Green-Schwarz Superstring with Conformal Symmetry

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With **Naoto Yokoi**, Prog. Theor. Phys. 125 (2011) 265 (arXiv:1008.4655)

1 Introduction and summary

1.1 Motivation

AdS/CFT

One of the most profound structures in physics

- ♦ **Many pieces of "evidence"**
- ♦ **Many "applications" (AdS/QCD, AdS/CMT, etc.)**

But still no real understanding

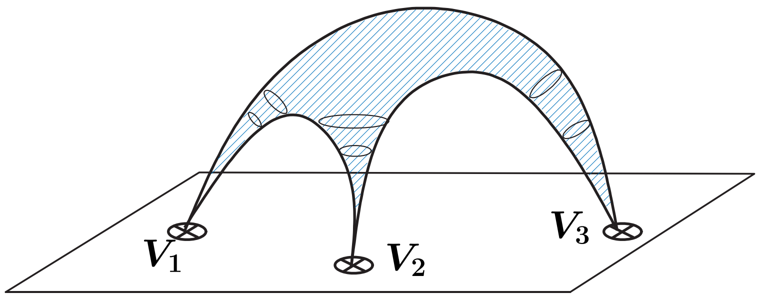
- ♦ **Strong/weak duality:**
  Open-closed duality cannot be the whole story
- ♦ **No basic dynamical picture has been identified**

Common tentative strategy:

- Postpone the dynamical understanding.
- Understand each side separately and find precise isomorphic structures.

Understanding of the string side is slow

Need to solve closed string theory in curved space with RR flux
Basic objects one wants to compute = boundary correlation functions



To be compared to SYM correlators of composite operators

- RR flux is crucially important $\Leftarrow$ D-branes
  $g_{YM}^2 N = 4\pi g_s N = R^4/\alpha'^2$ (balance between gravity and RR flux)
- RR (bispinor) fields are difficult to handle in RNS formalism

Advent of D-brane $\Rightarrow$ Decline of RNS, revival of GS, emergence of PS **(pure spinor)**

Papers after 1995 with title containing

RNS	26
Green-Schwarz	70
Pure spinor	96

♡ **"GS type" formalisms with increasing manifest symmetries**

$$\mathbf{GS}_{LC} \longrightarrow \underbrace{\mathbf{GS}_{SLC} \longrightarrow \mathbf{GS}_{DS} \longrightarrow \mathbf{PS}}_{\textbf{conformal inv}}$$

SLC gauge: $\gamma^+_{\alpha\beta}\theta^{A\beta} = 0$ (fix κ-symmetry only)

LC gauge: $\gamma^+_{\alpha\beta}\theta^{A\beta} = 0$ and $X^+(\tau,\sigma) = x^+ + p^+\tau$

DS=double spinor formalism (Aisaka and Kazama, 2005)

- PS has been powerful for higher loop amplitudes in flat space.

  Not sufficiently developed to handle curved background

- GS_{LC} is most physical: Suitable for analyzing the physical spectrum, both in flat space and in curved space (e.g. PP-wave background (Metsaev))

Lack of conformal symmetry $\Rightarrow$ Not suited for correlation functions

- $\boxed{\mathrm{GS}_{SLC}}$

  Has conformal symmetry lacking in GS_{LC}.

  Can be used for curved background

  —**Superstring in PP-wave background** (Kazama and Yokoi, (2008))

  Conformal symmetry is non-trivial: Left- and right-moving modes are coupled on the worldsheet, as in $AdS_5 \times S^5$.

  Quantum Virasoro algebra is established.

  Exact spectrum is reproduced.

But we found that only surprisingly little has been known about this theory, even in flat spacetime !

- Structures of quatum symmetries of the theory have not been clarified

- Vertex operators have not been constructed

— GS_{SLC} appears to be useful for Super SFT[1]

It should be worthwhile to

lay the systematic and comprehensive foundation of the Green-Schwarz superstring with conformal symmetry

the knowledge of which should be useful in future applications.

[1] Baba, Ishibashi, Murakami (2009 ∼)

1.2 Brief summary of results

1. Clarification of **complete gauge fixing procedure with compensating transformations**

2. **Systematic phase space quantization** which automatically incorporates the effect of compensating transformations

3. Clarification of **the structure of the quantum Virasoro algebra and its relation to the supersymmetry algebra**

$$T(z) = \Pi^+(z)\Pi^-(z) + \frac{1}{2}(\Pi^I(z))^2 - \frac{1}{2}S_a\partial S_a(z) + \frac{1}{2}\partial^2 \ln \Pi^+$$

$$Q \equiv \int [dz]\,(cT + bc\partial c)$$

$$\{Q_a, Q_b\} = 2\sqrt{2}\delta_{ab}p^+\,, \qquad \{Q_a, Q_{\dot{b}}\} = 2\bar{\gamma}^I_{a\dot{b}}p^I$$

$$\{Q_{\dot{a}}, Q_{\dot{b}}\} = -2\sqrt{2}\delta_{\dot{a}\dot{b}}p^- + \left\{Q, \frac{2\sqrt{2}}{\ell_s}\delta_{\dot{a}\dot{b}}\int [dz]\frac{b}{\Pi^+}(z)\right\}$$

4. Clarification of the **quantum super-Poincaré algebra**

In particular

$$[\mathcal{M}^{I-}, \mathcal{M}^{J-}] = \left\{ Q, \frac{1}{2} \int [dw] \left(\frac{b(w)(\bar{\gamma}^I S)_{\dot{a}} (\bar{\gamma}^J S)_{\dot{a}}(w)}{(\Pi^+(w))^2} \right) \right\}$$

$$[\mathcal{M}^{I-}, Q_{\dot{a}}] = \left[Q, (-i\, 2^{1/4}) \int [dw] \left\{ \frac{b(w)(\bar{\gamma}^I S)_{\dot{a}}(w)}{(\Pi^+(w))^{3/2}} \right\} \right]$$

5. Construction of the **vertex operators for the super-Maxwell multiplet**

from first principle

$$
\begin{aligned}
V_F(u) = \int [dz]\, e^{ik\cdot X(z)} \Big\{ & u^a \left(-i\, 2^{-1/4}\sqrt{\Pi^+}\, S_a(z)\right) \\
& + u^{\dot a} \left(-i\, 2^{-3/4} \frac{\left(\bar\gamma^I S\right)_{\dot a}\, \Pi_I(z)}{\sqrt{\Pi^+}} + i \left(\frac{2^{-3/4}}{12}\right) \frac{\left(\bar\gamma^I S\right)_{\dot a}\, R_I(z)}{\sqrt{\Pi^+}} \right) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
V_B(\zeta) = \int [dz]\, e^{ik\cdot X(z)} \Big[& \zeta^- \,\Pi^+(z) + \zeta^I \left(\Pi_I(z) - \frac{1}{4} R_I(z) \right) \\
& + \zeta^+ \left(\hat\Pi^-(z) + \frac{1}{4}\frac{\Pi^I\, R_I(z)}{\Pi^+} - \frac{1}{96}\frac{R^I\, R_I(z)}{\Pi^+} \right. \\
& \left. - k^-\, k_I \Pi^I(z) - \frac{\left(k_I \Pi^I\right)\left(k_J \Pi^J\right)(z)}{\Pi^+} \right) \Big] \qquad \left(R^I \equiv k_J S \gamma^{IJ} S\right)
\end{aligned}
$$

6. Construction of **exact quantum similarity transformation connecting the LC gauge and the SLC gauge** quantities

   An application: Construction of **fermionic DDF operator** for the first time

Plan of the talk and topics discussed

1. Introduction and summary

2. Classical action and the symmetries of the GS superstring

3. Gauge-fixing and compensating transformation

4. Phase space formulation and quantization

5. Structure of the quantum symmetry algebras

6. Vertex operators for massless states

7. Similarity transformation to the LC gauge and construction of the DDF operators

8. Discussions

2 Classical action and the symmetries of the GS superstring

□ **Classical Lagrangian for type IIB GS string :**

$$\mathcal{L}_{GS} = \mathcal{L}_K + \mathcal{L}_{WZ}$$

$$\mathcal{L}_K = -\frac{T}{2}\sqrt{-g}g^{ij}\Pi_i^\mu \Pi_{\mu j}\,, \quad \mathcal{L}_{WZ} = T\epsilon^{ij}\left(\Pi_i^\mu \widetilde{W}_{\mu j} + \frac{1}{2}W_i^\mu \widetilde{W}_{\mu j}\right)$$

Building blocks

$$\Pi_i^\mu = \partial_i X^\mu - W_i^\mu \qquad (i = 1, 2\,, \quad \mu = 0 \sim 9)$$

$$W_i^{A\mu} = i\theta^A \bar{\gamma}^\mu \partial_i \theta^A\,, \quad (A = 1, 2)$$

$$W_i^\mu = W_i^{1\mu} + W_i^{2\mu}\,, \qquad \widetilde{W}_i^\mu = W_i^{1\mu} - W_i^{2\mu}$$

$$\Gamma^\mu = \begin{pmatrix} 0 & (\gamma^\mu)^{\alpha\beta} \\ (\bar{\gamma}^\mu)_{\alpha\beta} & 0 \end{pmatrix}\,, \qquad \alpha, \beta = 1 \sim 16$$

Symmetries

- Worldsheet reparametrization
- Target space Lorentz invariance

- **Supersymmetry**

$$\delta_\chi \theta^A = \chi^A\,, \qquad \delta_\chi X^\mu = \sum_A i\chi^A \bar{\gamma}^\mu \theta^A$$

$\mathcal{L}_K$ is invariant but $\mathcal{L}_{WZ}$ transforms into a total derivative ($\Leftarrow$ Fierz)

$$\delta_\chi \mathcal{L}_{WZ} = \partial_i \left(\chi^{1\alpha} \Lambda^{1i}_\alpha + \chi^{2\alpha} \Lambda^{2i}_\alpha \right)$$

$$\Lambda^{1i}_\alpha = -iT\epsilon^{ij} \left(\Pi^\mu_j + W^{2\mu}_j + \frac{2}{3} W^{1\mu}_j \right) (\bar{\gamma}_\mu \theta^1)_\alpha$$

$$\Lambda^{2i}_\alpha = iT\epsilon^{ij} \left(\Pi^\mu_j + W^{1\mu}_j + \frac{2}{3} W^{2\mu}_j \right) (\bar{\gamma}_\mu \theta^2)_\alpha$$

These formulas are needed for construction of supercurrents.

- κ **symmetry** (off-shell)

$$\delta_\kappa \theta^{A\alpha} = (\gamma_i)^{\alpha\beta} \kappa^{Ai}_\beta \,, \qquad \delta_\kappa X^\mu = \sum_A i\theta^A \bar{\gamma}^\mu \delta_\kappa \theta^A$$

$$\delta_\kappa (\sqrt{-g}\, g^{ij}) = \sqrt{-g}\, h^{ij}$$

where $\quad h^{ij} = 8i \left(P_+^{ki} \partial_k \theta^1 \kappa^{1j} + P_-^{ki} \partial_k \theta^2 \kappa^{2j} \right)$

$P_\pm^{ij}$ are projection operators

$$P_\pm^{ij} = \frac{1}{2} \left(g^{ij} \pm \frac{\epsilon^{ij}}{\sqrt{-g}} \right)$$

κ parameters must satisfy the conditions

$$P_+^{ij} \kappa_j^1 = 0 \,, \qquad P_-^{ij} \kappa_j^2 = 0$$

3 Gauge-fixing and compensating transformation

Wish to keep conformal invariance intact.

3.1 Conformal gauge-fixing

Fix reparametrization invariance by the conformal gauge condition

$$\sqrt{-g}g^{ij} = \eta^{ij}$$

This breaks κ-invariance

$\Rightarrow$ Modify κ-transformation by a compensating reparametrization $\delta_f \xi^i = f^i(\xi)$ such that

$$(\delta_\kappa + \delta_f)\sqrt{-g}\, g^{ij} = 0$$

This is achieved by the choice

$$f^j = \Box^{-1}\partial_i h^{ij} \, .$$

Modified κ-transformations are

$$\delta_\kappa \theta^A = \delta^0_\kappa \theta^A + f^i \partial_i \theta^A \,,$$
$$\delta_\kappa X^\mu = \sum_A i\theta^A \bar{\gamma}^\mu \delta^0_\kappa \theta^A + f^i \partial_i X^\mu$$

3.2 Semi-light-cone (SLC) gauge fixing

Fix κ symmetry by SLC gauge conditions

$$\bar{\gamma}^+_{\alpha\beta}\theta^{A\beta} = 0 \quad \Leftrightarrow \quad \theta^{A\dot{a}} = 0 \,, \qquad \left(\theta^\alpha = \begin{pmatrix} \theta^{Aa} \\ \theta^{A\dot{a}} \end{pmatrix} \right)$$

Lagrangian simplifies drastically

$$\mathcal{L}_K = -\frac{T}{2}\left[2\partial_i X^+ \partial^i X^- + \partial_i X^I \partial^i X^I - 2\partial_i X^+ \sum_A i\theta^A \bar{\gamma}^- \partial^i \theta^A\right]$$

$$\mathcal{L}_{WZ} = iT\epsilon^{ij}\partial_i X^+ \sum_A \eta_A \theta^A \bar{\gamma}^- \partial_j \theta^A \,, \qquad (\eta_1 = -\eta_2 = 1)$$

- Unlike in the LC gauge, there is no fermion kinetic term.
- Nevertheless, X^μ and θ^A satisfy free field equations of motion.

3.2.1 Supersymmetry in SLC gauge

Write SUSY transformation for θ^A in $SO(8)$ basis as

"η-SUSY"	$\delta_\eta \theta^{Aa} = \eta^{Aa}$
"ϵ-SUSY"	$\delta_\epsilon \theta^{A\dot{a}} = \epsilon^{A\dot{a}}$

SLC gauge is violated by ϵ-SUSY $\Rightarrow$ Keep SLC gauge by additional κ-transf.

Parameter for the compensating κ-transformation should be determined by the requirement

$$\delta_\epsilon \theta^{A\dot{a}} \equiv (\delta^0_\epsilon + \delta_\kappa)\theta^{A\dot{a}} = \epsilon^{A\dot{a}} + (\Pi^\mu_i \gamma_\mu \kappa^{Ai})^{\dot{a}} = 0$$

Solution:

$$\kappa^{1,0}_{\dot{a}} = \kappa^{1,1}_{\dot{a}} = \frac{\delta_{\dot{a}\dot{b}}\epsilon^{1\dot{b}}}{2\sqrt{2}\partial_+ X^+}\,,$$

$$\kappa^{2,0}_{\dot{a}} = -\kappa^{2,1}_{\dot{a}} = \frac{\delta_{\dot{a}\dot{b}}\epsilon^{2\dot{b}}}{2\sqrt{2}\partial_- X^+}$$

Modified ϵ-SUSY transformations:

$$\delta_\epsilon \theta^{1a} = (\gamma^I)^{a\dot{b}}\delta_{\dot{b}\dot{c}}\frac{\partial_+ X^I}{\sqrt{2}\partial_+ X^+}\epsilon^{1\dot{c}}\,,$$

$$\delta_\epsilon \theta^{2a} = (\gamma^I)^{a\dot{b}}\delta_{\dot{b}\dot{c}}\frac{\partial_- X^I}{\sqrt{2}\partial_- X^+}\epsilon^{2\dot{c}}$$

$$\delta_\epsilon X^I = i\epsilon^A \bar{\gamma}^I \theta^A\,,$$

$$\delta_\epsilon X^- = i(\theta^1 \bar{\gamma}^I \epsilon^1)\frac{\partial_+ X^I}{\partial_+ X^+} + i(\theta^2 \bar{\gamma}^I \epsilon^2)\frac{\partial_- X^I}{\partial_- X^+}$$

Classical supercharges in SLC gauge take the form

$$Q_a^1 = -4\sqrt{2}\, i \int d\sigma \theta_a^1 T \partial_- X^+$$
$$Q_a^2 = -4\sqrt{2}\, i \int d\sigma \theta_a^2 T \partial_+ X^+$$
$$Q_{\dot{a}}^1 = -4i(\bar{\gamma}^I)_{\dot{a}b} \int d\sigma \theta^{1b} T \partial_- X^I$$
$$Q_{\dot{a}}^2 = -4i(\bar{\gamma}^I)_{\dot{a}b} \int d\sigma \theta^{2b} T \partial_+ X^I$$

3.2.2 Lorentz symmetry in SLC gauge

Before gauge-fixing the Lorentz transformations are of the familiar form:

$$\delta X^\mu = \frac{1}{2}\xi_{\rho\sigma}\left(\eta^{\mu\rho} X^\sigma - \eta^{\mu\sigma} X^\rho\right)$$
$$\delta\theta^{A\alpha} = \frac{1}{4}\xi_{\rho\sigma}(\gamma^{\rho\sigma})^\alpha{}_\beta \theta^{A\beta}$$

Transformation with the parameter ξ_{I-} breaks SLC gauge condition

$\Rightarrow$ Compensate with a κ-transformation

$$0 = \delta_{\xi_{I-}} \theta^{A\dot{a}} = (\delta^0_{\xi_{I-}} + \delta^\kappa_{\xi_{I-}})\theta^{A\dot{a}} = \xi_{I-}\frac{1}{2}(\gamma^{I-})^{\dot{a}}{}_b\theta^{Ab} - \Pi^+_i \delta^{\dot{a}\dot{b}}\kappa^{Ai}_{\dot{b}}$$

Solution

$$\kappa^{1,0}_{\dot{a}}(\xi_{I-}) = \frac{\delta_{\dot{a}\dot{b}}\xi_{I-}(\gamma^{I-})^{\dot{b}}{}_c\theta^{1c}}{4\partial_+ X^+},$$

$$\kappa^{2,0}_{\dot{a}}(\xi_{I-}) = \frac{\delta_{\dot{a}\dot{b}}\xi_{I-}(\gamma^{I-})^{\dot{b}}{}_c\theta^{2c}}{4\partial_- X^+}$$

Modified ξ_{I-} transformations for θ^{Aa} and X^-

$$\delta^\kappa_{\xi_{I-}}\theta^{1a} = \frac{1}{\sqrt{2}}\frac{\partial_+ X^J}{\partial_+ X^+}(\gamma^J\bar{\gamma}^I\theta^1)^a\xi_{I-},$$

$$\delta^\kappa_{\xi_{I-}}\theta^{2a} = \frac{1}{\sqrt{2}}\frac{\partial_- X^J}{\partial_- X^+}(\gamma^J\bar{\gamma}^I\theta^2)^a\xi_{I-}$$

$$\delta^\kappa_{\xi_{I-}} X^- = \frac{i}{\sqrt{2}}\left(\frac{\partial_+ X^J}{\partial_+ X^+}\theta^1\bar{\gamma}^{JI}\bar{\gamma}^-\theta^1 + \frac{\partial_- X^J}{\partial_- X^+}\theta^2\bar{\gamma}^{JI}\bar{\gamma}^-\theta^2\right)\xi_{I-}$$

The SLC-conformal gauge fixed action is still fully invariant under the modified super-Poincaré transformations

But in the Lagrangian formulation they are rather complicated and not easy to deal with.

Situation is much better in the phase space formulation

4 Phase space formulation

4.1 Poisson-Dirac backet and quantization

Bosonic momenta

$$\begin{aligned} P^+ &= T\partial_0 X^+ \\ P^- &= T\left[\partial_0 X^- - 2\sqrt{2}i(\theta_a^1\partial_+\theta_a^1 + \theta_a^2\partial_-\theta_a^2)\right] \\ P^I &= T\partial_0 X^I \end{aligned}$$

Fermionic momenta

$$p_a^A = i\sqrt{2}T(\partial_0 X^+ - \eta_A\partial_1 X^+)\theta_a^A = i\pi^{+A}\theta_a^A$$

where $\quad \pi^{+A} \equiv \sqrt{2}(P^+ - \eta_A T\partial_1 X^+)$

$\Leftrightarrow$ constraints

$$d_a^A \equiv p_a^A - i\pi^{+A}\theta_a^A = 0$$

Poisson brackets:

$$
\begin{aligned}
\{X^I(\sigma,t), P^J(\sigma',t)\}_P &= \delta^{IJ}\delta(\sigma-\sigma') \\
\{X^\pm(\sigma,t), P^\mp(\sigma',t)\}_P &= \delta(\sigma-\sigma') \\
\{\theta_a^A(\sigma,t), p_b^B(\sigma',t)\}_P &= -\delta^{AB}\delta_{ab}\delta(\sigma-\sigma') \\
\text{rest} &= 0
\end{aligned}
$$

d_a^A form the second class algebra

$$\{d_a^A(\sigma,t), d_b^B(\sigma',t)\}_P = 2i\delta^{AB}\delta_{ab}\pi^{+A}(\sigma,t)\delta(\sigma-\sigma')$$

Define Dirac bracket in the usual way. Then, θ_a^A's become self-conjugate

$$\{\theta_a^A(\sigma,t), \theta_b^B(\sigma',t)\}_D = \frac{i\delta^{AB}\delta_{ab}}{2\pi^{+A}(\sigma,t)}\delta(\sigma-\sigma')$$

Apparent difficulty:

$$\{X^-(\sigma,t), \theta_a^A(\sigma',t)\}_D = -\frac{1}{\sqrt{2}\pi^{+A}(\sigma,t)}\theta_a^A\delta(\sigma-\sigma') \neq 0$$

$$\{P^-(\sigma,t), \theta_a^A(\sigma',t)\}_D = -\frac{1}{\sqrt{2}\pi^{+A}(\sigma',t)}\theta_a^A(\sigma',t)\delta'(\sigma-\sigma') \neq 0$$

Cured by the use of Θ_a^A defined by

$$\Theta_a^A \equiv \sqrt{2\pi^{+A}}\,\theta_a^A$$

$\Rightarrow$ Dirac brackets become canonical for $(X^\mu, P^\mu, \Theta_a^A)$

Quantization at equal time is straight-forward: $[A, B\} = i\,\{A, B\}_D$

$$X^\mu(\sigma, 0) = \sum_n X_n^\mu e^{-in\sigma} = x^\mu + i\ell_s \sum_{n\neq 0}\left(\frac{1}{n}\alpha_n^\mu e^{-in\sigma} + \frac{1}{n}\bar{\alpha}_n^\mu e^{in\sigma}\right)$$

$$P^\mu(\sigma, 0) = \sum_n P_n^\mu e^{-in\sigma} = \frac{p^\mu}{2\pi} + \frac{1}{4\pi\ell_s}\sum_{n\neq 0}\left(\alpha_n^\mu e^{-in\sigma} + \bar{\alpha}_n^\mu e^{in\sigma}\right)$$

$$S_a(\sigma, 0) = \sum_n S_{a,n} e^{-in\sigma}\,, \qquad \bar{S}_a(\sigma, 0) = \sum_n \bar{S}_{a,n} e^{in\sigma}$$

where

$$S_a(\sigma, 0) = i\sqrt{2\pi}\,\Theta_a^2(\sigma, 0)\,, \qquad \bar{S}_a(\sigma, 0) = i\sqrt{2\pi}\,\Theta_a^1(-\sigma, 0)$$

Phase space fields are related to those in the canonical quantization scheme by

$$\phi_{can}(\sigma, t) = e^{iHt}\phi_{phase}(\sigma, 0)e^{-iHt}$$

This holds even for non-linear theory. For the present case,

$$H = \ell_s^2 p^2 + \sum_{n\geq 1}(\alpha^\mu_{-n}\alpha_{\mu,n} + \bar{\alpha}^\mu_{-n}\bar{\alpha}_{\mu,n} + nS_{a,-n}S_{a,n} + n\bar{S}_{a,-n}\bar{S}_{a,n})$$

We will use canonical fields in the Euclidean worldsheet ($\tau = it, z \equiv e^{\tau+i\sigma}, \bar{z} \equiv e^{\tau-i\sigma}$) such as

$$X^\mu(z, \bar{z}) = x^\mu - i\ell_s^2 p^\mu(\ln z + \ln \bar{z}) + i\ell_s \sum_{n\neq 0}\left(\frac{1}{n}\alpha_n z^{-n} + \frac{1}{n}\bar{\alpha}_n \bar{z}^{-n}\right)$$

Chiral fields:

$$X^\mu(z) \equiv x^\mu - i\ell_s^2 p^\mu \ln z + i\ell_s \sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n z^{-n}$$

$$\Pi^\mu(z) \equiv \sum_n \alpha^\mu_n z^{-n-1} = i\ell_s^{-1}\partial X^\mu(z)\,, \qquad S_a(z) \equiv \sum_n S_{a,n} z^{-n-1/2}$$

4.2 Compensating transformation in the phase space formulation

Compensating transformations $\Rightarrow$ **Stay on the gauge slice chosen**

Phase space formulation: **Dirac bracket does this automatically**

Pedagogical demonstration

Consider a system with a conjugate pair: $\{\phi(x), \pi(y)\}_P = \delta(x - y)$.

Assume

- Invariance under a gauge transf. generated by a first class constraint $\Phi_1(\phi, \pi)(x)$
- $^{\exists}$ gauge-invariant global sym. generator U: $\Leftrightarrow$ $\{\Phi_1, U\}_P = 0$
- Impose gauge condition $\Phi_2(x) = 0$ such that $\{\Phi_i(x), \Phi_j(y)\}_P = \epsilon_{ij} C(x) \delta(x - y) \neq 0$

- U breaks the gauge condition $\delta_\epsilon \Phi_2(x) \equiv \{\Phi_2(x), \epsilon U\}_P \neq 0$

To preserve the gauge condition, modify $U \to U + \Delta U$: $\Delta U =$ compensating gauge generator

$$\Delta U = \int dy \alpha(y) \Phi_1(y)$$

Must choose α such that

$$\underbrace{(\delta_\epsilon + \delta_\epsilon^{gauge})}_{\delta_\epsilon^{total}} \Phi_2(x) = \epsilon \{\Phi_2(x), U + \Delta U\}_P$$

$$= \epsilon \left(\{\Phi_2(x), U\}_P - \alpha(x) C(x)\right) = 0$$

$\Rightarrow \alpha(x) = C^{-1}(x) \{\Phi_2(x), U\}_P$. Then, (using $\{\Phi_1, U\}_P = 0$)

$$\delta_\epsilon^{total} F(x) = \epsilon \{F(x), U\}_P + \epsilon \int dy \{F(x), \Phi_1(y)\}_P \underbrace{C^{-1}(y) \{\Phi_2(y), U\}_P}_{\alpha(y)}$$

$$= \epsilon \left[\{F(x), U\}_P - \int dy \{F(x), \Phi_i(y)\}_P \underbrace{C^{-1}(y)_{ij}}_{-\epsilon_{ij} C^{-1}(y)} \{\Phi_j(y), U\}_P \right]$$

$$= \epsilon \{F(x), U\}_D \quad //$$

5 Structure of the quantum symmetry algebras

5.1 Virasoro algebra and BRST symmetry

5.1.1 Classical Virasoro algebra

By a standard procedure we obtain

$$T_+ = \frac{1}{2}(\mathcal{H} + \mathcal{P}) = \frac{1}{2\pi}\left(\Pi^+\Pi^- + \frac{1}{2}\Pi_I^2 + \frac{i}{2}S^2\partial_1 S^2\right)$$
$$T_- = \frac{1}{2}(\mathcal{H} - \mathcal{P}) = \frac{1}{2\pi}\left(\widetilde{\Pi}^+\widetilde{\Pi}^- + \frac{1}{2}\widetilde{\Pi}_I^2 - \frac{i}{2}S^1\partial_1 S^1\right)$$

Modes of $T_\pm$ satisfy the classical Virasoro algebras

$$T_\pm = \frac{1}{2\pi}\sum_n L_n^\pm e^{-in(t\pm\sigma)}$$

$$\{L_m^\pm, L_n^\pm\}_D = \frac{1}{i}(m-n)L_{m+n}^\pm\,, \qquad \{L_m^\pm, L_n^\mp\}_D = 0$$

5.1.2 Quantum Virasoro algebra and BRST operator

Naive Virasoro operator in the holomorphic sector:

$$T(z) = \Pi^+(z)\Pi^-(z) + \frac{1}{2}(\Pi^I(z))^2 - \frac{1}{2}S_a\partial S_a(z)$$

Basic OPE's

$$X^\mu(z)X^\nu(w) \sim -\eta^{\mu\nu}\ell_s^2 \ln(z-w)$$

$$X^\mu(z)\Pi^\nu(w) \sim \frac{i\ell_s\eta^{\mu\nu}}{z-w}, \qquad \Pi^\mu(z)\Pi^\nu(w) \sim \frac{\eta^{\mu\nu}}{(z-w)^2}$$

$$S_a(z)S_b(w) \sim \frac{\delta_{ab}}{z-w}$$

Central charge: $10_{boson} + \frac{1}{2} \times 8_{fermion} = 14$

$$T(z)T(w) = \frac{14/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

Modify $T(z)$ so that we get $c = 26$ (Berkovits and Marchioro: looks ad hoc)

$$T(z) = \Pi^{+}(z)\Pi^{-}(z) + \frac{1}{2}(\Pi^{I}(z))^{2} - \frac{1}{2}S_{a}\partial S_{a}(z) + \frac{1}{2}\partial^{2}\ln\Pi^{+}$$

$$\partial^{2}\ln\Pi^{+} = \frac{\partial^{2}\Pi^{+}}{\Pi^{+}} - \left(\frac{\partial\Pi^{+}}{\Pi^{+}}\right)^{2}$$

- **Π^{-} is no longer a primary**

Genuine primary of dimension 1 is

$$\hat{\Pi}^{-} \equiv \Pi^{-} + \partial^{2}\left(\frac{1}{2\Pi^{+}}\right) = \Pi^{-} - \frac{1}{2}\frac{\partial^{2}\Pi^{+}}{(\Pi^{+})^{2}} + \frac{(\partial\Pi^{+})^{2}}{(\Pi^{+})^{3}}$$

$\Rightarrow$ Nilpotent BRST operator

$$Q \equiv \int [dz]\,(cT + bc\partial c)$$

5.2 Quantum Super-Poincaré algebra

5.2.1 Quantum SUSY algebra

Noether charges in SLC gauge (for the left-sector)

$$Q_a = -\rho \int [dz] \sqrt{\Pi^+(z)}\, S_a(z)\,, \qquad Q_{\dot{a}} = -\rho \int [dz] \frac{\Pi^I(z)}{\sqrt{2\Pi^+(z)}} \bar{\gamma}^I_{\dot{a}b} S^b(z)$$

$$(\rho = 2^{3/4} \ell_s^{-1/2})$$

Quantum SUSY algebra:

$$\{Q_a, Q_b\} = 2\sqrt{2}\delta_{ab} p^+\,, \qquad \{Q_a, Q_{\dot{b}}\} = 2\bar{\gamma}^I_{a\dot{b}} p^I$$

$$\{Q_{\dot{a}}, Q_{\dot{b}}\} = \rho^2 \delta_{\dot{a}\dot{b}} \int [dw] \frac{1}{\Pi^+} \left(\frac{1}{2}(\Pi^I)^2 - \frac{1}{2} S^a \partial S^a - \frac{1}{2} \partial^2 \ln \Pi^+ \right)$$

$$= -2\sqrt{2}\delta_{\dot{a}\dot{b}} p^- + \rho^2 \delta_{\dot{a}\dot{b}} \int [dw] \mathcal{T}$$

$$\mathcal{T} \equiv \Pi^- + \frac{1}{\Pi^+} \left(\frac{1}{2}(\Pi^I)^2 - \frac{1}{2} S^a \partial S^a - \frac{1}{2} \partial^2 \ln \Pi^+ \right)$$

- $\Pi^+ \mathcal{T}$ is not quite equal to T and $\mathcal{T}(z)\mathcal{T}(w) \sim 0$ (regular)

Introduce **new BRST operator**

$$\hat{Q} \equiv \int [dz] \mathcal{T}(z) c(z) \,, \qquad \hat{Q}^2 = 0 \quad (\Leftarrow \mathcal{T}(z)\mathcal{T}(w) \sim 0)$$

Then, $\int [dw]\mathcal{T}$ is "BRST-exact"

$$\int [dw]\mathcal{T} = \left\{ \hat{Q}, \int [dw] b(w) \right\}, \quad b(z)c(w) \sim \frac{1}{z-w}, \quad \dim (b,c) = (1,0)$$

Relation to the usual Q:

$$e^R \hat{Q} e^{-R} = Q = \int [dz](cT + bc\partial c)$$

$$\text{where} \qquad R \equiv \int [dz] cb \ln \Pi^+$$

Under this transformation: $-\frac{1}{2}\partial^2 \ln \Pi^+ \in \mathcal{T} \longrightarrow +\frac{1}{2}\partial^2 \ln \Pi^+ \in T$

So we **UNDERSTAND** the origin of $+\frac{1}{2}\partial^2 \ln \Pi^+$ in T

The extra term is Q-exact

$$\{Q_{\dot{a}}, Q_{\dot{b}}\} = -2\sqrt{2}\delta_{\dot{a}\dot{b}} p^- + \left\{ Q, \frac{2\sqrt{2}}{\ell_s} \delta_{\dot{a}\dot{b}} \int [dz] \frac{b}{\Pi^+}(z) \right\}$$

5.2.2 Quantum Lorentz algebra

□ **A technical trick for using OPE method with chiral fields**:

Structure of the bosonic part of the Lorentz generator $M_B^{\mu\nu}$

$$M_B^{\mu\nu} = M_{0,B}^{\mu\nu} + \check{M}_L^{\mu\nu} + \check{M}_R^{\mu\nu}$$

where

$$\begin{aligned} M_{0,B}^{\mu\nu} &= x^\mu p^\nu - x^\nu p^\mu \\ \check{M}_L^{\mu\nu} &= \frac{1}{i}\sum_{n\geq 1}\frac{1}{n}(\alpha_{-n}^\mu \alpha_n^\nu - \alpha_{-n}^\nu \alpha_n^\mu) \\ \check{M}_R^{\mu\nu} &= \frac{1}{i}\sum_{n\geq 1}\frac{1}{n}(\bar{\alpha}_{-n}^\mu \bar{\alpha}_n^\nu - \bar{\alpha}_{-n}^\nu \bar{\alpha}_n^\mu) \end{aligned}$$

- $M_{0,B}^{\mu\nu}$, $\check{M}_L^{\mu\nu}$ and $\check{M}_R^{\mu\nu}$ separately satisfy the Lorentz algebra.
- "Chiral" expression like $\frac{1}{2}\int [dz](X^\mu \Pi^\nu - X^\nu \Pi^\mu)(z)$ is bad because
  - $X^\mu(z) \ni \ln z$: not a genuine conformal field
  - Regularization $\ln z \longrightarrow \ln(z+\epsilon)$ gives $\frac{1}{2}M_{0,B}^{\mu\nu} + \check{M}_L^{\mu\nu}$

    This does not satisfy the Lorentz algebra

Trick

Use a new coordinate field $\mathring{X}^\mu(z)$ in place of $X^\mu(z)$:

$$\mathring{X}^\mu(z) \equiv 2x^\mu + \check{X}^\mu(z)\,, \qquad \check{X}^\mu(z) = i \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu z^{-n}$$

Then, we can make use of much of the chiral OPE technique (details omitted).

Quantum generators in the "left sector"

- M^{IJ} and $M^{\mu +}$ are simple:

$$M^{IJ} = \int [dz] \left\{ \frac{1}{2} (\mathring{X}^I \Pi^J(z) - \mathring{X}^J \Pi^I(z)) - \frac{i}{4} S^a \left(\gamma^{IJ}\right)_{ab} S^b(z) \right\}$$

$$M^{\mu +} = \int [dz] \left\{ \frac{1}{2} (\mathring{X}^\mu \Pi^+(z) - \mathring{X}^+ \Pi^\mu(z)) \right\}$$

- M^{I-} **receives quantum correction**[2]

$$\mathcal{M}^{I-} = M^{I-} + \Delta M^{I-}$$

$$M^{I-} = \int [dz] \left\{ \frac{1}{2}(\mathring{X}^I \Pi^-(z) - \mathring{X}^- \Pi^I(z)) + \frac{i}{4}\frac{(\bar{\gamma}^I S)_{\dot{a}}(\bar{\gamma}^K S)_{\dot{a}}\Pi^K(z)}{\Pi^+(z)} \right\}$$

$$\Delta M^{I-} = -\int [dz] \frac{i}{2}\frac{\partial \Pi^I(z)}{\Pi^+(z)}$$

ΔM^{I-} can be understood as coming from the replacement

$$\Pi^- \longrightarrow \hat{\Pi}^- = \Pi^- + \partial^2 \left(\frac{1}{2\Pi^+}\right) = \text{genuine primary field}$$

$$\mathring{X}^- \longrightarrow \hat{\mathring{X}}^- = \mathring{X}^- - i\partial \left(\frac{1}{2\Pi^+}\right)$$

[2] Kunitomo-Mizoguchi (2007) for $D = 4, 6$.

Most non-trivial check

$$
\begin{aligned}
[\mathcal{M}^{I-}, \mathcal{M}^{J-}] = \int [dw] \Bigg[& \frac{1}{2} \frac{\Pi^+\Pi^-(w)(\bar{\gamma}^I S)_{\dot{a}}(\bar{\gamma}^J S)_{\dot{a}}(w)}{(\Pi^+(w))^2} \\
& - \frac{1}{4} \frac{\Pi^I\Pi^K(w)(\bar{\gamma}^J S)_{\dot{a}}(\bar{\gamma}^K S)_{\dot{a}}(w)}{(\Pi^+(w))^2} + \frac{1}{4} \frac{\Pi^J\Pi^K(w)(\bar{\gamma}^I S)_{\dot{a}}(\bar{\gamma}^K S)_{\dot{a}}(w)}{(\Pi^+(w))^2} \\
& + \frac{1}{2} \frac{\Pi^I \partial \Pi^J(w) - \Pi^J \partial \Pi^I(w)}{(\Pi^+(w))^2} + \frac{1}{4} \left\{ \partial^2 \left(\frac{1}{\Pi^+(w)} \right) \right\} \left(\frac{(\bar{\gamma}^I S)_{\dot{a}}(\bar{\gamma}^J S)_{\dot{a}}(w)}{\Pi^+(w)} \right) \\
& - \frac{1}{16} \left\{ \partial \left(\frac{(\bar{\gamma}^I S)_{\dot{a}}(\bar{\gamma}^K S)_{\dot{a}}(w)}{\Pi^+(w)} \right) \right\} \left(\frac{(\bar{\gamma}^J S)_{\dot{b}}(\bar{\gamma}^K S)_{\dot{b}}(w)}{\Pi^+(w)} \right) \\
& - \frac{1}{4} \left(\frac{\Pi^K\Pi^L(w)}{(\Pi^+(w))^2} \right) \left\{ S_a(w) \left(\gamma^{IK} \gamma^{JL} \right)^a{}_b S^b(w) \right\} \\
& + \frac{1}{8} \left\{ \partial \left(\frac{S_a(w)}{\Pi^+(w)} \right) \right\} \left(\gamma^{IK} \gamma^{JK} \right)^a{}_b \left\{ \partial \left(\frac{S^b(w)}{\Pi^+(w)} \right) \right\} \\
& - \frac{1}{8} \left\{ \partial \left(\frac{\Pi^K(w)}{\Pi^+(w)} \right) \right\} \left(\frac{\Pi^L(w)}{\Pi^+(w)} \right) \mathrm{Tr} \left(\gamma^{IK} \gamma^{JL} \right) \Bigg]
\end{aligned}
$$

$$= \cdots$$

Needs a lot of Fierz id's.

$$\left[\mathcal{M}^{I-},\, \mathcal{M}^{J-}\right] = 0 + \{Q, \Psi\}$$

$$\Psi \equiv \frac{1}{2}\int [dw]\ \left(\frac{b(w)(\bar{\gamma}^I S)_{\dot{a}}(\bar{\gamma}^J S)_{\dot{a}}(w)}{(\Pi^+(w))^2}\right)$$

5.2.3 Spinorial property of supercharges ([Lorentz, SUSY])

Must check that the supercharges transform like spinors, possibly up to a BRST exact term.

Most non-trivial is $\left[\mathcal{M}^{I-}, Q_{\dot{a}}\right]$, which should vanish.

$$\begin{aligned}\left[\mathcal{M}^{I-},\, Q_{\dot{a}}\right] &= (-i\, 2^{1/4}) \int [dw] \left\{ \frac{\Pi^- \left(\bar{\gamma}^I S\right)_{\dot{a}}(w)}{\sqrt{\Pi^+}} + \frac{1}{2}\frac{\Pi^K \Pi^K \left(\bar{\gamma}^I S\right)_{\dot{a}}(w)}{(\Pi^+)^{3/2}} \right. \\ &+ \frac{21}{16}\frac{(\partial\Pi^+)^2 \left(\bar{\gamma}^I S\right)_{\dot{a}}(w)}{(\Pi^+)^{7/2}} - \frac{7}{8}\frac{(\partial^2\Pi^+) \left(\bar{\gamma}^I S\right)_{\dot{a}}(w)}{(\Pi^+)^{5/2}} + \frac{1}{2}\partial^2\left(\frac{1}{\Pi^+(w)}\right)\frac{(\bar{\gamma}^I S)_{\dot{a}}(w)}{\sqrt{\Pi^+}} \\ &\left. + \frac{1}{4}\partial\left(\frac{(\bar{\gamma}^I S)_{\dot{b}}(\bar{\gamma}^K S)_{\dot{b}}(w)}{\Pi^+}\right)\left(\frac{(\bar{\gamma}^K S)_{\dot{a}}(w)}{\sqrt{\Pi^+}}\right)\right\} = \cdots\end{aligned}$$

$$\left[\mathcal{M}^{I-},\, Q_{\dot{a}}\right] = 0 + [Q,\, \Phi]$$

$$\Phi = (-i\, 2^{1/4}) \int [dw] \; \left\{ \frac{b(w)(\bar{\gamma}^I S)_{\dot{a}}(w)}{\left(\Pi^+(w)\right)^{3/2}} \right\}$$

So we have now understood how quantum super-Poincaré algebra is realized in the SLC-conformal gauge.

6 Vertex operators for massless open string states

$\Leftrightarrow$ on-shell super-Maxwell multiplet in10D (A^μ, ψ^α)

Principle for the construction of vertex operators

- ♦ **BRST invariance**
- ♦ **Form appropriate representation of the super-Poincaré algebra up to BRST-exact terms**

These requirements will indeed fix the vertex operators, albeit in fairly intricate manner

6.1 General form of the BRST invariant vertex operators

We will construct the integrated vertex operators $V = \int [dz] U(z)$:

Requirement

- $U(z)$ = primary operator of dimension 1
- Manifest $SO(8)$ covariance
- M^{+-} boost symmetry

boost charges

$$X^+, k^+, \zeta^+ \ : \ +1$$
$$X^-, k^-, \zeta^- \ : \ -1$$
$$X^I, k^I, \zeta^I \ : \ 0$$
$$u^a \ : \ -1/2$$
$$u^{\dot{a}} \ : \ +1/2$$
$$S_a \ : \ 0$$

Most general form of the boson emission vertex

$$V_B(\zeta) = \int [dz]\, e^{ik_\mu X^\mu(z)} \Big\{ \zeta^- \, A\, \Pi^+(z) + \zeta^I \, \left(B\, \Pi^I(z) + C\, R^I(z) \right)$$

$$+ \; \zeta^+ \bigg(D\, \hat{\Pi}^-(z) + E\, \frac{\Pi^I\, R^I(z)}{\Pi^+} + F\, \frac{R^I\, R^I(z)}{\Pi^+}$$

$$+ \, Y\, k^-\, k_I \Pi^I(z) + Z\, \frac{\left(k_I \Pi^I\right) \left(k_J \Pi^J\right)(z)}{\Pi^+} \bigg) \Big\}$$

where

$$R^I \equiv k_J S \gamma^{IJ} S$$

Most general form of the fermion emission vertex

$$V_F(u) = \int [dz]\, e^{ik_\mu X^\mu(z)} \Big\{ u^a \, \left(G\, \sqrt{\Pi^+}\, S_a(z) \right)$$

$$+ \; u^{\dot{a}} \, \left(K\, \frac{\left(\bar{\gamma}^I S\right)_{\dot{a}}\, \Pi^I(z)}{\sqrt{\Pi^+}} + L\, \frac{\left(\bar{\gamma}^I S\right)_{\dot{a}}\, R^I(z)}{\sqrt{\Pi^+}} \right) \Big\}$$

V_B and V_F must transform into each other under SUSY as

$$[\eta^a Q_a,\, V_B(\zeta)] = V_F(\tilde{u}), \quad [\eta^a Q_a,\, V_F(u)] = -V_B(\tilde{\zeta})$$
$$\left[\epsilon^{\dot{a}} Q_{\dot{a}},\, V_B(\zeta)\right] = V_F(\tilde{\tilde{u}}), \quad \left[\epsilon^{\dot{a}} Q_{\dot{a}},\, V_F(u)\right] = -V_B(\tilde{\tilde{\zeta}})$$

up to possible BRST exact terms.

- $\tilde{\zeta}, \tilde{u}$, etc : SUSY-transformed wave functions
- V_B and V_F are both bosonic operators
- Minus signs on V_B on the RHS are important for consistency [3]

- Existence of $\frac{1}{2}\partial^2 \ln \Pi^+$ in $T(z) \Rightarrow \exp(ik^+X^-)$ is not a primary.

At present, we need to impose $k^+ = 0$ to avoid this complication, as in LC gauge ($\Rightarrow$ discussion at the end.)

[3] [GSW] (for LC vertices) misses this point.

6.2 SUSY transformation of the wave functions ζ^μ, $u^a, u^{\dot{a}}$

It is dictated by the SUSY transformation for 10D super-Maxwell fields:

$$\delta A^\mu = i\bar{\epsilon}\Gamma^\mu\psi = i\epsilon^\alpha \left(\bar{\gamma}^\mu\right)_{\alpha\beta}\psi^\beta$$
$$\delta\psi^\alpha = \frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu}\epsilon = \frac{1}{2}F_{\mu\nu}\left(\gamma^{\mu\nu}\right)^\alpha{}_\beta\epsilon^\beta$$

Make Fourier transforms

$$A^\mu(x) = \int [dk]\ \zeta^\mu(k)e^{ikx}$$
$$\psi^\alpha(x) = \int [dk]\ u^\alpha(k)e^{ikx} = \text{Grassmann odd}$$

Transformations for $SO(8)$ components

η-SUSY

$$\delta_\eta\zeta^+ = 0, \quad \delta_\eta\zeta^- = i\sqrt{2}\eta^a u_a, \quad \delta_\eta\zeta^I = i\eta^a\bar{\gamma}^I_{a\dot{b}}u^{\dot{b}}$$
$$\delta_\eta u^a = ik_I\zeta_J\left(\gamma^{IJ}\right)^a{}_b\eta^b + i\left(k^-\zeta^+ - k^+\zeta^-\right)\eta^a$$
$$\delta_\eta u^{\dot{a}} = i\sqrt{2}\left(k^I\zeta^+ - k^+\zeta^I\right)\left(\gamma^I\right)^{\dot{a}}{}_b\eta^b$$

ϵ-SUSY

$$\delta_\epsilon \zeta^+ = -i\sqrt{2}\epsilon^{\dot{a}} u_{\dot{a}}\,, \quad \delta_\epsilon \zeta^- = 0\,, \quad \delta_\epsilon \zeta^I = i\epsilon^{\dot{a}} \bar{\gamma}^I_{\dot{a}b} u^b$$
$$\delta_\epsilon u^a = i\sqrt{2}\left(k^- \zeta^I - k^I \zeta^-\right)\left(\gamma^I\right)^a{}_{\dot{b}} \epsilon^{\dot{b}}$$
$$\delta_\epsilon u^{\dot{a}} = i k_I \zeta_J \left(\gamma^{IJ}\right)^{\dot{a}}{}_{\dot{b}} \epsilon^{\dot{b}} - i\left(k^- \zeta^+ - k^+ \zeta^-\right)\epsilon^{\dot{a}}$$

On-shell conditions

$$k_\mu k^\mu = 2k^+ k^- + k^I k^I = 0$$
$$k_\mu \zeta^\mu = k^+ \zeta^- + k^- \zeta^+ + k^I \zeta^I = 0$$

$$\sqrt{2} k^+ u_a + k^I \bar{\gamma}^I_{a\dot{b}} u^{\dot{b}} = 0$$
$$-\sqrt{2} k^- u_{\dot{a}} + k^I \bar{\gamma}^I_{\dot{a}b} u^b = 0$$

In the frame where $k^+ = 0$, these equations become

$$k^I k^I = 0, \quad k^- \zeta^+ + k^I \zeta^I = 0\,,$$
$$k^I \bar{\gamma}^I_{a\dot{b}} u^{\dot{b}} = 0, \qquad \sqrt{2} k^- u_{\dot{a}} = k^I \bar{\gamma}^I_{\dot{a}b} u^b$$

6.3 η-SUSY

First, study $[\eta^a Q_a, V_B(\zeta)] = V_F(\tilde{u})$

This gives the relations

$$2^{\frac{7}{4}}C = -iG, \quad 2^{-\frac{1}{4}}D = iG, \quad 2^{-\frac{1}{4}}D = i\sqrt{2}K,$$
$$2^{\frac{7}{4}}E = i\sqrt{2}K, \quad 2^{\frac{11}{4}}F = i\sqrt{2}L$$

Next, study $[\eta^a Q_a, V_F(u)] = -V_B(\tilde{\zeta})$. This gives the relations

$$-i\sqrt{2}A = 2^{\frac{3}{4}}G, \quad -iB = 2^{\frac{3}{4}}K, \quad -iC = 2^{\frac{3}{4}}\, 3\, L$$

Fix overall normalization by $B = 1$. Then, the relations above give

$$A = 1, \quad B = 1, \quad C = -\frac{1}{4}, \quad D = 1, \quad E = \frac{1}{4}, \quad F = -\frac{1}{96},$$
$$G = -i\, 2^{-\frac{1}{4}}, \quad K = -i\, 2^{-\frac{3}{4}}, \quad L = i\,\frac{2^{-\frac{3}{4}}}{12}$$

- $V_F(u)$ **is completely fixed.**
- **The coefficients Y and Z in $V_B(\zeta)$ are not yet determined.**

6.4 ϵ-SUSY

Next examine the action of ϵ-SUSY

$$\left[\epsilon^{\dot{a}} Q_{\dot{a}},\, V_F(u)\right] \stackrel{?}{=} -V_B(\tilde{\tilde{\zeta}})$$
$$\left[\epsilon^{\dot{a}} Q_{\dot{a}},\, V_B(\zeta)\right] \stackrel{?}{=} V_F(\tilde{\tilde{u}})$$

After considerable computation using various non-trivial spinor identities, we can bring $[\epsilon Q,\, V_F(u)]$ into the form

$$[\epsilon Q,\, V_F(u)] = \int [dw]\, e^{ik\cdot X} \Bigg[-i\,(\epsilon\bar{\gamma}^I u)\left(\Pi_I - \frac{1}{4}R_I\right) + i\sqrt{2}\,(\epsilon u)\left(\frac{1}{4}\frac{\Pi_I R^I}{\Pi^+} - \frac{1}{96}\frac{R_I R^I}{\Pi^+}\right)$$
$$- \; i\sqrt{2}\,(\epsilon u)\left\{\frac{1}{2}\left(\frac{\Pi_I \Pi^I}{\Pi^+} - \frac{S\partial S}{\Pi^+}\right) + \partial\left(\frac{1}{\sqrt{\Pi^+}}\right)\frac{k^-\Pi^+ + k_I \Pi^I}{\sqrt{\Pi^+}} + \frac{2}{\sqrt{\Pi^+}}\partial^2\left(\frac{1}{\sqrt{\Pi^+}}\right)\right\}\Bigg]$$

This is not equal to $-V_B(\tilde{\tilde{\zeta}})$

Take

$$\Psi_F(\epsilon, u) = \int [dw] \left(-i\sqrt{2}\, \frac{b(w)}{\Pi^+(w)}\, e^{ik\cdot X(w)} \right) (\epsilon u)$$

Then, after some computation we get

$$[\epsilon Q,\, V_F(u)] + V_B(\tilde{\tilde{\zeta}}) - \{Q, \Psi_F(\epsilon, u)\}$$
$$= \left(-i\sqrt{2}\right) \int [dw]\; e^{ik\cdot X}\, (\epsilon u) \left[(Y+1) k^- \, k_I \Pi^I + (Z+1) \frac{\left(k_I \Pi^I\right) \left(k_J \Pi^J\right)}{\Pi^+} \right]$$

This vanishes if $Y = Z = -1$

$\Rightarrow$ $V_B(\zeta)$ **is completely fixed** and we have

$$\left[\epsilon^{\dot{a}} Q_{\dot{a}},\, V_F(u)\right] = -\, V_B(\tilde{\tilde{\zeta}}) + \{Q,\; \Psi_F(\epsilon, u)\}$$

Final results for $V_F(u)$ and $V_B(\zeta)$:

$$
\begin{aligned}
V_F(u) &= \int [dz]\; e^{ik\cdot X(z)} \Big\{ u^a \left(-i\, 2^{-1/4} \sqrt{\Pi^+}\, S_a(z) \right) \\
&\qquad + u^{\dot a} \left(-i\, 2^{-3/4} \frac{\left(\bar\gamma^I S\right)_{\dot a} \Pi_I(z)}{\sqrt{\Pi^+}} + i \left(\frac{2^{-3/4}}{12} \right) \frac{\left(\bar\gamma^I S\right)_{\dot a} R_I(z)}{\sqrt{\Pi^+}} \right) \Big\} \\
V_B(\zeta) &= \int [dz]\; e^{ik\cdot X(z)} \Bigg[\zeta^- \Pi^+(z) + \zeta^I \left(\Pi_I(z) - \frac{1}{4} R_I(z) \right) \\
&\qquad + \zeta^+ \Bigg(\hat\Pi^-(z) + \frac{1}{4} \frac{\Pi^I R_I(z)}{\Pi^+} - \frac{1}{96} \frac{R^I R_I(z)}{\Pi^+} \\
&\qquad - k^- k_I \Pi^I(z) - \frac{\left(k_I \Pi^I\right)\left(k_J \Pi^J\right)(z)}{\Pi^+} \Bigg) \Bigg]
\end{aligned}
$$

- They reuduce to LC gauge vertex operators upon

$$
\Pi^+(z) \to p^+\,, \qquad \zeta^+ \to 0
$$

Consistency check:

We still have to check $\left[\epsilon^{\dot{a}} Q_{\dot{a}},\, V_B(\zeta)\right] \stackrel{?}{=} V_F(\tilde{\tilde{u}})$

After some non-trivial computation, we find

$$\left[\epsilon^{\dot{a}} Q_{\dot{a}},\, V_B(\zeta)\right] = V_F(\tilde{\tilde{u}}) \; + \; \{Q,\, \Psi_B(\epsilon, \zeta)\}$$

with

$$\Psi_B(\epsilon, \zeta) = -2^{1/4}\, \zeta^{+} \oint dw \left(\frac{k_I \left(\epsilon\, \bar{\gamma}^I S\right) b(w)}{\left(\Pi^{+}\right)^{3/2}}\, e^{ik\cdot X(w)} \right)$$

7 Similarity transformation to the LC gauge and construction of the DDF operators

7.1 Similarity transformation to the LC gauge

We will show: cohomology of Q of SLC gauge = LC gauge states

by constructing **explicit quantum similarity transformation connecting the two.**

7.1.1 Method

Recall

$$Q \equiv \int [dz] \left(cT + bc\partial c \right)(z)$$

$$T(z) = \Pi^+(z)\Pi^-(z) + \frac{1}{2}(\Pi^I(z))^2 - \frac{1}{2}S_a \partial S_a(z) + \frac{1}{2}\partial^2 \ln \Pi^+$$

$Q \ni \int [dz] c \Pi^+ \Pi^-$ part contains the simple nilpotent operator δ

$$\delta \equiv p^+ \sum_{n \neq 0} c_{-n} \alpha_n^-$$

which satisfies the relations

$$\begin{aligned} \{\delta, \delta\} &= 0 \\ \left[\delta, \alpha_{-n}^+\right] &= p^+ n c_{-n}\,, \qquad \{\delta, c_{-n}\} = 0 \\ \{\delta, b_{-n}\} &= p^+ \alpha_{-n}^-\,, \qquad \left[\delta, \alpha_{-n}^-\right] = 0 \end{aligned}$$

$\Leftrightarrow$ Unphysical modes $(b_n, c_n, \alpha_n^+, \alpha_n^-)_{n \neq 0}$ form a quartet with respect to δ

We will construct a quantum similarity transformation

$$Q = e^{-S}(\delta + Q_{lc})e^{S}$$

$$Q_{lc} = c_0 \left(\frac{1}{2} p^\mu p_\mu + \sum_{n \geq 1} \alpha_{-n}^I \alpha_n^I + \sum_{n \geq 1} n S_{-n}^a S_n^a \right)$$

Separate the zero-mode and the non-zero mode parts of the unphysical fields as

$$\Pi^+(z) = \frac{p^+}{z} + \check{\Pi}(z)\,, \qquad \Pi^-(z) = \frac{p^-}{z} + \check{\Pi}^-(z)$$

$$c(z) = c_0 z + \check{c}(z)\,, \qquad b(z) = \frac{b_0}{z^2} + \check{b}(z)$$

Assign no-zero degrees to unphysical parts

$$\mathrm{deg}(\check{\Pi}^+) = 2\,, \qquad \mathrm{deg}(\check{\Pi}^-) = -2$$

$$\mathrm{deg}(\check{c}) = 1\,, \qquad \mathrm{deg}(\check{b}) = -1\,, \qquad \mathrm{deg}(\mathrm{rest}) = 0$$

Decompose Q according to the degree:

$$Q = \delta_{(-1)} + Q_0 + d_1 + d_2 + d_3 + e_{\geq 3}$$

Physical dof's are in Q_0

$$\delta = p^+ \int [dz] \frac{1}{z} \check{c} \check{\Pi}^- = p^+ \sum_{n \neq 0} c_{-n} \alpha_n^-$$

$$Q_0 = c_0 \left(\frac{1}{2} + \int [dz] z (T^{(0)} - \check{b} \partial \check{c}) \right)$$

$$d_1 = \int [dz] (\check{c} T^{(0)} + \check{b} \check{c} \partial \check{c})$$

$$d_2 = b_0 \int [dz] \frac{1}{z^2} \check{c} \partial \check{c}$$

$$d_3 = p^- \int [dz] \frac{1}{z} \check{c} \check{\Pi}^+$$

$$e_{\geq 3} \equiv \sum_{n \geq 1} e_{2n+1}$$

where

$$\int [dz] c \frac{1}{2} \partial^2 \ln \Pi^+ = e_0 + \sum_{n=1}^{\infty} e_{2n+1}$$

$$e_0 = \int [dz] c_0 z \varpi = \frac{1}{2} c_0 , \quad e_{2n+1} = \frac{(-1)^{n-1}}{2n} \int [dz] \partial^2 \check{c} \left(\frac{z \check{\Pi}^+}{p^+} \right)^n , \qquad (n \geq 1)$$

Seek similarity transformation to remove the parts with positive degrees as

$$(\star) \quad Q = e^{-\mathfrak{R}}(\delta + Q_0)e^{\mathfrak{R}} = \delta + Q_0 + [\delta + Q_0, \mathfrak{R}] + \frac{1}{2}[[\delta + Q_0, \mathfrak{R}], \mathfrak{R}] + \cdots$$

Decompose the operator $\mathfrak{R}$ according to the degree:

$$\mathfrak{R} = R_2 + R_3 + R_4 + \cdots$$

Equation $(\star)$ above becomes (AB means $[A, B\}$)

$$Q = \delta + Q_0 + d_1 + d_2 + (d_3 + e_3) + e_4 + \cdots$$
$$= \delta + Q_0 + \underbrace{\delta R_2}_{1} + \underbrace{\delta R_3 + Q_0 R_2}_{2} + \underbrace{\delta R_4 + Q_0 R_3 + \frac{1}{2}(\delta R_2)R_2}_{3} + \cdots$$

Problem: Find R_n's which solve the infinite number of equations

$$d_1 = \delta R_2$$
$$d_2 = \delta R_3 + Q_0 R_2$$
$$d_3 + e_3 = \delta R_4 + Q_0 R_3 + \frac{1}{2}(\delta R_2)R_2 \qquad etc.$$

Two basic ingredients for the solution:

♦ δ has the homotopy operator $\hat{K} \equiv \frac{1}{p^+} \sum_{n \neq 0} \frac{1}{n} \alpha^+_{-n} b_n$

$$\hat{N} \equiv \delta\hat{K} = \sum_{n \neq 0} :(c_{-n} b_n + \frac{1}{n} \alpha^+_{-n} \alpha^-_n):$$

For $\mathcal{O}$ which satisfies $\delta\mathcal{O} = 0$ and $\hat{N}\mathcal{O} = n\mathcal{O}, n \neq 0$,

$$\mathcal{O} = \frac{1}{n}\hat{N}\mathcal{O} = \frac{1}{n}(\delta\hat{K})\mathcal{O} = \delta\left(\frac{1}{n}\hat{K}\mathcal{O}\right)$$

δ-homology is trivial for non-zero $\hat{N}$-number sector.

♦ **Degree-wise relations from the nilpotency $Q^2 = 0$** (shown up to degree 2)

$$
\begin{aligned}
&(E_{-2}) \quad \delta^2 = 0 \\
&(E_{-1}) \quad \delta Q_0 = 0 \\
&(E_0) \quad \frac{1}{2}\underbrace{Q_0^2}_{0} + \delta d_1 = 0 \\
&(E_1) \quad \underbrace{Q_0 d_1}_{0} + \delta d_2 = 0 \\
&(E_2) \quad \delta(d_3 + e_3) + Q_0 d_2 + \frac{1}{2} d_1 d_1 = 0\,, \qquad etc.
\end{aligned}
$$

Solution at low degrees

- $\delta d_1 = 0 \Rightarrow d_1 = \delta(\hat{K} d_1)$. Compare with $d_1 = \delta R_2$. We get $R_2 = \hat{K} d_1 + \delta X_3$.
- $\delta d_2 = 0 \Rightarrow d_2 = \delta\left(\frac{1}{2}\hat{K} d_2\right)$. One can show: $Q_0 R_2 = 0$ for $X_3 = 0$. Hence $d_2 = \delta R_3$. Thus we get $R_3 = \frac{1}{2}\hat{K} d_2 + \delta X_4$.

7.1.2 Solution up to degree 10

$$R_2 + R_3 = \hat{K} d_1 + \frac{1}{2}\hat{K} d_2 = \frac{1}{p^+}\sum_{k\neq 0}\frac{1}{k}\alpha^+_{-k}\widetilde{L}^{tot}_k$$

$$R_4 = r_4\,, \quad R_5 = 0\,, \quad R_6 = \frac{1}{2} r_6\,, \quad R_7 = 0\,,$$

$$R_8 = 0\,, \quad R_9 = 0\,, \quad R_{10} = -\frac{1}{6} r_{10}$$

where

$$r_{2n} \equiv \frac{(-1)^n}{2n(n-1)(p^+)^n}[(\alpha^+)^n]_0\,, \qquad n \geq 2$$

$$[(\alpha^+)^n]_m \equiv \sum_{\sum_i^n k_i = m} \alpha^+_{k_1}\alpha^+_{k_2}\cdots\alpha^+_{k_n}$$

- One can prove $R_{2n+1} = 0$ for $n \geq 2$
- Very hard to guess the pattern for R_{2n} for $n \geq 2$.

7.1.3 Exact form of the similarity transformation

Consider an ansatz of the form

$$Q = \underbrace{e^{-R_2 - R_3} e^{-\widetilde{R}}}_{e^{-\mathfrak{R}}} (\delta + Q_0) \underbrace{e^{\widetilde{R}} e^{R_2 + R_3}}_{e^{\mathfrak{R}}}$$

where $\widetilde{R}$ is taken to be of the form

$$\widetilde{R} = \sum_{n \geq 2} \xi_{2n} r_{2n}$$

<u>**Exact answer:**</u> $\xi_{2n} = (-1)^n$

$$\widetilde{R} = \sum_{n \geq 2} (-1)^n r_{2n} = \int [dz] \sum_{n \geq 2} \frac{1}{2n(n-1)} \left(\frac{z \check{\Pi}^+}{p^+} \right)^n$$
$$= \frac{1}{2} \int [dz] \left[f(z) + (1 - f(z)) \ln(1 - f(z)) \right]$$

where $f(z) \equiv z\check{\Pi}^+(z)/p^+$.

Check of the formula

Use general Baker-Campbell-Hausdorff formula

$$e^{\lambda}e^{\mu}= e^{E}$$

$$E= \lambda + \int_0^1 dt\psi\left(e^{\mathrm{ad}_\lambda}e^{t\,\mathrm{ad}_\mu}\right)\mu\,, \qquad \psi(z) = \frac{z\ln z}{z-1}$$

We can then show

$$e^{\widetilde{R}}e^{R_2+R_3} = e^{\mathfrak{R}} = e^{R_2+R_3+\hat{R}}$$

$$\hat{R} = r_4 + \sum_{n\geq 1}(-1)^{n-1}(2n+1)B_n r_{2(2n+1)}$$

B_n = Bernoulli numbers : $B_1 = 1/6, B_2 = 1/30, B_3 = 1/42$, etc.

Then we get

$$\hat{R} = r_4 + \frac{1}{2}r_6 - \frac{1}{6}r_{10} + \frac{1}{6}r_{14} + \cdots$$

which reproduces the degree-wise computation.

Remaining similarity transformation

We must further remove the unphysical part still remaining in Q_0:

$$Q_0 \ni c_0 \tilde{N} = c_0 \sum_{n \geq 1} \left(\alpha_{-n}^{+} \alpha_{n}^{-} + \alpha_{-n}^{-} \alpha_{n}^{+} + n c_{-n} b_n + n b_{-n} c_n \right)$$

This is achieved by an additional similarity transformation:

$$e^{-c_0 \tilde{K}} (\delta + Q_0) e^{c_0 \tilde{K}} = \delta + Q_{cl}$$

$$\tilde{K} \equiv \frac{1}{p^{+}} \sum_{n \neq 0} \alpha_{-n}^{+} b_n \,, \qquad \tilde{N} = \left\{ \tilde{K}, \delta \right\}$$

$$Q_{lc} = c_0 \left(\frac{1}{2} p^{\mu} p_{\mu} + \sum_{n \geq 1} \alpha_{-n}^{I} \alpha_{n}^{I} + \sum_{n \geq 1} n S_{-n}^{a} S_{n}^{a} \right)$$

Cohomology of Q_{lc} = LC physical states

7.2 An Application of the similarity transformation — Construction of the DDF operators—

Physical oscillators of LC gauge (α_n^I, S_n^a)

$\Downarrow$ **similarity transformation**

DDF oscillators of SLC gauge $(A_n^I, \mathbb{S}_n^a)$

$$[L_m, A_n^I] = [L_m, \mathbb{S}_n^a] = 0$$
$$[A_n^I, A_m^J] = \delta^{IJ}\delta_{n+m,0}\,, \quad \{\mathbb{S}_n^a, \mathbb{S}_m^b\} = \delta^{ab}\delta_{n+m,0}$$

7.2.1 Basic idea

For bosonic string, DDF operator A_n^I was shown (Aisaka and Kazama, 2004) to be connected to LC gauge basic oscillator by a similarity transformation as

$$
\begin{aligned}
A_n^I &= e^{inx^+/p^+} \check{A}_n^I \,, \\
\check{A}_n^I &= e^{-R} \alpha_n^I e^{R} = \int [d\tau] e^{in\tau} \partial_\tau X^I(\tau) e^{i(n/p^+)\check{X}^+(\tau)} \\
R &= R_2 + R_3 = \frac{1}{p^+} \sum_{k \neq 0} \frac{1}{k} \alpha_{-k}^+ \widetilde{L}_k^{tot}
\end{aligned}
$$

(Zero-mode phase is necessary for A_n^I to commute with Virasoro.)

$$
\begin{aligned}
\widetilde{L}_k^{tot} &= \widetilde{L}_k^b + \widetilde{L}_k^g + \widetilde{L}_k^f \\
\widetilde{L}_k^b &= p^I \alpha_k^I + \frac{1}{2} \sum_{n \neq 0} \alpha_{k-n}^\mu \alpha_{\mu,n} \\
\widetilde{L}_k^g &= \sum_{n \neq 0} n c_{-n} b_{k+n} \\
\widetilde{L}_k^f &= \frac{1}{2} \sum_{n \neq 0} n S_{k-n}^a S_n^a
\end{aligned}
$$

This formula is valid also for GS in SLC gauge.

For GS superstring in SLC gauge, we should be able to construct fermionic DDF operator as well by the same similarity transformation (including $\widetilde{L}^f_k$)

$$\mathbb{S}^a_n = e^{inx^+/p^+}\check{\mathbb{S}}^a_n = e^{inx^+/p^+}e^{-R}S^a_n e^R$$

Explicit form of $\check{\mathbb{S}}^a_n$?

Low order calculation:

$$\check{\mathbb{S}}^a_n = \int [d\tau] e^{in\tau}\left(S^a(\tau) - [R, S^a(\tau)] + \frac{1}{2}[R,[R,S^a(\tau)]] + \cdots\right)$$
$$= \int [d\tau] e^{in\tau} S^a(\tau)\left(1 + \frac{in}{p^+}\check{X}^+ + \frac{1}{2p^+}\partial_\tau \check{X}^+ - \frac{1}{2}\left(\frac{n}{p^+}\right)^2 (\check{X}^+)^2 - \frac{1}{8{p^+}^2}(\partial_\tau \check{X}^+)^2 + \frac{in}{4{p^+}^2}\partial_\tau(\check{X}^+)^2 + \cdots\right)$$

Much more complicated than the structure of $\check{A}^I_n$.

Guess for the exact result:

$$(\star) \quad \check{\mathbb{S}}^a_n = e^{-R} S^a_n e^{R} = \int [d\tau] e^{in\tau} e^{i(n/p^+)\check{X}^+} \left(1 + \frac{\partial_\tau \check{X}^+}{p^+}\right)^{1/2} S^a(\tau)$$

Including the zero-mode phase part,

$$\mathbb{S}^a_n = \frac{1}{\sqrt{p^+}} \int [d\tau] e^{i(n/p^+)X^+} \sqrt{\partial_\tau X^+} S^a(\tau)$$

7.2.2 Proof of the formula $(\star)$

The following simple but new powerful theorem is crucial in proving the guess above.

Set-up of the theorem:

Let the mode operators ϕ_n and χ_n enjoy the following commutation relations with a set of operators L_k:

$$[L_k, \phi_n] = -(n + (1-h)k)\phi_{n+k}$$
$$[L_k, \chi_n] = -(n + k)\chi_{n+k}$$

These relations are isomorphic to those for

L_k = Virasoro operators

ϕ_n = mode of a primary field $\phi(\tau)$ of dimension h

χ_n = mode of a primary field $\chi(\tau)$ of dimension 0

But we do not require the algebra of L_k themselves.

We will consider the fields $\phi(\tau)$ and $\chi(\tau)$ defined by

$$\phi(\tau) \equiv \sum_n \phi_n e^{-in\tau}\,, \qquad \chi(\tau) \equiv \sum_n{}' \chi_n e^{-in\tau}$$

Now define the operator T_χ, with finite $\chi(\tau)$, as

$$\boxed{T_\chi = i \sum_k{}' \chi_{-k} L_k}$$

Theorem:

T_χ generates a finite operator-dependent conformal transformation of $\phi(\tau)$ associated with $\tau \longrightarrow \tau' = \tau - \chi(\tau)$, as

$$e^{T_\chi} \phi(\tau) e^{-T_\chi} = \phi'(\tau)$$

where $\phi'(\tau')(d\tau')^h = \phi(\tau)(d\tau)^h$.

Remarks:

- It is crucial that $\chi(\tau)$ is an operator, not a c-number function.
- Proof is rather involved. Idea is to introduce a parameter λ and study the cou-

pled non-linear differential equations, with respect to τ and λ, for the quantities

$$f(\lambda,\tau) \equiv e^{\lambda T_\chi}\phi(\tau)e^{-\lambda T_\chi}\,, \qquad g(\lambda,\tau) \equiv e^{\lambda T_\chi}\chi(\tau)e^{-\lambda T_\chi}$$

We can apply the theorem above with

$$L_k = \widetilde{L}_k^{tot}\,, \qquad \phi(\tau) = S^a(\tau)\,, \qquad \chi(\tau) = -\check{X}^+(\tau)/p^+\,, \qquad h = \frac{1}{2}$$

$$\therefore \tau' = \tau - \chi(\tau) = \tau + \frac{\check{X}(\tau)}{p^+} \quad \Rightarrow \quad \frac{d\tau'}{d\tau} = 1 + \frac{\partial_\tau \check{X}^+}{p^+}$$

Then the theorem immediately gives us the result of the exact similarity transformation as

$$e^{-R}S^a(\tau)e^R = S'^a(\tau)$$

It is then not difficult to show that $S'^a(\tau)$ is exactly the same function as $\check{\mathbb{S}}^a(\tau) \equiv \sum_n \check{\mathbb{S}}^a_n e^{-in\tau}$, where $\check{\mathbb{S}}^a_n$ was defined previously in ($\star$).

$$(\star) \quad \check{\mathbb{S}}^a_n = e^{-R}S^a_n e^R = \int [d\tau] e^{in\tau} e^{i(n/p^+)\check{X}^+} \left(1 + \frac{\partial_\tau \check{X}^+}{p^+}\right)^{1/2} S^a(\tau)$$

8 Discussions

We have layed the foundation of the quantum GS superstring in the SLC gauge, where the conformal invariance is retained.

- ♦ Structures of the quantum symmetry algebras are clarified
- ♦ Vertex operators for massless states are constructed
- ♦ Similarity transformation connecting LC and SLC gauges is constructed

Important remaining problem

□ **Vertex operator for $k^+ \neq 0$**

Relation to the work of Baba-Ishibashi-Murakami [BIM] [4]

[BIM] wished to realize "dimensional regularization" for LC-SFT.

$\Leftrightarrow$ Non-critical LC-SFT $\Leftrightarrow$ Lorentz non-invariant

[4] Baba-Ishibashi-Murakami, 0909.4675

$\Rightarrow$ Non-standard worldsheet theory for $X^\pm$ when covariantized: **"$X^\pm$-CFT"**

$$T^{BIM}_{X^\pm} = \Pi^+\Pi^-_{BIM} - \frac{d-26}{12}\{X^+, z\} \qquad \text{(Schwarzian derivative)}$$
$$= \Pi^+\Pi^-_{BIM} - \frac{d-26}{12}\left(\frac{\partial^3 X^+}{\partial X^+} - \frac{3}{2}\left(\frac{\partial^2 X^+}{\partial X^+}\right)^2\right)$$

- For $d = 14$ (effective dimension for GS superstring),

  $\Pi^-_{BIM} = \hat{\Pi}^- = \Pi^- + \partial^2(1/2\Pi^+)$ = primary of dimension 1

  and $T^{BIM}_{X^\pm} = T^{GS}_{X^\pm} = \Pi^+\Pi^- + \frac{1}{2}\partial^2 \ln \Pi^+$

- [BIM] claims that they can compute the amplitudes containing the vertex op $e^{ik^+\hat{X}^-}$.

In operator formalism, it is hard to define this vertex, because the OPE of $\hat{X}^-$ with itself is singular and operator-valued:

$$\hat{X}^-(z)\hat{X}^-(w) \sim \frac{1}{(z-w)^2}\frac{1}{\partial X^+(z)\partial X^+(w)}$$

- Important to understand **how the BIM procedure can be imple-**

mented in operator formalism.

Hope to report progress on this and related matters in the near future