On Multiple-Brane Solutions in OSFT

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Fundamental Question

What is the origin of

COLOR

(non-abelian gauge symmetry)

in our universe ?

Plan of talk

1. What is String Field Theory
2. Review of known solutions
3. Multiple D-brane solutions

   (work in collaboration with Masaki Murata)

What is String Field Theory?

- Field theoretic description of all excitations of a string (open or closed) at once.
- Useful especially for physics of backgrounds: tachyon condensation or instanton physics
- Single Lagrangian field theory which should around its various critical points describe physics of diverse D-brane backgrounds, and hence possibly also **COLOR.**

Lightning review of OSFT

Open string field theory uses the following data

$$\mathcal{H}_{BCFT}, \quad *, \quad Q_B, \quad \langle . \rangle.$$

Let all the string degrees of freedom be assembled in

$$|\Psi\rangle = \sum_i \int d^{p+1}k\, \phi_i(k)|i,k\rangle,$$

Witten (1986) proposed the following action

$$S = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle\, \Psi * Q_B \Psi \,\rangle + \frac{1}{3}\langle\, \Psi * \Psi * \Psi \,\rangle\right],$$

Gauge symmetry

This action has a huge gauge symmetry

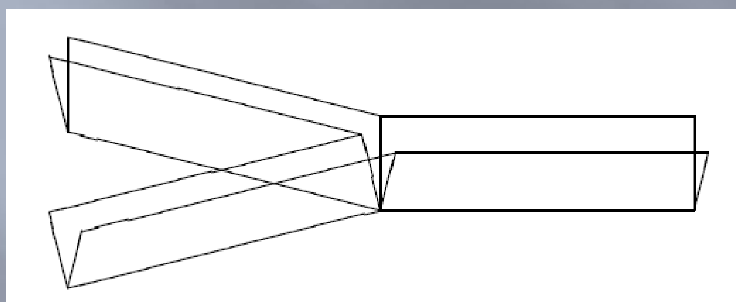
$$\delta\Psi = Q_B\Lambda + \Psi * \Lambda - \Lambda * \Psi,$$

provided that the star product is associative, Q_B acts as a graded derivation and < . > has properties of integration.

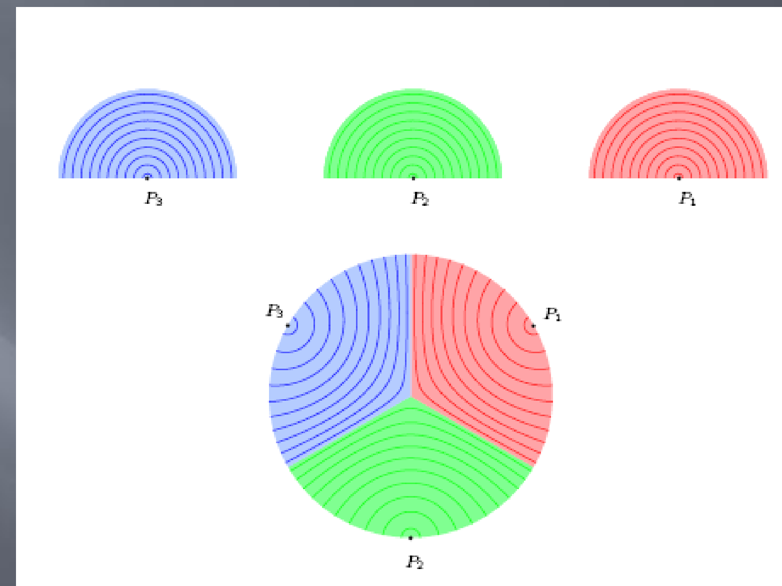
Note that there is a gauge symmetry for gauge symmetry so one expects infinite tower of ghosts – indeed they can be naturally incorporated by lifting the ghost number restriction on the string field.

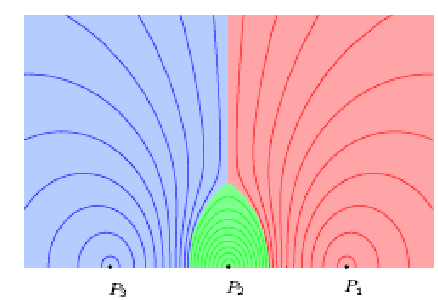
Witten's star product

Defined by gluing three strings:

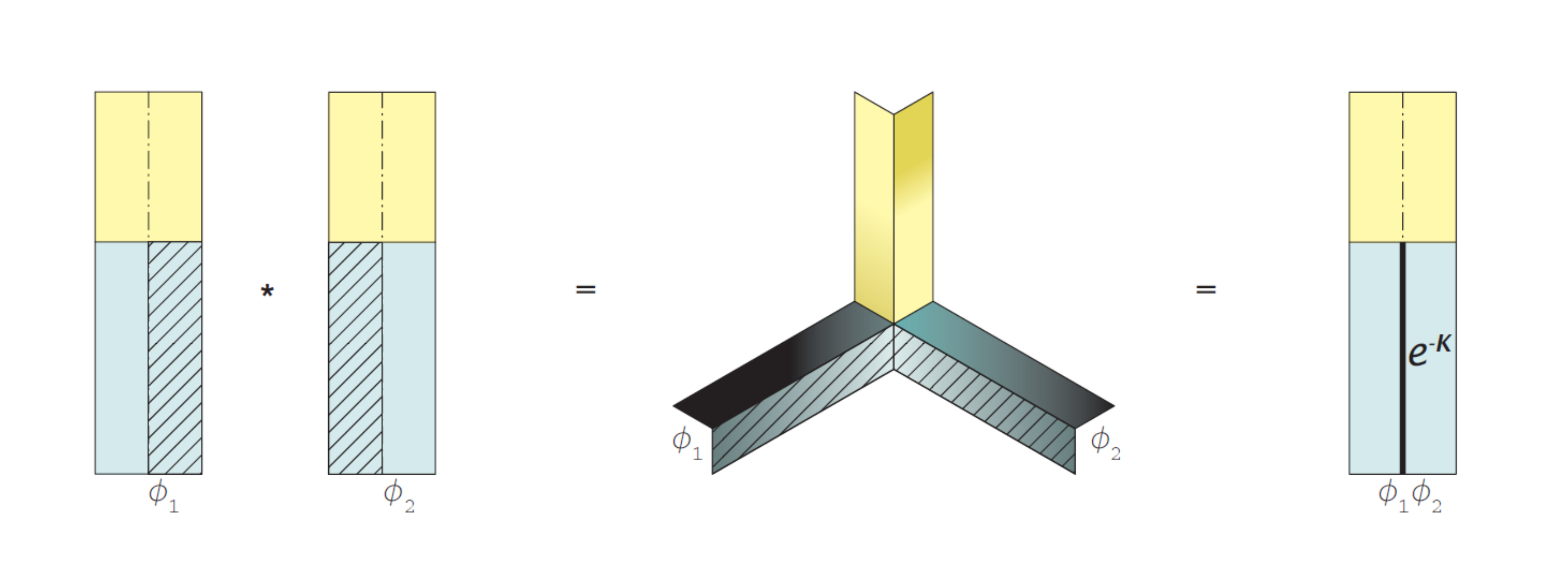


$$(\Psi_1 \star \Psi_2)\,[X(\sigma)] = \int [\mathcal{D}X_{\text{overlap}}]\,\Psi_1[\hat{X}(\sigma)]\Psi_2\,[\check{X}(\sigma)]$$





It used to be a very complicated definition…



- The elements of string field star algebra are states in the BCFT, they can be identified with a piece of a worldsheet.
- By performing the path integral on the glued surface in two steps, one sees that in fact:

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle.$$

Witten's star product as operator multiplication

We have just seen that the star product obeys

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle.$$

And therefore states $\hat{\phi} = e^{K/2} \phi e^{K/2}$ obey

$$|\hat{\phi}_1\rangle * |\hat{\phi}_2\rangle = |\widehat{\phi_1 \phi_2}\rangle$$

The star product and operator multiplication are thus isomorphic!

Survey of known solutions

$$Q_B\Psi + \Psi * \Psi = 0$$

- **Numerically:** tachyon vacuum, marginal deformations (Wilson line and rolling tachyon), lump solutions
- **Analytically:** tachyon vacuum in various gauges, marginal deformations (one simple type for regular OPE, another for general case).

  Lump solution still missing (cf. recent work by Erler, Maccaferri and Bonora et al.)

- Other solutions missing completely, such as flux solutions.

Solution for every BCFT?

- Let us assume that we have a pair of boundary condition changing operators σ_L, σ_R such that

$$\sigma_L \sigma_R = 1$$

Then the simplest possible solution of string field theory equations of motion is $\Psi = \sigma_L Q_B \sigma_R$

provided it is non-singular. In fact it is almost always singular, an example when it is non-singular would be a light-like Wilson line $\lambda \partial X^+$ with

$$\sigma_{L,R} = e^{\pm i \lambda X^+}$$

[Kiermaier,Okawa]

Solution for every BCFT?

- For more general marginal deformations more complicated version is needed. Such solutions were constructed by Kiermaier and Okawa. (See also their paper with Soler.)
- All known solutions of string field theory are of the form
$$\Psi = UQ_BU^{-1}$$
where either U or U^{-1} are not quite allowed, or well defined operators.
- In this talk we will present some new solutions of this form where U and U^{-1} are formed from Virasoro generators and b,c ghosts.

No positive energy solutions?

- Over the past 11 years many classical solutions of open string field theory (OSFT) have been constructed, numerically, or analytically.

  The common feature of all these solutions is that their energy relative to the perturbative vacuum is always negative.

  Is there a fundamental reason for this?
  Is the string field big enough?

Simple subsector of the star algebra

- The star algebra is formed by vertex operators and the operator *K*. The simplest subalgebra relevant for tachyon condensation is therefore spanned by *K* and *c*. Let us be more generous and add an operator *B* such that *QB*=*K*.
- The building elements thus obey

$$c^2 = 0, \quad B^2 = 0, \quad \{c, B\} = 1$$
$$[K, B] = 0, \quad [K, c] = \partial c$$

- The derivative *Q* acts as $$Q_B K = 0, \quad Q_B B = K, \quad Q_B c = cKc.$$

Classical solutions

This new understanding lets us construct solutions to OSFT equations of motion $\boxed{Q_B\Psi + \Psi * \Psi = 0}$ easily.

It does not take much trying to find the simplest solution is $\Psi = \alpha c - cK$

$$Q\Psi = \alpha(cKc) - (cKc)K$$

$$\Psi * \Psi = \cancel{\alpha^2 c^2} - \cancel{\alpha c^2 K} - \alpha cKc + (cK)(cK)$$

More general solutions are of the form

$$\boxed{\Psi = Fc\frac{KB}{1-F^2}cF,}$$

Here *F*=*F(K)* is arbitrary

Okawa 2006 (generalizing M.S. 2005)

Where to look for solutions?

- The space of all such solutions has not been completely classified yet, although we are getting closer (Rastelli; Erler; M.S.).
- Let us restrict our attention to different choices of *F(K)* only.
- Let us call a state *geometric* if *F(K)* is of the form

$$F(K) = \int_0^\infty d\alpha f(\alpha) e^{-\alpha K}$$

where $f(\alpha)$ is a tempered distribution.
- Restricting to "absolutely integrable" distributions one gets the notion of L_0–safe geometric states.

Where to look for solutions?

- Some nice spaces of distributions are (Schwartz, 1950)

$$
\begin{array}{ccccccccccccccc}
\mathcal{D} & \subset & \mathcal{S} & \subset & \mathcal{D}_{L^p} & \subset & \mathcal{D}_{L^q} & \subset & \dot{\mathcal{B}} & \subset & \mathcal{B} & \subset & \mathcal{O}_M & \subset & \mathcal{E} \\
\cap & & \cap & & \cap & & \cap & & \cap & & \cap & & \cap & & \cap \\
\mathcal{E}' & \subset & \mathcal{O}'_c & \subset & \mathcal{D}'_{L^p} & \subset & \mathcal{D}'_{L^q} & \subset & \dot{\mathcal{B}}' & \subset & \mathcal{B}' & \subset & \mathcal{S}' & \subset & \mathcal{D}'
\end{array}
$$

$$1 \le p < q < \infty$$

- The space $\mathcal{D}'_{L_1}$ guarantees that
  - the Fock space coefficients are finite
  - is closed under convolution
- In terms of Laplace transform: *F(K)* must be holomorphic for *Re(K)>0* and bounded by a polynomial there. Demanding that the polynomial is just a constant we would get a nice Banach algebra: the so called Hardy space H^∞.

What makes solutions nontrivial ?

- Since formally $\Psi = (1 - FBcF)Q(1 - FBcF)^{-1},$

  and $(1 - FBcF)^{-1} = 1 + \frac{F}{1 - F^2}BcF$

  the state is trivial if $F/(1 - F^2)$ is well defined

- There is another useful criterion. One can look at the cohomology of the theory around a given solution. It is given by an operator $Q_\Psi = Q_B + \{\Psi, \cdot\}_*.$

- The cohomology is formally trivialized by an operator $A = \frac{1 - F^2}{K}B,$ which obeys $\{Q_\Psi, A\} = 1.$

Classes of universal solutions

- Therefore in this class of solutions, the trivial ones are those for which $F^2(0) \neq 1$.
- Tachyon vacuum solutions are those for which $F^2(0) = 1$ but the zero of $1-F^2$ is first order

Old conjectures:

- When the order of zero of $1-F^2$ at $K=0$ is of higher order the solution is not quite well defined, formally it would correspond to negative number of D-branes.
- When $1-F^2$ blows up at $K=0$ the solution describes multi-brane configuration as we will demonstrate.

Examples so far

$$F(K) = a \quad (const.)$$

… trivial solution

$$F(K) = \sqrt{1 - \beta K}$$

…. 'tachyon vacuum' only *c* and *K* turned on

$$F(K) = e^{-K}$$

…. M.S. '05

$$F(K) = \frac{1}{\sqrt{1+K}}$$

… Erler, M.S. '09 – the simplest solution so far

On multiple-brane solutions

- Can one compute an energy of the general solution as a function of F?

$$\Psi = Fc\frac{KB}{1-F^2}cF,$$

- The technology has been here for quite some time, but the task seemed daunting. Recently we addressed the problem with M. Murata.

On multiple-brane solutions

- Let us compute an energy of such a general solution

$$\begin{aligned}\langle \Psi, Q\Psi \rangle &= \left\langle \frac{K}{G}, (1-G), \frac{K}{G}, KG \right\rangle - \left\langle K, (1-G), \frac{K}{G}, K \right\rangle \\ &\quad - \left\langle \frac{K}{G}, (1-G), K, K \right\rangle + \left\langle K, (1-G), K, \frac{K}{G} \right\rangle,\end{aligned}$$

$$G = 1 - F^2$$

where

$$\langle F_1, F_2, F_3, F_4 \rangle = \langle F_1(K)cF_2(K)cF_3(K)cF_4(K)cB \rangle$$

On multiple-brane solutions

- Assuming that all F_i are L_0-safe and geometric, we can compute this basic correlator by writing

$$F_i(K) = \int_0^\infty f_i(\alpha) e^{-\alpha K},$$

and using

$$\begin{aligned}
\langle e^{-\alpha_1 K} c e^{-\alpha_2 K} c e^{-\alpha_3 K} c e^{-\alpha_4 K} c B \rangle &= \langle c(\alpha_1) c(\alpha_1+\alpha_2) c(\alpha_1+\alpha_2+\alpha_3) c(\alpha_1+\alpha_2+\alpha_3+\alpha_4) B \rangle_{C_s} \\
&= \frac{s^2}{4\pi^3} \Bigg[\alpha_4 \sin\frac{2\pi\alpha_2}{s} - (\alpha_3+\alpha_4)\sin\frac{2\pi(\alpha_2+\alpha_3)}{s} \\
&\quad +\alpha_2 \sin\frac{2\pi\alpha_4}{s} - (\alpha_2+\alpha_3)\sin\frac{2\pi(\alpha_3+\alpha_4)}{s} \\
&\quad +\alpha_3 \sin\frac{2\pi(\alpha_2+\alpha_3+\alpha_4)}{s} + (\alpha_2+\alpha_3+\alpha_4)\sin\frac{2\pi\alpha_3}{s} \Bigg],
\end{aligned}$$

where $s = \sum_{i=1}^{4} \alpha_i.$

On multiple-brane solutions

- To re-express <F_1, F_2, F_3, F_4 > in terms of F's one can use a simple trick of writing

$$1 = \int_0^\infty ds\, \delta\left(s - \sum_{i=1}^{4} \alpha_i\right) = \int_0^\infty ds \int_{-i\infty}^{+i\infty} \frac{dz}{2\pi i}\; e^{sz}\, e^{-z\sum_{i=1}^{4}\alpha_i},$$

and find

$$\begin{aligned}\langle F_1, F_2, F_3, F_4 \rangle \;=\; & \int_0^\infty ds \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \frac{s^2}{4\pi^3} e^{sz} \frac{1}{2i}\Big[-F_1 \Delta F_2 F_3 F_4' + F_1 \Delta(F_2 F_3') F_4 \\ & + F_1 \Delta(F_2 F_3) F_4' - F_1 F_2' F_3 \Delta F_4 + F_1 F_2 \Delta(F_3' F_4) + F_1 F_2' \Delta(F_3 F_4) \\ & + \Delta F_1 F_2 F_3' F_4 + (s F_1 + F_1') F_2 \Delta F_3 F_4\Big], \qquad (2.10)\end{aligned}$$

where $$(\Delta F)(z) = F\left(z - \frac{2\pi i}{s}\right) - F\left(z + \frac{2\pi i}{s}\right).$$

On multiple-brane solutions

- The kinetic term after some contour manipulations (and computer algebra) becomes

$$\langle \Psi, Q\Psi \rangle = \frac{3}{\pi^2}\int_0^\infty ds \oint \frac{dz}{2\pi i}\, e^{sz}\left[z^2 s \frac{G'(z)}{G(z)} + (z\partial_z - s\partial_s)\left(\frac{izs^2}{8\pi}\frac{\Delta(zG)(z)}{G(z)}\right)\right].$$

The second term is effectively a total derivative which vanishes for *G ~ 1 +O(1/z)*. The final result for the energy is thus $\frac{E}{\mathrm{Vol}} = \frac{1}{2\pi^2}\oint_C \frac{dz}{2\pi i}\frac{G'(z)}{G(z)},$

where C runs up the imaginary axis, bypassing possible singularity at the origin on the left, and closes at infinity in the *Re z < 0* plane.

On multiple-brane solutions

To get further support let us look at the Ellwood invariant (or Hashimoto-Itzhaki-Gaiotto-Rastelli-Sen-Zwiebach invariant) . Ellwood proposed that for general OSFT solutions it would obey

$$\langle I |\, c\bar{c}V_{\rm cl}(i) | \Psi \rangle = \mathcal{A}^{\rm disk}_{\Psi}(V_{\rm cl}) - \mathcal{A}^{\rm disk}_{0}(V_{\rm cl})$$

let us calculate the LHS:

$$\begin{aligned} \left\langle c\bar{c}V_{\rm cl}(i) c\tilde{F}BcF^2 \right\rangle &= \int_0^\infty d\alpha \int_0^\infty d\beta \, \frac{2i}{\pi} \beta \tilde{f}(\alpha) f(\beta) \, \langle V_{\rm cl}(i) \rangle^{\rm matter}_{UHP} \\ &= \tilde{F}(0) \partial F^2(0) \, \mathcal{A}^{\rm disk}_0(V_{\rm cl}) \end{aligned}$$

For $F^2 = 1 + a z^n + O(z^{n+1})$ we find $\tilde{F}(0)\partial F^2(0) = -n$!

On multiple-brane solutions

- In particular for the energy (as measured by the graviton coupling) we find

$$\frac{E}{\mathrm{Vol}} = -\frac{1}{2\pi^2}\lim_{z\to 0} z\frac{G'(z)}{G(z)},$$

- How can this be compatible with our previous result? Assuming holomorphicity at infinity and *Re z>0* half plane we can prove that

$$\frac{E}{\mathrm{Vol}} = \frac{1}{2\pi^2}\oint_C \frac{dz}{2\pi i}\,\frac{G'(z)}{G(z)} = -\frac{1}{2\pi^2}\lim_{z\to 0} z\frac{G'(z)}{G(z)}$$

On multiple-brane solutions

- Let us summarize our assumptions:

  1) G has no zeros or poles in $Re\ z > 0$. This actually follows automatically from:

  2) $|G|$ and $1/|G|$ are bounded by a polynomial in $Re\ z > \varepsilon$ for all $\varepsilon > 0$. This is a weaker condition than L_0–safety.

  3) $G \to 1$ as $z \to \infty$. This is a 'no identity' criterion.

$$\Psi = c\,\frac{K}{G}\,Bc\,(1-G)$$

Comments on level expansion

- Is it really sensible to have inverse powers of K ?
  To test that we have computed several lowest level coefficients of

$$\Psi = cK^{n+1}Bc\left(1-\frac{1}{K^n}\right)$$

  even though $G(z)= z^n$ violates the no-identity criterion.

- The coefficient of $c_1|0>$ diverges for negative $n\leq-4$. This is good, as we do not like 'ghost branes' as solutions of OSFT. Cases n=-2 and n=-3 should be ruled out separately, n=-1 is allowed (tachyon vacuum).

Comments on level expansion

- To study other coefficients we can easily compute the overlap

$$\left\langle \phi \left| cK^{n+1}Bc\frac{1}{K^n} \right. \right\rangle = \int_0^\infty dt \frac{t^{n-1}}{(n-1)!} \mathrm{Tr}\left(e^{-K/2}\phi\, e^{-K/2} cK^{n+1}Bc\, e^{-tK}\right)$$
$$= \int_0^\infty dt \frac{t^{-h_V}}{(n-1)!} \mathrm{Tr}\left(e^{-\frac{K}{2t}} c\partial cV\, e^{-\frac{K}{2t}} cK^{n+1}Bc\, e^{-K}\right)$$

where $\phi = c\partial cV$, and V is a matter field of scaling dimension h_V. The ghosts contribute $1/t^2$, so the integral is perfectly convergent. For $\phi =: c\partial c\partial^2 cb :$ the integral however diverges for n≥3 as $\int_0^\infty dt t^{n-4}$ so that ghost coefficients starting with $c_{-2}\,|0>$ are divergent.

Comments on level expansion

- One of the most promising candidate for the n+1 brane solution is obtained by taking a ‘power’ of the simple solution of Erler and M.S.’09:

$$G(z) = \left(\frac{z+1}{z}\right)^n$$

- In level truncation we find for the tachyon coefficient by doing the integrals over wedge lengths α, β
  $t = 0.372994$
- Applying the s-z trick the same way as for the energy we get an extra contribution from the pole at zero:
  $t = 0.372994 - 0.588638 = -0.215644$. The s-z trick suggests an existence of a ‘phantom’.

Historical slide (2006)

Ordinarily the Chern-Simons action gives quantized values for pure (large) gauge configurations. Similar property can be shown to hold more generally

Let $\Psi = UQ_BV$

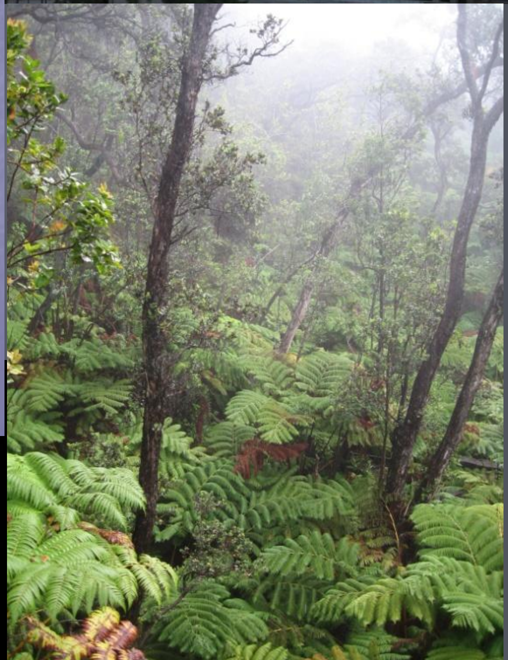
$$S[\Psi] = -\tfrac{1}{6}\langle UQ_BV\, UQ_BV\, UQ_BV \rangle = S_0$$

Then $S[U^nQ_BV^n] = nS_0$

In particular, formally $S[VQ_BU] = -S_0$

This suggests that VQ_BU should be interpreted as two D-brane solution ! Currently we are trying to test this conjecture by computing the energy rigorously and by working out the spectrum.

Ian Ellwood, M.S. in progress



Hawaii, APS-JPS joint meeting, 2006

Some newest developments

- New regularization: shift K → K+ε

$$\Psi = F(K+\varepsilon)\, c\, \frac{K+\varepsilon}{1-F^2(K+\varepsilon)}\, Bc\, F(K+\varepsilon)$$

- We get a phantom:

$$\Psi^{\text{phantom}} = \varepsilon\, F(K+\varepsilon)\, c\, \frac{1}{1-F^2(K+\varepsilon)}\, Bc\, F(K+\varepsilon)$$

- For the B_0 gauge vacuum this gives correct: energy, Ellwood invariant, curly $\mathcal{L}_0$ and square L_0 level expansion coefficients !
- But it also works for $G = (1-e^{-K})^{-1}$!!

What next ?

- Understand in detail level expansion.
  - Are we still missing a phantom term?
  - Is there some F for which the level expansion is fully regular for all $n>0$?
  - Is it possible to find regular twist symmetric solutions?

- Work out the cohomology around these solutions. (We have seen some hints that we indeed get $(n+1)^2$ copies of the original cohomology.) But more work is needed!

- Find other solutions which increase the energy !