frac{1}{\lambda^2}\int dt dx \left[\mathrm{Tr}(J_\mu J^\mu) - 2C\,\mathrm{Tr}(T_3 J_\mu)\mathrm{Tr}(T_3 J^\mu)\right]$$

coupling const.

$x^\mu = (t, x), \quad \eta_{\mu\nu} = (-1, 1)$: 2D Minkowski spacetime

NOTE We will not consider the Virasoro conditions

Global Symmetry: $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{R}}$

SU(2)$_{\mathrm{L}}$ Noether current: $j_\mu = \partial_\mu g\, g^{-1} - 2C\,\mathrm{Tr}(T_3 J_\mu) g T_3 g^{-1}$

Conserved ⬅ equation of motion

Remember

BIZZ construction

If the conserved current j_μ satisfies the flatness condition,

$$\epsilon^{\mu\nu}(\partial_\mu j_\nu - j_\mu j_\nu) = 0$$

then an infinite number of conserved non-local charges can be constructed.

Check the flatness for the *SU(2)*$_L$ current :

$$\epsilon^{\mu\nu}(\partial_\mu j_\nu - j_\mu j_\nu) = \underline{-C\epsilon^{\mu\nu}\partial_\mu(gT_3g^{-1})\partial_\nu(gT_3g^{-1})}$$

$$\Downarrow$$

Non-vanishing, because $C \neq 0$. But total derivative!

Current improvement

$$j_\mu^{\rm imp} = j_\mu + A\,\underbrace{\epsilon_{\mu\nu}\partial^\nu(gT_3g^{-1})}_{}$$

Improvement term

const.

Ambiguity to add a topological term

New flatness condition:

$$\epsilon^{\mu\nu}(\partial_\mu j_\nu^{\rm imp} - j_\mu^{\rm imp} j_\nu^{\rm imp}) = (A^2 - C)\epsilon^{\mu\nu}\partial_\mu(gT_3g^{-1})\partial_\nu(gT_3g^{-1})$$

If we take $\boxed{A = \pm\sqrt{C}}$, (assume $C \geq 0$)

then the improved current satisfies the flatness condition.

BIZZ

An infinite number of non-local charges (straightforward)

Current algebra

with $A^2 = C$ (The symbol "imp" is omitted below)

$$\{j_t^A(x), j_t^B(y)\}_{\mathrm{P}} = \varepsilon^{AB}{}_C j_t^C(x)\delta(x-y)$$

$$\{j_t^A(x), j_x^B(y)\}_{\mathrm{P}} = \varepsilon^{AB}{}_C j_x^C(x)\delta(x-y) + \underline{(1+C)}\delta^{AB}\partial_x\delta(x-y)$$

$$\{j_x^A(x), j_x^B(y)\}_{\mathrm{P}} = \underline{-C}\,\varepsilon^{AB}{}_C j_t^C(x)\delta(x-y)$$

The current algebra is deformed due to the improvement.

→ Is Yangian algebra still realized?

(non-trivial question)

$SU(2)_L$ Yangian algebra

$$\{Q^A_{(0)}, Q^B_{(0)}\}_{\rm P} = \varepsilon^{AB}{}_C Q^C_{(0)}$$

Non-trivial modification only for this part.

$$\{Q^A_{(0)}, Q^B_{(1)}\}_{\rm P} = \varepsilon^{AB}{}_C Q^C_{(1)}$$

$$\{Q^A_{(1)}, Q^B_{(1)}\}_{\rm P} = \varepsilon^{AB}{}_C [Q^C_{(2)} + \frac{1}{12} Q^C_{(0)} Q^D_{(0)} Q_{(0)D} - \underline{C Q^C_{(0)}}]$$

Serre relations are also satisfied, although the current algebra is modified.

In summary,

Yangian algebra is realized even after S^3 has been squashed.

Classical *r*-matrix?

Yangian ➡ rational type of *r*-matrix (naïve expectation)

Lax pair: $L_\mu(\lambda) \equiv \frac{\lambda}{\lambda^2-1}\left[\lambda j_\mu + \epsilon_{\mu\nu} j^\nu\right]$ λ : spectral parameter

$$[\partial_t - L_t, \partial_x - L_x] = 0$$

due to e.o.m. and flat condition

<u>Poisson bracket</u>

(up to non-ultra local term)

$$\{L_x^A(x;\lambda), L_x^B(y;\mu)\}_\mathrm{P}$$

$$= \frac{\lambda\mu}{\lambda-\mu}\varepsilon^{AB}{}_C\left[\frac{1}{\mu^2-1}L_x^C(x;\lambda) - \frac{1}{\lambda^2-1}L_x^C(x;\mu)\right]\delta(x-y)$$

$$-C\frac{\lambda^2\mu^2}{(\lambda^2-1)(\mu^2-1)}\varepsilon^{AB}{}_C\, \underline{j_t^C(x)}\delta(x-y)$$

because of $\{j_x, j_x\}_\mathrm{P} \neq 0$

Problem: How can we read off the classical r-matrix?

But it's possible in 3D Schrodinger case (Io's talk in lunch seminar)

3. squashed WZNW model

I. Kawaguchi, D. Orlando, K.Y., arXiv: 1104.0738.

A simple generalization is to add the Wess-Zumino term.

→ Squashed Wess-Zumino-Novikov-Witten model (SqWZNW model)

The classical action

$$S_{\mathrm{SqWZNW}} = S_{\sigma\mathrm{M}} + S_{\mathrm{WZ}} \,,$$

$$S_{\sigma\mathrm{M}} = \frac{1}{\lambda^2}\iint dt dx \left[\mathrm{Tr}(J_\mu J^\mu) - 2C\mathrm{Tr}\left(T_3 J_\mu\right)\mathrm{Tr}\left(T_3 J^\mu\right)\right] ,$$

$$S_{\mathrm{WZ}} = \frac{n}{12\pi}\int_0^1 ds \iint dt dx \, \epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}} \mathrm{Tr}\left(J_s^{\hat{\mu}} J_s^{\hat{\nu}} J_s^{\hat{\rho}}\right) \quad , \quad n \in \mathbb{Z} \,.$$

$$J_s \equiv g(x^\mu, s)^{-1} dg(x^\mu, s), \quad g(x^\mu, 1) = g(x^\mu), \quad g(x^\mu, 0) = 1$$

The coefficient is discretized from the consistency to the path integral.

$SU(2)_L$ current:

$$K \equiv \frac{n\lambda^2}{8\pi}$$

$$j_\mu = \partial_\mu g\, g^{-1} - 2C\,\mathrm{Tr}(T_3 J_\mu) g T_3 g^{-1} - K\epsilon_{\mu\nu}\partial^\nu g\, g^{-1}$$

Improved current:

$$j_\mu^{\mathrm{imp}} = j_\mu + A\epsilon_{\mu\nu}\partial^\nu(gT_3g^{-1})$$

Check the flatness condition:

$$\begin{aligned} &\epsilon^{\mu\nu}(\partial_\mu j_\nu^{\mathrm{imp}} - j_\mu^{\mathrm{imp}} j_\nu^{\mathrm{imp}}) \\ = \;& \left[A^2 - C + \frac{CK^2}{1+C}\right]\epsilon^{\mu\nu}\partial_\mu(gT_3g^{-1})\partial_\nu(gT_3g^{-1}) \end{aligned}$$

Flat current condition

$$A = \pm\sqrt{C\left(1 - \frac{K^2}{1+C}\right)}$$

In particular, when

$$K = \pm\sqrt{1+C} \qquad \text{(i.e., } A = 0\text{)}$$

the current improvement is not needed!

Current algebra

with $A^2 = C\left(1 - \dfrac{K^2}{1+C}\right)$

$$
\begin{aligned}
&\{j_t^A(x), j_t^B(y)\}_{\mathrm{P}} \\
&= \varepsilon^{AB}{}_C j_t^C(x)\delta(x-y) \underline{- 2K\delta^{AB}}\partial_x\delta(x-y)
\end{aligned}
$$

$$
\begin{aligned}
&\{j_t^A(x), j_x^B(y)\}_{\mathrm{P}} \\
&= \varepsilon^{AB}{}_C j_x^C(x)\delta(x-y) + \underline{\left(1 + C + \frac{K^2}{1+C}\right)\delta^{AB}}\partial_x\delta(x-y)
\end{aligned}
$$

$$
\begin{aligned}
&\{j_x^A(x), j_x^B(y)\}_{\mathrm{P}} \\
&= -\underline{\left(C + \frac{K^2}{1+C}\right)}\varepsilon^{AB}{}_C j_t^C(x)\delta(x-y) \\
&\quad \underline{- 2K\varepsilon^{AB}{}_C} j_x^C(x)\delta(x-y) \underline{- 2K\delta^{AB}}\partial_x\delta(x-y)
\end{aligned}
$$

The current algebra is fairly modified!

Yangian algebra

$$\{Q^A_{(0)}, Q^B_{(0)}\}_{\mathrm{P}} = \varepsilon^{AB}{}_C Q^C_{(0)}$$

$$\{Q^A_{(0)}, Q^B_{(1)}\}_{\mathrm{P}} = \varepsilon^{AB}{}_C Q^C_{(1)}$$

$$\{Q^A_{(1)}, Q^B_{(1)}\}_{\mathrm{P}} = \varepsilon^{AB}{}_C \left[Q^C_{(2)} + \frac{1}{12} Q^C_{(0)} Q^D_{(0)} Q_{(0)D} \right.$$
$$\left. \underline{+2K Q^C_{(1)} - \left(C + \frac{K^2}{1+C} \right) Q^C_{(0)}} \right]$$

+ Serre relations are also satisfied

Yangian algebra is realized even after adding the WZ term

Renormalization Group (RG) flow

Renormalized coupling and squashing parameter at 1-loop level

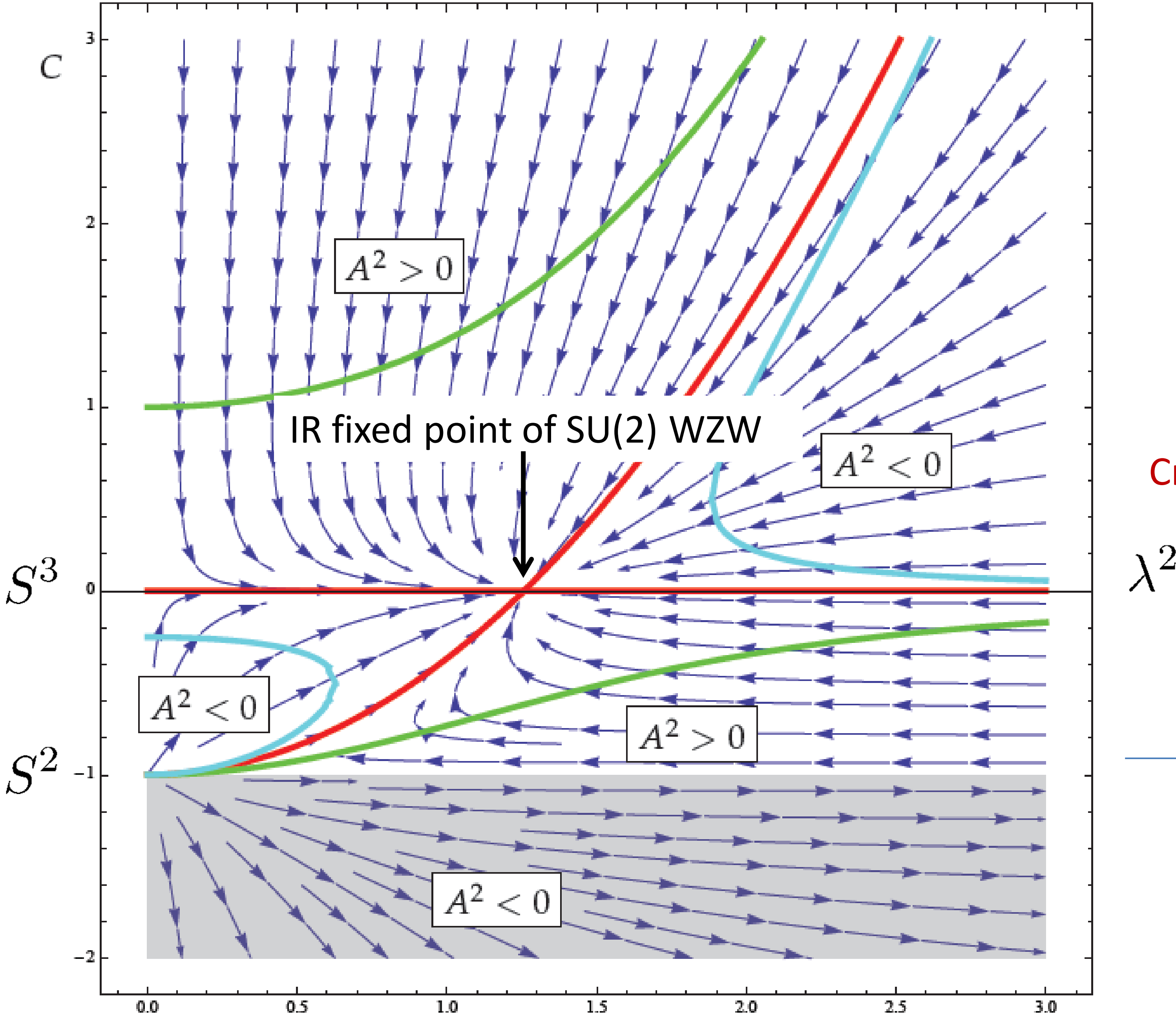
$$
\begin{cases}
\lambda_{\mathrm{R}}^2 = \lambda^2 + \dfrac{\lambda^4}{8\pi}\left(1 - C - \dfrac{1}{1+C}\left(\dfrac{\lambda^2 n}{8\pi}\right)^2\right)\log\dfrac{\Lambda^2}{\mu^2}\,, \\
C_{\mathrm{R}} = C - \dfrac{\lambda^2}{4\pi}C(1+C)\log\dfrac{\Lambda^2}{\mu^2}\,,
\end{cases}
$$

$\Lambda :$ UV cut-off

$\mu :$ IR cut-off

1-loop beta functions:

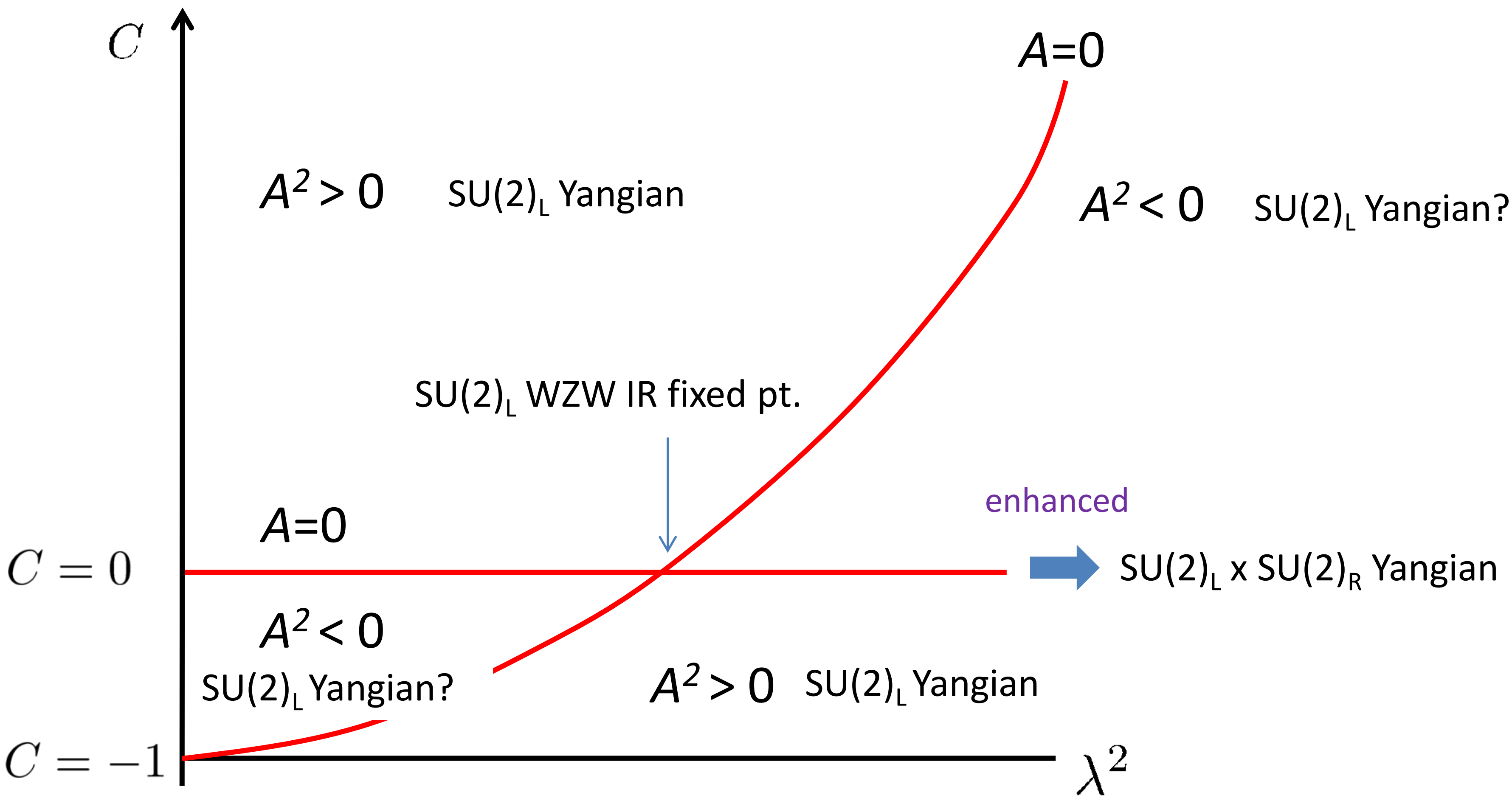
$$
\begin{cases}
\mu\dfrac{\partial\lambda_{\mathrm{R}}^2}{\partial\mu} = -\dfrac{\lambda_{\mathrm{R}}^4}{4\pi}\left\{1 - C_{\mathrm{R}} - \dfrac{1}{1+C_{\mathrm{R}}}\left(\dfrac{\lambda_{\mathrm{R}}^2 n}{8\pi}\right)^2\right\} \\
\mu\dfrac{\partial C_{\mathrm{R}}}{\partial\mu} = \dfrac{\lambda_{\mathrm{R}}^2}{2\pi}C_{\mathrm{R}}\,(1+C_{\mathrm{R}})
\end{cases}
$$



n=20

Critical surface

The relation between RG flow and current improvement



On the red line, a flat conserved current is obtained without improvement.

A^2<0 is formally possible → SU(2)$_L$ Yangian?

4. Summary & Discussions

Summary

- ✓ Yangian symmetry is realized for the squashed S^3 .
- ✓ This is the case even after the Wess-Zumino term has been added.
- ✓ RG flow of the squashed WZNW model

  ⇒ IR fixed point is the same as the SU(2) WZW model

Discussions

- Quantum non-local charges?

  ⇒ Check whether anomaly is forbidden from the coset structure.

  [Goldschmidt-Witten]

- String theory embedding?

  ⇒ An exact marginal deformation of heterotic string background

  [Israel, Kounnas, Orlando, Petropoulos, hep-th/0405213]

- How about the SU(2)R symmetry? [Orlando-Reffert-Uruchurtu, 1011.1771]

Thank you!