

A special Lagrangian subvariety of a Calabi-Yau manifold is a Lagrangian subvariety $S \subset M$ with its Riemannian volume proportional to the holomorphic volume form of M restricted to S . Such a subvariety is always minimal, and its deformation space is smooth and identified with $H^1(S)$. The SYZ conjecture, due to Strominger, Yau, Zaslow, is an attempt to explain the Mirror Symmetry in terms of fibrations by special Lagrangian tori. The only way to construct such fibrations known so far is the one using hyperkaehler geometry.

A hyperkaehler manifold M is a Riemannian manifold equipped with an action by quaternions I, J, K on its tangent bundle, such that I, J, K are parallel with respect to the Levi-Civita connection. Then (M, I) is a holomorphic symplectic Kaehler manifold. Converse is also true, by Calabi-Yau theorem. It is easy to see that any holomorphic Lagrangian subvariety of (M, I) is special Lagrangian in (M, J) . The SYZ conjecture predicts that any hyperkaehler manifold can be deformed to one which admits a holomorphic Lagrangian fibration. This would follow if one can prove a form of "Abundance Conjecture", which is one of the standard conjectures in algebraic geometry. I will explain the proposed strategy of the proof of abundance conjecture for hyperkaehler manifolds and some partial results obtained so far (arXiv:0811.0639).