

# ABCD of 4d/2d Correspondence

Aug 30, 2011 IPMU

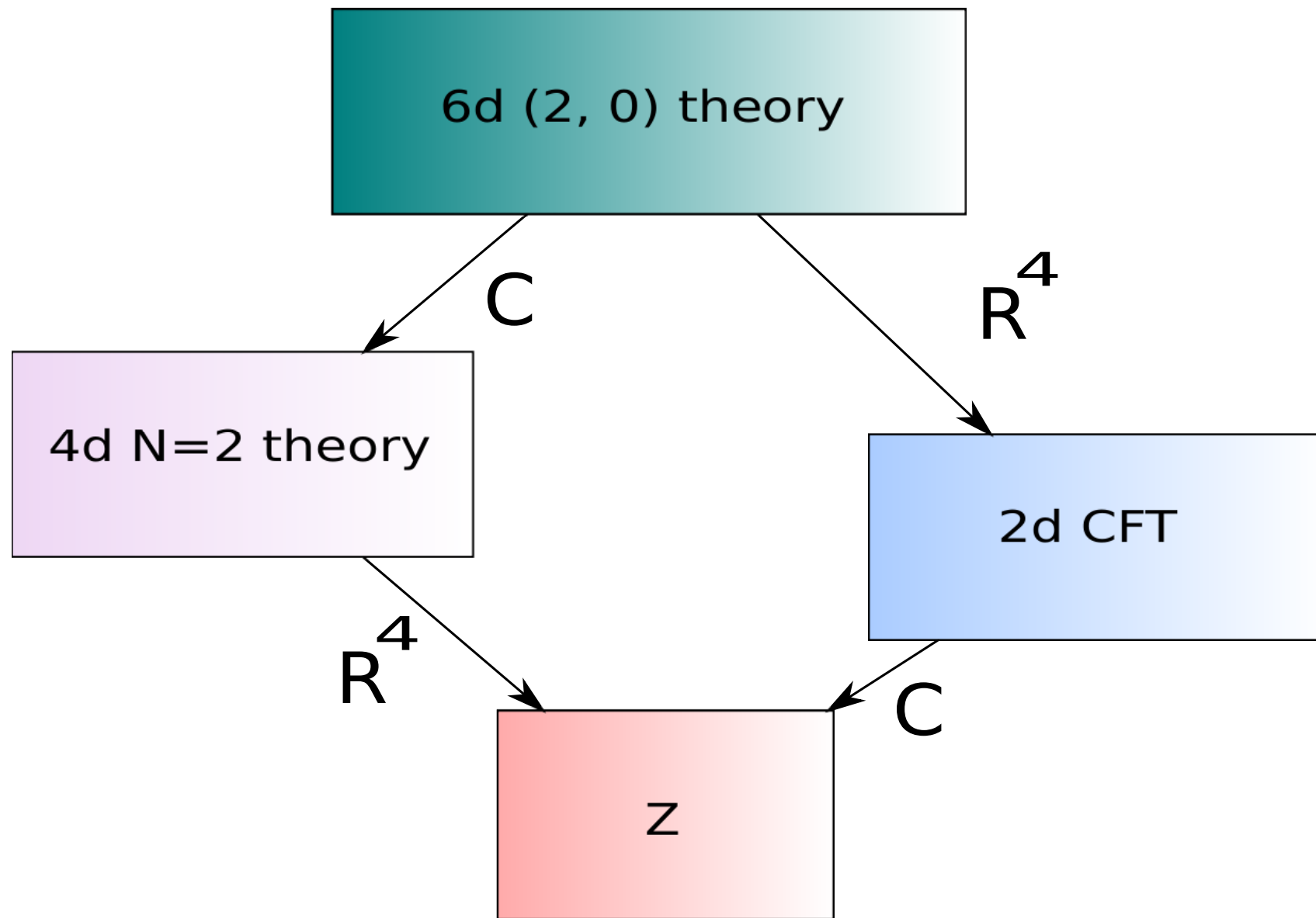
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based on arXiv:1012.4468, 1107.0973  
with L. Hollands and C. Keller

# AGT correspondence

- $Z_{4d}(\mathcal{C}, a, m, \epsilon) = Z_{2d}(\mathcal{C}, h_a, h_m, c)$
- 4d: N=2 Generalized quiver gauge theories  
T(C) of  $A_N$  type
- 2d: Liouville/Toda theory on C
- Nekrasov Partition function on  $\mathbb{R}_{\epsilon_1, \epsilon_2}^4$  or  $S^4$   
= Correlation function
- Chiral/Full

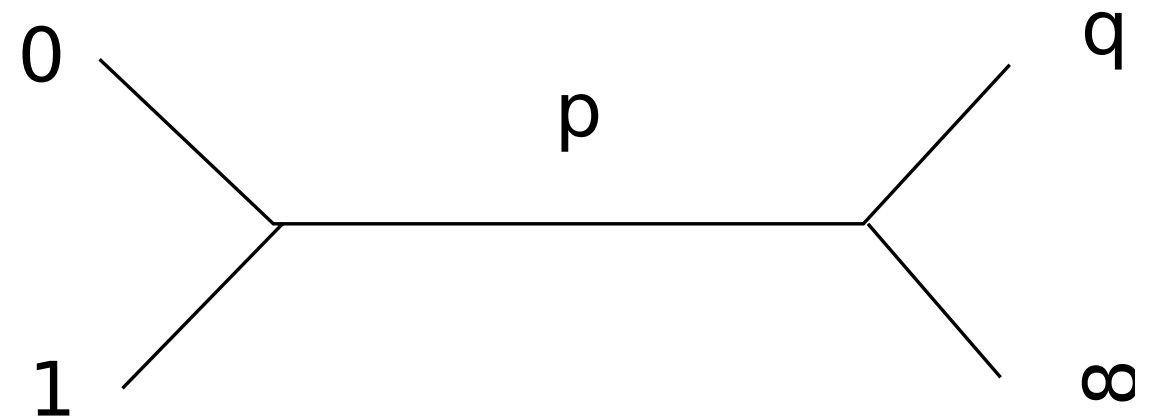
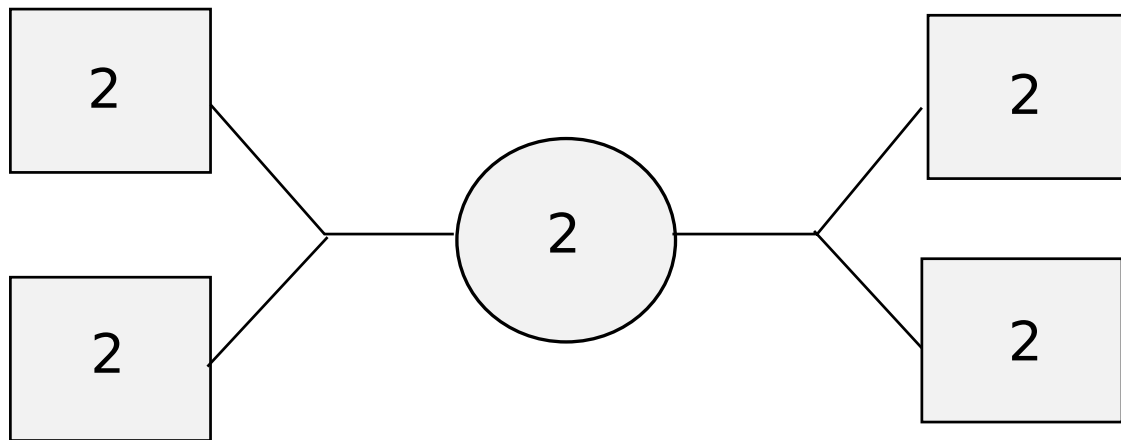
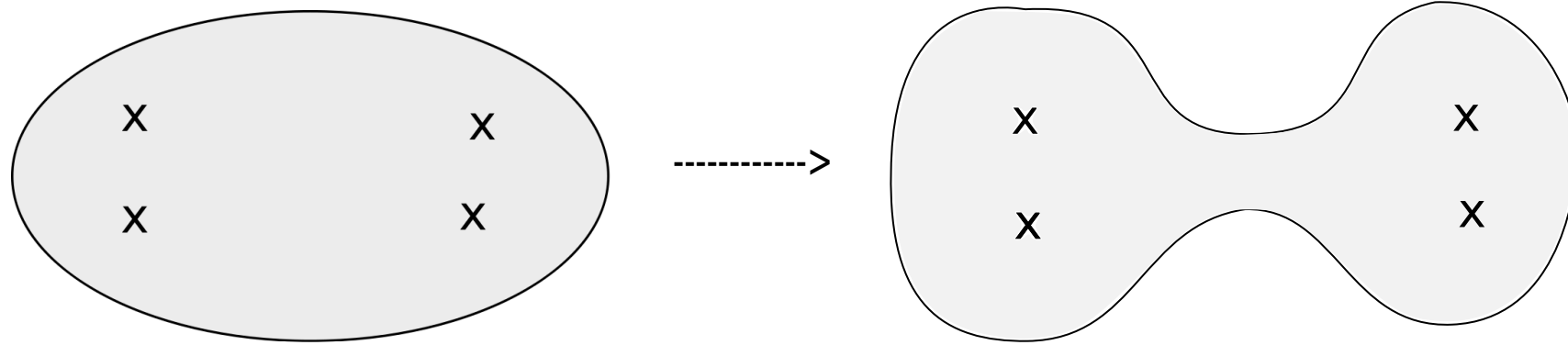
# Why?



# The theories of class $S$

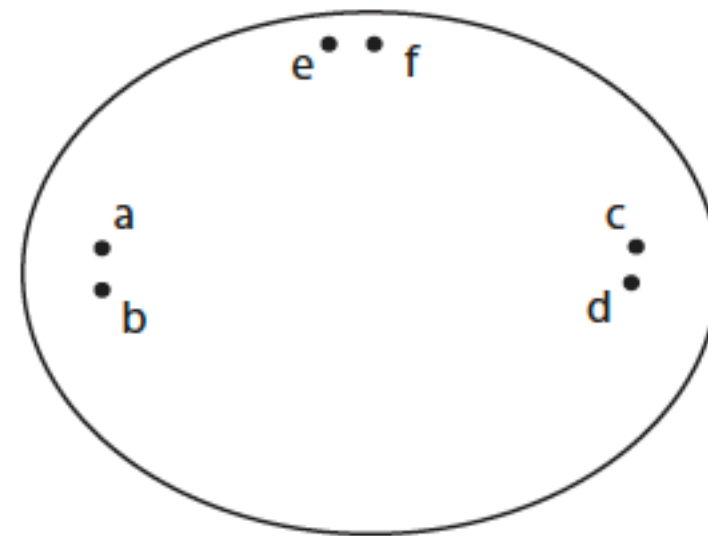
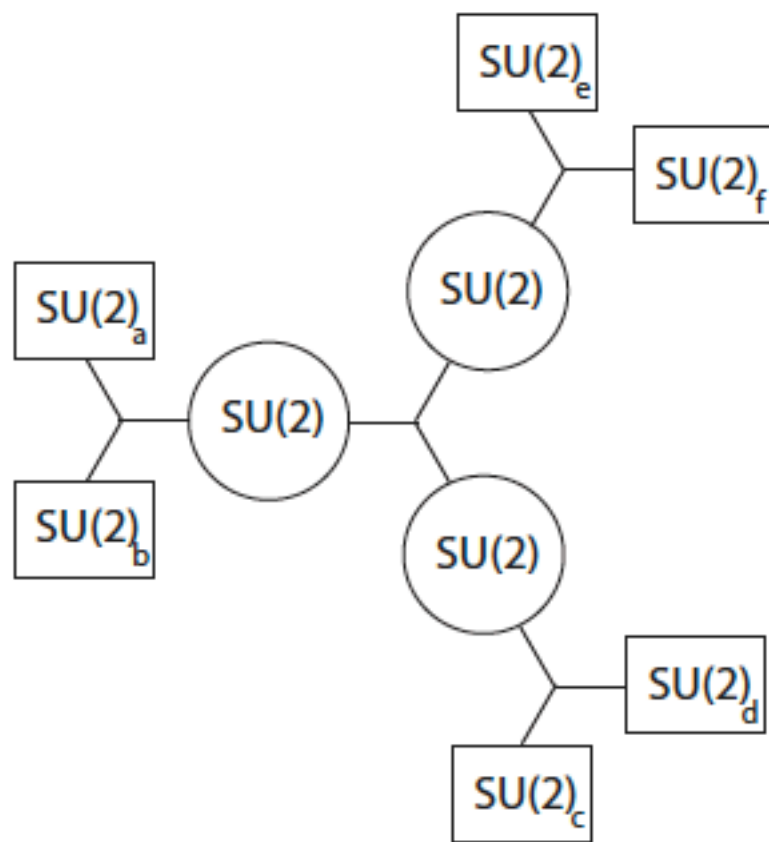
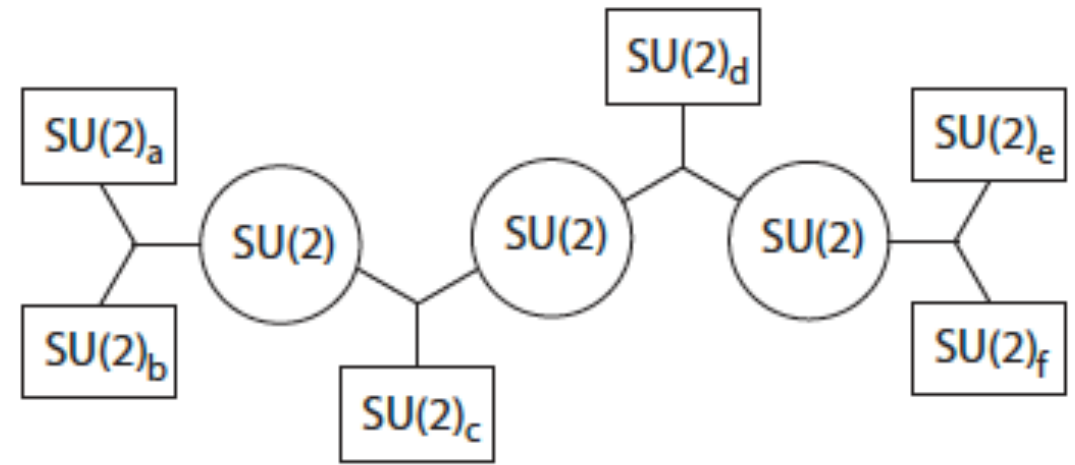
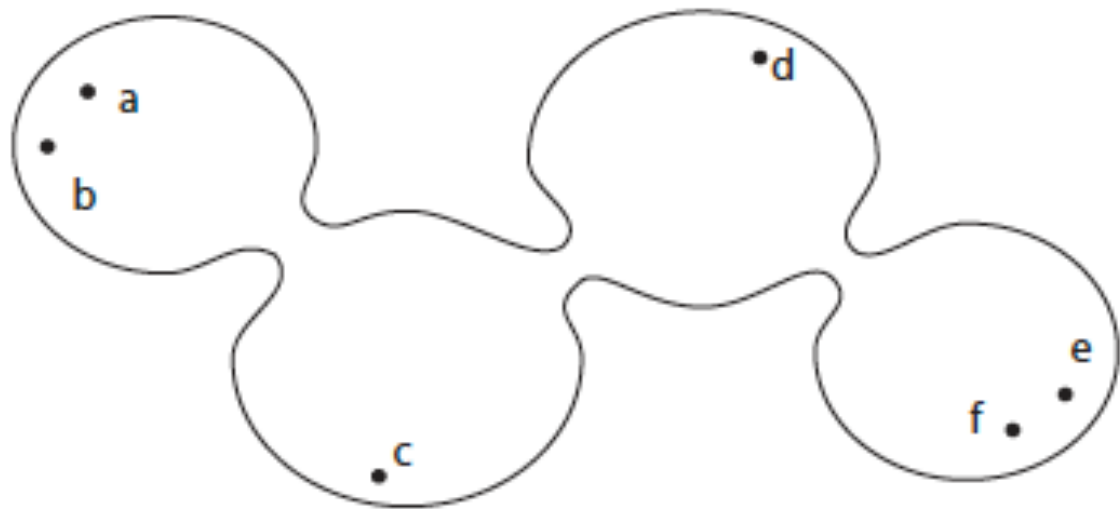
- Pick a 'gauge' group  $G=A, D, E$ .
- Riemann surface  $C$ , possibly with punctures.
- Punctures are decorated according to the choice of  $G$ .
- A pair of pants decomposition gives a weakly coupled description of the theory.

# Riemann surface to quiver/conformal block



# Generalizing AGT

- $A_N$  ( $N > 1$ ) theories [Mironov-Morozov<sup>2</sup>], [Wyllard]
- Non-conformal cases [Gaiotto], [Taki]
- Insertion of non-local BPS operators [AGGTV], [DGG], ...
- $D_N$  theories [Hollands-Keller-JS]
- $A_1$  Sicilian theories [Hollands-Keller-JS]



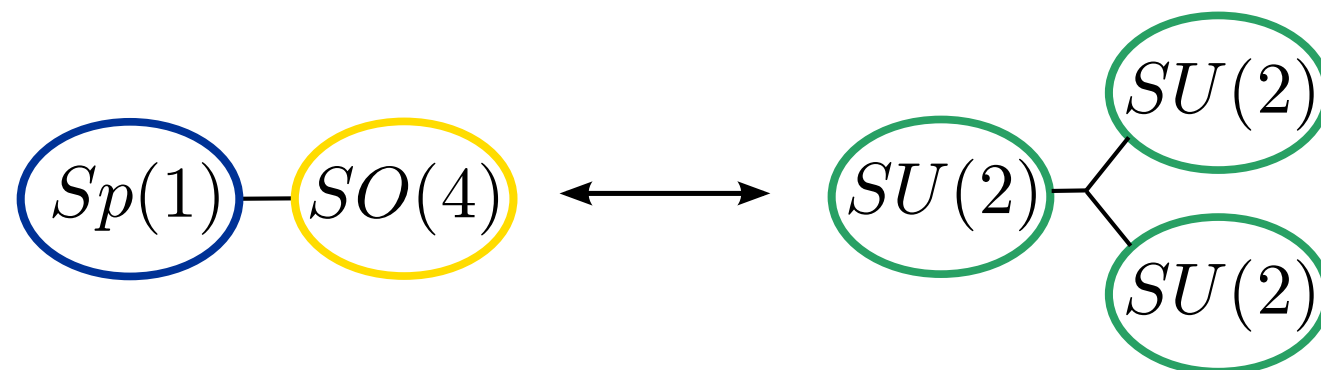
# Trifundamental (T2)

- It's a Half-hypermultiplet.
- Same matter content as a  $N=1$  chiral multiplet
- Has to be massless
- Consistent only if it's in a pseudo-real representation of the gauge group

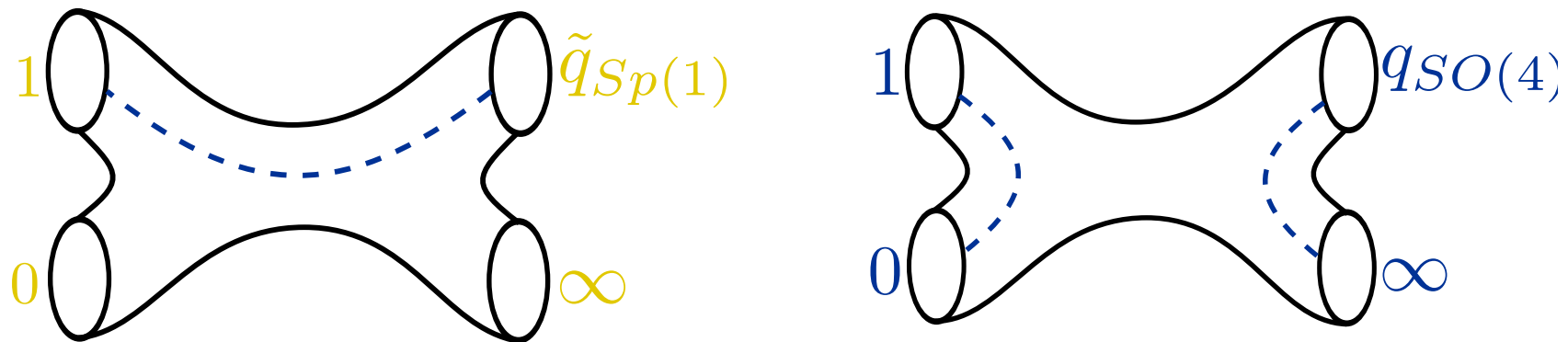


# The three roads to trifundamentals

- $U(2)^3$  : Not pseudo-real. Can't use it.
- $Sp(1)^3$  : No direct G-curve realization
- $Sp(1) \times SO(4)$ : Can't probe full parameter space



# $D_N$ -Theories of class S



- $G=SO(2N)$  theories
- Can also realize  $G=Sp(N-1), SO(2N-1)$
- $SO$ - $Sp$  quivers
- $D_2: SO(4), Sp(1), SO(3)$

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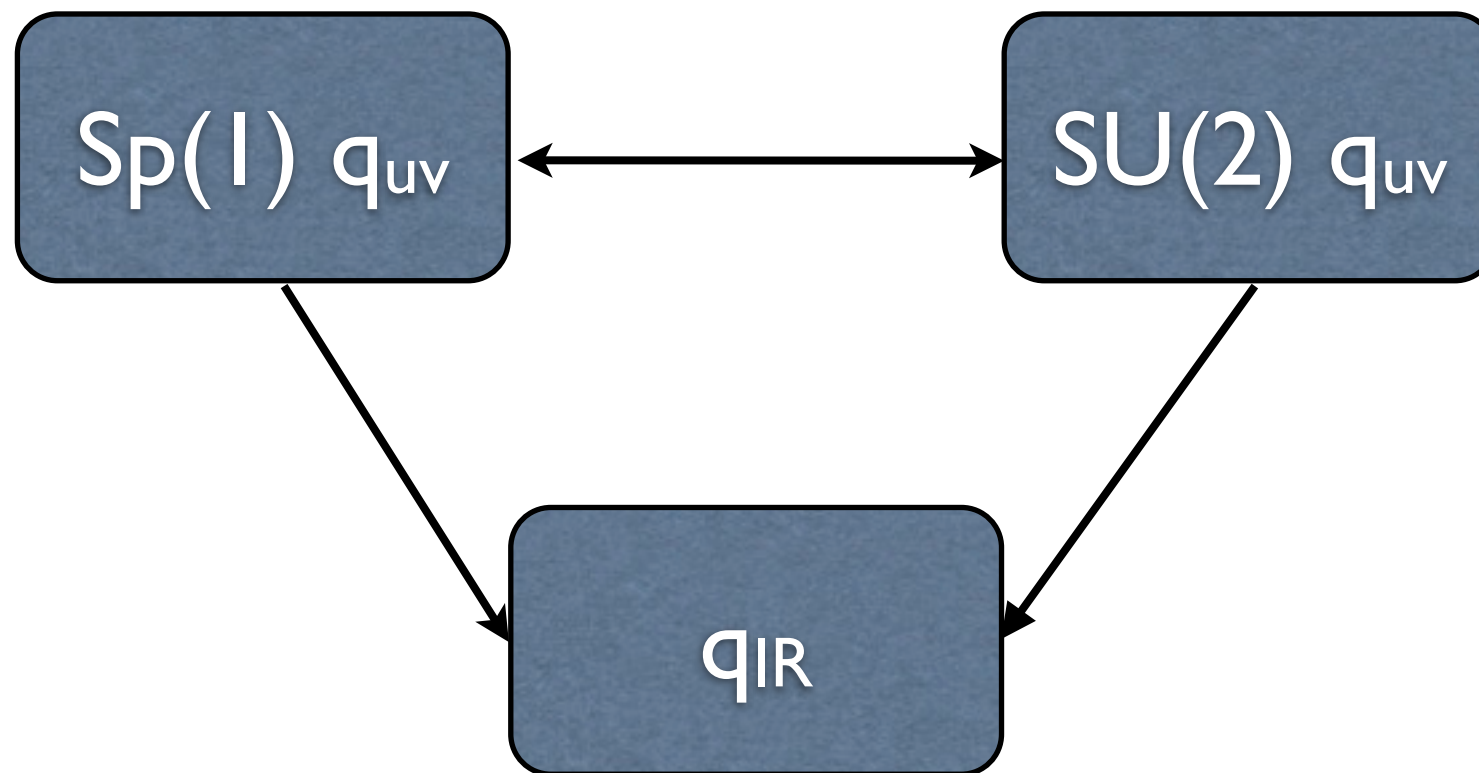
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- Instanton counting with  $Sp$ - $SO$ ,  $Sp(l)^3$  Half-hypermultiplets
- $Z_2$ -automorphism of  $W(D_n)$ -algebra
- Asymmetry of the conformal block for the trifundamental

# Summary

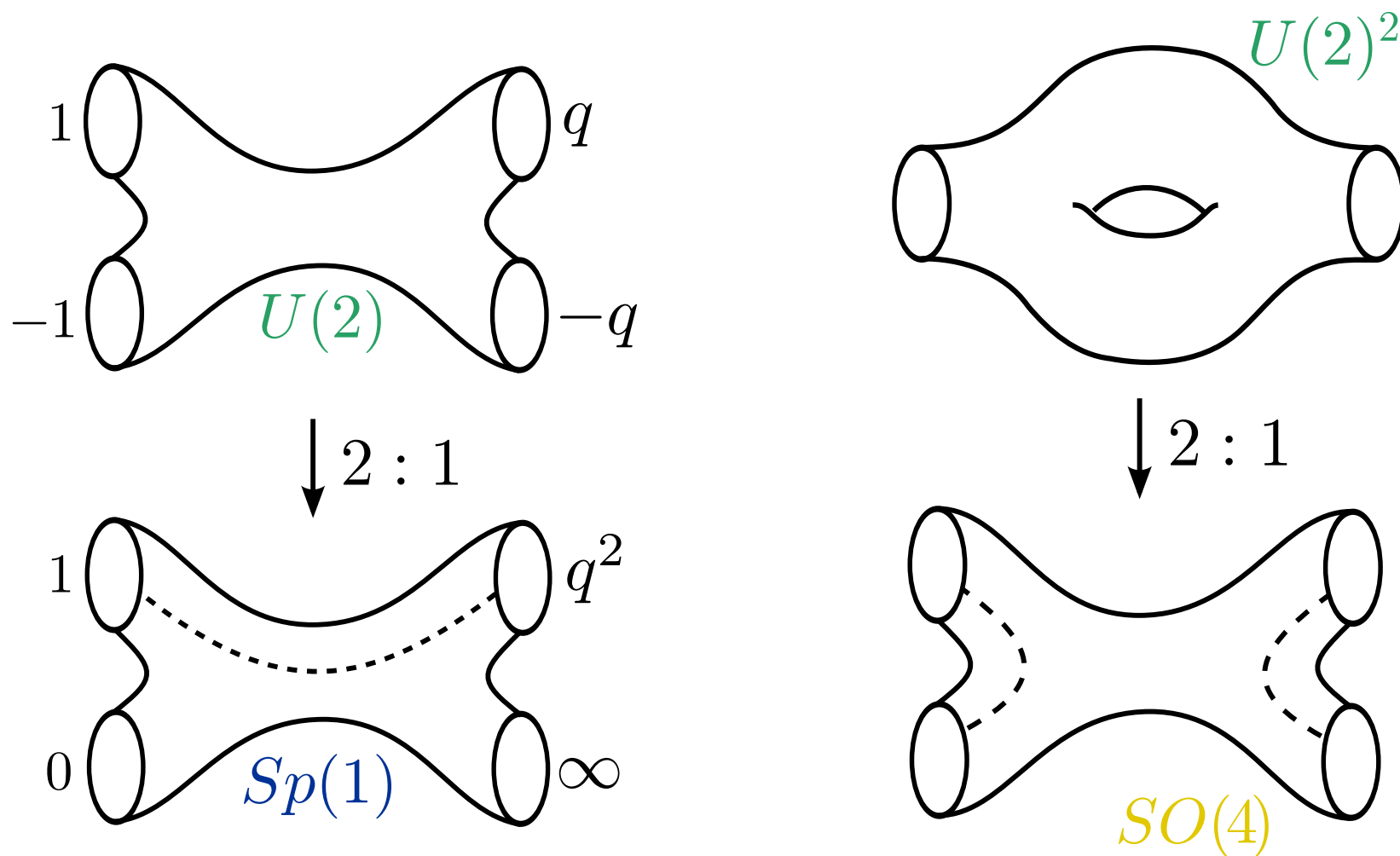
- The Nekrasov partition function for the different realization of the same theory agrees once it's written in terms of IR variables. (QFT lesson 101)





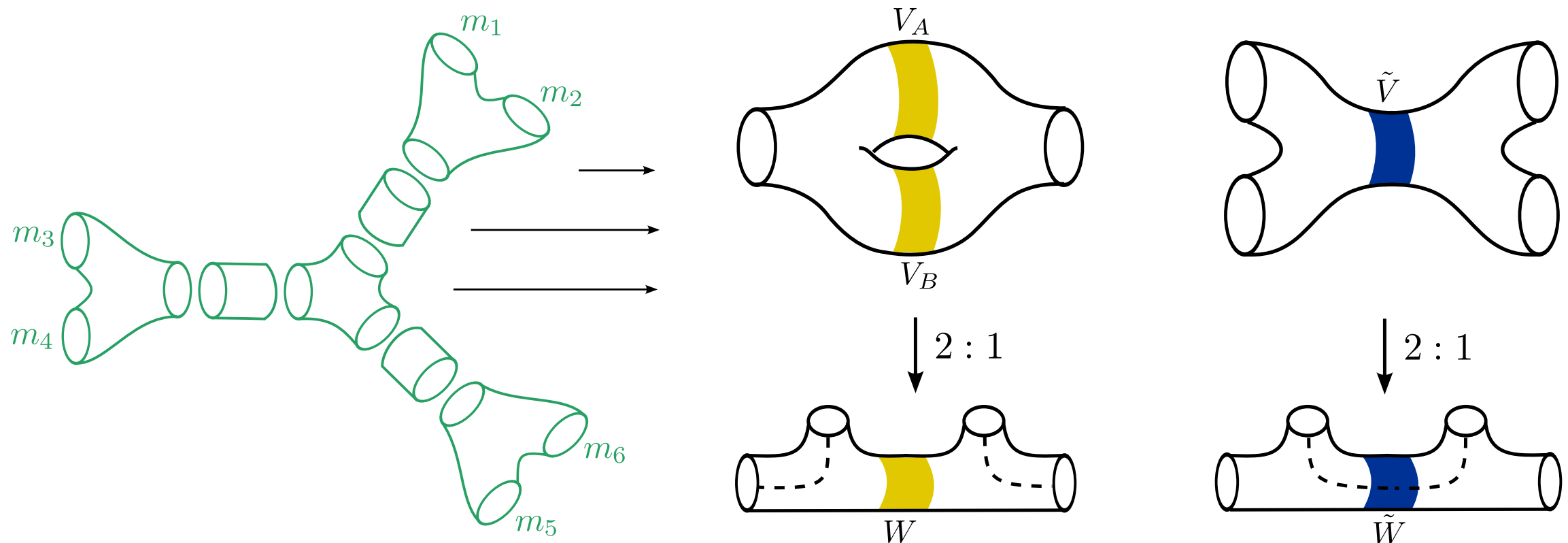
# Summary

- UV-UV map between parameters can be understood geometrically in terms of a map between the moduli spaces of Gaiotto curves.



# Summary

- AGT correspondence is extended and tested to the D-series and Sicilian quivers with bi/trifundamental half-hypers.



**break**

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- 4d story
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  - UV vs IR and Geometry
  - half-hypermultiplets
- 2d story
  - $Z_2$  -Twist of  $W(D_n)$  algebra
  - Asymmetry of trifund conformal block

**blackboard**